IGCSEGCSEMATHS.CO.UK

Cambridge IGCSE 0580

0580/41 (Extended) – May/June 2024

130 marks · 2 hours 30 minutes · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Cambridge IGCSE Mathematics, Paper 0580/41 (Extended), May/June 2024 series, sat Tuesday 7 May 2024 in Cambridge administrative zone 2 –130 marks, 2 hours 30 minutes, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Cambridge Assessment International Education. This resource reproduces neither the exam paper nor the official mark scheme.

Cambridge sits this paper on different days in different administrative zones. The date above is the one published for Zone 2; timetables for the other zones of this series are no longer available from Cambridge.

Both are PDF files hosted by Cambridge: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. (a) The areas, in km², of the world's four largest rainforests are given in this table.

Rainforest	**Area** (km^2)
Amazon	5 500 000
Congo	2 000 000
Atlantic	1 315 000
Valdivian	250 000

(i) What is the area of the Valdivian rainforest, as a percentage of the area of the Amazon rainforest? *[1 mark]*

(ii) Give the ratio of the rainforest areas Valdivian : Atlantic : Congo in its simplest form. *[2 marks]*

(iii) Of the area of the Amazon rainforest, 60% lies in Brazil and 10% lies in Colombia.

Peru holds $43\frac{1}{3}\%$ of the remaining area of the rainforest.

What percentage of the Amazon rainforest is in Brazil, Colombia and Peru? *[3 marks]*

(iv) $\dfrac{27}{50}$ of the total area of rainforest in the world is the area of the Amazon rainforest.
What is the total area of rainforest in the world?
Give your answer correct to the nearest $100\,000$ km². *[3 marks]*

(v) Every minute, the world loses 60.7 hectares of rainforest.
What is the total area, in hectares, of rainforest lost in 365 days?
Give your answer in standard form. *[3 marks]*

(b) Correct to the nearest 10 km, the Amazon river is 6440 km long.
Correct to the nearest 100 km, the Congo river is 4400 km long.
What is the upper bound of the difference between the lengths of the Amazon river and the Congo river? *[3 marks]*

(a)(i) % ....

(a)(ii) ....

(a)(iii) % ....

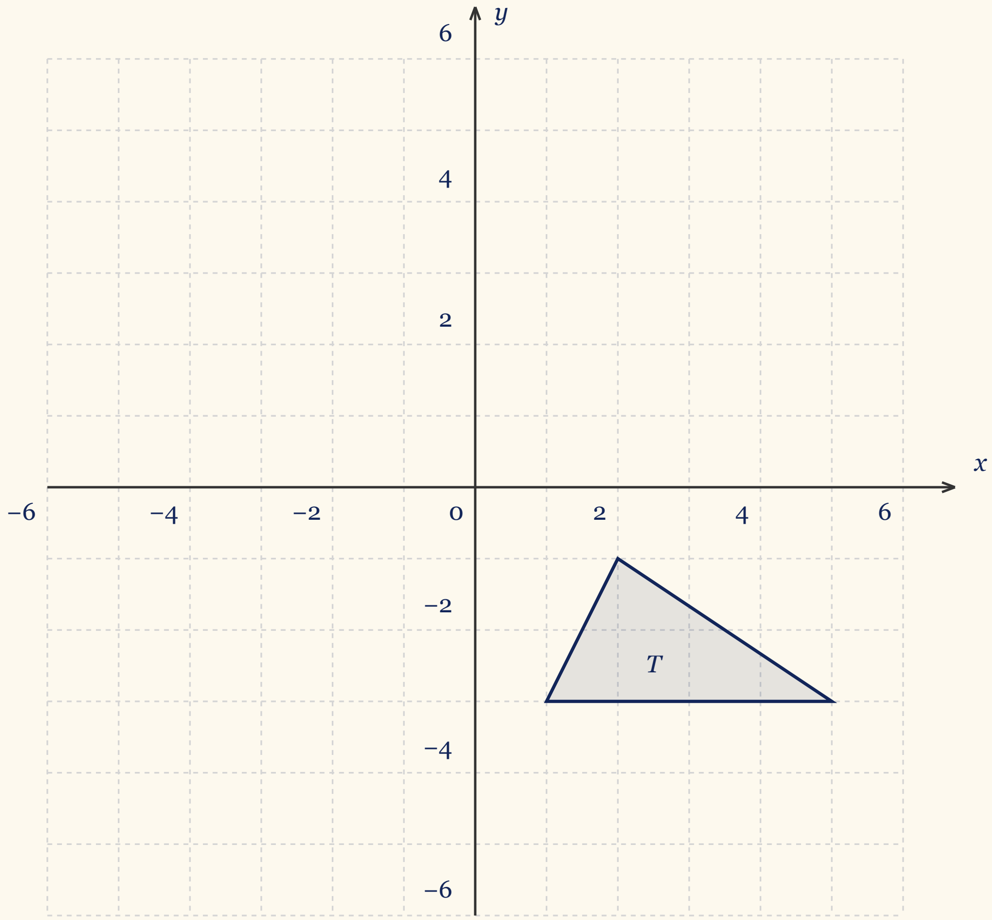
(a)(iv) km² ....

(a)(v) hectares ....

(b) km ....

[Total 15 marks]

2. (a) Draw each of these images on the grid.



(i) The image of triangle T under a reflection in the x-axis
[1 mark]

(ii) The image of triangle T under a translation by the vector $\binom{-5}{-2}$
[2 marks]

(iii) The image of triangle T under an enlargement with scale factor $-\dfrac{1}{2}$ and centre $(-1,\ 1)$. *[2 marks]*

(b) An enlargement with scale factor 3 maps shape P onto shape Q.
Shape Q is then mapped onto shape R by an enlargement with scale factor $\dfrac{2}{5}$.
Shape P has an area of 10 cm².
What is the area of shape R? *[3 marks]*

(b) cm² ....

[Total 8 marks]

3. (a) $C = \dfrac{1}{4}xy^2$

(i) When $x = 5$ and $y = 8$, what is the value of C?

[2 marks]

(ii) Given that $C = 15$ and $x = 2.4$, what positive value does y take?

[2 marks]

(b) Write the expression below as a single fraction, giving it in its simplest form.

$\dfrac{4}{x-1} - \dfrac{3}{2x+5}$ *[3 marks]*

(c) Multiply out the brackets below and simplify your answer.

$(2x+3)(4-x)^2$ *[3 marks]*

(d) Give the expression below in its simplest form.

$\left(\dfrac{y^8}{16x^{16}}\right)^{-\frac{3}{4}}$ *[3 marks]*

(a)(i) C = ...

(a)(ii) y = ...

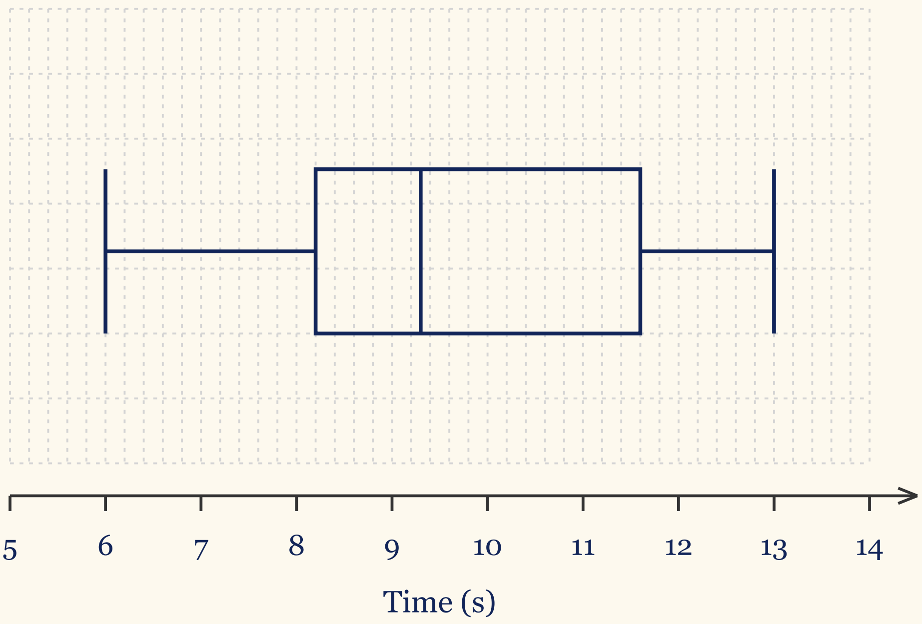
(b) ....

(c) ....

(d) ....

[Total 13 marks]

4. (a) Some motorbikes each travel 195 m, and Haoran records the time, in seconds, that each one takes.
This information is shown in the box and whisker plot.



(i) What is the median time? *[1 mark]*

(ii) What is the interquartile range? *[1 mark]*

(iii) How much greater is the average speed of the fastest motorbike than the average speed of the slowest motorbike?
Give your answer in kilometres per hour. *[5 marks]*

(b) For each of 80 different vans, Rosalind records the distance it can travel on a full tank of fuel.
These distances are shown in the table.

Distance (d km)	$250 < d \leqslant 300$	$300 < d \leqslant 400$	$400 < d \leqslant$
Frequency	7	13	19

(i) Which class interval contains the median? *[1 mark]*

(ii) What is an estimate of the mean? *[4 marks]*

(iii) This information is drawn as a histogram.
On that histogram the bar for $250 < d \leqslant 300$ has a height of 2.8 cm.
What is the height of the bar for each of the intervals below? *[3 marks]*

(iv) From the 80 vans, two are chosen at random.
Work out the probability that, on a full tank of fuel, one of these vans can travel more than 450 km while the other can travel not more than 300 km.
[3 marks]

(a)(i) s ....

(a)(ii) s ....

(a)(iii) km/h ....

(b)(i) ....

(b)(ii) km ....

(b)(iii) $300 < d \leqslant 400$ cm ....

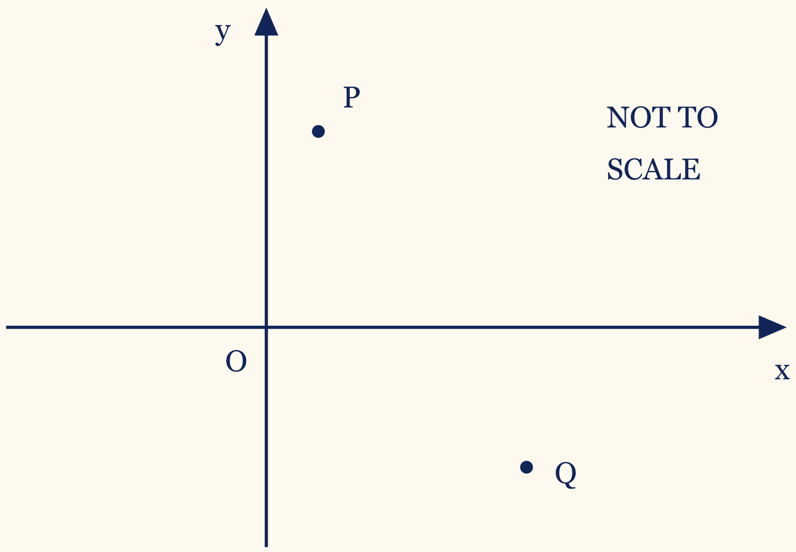
(b)(iii) $400 < d \leqslant 420$ cm ....

(b)(iii) $420 < d \leqslant 450$ cm ....

(b)(iv) ....

[Total 18 marks]

5. (a) The point P has coordinates $(1, 7)$.
The point Q has coordinates $(5, -5)$.



(i) What is $\overrightarrow{PQ}$? *[2 marks]*

(ii) Show that $|\overrightarrow{OP}|$ and $|\overrightarrow{OQ}|$ are equal. *[3 marks]*

(iii) A circle with centre O has PQ as a chord.
Work out this circle's circumference. *[2 marks]*

(iv) A different circle, with centre R, has PQ as its diameter.
What are the coordinates of R? *[2 marks]*

(v) What is the equation of the perpendicular bisector of PQ?
Give your answer in the form $y = mx + c$. *[4 marks]*

(b) Point A has position vector $\mathbf{a}$.
Point B has position vector $\mathbf{b}$.

M lies on AB, and $AM : MB = 2 : 3$.

In terms of $\mathbf{a}$ and $\mathbf{b}$, what is the position vector of M?
Give your answer in its simplest form. *[4 marks]*

(a)(i) ....

(a)(iii) ....

