IGCSEGCSEMATHS.CO.UK

Cambridge IGCSE 0580

0580/21 (Extended) – May/June 2024

70 marks · 1 hour 30 minutes · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Cambridge IGCSE Mathematics, Paper 0580/21 (Extended), May/June 2024 series, sat Friday 3 May 2024 in Cambridge administrative zone 2 –70 marks, 1 hour 30 minutes, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Cambridge Assessment International Education. This resource reproduces neither the exam paper nor the official mark scheme.

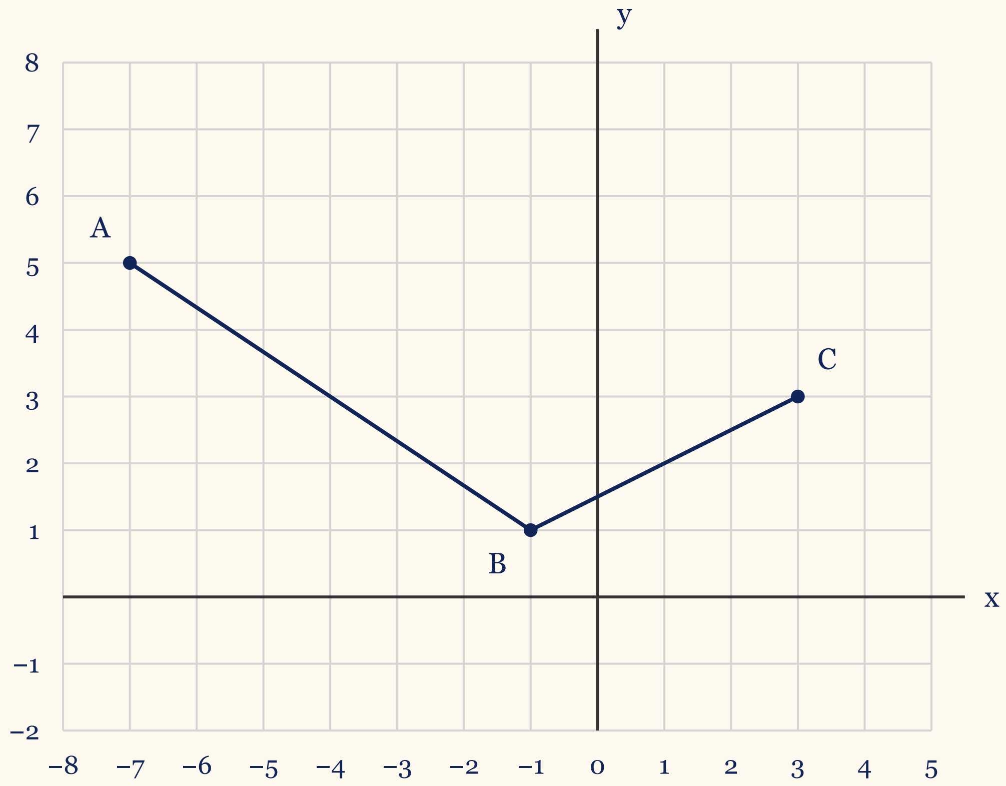
Cambridge sits this paper on different days in different administrative zones. The date above is the one published for Zone 2; timetables for the other zones of this series are no longer available from Cambridge.

Both are PDF files hosted by Cambridge: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Two sides of parallelogram $ABCD$ are drawn on the grid.



Work out the coordinates of the point D. *[2 marks]*

....

[Total 2 marks]

2. A jar of beads belongs to Meera.
From the jar she takes one bead at random.
There is a probability of 0.6 that the bead she takes is glass.

(a) Find the probability that the bead she takes is not glass. *[1 mark]*

(b) Three types of bead are in the jar: glass, clay and metal.

Type of bead	Glass	Clay	Metal
Number of beads		14	14
Probability	0.6		

Fill in the missing values in the table. *[2 marks]*

....

[Total 3 marks]

3. Some information about two sequences is given in the table.

	nth term	5th term
Sequence A	$60 - 4n$	
Sequence B	$n^2 - 300$	

(a) Fill in the missing values in the table. *[2 marks]*

(b) Work out the smallest positive number in sequence B. *[2 marks]*

....

[Total 4 marks]

4. An odd number is a factor of 140 and a factor of 210.
What is the greatest value this number can have? *[2 marks]*

....

[Total 2 marks]

5. Work out the value of each of the following.

(a) $\sqrt[3]{343} - \sqrt{40.96}$ *[1 mark]*

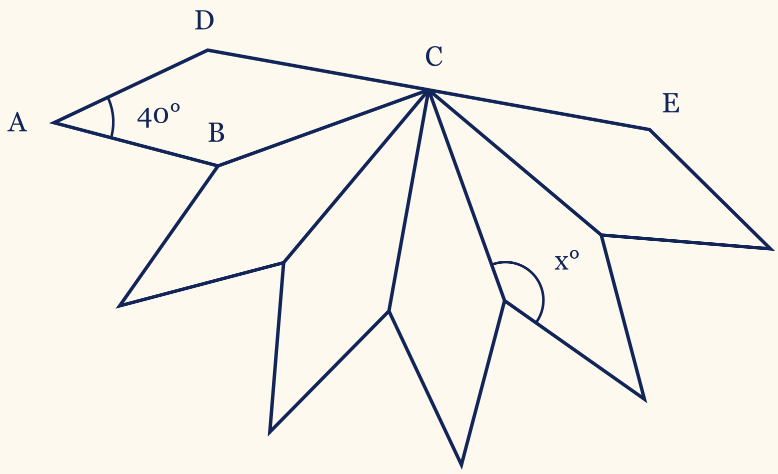
(b) $(192 + 4 \times 16)^{1.25}$ *[1 mark]*

(a) ....

(b) ....

[Total 2 marks]

6. In the diagram, 5 kites are congruent to kite $ABCD$.
Each pair of neighbouring kites shares one edge.
DCE is a straight line, and angle $DAB = 40°$.



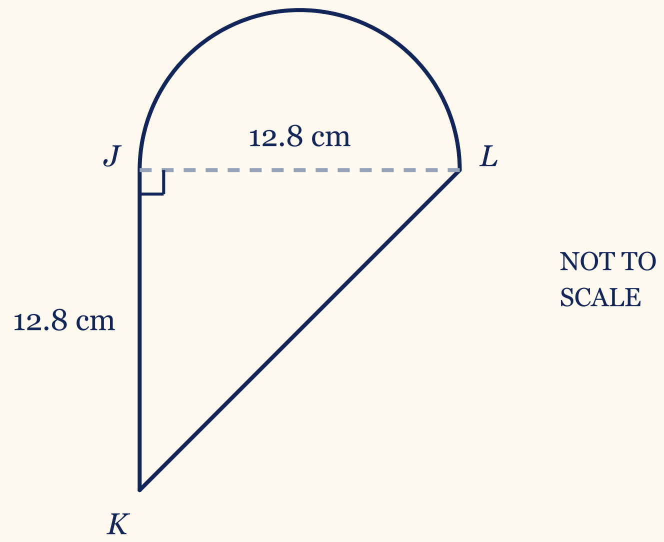
NOT TO SCALE

What is the value of x? *[3 marks]*

x = ...

[Total 3 marks]

7. Triangle JKL and a semicircle with diameter JL are joined to make the shape in the diagram.
$JK = JL = 12.8$ cm, and triangle JKL is both isosceles and right-angled.



(a) What is the area of this shape? *[3 marks]*

(b) What is the perimeter of this shape? *[4 marks]*

(a) cm² ....

(b) cm ....

[Total 7 marks]

8. A sequence starts with the five terms below.

11 18 25 32 39

What is an expression for the nth term of this sequence? *[2 marks]*

....

[Total 2 marks]

9. A delivery van is worth $\$8000$.
Its value falls exponentially by 25% each year.

What is the value of the van after 3 years? *[2 marks]*

$ ....

[Total 2 marks]

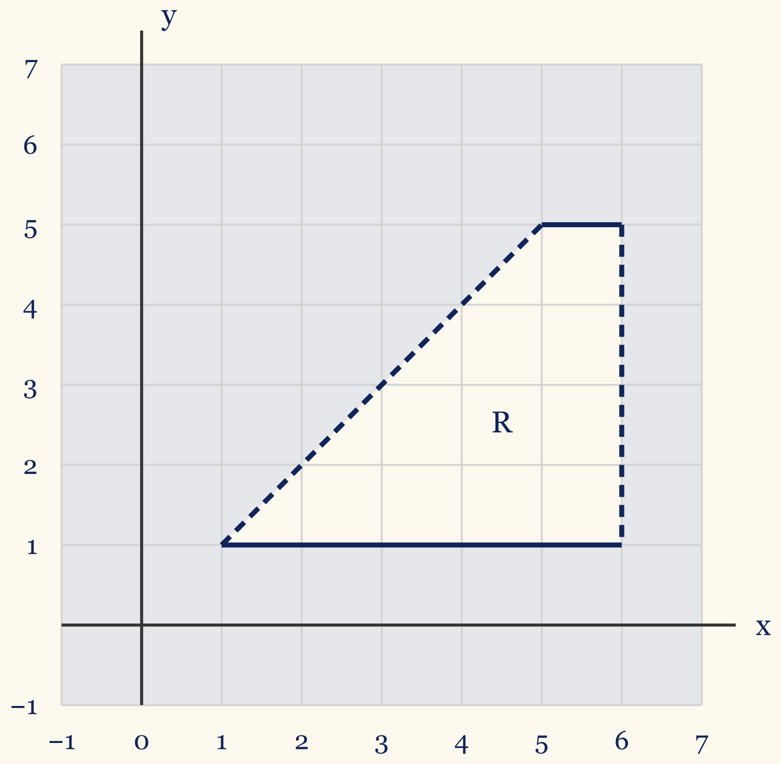
10. Bilal's $\$1500$ is paid into a savings account.
Compound interest is added each year at a rate of r %.
His investment is worth $\$1656.73$ when 8 years have passed.

What is the value of r? *[3 marks]*

r = ...

[Total 3 marks]

11. Which inequalities define R, the region that is not shaded? *[4 marks]*



....

[Total 4 marks]

12. What values of x and y satisfy both of these equations?
You must show all your working.

$$\frac{3x}{2} + 5y = 5$$

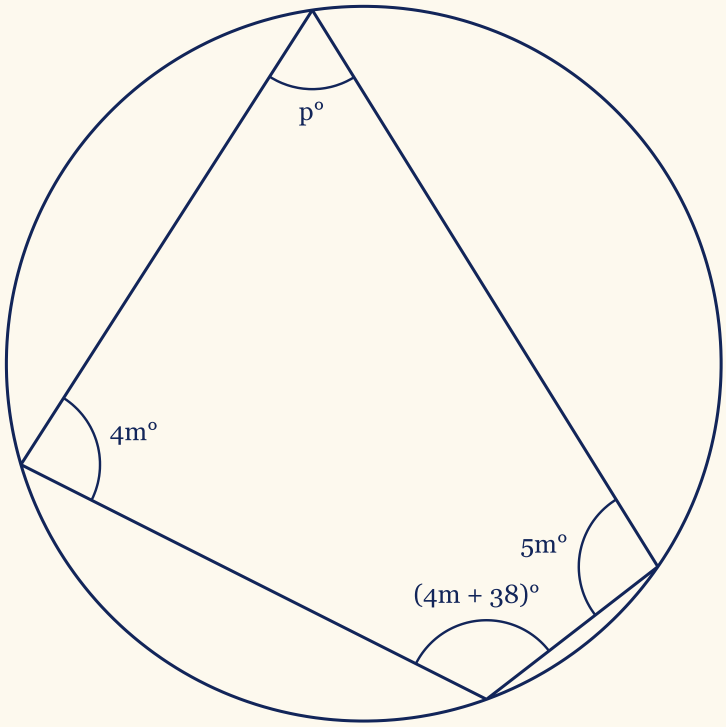
$4x - 3y = 46$ *[4 marks]*

x = ...

y = ...

[Total 4 marks]

13. A cyclic quadrilateral is drawn in the diagram.



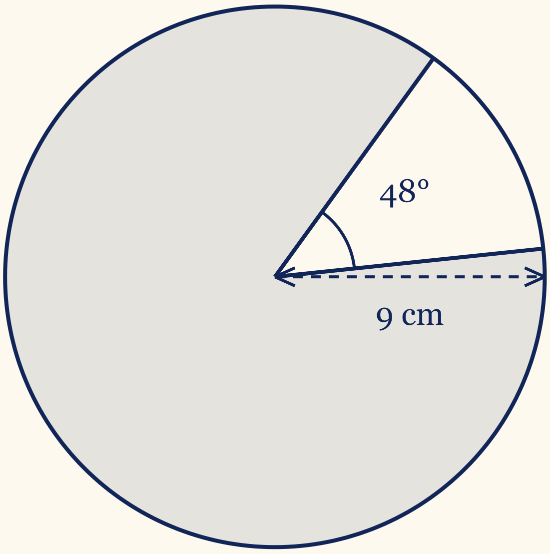
NOT TO SCALE

What is the value of p? *[3 marks]*

$p =$ ....

