

Cambridge IGCSE 0580

0580/21 (Extended) – May/June 2025

100 marks · 2 hours · Non-calculator

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Cambridge IGCSE Mathematics, Paper 0580/21 (Extended), May/June 2025 series, sat Friday 2 May 2025 in Cambridge administrative zone 2 – 100 marks, 2 hours, calculator not allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are Cambridge Assessment International Education's. This resource reproduces neither the exam paper nor the official mark scheme.

Cambridge publishes a timetable for each administrative zone, and the date above is the one published for Zone 2.

Cambridge has not yet published this paper on its public website. We will link the official question paper and mark scheme here as soon as they are released.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

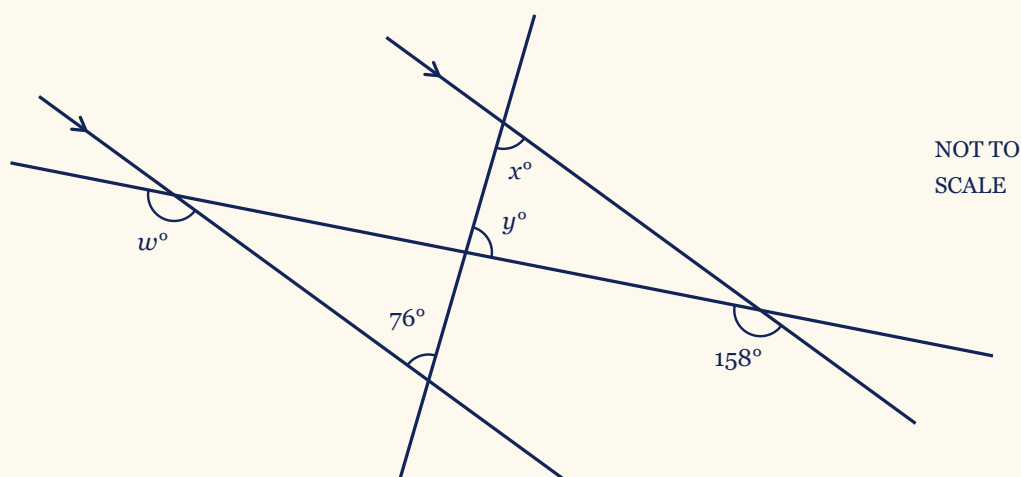
1. What is the simplest form of the expression below?



$$7c - 5d + c + 3d \text{ [2 marks]}$$

.....
[Total 2 marks]

2. In the diagram, a pair of parallel lines is crossed by two straight lines.



What are the values of w , x and y ? [4 marks]

$w =$

$x =$

$y =$

[Total 4 marks]

- 3.** Laura puts \$1500 into a savings account that pays simple interest at 3% per year.



How much are her savings worth altogether at the end of 6 years?
[3 marks]

\$

[Total 3 marks]

- 4.** What is the value of this calculation?



$$\frac{5}{6} - \frac{2}{3} \times \frac{3}{8} \text{ [3 marks]}$$

.....

[Total 3 marks]

- 5.** A regular polygon has interior angles of 150° .

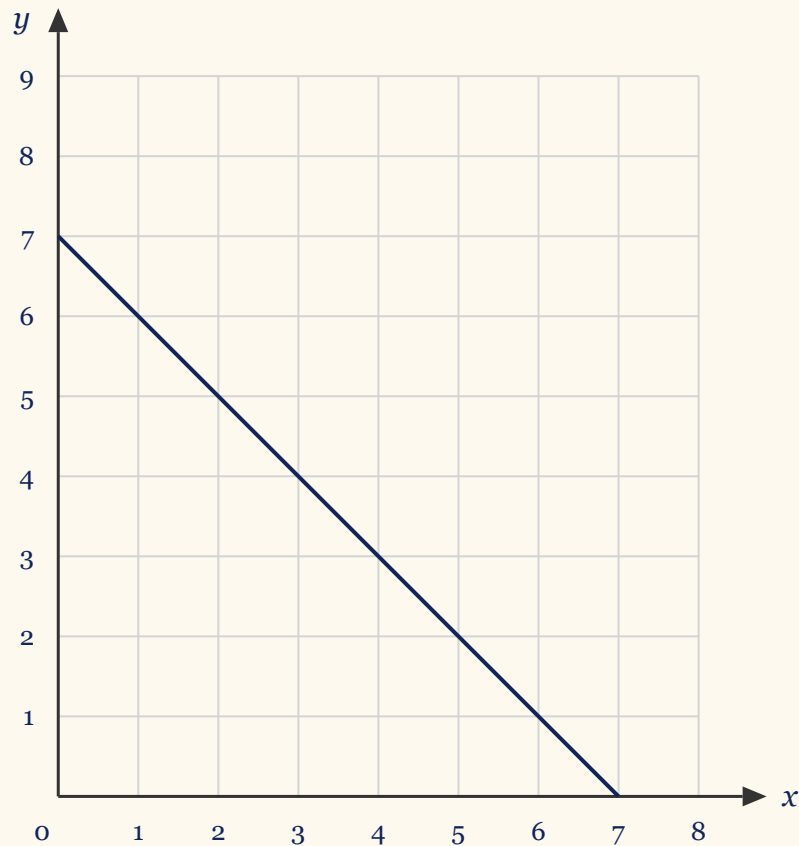


How many sides does this polygon have? [2 marks]

.....

[Total 2 marks]

6. The grid shows the line $x + y = 7$.



(a) Draw the line $y = 2x + 1$ on the grid. [2 marks]

(b) Solve this pair of simultaneous equations using your graph.

$$x + y = 7$$

$$y = 2x + 1 \text{ [1 mark]}$$

(b) $x =$

(b) $y =$

[Total 3 marks]

7. What fraction is equal to the recurring decimal $0.2\dot{6}$?
Give your answer in its simplest form. [3 marks]



.....

[Total 3 marks]

8. $\mathbf{m} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$



(a) What is the column vector $2\mathbf{m} - \mathbf{n}$? [2 marks]

(b) The magnitude of the vector $\begin{pmatrix} 5 \\ \sqrt{y} \end{pmatrix}$ is 7.

What is the value of y ? [2 marks]

(a)

(b) $y =$

[Total 4 marks]

9. Some information about the scores a group of friends got in a quiz is given in the table.



Score	4	5	8
Frequency	2	4	n

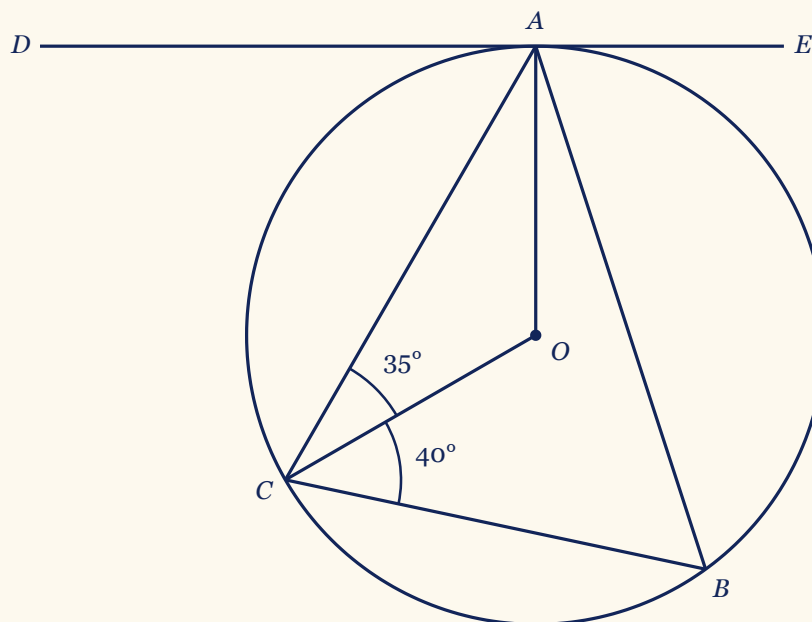
The mean of their scores is 6.

What is the value of n ? [3 marks]

$n =$

[Total 3 marks]

- 10.** The three points A , B and C lie on a circle with centre O .
 The tangent to the circle at A is the line DE .
 The diagram marks two angles at C : angle $ACO = 35^\circ$ and angle $BCO = 40^\circ$.



NOT TO
SCALE

What is the size of each of these angles?

(a) Angle AOC [1 mark]

(b) Angle ABC [1 mark]

(c) Angle DAC [1 mark]

(d) Angle OAB [1 mark]

(a) Angle AOC =

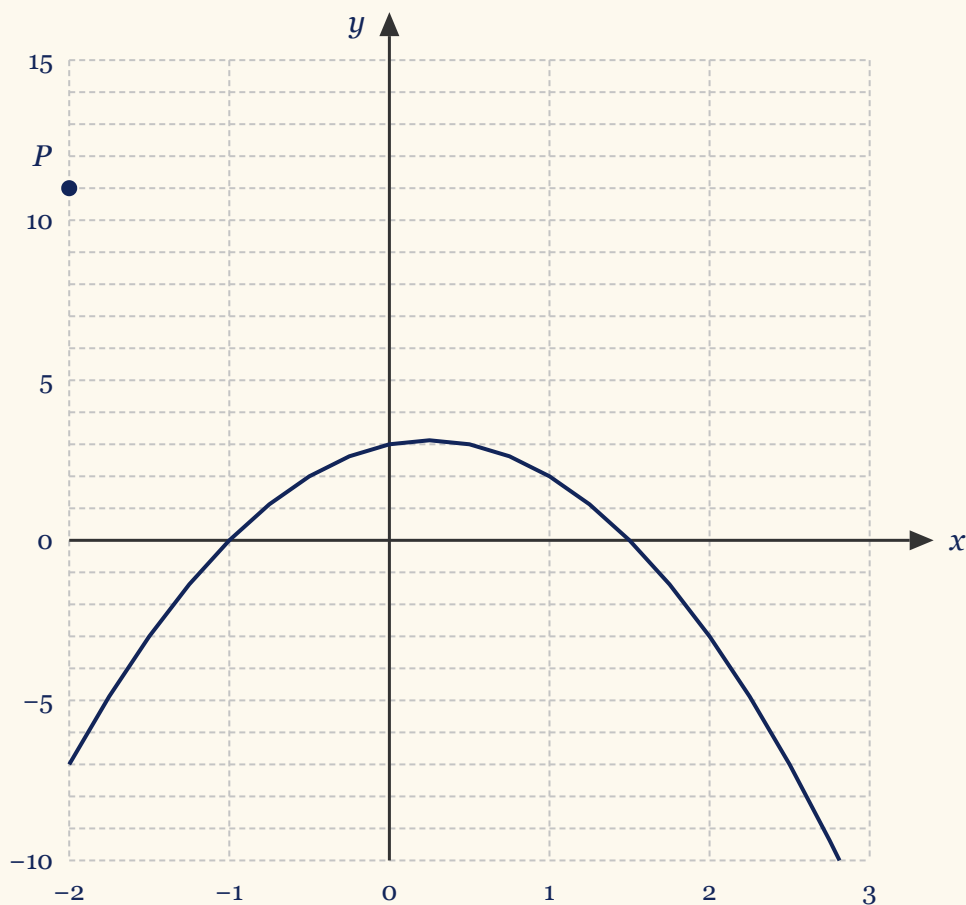
(b) Angle ABC =

(c) Angle DAC =

(d) Angle OAB =

[Total 4 marks]

11. The graph of $y = f(x)$ and the point $P(-2, 11)$ are drawn on the grid below.



The graph of $y = f(x)$ is touched at the point (a, b) by the tangent from P .

Both a and b are integers.

(a) Draw this tangent, and use it to find the value of a and the value of b .
[2 marks]

(b) What is the equation of the tangent?

Give your answer in the form $y = mx + c$. [3 marks]

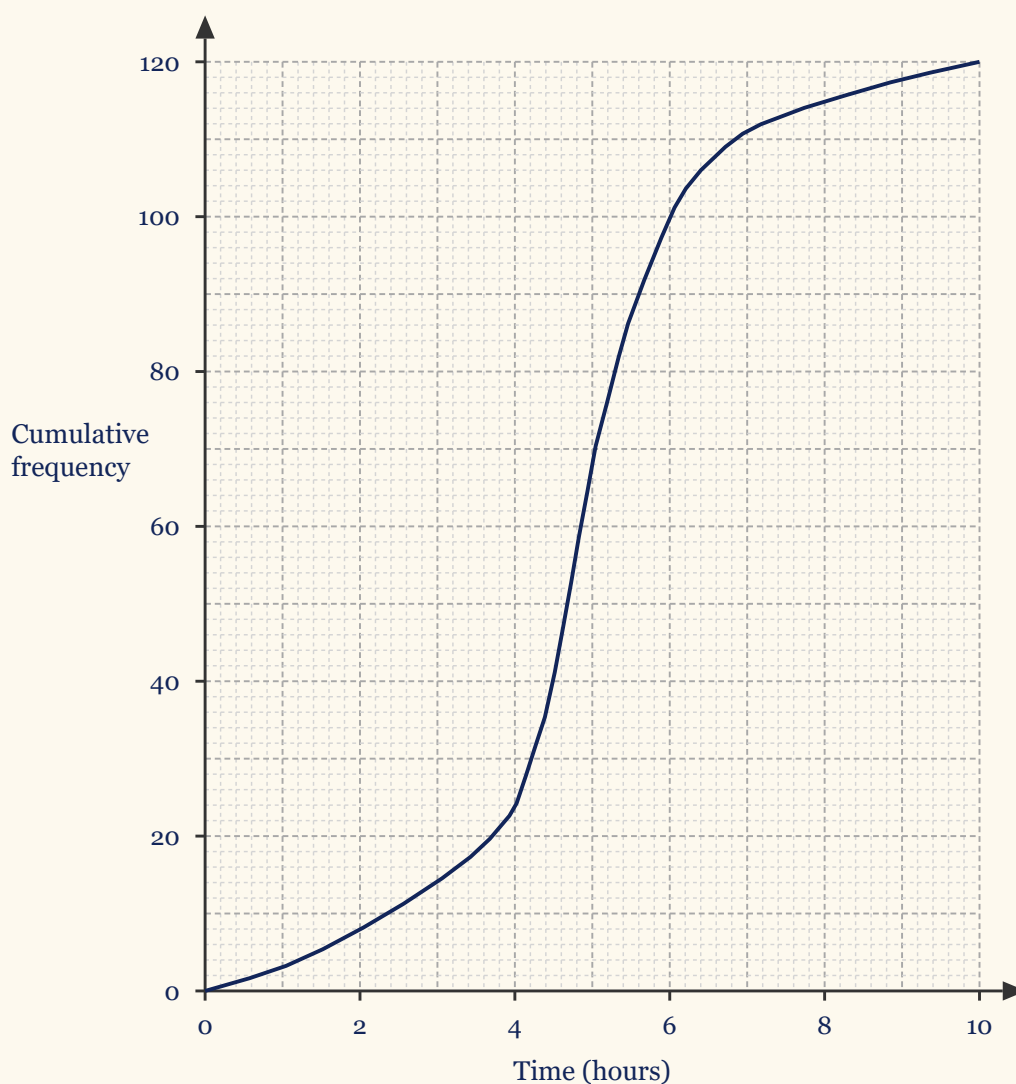
(a) $a =$

(a) $b =$

(b) $y =$

[Total 5 marks]

- 12.** For one day, a record was kept of the time each of 120 office workers spent working at a computer. 
- The results are displayed in the cumulative frequency diagram.



- (a) What estimate of the interquartile range does the cumulative frequency diagram give? [2 marks]
- (b) For 70% of the office workers, the time spent at a computer was less than k hours.
- What estimate of the value of k does the cumulative frequency diagram give? [2 marks]

(a) h

(b) $k =$

[Total 4 marks]

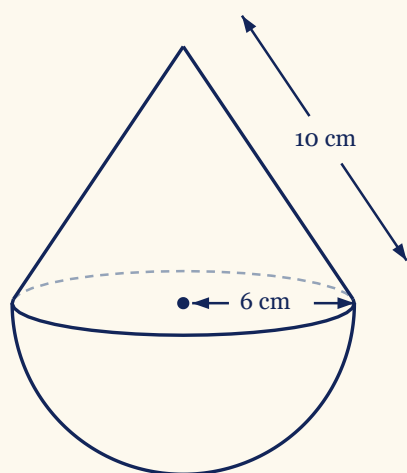
13. Solid A and solid B are drawn above.



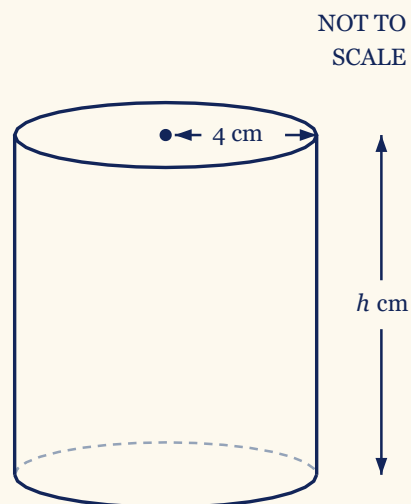
A cone and a hemisphere, each of radius 6 cm, together make up solid A .

The sloping edge of the cone is 10 cm long.

Solid B is a cylinder. Its radius is 4 cm and its height is h cm.



Solid A



Solid B

Solid A and solid B have the same **total** surface area.

- (a) What is the value of h ? [5 marks]
(b) What is the height of solid A ? [3 marks]

(a) $h =$

(b) cm

[Total 8 marks]

14. The functions f and g are given by $f(x) = 3x - 4$ and $g(x) = 4x + 1$.



(a) What is the value of $f(-2)$? [1 mark]

(b) What is $f^{-1}(x)$? [2 marks]

(c) When $fg(x)$ is written in the form $ax + b$, what are the values of a and b ? [2 marks]

(d) The expression below is to be simplified.

$$\frac{2}{f(x)} - \frac{5}{g(x)}$$

Give your answer as a single fraction in terms of x . [3 marks]

(a)

(b) $f^{-1}(x) =$

(c) $a =$

(c) $b =$

(d)

[Total 8 marks]

15. (a) The expression below is to be expanded and simplified.



$$(2 - \sqrt{5})(1 - 3\sqrt{5})$$
 [2 marks]

(b) The denominator of the fraction below is to be rationalised.