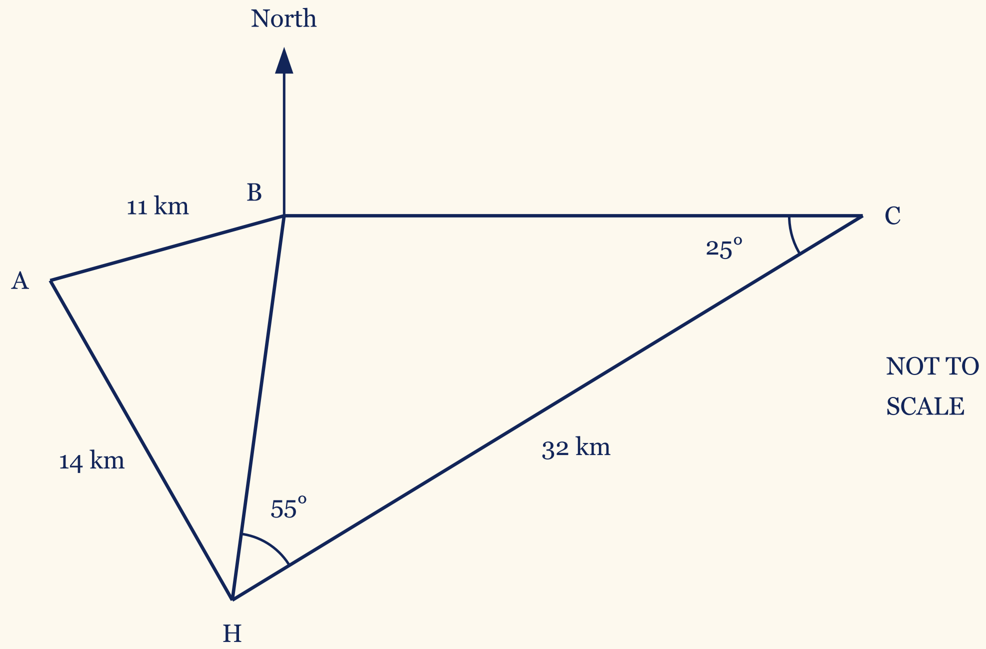
(a)(iv) ....

(a)(v) y = ...

(b) ....

[Total 17 marks]

6. The positions of two beacons A and B, a yacht C and a marina H are marked on the diagram.
B lies due west of C.



(a) From yacht C, what is the bearing of the marina? *[1 mark]*

(b) (i) Show that the size of angle CBH is $100°$. *[1 mark]*

(ii) Show that the length of BH is 13.7 km, correct to 1 decimal place. *[3 marks]*

(c) What is the bearing of A from B? *[5 marks]*

(d) Yacht C leaves at 1 pm and sails the 32 km straight to the marina, holding a speed of 10 knots.

(i) At what time does yacht C reach the marina?
Give this time correct to the nearest minute.
$[1 \text{ knot} = 1.852 \text{ km/h}]$ *[4 marks]*

(ii) How far is yacht C from the marina at the moment when it is closest to beacon B? *[3 marks]*

(a) ....

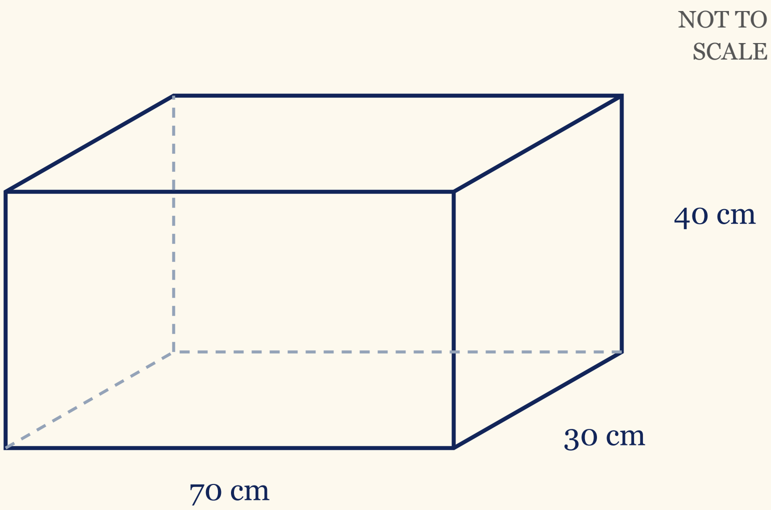
(c) ....

(d)(i) ....

(d)(ii) km ....

[Total 17 marks]

7. A crate in the shape of a cuboid is drawn above.
The crate has no top.



(a) (i) What is the surface area of the inside of the open crate? *[3 marks]*

(ii) Cylindrical tins, each of height 20 cm and diameter 15 cm, are put into the crate.
What is the greatest number of these tins that fit completely inside the crate? *[3 marks]*

(b) The mass of a solid bronze cone is 750 g.
The bronze has a density of 8.9 g/cm³.
The radius and the height of the cone are in the ratio $1 : 3$.

(i) Show that the cone has a radius of 2.99 cm, correct to 3 significant figures.

[$\text{Density} = \dfrac{\text{mass}}{\text{volume}}$]

[The volume, V, of a cone with radius r and height h is $V = \dfrac{1}{3}\pi r^2 h$.]

[4 marks]

(ii) What is the total surface area of the cone?
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.] *[5 marks]*

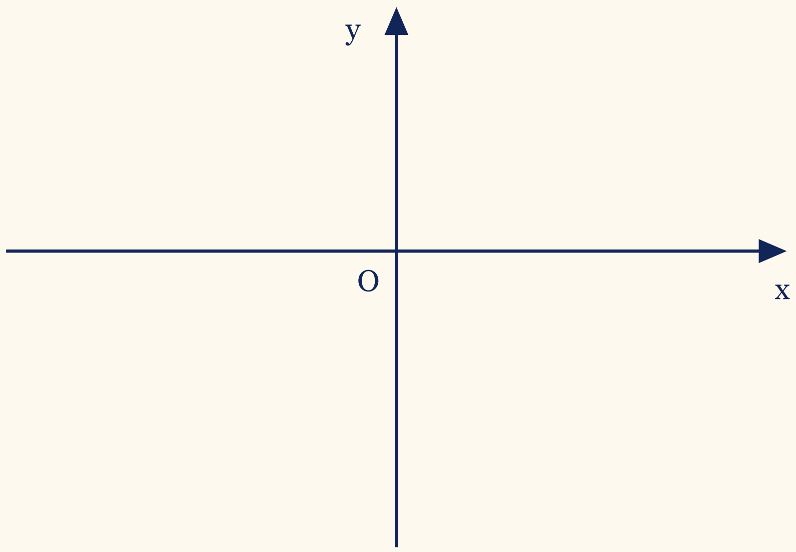
(a)(i) cm² ....

(a)(ii) ....

(b)(ii) cm² ....

[Total 15 marks]

8. (a) Sketch the graph of $y = x^2 + 7x - 18$ on the axes below.
The values where the graph meets the x-axis and the y-axis must be written on your sketch. *[4 marks]*



(b) (i) What is the derivative of $y = x^2 - 3x - 28$? *[2 marks]*

(ii) For $y = x^2 - 3x - 28$, give the coordinates of the turning point.
[3 marks]

(c) The graph of $y = x^2 - 3x - 28$ is intersected by the line $y = 5 - 2x$ at point P and point Q.

What are the coordinates of P and Q?
You must show all your working and give your answers correct to 2 decimal places. *[6 marks]*

(b)(i) ....

(b)(ii) ....

(c) ....

(c) ....

[Total 15 marks]

9. $f(x) = 4x + 1$ $g(x) = 6 - 2x$ $h(x) = 3^{x-2}$

(a) Work out the value of each of the following.

(i) $f(3)$ *[1 mark]*

(ii) $gf(3)$ *[1 mark]*

(b) What is $g^{-1}(x)$? *[2 marks]*

(c) When $f(x) = g(2x - 7)$, what is the value of x? *[4 marks]*

(d) What is the value of $hh(2)$? *[2 marks]*

(e) For which value of x is $h^{-1}(x) = 10$? *[2 marks]*

(a)(i) ....

(a)(ii) ....

(b) $g^{-1}(x)$ = ...

(c) x = ...

(d) ....

(e) x = ...

[Total 12 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Areas in km²: Amazon $5\,500\,000$, Congo $2\,000\,000$, Atlantic $1\,315\,000$, Valdivian $250\,000$.

Of the Amazon: 60% in Brazil, 10% in Colombia, and $43\frac{1}{3}\%$ of what is left in Peru.

$\frac{27}{50}$ of the world's rainforest area is the Amazon.

60.7 hectares are lost every minute.

Amazon river 6440 km to the nearest 10 km, Congo river 4400 km to the nearest 100 km.

Find

(a)(i) The Valdivian area as a percentage of the Amazon area. (a)(ii) Valdivian : Atlantic : Congo in its simplest form. (a)(iii) The percentage of the Amazon in Brazil, Colombia and Peru. (a)(iv) The world total, to the nearest $100\,000$ km². (a)(v) The hectares lost in 365 days, in standard form. (b) The upper bound of the difference between the two river lengths.

Plan the Solution

- A part as a percentage of a whole is $\frac{\text{part}}{\text{whole}} \times 100$.
- A ratio is simplest once every term has been divided by the highest common factor of all three.
- For (iii), find what is left after Brazil and Colombia, take Peru's share of that, then add the three shares.
- For (iv), dividing by $\frac{27}{50}$ is the same as multiplying by $\frac{50}{27}$.
- For (v), turn 365 days into minutes, multiply by the rate, then write the result in standard form.

- For (b), a difference is largest when the first length is as big as it can be and the second as small as it can be.

Worked Solution [15 marks]

> Rule - Part of a whole, and bounds: a part as a percentage of a whole is $\frac{\text{part}}{\text{whole}} \times 100$; a ratio is in its simplest form once every term has been divided by their highest common factor; and the largest value of $A - C$ is the upper bound of A minus the lower bound of C.

Step 1: Write the Valdivian area over the Amazon area, then multiply by 100

$$\frac{250\,000}{5\,500\,000} \times 100 = \frac{100}{22}$$
$$\frac{100}{22} = 4.545$$

(Reason: Both areas divide by $250\,000$, so the fraction cancels to $\frac{1}{22}$. The division gives the recurring decimal $4.5454...$, which is 4.55 to 3 significant figures.)

Step 2: Divide all three areas by their highest common factor

$$\frac{250\,000}{5000} = 50$$
$$\frac{1\,315\,000}{5000} = 263$$
$$\frac{2\,000\,000}{5000} = 400$$

(Reason: The largest number that divides all three areas is 5000. The three answers cannot cancel again, because 263 is prime.)

Step 3: Find the percentage of the Amazon left after Brazil and Colombia

$$100 - 60 - 10 = 30$$

(Reason: Brazil and Colombia take 70% between them, so the remaining area Peru's share is measured against is 30% of the rainforest.)

Step 4: Take Peru's share of that 30%

$$\frac{130}{3} \times \frac{30}{100} = 13$$

(Reason: $43\frac{1}{3}$ is $\frac{130}{3}$, and a percentage of a percentage is found by multiplying. So Peru holds 13% of the whole rainforest.)

Step 5: Add the three shares

$60 + 10 + 13 = 83$

(Reason: Brazil, Colombia and Peru together, as a percentage of the whole Amazon rainforest.)

Step 6: Divide the Amazon area by $\frac{27}{50}$

$$\frac{5\,500\,000 \times 50}{27} = 10\,185\,185.19$$

(Reason: The Amazon area is $\frac{27}{50}$ of the world total, so the world total is the Amazon area divided by $\frac{27}{50}$, which is the same as multiplying by $\frac{50}{27}$.)

Step 7: Round to the nearest $100\,000$

$10\,200\,000$

(Reason: $10\,185\,185.19$ lies between $10\,100\,000$ and $10\,200\,000$, and it is much nearer the second of those, so that is the answer in km².)

Step 8: Count the minutes in 365 days

$365 \times 24 \times 60 = 525\,600$

(Reason: Each day holds 24 hours and each hour holds 60 minutes.)

Step 9: Multiply the minutes by the rate

$60.7 \times 525\,600 = 31\,903\,920$

(Reason: 60.7 hectares go every minute, so this is the whole year's loss in hectares.)

Step 10: Write the total in standard form

$31\,903\,920 = 3.190392 \times 10^7$

(Reason: Standard form needs one non-zero digit in front of the point, and the point moves 7 places. To 3 significant figures the number in front of the point is 3.19.)

Step 11: Write the bounds of the Amazon river length

$6440 - 5 = 6435$

$6440 + 5 = 6445$

(Reason: A length given to the nearest 10 km is within 5 km of the value stated, so the largest the Amazon river can be is 6445 km.)

Step 12: Write the bounds of the Congo river length

$4400 - 50 = 4350$

$4400 + 50 = 4450$

(Reason: A length given to the nearest 100 km is within 50 km of the value stated, so the smallest the Congo river can be is 4350 km.)

Step 13: Subtract the smallest Congo length from the largest Amazon length

$6445 - 4350 = 2095$

(Reason: The difference is greatest when the first length is as large as it can be and the second as small as it can be, so this takes the upper bound of the Amazon length with the lower bound of the Congo length.)

(a)(i) 4.55%

(a)(ii) $50 : 263 : 400$

(a)(iii) 83%

(a)(iv) $10\,200\,000$ **km²**

(a)(v) 3.19×10^7 **hectares**

(b) 2095 **km**

Verification

✓ **Check 1:** Divide the Amazon area by 22 and see whether the Valdivian area comes back. $\dfrac{5\,500\,000}{22} = 250\,000$, the Valdivian area exactly, so the percentage really is one part in 22.