[Total 3 marks]

14. A circle of radius 9 cm is shown in the diagram.



NOT TO SCALE

What is the area of the shaded major sector? *[3 marks]*

.... cm^2

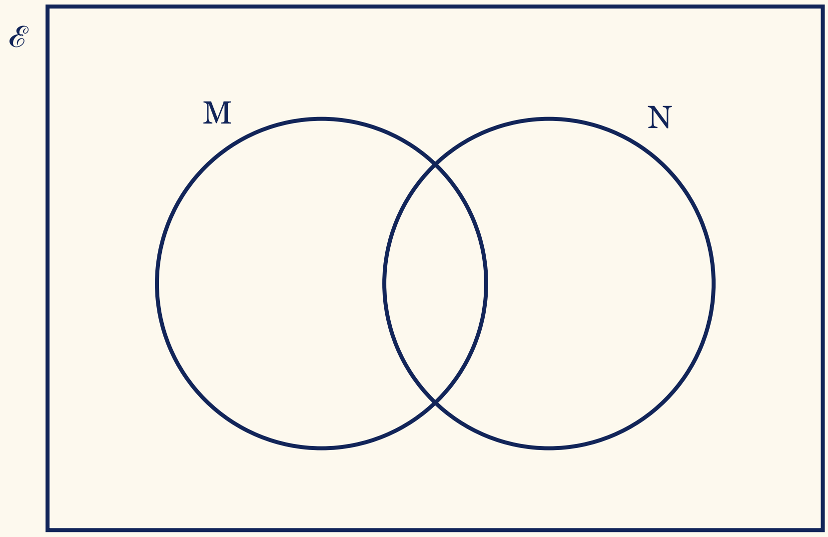
[Total 3 marks]

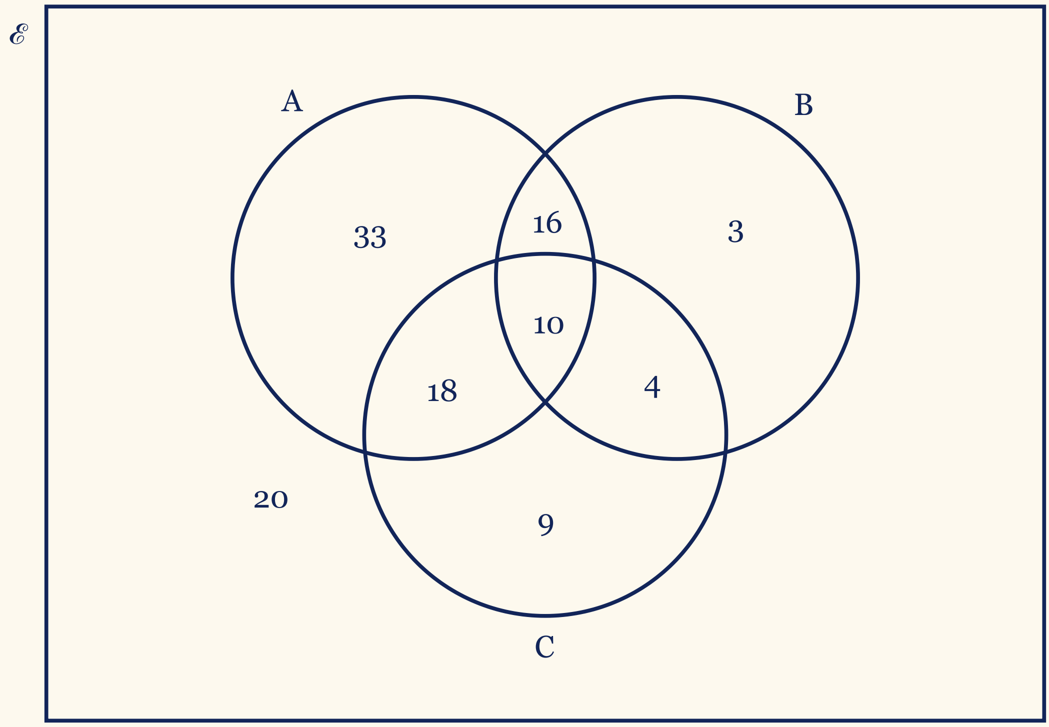
15. What fraction, in its simplest form, is equal to $0.1\dot{4}\dot{6}$?
You must show all your working. *[3 marks]*

....

[Total 3 marks]

16. (a) Shade the region $M' \cap N'$ on the Venn diagram. *[1 mark]*



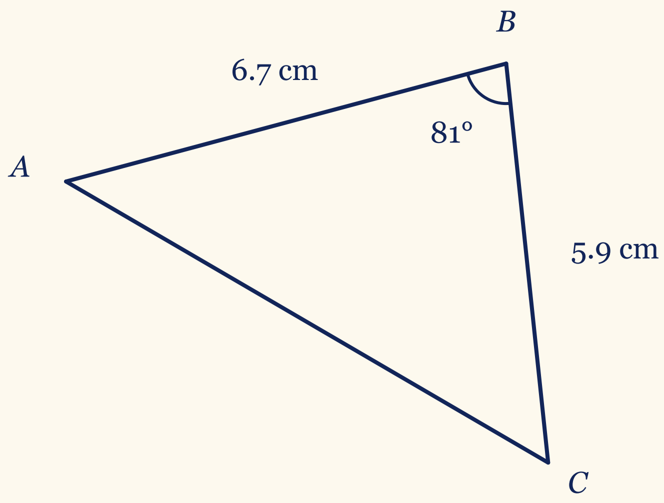


(b) What is $\mathrm{n}(B \cap (A' \cup C))$? *[1 mark]*

(b) ....

[Total 2 marks]

17. What is the area of triangle ABC? *[2 marks]*

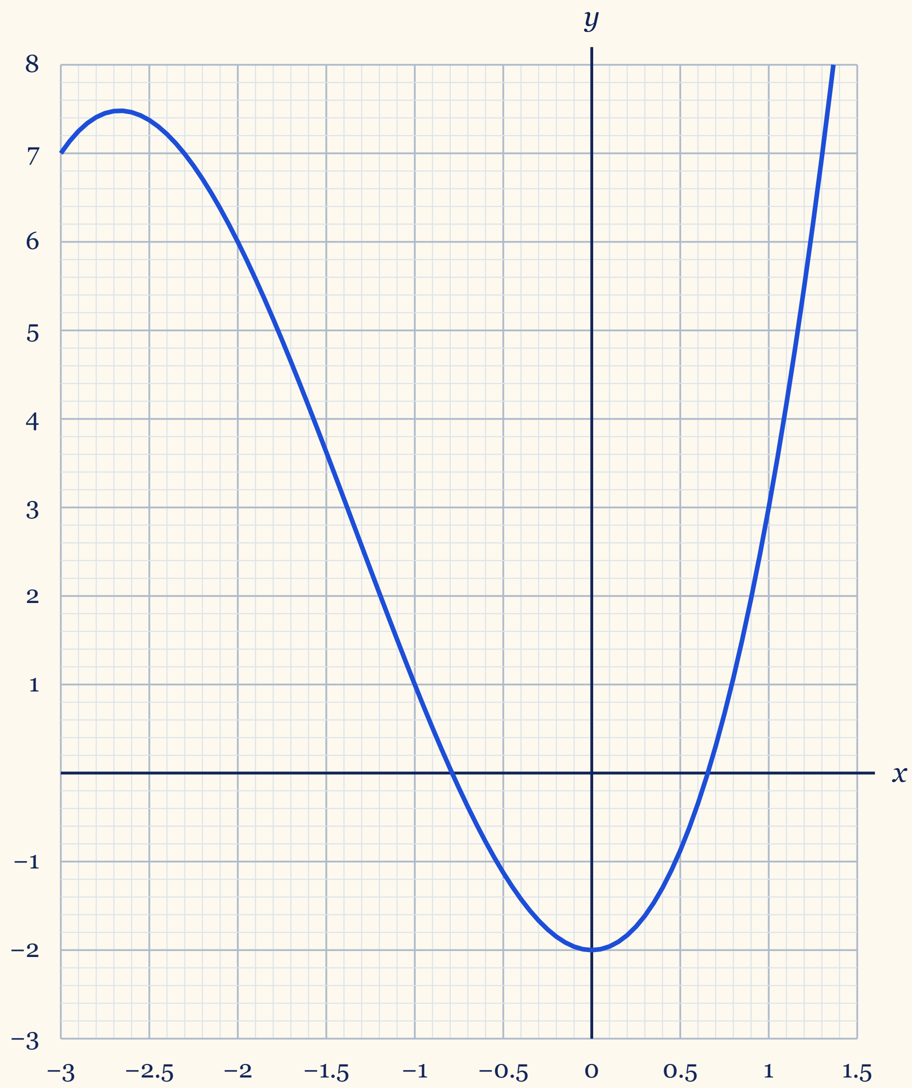


NOT TO SCALE

.... cm²

[Total 2 marks]

18. The graph of $y = x^3 + 4x^2 - 2$ is drawn in the diagram for $-3 \leqslant x \leqslant 1.5$.



Draw a suitable straight line to solve the equation $x^3 + 4x^2 - 2 = 2x$ for $-3 \leqslant x \leqslant 1.5$. *[3 marks]*

x = ...

x = ...

[Total 3 marks]

19. Each expression below is to be factorised completely.

(a) $12m^2 - 75t^2$ *[3 marks]*

(b) $xy + 15 + 3y + 5x$ *[2 marks]*

(a) ....

(b) ....

[Total 5 marks]

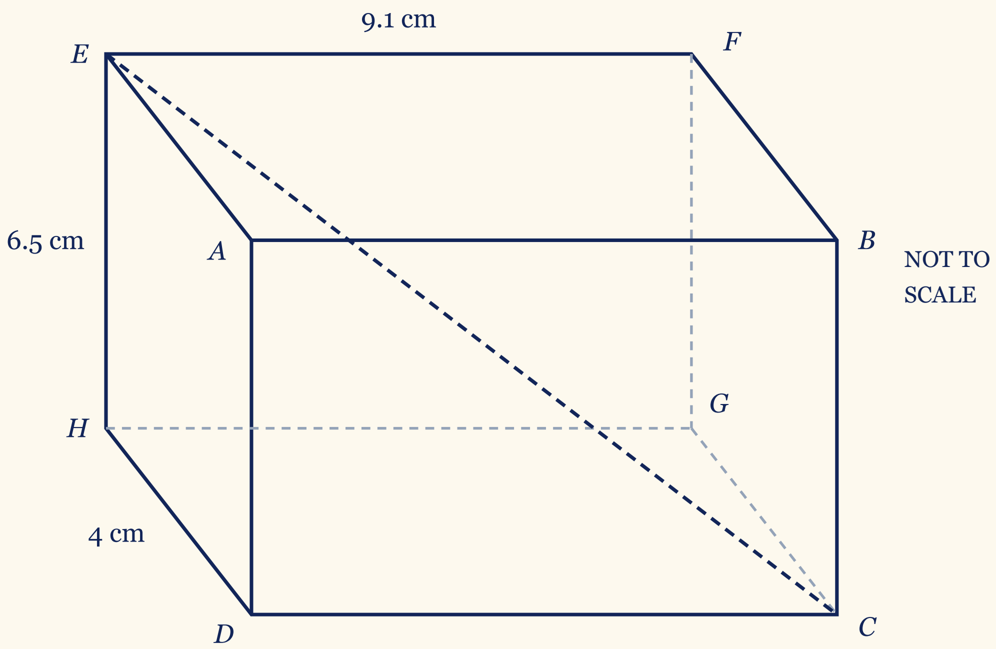
20. Find every value of x in the interval $0^\circ \leqslant x \leqslant 360^\circ$ that satisfies $8 \sin x + 6 = 1$. *[3 marks]*

x = ...

or x = ...

[Total 3 marks]

21. The solid in the diagram is a cuboid.
In it, $EF = 9.1$ cm, $EH = 6.5$ cm and $HD = 4$ cm.



What is the angle between CE and the base $CDHG$? *[4 marks]*

....

[Total 4 marks]

22. Only red beads and blue beads are kept in tin A and in tin B.
At random, Curtis takes one bead from tin A and Heather takes one bead from tin B.

For Curtis, the probability of taking a red bead is 0.4.
There is a probability of 0.25 that both beads taken are red.

What is the probability that both beads taken are blue? *[4 marks]*

....

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Two sides of a parallelogram $ABCD$ are drawn on the grid.
Reading the three plotted vertices off the grid: $A = (-7, 5)$, $B = (-1, 1)$ and $C = (3, 3)$.

Find

The coordinates of the fourth vertex D.

Plan the Solution

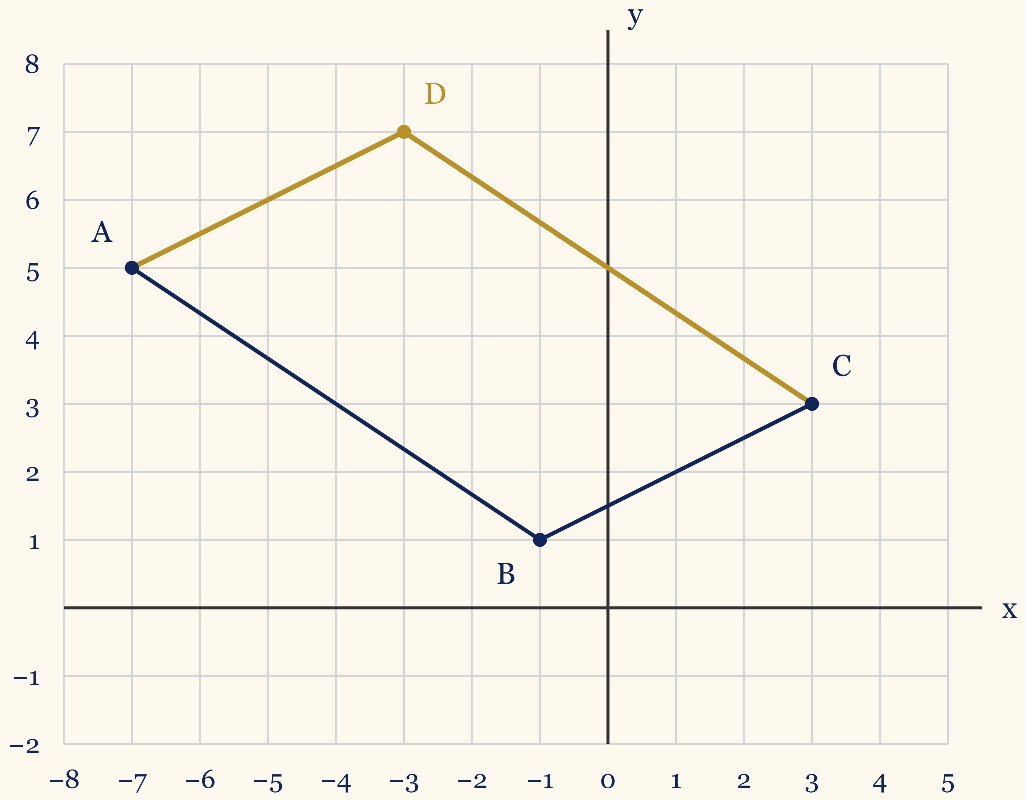
- The letters run round the shape in order, so the four sides are AB, BC, CD and DA. That makes AD the side opposite BC.
- Opposite sides of a parallelogram are parallel and the same length, so the move from A to D is the very same move as from B to C.
- So work that move out from the two points that are given, then start at A and make the same move.
- Confirm the answer a different way with the diagonals, which cut each other in half in any parallelogram.