Give your answer in its simplest form.

$$\frac{6}{\sqrt{10}}$$
 [2 marks]

(a)

(b)

[Total 4 marks]

16. The expression below is to be expanded and simplified.

$$(x + 4)(x - 3)(3x + 2) [3 \text{ marks}]$$

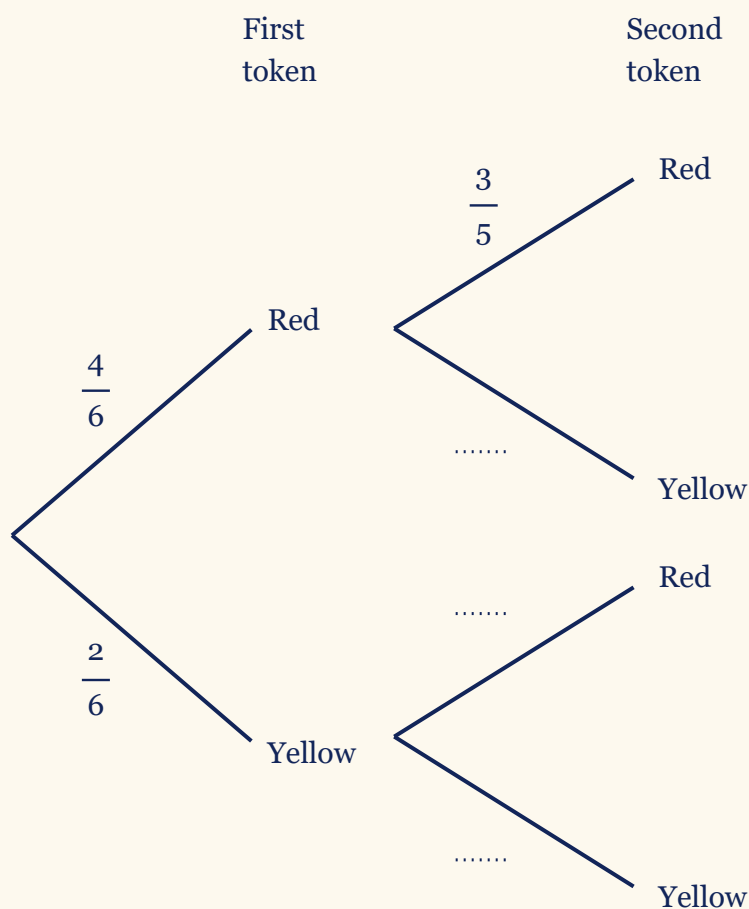


.....
[Total 3 marks]

17. (a) In a jar there are 6 red beads, 3 green beads and 1 blue bead, and two of them are taken at random with replacement.



What is the probability that the two beads taken are both green? [2 marks]



(b) In a second jar there are 4 red tokens and 2 yellow tokens, and two of them are taken at random without replacement.

(i) Write the missing probabilities on the tree diagram. [2 marks]

(ii) What is the probability that one of the two tokens is yellow? [3 marks]

(a)

(b)(ii)

[Total 7 marks]

- 18.** On Saturday, Tanya paddles a kayak 12 km across a lake at a speed of x km/h.



On Sunday she hikes 10 km at a speed of $(x - 4)$ km/h.

(a) In terms of x , how long does Tanya spend paddling? [1 mark]

(b) In terms of x , how long does she spend hiking? [1 mark]

(c) Tanya's hike takes 1 hour longer than her paddle.

Form an equation in terms of x from this fact, and show that it can be simplified to $x^2 - 2x - 48 = 0$. [4 marks]

(d) Solve $x^2 - 2x - 48 = 0$ by factorising. [3 marks]

(e) How many hours does Tanya spend paddling? [1 mark]

(a) h

(b) h

(d) $x =$

(d) or $x =$

(e) h

[Total 10 marks]

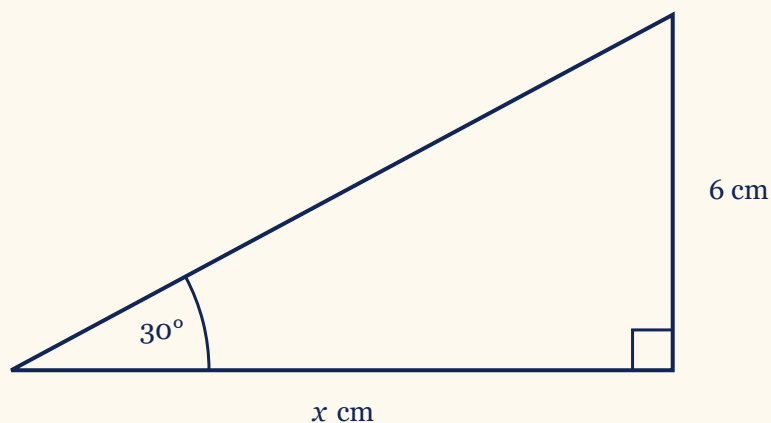
- 19.** What is the value of $27^{-\frac{2}{3}}$? [2 marks]



.....

[Total 2 marks]

20. What is the exact value of x ? [4 marks]

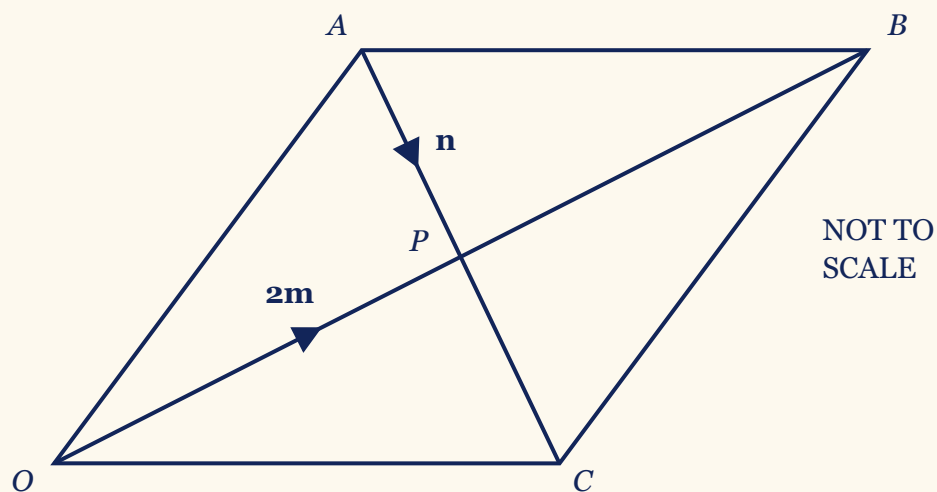


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SCALE

$x =$

[Total 4 marks]

- 21.** In the diagram, $OABC$ is a rhombus whose vertex O is at the origin, and P is the point where its two diagonals cross. 
- It is given that $\overrightarrow{AP} = \mathbf{n}$ and $\overrightarrow{OP} = 2\mathbf{m}$.



(a) Express each of these vectors in terms of $\mathbf{m}$ and $\mathbf{n}$, in its simplest form.

(i) $\overrightarrow{OA}$ [1 mark]

(ii) $\overrightarrow{OC}$ [1 mark]

(b) The point D is placed so that $\overrightarrow{AD} = 10\mathbf{m} - 3\mathbf{n}$.

The quadrilateral $OADC$ is a trapezium. Show that this is true. [3 marks]

(a)(i) $OA =$

(a)(ii) $OC =$

[Total 5 marks]

22. The equation of a curve is $y = x^n + qx^2 + 9x$.



For this curve, $\frac{dy}{dx} = 3x^2 - 12x + 9$.

(a) What are the values of n and q ? [2 marks]

(b) What are the coordinates of the curve's turning points? [4 marks]

(a) $n =$

(a) $q =$

(b)

[Total 6 marks]

23. The expression below is to be simplified.



$$\frac{2x^2 + 10x}{x^2 - 25} \text{ [3 marks]}$$

.....

[Total 3 marks]

Worked Solutions & Mark Schemes

Every mark (*M* for method, *A* for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The expression $7c - 5d + c + 3d$.
It has four terms: two in c and two in d .

Find

The expression in its simplest form.

Plan the Solution

- Put the c terms together and the d terms together, each term keeping the sign in front of it.
- Add the coefficients within each group, remembering that c on its own means $1c$.
- Stop when one c term and one d term are left, because unlike terms do not combine.

Worked Solution [2 marks]

Rule - Collecting like terms: terms with the same letter combine by adding their coefficients, and the sign in front of a term travels with it.

Step 1: Group the like terms

$$7c - 5d + c + 3d = 7c + c - 5d + 3d$$

(Reason: Each term moves together with the sign in front of it, so the $5d$ keeps its minus sign while c and $3d$ stay positive.)

Step 2: Combine the c terms

$$7c + c = 8c$$

(Reason: A c on its own means $1c$, so the coefficients add to $7 + 1 = 8$.)

Step 3: Combine the d terms

$$-5d + 3d = -2d$$

(Reason: Start at -5 and add 3 : $-5 + 3 = -2$. The minus sign belongs to the $5d$ alone, so the $3d$ is added, not taken away.)

Step 4: Write the simplified expression

$$7c - 5d + c + 3d = 8c - 2d$$

(Reason: $8c$ and $2d$ have different letters, so they are unlike terms and cannot be combined. This is the final answer: going on to merge them into one term would spoil it.)

$$8c - 2d$$

Verification

- ✓ **Check 1:** Put $c = 2$ and $d = 3$ into the original expression and into the answer.
 $7 \times 2 - 5 \times 3 + 2 + 3 \times 3 = 10$ and $8 \times 2 - 2 \times 3 = 10$, so the two forms give the same value.
- ✓ **Check 2:** Put $c = 1$ and $d = 0$ into the original, then $c = 0$ and $d = 1$. Each choice leaves only one letter's terms. $7 + 1 = 8$ is the coefficient of c and $-5 + 3 = -2$ is the coefficient of d , matching $8c - 2d$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The simplified expression	Note	The Answer column gives $8c - 2d$ final answer, and the Marks column gives 2 for it.	✓
An answer of $8c - kd$ or $kc - 2d$, or the correct answer spoilt	B1	For answer $8c - kd$ or $kc - 2d$ or for correct answer seen and spoilt.	✓

Full marks: 2/2

Question 2 - Exam Solution

Understanding the Question

Given

Two parallel lines, each marked with an arrow, are crossed by a long straight line and a steep straight line.

The long line makes an angle of 158° where it crosses the right-hand parallel

Find

The values of w , x and y .

line, and the steep line makes an angle of 76° where it crosses the left-hand parallel line.

The diagram is not to scale, so no angle can be measured from it.

Plan the Solution

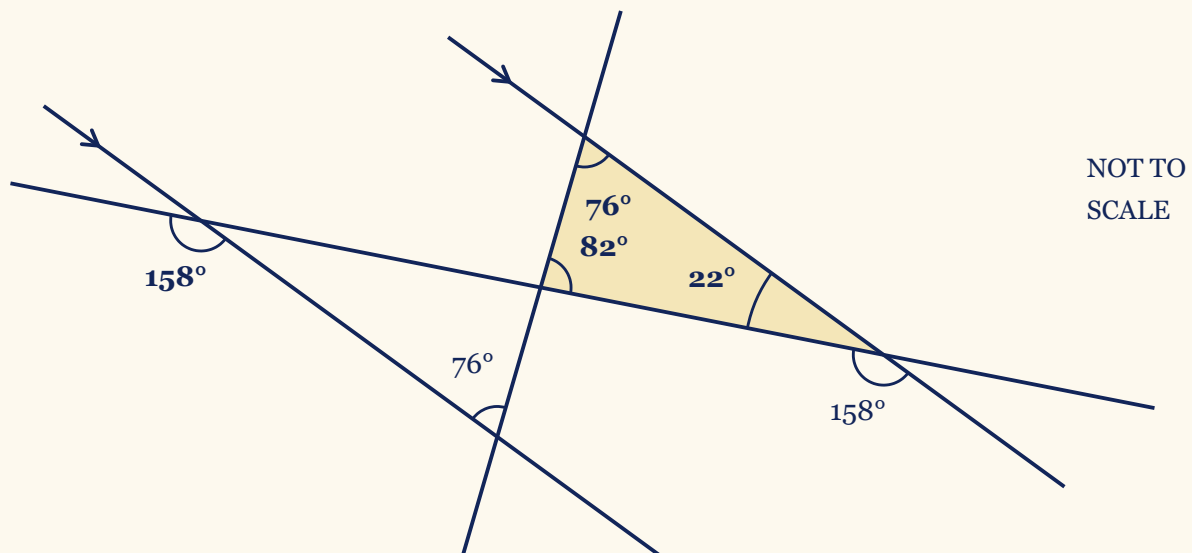
- w sits in the same position as the 158° angle, but at the other parallel line: corresponding angles.
- x and the 76° angle make a Z shape with the steep line: alternate angles.
- For y , use the triangle made by the long line, the steep line and the right-hand parallel line. Its angles are x , y and the angle beside the 158° on the straight parallel line.

Worked Solution [4 marks]

Rule - Angles and parallel lines: corresponding angles are equal and alternate angles are equal; angles on a straight line add up to 180° , and so do the angles in a triangle.

Step 1: Find w with corresponding angles

$$w = 158$$



(Reason: The long line crosses both parallel lines. At each crossing the marked angle is in the same position: below the crossing, between the part of the long line running to the left and the part of the parallel line running down to the right. So w and the 158° angle are corresponding angles, and corresponding angles are equal.)

Step 2: Find x with alternate angles

$$x = 76$$

(Reason: The steep line crosses both parallel lines. x is at the upper crossing, to the right of the steep line; the 76° angle is at the lower crossing, to the left of it. Both lie between the parallel lines, on opposite sides of the steep line, so they form a Z shape: they are alternate angles, and alternate angles are equal.)

Step 3: Use the straight line at the right-hand crossing

$$180 - 158 = 22$$

(Reason: The 158° angle and the angle beside it, above the long line, together make the straight right-hand parallel line, so they add up to 180° . This 22° angle is one corner of the triangle made by the long line, the steep line and the right-hand parallel line.)

Step 4: Find y from the angles in the triangle

$$y = 180 - 76 - 22$$

$$y = 82$$

(Reason: The triangle's corners are the three points where its lines cross. Its angles are $x = 76^\circ$ at the top, 22° on the right, and y where the steep line meets the long line. Angles in a triangle add up to 180° .)

$$w = 158$$

$$x = 76$$

$$y = 82$$

Verification

- ✓ **Check 1:** The 158° angle lies outside the triangle, beside its 22° corner, so it is an exterior angle of the triangle. An exterior angle equals the sum of the two interior angles opposite it, which are x and y . $76 + 82 = 158$, which is the marked angle.
- ✓ **Check 2:** Use the other triangle, made by the long line, the steep line and the left-hand parallel line. Its corners are the given 76° , the angle beside w on the straight long line, which is $180 - 158 = 22$ degrees, and the angle vertically opposite y .
 $180 - 76 - 22 = 82$, and vertically opposite angles are equal, so $y = 82$ again.
- ✓ **Check 3:** Co-interior angles between parallel lines add up to 180° . At the upper crossing of the steep line, the angle beside x on the right-hand parallel line is co-interior with the 76° angle: both are to the left of the steep line, between the parallels. The angle beside x is $180 - 76 = 104$, and $104 + 76 = 180$, as co-interior angles must.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The values of w , x and y	Note	The Answer column gives $[w =] 158$, $[x =] 76$ and $[y =] 82$, and the Marks column gives 4 for it.	✓
The value of w	B1	For w correct.	✓
The value of x	B1	For x correct.	✓
The value of y	B2FT	For $y = 158 - \text{their } x$ correctly evaluated.	✓
The value of y : the alternative to the B2FT	B1	Or for 22 (identified), or 82 in position vertically opposite to y , or for $y = 158 - \text{their } x$.	✓

Full marks: 4/4

Question 3 - Exam Solution

Understanding the Question

Given

Laura puts \$1500 into a savings account.
The account pays simple interest at 3% per year.
The value is wanted at the end of 6 years.
No calculator: every step is done by hand.

Find

The total value of her savings, in dollars, at the end of 6 years: the \$1500 she put in together with all the interest.

Plan the Solution

- Work out 3% of \$1500 by finding 1% first and then multiplying by 3. That is the interest for one year.
- Simple interest is the same amount every year, so multiply one year's interest by 6.
- Add the interest to the \$1500: the question asks for the total value, not just the interest.

Worked Solution [3 marks]

Rule - Simple interest: $I = \frac{P \times R \times T}{100}$, where P is the amount invested, R is the rate per cent per year and T is the number of years. The total value is $P + I$.

Step 1: Find 3% of \$1500

$$\frac{1500}{100} = 15$$

$$3 \times 15 = 45$$

(Reason: Per cent means out of 100, so 1% of 1500 is 1500 divided by 100, which is 15. Three lots of that make 3%, so one year's interest is \$45.)

Step 2: Find the interest for all 6 years

$$45 \times 6 = 270$$

(Reason: Simple interest is always worked out on the original \$1500, so every year earns the same \$45. Interest already paid into the account earns nothing more; that would be compound interest.)

Step 3: Add the interest to the amount she put in

$$1500 + 270 = 1770$$

(Reason: The total value at the end of 6 years is the \$1500 she put in plus the interest from every one of the 6 years.)

\$1770

Verification

✓ **Check 1:** Use the formula in one go, with $P = 1500$, $R = 3$ and $T = 6$. Cancel the 100 into the 1500 first, which leaves 15, and $3 \times 6 = 18$. $\frac{1500 \times 3 \times 6}{100} = 15 \times 18 = 270$, and $1500 + 270 = 1770$.