✓ **Check 2:** Multiply every term of the simplified ratio back up by 5000. $50 \times 5000 = 250\,000$, $263 \times 5000 = 1\,315\,000$ and $400 \times 5000 = 2\,000\,000$, the three areas in the table.

✓ **Check 3:** Work out the percentage of the Amazon that is in none of the three countries, two ways. From the answer, $100 - 83 = 17$; from the remaining area, $30 - 13 = 17$. The two agree.

✓ **Check 4:** Take $\dfrac{27}{50}$ of the rounded world total and compare it with the Amazon area. $10\,200\,000 \times \dfrac{27}{50} = 5\,508\,000$, which is $5\,500\,000$ to the nearest $100\,000$, as it should be after rounding.

✓ **Check 5:** Divide the year's loss by the number of minutes in the year, and compare the standard form with the total. $\dfrac{31\,903\,920}{525\,600} = 60.7$, the rate given, and $3.19 \times 10^7 = 31\,900\,000$, which matches $31\,903\,920$ to 3 significant figures.

✓ **Check 6:** Reach the upper bound a different way: take the difference of the two stated lengths and add both rounding slacks. $6440 - 4400 = 2040$ and $2040 + 5 + 50 = 2095$, the same upper bound.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The percentage	**Note**	The Answer column gives 4.55 or $4.545...$, and the Marks column gives 1 for it.	✓
(a)(ii) The simplest ratio	**Note**	The Answer column gives $50 : 263 : 400$ cao, and the Marks column gives 2 for it.	✓
A correct simplification of the three areas	**M1**	For a correct simplification from $250\,000 : 1\,315\,000 : 2\,000\,000$.	✓
(a)(iii) The percentage in the three countries	**Note**	The Answer column gives 83 cao, and the Marks column gives 3 for it.	✓
Peru's share of the area left, in one expression	**M2**	For $\dfrac{43\frac{1}{3}}{100} \times (100 - 60 - 10)$ oe.	✓
The area left after Brazil and Colombia, seen	**M1**	Or for $100 - 60 - 10$ seen.	✓
(a)(iv) The world total	**Note**	The Answer column gives $10\,200\,000$ cao, and the Marks column gives 3 for it.	✓
The total before rounding	**B2**	For $10\,185\,185$ to $10\,185\,200$.	✓
The division that gives the total	**M1**	Or for $\dfrac{5\,500\,000}{27}$ $[\times 50]$.	✓
Where the partial range comes from	**Note**	The range $10\,185\,185$ to $10\,185\,200$ is the total before it is rounded, and it rounds to the answer $10\,200\,000$.	✓
(a)(v) The area lost, in standard form	**Note**	The Answer column gives 3.19×10^7 or $3.190\ldots \times 10^7$, and the Marks column gives 3 for it.	✓
The total before it is written in standard form	**B2**	For $31\,903\,920$.	✓

Step	Mark	Description	Got it?
The product that gives the total	**M1**	Or for $60.7 \times 60 \times 24 \times 365$.	✓
A candidate's own number converted to standard form	**SC1**	If B0 scored, for correctly converting their number seen to standard form to 3sf or better.	✓
(b) The upper bound of the difference	**Note**	The Answer column gives 2095 nfww, and the Marks column gives 3 for it.	✓
The subtraction, with the other length inside its stated range	**M2**	For $6445 - C$ where $4300 \leqslant C < 4400$ oe, or $A - 4350$ where $6440 < A \leqslant 6450$ oe.	✓
One of the four bounds seen	**M1**	Or for $6440 + 5$ or $6440 - 5$ or $4400 + 50$ or $4400 - 50$ seen oe.	✓
Where the two bounds in the M2 row come from	**Note**	The Amazon length is at most 6445 and the Congo length is at least 4350, and $6445 - 4350 = 2095$.	✓

Full marks: 15/15

Question 2 - Exam Solution

Understanding the Question

Given

Triangle T is drawn on the grid, with vertices $(2,\ -1)$, $(1,\ -3)$ and $(5,\ -3)$. An enlargement with scale factor 3 maps P onto Q, and one with scale factor $\dfrac{2}{5}$ maps Q onto R.
Shape P has an area of 10 cm².

Find

(a)(i) The image of T after a reflection in the x-axis. (a)(ii) The image of T after a translation by $\binom{-5}{-2}$. (a)(iii) The image of T after an enlargement with scale factor $-\dfrac{1}{2}$ and centre $(-1,\ 1)$. (b) The area of shape R.

Plan the Solution

- Transform one vertex at a time, then join the three images up in the same order.

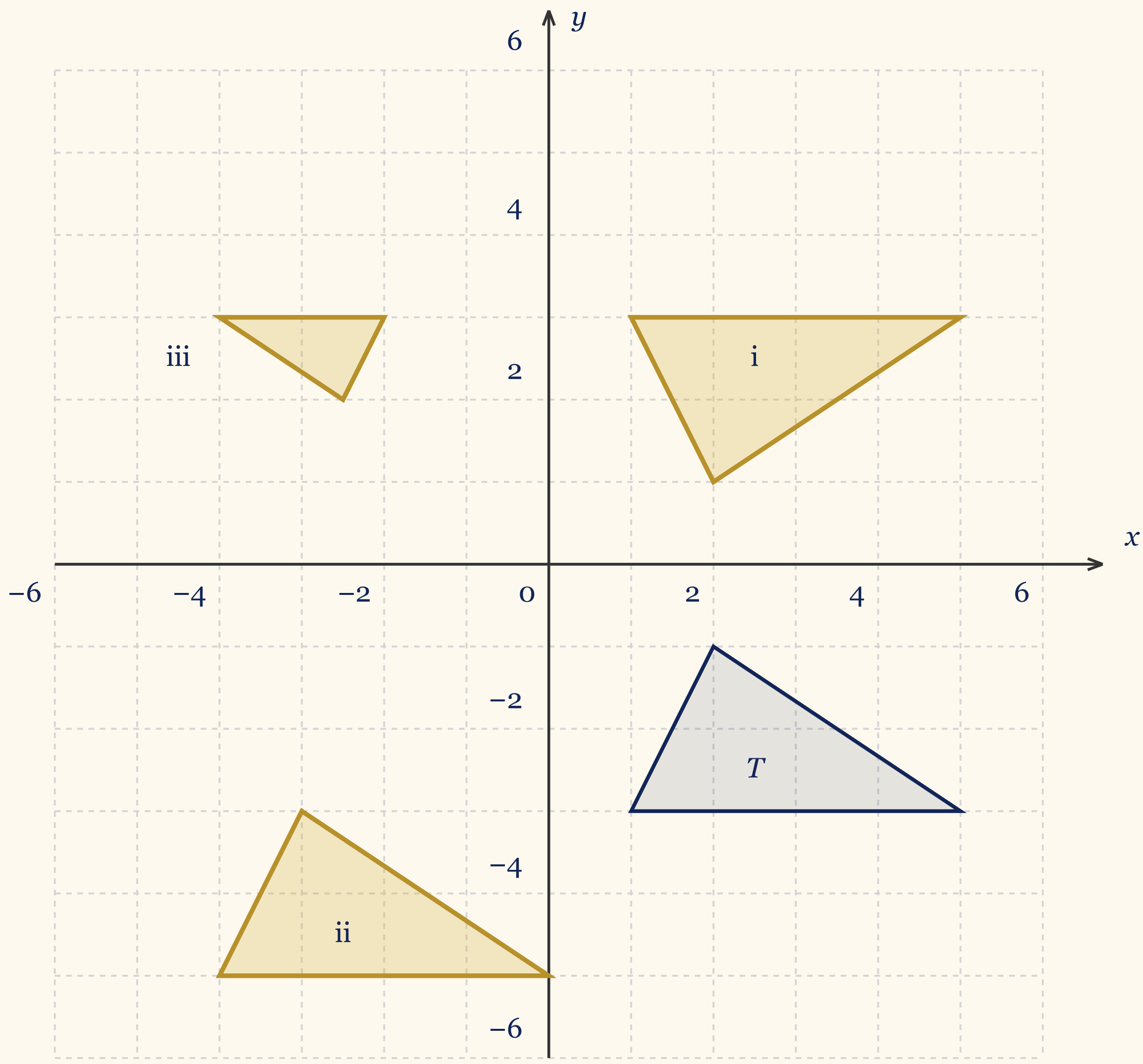
- The x-axis is the line $y = 0$, so a reflection in it leaves x alone and turns y into $-y$.
- A translation adds the top number of the vector to every x and the bottom number to every y.
- For an enlargement, measure the step from the centre to the vertex, multiply that step by the scale factor, and set off from the centre again.
- For (b), an enlargement with scale factor k multiplies every area by k^2, so do it once for each enlargement.

Worked Solution [8 marks]

> Rule - Transformations of points, and area: a reflection in the x-axis sends $(x,\ y)$ to $(x,\ -y)$; a translation by $\binom{a}{b}$ sends it to $(x + a,\ y + b)$; an enlargement with scale factor k and centre C sends a point X to $C + k(X - C)$; and that enlargement multiplies every area by k^2.

Step 1: Read the three vertices of triangle T off the grid

$(2,\ -1) \qquad (1,\ -3) \qquad (5,\ -3)$



(Reason: The apex sits one square below the x-axis and two to the right of the y-axis; the flat edge runs along $y = -3$ from $x = 1$ to $x = 5$.)

Step 2: Reflect each vertex in the x-axis

$(2,\ -1) \to (2,\ 1)$
$(1,\ -3) \to (1,\ 3)$
$(5,\ -3) \to (5,\ 3)$

(Reason: A point 3 below the mirror goes 3 above it, so each x-coordinate stays where it is and each y-coordinate changes sign.)

Step 3: Add the translation vector to each vertex

$(2,\ -1) \to (-3,\ -3)$
$(1,\ -3) \to (-4,\ -5)$
$(5,\ -3) \to (0,\ -5)$

(Reason: The vector $\binom{-5}{-2}$ moves every point 5 squares left and 2 squares down, so 5 comes off each x-coordinate and 2 off each y-coordinate.)

Step 4: Measure the step from the centre $(-1,\ 1)$ to each vertex

$(2,\ -1) \to (3,\ -2)$
$(1,\ -3) \to (2,\ -4)$
$(5,\ -3) \to (6,\ -4)$

(Reason: Each step is the vertex take away the centre. For the first one that is $2 - (-1) = 3$ across and $-1 - 1 = -2$ up.)

Step 5: Multiply each step by $-\frac{1}{2}$ and set off from the centre again

$(3,\ -2) \to (-2.5,\ 2)$
$(2,\ -4) \to (-2,\ 3)$
$(6,\ -4) \to (-4,\ 3)$

(Reason: The minus sign turns the step round, so the image lands on the far side of the centre, and the half makes it half as long. For the first vertex that gives $-1 - \frac{1}{2} \times 3 = -2.5$ and $1 - \frac{1}{2} \times (-2) = 2$.)

Step 6: Turn the first scale factor into an area scale factor

$10 \times 3^2 = 90$

(Reason: An enlargement with scale factor 3 makes every length 3 times as long, so every area is multiplied by $3^2 = 9$.)

Step 7: Do the same for the second enlargement

$$90 \times \left(\frac{2}{5}\right)^2 = 90 \times \frac{4}{25} = 14.4$$

(Reason: Squaring $\frac{2}{5}$ gives the area scale factor $\frac{4}{25}$, and that is applied to the area of Q to reach the area of R in cm².)

(a)(i) $(2,\ 1), (1,\ 3), (5,\ 3)$
(a)(ii) $(-3,\ -3), (-4,\ -5), (0,\ -5)$
(a)(iii) $(-2.5,\ 2), (-2,\ 3), (-4,\ 3)$
(b) 14.4 **cm²**

Verification

✓ **Check 1:** A reflection and a translation both move a shape without changing it, so compare the squared side lengths of T with those of the two images. Triangle T gives 5, 13 and 16, and so do both images, so neither one has been stretched or turned into a different shape.

✓ **Check 2:** Take the far vertex $(5,\ -3)$ and check that its image really is on the opposite side of the centre and half as far away. The step from the centre is $(6,\ -4)$ and the step to the image is $(-3,\ 2)$, because $-\dfrac{1}{2} \times 6 = -3$ and $-\dfrac{1}{2} \times (-4) = 2$. The two steps point opposite ways, and the second is half the length of the first.

✓ **Check 3:** Count the area of T and of its enlarged image by the base-and-height rule, and compare them with the area scale factor. T has base 4 and height 2, so its area is 4; the image has base 2 and height 1, so its area is 1. That is a quarter, which is what $\left(-\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ demands.