Worked Solution [2 marks]

> Rule - Opposite sides of a parallelogram: in $ABCD$ the move from A to D equals the move from B to C, so $D = A + (C - B)$.

Step 1: read the three given vertices off the grid

$A = (-7, 5)$
$B = (-1, 1)$
$C = (3, 3)$



(Reason: Every one of the three points sits on a crossing of the grid lines, so each coordinate is a whole number. A is seven squares left of the y-axis and five up, B is one left and one up, and C is three right and three up.)

Step 2: work out the move from B to C

$$3-(-1)=4$$
$$3-1=2$$

(Reason: Subtract B from C, the across coordinates first and then the up ones. Take care with the first line: B has a negative x-coordinate, and subtracting -1 adds one.)

Step 3: make that same move, starting from A

$$-7+4=-3$$
$$5+2=7$$

(Reason: The move runs from A to D in the same direction it runs from B to C, which is four squares to the right and two squares up. Apply it to each of A's coordinates in turn.)

Step 4: write down the coordinates of D

$$D=(-3,7)$$

(Reason: Plotting the point three squares left of the y-axis and seven up closes the shape, and AD comes out parallel to BC with CD parallel to AB.)

$$D=(-3,7)$$

Verification

✓ **Check 1:** Test the OTHER pair of opposite sides. The move from A to B must equal the move from D to C. $B - A = (6, -4)$ and $C - D = (6, -4)$, the same move

✓ **Check 2:** The diagonals of a parallelogram cut each other in half, so AC and BD must have the same midpoint. This uses D in a completely different way from the working. the midpoint of AC is $(-2, 4)$ and the midpoint of BD is $(-2, 4)$

✓ **Check 3:** Opposite sides must also come out the same LENGTH. Use Pythagoras on the across and up steps of each side. $AD = BC = \sqrt{20}$ and $AB = DC = \sqrt{52}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The coordinates of D	**Note**	The Answer column gives $(-3, 7)$, and the Marks column gives 2 for it.	✓
Partial Marks	**B1**	For correct diagram, or correct coordinates for their point D, or for $(-3, k)$ or $(k, 7)$.	✓

Full marks: 2/2

Question 2 - Exam Solution

Understanding the Question

Given

One bead is taken at random from Meera's jar.
The probability that the bead is glass is 0.6.
Of the three types, the jar holds 14 clay beads and 14 metal beads.

Find

(a) The probability that the bead is not glass. (b) The number of glass beads, and the probability for clay and the probability for metal.

Plan the Solution

- Part (a) is a complement. Glass and not glass are the only two things that can happen, so the two probabilities add up to 1.

- In part (b), clay and metal are the only types that are not glass, so the 14 and the 14 together are the beads that are not glass. Part (a) has already given the probability of that outcome.
- Knowing how many beads carry a probability of 0.4 is enough to scale back up to the whole jar, and the glass beads are then whatever is left.
- Each missing probability is that type's count out of the total number of beads.

Worked Solution [3 marks]

> Rule - Probability from a count: the probabilities of all the outcomes add up to 1, and the probability of one type is $\dfrac{\text{number of that type}}{\text{total number of beads}}$.

Step 1: part (a), take the given probability away from 1

$1 - 0.6 = 0.4$

(Reason: Glass and not glass are the only two things that can happen, so their probabilities add up to 1*. Taking* 0.6 *away from* 1 *leaves* 0.4*.)*

Step 2: part (b), count the beads that are not glass

$14 + 14 = 28$

(Reason: Clay and metal are the only other types, so every bead that is not glass is one of these 28*. Part (a) has already priced that outcome at* 0.4*.)*

Step 3: scale up to the whole jar

$$\frac{28}{0.4} = 70$$

(Reason: Those 28 *beads make up* 0.4 *of the jar. Dividing by* 0.4 *undoes that scaling and gives the whole jar, so there are* 70 *beads in it altogether.)*

Step 4: find the number of glass beads

$70 - 28 = 42$

(Reason: Every bead in the jar is glass, clay or metal, so the glass beads are whatever is left once the other two types are taken away from the total.)

Step 5: find the two missing probabilities

$$\frac{14}{70} = 0.2$$

(Reason: Clay and metal each have 14 *beads out of the* 70 *in the jar, so each probability is* 14 *out of* 70*. The two come out the same because the two counts are the same.)*

(a) 0.4

(b) number of glass beads 42

(b) probability of clay 0.2**, probability of metal** 0.2

Verification

- ✓ **Check 1:** Add the three counts together. They must come to the total worked out in step 3. $42 + 14 + 14 = 70$
- ✓ **Check 2:** Add the three probabilities. Every bead is one of the three types, so they must come to 1. $0.6 + 0.2 + 0.2 = 1$
- ✓ **Check 3:** Read the given probability back off the completed table. The glass beads out of the whole jar must return the 0.6 the question states, which uses the finished table in the opposite direction from the working. $\frac{42}{70} = 0.6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The probability of not glass	**Note**	The Answer column gives 0.4 oe, and the Marks column gives 1 for it.	✓
(b) The completed table	**Note**	The Answer column gives 42, then 0.2 and 0.2, and the Marks column gives 2 for it.	✓
Partial Marks	**B1**	For 42.	✓
Partial Marks	**B1**	For 0.2 and 0.2.	✓
Partial Marks	**SC1**	If B0 scored, for their two probabilities being half their (a).	✓
Where the special case comes from	**Note**	Both of the other two types have the same count, so each of their probabilities comes out as half of the part (a) answer. A candidate who halves their own part (a) has used that method, whatever their part (a) was.	✓

Full marks: 3/3

Question 3 - Exam Solution

Understanding the Question

Given

Sequence A has nth term $60 - 4n$. Sequence B has nth term $n^2 - 300$. The table leaves the 5th term of each sequence blank.

Find

(a) The 5th term of sequence A and the 5 th term of sequence B. (b) The smallest positive number in sequence B.

Plan the Solution

- Part (a) is substitution. The 5th term is the term at $n = 5$, so 5 goes in place of n in each rule.
- Sequence B starts far below zero and climbs, because n^2 grows while the 300 stays put. Its terms stay negative until n^2 passes 300.
- So part (b) is asking where that crossing happens. Find the smallest whole number n whose square is greater than 300, then work out that term.
- The terms only get bigger after the crossing, so the first positive term is also the smallest positive one.

Worked Solution [4 marks]

> Rule - The nth term rule gives any term you ask it for: put that term's position in for n. A number is positive when it is greater than zero, and a position is always one of $1, 2, 3, \ldots$.

Step 1: the 5th term of sequence A

$60 - 4 \times 5 = 60 - 20 = 40$

(Reason: The 5th term is the one at $n = 5$, so 5 replaces n. Four lots of 5 is 20, and 20 taken from 60 leaves 40.)

Step 2: the 5th term of sequence B

$5^2 - 300 = 25 - 300 = -275$

(Reason: The same position goes into the second rule, but here it is squared first. 25 is far smaller than 300, so the term lands well below zero and the answer is negative.)

Step 3: part (b), say what positive means here

$n^2 - 300 > 0$

$n^2 > 300$

(Reason: A positive term is one greater than zero. Adding 300 to both sides leaves n^2 on its own, so the question has turned into: which squares beat 300?)

Step 4: find the first whole number n that works

$17^2 = 289$
$18^2 = 324$

(Reason: The square root marks the crossing point: $\sqrt{300} = 17.32\ldots$, so the first whole number past it is 18. The squares either side settle it - 289 is still below 300 and 324 is above it.)

Step 5: work out that term of sequence B

$18^2 - 300 = 324 - 300 = 24$

(Reason: Putting that value of n into the rule for sequence B gives the first term above zero. Every term after it is bigger, because a bigger n has a bigger square, so this one is the smallest positive number in the sequence.)

(a) 5th term of sequence A: 40

(a) 5th term of sequence B: -275

(b) 24

Verification

✓ **Check 1:** Count sequence A out term by term instead of using the rule. It starts at 56 and drops by 4 every time, so the fifth number in the list is part (a)'s first answer.
$56, 52, 48, 44, 40$

✓ **Check 2:** Reach the 5th term of sequence B from the 4th rather than from the rule. Squaring 5 instead of 4 adds 9, so the 5th term must sit 9 above the 4th.
$4^2 - 300 = -284$, then $-284 + 9 = -275$

✓ **Check 3:** List the terms of sequence B on either side of the answer. If the term before it were positive too, 24 could not be the smallest positive one. $16^2 - 300 = -44$, $17^2 - 300 = -11$, $18^2 - 300 = 24$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The two 5th terms	**Note**	The Answer column gives 40 and -275, and the Marks column gives 2 for it.	✓

Step	Mark	Description	Got it?
Partial Marks	**B1**	For each of the two correct terms.	✓
(b) The smallest positive number	**Note**	The Answer column gives 24, and the Marks column gives 2 for it.	✓
Partial Marks	**B1**	For 324 or 289 or $\sqrt{300}$ or $17.3\ldots$.	✓
Where those four values come from	**Note**	The four values the Partial Marks cell names are the landmarks on the way to the answer: the square just below 300, the square just above it, the square root of 300 itself, and that root written as a decimal. Any one of them shows the candidate has located where the sequence turns positive.	✓

Full marks: 4/4

Question 4 - Exam Solution

Understanding the Question

Given

The number is a factor of 140 and a factor of 210.

The number is odd.

Find

The greatest number that meets both of those conditions.

Plan the Solution

- Write 140 and 210 as products of prime factors.
- A number that divides both can be built only from primes that appear in both lists.
- Odd means no factor of 2, so multiply the shared primes with every 2 left out.

Worked Solution [2 marks]

> Rule - Common factors from primes: a number divides both only if every prime inside it appears in both factorisations, and an odd number contains no factor of 2.

Step 1: Write 140 as a product of primes

$140 = 2 \times 2 \times 5 \times 7$

(Reason: Halving twice gives $140 = 2 \times 70$ then $70 = 2 \times 35$, and $35 = 5 \times 7$.)

Step 2: Write 210 as a product of primes

$210 = 2 \times 3 \times 5 \times 7$

(Reason: Halving once gives $210 = 2 \times 105$, then $105 = 3 \times 35$ and $35 = 5 \times 7$.)

Step 3: Multiply the primes that appear in both lists

$2 \times 5 \times 7 = 70$

(Reason: A factor of both numbers can use only the primes the two lists share, so that product is the highest common factor.)

Step 4: Leave the 2 out to make it odd

$70 = 2 \times 5 \times 7$

$5 \times 7 = 35$

(Reason: Any factor that still contains a 2 is even, so the 2 is dropped. What is left is the largest odd number that divides both.)

35

Verification

✓ **Check 1:** Divide each of the two numbers by 35. $\frac{140}{35} = 4$ and $\frac{210}{35} = 6$, both whole numbers, and 35 is odd.

✓ **Check 2:** List every odd factor of 140 and test the largest of them. 1, 5, 7 and 35. The largest is 35, and it divides 210 as well, so nothing bigger can work.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The greatest odd number	**Note**	The Answer column gives 35, and the Marks column gives 2 for it.	✓
An answer of 5, 7 or 70	**B1**	For answer 5, 7 or 70.	✓
Both prime factorisations, or factor trees or tables, or the shared odd primes	**M1**	Or for $2 \times 2 \times 5 \times 7$ and $2 \times 3 \times 5 \times 7$, or two correct factor trees or tables, or $5 \times 7 \times k$ seen.	✓

Step	Mark	Description	Got it?
Where those values come from	**Note**	70 is the highest common factor of the two numbers, and it is even. 5 and 7 are the odd primes shared by the two factorisations, and $5 \times 7 = 35$.	✓

Full marks: 2/2

Question 5 - Exam Solution

Understanding the Question

Given

Part (a) is $\sqrt[3]{343} - \sqrt{40.96}$.
Part (b) is $(192 + 4 \times 16)^{1.25}$.
A calculator is allowed on this paper.
Every number here is exact, so each part also comes out by hand.

Find

(a) The value of $\sqrt[3]{343} - \sqrt{40.96}$. (b) The value of $(192 + 4 \times 16)^{1.25}$.

Plan the Solution

- Take part (a) one root at a time. 343 is a cube number and 40.96 is a square, so both roots are whole or exact and the subtraction is exact too.
- In part (b) the bracket is worked out first, and inside the bracket the multiplication comes before the addition.
- Then turn the index into a fraction: $1.25 = \frac{5}{4}$. A denominator of 4 is a fourth root and a numerator of 5 is a fifth power.

Worked Solution [2 marks]

> Rule - A fractional index is a root and a power together: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$, where the denominator gives the root and the numerator gives the power.

Step 1: part (a), the cube root

$7 \times 7 \times 7 = 343$

$\sqrt[3]{343} = 7$

(Reason: A cube root asks which number multiplied by itself three times gives 343. Trying 7 gives 343 exactly, so this root is a whole number.)