✓ **Check 2:** Over 6 years the interest adds up to $6 \times 3 = 18$ per cent of the \$1500, so the total value is 118% of it. Since 1% of 1500 is 15: $118 \times 15 = 1770$

✓ **Check 3:** Work backwards from the answer: take away the \$1500 to leave the interest, share it over the 6 years, and compare one year's interest with the \$1500.

$$1770 - 1500 = 270, \frac{270}{6} = 45 \text{ and } \frac{45}{1500} = \frac{3}{100}, \text{ which is } 3\% \text{ per year.}$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The total value at the end of 6 years	Note	The Answer column gives 1770, and the Marks column gives 3 for it.	✓
The interest for the 6 years	B2	For 270.	✓

Step	Mark	Description	Got it?
The total value: the alternative to the B2	M2	Or for $1500 + \frac{1500 \times 3 \times 6}{100}$.	✓
The interest calculation: the alternative to the B2 and the M2	M1	Or for $\frac{1500 \times 3[\times 6]}{100}$.	✓
Full marks: 3/3			

Question 4 - Exam Solution

Understanding the Question

Given

The calculation $\frac{5}{6} - \frac{2}{3} \times \frac{3}{8}$, which has no brackets.
No calculator: every step is done by hand.

Find

The value of $\frac{5}{6} - \frac{2}{3} \times \frac{3}{8}$.

Plan the Solution

- There are no brackets, so the order of operations (BIDMAS) says the multiplication $\frac{2}{3} \times \frac{3}{8}$ is done before the subtraction.
- Multiply the two fractions: numerator times numerator, and denominator times denominator.
- Write $\frac{5}{6}$ over the same denominator as the product, subtract the numerators, then simplify.

Worked Solution [3 marks]

Rule - Order of operations with fractions: multiply before subtracting. To multiply, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$. To subtract, write both fractions over a common denominator first, then subtract the numerators.

Step 1: Do the multiplication first

$$\frac{2}{3} \times \frac{3}{8} = \frac{2 \times 3}{3 \times 8} = \frac{6}{24}$$

(Reason: Multiplication comes before subtraction, so $\frac{2}{3} \times \frac{3}{8}$ is worked out first: multiply the numerators together and the denominators together. Leave $\frac{6}{24}$ as it is for now, because 24 is a multiple of 6, which makes the subtraction easier. Working from left to right instead gives $\frac{5}{6} - \frac{2}{3} = \frac{1}{6}$ and then $\frac{1}{6} \times \frac{3}{8} = \frac{1}{16}$, which is the wrong answer the mark scheme's SC1 names.)

Step 2: Write $\frac{5}{6}$ over the denominator 24

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

(Reason: The smallest number that both 6 and 24 divide into is 24, because $6 \times 4 = 24$. Multiplying the top and the bottom of $\frac{5}{6}$ by the same number, 4, changes how the fraction looks but not its value.)

Step 3: Subtract the numerators

$$\frac{20}{24} - \frac{6}{24} = \frac{14}{24}$$

(Reason: Both fractions are now in twenty-fourths, so take the numerators away, $20 - 6 = 14$, and keep the denominator 24.)

Step 4: Simplify

$$\frac{14}{24} = \frac{7 \times 2}{12 \times 2} = \frac{7}{12}$$

(Reason: Both 14 and 24 are even, so the common factor 2 cancels from the top and the bottom. 7 and 12 have no common factor other than 1, so $\frac{7}{12}$ is in its simplest form.)

$$\frac{7}{12}$$

Verification

✓ **Check 1:** Cancel before multiplying: the 3 on top of $\frac{3}{8}$ cancels with the 3 underneath $\frac{2}{3}$.

Then subtract over the common denominator 12 instead of 24. $\frac{2}{3} \times \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$, then

$$\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12} = \frac{7}{12}$$

✓ **Check 2:** Adding undoes subtracting, so adding the product back on to the answer must give $\frac{5}{6}$. The product $\frac{6}{24}$ is $\frac{1}{4}$, which is $\frac{3}{12} \cdot \frac{7}{12} + \frac{3}{12} = \frac{10}{12} = \frac{5}{6}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The value of the calculation	Note	The Answer column gives $\frac{7}{12}$ oe, and the Marks column gives 3 for it.	✓
The product of the two fractions	M1	For $\frac{6}{24}$ oe.	✓
The subtraction over a common denominator	M1	For correct use of common denominator in subtraction $\frac{5}{6} - \text{their } \frac{6}{24}$, e.g. $\frac{20}{24}$ and $\frac{6}{24}$ oe.	✓
Special case: answer $\frac{1}{16}$	SC1	If 0 scored, for answer $\frac{1}{16}$ oe.	✓

Full marks: 3/3

Question 5 - Exam Solution

Understanding the Question

Given

A regular polygon, so all of its sides are equal and all of its interior angles are equal.
Each interior angle is 150° .

Find

The number of sides of the polygon, n .

Plan the Solution

- At each corner the interior angle and the exterior angle sit on a straight line, so one exterior angle is 180° take away the interior angle.
- The exterior angles of any polygon add up to 360° . In a regular polygon they are all equal, so 360° divided by one exterior angle gives the number of sides.

Worked Solution [2 marks]

$$\text{Rule - Exterior angles of a regular polygon: number of sides} = \frac{360^\circ}{\text{one exterior angle}}$$

Find one exterior angle

$$180 - 150 = 30$$

(Reason: The interior and exterior angles at a corner make a straight line, and angles on a straight line add up to 180° , so each exterior angle is 30° .)

Divide 360° by one exterior angle

$$\frac{360}{180 - 150} = \frac{360}{30} = 12$$

(Reason: Going once round the polygon turns through 360° in total, a turn of 30° at every corner. That takes 12 corners, and a polygon has as many sides as corners.)

12 sides

Verification

- ✓ **Check 1 - the interior angle sum:** The interior angles of a 12-sided polygon add up to $(12 - 2) \times 180 = 1800$ degrees. Shared equally between its 12 angles, each one is $\frac{1800}{12} = 150$ degrees, the interior angle given.
- ✓ **Check 2 - the interior angle equation:** Set the interior angle formula equal to 150 and solve: $\frac{180(n - 2)}{n} = 150$ gives $180n - 360 = 150n$, so $30n = 360$. $n = 12$, the same number of sides.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The number of sides	Note	The Answer column gives 12, and the Marks column gives 2 for it.	✓
Method for the number of sides	M1	For $\frac{360}{180 - 150}$ or for $\frac{180(n - 2)}{n} = 150$.	✓

Full marks: 2/2

Question 6 - Exam Solution

Understanding the Question

Given

The line $x + y = 7$ is already drawn on a grid numbered from 0 to 8 across and from 0 to 9 up.

Part (a) adds a second line, $y = 2x + 1$, to the same grid.

Find

(a) The line $y = 2x + 1$, drawn on the grid. (b) The values of x and y that make both equations true at once, read from the graph.

Plan the Solution

- (a) Choose some values of x , work out $y = 2x + 1$ for each one, then plot the points and rule one straight line through them.
- The grid stops at $y = 9$, so use x from 0 to 4: over that range y runs from 1 up to 9.
- (b) Every point on a line fits that line's equation, so the one point that is on both lines fits both equations. Read off the coordinates of the point where the lines cross.

Worked Solution [3 marks]

Rule - Solving simultaneous equations with a graph: draw both lines on the same axes. The point where they cross is the solution, its x -coordinate giving x and its y -coordinate giving y .

Step 1: work out five points on $y = 2x + 1$

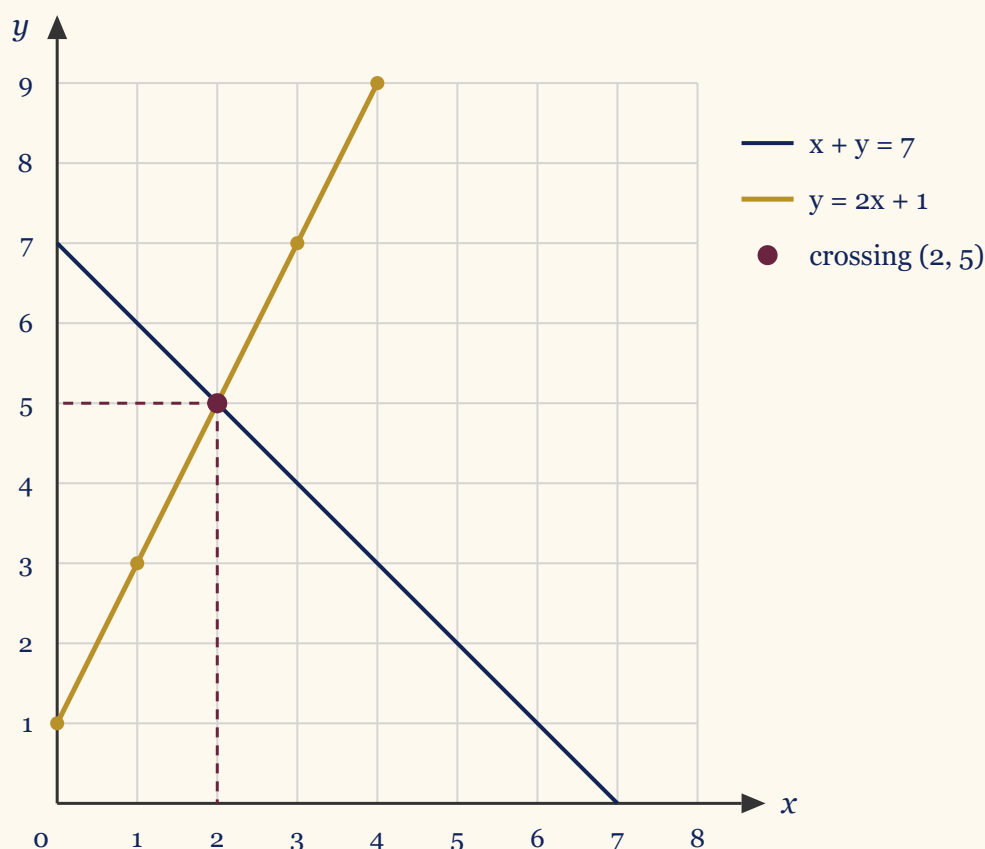
$$2 \times 0 + 1 = 1$$

$$2 \times 1 + 1 = 3$$

$$2 \times 2 + 1 = 5$$

$$2 \times 3 + 1 = 7$$

$$2 \times 4 + 1 = 9$$



(Reason: Put each chosen value of x into $2x + 1$: multiply it by 2, then add 1. The lines take $x = 0, 1, 2, 3$ and 4 in turn, and each answer is the y -value that goes with that x .)

Step 2: plot the points and rule the line

x	0	1	2	3	4
y	1	3	5	7	9

(Reason: Plot $(0, 1)$, $(1, 3)$, $(2, 5)$, $(3, 7)$ and $(4, 9)$, then join them with a ruler right across the grid, from the y -axis to the top edge. The points must lie in one straight line, which is the check that none of them is wrong: each step of 1 square to the right goes 2 squares up, so the gradient is 2, and the line meets the y -axis at 1. It is the gold line in the figure at Step 3.)

Step 3: read off where the two lines cross

$$x = 2$$

$$y = 5$$

(Reason: The gold line crosses the navy line $x + y = 7$ at a grid point two squares to the right of the y -axis and five squares up. The dashed lines from that point, down to the x -axis and across to the y -axis, give the two readings.)

(a) the line $y = 2x + 1$, ruled from $(0, 1)$ to $(4, 9)$

(b) $x = 2, y = 5$

Verification

- ✓ **Check 1:** Test the drawn line against the mark scheme's description. From $(0, 1)$ to $(4, 9)$ it rises 8 squares while it runs 4 across. $\frac{8}{4} = 2$, so the gradient is 2, and the line meets the y -axis at 1
- ✓ **Check 2:** Put $x = 2$ and $y = 5$ back into the first equation, $x + y = 7$. $2 + 5 = 7$
- ✓ **Check 3:** Put the same values into the second equation, $y = 2x + 1$. $2 \times 2 + 1 = 5$, which is the value of y
- ✓ **Check 4:** Solve the pair without the graph. Replace y in the first equation with $2x + 1$. $x + 2x + 1 = 7$, so $3x = 6$, giving $x = 2$ and then $y = 2 \times 2 + 1 = 5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The line drawn	Note	The Answer column gives "Correct line drawn", and the Marks column gives 2 for it.	✓
(a) Partial Marks	M1	For a line with gradient 2 or for a line with positive gradient and intercept at $y = 1$.	✓
(b) The solution	Note	The Answer column gives $[x =] 2, [y =] 5$, and the Marks column gives 1 for it.	✓
(b) Follow through	Note	FT the intersection of their (a) with the given line.	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

Find

The recurring decimal $0.2\dot{6}$, whose dot sits over the 6 alone, so only the 6 repeats

The decimal written as a fraction in its simplest form

Plan the Solution

- Call the decimal x .
- Multiply it by 10 and by 100, which gives two numbers with exactly the same recurring part after the point.
- Take the smaller from the larger. The recurring parts cancel and a whole number is left.
- Divide to get a fraction, then cancel it until nothing more will cancel.

Worked Solution [3 marks]

Rule - shift, then subtract: multiply a recurring decimal by two powers of 10 that leave the same recurring tail after the point. Subtracting one line from the other removes the tail, and dividing turns what is left into a fraction.

Step 1: Give the decimal a name

$$x = 0.2\dot{6} = 0.2666\dots$$

(Reason: The dot is over the 6 only, so the 2 appears once and every digit after it is a 6. Calling the decimal x lets it be handled with algebra.)

Step 2: Multiply by 10 and by 100

$$10x = 2.\dot{6}$$

$$100x = 26.\dot{6}$$

(Reason: One digit comes before the repeating part, so multiplying by 10 moves the point just past it. The repeating part is one digit long, so multiplying by 100 moves the point one place further. Both lines now end in the same tail, $0.666\dots$, after the point.)

Step 3: Subtract to clear the recurring tail

$$100x - 10x = 26.\dot{6} - 2.\dot{6}$$

$$90x = 24$$

(Reason: The two tails match digit for digit, so they cancel exactly and only the whole numbers are left on the right, 26 take away 2. On the left, $100x - 10x$ collects to $90x$.)

Step 4: Divide, then cancel to the simplest form

$$x = \frac{24}{90}$$

$$\frac{24}{90} = \frac{4}{15}$$

(Reason: Dividing both sides by 90 gives the fraction. The highest common factor of 24 and 90 is 6, because $24 = 6 \times 4$ and $90 = 6 \times 15$. After dividing it out, $4 = 2 \times 2$ and $15 = 3 \times 5$ have no factor in common, so the fraction is in its simplest form.)

$$\frac{4}{15}$$

Verification

- ✓ **Check 1 - put the fraction back into the subtraction:** The subtraction ends at $90x = 24$, so multiplying the answer by 90 has to give 24 again. $\frac{4}{15} \times 90 = 24$
- ✓ **Check 2 - divide it out by hand:** Short division of 4 by 15: 15 goes into 40 twice with 10 left over, then into 100 six times with 10 left over again. The same remainder keeps coming back, so every digit after that is a 6. The division gives $0.2666\dots$, which is $0.2\dot{6}$
- ✓ **Check 3 - split the decimal into two parts:** Write the decimal as 0.2 plus $0.0\dot{6}$. The first part is $\frac{2}{10}$. The second is one tenth of $0.\dot{6}$, and a single repeating digit is that digit over 9, so it is one tenth of $\frac{6}{9}$, which is $\frac{6}{90}$. $\frac{2}{10} + \frac{6}{90} = \frac{18}{90} + \frac{6}{90} = \frac{24}{90} = \frac{4}{15}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The fraction in its simplest form	Note	The Answer column gives $\frac{4}{15}$, and the Marks column gives 3 for it.	✓
The right fraction, not yet cancelled	B2	For $\frac{24}{90}$ oe.	✓
A subtraction, an equation or a sum that leads to that fraction	M1	Or for $26.66\dots - 2.66\dots$ oe, or for $90x = 24$ oe, or for $\frac{2}{10} + \frac{6}{90}$ oe.	✓
Where the B2 value comes from	Note	The $\frac{24}{90}$ in the B2 row is $\frac{4}{15}$ before it is cancelled.	✓

Full marks: 3/3

Question 8 - Exam Solution

Understanding the Question

Given

The column vectors $\mathbf{m} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$ and

$$\mathbf{n} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

(b) The vector $\begin{pmatrix} 5 \\ \sqrt{y} \end{pmatrix}$ has a magnitude of 7

Find

(a) The column vector $2\mathbf{m} - \mathbf{n}$ (b) The value of y

Plan the Solution

- (a) Double each component of $\mathbf{m}$, then take away each component of $\mathbf{n}$, top from top and bottom from bottom.
- (b) The magnitude is the length of the vector. Its two components are the two shorter sides of a right-angled triangle, so Pythagoras' theorem links them to the length 7.
- Square both sides so that no square root is left, then solve for y .

Worked Solution [4 marks]

Rule - Vectors work component by component: a number multiplies every component, and subtracting pairs the top entries together and the bottom entries together. The magnitude of $\begin{pmatrix} a \\ b \end{pmatrix}$ is $\sqrt{a^2 + b^2}$.

Step 1: Double $\mathbf{m}$

$$2\mathbf{m} = 2 \begin{pmatrix} 11 \\ 5 \end{pmatrix} = \begin{pmatrix} 22 \\ 10 \end{pmatrix}$$

(Reason: Multiplying a vector by 2 multiplies each of its components by 2: $2 \times 11 = 22$ on top and $2 \times 5 = 10$ underneath.)

Step 2: Take $\mathbf{n}$ away, one component at a time

$$2\mathbf{m} - \mathbf{n} = \begin{pmatrix} 22 \\ 10 \end{pmatrix} - \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 22 - 8 \\ 10 - (-3) \end{pmatrix} = \begin{pmatrix} 14 \\ 13 \end{pmatrix}$$

(Reason: The top entry of $\mathbf{n}$ comes off the top entry of $2\mathbf{m}$, and the bottom entry off the bottom entry. The bottom entry of $\mathbf{n}$ is negative, and taking away a negative number is the same as adding the positive one, so $10 - (-3)$ is $10 + 3$.)

Step 3: Write the magnitude with Pythagoras' theorem

$$\sqrt{5^2 + (\sqrt{y})^2} = 7$$

(Reason: Drawn as an arrow, the vector goes 5 across and $\sqrt{y}$ up. Those two moves are the shorter sides of a right-angled triangle, and the arrow itself is the hypotenuse, whose length is the magnitude 7.)