✓ **Check 4:** Reach (b) the other way round: combine the two enlargements into one, then square only once. The two together multiply every length by $3 \times \dfrac{2}{5} = 1.2$, so the area is multiplied by $1.2^2 = 1.44$ and $10 \times 1.44 = 14.4$, the same answer.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The image after the reflection	**Note**	The Answer column gives Triangle at $(2,\ 1)$ $(1,\ 3)$ $(5,\ 3)$, and the Marks column gives 1 for it.	✓
(a)(ii) The image after the translation	**Note**	The Answer column gives Triangle at $(-4,\ -5)$ $(-3,\ -3)$ $(0,\ -5)$, and the Marks column gives 2 for it.	✓
A translation by a vector with one component correct	**B1**	For translation by $\binom{-5}{k}$ or $\binom{k}{-2}$.	✓
(a)(iii) The image after the enlargement	**Note**	The Answer column gives Triangle at $(-2.5,\ 2)$ $(-4,\ 3)$ $(-2,\ 3)$, and the Marks column gives 2 for it.	✓
An enlargement with the correct scale factor from any centre	**B1**	For enlargement by sf $-\dfrac{1}{2}$ with any centre.	✓
(b) The area of shape R	**Note**	The Answer column gives 14.4, and the Marks column gives 3 for it.	✓

Step	Mark	Description	Got it?
Both area scale factors applied to the area of P, in one expression	**M2**	For $[10\times]\ 3^2 \times \left(\frac{2}{5}\right)^2$ oe.	✓
One of the two area scale factors seen or implied	**M1**	Or for 3^2 or $\left(\frac{2}{5}\right)^2$ soi.	✓

Full marks: 8/8

Question 3 - Exam Solution

Understanding the Question

Given

$C = \frac{1}{4}xy^2$

Part (b): $\frac{4}{x-1} - \frac{3}{2x+5}$

Part (c): $(2x+3)(4-x)^2$

Part (d): $\left(\frac{y^8}{16x^{16}}\right)^{-\frac{3}{4}}$

Find

(a)(i) the value of C when $x = 5$ and $y = 8$ (a)(ii) the positive value of y when $C = 15$ and $x = 2.4$ (b) one fraction, in its simplest form (c) the expansion, simplified (d) the expression in its simplest form

Plan the Solution

- (a) is substitution both ways round: put the numbers straight in for (i), and undo the formula one operation at a time for (ii).
- (b) needs one common denominator. The two denominators share no factor, so their product $(x-1)(2x+5)$ is the smallest one available.
- (c) squares the bracket first, then multiplies the two brackets together and collects like terms.
- (d) is index laws: a negative index turns the fraction over, and a power of a power multiplies the indices.

Worked Solution [13 marks]

Index laws and a common denominator: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$, $(a^m)^n = a^{mn}$, and two fractions are subtracted over the product of their denominators when those denominators share no factor.

(a)(i) Put the numbers into the formula

$$C = \frac{1}{4} \times 5 \times 8^2$$
$$8^2 = 64$$
$$C = \frac{1}{4} \times 5 \times 64 = 80$$

(Reason: The index is worked out before the multiplication, so 8^2 becomes 64 first, and a quarter of 320 is 80.)

(a)(ii) Substitute, then clear the quarter

$$15 = \frac{1}{4} \times 2.4 \times y^2$$
$$y^2 = \frac{15 \times 4}{2.4} = 25$$

(Reason: Multiplying both sides by 4 and then dividing by 2.4 leaves y^2 by itself.)

(a)(ii) Take the positive square root

$$y = \sqrt{25} = 5$$

(Reason: 25 has two square roots, 5 and -5, and only the positive one is asked for.)

(b) Write both fractions over one denominator

$$\frac{4}{x-1} - \frac{3}{2x+5} = \frac{4(2x+5) - 3(x-1)}{(x-1)(2x+5)}$$

(Reason: The denominators share no factor, so the common denominator is their product. Each numerator is multiplied by the denominator that the other fraction brought.)

(b) Expand the numerator and collect

$$4(2x+5) - 3(x-1) = 8x + 20 - 3x + 3 = 5x + 23$$

(Reason: Each bracket is multiplied out, and then the x terms and the number terms are collected separately.)

(b) Write the single fraction

$$\frac{4}{x-1} - \frac{3}{2x+5} = \frac{5x+23}{(x-1)(2x+5)}$$

(Reason: Nothing cancels: $5x + 23$ shares no factor with $x - 1$ or with $2x + 5$, so this is already in its simplest form. The denominator may be left expanded as $2x^2 + 3x - 5$ instead.)

(c) Square the bracket first

$(4 - x)^2 = (4 - x)(4 - x) = 16 - 8x + x^2$

(Reason: A squared bracket is the bracket multiplied by itself, never each term squared on its own.)

(c) Multiply by the remaining bracket

$(2x + 3)(16 - 8x + x^2) = 32x - 16x^2 + 2x^3 + 48 - 24x + 3x^2$

(Reason: Every term in the first bracket multiplies every term in the second, which gives six terms.)

(c) Collect like terms

$2x^3 - 16x^2 + 3x^2 + 32x - 24x + 48 = 2x^3 - 13x^2 + 8x + 48$

(Reason: $-16x^2 + 3x^2 = -13x^2$ and $32x - 24x = 8x$, and the 48 has nothing to pair with.)

(d) Use the negative index to turn the fraction over

$$\left(\frac{y^8}{16x^{16}}\right)^{-\frac{3}{4}} = \left(\frac{16x^{16}}{y^8}\right)^{\frac{3}{4}}$$

(Reason: A negative index means the reciprocal, so turning the fraction upside down makes the index positive.)

(d) Apply the three-quarter power to each part

$16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

$\left(x^{16}\right)^{\frac{3}{4}} = x^{12}$

$\left(y^8\right)^{\frac{3}{4}} = y^6$

(Reason: A power of $\frac{3}{4}$ is the fourth root followed by the cube. For the letters the indices multiply: $16 \times \frac{3}{4} = 12$ and $8 \times \frac{3}{4} = 6$.)

(d) Put the three results back together

$$\left(\frac{16x^{16}}{y^8}\right)^{\frac{3}{4}} = \frac{8x^{12}}{y^6}$$

(Reason: Each part stays where it started, so the answer is $\frac{8x^{12}}{y^6}$, which may also be written $8x^{12}y^{-6}$.)

$$\textbf{(a)(i) } C = 80$$

$$\textbf{(a)(ii) } y = 5$$

$$\textbf{(b) } \frac{5x + 23}{(x-1)(2x+5)}$$

$$\textbf{(c) } 2x^3 - 13x^2 + 8x + 48$$

$$\textbf{(d) } \frac{8x^{12}}{y^6}$$

Verification

✓ **Check 1:** Part (a) run backwards. Put $x = 2.4$ and $y = 5$ into the formula and see whether C comes back out. $\frac{1}{4} \times 2.4 \times 25 = 15$, the value the question gives.

✓ **Check 2:** Part (b) at a number. Put $x = 3$ into the printed expression and into the single fraction. $\frac{4}{2} - \frac{3}{11} = \frac{19}{11}$ and $\frac{38}{2 \times 11} = \frac{19}{11}$.

✓ **Check 3:** Part (c) at a number. Put $x = 1$ into $(2x+3)(4-x)^2$ and into the expansion. $5 \times 9 = 45$ and $2 - 13 + 8 + 48 = 45$.

✓ **Check 4:** Part (d) at numbers. Put $x = 1$ and $y = 2$ into the printed expression and into the simplified one. $\left(\frac{256}{16}\right)^{-\frac{3}{4}} = 16^{-\frac{3}{4}} = \frac{1}{8}$ and $\frac{8 \times 1}{64} = \frac{1}{8}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The value of C	**Note**	The Answer column gives 80, and the Marks column gives 2 for it.	✓
(a)(i) Substituting into the formula	**M1**	For $\frac{1}{4} \times 5 \times 8^2$.	✓
(a)(ii) The positive value of y	**Note**	The Answer column gives 5, and the Marks column gives 2 for it.	✓
(a)(ii) Rearranging to reach y squared	**M1**	For $[y^2 =] \frac{15 \times 4}{2.4}$ oe.	✓

Step	Mark	Description	Got it?
(b) The single fraction	**Note**	The Answer column gives $\dfrac{5x+23}{(x-1)(2x+5)}$ or $\dfrac{5x+23}{2x^2+3x-5}$ as the final answer, and the Marks column gives 3 for it.	✓
(b) The numerator over the common denominator	**B1**	For $4(2x+5)-3(x-1)$ oe isw.	✓
(b) The common denominator	**B1**	For common denominator $=(x-1)(2x+5)$ oe isw.	✓
(c) The expansion, simplified	**Note**	The Answer column gives $2x^3-13x^2+8x+48$ as the final answer, and the Marks column gives 3 for it.	✓
(c) Expanding all three brackets	**B2**	For correct expansion of 3 brackets but unsimplified, or for simplified four-term expression of correct form with 3 terms correct.	✓
(c) Expanding two brackets only	**B1**	Or for correct expansion of two brackets with at least 3 terms out of 4 correct.	✓
(d) The expression in its simplest form	**Note**	The Answer column gives $\dfrac{8x^{12}}{y^6}$ or $8x^{12}y^{-6}$ as the final answer, and the Marks column gives 3 for it.	✓
(d) Two elements of the final answer correct	**B2**	For two elements correct in final answer, or for correct answer seen then spoiled, or for correct expression where all parts of the power have been dealt with, or for $(\ )^{-1}$ or $\left(\dfrac{2x^4}{y^2}\right)^3$.	✓
(d) One element of the final answer correct	**B1**	Or for 8 or y^6 or y^{-6} or x^{12} correct in final answer, or for $\left(\dfrac{16x^{16}}{y^8}\right)^{\frac{3}{4}}$ or $\left(\dfrac{y^2}{2x^4}\right)^{-3}$.	✓

Full marks: 13/13

Question 4 - Exam Solution

Understanding the Question

Given

(a) Each motorbike travels 195 m, and the box and whisker plot shows the times in seconds.
(b) 80 vans, with the distances grouped in the table, and a histogram whose bar for $250 < d \leqslant 300$ is 2.8 cm tall.

Find

(a)(i) The median time. (a)(ii) The interquartile range. (a)(iii) The difference between the two average speeds, in km/h. (b)(i) The class interval that the median falls in. (b)(ii) An estimate of the mean. (b)(iii) The heights of three more bars. (b)(iv) The probability for the two vans chosen.

Plan the Solution

- Read the five values off the box plot: the two whisker ends, the two box edges and the line inside the box.
- Average speed is distance divided by time, so the shortest time gives the fastest motorbike.
- To turn m/s into km/h, multiply by $\dfrac{3600}{1000}$.
- For grouped data, treat every value in an interval as the midpoint of that interval.
- On a histogram the height of a bar is the frequency density, $\dfrac{\text{frequency}}{\text{class width}}$, drawn to one fixed scale.
- For two vans chosen without replacement, the second is chosen from 79, and the two orders both count.

Worked Solution [18 marks]

Rule - Reading a box plot and summarising grouped data: the interquartile range is the upper quartile minus the lower quartile; average speed is $\dfrac{\text{distance}}{\text{time}}$, and 1 m/s is $\dfrac{3600}{1000}$ km/h; an estimate of the mean of grouped data is $\dfrac{\Sigma fx}{\Sigma f}$ with x the midpoint of each class; and the height of a histogram bar is its frequency density, $\dfrac{\text{frequency}}{\text{class width}}$.

Step 1: Read the five values off the box and whisker plot

minimum $= 6$

lower quartile $= 8.2$

median $= 9.3$

upper quartile $= 11.6$

maximum $= 13$

(Reason: The two end bars give the smallest and largest times, the two edges of the box give the quartiles, and the line inside the box gives the median. The scale runs from 5 to 14 with five small squares to the second, so each square is 0.2 s.)

Step 2: Write down the median

median $= 9.3$

(Reason: The line drawn inside the box marks the median, and it stands 4.3 s to the right of the 5 on the scale.)

Step 3: Take the lower quartile from the upper quartile

$$11.6 - 8.2 = 3.4$$

(Reason: The interquartile range is the width of the box: the upper quartile minus the lower quartile.)

Step 4: Turn the shortest and the longest times into speeds in metres per second

$$\frac{195}{6} = 32.5$$

$$\frac{195}{13} = 15$$

(Reason: The fastest motorbike is the one that takes the least time, 6 s, and the slowest takes 13 s. Average speed is distance divided by time, and both cover 195 m.)

Step 5: Change each speed to kilometres per hour

$$32.5 \times \frac{3600}{1000} = 117$$

$$15 \times \frac{3600}{1000} = 54$$

(Reason: There are 3600 seconds in an hour and 1000 metres in a kilometre, so multiplying by $\frac{3600}{1000}$ turns metres per second into kilometres per hour.)

Step 6: Subtract the two speeds

$$117 - 54 = 63$$

(Reason: The question asks how much greater the faster average speed is, so it is the larger speed minus the smaller.)

Step 7: Build the frequencies up until the middle of the data is reached

$7,\ 20,\ 39,\ 60,\ 80$

$$\frac{80}{2} = 40$$

(Reason: With 80 distances the median lies between the 40th and the 41st. Only 39 of them are at most 420 km, while 60 are at most 450 km, so both of those two lie in $420 < d \leqslant 450$.)