Step 2: part (a), the square root

$6.4 \times 6.4 = 40.96$

$\sqrt{40.96} = 6.4$

(Reason: Because 40.96 has two decimal places, its square root has one. $64 \times 64 = 4096$ fixes the digits, and placing the decimal point gives 6.4.)

Step 3: part (a), subtract the square root from the cube root

$7 - 6.4 = 0.6$

(Reason: Both roots are exact, so no rounding enters the subtraction and the answer is exact as well.)

Step 4: part (b), work out the bracket

$4 \times 16 = 64$

$192 + 64 = 256$

(Reason: Inside the bracket the multiplication is done before the addition, so 4×16 becomes 64 first. Adding that to 192 gives 256. Adding first would give 3136 instead.)

Step 5: part (b), write the index as a fraction

$1.25 = \dfrac{5}{4}$

$256^{1.25} = \left(\sqrt[4]{256}\right)^5$

(Reason: A fractional index splits into a root and a power. The denominator 4 names the fourth root and the numerator 5 names the fifth power.)

Step 6: part (b), take the root, then the power

$\sqrt[4]{256} = 4$

$4^5 = 1024$

(Reason: Rooting first keeps the numbers small: $4 \times 4 \times 4 \times 4 = 256$, and then 4^5 is 1024.)

(a) 0.6

(b) 1024

Verification

✓ **Check 1 - part (a), cube and square back:** Reverse both roots. 7^3 should be the number under the cube root, and 6.4^2 the number under the square root. $7^3 = 343$ and $6.4^2 = 40.96$, so both roots are right and $7 - 6.4 = 0.6$.

✓ **Check 2 - part (a), the same subtraction in tenths:** Write both roots over 10, so nothing is a decimal: $7 = \frac{70}{10}$ and $6.4 = \frac{64}{10}$. $\frac{70}{10} - \frac{64}{10} = \frac{6}{10} = \frac{3}{5}$, which is the second spelling of this value in the Answer column.

✓ **Check 3 - part (b), in powers of two:** The bracket is a power of two, $256 = 2^8$, and raising a power to a power multiplies the indices. $(2^8)^{1.25} = 2^{10} = 1024$, reached without taking a root at all.

✓ **Check 4 - part (b), work backwards:** Start from the answer. If $1024 = 4^5$, then the fourth root used in step 6 must have come from 4^4. $4^4 = 256$, which is the value of the bracket, so the chain closes.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The two roots, subtracted	Note	The Answer column gives 0.6 or $\frac{3}{5}$, and the Marks column gives 1 for it.	✓
(b) The bracket, raised to the power	Note	The Answer column gives 1024, and the Marks column gives 1 for it.	✓
Where the marks sit	Note	Neither part has an entry in the Partial Marks column, so the whole of each mark is for the answer itself.	✓

Full marks: 2/2

Question 6 - Exam Solution

Understanding the Question

Given

In the diagram, 5 kites are congruent to kite $ABCD$, and each pair of neighbouring kites shares one edge.

Find

The value of x.

Angle $DAB = 40^\circ$, and DCE is a straight line.
The angle marked x° is inside one of the kites, at a corner where two of them meet.

Plan the Solution

- Count the kites with a corner at C: the kites congruent to $ABCD$, and $ABCD$ itself.
- Those corners lie along the straight line DCE, and congruent kites have equal angles, so divide 180° by that count to get the angle one kite has at C.
- Kite $ABCD$ then has two known angles, one at A and one at C. Its other two are equal to each other, because a kite is symmetrical about the diagonal joining those two.
- Use the angle sum of a quadrilateral to find that equal pair, then match the marked angle to it.

Worked Solution [3 marks]

> Rule - Angles on a straight line add to 180°, and the four angles of a quadrilateral add to 360°. Congruent shapes have equal angles, and a kite is symmetrical about one diagonal, so the angles at the two vertices that diagonal does not join are equal.

Step 1: count the kites that meet at C

$5 + 1 = 6$

(Reason: Every kite has one corner at C. The 5 congruent kites supply 5 of those corners, and kite $ABCD$ supplies one more.)

Step 2: share the straight angle DCE between them

$$\frac{180}{6} = 30$$

(Reason: Angles on a straight line add to 180°, so the corners at C fill 180° between them. The kites are congruent, so their angles at C are all equal and each one takes the same share.)

Step 3: use the angle sum of kite $ABCD$

$360 - 40 - 30 = 290$

$$\frac{290}{2} = 145$$

(Reason: The four angles of a quadrilateral add to 360°. Taking off the 40° at A and the 30° at C leaves the angles at B and D. The diagonal AC is the kite's line of symmetry, so those two are equal and each takes half of what is left.)

Step 4: read off x

$x = 145$

(Reason: The angle marked x° sits at a corner of one of the congruent kites, matching B and D in kite $ABCD$. Congruent kites have equal angles, so it takes the same value.)

$$x = 145$$

Verification

✓ **Check 1:** Add the four angles of kite $ABCD$ and see whether they come to 360°.
$40 + 30 + 145 + 145 = 360$

✓ **Check 2:** Put the angles at C back on the straight line: 6 kites, each giving the same angle.
$6 \times 30 = 180$

✓ **Check 3:** Reach the answer a second way. The diagonal AC halves both the angle at A and the angle at C, so work inside triangle ACD instead of inside the whole kite.
$180 - 20 - 15 = 145$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The answer	**Note**	The Answer column gives 145, and the Marks column gives 3 for it.	✓
Partial Marks	**M1**	For $\dfrac{180}{6}$, or any angle congruent to $BCD = 30$.	✓
Partial Marks	**M1**	For $\dfrac{360 - 40 - \text{their } 30}{2}$ oe.	✓
Where the first Partial Marks row's two values come from	**Note**	Kite $ABCD$ has a corner at C as well as the kites congruent to it, so 6 equal angles fill the straight line DCE and each of them is 30. An angle congruent to BCD is that same angle in one of the other kites.	✓
What the second Partial Marks row allows	**Note**	The second method mark is for the working rather than for the 30. A candidate who found a different angle at C still earns it by using their own value in the same way.	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

Triangle JKL is right-angled and isosceles, with $JK = JL = 12.8$ cm. A semicircle stands on JL as its diameter.

Find

(a) the area of the shape (b) the perimeter of the shape

Plan the Solution

- (a) Add the area of triangle JKL to the area of the semicircle.
- (b) Walk round the outside: JK, then KL, then the curved edge. JL is inside the shape, so it is left out.

Worked Solution [7 marks]

> For a triangle, area $\frac{1}{2} \times \text{base} \times \text{height}$. For a semicircle of radius r on a diameter d, area $\frac{1}{2}\pi r^2$ and curved edge $\frac{1}{2}\pi d$.

Part (a), step 1: the area of the triangle

$$\frac{1}{2} \times 12.8 \times 12.8 = 81.92$$

(Reason: The two equal sides of a right-angled isosceles triangle are the ones holding the right angle, so the right angle is at J and JK and JL are the base and the height.)

Part (a), step 2: the radius of the semicircle

$$\frac{12.8}{2} = 6.4$$

(Reason: JL is the diameter, so the radius is half of it.)

Part (a), step 3: the area of the semicircle

$$\frac{1}{2} \times \pi \times 6.4^2 = 64.3398\ldots$$

(Reason: A whole circle of radius 6.4 has area $\pi \times 6.4^2$, and a semicircle is half of that. Keep the unrounded value for the next step.)

Part (a), step 4: add the two areas

$81.92 + 64.3398\ldots = 146.2598\ldots$

(Reason: The triangle and the semicircle meet along JL and do not overlap, so the two areas simply add. To 3 significant figures this is 146.)

Part (b), step 1: the length of KL

$12.8^2 + 12.8^2 = 327.68$

$\sqrt{327.68} = 18.1019\ldots$

(Reason: KL is the hypotenuse of the right-angled triangle, so Pythagoras gives it from the two sides of 12.8 cm.)

Part (b), step 2: the length of the curved edge

$$\frac{1}{2} \times \pi \times 12.8 = 20.1061\ldots$$

(Reason: The whole circle on JL has circumference $\pi \times 12.8$, and the curved edge of this shape is half of it.)

Part (b), step 3: add the three edges of the boundary

$12.8 + 18.1019\ldots + 20.1061\ldots = 51.0081\ldots$

(Reason: The boundary is JK, then KL, then the arc back to J. JL is a line inside the shape, so it is not walked along. To 3 significant figures this is 51.0.)

(a) $146\ \text{cm}^2$

(b) 51.0 cm

Verification

✓ **Check 1:** A semicircle's area is also $\dfrac{\pi d^2}{8}$, which works from the diameter and never touches the radius. Here $12.8^2 = 163.84$ and $\dfrac{163.84}{8} = 20.48$. The semicircle comes to $20.48 \times \pi = 64.3398\ldots$, so the shape is $81.92 + 64.3398\ldots = 146.2598\ldots$ again.

✓ **Check 2:** The right angle is at J, so the other two angles are $45°$ each and trigonometry gives $KL = \dfrac{12.8}{\sin 45°}$ without using Pythagoras at all. $KL = 18.1019\ldots$ once more, and with the arc at $20.1061\ldots$ the boundary is $12.8 + 18.1019\ldots + 20.1061\ldots = 51.0081\ldots$ as before.

✓ **Check 3:** A size check on the curved edge. An arc joining the two ends of a diameter has to be longer than the diameter itself, and half of $\pi \times 12.8$ is roughly 1.57×12.8. The arc,

$20.1061\ldots$, is comfortably longer than 12.8, which is what an arc on a diameter must be.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The area of the shape	**Note**	The Answer column gives 146 or 146.2 to 146.3, and the Marks column gives 3 for it.	✓
(a) Area of the triangle	**M1**	For $\frac{1}{2} \times 12.8 \times 12.8$.	✓
(a) Area of the semicircle	**M1**	For $\left[\frac{1}{2}\times\right] \pi \times \left(\frac{12.8}{2}\right)^2$.	✓
(b) The perimeter of the shape	**Note**	The Answer column gives $51[.0]$ or 51.00 to $51.01\ldots$, and the Marks column gives 4 for it.	✓
(b) The semicircular arc	**M1**	For $\frac{1}{2} \times \pi \times 12.8$.	✓
(b) The length of KL	**M2**	For $\sqrt{12.8^2 + 12.8^2}$ or $\frac{12.8}{\sin 45}$ oe.	✓
(b) Part-way towards KL	**M1**	Or for $12.8^2 + 12.8^2$ oe, or $\sin 45 = \frac{12.8}{KL}$ oe.	✓

Full marks: 7/7

Question 8 - Exam Solution

Understanding the Question

Given

The first five terms of a sequence are 11, 18, 25, 32 and 39.

Find

An expression for the nth term.

Plan the Solution

- Check that the step from one term to the next is the same all the way along the list.
- Use that step as the number in front of n, then work out what has to be added to it.

Worked Solution [2 marks]

> A sequence that goes up by the same amount every time has nth term $kn + j$, where k is that common difference and j is what is left when the matching multiple of k is taken away from a term.

Step 1: is the step between terms always the same?

$18 - 11 = 7$

$25 - 18 = 7$

$32 - 25 = 7$

$39 - 32 = 7$

(Reason: The same 7 every time, so the terms climb at a steady rate and the expression is a multiple of n with a number added on. The multiplier is that common difference, so the expression begins $7n$.)

Step 2: write the 7 times table under the sequence

$7 \times 1 = 7$

$7 \times 2 = 14$

$7 \times 3 = 21$

(Reason: $7n$ runs $7, 14, 21, 28, 35$ as n runs from 1 to 5. These are not the sequence itself, but each one sits directly under a term of it.)

Step 3: how far above the times table each term sits

$11 - 7 = 4$

(Reason: Take the multiple of 7 away from the term standing over it. The same amount is left at every one of the five terms, so one pair settles it, and that amount is the number added to $7n$.)

Step 4: the nth term

$7n + 4$

(Reason: The two pieces go together: 7 for every step along the list, and 4 added on. At $n = 1$ that gives 11, which is where the sequence starts.)

$$7n + 4$$

Verification

✓ **Check 1:** Put $n = 1$ and $n = 5$ into the expression. They must give the first and the fifth numbers on the paper, 11 and 39. $7 \times 1 + 4 = 11$ and $7 \times 5 + 4 = 39$, the two ends of the printed list.

✓ **Check 2:** Take one term of the expression away from the next one. Whatever n is, the answer has to be the sequence's own step of 7. $7(n+1)+4-(7n+4)=7$, so the expression climbs by 7 a term, exactly as the list does.

✓ **Check 3:** Build the number to add from the other end of the list instead. The fifth term is 39 and the fifth multiple of 7 is 35. $39-35=4$, the same number to add as the first term gave.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
An expression for the nth term	**Note**	The Answer column gives $7n+4$ oe final answer, and the Marks column gives 2 for it.	✓
A partly correct expression	**B1**	For $7n+j$ or $kn+4$, $k \neq 0$, or $7n+4$ seen then spoilt.	✓

Full marks: 2/2

Question 9 - Exam Solution

Understanding the Question

Given

A delivery van is worth $\$8000$.
Its value falls exponentially by 25% each year.

Find

The value of the van after 3 years.

Plan the Solution

- Turn the 25% fall into the fraction of the value that is still there after one year.
- Use that multiplier once for each of the 3 years, then apply it to the starting value.

Worked Solution [2 marks]

> Exponential decay: a starting value V that falls by $r\%$ of itself every year is worth $V \times \left(1-\dfrac{r}{100}\right)^n$ after n years. Each year's fall is taken from the value at the start of that year, not from the original value, which is why the multiplier is used again and again rather than added up.