Step 4: Square both sides and solve for y

$$5^2 + (\sqrt{y})^2 = 7^2$$

$$25 + y = 49$$

$$y = 49 - 25 = 24$$

(Reason: Squaring both sides removes the outer square root. Squaring $\sqrt{y}$ gives back y , and $5^2 = 25$ and $7^2 = 49$, so taking 25 from both sides leaves y . The square root of y only exists when y is at least 0, and 24 is.)

$$\text{(a)} \begin{pmatrix} 14 \\ 13 \end{pmatrix}$$

$$\text{(b)} y = 24$$

Verification

✓ **Check 1 - (a) add $\mathbf{n}$ back on:** If the answer is $2\mathbf{m} - \mathbf{n}$, then adding $\mathbf{n}$ to it has to give

$$2\mathbf{m} \text{ again. } \begin{pmatrix} 14 \\ 13 \end{pmatrix} + \begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 22 \\ 10 \end{pmatrix}, \text{ which is } 2\mathbf{m}$$

✓ **Check 2 - (a) group it a different way:** $2\mathbf{m} - \mathbf{n}$ is also $\mathbf{m} + (\mathbf{m} - \mathbf{n})$. First

$$\mathbf{m} - \mathbf{n} = \begin{pmatrix} 11 - 8 \\ 5 + 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}, \text{ then add } \mathbf{m} \text{ to it. } \begin{pmatrix} 11 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 14 \\ 13 \end{pmatrix}$$

✓ **Check 3 - (b) put $y = 24$ back in:** With $y = 24$ the vector is $\begin{pmatrix} 5 \\ \sqrt{24} \end{pmatrix}$. Work out its

$$\text{magnitude from the start. } \sqrt{5^2 + (\sqrt{24})^2} = \sqrt{25 + 24} = \sqrt{49} = 7$$

✓ **Check 4 - (b) difference of two squares:** From $y + 5^2 = 7^2$, the value of y is $7^2 - 5^2$, and a difference of two squares factorises, so neither square has to be worked out.
 $7^2 - 5^2 = (7 - 5)(7 + 5) = 2 \times 12 = 24$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The column vector $2\mathbf{m} - \mathbf{n}$	Note	The Answer column gives $\begin{pmatrix} 14 \\ 13 \end{pmatrix}$, and the Marks column gives 2 for it.	✓
(a) One entry right, or $2\mathbf{m}$	B1	For $\begin{pmatrix} 14 \\ k \end{pmatrix}$, or for $\begin{pmatrix} k \\ 13 \end{pmatrix}$, or for $\begin{pmatrix} 22 \\ 10 \end{pmatrix}$.	✓
(a) Where the B1 forms come from	Note	In the first two forms one entry matches the answer, 14 on top or 13 underneath, with k in the other place. The third form, $\begin{pmatrix} 22 \\ 10 \end{pmatrix}$, is $2\mathbf{m}$ before $\mathbf{n}$ is taken away.	✓
(b) The value of y	Note	The Answer column gives 24, and the Marks column gives 2 for it.	✓
(b) Pythagoras' theorem with the magnitude	M1	For $y + 5^2 = 7^2$ or better.	✓
(b) Why the M1 has y and no square root	Note	The y in the M1 form is $(\sqrt{y})^2$, the square of the vector's lower entry, so the square root has already gone.	✓

Full marks: 4/4

Question 9 - Exam Solution

Understanding the Question

Given

A frequency table of quiz scores: the scores 4, 5 and 8 have frequencies 2, 4

Find

The value of n , the number of friends who scored 8

and n

The mean of the scores is 6. No calculator: every step is done by hand.

Plan the Solution

- The total of all the scores is each score times its frequency, added up:
 $2 \times 4 + 4 \times 5 + n \times 8$.
- The number of friends is the total of the frequencies, $2 + 4 + n$.
- The total divided by the number of friends is the mean, so set that fraction equal to 6.
- Multiply both sides by the number of friends to clear the fraction, then solve the linear equation that is left.

Worked Solution [3 marks]

Rule - mean from a frequency table: $\text{mean} = \frac{\text{total of all the scores}}{\text{number of people}}$, where the total is each score times its frequency, added up, and the number of people is the total of the frequencies.

Step 1: Write the mean as a fraction

$$\text{mean} = \frac{2 \times 4 + 4 \times 5 + n \times 8}{2 + 4 + n}$$
$$\frac{2 \times 4 + 4 \times 5 + n \times 8}{2 + 4 + n} = 6$$

(Reason: Each score counts as many times as its frequency: 2 friends scored 4, 4 scored 5 and n scored 8, so the top is the total of all the scores. The bottom adds up the frequencies, which counts the friends. The mean is given as 6.)

Step 2: Simplify the top and the bottom

$$2 \times 4 + 4 \times 5 + n \times 8 = 8 + 20 + 8n = 28 + 8n$$
$$2 + 4 + n = 6 + n$$

(Reason: $2 \times 4 = 8$ and $4 \times 5 = 20$, while $n \times 8$ stays as $8n$ because n is not known yet. Collecting the numbers gives 28 on the top and 6 on the bottom.)

Step 3: Multiply both sides by the number of friends

$$8 + 20 + 8n = 6(2 + 4 + n)$$
$$28 + 8n = 36 + 6n$$

(Reason: The total divided by the number of friends gives the mean, so the total equals the mean times the number of friends. Multiplying both sides by $2 + 4 + n$ clears the fraction. On the right the 6

multiplies every term in the bracket: $6 \times 2 + 6 \times 4 + 6 \times n = 12 + 24 + 6n$.)

Step 4: Solve for n

$$8n - 6n = 36 - 28$$

$$2n = 8$$

$$n = 4$$

(Reason: Taking $6n$ and 28 from both sides puts the n terms on the left and the numbers on the right. Dividing both sides by 2 then leaves n on its own.)

$$n = 4$$

Verification

✓ **Check 1 - put $n = 4$ back into the table:** With 4 friends scoring 8, the total of the scores is $8 + 20 + 4 \times 8$ and there are $2 + 4 + 4$ friends. Dividing the total by the number of friends must give the mean. $\frac{8 + 20 + 32}{2 + 4 + 4} = \frac{60}{10} = 6$

✓ **Check 2 - balance the scores around the mean:** At the mean, the amounts by which scores fall below it and rise above it are equal. Each score of 4 is 2 below 6 and each score of 5 is 1 below; each score of 8 is 2 above, and there are 4 of them. Below: $2 \times 2 + 4 \times 1 = 8$. Above: $4 \times 2 = 8$. The two balance.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The value of n	Note	The Answer column gives 4 nfw, and the Marks column gives 3 for it.	✓
The equation for the total of the scores	M2	For $8 + 20 + 8n = 6(2 + 4 + n)$ or better.	✓
The mean as a fraction, or one side of the equation, instead of the M2	M1	Or for $\frac{2 \times 4 + 4 \times 5 + n \times 8}{2 + 4 + n} [= 6]$ oe, or $8 + 20 + 8n$ or $6(2 + 4 + n)$.	✓
Where the M2 equation comes from	Note	$8 + 20 + 8n$ is the total of all the scores, $2 \times 4 + 4 \times 5 + n \times 8$, and $6(2 + 4 + n)$ is the	✓

Step	Mark	Description	Got it?
		mean times the number of people, so the equation says that the two ways of finding the total agree.	
What nfww means	Note	nfww is the scheme's abbreviation for not from wrong working: the answer 4 must not come from wrong working.	✓
Full marks: 3/3			

Question 10 - Exam Solution

Understanding the Question

Given

A , B and C lie on a circle with centre O , so $OA = OB = OC$, because all three are radii

The line DE is the tangent to the circle at A

Angle $ACO = 35^\circ$ and angle $BCO = 40^\circ$

The diagram is not to scale, so no angle can be measured off it, and there is no calculator: every step is done by hand.

Find

(a) Angle AOC (b) Angle ABC (c) Angle DAC (d) Angle OAB

Plan the Solution

- (a) OA and OC are radii, so triangle AOC is isosceles. Its base angles are equal, and the angle sum of the triangle gives the angle at O .
- (b) Angle AOC at the centre and angle ABC at the circumference stand on the same arc AC , so halve the answer to (a).
- (c) Angle DAC lies between the tangent and the chord AC , so the alternate segment theorem makes it equal to angle ABC .
- (d) Add the two marked angles to get angle ACB , use the angle sum of triangle ABC to find angle BAC , then take away angle OAC from (a).

Worked Solution [4 marks]

Rule - Circle theorems: two radii make an isosceles triangle; the angle at the centre is twice the angle at the circumference standing on the same arc; the angle between a tangent and a chord equals the angle in the alternate segment.

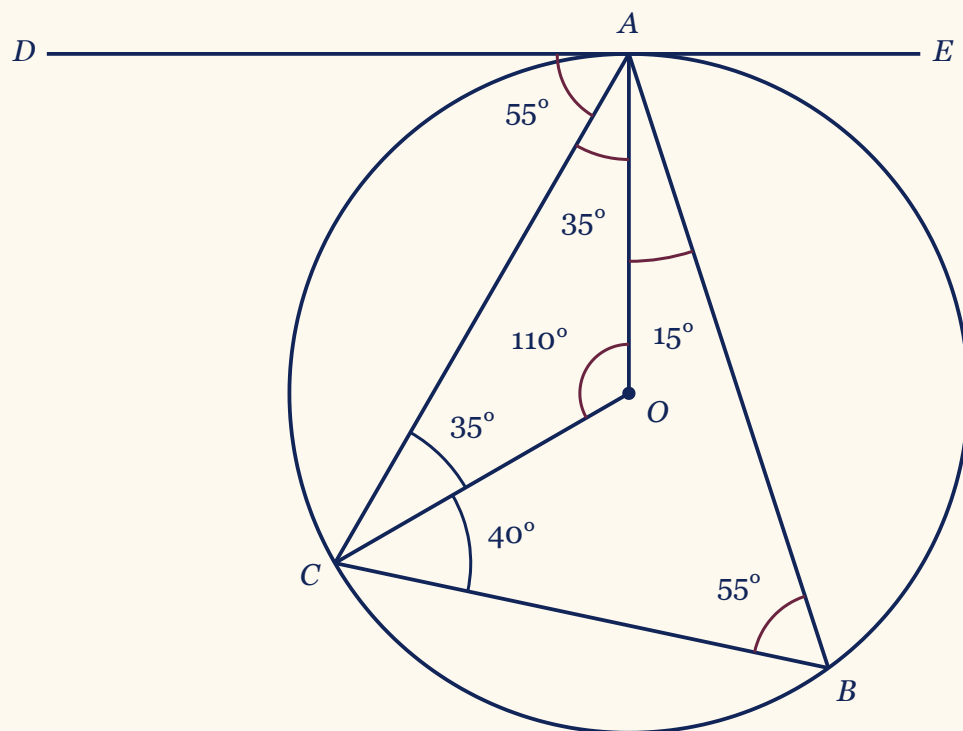
Step 1: (a) Angle AOC from the isosceles triangle AOC

$$OA = OC$$

$$OAC = OCA = 35^\circ$$

$$AOC = 180 - 2 \times 35$$

$$180 - 70 = 110$$



NOT TO
SCALE

(Reason: Both OA and OC are radii, so they are the same length and triangle AOC is isosceles. The angles opposite the equal sides are equal, so angle OAC is also 35° . The three angles of a triangle add to 180° , and the two base angles take 70° of that, which leaves 110° at O .)

Step 2: (b) Angle ABC , half the angle at the centre

$$ABC = \frac{1}{2} \times AOC$$

$$\frac{110}{2} = 55$$

(Reason: Angle AOC is at the centre and angle ABC is at the circumference, and both stand on the same arc AC , the arc that does not contain B . The angle at the centre is twice the angle at the circumference, so angle ABC is half of the 110° found in (a).)

Step 3: (c) Angle DAC , by the alternate segment theorem

$$DAC = ABC$$

$$DAC = 55^\circ$$

(Reason: The line DA is part of the tangent at A , and AC is a chord. The angle between a tangent and a chord equals the angle in the alternate segment: the angle the chord makes at any point of the circle on the other side of AC from D . B is such a point, so angle DAC equals angle ABC , which is 55° from (b).)

Step 4: (d) Angle OAB , from the angles of triangle ABC

$$ACB = ACO + BCO$$

$$35 + 40 = 75$$

$$BAC = 180 - ACB - ABC$$

$$180 - 75 - 55 = 50$$

$$OAB = BAC - OAC$$

$$50 - 35 = 15$$

(Reason: The radius OC splits angle ACB into the two marked angles, so angle ACB is 75° . The angles of triangle ABC add to 180° , which leaves 50° for angle BAC . The radius OA splits angle BAC into angle OAC , which is 35° from step 1, and angle OAB , the angle wanted. The figure at this step marks every angle the four parts found.)

$$(a) AOC = 110^\circ$$

$$(b) ABC = 55^\circ$$

$$(c) DAC = 55^\circ$$

$$(d) OAB = 15^\circ$$

Verification

- ✓ **Check 1 - (c) by the tangent and the radius:** The radius OA meets the tangent at A at a right angle, so angle DAO is 90° . C is on the same side of OA as D , so angle DAC is what is left of that right angle after angle OAC . $90 - 35 = 55$, the same as (c)
- ✓ **Check 2 - (b) and (d) from the three triangles at O :** Join OB . Triangle OBC is isosceles too, so angle OBC is 40° and angle BOC is $180 - 80 = 100$. The three angles at O make a full turn, so angle AOB is $360 - 110 - 100 = 150$, and the isosceles triangle OAB shares what is left of its angle sum equally between its two base angles.

$$\frac{180 - 150}{2} = 15$$
for angle OAB and for angle OBA , so angle ABC is $15 + 40 = 55$
- ✓ **Check 3 - (d) by the tangent at A :** The angle between the tangent AE and the chord AB equals the angle in the alternate segment, angle ACB , which is 75° . Angle OAE is a

right angle, and angle OAB is what is left of it. $90 - 75 = 15$, the same as (d)

✓ **Check 4 - the angle sum of triangle ABC :** Angle BAC is $35 + 15$, angle ABC is 55 and angle ACB is $35 + 40$. The three must add to 180 . $50 + 55 + 75 = 180$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Angle AOC	Note	The Answer column gives 110, and the Marks column gives 1 for it.	✓
(b) Angle ABC	Note	The Answer column gives 55, and the Marks column gives 1 for it.	✓
(b) Follow-through from (a)	Note	FT $\frac{\text{their } 110}{2}$.	✓
(c) Angle DAC	Note	The Answer column gives 55, and the Marks column gives 1 for it.	✓
(c) Follow-through from (b)	Note	FT their (b).	✓
(c) The condition on that follow-through	Note	Provided their (b) < 90 .	✓
(d) Angle OAB	Note	The Answer column gives 15, and the Marks column gives 1 for it.	✓
(d) Follow-through from (b)	Note	FT $70 - \text{their (b)}$.	✓
(d) The condition on that follow-through	Note	Provided their (b) < 70 .	✓

Step	Mark	Description	Got it?
What FT and their mean	Note	FT is the scheme's abbreviation for follow through after error. Their (b) means the candidate's own answer to (b), and their 110 means the candidate's own answer to (a), used where the 110 would be.	✓
Where the 70 comes from	Note	The angle sum of triangle ABC , less angle ACB (the two marked angles at C) and less angle OAC (equal to angle OCA), leaves 70 for angle OAB and angle ABC together. So angle OAB is 70 minus angle ABC , which is why (d) follows through from (b).	✓
Full marks: 4/4			

Question 11 - Exam Solution

Understanding the Question

Given

The graph of $y = f(x)$, drawn on a grid with x from -2 to 3 and y from -10 to 15 .

The point $P(-2, 11)$, marked on the left edge of the grid, above the curve.

The tangent from P touches the curve at (a, b) , where a and b are integers.

Find

(a) The values of a and b , by drawing the tangent. (b) The equation of the tangent, in the form $y = mx + c$.

Plan the Solution

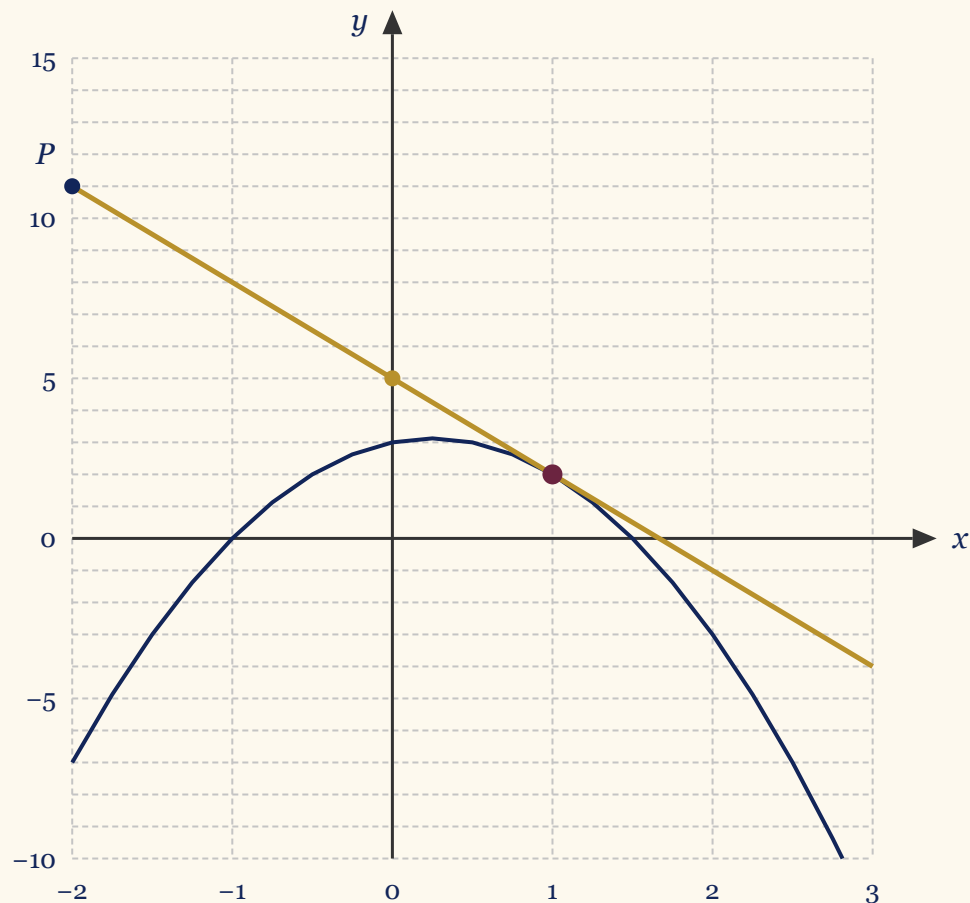
- (a) Lay a ruler through P and turn it until it just touches the curve: it meets the curve at one point, with the whole curve on one side of it. Rule the line and read off the point where it touches.
- (b) The gradient m is rise over run between two points on the ruled line, P and the touching point. Then put the touching point into $y = mx + c$ to find c , and check it against where the line crosses the y -axis.