Step 8: Take the midpoint of each interval

$$\frac{250 + 300}{2} = 275$$

$275,\ 350,\ 410,\ 435,\ 475$

(Reason: The individual distances are not known, so every van in an interval is treated as travelling the midpoint of that interval. The other four midpoints are found the same way.)

Step 9: Multiply each midpoint by its frequency and add

$$275 \times 7 + 350 \times 13 + 410 \times 19 + 435 \times 21 + 475 \times 20 = 32\,900$$

(Reason: This is Σfx: the total distance the 80 vans would travel if every one of them managed its own interval's midpoint.)

Step 10: Divide by the number of vans

$$\frac{32\,900}{80} = 411.25$$

(Reason: It is only an estimate because each distance has been replaced by the midpoint of the interval it falls in.)

Step 11: Use the bar that is given to find the scale of the histogram

$$\frac{7}{50} = 0.14$$

$$\frac{2.8}{0.14} = 20$$

(Reason: The interval $250 < d \leqslant 300$ has width 50 and frequency 7, so its frequency density is 0.14. That bar is drawn 2.8 cm tall, so one unit of frequency density is drawn as 20 cm.)

Step 12: Work out the other three frequency densities and scale each one

$$\frac{13}{100} = 0.13$$

$$0.13 \times 20 = 2.6$$

$$\frac{19}{20} = 0.95$$

$$0.95 \times 20 = 19$$

$$\frac{21}{30} = 0.7$$

$0.7 \times 20 = 14$

(Reason: The three class widths are 100, 20 and 30. The narrow interval $400 < d \leqslant 420$ holds almost as many vans as the wide one, which is why its bar is so much taller.)

Step 13: Count the vans in the two intervals the last part asks about

$450 < d \leqslant 500 \rightarrow 20$

$250 < d \leqslant 300 \rightarrow 7$

(Reason: More than 450 km is the top interval, which holds 20 vans. Not more than 300 km is the bottom interval, which holds 7.)

Step 14: Multiply the two probabilities, then double for the two orders

$$2 \times \frac{20}{80} \times \frac{7}{79} = \frac{7}{158}$$

(Reason: The first van is not put back, so the second is chosen from 79. The van that travels furthest could be picked first or second, so there are two ways round and the product is doubled.)

(a)(i) 9.3 **s**

(a)(ii) 3.4 **s**

(a)(iii) 63 **km/h**

(b)(i) $420 < d \leqslant 450$

(b)(ii) 411.25 **km**

(b)(iii) 2.6 **cm,** 19 **cm,** 14 **cm**

(b)(iv) $\frac{7}{158}$

Verification

✓ **Check 1:** Subtract the two speeds while they are still in metres per second, and convert only once at the end. $\frac{195}{6} - \frac{195}{13} = 17.5$ m/s, and $17.5 \times 3.6 = 63$ km/h, the same answer by a different order of operations.

✓ **Check 2:** Estimate the mean from an assumed mean of 400 instead. The weighted total of the deviations is $7 \times (-125) + 13 \times (-50) + 19 \times 10 + 21 \times 35 + 20 \times 75 = 900$. $400 + \frac{900}{80} = 411.25$, the same estimate, and it sits inside the table's range of 250 to 500 km as it must.

✓ **Check 3:** On a histogram the AREAS of the bars are in the same ratio as the frequencies, so compare the given bar with the tall one. $2.8 \times 50 = 140$ and $19 \times 20 = 380$, and $\dfrac{380}{140} = \dfrac{19}{7}$ - exactly the ratio of the frequencies 19 and 7.

✓ **Check 4:** Count ordered pairs of vans instead of using probabilities: $20 \times 7 + 7 \times 20 = 280$ favourable ordered pairs out of $80 \times 79 = 6320$. $\dfrac{280}{6320} = \dfrac{7}{158}$, which is the same probability.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The median time	**Note**	The Answer column gives 9.3, and the Marks column gives 1 for it.	✓
(a)(ii) The interquartile range	**Note**	The Answer column gives 3.4, and the Marks column gives 1 for it.	✓
(a)(iii) The difference between the two average speeds	**Note**	The Answer column gives 63, and the Marks column gives 5 for it.	✓
(a)(iii) Both speeds converted, then subtracted	**M4**	For $\dfrac{195}{6} \times \dfrac{3600}{1000} - \dfrac{195}{13} \times \dfrac{3600}{1000}$ oe.	✓
(a)(iii) One speed converted, or the difference taken before converting	**M3**	Or for $\dfrac{195}{6} \times \dfrac{3600}{1000}$ oe or $\dfrac{195}{13} \times \dfrac{3600}{1000}$ oe, or for $\left(\dfrac{195}{6} - \dfrac{195}{13}\right)$ $[\times k]$ oe.	✓
(a)(iii) Second method: a speed with the conversion seen	**M1**	Or for $\dfrac{195}{6}$ or $\dfrac{195}{13}$ or their speed $\times \dfrac{3600}{1000}$ seen.	✓
(a)(iii) Second method: selecting the two times	**M1**	For selecting 6 and 13.	✓
(b)(i) The class interval containing the median	**Note**	The Answer column gives $420 < d \leqslant 450$, and the Marks column gives 1 for it.	✓

Step	Mark	Description	Got it?
(b)(ii) The estimate of the mean	**Note**	The Answer column gives 411.25, and the Marks column gives 4 for it.	✓
(b)(ii) The midpoints of the five intervals	**M1**	For 275, 350, 410, 435, 475 soi.	✓
(b)(ii) The total of frequency times midpoint	**M1**	For Σfx.	✓
(b)(ii) Dividing that total by the number of vans	**M1 dep**	For their $\frac{\Sigma fx}{80}$.	✓
(b)(iii) The three bar heights	**Note**	The Answer column gives 2.6, 19 and 14, and the Marks column gives 3 for it.	✓
(b)(iii) Each correct height	**B1**	For each.	✓
(b)(iii) Three correct frequency densities	**SC1**	If 0 scored, for 3 of 0.14, 0.13, 0.95 or 0.7 oe.	✓
(b)(iv) The probability	**Note**	The Answer column gives $\frac{7}{158}$ oe, and the Marks column gives 3 for it.	✓
(b)(iv) Both fractions multiplied, doubled for the two orders	**M2**	For $[2\times]\frac{20}{80} \times \frac{7}{79}$ oe.	✓
(b)(iv) One of the four fractions seen	**M1**	Or for $\frac{20}{80}$ or $\frac{7}{79}$ or $\frac{7}{80}$ or $\frac{20}{79}$ oe seen.	✓
(b)(iv) The answer worked out with replacement	**SC1**	After 0 scored, for $\frac{7}{160}$ oe.	✓

Full marks: 18/18

Question 5 - Exam Solution

Understanding the Question

Given

Find

$P(1, 7)$ and $Q(5, -5)$, with O the origin.
A circle centred on O with PQ as a chord, and a second circle with PQ as its diameter and R as its centre.
Position vectors $\mathbf{a}$ and $\mathbf{b}$, with M on AB and $AM : MB = 2 : 3$.

$\overrightarrow{PQ}$, and a demonstration that $|\overrightarrow{OP}|$ and $|\overrightarrow{OQ}|$ are the same length. The circumference of the circle centred on O, the coordinates of R, and the perpendicular bisector of PQ in the form $y = mx + c$. $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Plan the Solution

- Subtract position vectors for $\overrightarrow{PQ}$, then use $\sqrt{x^2 + y^2}$ for each distance from the origin.
- P and Q both lie on the circle centred O, so that common distance is its radius and the circumference follows.
- The centre of a circle is halfway along any diameter, so R is the midpoint of PQ.
- The perpendicular bisector goes through that midpoint with the negative reciprocal of the gradient of PQ.
- For part (b), go from O to A and then part of the way along AB; a ratio of $2 : 3$ makes that part $\frac{2}{5}$.

Worked Solution [17 marks]

> Rule - Vectors and coordinate geometry: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$, a point's distance from the origin is $\sqrt{x^2 + y^2}$, the midpoint of a segment is the average of its ends, and perpendicular gradients multiply to -1.

(a)(i) Subtract the position vectors

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$$

(Reason: The vector from P to Q is where you finish minus where you start, taken one component at a time.)

(a)(ii) The distance of P from the origin

$$|\overrightarrow{OP}| = \sqrt{1^2 + 7^2} = \sqrt{1 + 49} = \sqrt{50}$$

(Reason: Pythagoras on the right-angled triangle whose two shorter sides are the coordinates of P.)

(a)(ii) The distance of Q from the origin

$|\overrightarrow{OQ}| = \sqrt{5^2 + (-5)^2} = \sqrt{25 + 25} = \sqrt{50}$

(Reason: Squaring removes the sign, so a y-coordinate of -5 contributes 25 just as 5 would. Both magnitudes come out as $\sqrt{50}$, which is what this part asks you to show.)

(a)(iii) The radius of the circle centred O

$r = |\overrightarrow{OP}| = |\overrightarrow{OQ}| = \sqrt{50}$

(Reason: PQ is a chord, so P and Q sit on the circle and OP and OQ are both radii. Part (ii) has already shown they are equal.)

(a)(iii) Apply the circumference formula

$C = 2 \times \pi \times r$

$C = 2 \times \pi \times \sqrt{50} = 44.4288\ldots$

(Reason: Keep the surd in the calculator instead of rounding the radius first, or the last figure drifts.)

(a)(iii) Round the circumference

$C = 44.4$

(Reason: A calculated length is given to three significant figures unless the question says otherwise, and $44.4288\ldots$ rounds to 44.4.)

(a)(iv) R is the midpoint of PQ

$$R = \left(\frac{1+5}{2}, \frac{7+(-5)}{2}\right) = (3, 1)$$

(Reason: The centre of a circle sits halfway along any diameter, so average the x-coordinates and average the y-coordinates.)

(a)(v) The gradient of PQ

$$\frac{7-(-5)}{1-5} = \frac{12}{-4} = -3$$

(Reason: Rise over run, taking P first on the top and on the bottom so the two subtractions match.)

(a)(v) The perpendicular gradient

$$\frac{-1}{-3} = \frac{1}{3}$$

(Reason: Perpendicular gradients multiply to -1, so turn the gradient upside down and change its sign.)

(a)(v) Fit the line through R

$$1 = \frac{1}{3} \times 3 + c$$

$c = 0$

$y = \dfrac{1}{3}x$

(Reason: A bisector cuts PQ at its midpoint, so $R(3, 1)$ lies on it. Here the constant works out as zero, so the line passes through the origin.)

(b) The step from A to M

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$\overrightarrow{AM} = \dfrac{2}{2+3}\overrightarrow{AB} = \dfrac{2}{5}(\mathbf{b} - \mathbf{a})$

(Reason: A ratio of $2 : 3$ splits AB into 5 equal parts, and M stands after 2 of them.)

(b) Travel from O to M

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$\overrightarrow{OM} = \mathbf{a} + \dfrac{2}{5}(\mathbf{b} - \mathbf{a}) = \dfrac{3}{5}\mathbf{a} + \dfrac{2}{5}\mathbf{b}$

(Reason: Collecting the $\mathbf{a}$ terms gives $1 - \frac{2}{5} = \frac{3}{5}$, and that collecting is what makes it the simplest form.)

$$\textbf{(a)(i) } \overrightarrow{PQ} = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$$

$$\textbf{(a)(ii) } |\overrightarrow{OP}| = |\overrightarrow{OQ}| = \sqrt{50}$$

$$\textbf{(a)(iii) } 44.4$$

$$\textbf{(a)(iv) } R = (3, 1)$$

$$\textbf{(a)(v) } y = \frac{1}{3}x$$

$$\textbf{(b) } \overrightarrow{OM} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$$

Verification

✓ **Check 1:** Put $x = 3$ into the bisector and see whether it reaches R. $\dfrac{3}{3} = 1$, so $(3, 1)$ is on the line.

✓ **Check 2:** Measure R against both ends of the diameter. $RP = \sqrt{2^2 + 6^2} = \sqrt{40}$ and $RQ = \sqrt{2^2 + 6^2} = \sqrt{40}$, equal as a centre requires.

✓ **Check 3:** Work the circumference the other way round, from the diameter. $2 \times \sqrt{50} = 14.1421\ldots$ and $\pi \times 14.1421\ldots = 44.4288\ldots$, which matches.