Step 1: what is left after one year

$$1-\frac{25}{100}=0.75$$

(Reason: Losing 25% of the value leaves 75% of it, and 75% as a decimal is 0.75. So after any one year the van is worth 0.75 of what it was worth a year earlier.)

Step 2: what is left after three years

$$0.75\times0.75\times0.75=0.421875$$

(Reason: The fall happens once a year for 3 years, and each one acts on the value left by the year before, so the multiplier is used 3 times over. That is 0.75^3, the three-year multiplier.)

Step 3: apply the multiplier to the starting value

$$8000\times0.421875=3375$$

(Reason: The van started at \$8000, so multiplying by the three-year multiplier gives what it is worth once the 3 years have passed.)

\$3375

Verification

✓ **Check 1:** Do it in three separate steps, with no power anywhere. A quarter of 8000 is 2000, and each later year loses a quarter of whatever is left at the time. $8000-2000=6000$, then $6000-1500=4500$, then $4500-1125=3375$, the same value the multiplier gave.

✓ **Check 2:** Work in fractions instead of decimals. Taking a quarter off leaves $\frac{3}{4}$, so three years leave $\left(\frac{3}{4}\right)^3=\frac{27}{64}$ of the value. $\frac{8000}{64}=125$, and $125\times27=3375$, reached without a decimal at any point.

✓ **Check 3:** Run the years backwards. Dividing by 0.75 undoes one year's fall, so doing it three times has to bring the original value back. $\frac{3375}{0.75}=4500$, $\frac{4500}{0.75}=6000$, $\frac{6000}{0.75}=8000$, the van's value at the start.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The value after three years	**Note**	The Answer column gives 3375, and the Marks column gives 2 for it.	✓
The method	**M1**	For $8000 \times \left(1 - \frac{25}{100}\right)^3$ oe.	✓
Where the bracket in the Partial Marks row comes from	**Note**	A fall of 25 per cent leaves $1 - \frac{25}{100}$ of the value, so the bracket is one year's multiplier, and the index 3 is the number of years it is applied over.	✓

Full marks: 2/2

Question 10 - Exam Solution

Understanding the Question

Given

Paid into the account: $\$1500$
Compound interest each year: r %
Value after 8 years: $\$1656.73$

Find

The value of r

Plan the Solution

- Set the 8-year compound interest expression equal to $\$1656.73$.
- Divide by 1500 so that the growth factor stands on its own.
- Take the eighth root for one year's multiplier, then read r off it.

Worked Solution [3 marks]

> Compound interest multiplies the amount by the same factor every year, so after n years an investment of P is worth $P \times \left(1 + \frac{r}{100}\right)^n$.

Step 1: write the 8-year equation

$$1500 \times \left(1 + \frac{r}{100}\right)^8 = 1656.73$$

(Reason: Each year the balance is multiplied by the same factor, so 8 years multiply by it 8 times over.)

Step 2: divide by 1500

$$\left(1+\frac{r}{100}\right)^8 = \frac{1656.73}{1500} = 1.1044866\ldots$$

(Reason: Dividing the final value by the starting value leaves the whole 8-year growth factor and nothing else.)

Step 3: take the eighth root

$$1+\frac{r}{100} = \sqrt[8]{\frac{1656.73}{1500}} = 1.0125000\ldots$$

(Reason: The eighth root undoes the eighth power, so what is left is one year's multiplier.)

Step 4: turn the multiplier into a percentage

$1.0125 - 1 = 0.0125$

$0.0125 \times 100 = 1.25$

(Reason: Taking 1 away leaves the interest part of the multiplier, and multiplying by 100 writes that decimal as a percentage, which is r.)

$$r = 1.25$$

Verification

✓ **Check 1:** Put the answer back into the question and grow the money with the power: multiply $\$1500$ by 1.0125 eight times over. $1500 \times 1.0125^8 = 1656.73$ to the nearest cent, which is the value the question gives.

✓ **Check 2:** Drop the power and do it the long way, one year at a time:
$1500 \to 1518.75 \to 1537.73 \to 1556.96 \to 1576.42 \to 1596.12 \to 1616.07 \to 1636.28 \to 1656.73$
. The eighth multiplication lands on $\$1656.73$, so eight years at 1.25 % really does turn $\$1500$ into that amount.

✓ **Check 3:** Weigh the answer against simple interest at the same rate, which would give $1500 \times 0.0125 \times 8 = 150$. The account actually gained $1656.73 - 1500 = 156.73$, a little more than the 150 simple interest would give, which is what compounding should do. A much larger rate would overshoot badly.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The value of r	**Note**	The Answer column gives 1.25 or $1.250\ldots$, and the Marks column gives 3 for it.	✓
Taking the eighth root	**M2**	For $\sqrt[8]{\dfrac{1656.73}{1500}}$ oe.	✓
Setting up the equation	**M1**	Or for $1656.73 = 1500(k)^8$ oe for any k.	✓
What k stands for	**Note**	Here k is the yearly multiplier, the number the amount is multiplied by each year, and the 8 is the number of years.	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

The grid is shaded everywhere except inside R.
The region has four straight edges. Two of them are drawn solid and two are drawn dashed.

Find

Every inequality a point has to satisfy in order to lie inside R.

Plan the Solution

- Read the four corners of the region off the grid.
- Turn each edge into the equation of the line it lies along.
- Fix the direction of each inequality by testing one point taken from well inside the region.
- Fix the strictness from the drawing, then gather the four statements together.

Worked Solution [4 marks]

> Rule - A boundary drawn solid belongs to its region, so it gives $\leqslant$ or $\geqslant$. A boundary drawn dashed does not belong to it, so it gives $<$ or $>$. Which way an inequality points is settled by one test point taken from inside the region.

Step 1: read the corners of the region

$$(1,1), \quad (5,5), \quad (6,5), \quad (6,1)$$

(Reason: Every corner sits on a crossing of the grid lines, so all four can be read straight off the diagram. One edge slopes, one is upright, and the other two are level.)

Step 2: the equation of the sloping edge

$$\frac{5-1}{5-1} = 1$$

$$y = x$$

(Reason: The sloping edge runs from $(1,1)$ to $(5,5)$. Its gradient is the rise divided by the run, and it passes through $(1,1)$, so every point on it has its two coordinates equal.)

Step 3: which side of the sloping edge the region is on

$$(4,2)$$

$$2 < 4$$

$$y < x$$

(Reason: The point $(4,2)$ lies well inside R, and there the y coordinate is the smaller of the two. So the region is the side on which y is below x. This edge is dashed, so the line itself is left out and the inequality is strict.)

Step 4: the upright edge

$$x = 6$$

$$4 < 6$$

$$x < 6$$

(Reason: The upright edge joins $(6,1)$ to $(6,5)$, so every point on it has $x = 6$. The test point has $x = 4$, which puts the region on the left of that line, and this edge is dashed as well.)

Step 5: the two level edges

$$y = 1$$

$$y = 5$$

$$1 \leqslant y \leqslant 5$$

(Reason: The lower edge is $y = 1$ and the upper edge is $y = 5$, and the region lies between them, since the test point has $y = 2$. Both are drawn solid, so points on either line do belong to R, giving $y \geqslant 1$ and $y \leqslant 5$, which are written together as one double inequality.)

$$y < x$$

$$x < 6$$

$$1 \leqslant y \leqslant 5$$

Verification

✓ **Check 1:** Put a point that is plainly inside the region, $(5, 2)$, into all four statements, then do the same with a point that is plainly outside, $(2, 4)$. $(5, 2)$ satisfies every one of them, while $(2, 4)$ already fails $y < x$

✓ **Check 2:** Test the middle of each edge in turn: $(3, 3)$ on the sloping edge, $(6, 3)$ on the upright edge, $(3, 1)$ on the lower edge and $(5.5, 5)$ on the upper edge. Each one should pass exactly when the paper draws that edge solid. The two dashed midpoints fail, on $y < x$ and on $x < 6$, and the two solid midpoints pass

✓ **Check 3:** Sweep the whole grid at quarter-unit steps. For each point decide from the picture alone whether it is in the unshaded region, then decide again from the four inequalities, and compare the two answers. The picture and the inequalities agree at all 1089 points tested

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The answer	**Note**	The Answer column gives $y < x$, $x < 6$ and $1 \leqslant y \leqslant 5$ oe, and the Marks column gives 4 for it.	✓
Partial Marks	**B1**	For $y < x$.	✓
Partial Marks	**B1**	For $x < 6$.	✓
Partial Marks	**B2**	For $1 \leqslant y \leqslant 5$.	✓
Partial Marks	**B1**	Or for $y \geqslant 1$ or $y \leqslant 5$.	✓
Partial Marks	**SC2**	If B0 scored, for $y \leqslant x$, $x \leqslant 6$ and $1 < y < 5$ oe.	✓
Partial Marks	**SC1**	If B0 scored, or for three correct from $y = x$, $x = 6$, $y = 1$ and $y = 5$.	✓
Where the special case comes from	**Note**	The special case is the two kinds of boundary read the wrong way round: each dashed edge treated as part of the region and each solid edge treated as outside it. All four lines are then right and only the inequality signs are wrong.	✓

Full marks: 4/4

Question 12 - Exam Solution

Understanding the Question

Given

First equation: $\dfrac{3x}{2} + 5y = 5$

Second equation: $4x - 3y = 46$

Two linear equations in the same two unknowns, x and y, one of them carrying a fraction.

Find

The value of x and the value of y. One pair of values has to fit both equations at once, so expect a single solution pair.

Plan the Solution

- Multiply the first equation by 2 so that no fraction is left in it.
- Scale both equations until one letter carries the same coefficient in each.
- Subtract to remove that letter, then solve the single equation that is left.
- Substitute the value found back into one of the equations to get the other letter.

Worked Solution [4 marks]

> Rule - Elimination: multiply one or both equations so that one letter has the same size of coefficient in both, then add or subtract the equations to remove that letter.

Step 1: Clear the fraction

$$\frac{3x}{2} + 5y = 5$$

$$2 \times \frac{3x}{2} + 2 \times 5y = 2 \times 5$$

$$3x + 10y = 10$$

(Reason: Multiplying every term by 2 clears the denominator, so both equations now have whole-number coefficients and are easier to scale.)

Step 2: Match the coefficients of x

$$\text{Cleared equation} \times 4: \quad 12x + 40y = 40$$

$$\text{Second equation} \times 3: \quad 12x - 9y = 138$$

(Reason: The lowest common multiple of 3 and 4 is 12, so both equations can be written starting with $12x$. Every term on both sides is multiplied, the constant included.)

Step 3: Subtract to eliminate x

$(12x + 40y) - (12x - 9y) = 40 - 138$

$49y = -98$

$$y = \frac{-98}{49} = -2$$

(Reason: Both equations now start with $12x$, so subtracting one from the other removes x completely. Subtracting $-9y$ adds $9y$, which is where the $49y$ comes from.)

Step 4: Substitute back to find x

$3x + 10 \times (-2) = 10$

$3x - 20 = 10$

$3x = 30$

$$x = \frac{30}{3} = 10$$

(Reason: Putting $y = -2$ into the cleared equation leaves one unknown, which solves in one line.)

$$x = 10$$
$$y = -2$$

Verification

✓ **Check 1:** Put $x = 10$ and $y = -2$ into the first equation in the form the question prints it, fraction and all. $\frac{3 \times 10}{2} + 5 \times (-2) = 15 - 10 = 5$

✓ **Check 2:** Put the same pair into the second equation. $4 \times 10 - 3 \times (-2) = 40 + 6 = 46$

✓ **Check 3:** Reach the pair a different way: rearrange the second equation for y and put $x = 10$ into it. $\frac{4 \times 10 - 46}{3} = \frac{-6}{3} = -2$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Equating a set of coefficients	M1	Correctly equating one set of coefficients.	✓

Step	Mark	Description	Got it?
Eliminating a variable	**M1**	Correct method to eliminate one variable.	✓
Both values	**A2**	For $x = 10$ and $y = -2$.	✓
The value of x	**A1**	For $x = 10$.	✓
The value of y	**A1**	For $y = -2$.	✓
Special case	**SC1**	If M0 scored, for 2 values satisfying one of the original equations.	✓
About the special case	**Note**	M0 records that neither method mark was earned. The special case then covers a pair of values that satisfies one of the two printed equations.	✓

Full marks: 4/4

Question 13 - Exam Solution

Understanding the Question

Given

A quadrilateral drawn inside a circle with all four of its vertices on the circle, so it is a cyclic quadrilateral.
Going round the shape the marked angles are p°, $5m^\circ$, $(4m+38)^\circ$ and $4m^\circ$.

Find

The value of p.

Plan the Solution

- Opposite angles of a cyclic quadrilateral add to 180°. Read off which vertex faces which before writing anything down.
- Start with the pair that carries only m, the $5m^\circ$ and $4m^\circ$ angles, because that equation has one unknown in it.
- Then bring m to the other pair, p° and $(4m+38)^\circ$, which is where p lives.

Worked Solution [3 marks]

> Rule - Cyclic quadrilateral: opposite angles add to $180°$.

Step 1: use the pair of opposite angles that carries only m

$5m + 4m = 180$

$9m = 180$

$m = 20$

(Reason: The $5m°$ and $4m°$ angles are at opposite vertices, so they add to 180. No other letter appears, so this equation gives m on its own.)

Step 2: work out the angle that faces $p°$

$4 \times 20 + 38 = 118$

(Reason: The bottom vertex is the one opposite $p°$, and its angle is $(4m + 38)°$. Putting $m = 20$ into that expression turns it into a number.)