Worked Solution [5 marks]

Rule - Straight line through two points: $y = mx + c$, where the gradient $m = \frac{\text{rise}}{\text{run}}$ and c is where the line crosses the y -axis. A tangent touches a curve at one point without crossing it there.

Step 1: rule the tangent from P and find where it touches

$$(a, b) = (1, 2)$$



- tangent $y = 5 - 3x$
- touches the curve at $(1, 2)$
- crosses the y -axis at $(0, 5)$

(Reason: Put the ruler's edge on P and turn it about P until it rests against the curve, touching it at one point with the whole curve below the ruler. Rule that line. It touches the curve at the grid point where $x = 1$ and $y = 2$, which fits the question's clue that a and b are integers. The solution figure shows the ruled line in gold.)

Step 2: write down a and b

$$a = 1$$

$$b = 2$$

(Reason: The touching point is (a, b) , so a is its x -coordinate and b is its y -coordinate. A correct ruled tangent together with both values earns both marks for (a).)

Step 3: find the gradient m

$$\frac{2 - 11}{1 - (-2)} = \frac{-9}{3} = -3$$

$$m = -3$$

(Reason: Use rise over run from $P(-2, 11)$ to the touching point $(1, 2)$. The rise is the change in y and the run is the change in x , both taken in the same order. The line falls from left to right, so its gradient is negative.)

Step 4: find the y -intercept c

$$2 = -3 \times 1 + c$$

$$c = 2 + 3 = 5$$

(Reason: Put the touching point $(1, 2)$ into $y = -3x + c$ and solve for c . On the graph the ruled line crosses the y -axis at 5, which agrees.)

Step 5: write the equation

$$y = -3x + 5$$

$$y = 5 - 3x$$

(Reason: Put $m = -3$ and $c = 5$ into $y = mx + c$. The two lines are the same equation, and the mark scheme writes it as $5 - 3x$.)

$$(a) a = 1, b = 2$$

$$(b) y = 5 - 3x$$

Verification

- ✓ **Check 1:** Put $x = -2$ into $y = 5 - 3x$. The tangent must pass through P .
 $5 - 3 \times (-2) = 11$, the y -coordinate of P
- ✓ **Check 2:** Put $x = 1$ into $y = 5 - 3x$. The tangent must pass through the touching point.
 $5 - 3 \times 1 = 2$, which is b
- ✓ **Check 3:** Compare the line with the curve one unit either side of the touching point. The curve passes through the grid points $(0, 3)$ and $(2, -3)$. $5 - 3 \times 0 = 5$ and $5 - 3 \times 2 = -1$, so the line is above the curve on both sides: it touches at $(1, 2)$ without crossing

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The tangent and the values	Note	The Answer column gives “For correct ruled tangent and $[a =] 1$, $[b =] 2$ ”, and the Marks column gives 2 for it.	✓
(a) Partial Marks	B1	For correct ruled tangent or both values correct without a correct tangent.	✓
(b) The equation	Note	The Answer column gives $[y =] 5 - 3x$, and the Marks column gives 3 for it.	✓
(b) Full marks FT their (a)	3FT	Full marks FT their (a) provided $m < 0$, $c \neq 0$.	✓
(b) Without the 3FT: the gradient term	B1	Otherwise, for (their -3) $x + c$.	✓
(b) Without the 3FT: the gradient term, by rise over run	M1	Or for correct $\frac{\text{rise}}{\text{run}}$ for their line.	✓
(b) Without the 3FT: the intercept	B1	For $mx + c$ where c is the correct intercept for their graph, $m \neq 0$.	✓

Full marks: 5/5

Question 12 - Exam Solution

Understanding the Question

Given

A cumulative frequency curve for 120 office workers, showing the time each of them spent working at a computer on one day.

Time runs from 0 to 10 hours along the bottom, and cumulative frequency runs from 0 to 120 up the side.

One small square of the grid is 0.2 hours across and 2 workers up.

Find

(a) An estimate of the interquartile range, in hours. (b) An estimate of k .

(b) 70% of the workers spent less than k hours at a computer.

Plan the Solution

- (a) The lower quartile is a quarter of the way through the 120 workers, and the upper quartile three quarters of the way. Work out those two positions, read each one across to the curve and down to the time axis, then subtract.
- (b) Turn 70% into a number of workers, then read across from that cumulative frequency to the curve and down in the same way.

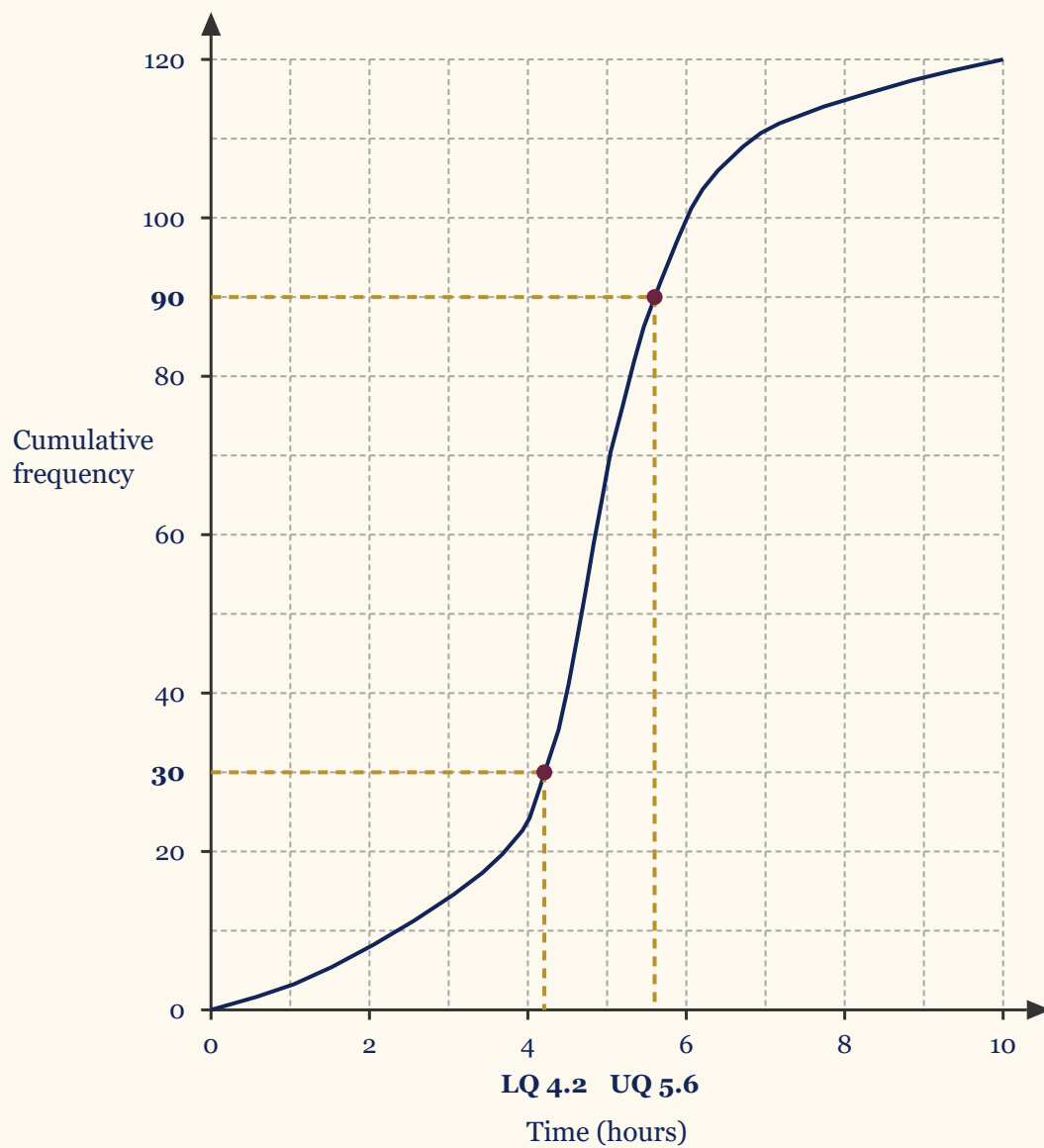
Worked Solution [4 marks]

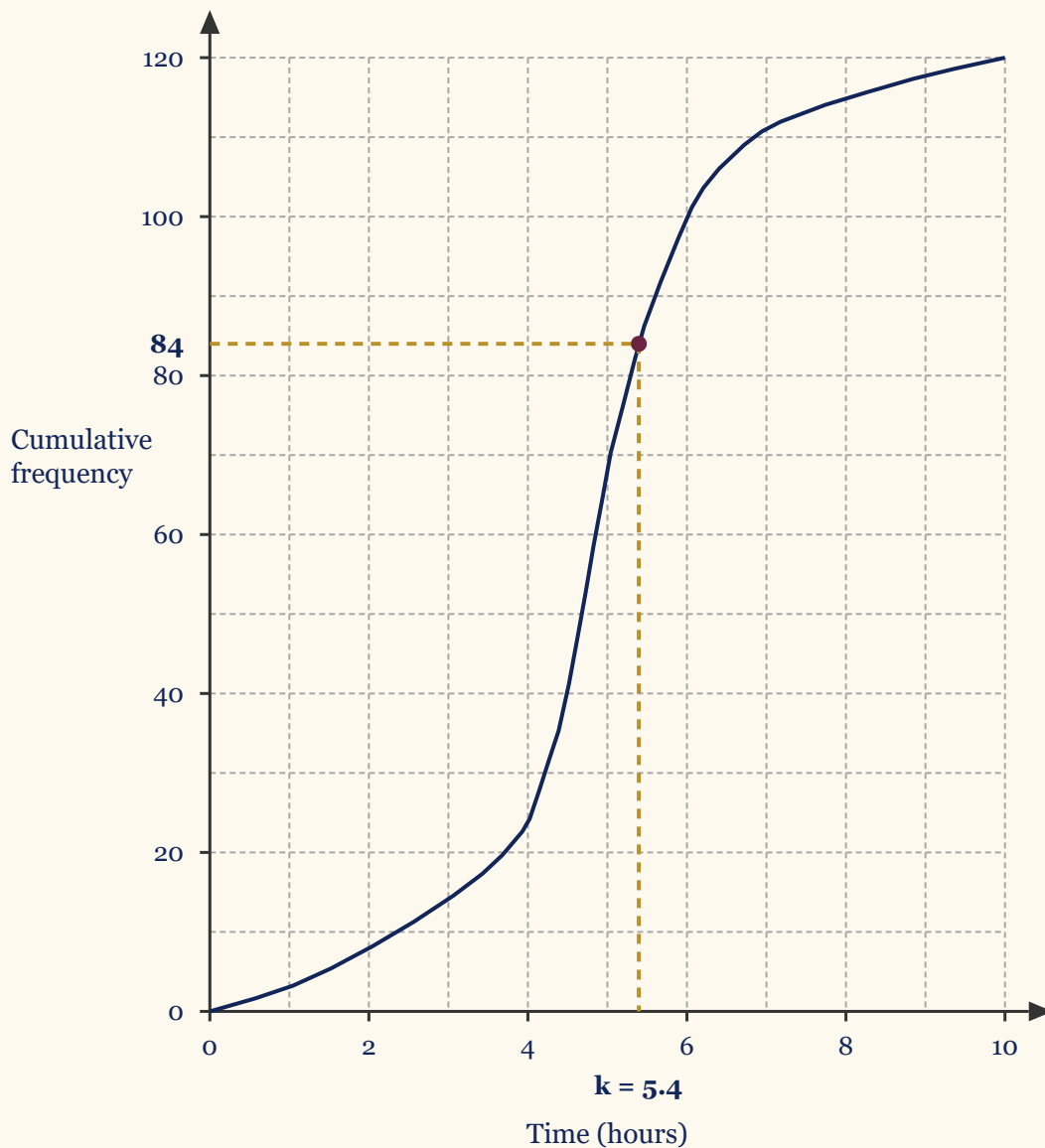
Rule - Quartiles from a cumulative frequency curve: for n values, read the lower quartile at $\frac{1}{4}n$ and the upper quartile at $\frac{3}{4}n$ on the cumulative frequency axis. Then
$$\text{IQR} = \text{UQ} - \text{LQ}.$$

Step 1: find where the quartiles are on the cumulative frequency axis

$$\frac{1}{4} \times 120 = 30$$

$$\frac{3}{4} \times 120 = 90$$





(Reason: The lower quartile belongs to the worker a quarter of the way up the ordered list, and the upper quartile to the worker three quarters of the way up. With 120 workers, these are at cumulative frequencies 30 and 90.)

Step 2: read both quartiles off the curve

$$LQ = 4.2$$

$$UQ = 5.6$$

(Reason: From 30 on the cumulative frequency axis, go across to the curve and straight down to the time axis: the line lands one small square past 4, at 4.2 hours. From 90 the line lands two small squares short of 6, at 5.6 hours. The solution figure shows both reading lines. Either quartile read correctly is worth a mark on its own.)

Step 3: subtract the lower quartile from the upper quartile

$$IQR = 5.6 - 4.2 = 1.4$$

(Reason: The interquartile range is the whole gap between the two quartiles, in hours. Halving it gives 0.7, which the mark scheme credits only with a special-case mark, SC1.)

Step 4: find 70% of the 120 workers

$$\frac{70}{100} \times 120 = 84$$

(Reason: The cumulative frequency axis counts workers, not percentages, and it runs up to 120, so 70% of the workers means 70% of 120.)

Step 5: read across from 84

$$k = 5.4$$

(Reason: From 84 on the cumulative frequency axis, go across to the curve and straight down: the line lands two small squares past 5, at 5.4 hours. So 84 workers, which is 70% of them, spent less than 5.4 hours at a computer. The second solution figure shows this reading.)

(a) 1.4 hours

(b) $k = 5.4$

Verification

- ✓ **Check 1:** (a) Count the small squares along the time axis between the two reading lines, from 4.2 to 5.6 hours. Each small square is 0.2 hours wide. $7 \times 0.2 = 1.4$ hours, the same interquartile range
- ✓ **Check 2:** (b) Work backwards from 5.4 hours: go up to the curve and across, and the line meets the cumulative frequency axis at 84. Then write 84 as a fraction of the 120 workers. $\frac{84}{120} = \frac{7}{10}$, which is 70%
- ✓ **Check 3:** (b) 70% is more than 25% and less than 75%, so k must lie between the lower and upper quartiles found in (a). $4.2 < 5.4 < 5.6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The interquartile range	Note	The Answer column gives 1.4, and the Marks column gives 2 for it.	✓
(a) One quartile correct	B1	For [UQ =] 5.6 or for [LQ =] 4.2	✓
(a) Special case, instead of the B1	SC1	Or for 0.7	✓
(b) The value of k	Note	The Answer column gives 5.4, and the Marks column gives 2 for it.	✓

Step	Mark	Description	Got it?
(b) 70% of the workers	B1	For 84 seen.	✓
Full marks: 4/4			

Question 13 - Exam Solution

Understanding the Question

Given

Solid *A*: a cone and a hemisphere, both of radius 6 cm, with the cone's sloping edge 10 cm long

Solid *B*: a cylinder of radius 4 cm and height *h* cm

The total surface areas of *A* and *B* are equal

No calculator is allowed, so every area stays a multiple of π , and the π cancels by hand

Find

(a) The value of *h* (b) The height of solid *A*, from the tip of the cone to the bottom of the hemisphere

Plan the Solution

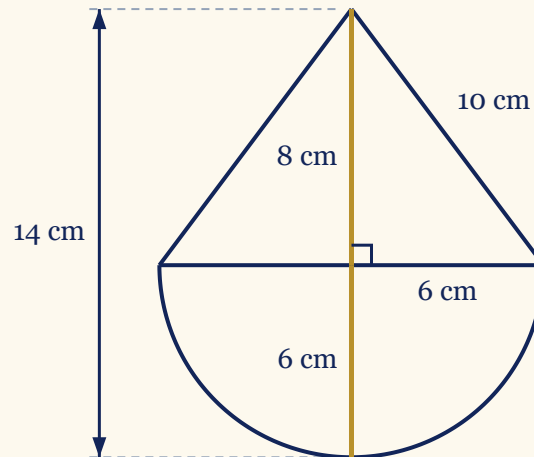
- (a) List the surfaces that make up each total. For *A*: the curved surface of the cone and the curved surface of the hemisphere, because the flat circle where they are joined is inside the solid. For *B*: two circular ends and the curved surface. Set the two totals equal and solve for *h*.
- (b) The height of *A* is the height of the cone plus the radius of the hemisphere. The cone's height, its radius and its sloping edge form a right-angled triangle, so Pythagoras gives the height of the cone.

Worked Solution [8 marks]

Rule - Surface areas: curved surface of a cone $\pi r l$; surface of a sphere $4\pi r^2$, so the curved surface of a hemisphere is $2\pi r^2$; curved surface of a cylinder $2\pi r h$; circle πr^2 .
Pythagoras: $a^2 + b^2 = c^2$.

Step 1: (a) the curved surface of the cone

$$\pi \times 6 \times 10 = 60\pi$$



(Reason: The sloping edge is the cone's slant height l , so $r = 6$ and $l = 10$ go into $\pi r l$. Only the curved surface counts: the cone's flat base is the circle where it is joined to the hemisphere.)

Step 2: (a) the curved surface of the hemisphere

$$\frac{1}{2} \times 4\pi r^2 = 2\pi r^2$$

$$2 \times \pi \times 6^2 = 72\pi$$

(Reason: A hemisphere is half of a sphere, and a sphere's surface is $4\pi r^2$, so the curved surface of the hemisphere is $2\pi r^2$. Its radius is the same 6 cm as the cone's. Its flat face is the same joined circle, inside the solid, so that does not count either.)