✓ **Check 4:** A point ON the line AB has coefficients adding to 1. $\frac{3}{5} + \frac{2}{5} = 1$, so M is on AB and not off it.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $\overrightarrow{PQ}$ as a column vector	**Note**	The Answer column gives the column vector with 4 above -12, and the Marks column gives 2 for it.	✓
(a)(i) Each component of $\overrightarrow{PQ}$	**B1**	For each.	✓
(a)(ii) The sum of squares for $\overrightarrow{OP}$	**M1**	For $1^2 + 7^2$.	✓
(a)(ii) The sum of squares for $\overrightarrow{OQ}$	**M1**	For $5^2 + ([-]5)^2$.	✓
(a)(ii) Both magnitudes equal $\sqrt{50}$	**A1**	Both $\sqrt{50}$ oe.	✓
(a)(ii) The condition on the accuracy mark	**Note**	With no errors seen.	✓
(a)(ii) The special case when nothing else is scored	**SC1**	If M0M0A0 scored, for $\sqrt{50}$ oe for each.	✓
(a)(iii) The circumference	**Note**	The Answer column gives 44.4 or $44.42[8\ldots]$ to 44.435, and the Marks column gives 2 for it.	✓
(a)(iii) The follow-through allowed	**Note**	FT their (a)(ii) correct to 3sf or better.	✓
(a)(iii) The method for the circumference	**M1**	For $2 \times \pi \times$ their $\sqrt{50}$ oe.	✓

Step	Mark	Description	Got it?
(a)(iv) The centre R	**Note**	The Answer column gives $(3, 1)$, and the Marks column gives 2 for it.	✓
(a)(iv) Each coordinate of R	**B1**	For each.	✓
(a)(v) The equation of the perpendicular bisector	**Note**	The Answer column gives $[y =]\dfrac{1}{3}x$, and the Marks column gives 4 for it.	✓
(a)(v) A correct equation in the wrong form	**B3**	For a correct equation in the wrong form as final answer.	✓
(a)(v) The perpendicular gradient on its own	**B2**	Or for $\dfrac{1}{3}$ stated or used as perpendicular gradient.	✓
(a)(v) Second method: the gradient of PQ	**M1**	Or for $[\text{grad } PQ] = \dfrac{7 - -5}{1 - 5}$ oe.	✓
(a)(v) Second method: the negative reciprocal	**M1**	For $\dfrac{-1}{\text{their grad } PQ}$.	✓
(a)(v) Second method: substituting into $y = mx + c$	**M1dep**	For substituting their (a)(iv) or $(0, 0)$ into $y = \text{their } mx + c$ oe, dep on the 2nd M1 or B2.	✓
(b) The position vector of M	**Note**	The Answer column gives $\dfrac{3}{5}\mathbf{a} + \dfrac{2}{5}\mathbf{b}$ final answer, and the Marks column gives 4 for it.	✓
(b) An unsimplified correct answer	**B3**	For an unsimplified correct answer.	✓
(b) $\overrightarrow{AM}$ or $\overrightarrow{BM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$	**B2**	Or for $AM = \dfrac{2}{5}(\mathbf{b} - \mathbf{a})$ soi, or $BM = \dfrac{3}{5}(\mathbf{a} - \mathbf{b})$ soi.	✓
(b) $\overrightarrow{AB}$, a correct route, or a correct diagram	**B1**	Or for $AB = \mathbf{b} - \mathbf{a}$ or $BA = \mathbf{a} - \mathbf{b}$, or for a correct route for OM, or for correct diagram.	✓

Full marks: 17/17

Question 6 - Exam Solution

Understanding the Question

Given

Two beacons A and B, a yacht C and a marina H, with B due west of C.
$AB = 11$ km, $AH = 14$ km and $HC = 32$ km.
Angle $BCH = 25^\circ$ and angle $BHC = 55^\circ$.
The yacht leaves at 1 pm at 10 knots, and 1 knot is 1.852 km/h.

Find

The bearing of the marina from the yacht, and a demonstration that angle CBH is 100° and that BH is 13.7 km. The bearing of A from B. The arrival time, and the distance still to run when the yacht is nearest to B.

Plan the Solution

- A bearing is measured clockwise from north, so start (a) from the 270° line that points from C back to B.
- The angles of triangle BCH add to 180°, which leaves the angle at B.
- Two angles and a side opposite one of them make triangle BCH a sine-rule triangle.
- Triangle ABH has all three sides known, so the cosine rule gives the angle at B. Adding it to the bearing of H from B turns it into a bearing.
- A knot is a nautical mile an hour, so convert the speed once and then divide distance by speed.
- The shortest distance from B to the yacht's track is the perpendicular, and that makes a right-angled triangle with BH as its hypotenuse.

Worked Solution [17 marks]

Rule - Bearings, the sine rule and the cosine rule: a bearing is measured clockwise from north and written with three figures, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$, and $a^2 = b^2 + c^2 - 2bc\cos A$.

(a) Turn round the line from C to B

$270 - 25 = 245$

(Reason: B is due west of C, so from C the beacon lies on a bearing of 270°. The marina sits 25° on the anticlockwise side of that line, so its bearing is 25 less.)

(b)(i) The angles of triangle BCH

$180 - (55 + 25) = 100$

(Reason: The angles of any triangle add to $180°$. The angles at H and C are given, so the one at B is whatever is left.)

(b)(ii) Set up the sine rule in triangle BCH

$$\frac{BH}{\sin 25} = \frac{32}{\sin 100}$$

(Reason: BH is opposite the $25°$ angle and HC is opposite the $100°$ angle, so those are the two pairs that go together.)

(b)(ii) Rearrange and work it out

$$BH = \frac{32 \times \sin 25}{\sin 100}$$

$$BH = 13.7324\ldots$$

(Reason: Multiply both sides by $\sin 25$. Keep every figure in the calculator instead of rounding here, or the first decimal place is not safe.)

(b)(ii) Round to 1 decimal place

$$BH = 13.7$$

(Reason: The digit after the first decimal place is 3, so the 7 stays as it is, and that is the 13.7 km the question asks to be shown.)

(c) The bearing of H from B

$$90 + 100 = 190$$

(Reason: C is due east of B, so BC points along $090°$. Turning clockwise through the $100°$ of angle CBH lands on BH.)

(c) The cosine rule in triangle ABH

$$\cos B = \frac{11^2 + 13.7^2 - 14^2}{2 \times 11 \times 13.7}$$

(Reason: All three sides are known, so the rearranged cosine rule gives the cosine of the angle at B. The side that is subtracted is the one opposite that angle.)

(c) Work out the top and the bottom

$$11^2 + 13.7^2 - 14^2 = 112.69$$

$$2 \times 11 \times 13.7 = 301.4$$

(Reason: Square each side first, then combine the squares the way the rearranged rule sets them out.)

(c) Divide, then take the inverse cosine

$$\frac{112.69}{301.4} = 0.3739$$

$\cos^{-1}(0.3739) = 68.0\ldots$

(Reason: The calculator must be in degree mode. Angle ABH comes out as $68.0°$ to 1 decimal place.)

(c) Add the angle to the bearing of H

$190 + 68.0 = 258.0$

(Reason: A lies on the far side of BH from C, so the turn carries on clockwise past $190°$. A bearing is written with three figures, so the answer is $258°$.)

(d)(i) Change the speed into km/h

$10 \times 1.852 = 18.52$

(Reason: A knot is one nautical mile an hour, and the question gives 1 knot as 1.852 km/h.)

(d)(i) Divide the distance by the speed

$$\frac{32}{18.52} \times 60 = 103.67$$

(Reason: Distance over speed gives the time in hours, and the 60 turns those hours straight into minutes.)

(d)(i) Change the minutes into hours and minutes

$60 + 44 = 104$

(Reason: 103.67 minutes is 104 minutes to the nearest minute, which is 1 hour and 44 minutes. Starting at 1 pm, the yacht ties up at 2 44 pm.)

(d)(ii) Find where the yacht passes closest to B

$$\frac{x}{13.7} = \cos 55$$

(Reason: The shortest distance from a point to a line is the perpendicular, so drop one from B onto CH and call the foot X. In right-angled triangle BXH the side HX is adjacent to the $55°$ angle and BH is the hypotenuse.)

(d)(ii) Work out that distance

$x = 13.7 \times \cos 55 = 7.858\ldots$

$x = 7.86$

(Reason: That is the stretch of CH still lying between the yacht and the marina, given to 3 significant figures.)

(a) 245°

(b)(i) angle CBH **is** 100°

(b)(ii) BH **is** 13.7 **km**

(c) 258°

(d)(i) $2\ 44$ **pm**

(d)(ii) 7.86 **km**

Verification

✓ **Check 1:** Walk the bearings round the figure. The reverse of the bearing of H from C should lead back through the 55° angle to the reverse of 190°. $245 - 180 = 65$, then $65 - 55 = 10$, and the reverse of 190° is $190 - 180 = 10$, which agrees.

✓ **Check 2:** Add all three angles of triangle BCH. $25 + 55 + 100 = 180$, as a triangle requires.

✓ **Check 3:** Put the unrounded BH back into the sine rule and compare the two ratios. $\dfrac{13.7324}{\sin 25} = 32.49\ldots$ and $\dfrac{32}{\sin 100} = 32.49\ldots$, the same ratio.

✓ **Check 4:** Run the cosine rule the other way round, using the angle just found to rebuild AH. $11^2 + 13.7^2 - 2 \times 11 \times 13.7 \times \cos 68.04^\circ = 196.0\ldots$, and $\sqrt{196} = 14$, which is AH.

✓ **Check 5:** Multiply the sailing time back by the speed. $18.52 \times 1.7279 = 32.0$ km, which is HC.

✓ **Check 6:** Test right-angled triangle BXH with Pythagoras, where $BX = 13.7 \times \sin 55$. $11.2224^2 + 7.858^2 = 187.69$, and $13.7^2 = 187.69$, so the two sides do make up the hypotenuse.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The bearing of the marina from the yacht	**Note**	The Answer column gives 245, and the Marks column gives 1 for it.	✓
(b)(i) The angle at B from the angle sum	**M1**	For $180 - (55 + 25)$ $[= 100]$.	✓

Step	Mark	Description	Got it?
(b)(ii) The sine rule rearranged for BH	**M2**	For $\dfrac{32 \times \sin 25}{\sin 100}$ oe.	✓
(b)(ii) The sine rule before it is rearranged	**M1**	For $\dfrac{\sin 25}{BH} = \dfrac{\sin 100}{32}$ oe.	✓
(b)(ii) The unrounded length of BH	**A1**	For $13.73\ldots$.	✓
(c) The bearing of A from B	**Note**	The Answer column gives 258 or 257.9 to $258.0\ldots$, and the Marks column gives 5 for it.	✓
(c) The angle at B on its own	**B4**	For 67.9 to $68.0\ldots$.	✓
(c) Second method: the cosine rule rearranged for $\cos B$	**M2**	Or for $[\cos =] \dfrac{11^2 + 13.7^2 - 14^2}{2 \times 13.7 \times 11}$.	✓
(c) Second method: the value of the cosine	**A1**	For 0.3738 to 0.376.	✓
(c) Second method: the cosine rule before it is rearranged	**M1**	Or for $14^2 = 11^2 + 13.7^2 - 2 \times 11 \times 13.7 \times \cos B$.	✓
(c) Turning the angle at B into a bearing	**M1dep**	For $190 + \text{their angle } B$, dep on at least M1.	✓
(d)(i) The arrival time	**Note**	The Answer column gives $2\ 44$ pm or $14\ 44$ cao, and the Marks column gives 4 for it.	✓
(d)(i) The journey time, as hours and minutes or as minutes	**B3**	For $1 \text{ hour } 44$ or $1 \text{ hour } 43.6$ to $1 \text{ hour } 43.8$ or 104 or 103.6 to 103.8.	✓
(d)(i) The journey time in hours	**B2**	Or for 1.727 to 1.73.	✓
(d)(i) The method, taken as far as minutes	**M2**	Or for $\dfrac{32}{10 \times 1.852} \times 60$.	✓

Step	Mark	Description	Got it?
(d)(i) The method, taken only as far as hours	M1	Or for $\dfrac{32}{10 \times 1.852}$.	✓
(d)(ii) The distance still to run	Note	The Answer column gives 7.857 to 7.88, and the Marks column gives 3 for it.	✓
(d)(ii) The trigonometry in the right-angled triangle	M2	For $\dfrac{x}{13.7} = \cos 55$ oe.	✓
(d)(ii) Recognising where the shortest distance falls	M1	Or for dist to H occurs when perpendicular from B meets CH soi.	✓

Full marks: 17/17

Question 7 - Exam Solution

Understanding the Question

Given

An open crate in the shape of a cuboid, 70 cm long, 30 cm deep and 40 cm high, with no top.
Tins that are cylinders of height 20 cm and diameter 15 cm.
A solid bronze cone of mass 750 g, made of bronze of density 8.9 g/cm³, whose radius and height are in the ratio $1 : 3$.

Find

The surface area of the inside of the open crate, and the greatest number of tins that fit inside it. A demonstration that the cone has radius 2.99 cm, and then the total surface area of that cone.

Plan the Solution

- An open box has five faces, not six: a base and two matching pairs of walls.
- A tin cannot be cut, so divide each edge of the crate by the measurement of the tin that lies along it and round every count down before multiplying.
- Density ties mass to volume, so turn the 750 g into a volume first.
- The ratio makes the height $3r$, and $\dfrac{1}{3}\pi r^2 \times 3r$ collapses to πr^3, which leaves one unknown in one equation.

- The radius, the height and the slant height make a right-angled triangle, so Pythagoras gives l before $A = \pi r l$ can be used.
- A cone standing on its base has a flat circle as well as a curved surface, so the total is $\pi r l + \pi r^2$.