Step 3: use the second pair of opposite angles

$p + 118 = 180$

$p = 180 - 118 = 62$

(Reason: Opposite angles add to 180, so taking the facing angle away from 180 is what is left for p.)

$$p = 62$$

Verification

✓ **Check 1:** Test BOTH pairs of opposite angles, not just the one that was solved. With $m = 20$ the four angles are 62, 100, 118 and 80. $62 + 118 = 180$ and $100 + 80 = 180$

✓ **Check 2:** The angles of any quadrilateral add to 360, cyclic or not, so adding all four is a test that does not use the circle at all. $62 + 100 + 118 + 80 = 360$

✓ **Check 3:** Work backwards. If $p = 62$ then the angle facing it has to be 118, which means $4m + 38 = 118$. $118 - 38 = 80$, so $4m = 80$ and $m = 20$, the value Step 1 found.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The value of p	**Note**	The Answer column gives 62, and the Marks column gives 3 for it.	✓
Finding m	**B2**	For $m = 20$.	✓

Step	Mark	Description	Got it?
Method for m, or method for p	**M1**	Or for $5m + 4m = 180$ soi, or $p + 4m + 38 = 180$ soi.	✓
Where the scheme's value comes from	**Note**	The scheme's $m = 20$ is the value that makes the $5m$ and $4m$ angles total 180.	✓

Full marks: 3/3

Question 14 - Exam Solution

Understanding the Question

Given

A circle of radius 9 cm
An angle of 48° at the centre, marking off the unshaded minor sector
The shaded region is the major sector, the larger of the two

Find

The area of the shaded major sector

Plan the Solution

- Take the 48° minor sector away from a full turn, to get the angle of the major sector.
- Write that angle over 360: that is the fraction of the circle the sector covers.
- Multiply the fraction by the area of the whole circle, πr^2.
- Round to 3 significant figures.

Worked Solution [3 marks]

> Rule - Area of a sector: $\dfrac{\theta}{360} \times \pi r^2$, where θ is the angle at the centre in degrees.

Step 1: Find the angle of the major sector

$360^\circ - 48^\circ = 312^\circ$

(Reason: The major sector and the 48° minor sector together make a full turn, so the major sector takes whatever the minor one leaves.)

Step 2: Write the sector as a fraction of the whole circle

$$\frac{312}{360}$$

(Reason: An angle of 312° out of a full turn of 360° is that fraction of the circle, so the sector's area is the same fraction of the circle's area.)

Step 3: Multiply by the area of the whole circle

$$9^2 = 81$$

$$\frac{312}{360} \times \pi \times 81 = 220.539\ldots$$

(Reason: The whole circle has area πr^2, and squaring the radius gives 81, so the circle is $\pi \times 81$ square centimetres.)

Step 4: Round the answer

$$220.539\ldots = 221 \text{ (3 s.f.)}$$

(Reason: An answer is given to 3 significant figures unless the question asks for something else, and the units are square centimetres because this is an area.)

$$221 \text{ cm}^2$$

Verification

✓ **Check 1 - take the minor sector away instead:** Work out the whole circle and the 48° minor sector separately. The circle is $\pi \times 81 = 254.469\ldots$ and the minor sector is $\frac{48}{360} \times 254.469 = 33.929\ldots$. $254.469 - 33.929 = 220.54$

✓ **Check 2 - the radian form of the sector formula:** The same sector by $\frac{1}{2}r^2\theta$, with the angle in radians. 312° is 5.4454 radians, and this formula never forms the fraction used above. $0.5 \times 81 \times 5.4454 = 220.54$

✓ **Check 3 - a size check:** The shaded sector is more than half the circle, because 312° is more than 180°, and it must be less than the whole circle. Half the circle is about 127 cm^2 and the whole circle about 254 cm^2, and 221 sits between them.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The area of the shaded major sector	**Note**	The Answer column gives 221 or 220.5 to 220.6, and the Marks column gives 3 for it.	✓

Step	Mark	Description	Got it?
The full method: the major sector's fraction of the circle, times the circle's area	**M2**	For $\dfrac{360-48}{360} \times \pi \times 9^2$	✓
Partial method: the sector formula with an angle that is not a full turn	**M1**	Or for $\dfrac{k}{360} \times \pi \times 9^2$ where $k < 360$	✓
The angle of the major sector on its own	**B1**	Or for 312	✓
Where the partial award's value comes from	**Note**	The 312 in the B1 row is $360-48$, the angle of the shaded major sector.	✓

Full marks: 3/3

Question 15 - Exam Solution

Understanding the Question

Given

The recurring decimal $0.1\dot{4}\dot{6}$, in which the block 46 repeats without end

All the working must be shown, so the fraction has to be reached by algebra

Find

The decimal written as a fraction in its simplest form

Plan the Solution

- Give the decimal a name, x.
- Multiply by 1000 and by 10, so that both lines end in the same recurring tail.
- Subtract the smaller line from the larger, which cancels the tail and leaves a whole number.
- Divide to reach a fraction, then cancel that fraction to its simplest form.

Worked Solution [3 marks]

> Rule - shift, then subtract: multiply a recurring decimal by two powers of 10 chosen so that both results carry the same recurring tail. Subtracting one from the other cancels that

tail and leaves a whole number, and dividing then turns the decimal into a fraction.

Step 1: Give the decimal a name

$x = 0.1\dot{4}\dot{6} = 0.1464646\ldots$

(Reason: Naming the decimal turns the question into algebra, which is what the working has to be. Written out, the 1 *happens once and then the block* 46 *repeats without end.)*

Step 2: Multiply by 1000 and by 10

$1000x = 146.\dot{4}\dot{6}$

$10x = 1.\dot{4}\dot{6}$

(Reason: One digit stands before the repeating block, so multiplying by 10 *moves the point just past it. The block is two digits long, so multiplying by* 1000 *moves the point two places further on. Both lines then break off at the same place in the pattern, leaving the same tail* $0.4646\ldots$ *on each.)*

Step 3: Subtract to clear the recurring tail

$1000x - 10x = 146.\dot{4}\dot{6} - 1.\dot{4}\dot{6}$

$990x = 145$

(Reason: The two tails are identical, so subtracting cancels them exactly and what is left is the whole number $146 - 1$*. On the left,* $1000x - 10x$ *collects to* $990x$*.)*

Step 4: Divide, then cancel to the simplest form

$$x = \frac{145}{990}$$

$$\frac{145}{990} = \frac{29}{198}$$

(Reason: Dividing both sides by 990 *gives the fraction. Both parts are multiples of* 5*, since* $145 = 5 \times 29$ *and* $990 = 5 \times 198$*. After that cancelling,* 29 *is prime and* 198 *is not a multiple of it, so nothing is left to cancel.)*

$$\frac{29}{198}$$

Verification

✓ **Check 1 - put the fraction back into the subtraction:** The working ends at $990x = 145$, so multiplying the answer by 990 must give that same whole number back.

$$\frac{29}{198} \times 990 = 145$$

✓ **Check 2 - divide it out again:** A calculator divides 29 by 198. The digits it shows must be the decimal the question prints, not merely something close to it. The display reads

$0.1464646...$, which is $0.1\dot{4}\dot{6}$

✓ **Check 3 - the fraction really is in its simplest form:** Break the denominator into prime factors and look for a factor it shares with 29. $198 = 2 \times 3^2 \times 11$, and 29 is prime, so the only factor they share is 1

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two multiples of the decimal, subtracted so the recurring tails cancel	M1	For $146.\dot{4}\dot{6} - 1.\dot{4}\dot{6}$ oe	✓
The fraction in its simplest form	A2	For $\frac{29}{198}$ cao	✓
The right fraction, not yet cancelled	A1	For $\frac{145}{990}$ oe	✓
A fraction over 990 reached without enough working	SC1	If M0 scored, for $\frac{k}{990}$ with insufficient working	✓
Where the partial award's value comes from	Note	The $\frac{145}{990}$ in the A1 row is $\frac{29}{198}$ before it is cancelled.	✓

Full marks: 3/3

Question 16 - Exam Solution

Understanding the Question

Given

Part (a): a rectangle for the universal set holding two overlapping circles, M and N.

Part (b): a rectangle holding three overlapping circles, A, B and C, with a number written in every one of its eight regions.

Find

(a) The region $M' \cap N'$, shaded on the first diagram. (b) The value of $\mathrm{n}(B \cap (A' \cup C))$.

Those numbers are 33 in A alone, 16 in A and B only, 3 in B alone, 10 in all three, 18 in A and C only, 4 in B and C only, 9 in C alone, and 20 outside all three.

Plan the Solution

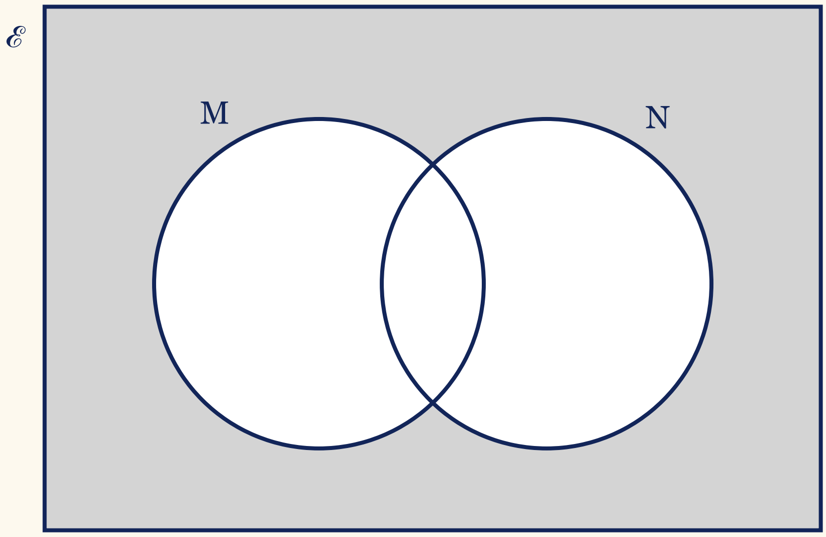
- A dash is the complement sign, so M' means everything outside M. Two complements joined by $\cap$ means outside both at once.
- In part (b) work from inside the brackets outwards. Build $A' \cup C$ first, then throw away anything that is not also in B.
- Do it one region at a time. Each of the eight regions is wholly inside a circle or wholly outside it, so every region gets a plain yes or no.

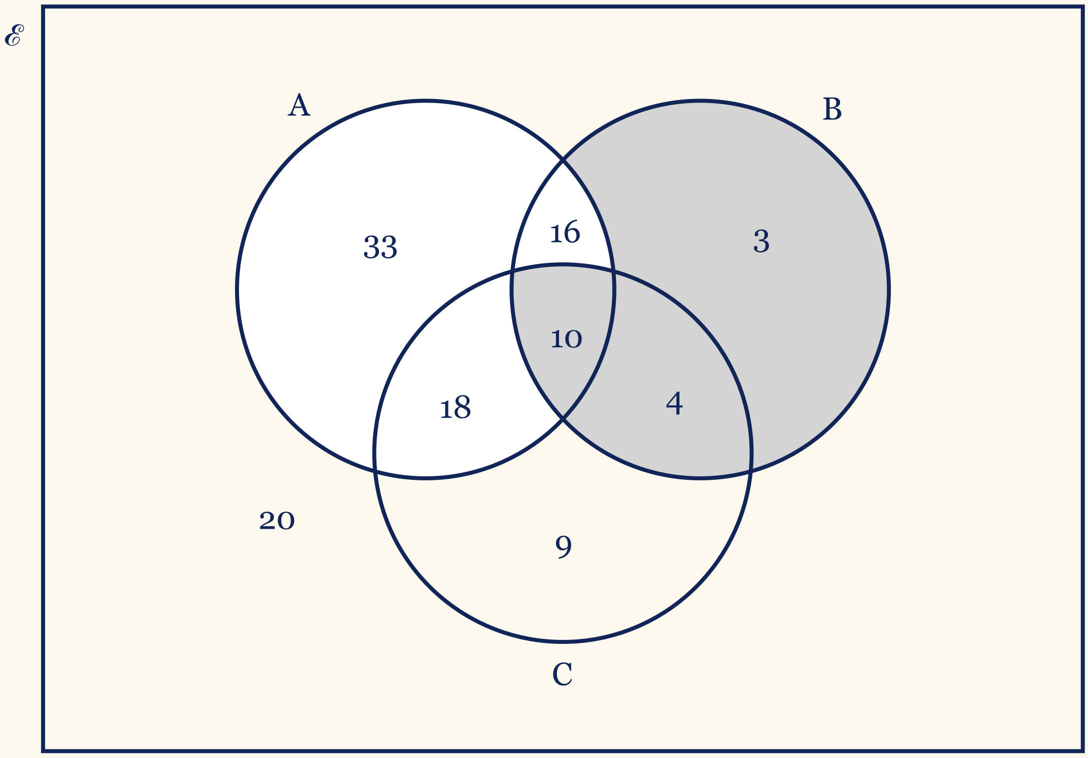
Worked Solution [2 marks]

> Rule - a dash is the complement, everything outside the set; $\cap$ means in both; $\cup$ means in one or the other or both; and $\mathrm{n}(X)$ is how many members X has.

Step 1: read the two dashes in $M' \cap N'$

$M' \cap N' = (M \cup N)'$





(Reason: A point in M' is outside M, and a point in N' is outside N. To be in both at once a point has to miss both circles, which is the same as sitting outside the whole of $M \cup N$. So the shading is every part of the rectangle that no circle covers, and the overlap stays clear along with the rest.)

Step 2: build $A' \cup C$

$A' \cup C$ holds $3, 10, 18, 4, 9, 20$

(Reason: A' is the four regions with no A over them, holding 3, 4, 9 and 20. C is the four regions inside the bottom circle, holding 18, 10, 4 and 9. A union keeps anything on either list, and the only two regions on neither are the ones holding 33 and 16.)

Step 3: keep only the regions that are also in B

B holds $16, 3, 10, 4$

$B \cap (A' \cup C)$ holds $3, 10, 4$

(Reason: There are four regions inside B. Of those, the one holding 16 is the only one Step 2 left out, because it lies inside A and outside C and so fails both halves of the union. The other three survive.)