Step 3: (a) the total surface area of solid A

$$60\pi + 72\pi = 132\pi$$

(Reason: The two curved surfaces are the whole outside of solid A. The circle where the cone meets the hemisphere is covered on both sides, so it is not part of the surface.)

Step 4: (a) the total surface area of solid B

$$2 \times \pi \times 4^2 = 32\pi$$

$$2 \times \pi \times 4 \times h = 8\pi h$$

$$32\pi + 8\pi h$$

(Reason: A cylinder has two circular ends, each πr^2 , and a curved surface $2\pi r h$. With $r = 4$ the two ends make 32π and the curved surface is $8\pi h$. A total surface area includes both ends.)

Step 5: (a) set the two totals equal and solve

$$32\pi + 8\pi h = 132\pi$$

$$8\pi h = 100\pi$$

$$8h = 100$$

$$h = \frac{100}{8} = 12.5$$

(Reason: Take 32π from both sides, then divide both sides by π . Every term carries π , so its value is never needed. Leaving out the cylinder's two ends would give $8\pi h = 132\pi$ and the answer 16.5 instead, which is the special case in the mark scheme.)

Step 6: (b) the height of the cone, by Pythagoras

$$x^2 + 6^2 = 10^2$$

$$x = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$$

(Reason: Call the height of the cone x cm. It runs straight down the middle of the cone, at right angles to the radius, so the height, the radius 6 and the sloping edge 10 make a right-angled triangle with the sloping edge as its hypotenuse. By hand, $100 - 36 = 64$, and $64 = 8^2$.)

Step 7: (b) the height of solid A

$$8 + 6 = 14$$

(Reason: Solid A stands from the tip of the cone to the lowest point of the hemisphere. Below the joined circle the hemisphere goes down by its radius, 6 cm, so the height is the cone's 8 cm plus 6 cm. The figure here is a cross-section through the middle of solid A, drawn to scale.)

$$\text{(a) } h = 12.5$$

$$\text{(b) } 14 \text{ cm}$$

Verification

- ✓ **Check 1 - (a) back into solid B:** Put $h = 12.5$ into the total surface area of solid B.
 $32\pi + 8\pi \times 12.5 = 132\pi$, the total surface area of solid A
- ✓ **Check 2 - (a) dividing by 2π first:** Every term of $60\pi + 72\pi = 32\pi + 8\pi h$ is a multiple of 2π . Divided through by 2π , the equation becomes $30 + 36 = 16 + 4h$.
 $4h = 66 - 16 = 50$, so $h = 12.5$ again
- ✓ **Check 3 - (b) Pythagoras the other way round:** Square the height of the cone and its radius and add them. The sum must be the square of the sloping edge.
 $8^2 + 6^2 = 64 + 36 = 100 = 10^2$
- ✓ **Check 4 - (b) the 3, 4, 5 triangle:** The radius 6 and the sloping edge 10 are 2×3 and 2×5 , so the triangle is the 3, 4, 5 right-angled triangle doubled, and its third side is 2×4 .
 $2 \times 4 = 8$ for the cone, and $8 + 6 = 14$ for solid A

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The value of h	Note	The Answer column gives 12.5 oe, and the Marks column gives 5 for it.	✓
(a) First method: the equation simplified	M4	For $8[\pi]h = 100[\pi]$.	✓
(a) Second method: the two totals set equal	M3	Or for $\pi \times 6 \times 10 + 2 \times \pi \times 6^2 = 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times h$ oe.	✓
(a) Third method: the surface area of solid A	M1	Or for $\pi \times 6 \times 10 + 2 \times \pi \times 6^2$ oe.	✓
(a) Third method: the surface area of solid B	M1	For $2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times h$ oe.	✓
(a) In place of the three methods: special case	SC2	Or for answer 16.5.	✓
(a) The square brackets in the first method	Note	The square brackets in $8[\pi]h = 100[\pi]$ mark the π as optional: $8\pi h = 100\pi$ and $8h = 100$ both earn that mark.	✓
(a) Where the special case comes from	Note	The special case is the answer 16.5. It comes from leaving out the two circular ends of the cylinder, so that only its curved surface is set equal to the surface area of solid A.	✓
(b) The height of solid A	Note	The Answer column gives 14, and the Marks column gives 3 for it.	✓
(b) The height of the cone	M2	For $\sqrt{10^2 - 6^2}$.	✓
(b) The height of the cone, by Pythagoras	M1	Or for $6^2 + x^2 = 10^2$.	✓

Step	Mark	Description	Got it?
(b) What the letter stands for	Note	In the Pythagoras line, x is the height of the cone.	✓

Full marks: 8/8

Question 14 - Exam Solution

Understanding the Question

Given

$f(x) = 3x - 4$ and $g(x) = 4x + 1$, two linear functions

There is no calculator, so every step is done by hand, and every value stays exact.

Find

(a) $f(-2)$ (b) $f^{-1}(x)$, the inverse of f (c) a and b , where $fg(x) = ax + b$ (d)

$\frac{2}{f(x)} - \frac{5}{g(x)}$ as a single fraction in terms of x

Plan the Solution

- (a) Put -2 in place of every x in $3x - 4$.
- (b) Write $y = 3x - 4$, make x the subject, then write the result in terms of x .
- (c) $fg(x)$ means g first and then f , so put $4x + 1$ into f , expand, and read off the number of x and the constant.
- (d) Replace $f(x)$ and $g(x)$ by their expressions, put both fractions over the common denominator $(3x - 4)(4x + 1)$, then expand and simplify the numerator.

Worked Solution [8 marks]

Rule - Functions: $f(-2)$ means put -2 in place of every x ; the inverse f^{-1} comes from making x the subject of $y = f(x)$; $fg(x) = f(g(x))$, with g applied first; and two fractions are subtracted over a common denominator.

Step 1: (a) Substitute $x = -2$ into $f(x)$

$$f(-2) = 3 \times (-2) - 4$$

$$-6 - 4 = -10$$

(Reason: Every x in $3x - 4$ is replaced by -2 , in a bracket so that the sign stays with it. Three lots of -2 is -6 , and taking away a further 4 goes further below zero, to -10 .)

Step 2: (b) Make x the subject of $y = 3x - 4$

$$y = 3x - 4$$

$$y + 4 = 3x$$

$$x = \frac{y + 4}{3}$$

$$f^{-1}(x) = \frac{x + 4}{3}$$

(Reason: The inverse undoes f . The function f multiplies by 3 and then subtracts 4, so its inverse adds 4 first and then divides by 3, the operations in reverse order. Adding 4 to both sides is the first step. Once x is the subject, the letter y is replaced by x so that f^{-1} is written in terms of x , as the question asks.)

Step 3: (c) Put $g(x)$ into f

$$fg(x) = f(4x + 1)$$

$$f(4x + 1) = 3(4x + 1) - 4$$

$$3(4x + 1) - 4 = 12x + 3 - 4$$

$$12x + 3 - 4 = 12x - 1$$

(Reason: In $fg(x)$ the function written next to the x acts first, so g turns x into $4x + 1$, and then f is applied to that whole bracket: three times it, then subtract 4. Matching $12x - 1$ with $ax + b$ term by term gives $a = 12$ and $b = -1$. The other order is a different function: $gf(x) = 4(3x - 4) + 1 = 12x - 15$, which gets a right and b wrong.)

Step 4: (d) Write both fractions in terms of x

$$\frac{2}{f(x)} - \frac{5}{g(x)} = \frac{2}{3x - 4} - \frac{5}{4x + 1}$$

(Reason: The expression is in terms of $f(x)$ and $g(x)$, and the answer has to be in terms of x , so each is replaced by the expression it stands for.)

Step 5: (d) Put both fractions over $(3x - 4)(4x + 1)$

$$\frac{2(4x + 1)}{(3x - 4)(4x + 1)} - \frac{5(3x - 4)}{(3x - 4)(4x + 1)}$$

$$\frac{2(4x + 1) - 5(3x - 4)}{(3x - 4)(4x + 1)}$$

(Reason: The two denominators have no factor in common, so the simplest common denominator is their product. The first fraction is multiplied top and bottom by $4x + 1$ and the second by $3x - 4$, which changes neither fraction's value. With one denominator the numerators can be subtracted, and the minus sign applies to the whole of $5(3x - 4)$, which is why it stays in its bracket.)

Step 6: (d) Expand and simplify the numerator

$$2(4x + 1) = 8x + 2$$

$$-5(3x - 4) = -15x + 20$$

$$8x + 2 - 15x + 20 = 22 - 7x$$

(Reason: Each bracket is multiplied out with the sign in front of it. In the second, -5 times $3x$ is $-15x$, and -5 times -4 is positive 20 , because two negatives multiply to give a positive. Then the x terms are collected, $8x - 15x = -7x$, and the numbers, $2 + 20 = 22$.)

Step 7: (d) Write the answer as a single fraction

$$\frac{2}{3x - 4} - \frac{5}{4x + 1} = \frac{22 - 7x}{(3x - 4)(4x + 1)}$$

$$(3x - 4)(4x + 1) = 12x^2 + 3x - 16x - 4$$

$$12x^2 + 3x - 16x - 4 = 12x^2 - 13x - 4$$

(Reason: The numerator $22 - 7x$ is zero only at $x = \frac{22}{7}$, where neither bracket of the denominator is zero, so it shares no factor with either bracket and nothing cancels. The denominator may be left as two brackets or multiplied out to $12x^2 - 13x - 4$; both give the same single fraction.)

$$(a) f(-2) = -10$$

$$(b) f^{-1}(x) = \frac{x + 4}{3}$$

$$(c) a = 12, b = -1$$

$$(d) \frac{22 - 7x}{(3x - 4)(4x + 1)}$$

Verification

✓ **Check 1 - (b) undoes (a):** The inverse must send the answer to (a) back to where it started, so $f^{-1}(-10)$ should be -2 . $\frac{-10 + 4}{3} = \frac{-6}{3} = -2$

✓ **Check 2 - (b) applying f to the inverse:** Put $\frac{x + 4}{3}$ into f : multiply by 3 and subtract 4 . The result should be x itself. $3 \times \frac{x + 4}{3} - 4 = x + 4 - 4 = x$

✓ **Check 3 - (c) with numbers:** Take $x = 1$: $g(1) = 5$ and $f(5) = 11$. Take $x = 0$: $g(0) = 1$ and $f(1) = -1$. The expression $12x - 1$ should give the same two values. $12 \times 1 - 1 = 11$ and $12 \times 0 - 1 = -1$

✓ **Check 4 - (d) at $x = 1$:** At $x = 1$, $f(1) = -1$ and $g(1) = 5$, so the original expression is $\frac{2}{-1} - \frac{5}{5} = -3$. The single fraction should give the same value.

$$\frac{22 - 7}{(-1)(5)} = \frac{15}{-5} = -3$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The value of $f(-2)$	Note	The Answer column gives -10 , and the Marks column gives 1 for it.	✓
(b) The inverse function $f^{-1}(x)$	Note	The Answer column gives $\frac{x+4}{3}$ oe, and the Marks column gives 2 for it.	✓
(b) The first step of the rearrangement	M1	For correct first step, $y + 4 = 3x$ or $x = 3y - 4$ or $\frac{y}{3} = x - \frac{4}{3}$.	✓
(c) The values of a and b	Note	The Answer column gives $a = 12$, $b = -1$, and the Marks column gives 2 for it.	✓
(c) One of the two values	B1	For either a or b correct.	✓
(c) In place of the B1: $fg(x)$ written out	M1	Or for $3(4x + 1) - 4$.	✓
(d) The single fraction	Note	The Answer column gives $\frac{22 - 7x}{(3x - 4)(4x + 1)}$ or $\frac{22 - 7x}{12x^2 - 13x - 4}$ final answer, and the Marks column gives 3 for it.	✓
(d) The numerator over the common denominator	B1	For $2(4x + 1) - 5(3x - 4)$ oe or better isw.	✓
(d) The common denominator	B1	For common denominator $(3x - 4)(4x + 1)$ oe isw.	✓
(d) What oe, or better and isw mean	Note	oe is the scheme's abbreviation for or equivalent, and isw for ignore subsequent working: once the expression has been written, a slip in the working after it does not take that mark away. Or better means that	✓

Step	Mark	Description	Got it?
		a simplified form of the same numerator, such as $22 - 7x$, is credited as well.	

Full marks: 8/8

Question 15 - Exam Solution

Understanding the Question

Given

$(2 - \sqrt{5})(1 - 3\sqrt{5})$, a product of two brackets, each holding a whole number and a multiple of $\sqrt{5}$

$\frac{6}{\sqrt{10}}$, a fraction with the surd $\sqrt{10}$ in its denominator

There is no calculator, so every surd is kept exact and nothing is written as a decimal.

Find

(a) $(2 - \sqrt{5})(1 - 3\sqrt{5})$ expanded and simplified (b) $\frac{6}{\sqrt{10}}$ written with a rational denominator, in its simplest form

Plan the Solution

- (a) Multiply each term of the first bracket by each term of the second, which gives four products. Then use $\sqrt{5} \times \sqrt{5} = 5$, and collect the whole numbers and the multiples of $\sqrt{5}$ separately.
- (b) Multiply the numerator and the denominator by $\sqrt{10}$, so that the denominator becomes $\sqrt{10} \times \sqrt{10} = 10$. Then cancel the common factor of the whole numbers.

Worked Solution [4 marks]

Rule - Surds: $\sqrt{a} \times \sqrt{a} = a$. Brackets holding surds are expanded term by term, exactly like brackets holding letters, and a denominator $\sqrt{a}$ is made rational by multiplying the fraction by $\frac{\sqrt{a}}{\sqrt{a}}$, which is equal to 1.

Step 1: (a) Multiply each term of the first bracket by each term of the second

$$2 \times 1 = 2$$

$$2 \times (-3\sqrt{5}) = -6\sqrt{5}$$

$$(-\sqrt{5}) \times 1 = -\sqrt{5}$$

$$(-\sqrt{5}) \times (-3\sqrt{5}) = 3 \times \sqrt{5} \times \sqrt{5}$$

(Reason: The first bracket holds the terms 2 and $-\sqrt{5}$, and the second holds 1 and $-3\sqrt{5}$, so there are four products, and each term keeps the sign in front of it. The last product is a negative times a negative, so it is positive, and the numbers in front of the two roots, 1 and 3, multiply to give 3.)

Step 2: (a) Write the four terms as one expression

$$(2 - \sqrt{5})(1 - 3\sqrt{5}) = 2 - 6\sqrt{5} - \sqrt{5} + 3 \times \sqrt{5} \times \sqrt{5}$$

(Reason: This is the expansion before anything is simplified. It is the line the mark scheme's B1 is for: any 3 of these four terms written correctly earn that mark.)

Step 3: (a) Simplify $\sqrt{5} \times \sqrt{5}$

$$3 \times \sqrt{5} \times \sqrt{5} = 3 \times 5 = 15$$

$$(2 - \sqrt{5})(1 - 3\sqrt{5}) = 2 - 6\sqrt{5} - \sqrt{5} + 15$$

(Reason: A square root multiplied by itself gives back the number under the root, so $\sqrt{5} \times \sqrt{5} = 5$. The last term is therefore a whole number, and it can join the 2 in the next step.)

Step 4: (a) Collect like terms

$$2 + 15 = 17$$

$$-6\sqrt{5} - \sqrt{5} = -7\sqrt{5}$$

$$(2 - \sqrt{5})(1 - 3\sqrt{5}) = 17 - 7\sqrt{5}$$

(Reason: Whole numbers are collected with whole numbers, and multiples of $\sqrt{5}$ with multiples of $\sqrt{5}$, just as x terms would be: 6 lots of $\sqrt{5}$ taken away and then 1 more taken away is 7 lots taken away. 17 and $7\sqrt{5}$ cannot be combined, because one is a whole number and the other a multiple of $\sqrt{5}$, so the expression is fully simplified.)

Step 5: (b) Multiply the numerator and the denominator by $\sqrt{10}$

$$\frac{6}{\sqrt{10}} = \frac{6}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$\sqrt{10} \times \sqrt{10} = 10$$

$$\frac{6}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{6\sqrt{10}}{10}$$

(Reason: Multiplying by $\frac{\sqrt{10}}{\sqrt{10}}$ is multiplying by 1, so the value of the fraction does not change. The denominator does: $\sqrt{10}$ times itself is 10, a whole number, and a whole-number denominator is what rationalising means. The multiplication written out in the first line is what the M1 is for.)