Worked Solution [15 marks]

> Rule - Surface area, density and the cone: an open box has five faces, density $= \dfrac{\text{mass}}{\text{volume}}$, $V = \dfrac{1}{3}\pi r^2 h$ and $A = \pi r l$.

(a)(i) The base of the crate

$70 \times 30 = 2100$

(Reason: The crate stands on a rectangle 70 cm by 30 cm, and that face is inside the crate, so it is one of the faces counted.)

(a)(i) The two end walls

$2 \times 30 \times 40 = 2400$

(Reason: The two ends stand on the 30 cm edges and rise the full 40 cm. They are a matching pair, so one area is doubled.)

(a)(i) The two long walls

$2 \times 40 \times 70 = 5600$

(Reason: The other two walls stand on the 70 cm edges and rise to the same height as the ends. They are a matching pair as well.)

(a)(i) Add the five faces

$2100 + 2400 + 5600 = 10\,100$

(Reason: A base and four walls, with no lid to add. Every term is a length times a length, so the answer is in square centimetres.)

(a)(ii) How many tins lie along the 70 cm edge

$$\frac{70}{15} = 4.66\ldots$$

(Reason: Each tin needs 15 cm of that edge. Four tins use 60 cm and a fifth would need 75 cm, so the count is 4 - part of a tin is no use.)

(a)(ii) Across the crate, and up it

$$\frac{30}{15} = 2$$

$$\frac{40}{20} = 2$$

(Reason: The tins stand upright, so it is the diameter that has to fit across the 30 cm and the height of a tin that has to fit up the 40 cm. Both of these divide exactly.)

(a)(ii) Multiply the three counts

$$4 \times 2 \times 2 = 16$$

(Reason: Eight tins cover the base in a 4 by 2 block, and a second layer of eight stands on top of them.)

(b)(i) Turn the mass into a volume

$$V = \frac{750}{8.9} = 84.2696\ldots$$

(Reason: Density is mass divided by volume, so volume is mass divided by density. Keep every figure in the calculator here - rounding now would move the third significant figure of the radius.)

(b)(i) Put the ratio into the cone formula

$$h = 3r$$

$$V = \frac{1}{3}\pi r^2 \times 3r = \pi r^3$$

(Reason: The ratio $1:3$ makes the height three times the radius. The 3 and the $\frac{1}{3}$ cancel, which leaves one unknown.)

(b)(i) Solve for r

$$\pi r^3 = 84.2696\ldots$$

$$r^3 = \frac{84.2696\ldots}{\pi} = 26.8238\ldots$$

$$r = \sqrt[3]{26.8238\ldots} = 2.9934\ldots$$

(Reason: Divide by π, then take the cube root. Every figure is carried until the last line.)

(b)(i) Round to 3 significant figures

$$r = 2.99$$

(Reason: The fourth significant figure is 4, so the third stays as it is. That is the 2.99 cm the question asks to be shown.)

(b)(ii) The height and the slant height

$$h = 3 \times 2.99 = 8.97$$

$$2.99^2 + 8.97^2 = 89.401$$

$$l = \sqrt{89.401} = 9.4552\ldots$$

(Reason: The radius, the height and the slant height make a right-angled triangle, with the slant height as its hypotenuse.)

(b)(ii) The curved surface

$\pi \times 2.99 \times 9.4552 = 88.816$

(Reason: This is the formula the question gives, $A = \pi r l$, with the radius and the slant height just found.)

(b)(ii) The base circle

$\pi \times 2.99^2 = 28.086$

(Reason: The cone is solid and stands on its base, so there is a flat circle of radius 2.99 cm to add as well.)

(b)(ii) Add the two areas

$88.816 + 28.086 = 116.902$

$A = 117$

(Reason: Rounded to 3 significant figures, the total surface area is 117 cm².)

(a)(i) $10\,100$ cm²

(a)(ii) 16 tins

(b)(i) the radius is 2.99 cm

(b)(ii) 117 cm²

Verification

✓ **Check 1:** Build the four walls as one band round the base instead of adding them face by face. $2 \times (70 + 30) = 200$ cm round the base, $200 \times 40 = 8000$ cm² of wall, and $8000 + 2100 = 10\,100$ cm² once the base is added.

✓ **Check 2:** Measure the space the tins take up inside the crate. $4 \times 15 = 60$ cm of the 70 cm, $2 \times 15 = 30$ cm of the 30 cm and $2 \times 20 = 40$ cm of the 40 cm. A fifth tin along the longest edge would need 75 cm.

✓ **Check 3:** Put the unrounded radius back and rebuild the mass it came from. With $r = 2.9934\ldots$ the volume πr^3 is $84.2696\ldots$ cm³, and $8.9 \times 84.2696\ldots = 750$ g.

✓ **Check 4:** Rebuild the surface area a different way. For this cone $l = r\sqrt{10}$, so the total is $\pi r^2(1 + \sqrt{10})$. $1 + \sqrt{10} = 4.1623$ and $28.086 \times 4.1623 = 116.902$.

✓ **Check 5:** Is the size sensible? This cone is three times as tall as its radius, so the curved surface should be a few times the base circle. $\dfrac{88.816}{28.086} = 3.162$, which is $\sqrt{10}$ to 4 significant figures - the ratio $\dfrac{l}{r}$ for this cone.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The surface area of the inside of the crate	**Note**	The Answer column gives $10\,100$, and the Marks column gives 3 for it.	✓
(a)(i) The five faces, all of them	**M2**	For $30 \times 70 + 2 \times 30 \times 40 + 2 \times 40 \times 70$.	✓
(a)(i) One face of the crate worked out	**M1**	Or for 30×40 or 30×70 or 40×70.	✓
(a)(ii) The greatest number of tins	**Note**	The Answer column gives 16, and the Marks column gives 3 for it.	✓
(a)(ii) How many tins fit along each edge	**M2**	For 2 fit width, 2 fit height and 4 fit length soi.	✓
(a)(ii) One edge divided by one tin	**M1**	Or for 70, 30 or 40 divided by 15 or 20.	✓
(b)(i) The volume equation, with the ratio already used	**M2**	For $\frac{1}{3}\pi r^2 \times 3r = \text{their } \frac{750}{8.9}$ oe.	✓
(b)(i) The density formula used on the two given values	**M1**	For using 750 and 8.9 correctly in $v = \frac{m}{d}$ oe or $\frac{750}{8.9}$.	✓
(b)(i) The cube of the radius made the subject	**M1dep**	For $r^3 = \dfrac{\text{their } \left(\frac{750}{8.9}\right)}{\pi}$ oe.	✓
(b)(i) The unrounded radius	**A1**	For $r = 2.993\ldots$.	✓
(b)(ii) The total surface area of the cone	**Note**	The Answer column gives 117 or 116.9 to 117.2, and the Marks column gives 5 for it.	✓
(b)(ii) Both surfaces in one expression	**M4**	For $\pi \times 2.99^2 + \pi \times 2.99 \times \sqrt{2.99^2 + (3 \times 2.99)^2}$ oe.	✓

Step	Mark	Description	Got it?
(b)(ii) The curved surface area on its own	**M3**	Or for $\pi \times 2.99 \times \sqrt{2.99^2 + (3 \times 2.99)^2}$.	✓
(b)(ii) The slant height	**M2**	Or for $\sqrt{2.99^2 + (3 \times 2.99)^2}$.	✓
(b)(ii) Pythagoras started, or the base circle alone	**M1**	Or for $2.99^2 + (3 \times 2.99)^2$, or for $\pi \times 2.99^2$.	✓

Full marks: 15/15

Question 8 - Exam Solution

Understanding the Question

Given

The curve $y = x^2 + 7x - 18$, and a blank pair of axes to sketch it on.
The curve $y = x^2 - 3x - 28$, used in parts (b) and (c).
The line $y = 5 - 2x$, which cuts that curve at P and Q.

Find

A sketch of $y = x^2 + 7x - 18$ carrying the values where it meets each axis. The derivative of $y = x^2 - 3x - 28$, and the coordinates of its turning point. The coordinates of P and Q, each correct to 2 decimal places.

Plan the Solution

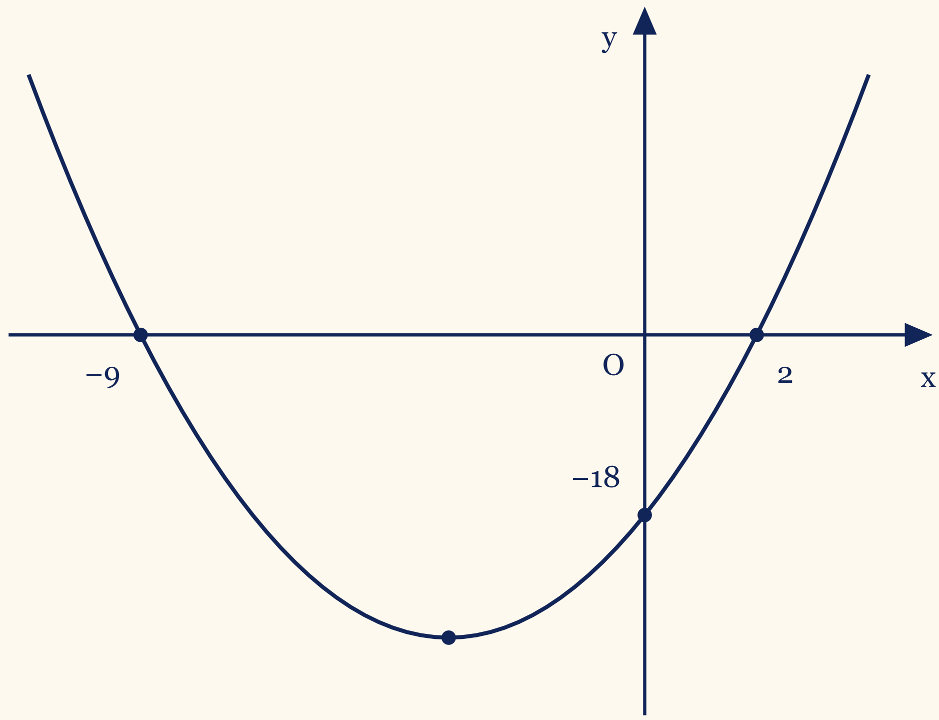
- A sketch needs three things: which way up the curve is, where it cuts each axis, and roughly where its lowest point sits.
- The x^2 term is positive, so the curve is U shaped, and it meets the x-axis where the right-hand side factorises to zero.
- Putting $x = 0$ in gives the y-axis value in one line, because it leaves only the constant term.
- The lowest point of a U shaped quadratic sits halfway between its two x-axis values.
- A turning point is where the gradient is zero, so differentiate first and then solve.
- Two graphs meet where their y-values agree, so set the two right-hand sides equal and collect every term on one side.
- The quadratic that comes out has no factors, so the formula is the way through, and the y-values then come from the simpler of the two equations.

Worked Solution [15 marks]

Rule - Sketching a quadratic, and where two graphs meet: factorise for the x-axis values, put $x = 0$ for the y-axis value, solve $\dfrac{dy}{dx} = 0$ for a turning point, and set the two expressions for y equal where two graphs cross.

(a) Factorise the right-hand side

$x^2 + 7x - 18 = (x + 9)(x - 2)$



(Reason: Two numbers that multiply to -18 and add to 7 are 9 and -2.)

(a) The values where the graph meets the x-axis

$(x + 9)(x - 2) = 0$

$x = -9 \quad \text{or} \quad x = 2$

(Reason: The graph meets the x-axis where $y = 0$, and a product is zero only when one of its brackets is zero.)

(a) The value where the graph meets the y-axis

$y = 0^2 + 7(0) - 18 = -18$

(Reason: Every point on the y-axis has $x = 0$, so only the constant term survives.)

(a) The lowest point, so the curve can be placed

$$x = \frac{-9+2}{2} = -3.5$$

$$y = (-3.5)^2 + 7(-3.5) - 18 = -30.25$$

(Reason: A U shaped quadratic is symmetrical, so its lowest point is halfway between the two x-axis values. Both coordinates are negative, which puts the lowest point in the third quadrant.)

(a) Draw the curve

$$y = x^2 + 7x - 18$$

(Reason: The coefficient of x^2 is positive, so the curve is U shaped. Mark -9 and 2 on the x-axis and -18 on the y-axis, write those three values beside the marks, and draw one smooth curve through them.)

(b)(i) Differentiate term by term

$$y = x^2 - 3x - 28$$

$$\frac{dy}{dx} = 2x - 3$$

(Reason: Bring each power down in front and drop it by one: x^2 gives $2x$, $-3x$ gives -3, and a constant has no gradient of its own.)

(b)(ii) The gradient is zero at a turning point

$$2x - 3 = 0$$

$$x = 1.5$$

(Reason: The curve is level for an instant at its turning point, so set the derivative from part (b)(i) equal to zero and solve.)

(b)(ii) The y-coordinate of the turning point

$$y = 1.5^2 - 3(1.5) - 28$$

$$y = 2.25 - 4.5 - 28 = -30.25$$

(Reason: The turning point lies on the curve, so put the x-value back into the equation of the curve rather than into the derivative.)