Step 4: count the members in those three regions

$3 + 10 + 4 = 17$

(Reason: n counts members, so add together the numbers written in the three regions Step 3 kept.)

(a) Every part of the rectangle outside both circles is shaded

(b) 17

Verification

✓ **Check 1:** Take the unwanted slice off the whole of B instead of building the answer up. All four regions of B together give $16 + 3 + 10 + 4 = 33$, and the only one of them that fails the condition is the region inside A and outside C. $33 - 16 = 17$

✓ **Check 2:** Split the union instead, so that $B \cap (A' \cup C)$ becomes $B \cap A'$ together with $B \cap C$. The first holds 7 members and the second holds 14, and the region holding 4 sits in both of them, so it must not be counted twice. $7 + 14 - 4 = 17$

✓ **Check 3:** For part (a), test one point from each of the four areas of the first diagram. A point in M alone fails M'; a point in the overlap fails both dashes; a point in N alone fails N'. Only a point that misses both circles passes, so the shading is the outside area and nothing else.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The shaded region	**Note**	The Answer column gives a diagram with the whole rectangle shaded except inside the two circles, and the Marks column gives 1 for it.	✓
(b) The number of members	**Note**	The Answer column gives 17, and the Marks column gives 1 for it.	✓
Neither part has a Partial Marks entry	**Note**	Both parts leave the Partial Marks column empty, so on each part the whole mark is for the answer itself and nothing is printed for a method.	✓

Full marks: 2/2

Question 17 - Exam Solution

Understanding the Question

Given

In triangle ABC, side AB is 6.7 cm and side BC is 5.9 cm.
The angle where those two sides meet, at vertex B, is marked 81 degrees.

Find

The area of triangle ABC, in cm^2.

Plan the Solution

- Two sides and the angle between them are given, so the area comes straight from $\frac{1}{2}ab\sin C$ and no perpendicular height has to be found first.
- Confirm that the marked angle is the one where the two given sides actually meet. That is the only angle this rule accepts.
- Multiply everything in one go, keep the unrounded value on the calculator, and round once at the very end.

Worked Solution [2 marks]

> Rule - Area from two sides and the angle between them: $\text{Area} = \frac{1}{2}ab\sin C$, where a and b are the two sides and C is the angle between them.

Choose the rule that fits what is given

$$\text{Area} = \frac{1}{2} \times AB \times BC \times \sin B$$

(Reason: Two sides and the angle between them are known, and that is exactly what this rule takes. A half-base-times-height calculation would need a perpendicular length the question never gives.)

Put in the two sides and the angle between them

$$\text{Area} = \frac{1}{2} \times 6.7 \times 5.9 \times \sin 81^\circ$$

(Reason: The sides 6.7 cm and 5.9 cm both meet at B, and the angle marked there is 81°.)

Work the product out

$$\frac{1}{2} \times 6.7 \times 5.9 = 19.765$$

$$19.765 \times \sin 81^\circ = 19.5216\ldots$$

(Reason: Halve the product of the two sides first, then multiply by the sine of the angle. Leave the value on the calculator display rather than rounding partway through.)

Round the answer

$$\text{Area} = 19.5 \text{ cm}^2$$

(Reason: $19.5216\ldots$ is not exact, so it is given to three significant figures. That is the 19.5 the mark scheme gives, and the scheme also accepts the longer $19.52\ldots$.)

$$19.5 \text{ cm}^2$$

Verification

✓ **Check 1 - undo the multiplication:** Divide the area back by $\frac{1}{2} \times 6.7 \times 5.9$. If the working is right, what is left has to be the sine of the marked angle.

$$\frac{19.5216\ldots}{19.765} = 0.98768\ldots = \sin 81^\circ$$

✓ **Check 2 - build it from a perpendicular height:** Drop a perpendicular from A on to BC. Its length is $6.7 \times \sin 81^\circ = 6.6175\ldots$ cm, so half the base times that height gives the area a second way. $\frac{1}{2} \times 5.9 \times 6.6175\ldots = 19.52\ldots$

✓ **Check 3 - is the size sensible:** With these two sides the area is at its largest when the angle between them is 90°, which would give $\frac{1}{2} \times 6.7 \times 5.9 = 19.765$. An angle of 81° is close to a right angle, so the area should land just below that ceiling. The answer 19.5 sits a little under 19.765, as expected.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The area of triangle ABC	**Note**	The Answer column gives 19.5 or $19.52\ldots$, and the Marks column gives 2 for it.	✓
Method: the two sides and the angle between them	**M1**	For $\frac{1}{2} \times 6.7 \times 5.9 \times \sin 81$ oe.	✓
Note: which angle the rule needs	**Note**	The rule needs the angle between the two sides being used. The sides 6.7 and 5.9 meet at B, and the angle marked there is 81, so both sides and the angle drop straight into the formula with nothing to work out first.	✓

Full marks: 2/2

Question 18 - Exam Solution

Understanding the Question

Given

Find

The curve $y = x^3 + 4x^2 - 2$ is already drawn for $-3 \leqslant x \leqslant 1.5$.

Every value of x in $-3 \leqslant x \leqslant 1.5$ that satisfies $x^3 + 4x^2 - 2 = 2x$, found by ruling one straight line.

Plan the Solution

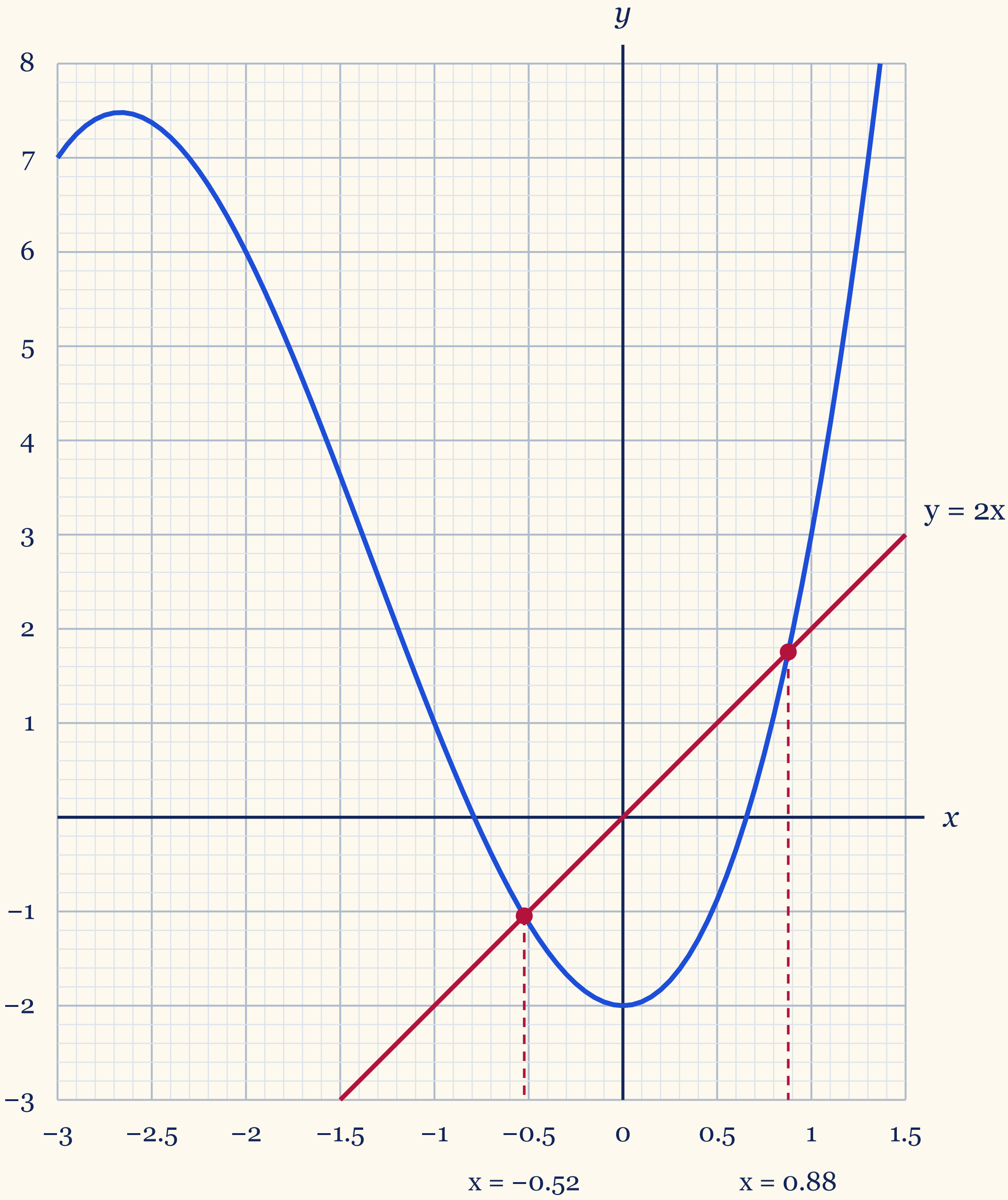
- Read the equation as two graphs: the curve that is already there, and one straight line.
- Rule that line right across the grid.
- Take the x-coordinate of each point where the line meets the curve.

Worked Solution [3 marks]

> Two graphs meet where their y-values are equal, so the x-coordinate of every crossing point solves the equation made by putting the two expressions equal to each other.

Step 1: Decide which straight line to draw

$x^3 + 4x^2 - 2 = 2x$



(Reason: The left-hand side is exactly the expression that has already been plotted, so the equation holds wherever the curve's y-value is equal to $2x$. The line to add is therefore $y = 2x$.)

Step 2: Work out two points on that line

$$2 \times (-1) = -2$$
$$2 \times 1.5 = 3$$

(Reason: Two points fix a straight line. Take $x = -1$ and $x = 1.5$, which are both inside the range shown. The line also passes through the origin, so rule it right across the grid.)

Step 3: Read the x-coordinate of each crossing

$$x = -0.52$$
$$x = 0.88$$

(Reason: The ruled line cuts the curve at two points inside $-3 \leqslant x \leqslant 1.5$. Dropping from each crossing to the x-axis gives a reading a small part of a square to the left of -0.5, and another the same distance to the left of 0.9.)

$$x = -0.52$$
$$x = 0.88$$

Verification

✓ **Check 1:** Rearranged, the equation is $x^3 + 4x^2 - 2x - 2 = 0$. Substitute the two ends of the square that the first crossing sits in.
$(-0.6)^3 + 4 \times (-0.6)^2 - 2 \times (-0.6) - 2 = 0.424$ and
$(-0.5)^3 + 4 \times (-0.5)^2 - 2 \times (-0.5) - 2 = -0.125$. The sign changes, so a solution lies between -0.6 and -0.5.

✓ **Check 2:** The same test on the square that the second crossing sits in.
$0.85^3 + 4 \times 0.85^2 - 2 \times 0.85 - 2 = -0.195875$ and
$0.9^3 + 4 \times 0.9^2 - 2 \times 0.9 - 2 = 0.169$. The sign changes again, so a solution lies between 0.85 and 0.9.

✓ **Check 3:** This cubic has three solutions, and they add to -4. Test whether the one that was not read off really is out of range. The two crossings add to about 0.35, so the third solution is about -4.35, well outside $-3 \leqslant x \leqslant 1.5$. The two read off the graph are all there are in the range.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The straight line	**B1**	For the line $y = 2x$ ruled.	✓
Both solutions	**B2**	For $x = -0.5$ to -0.55 and $x = 0.85$ to 0.9.	✓
The negative solution only	**B1**	For -0.5 to -0.55.	✓
The positive solution only	**B1**	For 0.85 to 0.9.	✓

Full marks: 3/3

Question 19 - Exam Solution

Understanding the Question

Given

(a) $12m^2 - 75t^2$

(b) $xy + 15 + 3y + 5x$

Two expressions, neither of them already written as a product.

Find

(a) $12m^2 - 75t^2$ written as a product, factorised completely. (b) $xy + 15 + 3y + 5x$ written as a product, factorised completely.

Plan the Solution

- (a) Two terms. Take out the number that divides both of them, and look at what is left inside the bracket.
- (a) $4m^2 - 25t^2$ is one square subtract another, so it splits into two brackets.
- (b) Four terms, with nothing dividing all four. Put them in pairs and take a factor out of each pair.
- (b) The same bracket should come out of both pairs; that bracket is the last factor to take out.
- Completely means keep going until no bracket has a factor left inside it.

Worked Solution [5 marks]

> Rule - Factorise completely: take out every common factor first, then factorise what is left. A difference of two squares splits as $a^2 - b^2 = (a+b)(a-b)$.

(a) Take out the factor common to both terms

$12m^2 - 75t^2 = 3(4m^2 - 25t^2)$

(Reason: The largest number dividing both 12 and 75 is 3, and no letter appears in both terms, so only the number comes out.)

(a) Write the bracket as a difference of two squares

$4m^2 - 25t^2 = (2m)^2 - (5t)^2$

(Reason: Both terms inside the bracket are perfect squares, so the bracket is one square subtract another.)

(a) Split the difference of two squares

$3(4m^2 - 25t^2) = 3(2m + 5t)(2m - 5t)$

(Reason: Using $a^2 - b^2 = (a+b)(a-b)$ with $a = 2m$ and $b = 5t$. Neither bracket has a factor left inside it, so the factorising stops here.)

(b) Put the four terms into pairs

$xy + 15 + 3y + 5x = xy + 5x + 3y + 15$

(Reason: Nothing divides all four terms, so they are paired off instead: xy with $5x$, and $3y$ with 15.)

(b) Take a factor out of each pair

$xy + 5x + 3y + 15 = x(y + 5) + 3(y + 5)$

(Reason: x comes out of the first pair and 3 out of the second, and each pair leaves $y + 5$ behind.)