Step 6: (b) Cancel to the simplest form

$$\frac{6}{10} = \frac{3}{5}$$

$$\frac{6\sqrt{10}}{10} = \frac{3\sqrt{10}}{5}$$

(Reason: The whole numbers 6 and 10 share a factor of 2, so both are divided by 2, and the $\sqrt{10}$ is left as it is. The root cannot be simplified either, because $10 = 2 \times 5$ has no square factor other than 1. So $\frac{3\sqrt{10}}{5}$ is the simplest form the question asks for.)

$$(a) 17 - 7\sqrt{5}$$

$$(b) \frac{3\sqrt{10}}{5}$$

Verification

- ✓ **Check 1 - (a) expanded the other way round:** Multiply the whole first bracket by each term of the second instead: $(2 - \sqrt{5}) \times 1 = 2 - \sqrt{5}$, and $(2 - \sqrt{5}) \times (-3\sqrt{5}) = -6\sqrt{5} + 15$. Adding the two should give the same answer. $2 - \sqrt{5} - 6\sqrt{5} + 15 = 17 - 7\sqrt{5}$
- ✓ **Check 2 - (a) with the partner brackets:** Changing the sign of every $\sqrt{5}$ in Steps 1 to 4 leaves the working true, so $(2 + \sqrt{5})(1 + 3\sqrt{5}) = 17 + 7\sqrt{5}$ too. Multiply the two results together, using $(p - q\sqrt{5})(p + q\sqrt{5}) = p^2 - 5q^2$ on each pair of partners. The brackets give $(2^2 - 5 \times 1^2)(1^2 - 5 \times 3^2)$, which is $(-1) \times (-44) = 44$, so the answer side must give 44 as well. $17^2 - 5 \times 7^2 = 289 - 245 = 44$
- ✓ **Check 3 - (b) by squaring:** Both fractions are positive, so they are equal if their squares are. Squaring a root gives the number under it, so $\left(\frac{6}{\sqrt{10}}\right)^2 = \frac{36}{10}$ and

$$\left(\frac{3\sqrt{10}}{5}\right)^2 = \frac{9 \times 10}{25} = \frac{90}{25} \cdot \frac{36}{10} = \frac{18}{5} \text{ and } \frac{90}{25} = \frac{18}{5}$$

✓ **Check 4 - (b) multiplying back:** If $\frac{3\sqrt{10}}{5}$ is equal to $\frac{6}{\sqrt{10}}$, then multiplying it by $\sqrt{10}$

must give back the numerator 6. $\frac{3\sqrt{10}}{5} \times \sqrt{10} = \frac{3 \times 10}{5} = 6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The expansion, simplified	Note	The Answer column gives $17 - 7\sqrt{5}$ final answer, and the Marks column gives 2 for it.	✓
(a) Three of the four terms of the expansion	B1	For 3 correct terms from $2 - 6\sqrt{5} - \sqrt{5} + 3 \times \sqrt{5} \times \sqrt{5}$ oe.	✓
(b) The fraction with a rational denominator	Note	The Answer column gives $\frac{3\sqrt{10}}{5}$ cao, and the Marks column gives 2 for it.	✓
(b) Multiplying top and bottom by $\sqrt{10}$	M1	For $\frac{6}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$ oe.	✓
(b) What cao and oe mean	Note	cao is the scheme's abbreviation for correct answer only, and oe for or equivalent. The answer asked for is the simplest form, $\frac{3\sqrt{10}}{5}$; the line $\frac{6\sqrt{10}}{10}$ before the cancelling is equal to it, but it is not yet in its simplest form.	✓

Full marks: 4/4

Question 16 - Exam Solution

Understanding the Question

Given

$(x + 4)(x - 3)(3x + 2)$, a product of three brackets, each holding a multiple of x and a number

There is no calculator, so every product is worked out by hand with whole numbers.

Find

The product multiplied out and simplified. Three brackets that each hold an x term multiply to give an x^3 term, so expect terms in x^3 , x^2 and x , and a number.

Plan the Solution

- Expand the first two brackets, $(x + 4)(x - 3)$, and collect like terms. That gives a quadratic with three terms.
- Multiply each of those three terms by each term of $(3x + 2)$. That gives $3 \times 2 = 6$ terms.
- Collect like terms: the two x^2 terms together, and the two x terms together.

Worked Solution [3 marks]

Rule - Expanding three brackets: multiply two of the brackets together and simplify, then multiply every term of that result by every term of the third bracket, and collect like terms.

Step 1: Expand the first two brackets

$$(x + 4)(x - 3) = x^2 - 3x + 4x - 12$$

$$-3x + 4x = x$$

$$(x + 4)(x - 3) = x^2 + x - 12$$

(Reason: Each term of $(x + 4)$ multiplies each term of $(x - 3)$: $x \times x = x^2$, $x \times (-3) = -3x$, $4 \times x = 4x$ and $4 \times (-3) = -12$. Those four terms are one pair of brackets expanded, which is the line the mark scheme's B1 describes, with at least 3 of the 4 correct. Then $-3x$ and $4x$ are like terms, and together they make x .)

Step 2: Multiply each term of $x^2 + x - 12$ by $(3x + 2)$

$$x^2(3x + 2) = 3x^3 + 2x^2$$

$$x(3x + 2) = 3x^2 + 2x$$

$$-12(3x + 2) = -36x - 24$$

(Reason: The quadratic has three terms and $(3x + 2)$ has two, so there are six products. Each term keeps the sign in front of it. The last term of the quadratic is -12 , so $-12 \times 3x = -36x$ and $-12 \times 2 = -24$.)

Step 3: Write the six terms as one expression

$$(x + 4)(x - 3)(3x + 2) = 3x^3 + 2x^2 + 3x^2 + 2x - 36x - 24$$

(Reason: This is the product fully multiplied out, before anything is collected. It is a correct expansion unsimplified, the first of the two forms the mark scheme's B2 accepts.)

Step 4: Collect like terms

$$2x^2 + 3x^2 = 5x^2$$

$$2x - 36x = -34x$$

$$(x + 4)(x - 3)(3x + 2) = 3x^3 + 5x^2 - 34x - 24$$

(Reason: The two x^2 terms add to $5x^2$, and $2x$ take away $36x$ leaves $-34x$. The x^3 term and the number have nothing to join with, so they stay as they are. No two of the four terms left are alike, so the expression is fully simplified, with the powers of x in descending order.)

$$3x^3 + 5x^2 - 34x - 24$$

Verification

- ✓ **Check 1 - put $x = 1$ into both:** An expansion is equal to the product it came from for every value of x , so the brackets and the answer must give the same number when $x = 1$.
The brackets give $(1 + 4) \times (1 - 3) \times (3 \times 1 + 2) = 5 \times (-2) \times 5 = -50$.
 $3 \times 1^3 + 5 \times 1^2 - 34 \times 1 - 24 = -50$
- ✓ **Check 2 - the brackets in a different order:** Expand $(x - 3)(3x + 2)$ first instead:
 $3x^2 + 2x - 9x - 6 = 3x^2 - 7x - 6$. Then multiply by $(x + 4)$:
 $x(3x^2 - 7x - 6) = 3x^3 - 7x^2 - 6x$ and $4(3x^2 - 7x - 6) = 12x^2 - 28x - 24$.
Adding the two should give the same answer.
 $3x^3 - 7x^2 + 12x^2 - 6x - 28x - 24 = 3x^3 + 5x^2 - 34x - 24$
- ✓ **Check 3 - the answer is 0 when $x = 3$:** When $x = 3$ the bracket $(x - 3)$ is 0, so the whole product is 0. The answer must be 0 there as well.
 $3 \times 3^3 + 5 \times 3^2 - 34 \times 3 - 24 = 81 + 45 - 102 - 24 = 0$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The expansion, simplified	Note	The Answer column gives $3x^3 + 5x^2 - 34x - 24$ final answer, and the Marks column gives 3 for it.	✓
The full expansion	B2	For correct expansion unsimplified, or simplified four-term expression of correct form with 3 terms correct.	✓
One pair of brackets expanded	B1	Or for one pair of brackets expanded with at least 3 terms out of 4 correct.	✓

Question 17 - Exam Solution

Understanding the Question

Given

First jar: 6 red, 3 green and 1 blue bead, so 10 beads in all. Two are taken with replacement.

Second jar: 4 red and 2 yellow tokens, so 6 tokens in all. Two are taken without replacement.

The tree already shows $\frac{4}{6}$ for a red first token, $\frac{2}{6}$ for a yellow first token, and $\frac{3}{5}$ for red after red.

Find

(a) The probability that both beads are green. (b)(i) The three probabilities missing from the tree. (b)(ii) The probability that one of the two tokens is yellow.

Plan the Solution

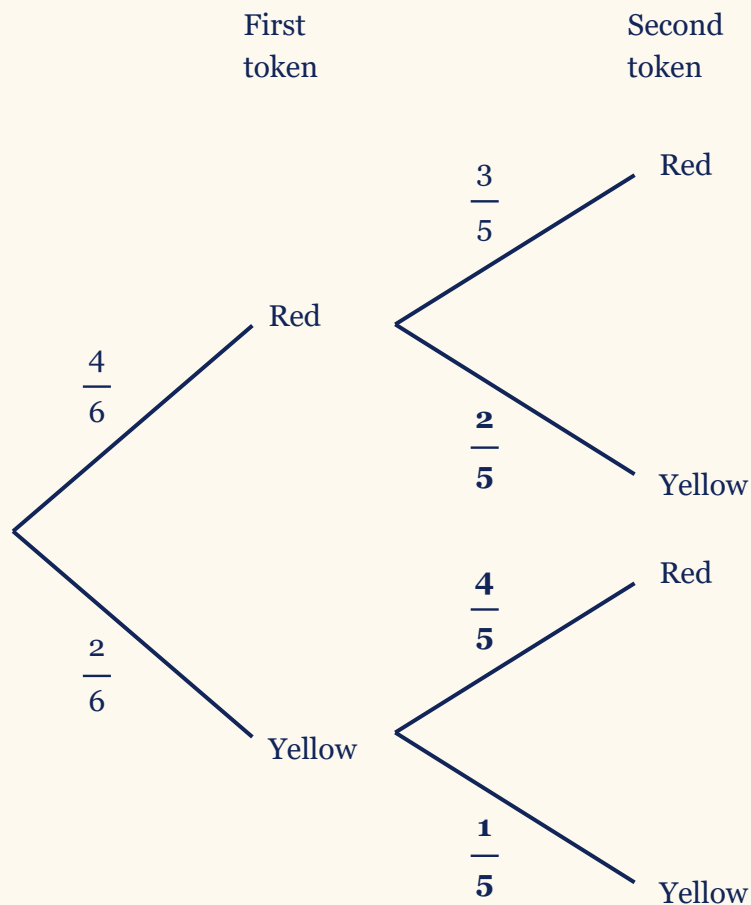
- (a) With replacement the jar is the same for both picks, so green has probability $\frac{3}{10}$ each time. Multiply the two.
- (b)(i) Without replacement the second pick is from one token fewer. Count what is left after each colour of first token.
- (b)(ii) One yellow can happen in two ways: red then yellow, or yellow then red. Multiply along each path, then add the two paths.

Worked Solution [7 marks]

Rule - Probability tree: multiply the probabilities along a path, then add the paths that give the outcome you want.

(a) Green on one pick

$$\frac{3}{6 + 3 + 1} = \frac{3}{10}$$



(Reason: The bead goes back after the first pick, so each pick is from all 10 beads, and 3 of them are green.)

(a) Green on both picks

$$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$$

(Reason: Putting the bead back makes the second pick independent of the first, so the two probabilities multiply.)

(b)(i) After a red first token

$$\frac{2}{6-1} = \frac{2}{5}$$

$$\frac{3}{5} + \frac{2}{5} = 1$$

(Reason: Taking a red token leaves 5 tokens: 3 red and both yellow ones. So the Red to Yellow branch is $\frac{2}{5}$, and with the printed $\frac{3}{5}$ it makes 1, as the two branches from one point must.)

(b)(i) After a yellow first token

$$\frac{4}{6-1} = \frac{4}{5}$$

$$\frac{2-1}{6-1} = \frac{1}{5}$$

$$\frac{4}{5} + \frac{1}{5} = 1$$

(Reason: Taking a yellow token leaves 5 tokens: all 4 red and only 1 yellow. These go on the Yellow to Red and Yellow to Yellow branches.)

(b)(ii) Red, then yellow

$$\frac{4}{6} \times \frac{2}{5} = \frac{8}{30}$$

(Reason: Multiply along the path: a red first token, then a yellow one from the tokens that are left.)

(b)(ii) Yellow, then red

$$\frac{2}{6} \times \frac{4}{5} = \frac{8}{30}$$

(Reason: Multiply along the other path: a yellow first token, then a red one from the tokens that are left.)

(b)(ii) Add the two paths

$$\frac{8}{30} + \frac{8}{30} = \frac{16}{30}$$

(Reason: The two paths cannot both happen, so their probabilities add. $\frac{16}{30}$ is the form the mark scheme gives, and it accepts an equivalent such as $\frac{8}{15}$.)

$$\begin{array}{l} \text{(a)} \quad \frac{9}{100} \\ \text{(b)(i) Red then Yellow } \frac{2}{5}, \text{ Yellow then Red } \frac{4}{5}, \text{ Yellow then} \\ \text{Yellow } \frac{1}{5} \\ \text{(b)(ii) } \frac{16}{30} \end{array}$$

Verification

✓ **Check 1 - (a):** Count ordered picks instead: there are $10 \times 10 = 100$ equally likely pairs, and $3 \times 3 = 9$ of them are green then green. $\frac{9}{100}$ again

- ✓ **Check 2 - (b)(i):** The two branches leaving any one point of the tree must add to 1.

$$\frac{3}{5} + \frac{2}{5} = 1 \text{ and } \frac{4}{5} + \frac{1}{5} = 1$$

- ✓ **Check 3 - (b)(ii):** Take the two same-colour paths away from 1. Both red is

$$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30} \text{ and both yellow is } \frac{2}{6} \times \frac{1}{5} = \frac{2}{30}. 1 - \frac{12}{30} - \frac{2}{30} = \frac{16}{30}$$

- ✓ **Check 4 - (b)(ii):** Count ordered pairs of different tokens: $6 \times 5 = 30$ in all, and

$$4 \times 2 + 2 \times 4 = 16 \text{ of them hold exactly one yellow token. } \frac{16}{30} \text{ again}$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Both beads green	Note	The Answer column gives $\frac{9}{100}$ oe, and the Marks column gives 2 for it.	✓
(a) The method	M1	For $\frac{3}{10} \times \frac{3}{10}$.	✓
(b)(i) The three missing probabilities	Note	The Answer column gives $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$, and the Marks column gives 2 for it.	✓
(b)(i) Partial Marks	B1	For $\frac{2}{5}$.	✓
(b)(ii) One of the two tokens yellow	Note	The Answer column gives $\frac{16}{30}$ oe, and the Marks column gives 3 for it.	✓
(b)(ii) Full marks FT their tree diagram	3FT	Full marks FT their tree diagram dep on probabilities < 1 .	✓
(b)(ii) Without the 3FT: both paths	M2FT	Otherwise, for $\frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5}$.	✓
(b)(ii) Without the 3FT: one path	M1FT	Or for $\frac{4}{6} \times \frac{2}{5}$ or for $\frac{2}{6} \times \frac{4}{5}$.	✓

Full marks: 7/7

Question 18 - Exam Solution

Understanding the Question

Given

Saturday: 12 km paddled at x km/h
Sunday: 10 km hiked at $(x - 4)$ km/h
The hike takes 1 hour longer than the paddle
There is no calculator, so every step is done by hand and every value stays exact.

Find

(a) the paddling time, in terms of x (b) the hiking time, in terms of x (c) an equation in x , shown to simplify to $x^2 - 2x - 48 = 0$ (d) the solutions of $x^2 - 2x - 48 = 0$; it is a quadratic, so expect two (e) the paddling time in hours

Plan the Solution

- (a) and (b) Time is distance divided by speed.
- (c) The hiking time minus the paddling time is 1. Multiply every term by $x(x - 4)$ to clear both fractions, multiply out the brackets, then collect every term on one side.
- (d) Find two numbers that multiply to -48 and add to -2 , split the middle term with them, factorise, and set each bracket equal to zero.
- (e) A speed cannot be negative, so keep the positive solution and put it into the answer to (a).

Worked Solution [10 marks]

Rule - Speed, then a quadratic: $\text{time} = \frac{\text{distance}}{\text{speed}}$; an equation with algebraic fractions is cleared by multiplying every term by a common denominator; and a quadratic equal to zero is solved by factorising and setting each bracket equal to zero.

Step 1: (a) The time spent paddling

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time paddling} = \frac{12}{x}$$

(Reason: Distance is speed multiplied by time, so time is distance divided by speed. Tanya paddles 12 km at x km/h, so the time is 12 divided by x , written as a fraction. The speed is in kilometres per hour and the distance in kilometres, so the time is in hours, the unit on the answer line.)

Step 2: (b) The time spent hiking

$$\text{time hiking} = \frac{10}{x-4}$$

(Reason: The same rule, with the hike's 10 km and its speed of $x - 4$ km/h. The whole of $x - 4$ is the speed, so all of it goes underneath the 10. Writing $\frac{10}{x} - 4$ instead would divide by x alone and then take 4 away, which is a different expression.)

Step 3: (c) Turn the sentence into an equation

$$\text{time hiking} - \text{time paddling} = 1$$

$$\frac{10}{x-4} - \frac{12}{x} = 1$$

(Reason: The hike takes 1 hour longer, so it is the longer of the two times, and taking the paddling time away from it leaves 1. The two times are the expressions from (b) and (a). The other order, paddling time minus hiking time, would equal -1 , not 1.)

Step 4: (c) Multiply every term by $x(x-4)$

$$x(x-4) \times \frac{10}{x-4} - x(x-4) \times \frac{12}{x} = x(x-4) \times 1$$

$$10x - 12(x-4) = x(x-4)$$

(Reason: Both denominators, $x-4$ and x , divide into $x(x-4)$, so multiplying every term by it clears both fractions. Every term means the 1 on the right as well: an equation stays true only when both sides are multiplied by the same thing. In the first fraction the $x-4$ cancels and leaves $10x$; in the second the x cancels and leaves $12(x-4)$, which stays in its bracket because the minus sign in front of it belongs to all of it.)

Step 5: (c) Multiply out the brackets

$$-12(x-4) = -12x + 48$$

$$x(x-4) = x^2 - 4x$$

$$10x - 12x + 48 = x^2 - 4x$$

(Reason: The -12 multiplies both terms inside its bracket: -12 times x is $-12x$, and -12 times -4 is $+48$, because two negatives multiply to give a positive. On the right, x times each term of $x-4$ gives $x^2 - 4x$.)

Step 6: (c) Collect every term on one side

$$-2x + 48 = x^2 - 4x$$

$$0 = x^2 - 4x + 2x - 48$$

$$0 = x^2 - 2x - 48$$

(Reason: On the left, $10x - 12x = -2x$. Adding $2x$ to both sides and subtracting 48 from both sides moves every term to the right, where the x^2 term is already positive. Then $-4x + 2x = -2x$, and the equation reads $x^2 - 2x - 48 = 0$, as the question asked. In a show-that question the working is the answer, so every line is written down.)

Step 7: (d) Two numbers with product -48 and sum -2

$$6 \times (-8) = -48$$

$$6 + (-8) = -2$$

(Reason: To factorise $x^2 - 2x - 48$, look for two numbers that multiply to the constant term, -48 , and add to the coefficient of x , which is -2 . The product is negative, so one number is positive and the other negative; the sum is negative, so the negative one is the larger in size. The factor pairs of 48 are 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8, and only 6 and 8 differ by 2.)