(c) The line and the curve share a y-value where they cross

$$x^2 - 3x - 28 = 5 - 2x$$

(Reason: At P and at Q the point lies on both graphs, so the two expressions for y must be equal there.)

(c) Collect every term on one side

$$x^2 - 3x - 28 - 5 + 2x = 0$$

$$x^2 - x - 33 = 0$$

(Reason: Take 5 from both sides and add $2x$ to both sides. The $-3x$ and the $2x$ leave $-x$, and $-28 - 5$ leaves -33.)

(c) No factors, so use the formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-33)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{133}}{2}$$

(Reason: Here $a = 1$, $b = -1$ and $c = -33$. Under the root, $1 + 132 = 133$, which is not a square number, so the answers will not be exact.)

(c) The two x-values

$$x = \frac{1 + 11.5325\ldots}{2} = 6.2662\ldots$$

$$x = \frac{1 - 11.5325\ldots}{2} = -5.2662\ldots$$

(Reason: Keep the whole calculator display at this stage. Rounding here would move the second decimal place of the y-values.)

(c) The matching y-values

$$y = 5 - 2(6.2662\ldots) = -7.5325\ldots$$

$$y = 5 - 2(-5.2662\ldots) = 15.5325\ldots$$

(Reason: Use the line, not the curve: $y = 5 - 2x$ is the shorter calculation and gives the same point. Watch the second one, where subtracting a negative adds.)

(c) Round to 2 decimal places

$$(-5.27,\ 15.53)$$

$$(6.27,\ -7.53)$$

(Reason: The third decimal place is 6 in both x-values, so each of those rounds up, and 2 in both y-values, so each of those stays as it is.)

(a) a U shaped curve through $(-9,\ 0)$, $(2,\ 0)$ and $(0,\ -18)$

(b)(i) $\dfrac{dy}{dx} = 2x - 3$

(b)(ii) $(1.5,\ -30.25)$

(c) $(-5.27,\ 15.53)$ **and** $(6.27,\ -7.53)$

Verification

✓ **Check 1:** Expand the brackets again and see the original expression come back.
$(x + 9)(x - 2) = x^2 - 2x + 9x - 18 = x^2 + 7x - 18$

✓ **Check 2:** Put both x-axis values back into $y = x^2 + 7x - 18$; each must give zero. $81 - 63 - 18 = 0$ and $4 + 14 - 18 = 0$.

✓ **Check 3:** Find the turning point of $y = x^2 - 3x - 28$ without differentiating, by completing the square. Completing the square gives $(x - 1.5)^2 - 30.25$, which is smallest when the bracket is zero, so the turning point is $(1.5,\ -30.25)$. Part (a)'s curve reaches the same depth, since $-18 - 12.25 = -30.25$.

✓ **Check 4:** Both points must sit on the line, so put the rounded x-values into $y = 5 - 2x$. $5 - 2(-5.27) = 15.54$ and $5 - 2(6.27) = -7.54$, each within 0.01 of the rounded y-values, which is why the mark scheme accepts either figure.

✓ **Check 5:** The two solutions of $x^2 - x - 33 = 0$ must add to 1, because the coefficient of x is -1. $-5.2662\ldots + 6.2662\ldots = 1$, and the two values sit either side of 0.5, the line of symmetry of that quadratic.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The sketch	**Note**	The Answer column gives correct sketch with roots indicated at $x = -9$ and $x = 2$ and y intercept = -18, minimum should be in 3rd quadrant, and the Marks column gives 4 for it.	✓
(a) The shape of the curve	**B1**	For a U shaped parabola.	✓
(a) The two values on the x-axis	**B2**	For roots at -9 and 2 on diagram.	✓
(a) The factorisation instead	**M1**	Or for $(x + 9)(x - 2)$ $[= 0]$.	✓
(a) The value on the y-axis	**B1**	For y – intercept at -18 on diagram.	✓
(a) The cap on the sketch	**Note**	Maximum 3 marks if sketch not fully correct.	✓
(b)(i) The derivative	**Note**	The Answer column gives $2x - 3$, and the Marks column gives 2 for it.	✓
(b)(i) One term differentiated correctly	**B1**	For $2x + k$ or $kx^{[p]} - 3$.	✓

Step	Mark	Description	Got it?
(b)(ii) The turning point	**Note**	The Answer column gives $(1.5,\ -30.25)$ oe, and the Marks column gives 3 for it.	✓
(b)(ii) The x-coordinate alone	**B2**	For $x = 1.5$.	✓
(b)(ii) The derivative set to zero, or the completed square	**M1**	Or for their $\text{(b)(i)} = 0$, or for $(x - 1.5)^2$.	✓
(c) The rearranged quadratic	**B1**	For $x^2 - x - 33\ [= 0]$ seen.	✓
(c) How the formula row follows through	**Note**	FT their quadratic dep on no factors.	✓
(c) The quadratic formula with the right values in it	**B2FT**	For $\dfrac{[--]1 \pm \sqrt{([-]1)^2 - 4(1)(-33)}}{2 \times 1}$ oe.	✓
(c) The discriminant under the root	**B1**	For $\sqrt{([-]1)^2 - 4(1)(-33)}$ or better.	✓
(c) One branch of the formula	**B1**	Or for $\dfrac{[--]1 + \sqrt{q}}{2(1)}$ oe or $\dfrac{[--]1 - \sqrt{q}}{2(1)}$ oe.	✓
(c) The two x-values	**B2**	For -5.27 or -5.267 to -5.266 and 6.27 or 6.266 to 6.267.	✓
(c) One of the two x-values	**B1**	For each.	✓
(c) The special case for two swapped signs	**SC1**	If 0 scored, for -6.27 and 5.27.	✓
(c) The two points	**B1**	For $(-5.27,\ 15.53 \text{ or } 15.54)$ and $(6.27,\ -7.53 \text{ or } -7.54)$.	✓

Full marks: 15/15

Question 9 - Exam Solution

Understanding the Question

Given

$\mathrm{f}(x) = 4x + 1$
$\mathrm{g}(x) = 6 - 2x$
$\mathrm{h}(x) = 3^{x-2}$
Two linear functions and one function with the variable in the index.

Find

(a)(i) $\mathrm{f}(3)$ and (ii) $\mathrm{gf}(3)$ (b) the inverse $\mathrm{g}^{-1}(x)$ (c) x when $\mathrm{f}(x) = \mathrm{g}(2x - 7)$ (d) $\mathrm{hh}(2)$ (e) x when $\mathrm{h}^{-1}(x) = 10$

Plan the Solution

- To evaluate a function at a number, replace every x in its formula by that number.
- In $\mathrm{gf}(3)$ the function written nearest the number acts first, so work out $\mathrm{f}(3)$ and feed the result into g.
- For the inverse, write $y = \mathrm{g}(x)$, make x the subject, then rename the letters.
- For (c), put $2x - 7$ into g first, then solve the linear equation that is left.
- For (d), work outwards: $\mathrm{h}(2)$ first, then h of that answer.
- For (e), an inverse undoes its function, so $\mathrm{h}^{-1}(x) = 10$ is the same statement as $x = \mathrm{h}(10)$.

Worked Solution [12 marks]

> Rule - Composite and inverse functions: $\mathrm{gf}(x)$ means apply f first and then g; and an inverse reverses a function, so $\mathrm{g}^{-1}(y) = x$ exactly when $\mathrm{g}(x) = y$.

Step 1: (a)(i) Put 3 in place of x in f

$\mathrm{f}(3) = 4 \times 3 + 1$

$4 \times 3 + 1 = 12 + 1 = 13$

(Reason: the formula $4x + 1$ is a rule for what to do to its input, so the 3 is multiplied by 4 before the 1 is added)

Step 2: (a)(ii) Apply f first, then g

$\mathrm{gf}(3) = \mathrm{g}(13)$

$\mathrm{g}(13) = 6 - 2 \times 13$

$6 - 2 \times 13 = 6 - 26 = -20$

(Reason: the function written nearest the number acts first, so the 13 from part (i) becomes the input of g)

Step 3: (b) Make x the subject of $y = 6 - 2x$

$y = 6 - 2x$

$2x = 6 - y$

$x = \dfrac{6 - y}{2}$

$\mathrm{g}^{-1}(x) = \dfrac{6 - x}{2}$

(Reason: the inverse sends an output of g back to the input it came from, so the working is that rearrangement; the letters are renamed at the end because an inverse is written as a function of x)

Step 4: (c) Feed $2x - 7$ into g

$\mathrm{g}(2x - 7) = 6 - 2(2x - 7)$

$6 - 2(2x - 7) = 6 - 4x + 14$

$6 - 4x + 14 = 20 - 4x$

(Reason: both terms inside the bracket are multiplied by -2, and a negative multiplied by a negative is positive, so the constant term grows rather than shrinks)

Step 5: (c) Collect the terms and solve

$4x + 1 = 20 - 4x$

$4x + 4x = 20 - 1$

$8x = 19$

$x = \dfrac{19}{8} = 2.375$

(Reason: the x terms go on one side and the numbers on the other, which leaves a single division by 8)

Step 6: (d) Work outwards for $\mathrm{hh}(2)$

$\mathrm{h}(2) = 3^{2-2} = 3^0 = 1$

$\mathrm{hh}(2) = \mathrm{h}(1)$

$3^{1-2} = 3^{-1} = \dfrac{1}{3}$

(Reason: the inner h acts first and gives 1, and that 1 is then the input of the outer h)

Step 7: (e) Turn the inverse statement round

$\mathrm{h}^{-1}(x) = 10$

$x = \mathrm{h}(10) = 3^{10-2}$

$3^{10-2} = 3^8 = 6561$

(Reason: an inverse undoes its function, so saying that h^{-1} sends x to 10 is the same as saying that h sends 10 to x)

(a)(i) 13

(a)(ii) -20

(b) $\mathrm{g}^{-1}(x) = \dfrac{6-x}{2}$

(c) $x = 2.375$

(d) $\dfrac{1}{3}$

(e) $x = 6561$

Verification

✓ **Check 1 - part (a)(ii):** Compose the two functions into one formula first, $\mathrm{gf}(x) = 6 - 2(4x+1) = 4 - 8x$, then put $x = 3$ into that instead. $4 - 8 \times 3 = 4 - 24 = -20$

✓ **Check 2 - part (b):** $\mathrm{g}(5) = 6 - 10 = -4$, so the inverse must send -4 back to 5. $\dfrac{6-(-4)}{2} = \dfrac{10}{2} = 5$

✓ **Check 3 - part (c):** Put $x = 2.375$ into each side of $\mathrm{f}(x) = \mathrm{g}(2x-7)$ separately. The bracket gives $2 \times 2.375 - 7 = -2.25$. $4 \times 2.375 + 1 = 10.5$ and $6 - 2 \times (-2.25) = 10.5$

✓ **Check 4 - part (d):** A negative index is a reciprocal, not a negative number, so multiplying the answer by the base must give 1. $\dfrac{1}{3} \times 3 = 1$

✓ **Check 5 - part (e):** Divide 6561 by 3 over and over: it takes 8 divisions to reach 1. $6561 = 3^8$, so the index $x - 2$ is 8 and $x = 10$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The value of $\mathrm{f}(3)$	**Note**	The Answer column gives 13, and the Marks column gives 1 for it.	✓

Step	Mark	Description	Got it?
(a)(ii) The value of $\mathrm{gf}(3)$	**Note**	The Answer column gives -20, and the Marks column gives 1 for it.	✓
(a)(ii) The instruction printed in the Partial Marks column	**Note**	FT $6-2(\textit{their}\ (\mathrm{a})(\mathrm{i}))$.	✓
(b) The expression for $\mathrm{g}^{-1}(x)$	**Note**	The Answer column gives $\dfrac{6-x}{2}$ oe final answer, and the Marks column gives 2 for it.	✓
(b) The first step of the rearrangement	**M1**	For the correct first step: $x = 6 - 2y$, $y - 6 = -2x$, $\dfrac{y}{2} = 3 - x$.	✓
(c) The value of x	**Note**	The Answer column gives 2.375 oe, and the Marks column gives 4 for it.	✓
(c) Substituting $2x-7$ into g	**B1**	For $6-2(2x-7)$ oe.	✓
(c) Expanding the bracket	**B1**	For $4x+1 = 6-4x+14$.	✓
(c) Rearranging to a single term in x	**M1**	For $8x = 19$ FT their linear equation rearranged correctly from $ax+b = cx+d$ to form $ex = f$.	✓
(d) The value of $\mathrm{hh}(2)$	**Note**	The Answer column gives $\dfrac{1}{3}$ or $0.333\ldots$, and the Marks column gives 2 for it.	✓
(d) Reaching the inner value, or writing the whole index expression	**M1**	For $\mathrm{h}(1)$ or $3^{(3^{x-2}-2)}$ or $3^{(3^{2-2}-2)}$, or better.	✓
(e) The value of x	**Note**	The Answer column gives 6561, and the Marks column gives 2 for it.	✓
(e) Turning the inverse statement round	**M1**	For 3^{10-2} or $x = \mathrm{h}(10)$.	✓

Full marks: 12/12

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