(b) Take out the bracket both parts share

$x(y + 5) + 3(y + 5) = (x + 3)(y + 5)$

(Reason: $y + 5$ is a factor of both parts, so it comes out and $x + 3$ is what is left.)

(a) $3(2m + 5t)(2m - 5t)$

(b) $(x + 3)(y + 5)$

Verification

✓ **Check 1:** Multiply part (a)'s brackets out, then put the 3 back. $(2m + 5t)(2m - 5t) = 4m^2 - 25t^2$, so $3(2m + 5t)(2m - 5t) = 12m^2 - 75t^2$, which is the expression in part (a).

✓ **Check 2:** Put $m = 2$ and $t = 1$ into part (a)'s expression and into its answer. $12 \times 4 - 75 = -27$ and $3 \times 9 \times (-1) = -27$.

✓ **Check 3:** Multiply $(x + 3)(y + 5)$ out, term by term. $xy + 5x + 3y + 15$, the same four terms as in part (b).

✓ **Check 4:** Put $x = 2$ and $y = 4$ into part (b)'s expression and into its answer. $8 + 15 + 12 + 10 = 45$ and $5 \times 9 = 45$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The complete factorisation	**Note**	The Answer column gives $3(2m + 5t)(2m - 5t)$ final answer, and the Marks column gives 3 for it.	✓
(a) Factorised into two brackets, one of them still holding a common factor	**B2**	For $(6m + 15t)(2m - 5t)$ or $(2m + 5t)(6m - 15t)$.	✓
(a) The common factor taken out, or the two brackets without	**B1**	Or for $3(4m^2 - 25t^2)$ or $(2m + 5t)(2m - 5t)$.	✓

Step	Mark	Description	Got it?
it			
(b) The complete factorisation	**Note**	The Answer column gives $(x+3)(y+5)$ final answer, and the Marks column gives 2 for it.	✓
(b) Either grouping of the four terms into pairs	**B1**	For $x(y+5)+3(y+5)$ or $y(x+3)+5(x+3)$.	✓

Full marks: 5/5

Question 20 - Exam Solution

Understanding the Question

Given

The equation $8\sin x + 6 = 1$
The interval $0^\circ \leqslant x \leqslant 360^\circ$, so the calculator is set to degrees

Find

Every value of x in that interval that satisfies the equation Expect two of them: one full turn takes every sine value strictly between -1 and 1 twice

Plan the Solution

- Rearrange the equation until $\sin x$ stands alone.
- Read the acute angle a from $\sin^{-1}\left(\frac{5}{8}\right)$, leaving the minus sign aside for the moment.
- The sine is negative, so take the two angles of the turn whose sine is negative: $180^\circ + a$ and $360^\circ - a$.
- Write each angle to one decimal place.

Worked Solution [3 marks]

Rule - Negative sine over one turn: if $\sin x = -k$ with $0 < k < 1$ and $a = \sin^{-1} k$, then the solutions in $0^\circ \leqslant x \leqslant 360^\circ$ are $x = 180^\circ + a$ and $x = 360^\circ - a$.

Step 1: get $\sin x$ on its own

$8\sin x + 6 = 1$

$8\sin x = 1 - 6 = -5$

$\sin x = -\dfrac{5}{8}$

(Reason: Take 6 from both sides, then divide both sides by 8. The result sits between -1 and 1, so the equation does have solutions.)

Step 2: find the acute angle behind the answer

$$\sin^{-1}\left(\frac{5}{8}\right) = 38.682\ldots^\circ$$

(Reason: The inverse sine of a positive value gives the acute angle a. The minus sign is not needed here - it settles which two angles are wanted, not how big a is.)

Step 3: the solution between 180° and 270°

$x = 180^\circ + 38.682^\circ = 218.682^\circ$

(Reason: Sine is negative for every angle between 180° and 270°, and the one wanted sits a past 180°.)

Step 4: the solution between 270° and 360°

$x = 360^\circ - 38.682^\circ = 321.318^\circ$

(Reason: Sine is negative across that quarter of the turn as well, and the second solution sits a short of 360°.)

Step 5: write each angle to one decimal place

$x = 218.7^\circ$

$x = 321.3^\circ$

(Reason: Neither decimal terminates, so each angle is given to one decimal place.)

$$x = 218.7^\circ \textbf{ or } x = 321.3^\circ$$

Verification

✓ **Check 1:** Both angles have the same sine, -0.625. Put that back into $8\sin x + 6$. $8 \times (-0.625) + 6 = 1$, which is the right-hand side

✓ **Check 2:** One solution sits a past 180° and the other a short of 360°, so the pair must add to 540° whatever a turns out to be. $218.682 + 321.318 = 540$

✓ **Check 3:** Count the crossings over the whole turn: $8\sin x + 6$ runs from 6 up to 14, down to -2 at 270° and back to 6. It passes 1 once on the way down and once on the way up, so there are exactly two solutions - the two found above.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The two values of x	**Note**	The Answer column gives 218.7, 321.3, and the Marks column gives 3 for it.	✓
One value correct	**B2**	For one correct.	✓
Method: sin x on its own	**M1**	Or for $\sin x = -\frac{5}{8}$ oe.	✓
Special case	**SC1**	If M1 or 0 scored, for two reflex angles with a sum of 540 or two non-reflex angles with a sum of 180.	✓

Full marks: 3/3

Question 21 - Exam Solution

Understanding the Question

Given

A cuboid with $EF = 9.1$ cm, $EH = 6.5$ cm and $HD = 4$ cm
EH is a vertical edge, so E is directly above H
The base $CDHG$ is a rectangle, with $HG = EF = 9.1$ cm and $GC = HD = 4$ cm

Find

The angle between the line CE and the plane of the base $CDHG$

Plan the Solution

- The line CE meets the base at C. Drop E straight down onto the base and it lands on H, so CH is the shadow of CE on the base.
- The angle wanted is therefore the angle at C in triangle ECH, and that triangle has its right angle at H because EH is perpendicular to the base.
- So find CH with Pythagoras in the base rectangle first, then use the tangent ratio.

Worked Solution [4 marks]

> Rule - the angle between a line and a plane: project the line onto the plane, then work in the right-angled triangle made by the line, its projection and the perpendicular between them.

Step 1: find the base diagonal CH

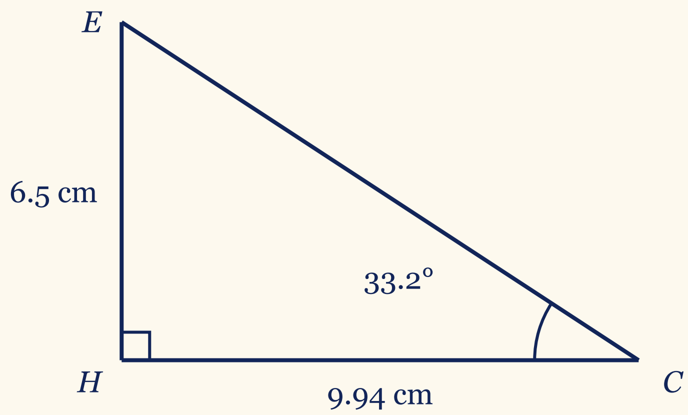
$HG = EF = 9.1 \text{ cm}$

$GC = HD = 4 \text{ cm}$

$CH^2 = HG^2 + GC^2$

$9.1^2 + 4^2 = 82.81 + 16 = 98.81$

$CH = \sqrt{98.81} = 9.9403\ldots \text{ cm}$



(Reason: (Reason: $CDHG$ is a rectangle measuring 9.1 cm by 4 cm, so its diagonal comes straight from Pythagoras. Keep the surd rather than a rounded value, because it is used again in the next step.))

Step 2: name the angle

$\angle ECH$

(Reason: E sits directly above H, so H is the point where E lands on the base. That makes CH the projection of CE, and the angle between CE and the base is the angle between CE and CH. (Reason: picking the right triangle is the whole difficulty here, and the mark scheme has a mark for this identification on its own.))

Step 3: use the tangent ratio in triangle ECH

$$\tan(\angle ECH) = \frac{EH}{CH} = \frac{6.5}{\sqrt{98.81}}$$

(Reason: (Reason: triangle ECH has its right angle at H. Seen from C, the vertical EH is the opposite side and the projection CH is the adjacent side, and opposite over adjacent is the tangent.))

Step 4: work out the angle

$$\angle ECH = \tan^{-1}\left(\frac{6.5}{\sqrt{98.81}}\right) = 33.18\ldots^\circ$$

$$\angle ECH = 33.2^\circ \text{ (3 s.f.)}$$

(Reason: (Reason: the calculator must be in degree mode. The unrounded value is $33.18\ldots$, which rounds to 33.2 to three significant figures.))

33.2°

Verification

✓ **Check 1:** Use the space diagonal instead of the base diagonal. $CE = \sqrt{4^2 + 9.1^2 + 6.5^2} = \sqrt{141.06}$, and from C the side EH is opposite while CE is the hypotenuse, so use the sine. $\sin^{-1}\left(\frac{6.5}{\sqrt{141.06}}\right) = 33.2^\circ$ to three significant figures, the same angle

✓ **Check 2:** Use the cosine on the same triangle: adjacent $CH = \sqrt{98.81}$ over hypotenuse $CE = \sqrt{141.06}$. This uses neither of the other two ratios. $\cos^{-1}\left(\frac{\sqrt{98.81}}{\sqrt{141.06}}\right) = 33.2^\circ$ to three significant figures, the same angle again

✓ **Check 3:** A size check. The height $EH = 6.5$ cm is smaller than the base diagonal $CH = 9.94\ldots$ cm, so the opposite side is shorter than the adjacent side and the angle has to be under 45°. 33.2° is under 45°

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The angle between CE and the base $CDHG$	Note	The Answer column gives 33.2 or $33.18\ldots$, and the Marks column gives 4 for it.	✓
Method - the tangent of angle ECH	M3	For $\tan = \frac{6.5}{\sqrt{4^2 + 9.1^2}}$ oe.	✓
Method - the squaring that gives a diagonal	M2	Or for $4^2 + 9.1^2$ oe, or $4^2 + 9.1^2 + 6.5^2$ oe.	✓

Step	Mark	Description	Got it?
Method - identifying the angle	M1	Or for recognising the angle ECH.	✓

Full marks: 4/4

Question 22 - Exam Solution

Understanding the Question

Given

Tin A and tin B hold red beads and blue beads only, and one bead is taken at random from each
$\mathrm{P}(\text{Curtis red}) = 0.4$
$\mathrm{P}(\text{both red}) = 0.25$

Find

$\mathrm{P}(\text{both blue})$, the probability that Curtis's bead and Heather's bead are both blue

Plan the Solution

- Each tin holds two colours only, so a blue probability is 1 minus the matching red probability.
- The two beads come from different tins and are taken at random, so the picks are independent and their probabilities multiply.
- Heather's red probability is not given, so take it out of the 0.25 first. Then change both reds into blues and multiply.

Worked Solution [4 marks]

> Rule - Independent events: $\mathrm{P}(X \text{ and } Y) = \mathrm{P}(X) \times \mathrm{P}(Y)$. Complement: $\mathrm{P}(\text{not } X) = 1 - \mathrm{P}(X)$.

Step 1: Heather's probability of a red bead

$$\mathrm{P}(\text{Heather red}) = \frac{0.25}{0.4} = 0.625$$

(Reason: The picks are independent, so the two red probabilities multiply to give 0.25*. Dividing by Curtis's* 0.4 *undoes that multiplication.)*

Step 2: Curtis's probability of a blue bead

$\mathrm{P}(\text{Curtis blue}) = 1 - 0.4 = 0.6$

(Reason: Tin A holds red beads and blue beads only, so its two probabilities add to 1.)

Step 3: Heather's probability of a blue bead

$\mathrm{P}(\text{Heather blue}) = 1 - 0.625 = 0.375$

(Reason: Tin B holds two colours only as well, so the same subtraction from 1 applies to it.)

Step 4: Multiply the two blue probabilities

$\mathrm{P}(\text{both blue}) = 0.6 \times 0.375 = 0.225$

(Reason: Independent again: the probability that both blue picks happen is the product of the two separate probabilities.)

$$0.225$$

Verification

- ✓ **Check 1:** Rebuild the figure the question gives. Multiply Curtis's red probability 0.4 by Heather's red probability 0.625. $0.4 \times 0.625 = 0.25$, which is the both-red probability the question states.
- ✓ **Check 2:** There are four colour pairs and they must total 1. Red then blue is $0.4 \times 0.375 = 0.15$, and blue then red is $0.6 \times 0.625 = 0.375$. $0.25 + 0.15 + 0.375 + 0.225 = 1$
- ✓ **Check 3:** Count whole beads instead of using probabilities. Tin A with 2 red and 3 blue gives 0.4, and tin B with 5 red and 3 blue gives both-red 0.25. That makes $5 \times 8 = 40$ equally likely pairs, of which 3×3 are blue with blue. $\frac{9}{40} = 0.225$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The probability that both beads taken are blue	**Note**	The Answer column gives 0.225 oe, and the Marks column gives 4 for it.	✓
Complete method in one expression	**M3**	For $\left(1 - \frac{0.25}{0.4}\right) \times (1 - 0.4)$ oe.	✓
Second method: Heather's probability of a red bead	**M2**	Or for $\frac{0.25}{0.4}$.	✓
Second method: an equation for Heather's	**M1**	Or for $0.4 \times p = 0.25$ oe.	✓

Step	Mark	Description	Got it?
probability of a red bead			
Second method: both blue from their probability of a red bead	**M1**	For $(1 - \text{their } \mathrm{P}(\text{Heather red})) \times (1 - 0.4)$ oe.	✓
What the letter p stands for	**Note**	The letter p in the second method stands for the probability that Heather takes a red bead.	✓

Full marks: 4/4

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