Step 8: (d) Split the middle term and factorise

$$x^2 - 2x - 48 = x^2 + 6x - 8x - 48$$

$$x^2 + 6x - 8x - 48 = x(x + 6) - 8(x + 6)$$

$$x(x + 6) - 8(x + 6) = (x + 6)(x - 8)$$

(Reason: The $-2x$ is written as $+6x - 8x$, using the pair just found. Taking x out of the first two terms and -8 out of the last two leaves the same bracket, $(x + 6)$, in both, and that common bracket is taken out as a factor. With practice the pair can be written straight into the brackets, $(x + 6)(x - 8)$.)

Step 9: (d) Solve

$$(x + 6)(x - 8) = 0$$

$$x + 6 = 0 \text{ or } x - 8 = 0$$

$$x = -6 \text{ or } x = 8$$

(Reason: Two numbers multiply to give zero only when one of them is zero, so one bracket or the other is zero. Setting each bracket equal to zero gives one solution each.)

Step 10: (e) Keep the value that can be a speed, then find the time

$$x = 8$$

$$\frac{12}{8} = \frac{3}{2}$$

$$\frac{3}{2} = 1.5$$

(Reason: The paddling speed is x km/h and a speed cannot be negative, so $x = -6$ is rejected: it would make the paddling time $\frac{12}{-6} = -2$ hours. With $x = 8$ the hiking speed is $8 - 4 = 4$ km/h, positive as it must be. Putting $x = 8$ into the answer to (a) gives 1.5 hours, which is $1\frac{1}{2}$ hours, or 1 hour and 30 minutes.)

$$(a) \frac{12}{x} \text{ hours}$$

$$(b) \frac{10}{x-4} \text{ hours}$$

$$(c) x^2 - 2x - 48 = 0, \text{ as required}$$

$$(d) x = -6 \text{ or } x = 8$$

$$(e) 1.5 \text{ hours}$$

Verification

- ✓ **Check 1 - (c) and (e) together:** With $x = 8$, the paddle takes $\frac{12}{8} = 1.5$ hours and the hike takes $\frac{10}{8-4} = 2.5$ hours. The hike should take exactly 1 hour longer. $2.5 - 1.5 = 1$
- ✓ **Check 2 - (d) multiplying the brackets back out:** Multiply out $(x+6)(x-8)$ term by term. It should give back the quadratic. $x^2 - 8x + 6x - 48 = x^2 - 2x - 48$
- ✓ **Check 3 - (d) both solutions in the equation:** Put each solution into $x^2 - 2x - 48$. Each should give 0. $8^2 - 2 \times 8 - 48 = 0$ and $(-6)^2 - 2 \times (-6) - 48 = 0$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The paddling time	Note	The Answer column gives $\frac{12}{x}$, and the Marks column gives 1 for it.	✓
(b) The hiking time	Note	The Answer column gives $\frac{10}{x-4}$, and the Marks column gives 1 for it.	✓
(c) The equation from the two times	M1	For their $\frac{10}{x-4}$ - their $\frac{12}{x} = 1$ oe.	✓
(c) Multiplying out and clearing the fractions	M2	For $10x - 12x + 48 = x^2 - 4x$: correctly multiplying their brackets and clearing algebraic fractions, e.g. $(x-4) \left(\frac{12}{x} + 1 \right) = 10$ leading to	✓

Step	Mark	Description	Got it?
		$12 - \frac{48}{x} + x - 4 = 10$ and then $12x - 48 + x^2 - 4x = 10x$.	
(c) In place of the M2: clearing the fractions only	M1	Or for correctly clearing, or correctly collecting into a single fraction, two fractions both with different algebraic denominators, e.g. $10x - 12(x - 4) = x(x - 4)$ or $\frac{10x - 12(x - 4)}{x(x - 4)} [= 1]$.	✓
(c) The quadratic	A1	For leading to $0 = x^2 - 2x - 48$, with no errors or omissions seen, dep on M3.	✓
(d) The factorisation	M2	For $(x + 6)(x - 8)$.	✓
(d) Without the M2: the grouped form or the pair of numbers	M1	Otherwise, for $x(x - 8) + 6(x - 8)$ or $x(x + 6) - 8(x + 6)$ or $(x + a)(x + b)$ where $ab = -48$ or $a + b = -2$.	✓
(d) The two solutions	B1	For $-6, 8$.	✓
(e) The paddling time in hours	Note	The Answer column gives 1.5 or $1\frac{1}{2}$, and the Marks column gives 1 for it.	✓
(e) Follow-through	Note	FT $\frac{12}{\text{their } 8}$.	✓

Full marks: 10/10

Question 19 - Exam Solution

Understanding the Question

Given

Find

The value of $27^{-\frac{2}{3}}$.

The number to evaluate is $27^{-\frac{2}{3}}$: base 27
 , index $-\frac{2}{3}$.

No calculator may be used on this paper,
 so every step is worked by hand.

Plan the Solution

- Each part of the index $-\frac{2}{3}$ has its own meaning. The 3 underneath asks for a cube root, the 2 on top asks for a square, and the minus sign turns the result upside down.
- Take the cube root before squaring. 27 is a cube number, so its cube root is a small whole number and the square that follows stays small too.
- Deal with the minus sign last, by writing one over the positive power.

Worked Solution [2 marks]

Rule - Fractional and negative indices: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ and $a^{-k} = \frac{1}{a^k}$

Cube root, for the 3 in the denominator

$$27 = 3 \times 3 \times 3$$

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$

(Reason: A power of $\frac{1}{3}$ is a cube root, and $3 \times 3 \times 3 = 27$, so the cube root of 27 is 3.)

Square, for the 2 in the numerator

$$27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2 = 3^2 = 9$$

(Reason: A power of $\frac{2}{3}$ means the cube root, then squared. Squaring the 3 from Step 1 gives 9.)

Reciprocal, for the minus sign

$$27^{-\frac{2}{3}} = \left(27^{\frac{2}{3}}\right)^{-1} = 9^{-1}$$

$$9^{-1} = \frac{1}{9}$$

(Reason: A negative power is one over the matching positive power, so the value is the reciprocal of 9.

Reaching 9^{-1} or $\frac{1}{3^2}$ on the way is the working the method mark rewards.)

$$\frac{1}{9}$$

Verification

✓ **Check 1 - 27 as a power of 3:** Replace 27 with 3^3 . A power of a power multiplies the indices, and $3 \times \left(-\frac{2}{3}\right) = -2$. $(3^3)^{-\frac{2}{3}} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$, which agrees.

✓ **Check 2 - cube the answer:** Cubing multiplies the index by 3, so the cube of $27^{-\frac{2}{3}}$ is 27^{-2} . The cube of $\frac{1}{9}$ must therefore be one over 27^2 : $\left(\frac{1}{9}\right)^3 = \frac{1}{729} = \frac{1}{27^2}$, as it should be.

✓ **Check 3 - square before the cube root:** Doing the square first gives $27^2 = 729$. Since $9^3 = 729$, the cube root of 729 is 9. $\frac{1}{\sqrt[3]{729}} = \frac{1}{9}$, the same value.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The answer	Note	The Answer column gives $\frac{1}{9}$, and the Marks column gives 2 for it.	✓
Method for the value	M1	For 9^{-1} or for $\frac{1}{3^2}$ or $\frac{1}{\sqrt[3]{729}}$.	✓

Full marks: 2/2

Question 20 - Exam Solution

Understanding the Question

Given

A right-angled triangle, with the angle 30° at one end of the side x cm and the right angle at the other end
The side of 6 cm, across the triangle from the 30° angle
The figure is not to scale, and no calculator may be used

Find

The exact value of x , so a square root stays as a root instead of becoming a rounded decimal

Plan the Solution

- Name each side from the 30° angle. The side of 6 cm is opposite it and the side x cm is adjacent to it, so $\tan$ is the ratio that links them.
- Find $\tan 30^\circ$ exactly, by hand, from half of an equilateral triangle.
- Rearrange to make x the subject, then divide by the exact value and simplify.

Worked Solution [4 marks]

Rule - The tangent ratio: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$, with the exact value $\tan 30^\circ = \frac{1}{\sqrt{3}}$ from half of an equilateral triangle.

Name the sides from the 30° angle

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 30^\circ = \frac{6}{x}$$

(Reason: The 6 cm side is across the triangle from the 30° angle, so it is the opposite side. The x cm side runs from the 30° angle to the right angle, so it is the adjacent side. The hypotenuse is neither given nor asked for, so the ratio to use is $\tan$, the one that links the opposite and adjacent sides.)

The exact value of $\tan 30^\circ$

$$2^2 - 1^2 = 4 - 1 = 3$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

(Reason: No calculator is allowed, so the value comes from an equilateral triangle with sides of 2, cut in half down its line of symmetry. Each half is a right-angled triangle with angles of 30° , 60° and 90° ,

a hypotenuse of 2 and a side of 1 opposite the 30° . Pythagoras gives the side next to the 30° as $\sqrt{3}$, so $\tan 30^\circ$ is 1 over $\sqrt{3}$. This exact value, or the equal form $\frac{\sqrt{3}}{3}$, is what the B1 is for.)

Make x the subject

$$6 = x \tan 30^\circ$$

$$x = \frac{6}{\tan 30^\circ}$$

(Reason: Multiply both sides of $\tan 30^\circ = \frac{6}{x}$ by x , then divide both sides by $\tan 30^\circ$. Reaching

$\frac{6}{\tan 30^\circ}$ is the working the M2 is for. The equation $\frac{6}{x} = \tan 30^\circ$ on its own earns the M1 instead.)

Divide by $\frac{1}{\sqrt{3}}$

$$\frac{1}{\tan 30^\circ} = \sqrt{3}$$

$$x = 6 \times \sqrt{3} = 6\sqrt{3}$$

(Reason: Dividing by a fraction is the same as multiplying by its reciprocal, and the reciprocal of $\frac{1}{\sqrt{3}}$ is $\sqrt{3}$. So dividing 6 by $\tan 30^\circ$ is the same as multiplying 6 by $\sqrt{3}$. The root cannot be simplified, because 3 has no square factor other than 1, so $6\sqrt{3}$ is the exact value.)

$$x = 6\sqrt{3}$$

Verification

- ✓ **Check 1 - through the hypotenuse:** The 6 cm side is opposite the 30° and $\sin 30^\circ = \frac{1}{2}$, so the hypotenuse is twice as long, 12 cm. Pythagoras then gives the square of the third side, which must equal the square of the answer.
 $12^2 - 6^2 = 144 - 36 = 108$ and $(6\sqrt{3})^2 = 36 \times 3 = 108$, which agree.
- ✓ **Check 2 - the sine rule:** The third angle is $180 - 90 - 30 = 60$ degrees, and it is opposite x . The sine rule pairs each side with the sine of the angle opposite it, which is the scheme's second form of the M2. $x = \frac{6 \sin 60^\circ}{\sin 30^\circ} = 6 \times \frac{\sqrt{3}}{2} \times 2 = 6\sqrt{3}$, the same value.

✓ **Check 3 - the size of the answer:** Without a calculator, $1.7^2 = 2.89$ and $1.8^2 = 3.24$, so $\sqrt{3}$ lies between 1.7 and 1.8. Multiplying by 6 gives $6 \times 1.7 = 10.2$ and $6 \times 1.8 = 10.8$. $10.2 < 6\sqrt{3} < 10.8$: shorter than the 12 cm hypotenuse, and longer than the 6 cm side because it faces the larger angle, 60° .

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The exact value of x	Note	The Answer column gives $6\sqrt{3}$ oe, and the Marks column gives 4 for it.	✓
An exact trigonometric value	B1	For $\tan 30 = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$, or for $\sin 60 = \frac{\sqrt{3}}{2}$ and $\sin 30 = \frac{1}{2}$.	✓
A calculation for x	M2	For $\frac{6}{\tan 30}$ or $\frac{6 \sin(60)}{\sin(30)}$.	✓
An equation for x	M1	Or for $\frac{6}{x} = \tan 30$ or $\frac{\sin 60}{x} = \frac{\sin 30}{6}$.	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

A rhombus $OABC$ with its vertex O at the origin; its diagonals cross at P

$$\overrightarrow{OP} = 2\mathbf{m} \text{ and } \overrightarrow{AP} = \mathbf{n}$$

$$\text{For (b): } \overrightarrow{AD} = 10\mathbf{m} - 3\mathbf{n}$$

The figure is not to scale, and no calculator may be used

Find

(a) $\overrightarrow{OA}$ and $\overrightarrow{OC}$ in terms of $\mathbf{m}$ and $\mathbf{n}$, each in its simplest form (b) a reason why $OADC$ is a trapezium, which means showing that two of its opposite sides are parallel

Plan the Solution

- (a)(i) There is no given vector from O to A , so go the long way round through P : out along $\overrightarrow{OP}$, then back along $\overrightarrow{AP}$.
- (a)(ii) The diagonals of a rhombus bisect each other, so P is the midpoint of AC and $\overrightarrow{PC}$ is the same vector as $\overrightarrow{AP}$. Then go from O to P to C .
- (b) Find $\overrightarrow{CD}$ by the route C to O to A to D , simplify it, and compare it with $\overrightarrow{OA}$. If one is a number times the other, the two sides are parallel.

Worked Solution [5 marks]

Rule - Vector routes: the vector from one point to another is the sum of the vectors along any route between them; going backwards along a vector changes its sign, so $\overrightarrow{PA} = -\overrightarrow{AP}$; and if $\overrightarrow{CD} = k\overrightarrow{OA}$ for a number k , then CD is parallel to OA .

Step 1: (a)(i) The route from O to A through P

$$\overrightarrow{OA} = \overrightarrow{OP} + \overrightarrow{PA}$$

$$\overrightarrow{PA} = -\overrightarrow{AP} = -\mathbf{n}$$

$$\overrightarrow{OA} = 2\mathbf{m} + (-\mathbf{n}) = 2\mathbf{m} - \mathbf{n}$$

(Reason: Both given vectors end at P , so the journey from O to A goes through it: first along $\overrightarrow{OP}$, then from P to A . The arrow labelled $\mathbf{n}$ points from A to P , and this part of the journey goes the other way, so it is $-\mathbf{n}$. The two terms are different vectors and cannot be combined, so $2\mathbf{m} - \mathbf{n}$ is already in its simplest form.)

Step 2: (a)(ii) P is the midpoint of AC

$$\overrightarrow{PC} = \overrightarrow{AP}$$

$$\overrightarrow{PC} = \mathbf{n}$$

(Reason: The diagonals of a rhombus bisect each other: they cut each other exactly in half. So P is halfway along AC , and the step from P to C carries on in the same direction as the step from A to P , for the same distance. Two steps with the same length and the same direction are the same vector.)

Step 3: (a)(ii) The route from O to C through P

$$\overrightarrow{OC} = \overrightarrow{OP} + \overrightarrow{PC}$$

$$\overrightarrow{OC} = 2\mathbf{m} + \mathbf{n}$$

(Reason: Out along $\overrightarrow{OP}$ as before, but this time on to C , which is forwards along $\mathbf{n}$ rather than backwards. Compare (a)(i): the only change is the sign of $\mathbf{n}$, because A and C lie on opposite sides of P along the same diagonal.)

Step 4: (b) A route from C to D

$$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AD}$$

$$\overrightarrow{CO} = -\overrightarrow{OC} = -(2\mathbf{m} + \mathbf{n})$$

$$\overrightarrow{CD} = -(2\mathbf{m} + \mathbf{n}) + 2\mathbf{m} - \mathbf{n} + 10\mathbf{m} - 3\mathbf{n}$$

(Reason: The vector $\overrightarrow{AD}$ is given, and the answers to (a) give the rest of a route: from C back to the origin, out to A , then on to D . Going from C to O is backwards along $\overrightarrow{OC}$, so the whole of $2\mathbf{m} + \mathbf{n}$ changes sign, and it stays in a bracket until that minus sign is dealt with.)

Step 5: (b) Expand the bracket and collect like terms

$$\overrightarrow{CD} = -2\mathbf{m} - \mathbf{n} + 2\mathbf{m} - \mathbf{n} + 10\mathbf{m} - 3\mathbf{n}$$

$$\overrightarrow{CD} = 10\mathbf{m} - 5\mathbf{n}$$

(Reason: The minus sign in front of the bracket multiplies both terms inside it, so $-(2\mathbf{m} + \mathbf{n})$ becomes $-2\mathbf{m} - \mathbf{n}$. Then the $\mathbf{m}$ terms and the $\mathbf{n}$ terms are collected separately: the numbers in front of $\mathbf{m}$ give $-2 + 2 + 10 = 10$, and the numbers in front of $\mathbf{n}$ give $-1 - 1 - 3 = -5$.)

Step 6: (b) Write $\overrightarrow{CD}$ as a multiple of $\overrightarrow{OA}$

$$\overrightarrow{CD} = 5(2\mathbf{m} - \mathbf{n})$$

$$\overrightarrow{CD} = 5\overrightarrow{OA}$$

(Reason: Both 10 and 5 are multiples of 5, so 5 is a common factor. Taking it out leaves $2\mathbf{m} - \mathbf{n}$, which is $\overrightarrow{OA}$ from (a)(i). When one vector is a number times another, the lines they run along are parallel. Here the number, 5, is positive, so the two vectors also point the same way.)

Step 7: (b) Why this makes $OADC$ a trapezium

$$CD \parallel OA$$

(Reason: Going round $OADC$ in order, its sides are OA , AD , DC and CO , so OA and DC are opposite sides. A quadrilateral with a pair of parallel opposite sides is a trapezium. The factor 5 also shows that CD is five times as long as OA , so the two parallel sides are different lengths and $OADC$ is not a parallelogram. In a show-that question the working is the answer, so every line is written down.)

$$(a)(i) \overrightarrow{OA} = 2\mathbf{m} - \mathbf{n}$$

$$(a)(ii) \overrightarrow{OC} = 2\mathbf{m} + \mathbf{n}$$

$$(b) \overrightarrow{CD} = 10\mathbf{m} - 5\mathbf{n} = 5\overrightarrow{OA}, \text{ so } CD \text{ is parallel to } OA \text{ and } OADC \text{ is a trapezium}$$

Verification

✓ **Check 1 - (a)(i) and (a)(ii) together:** The long diagonal OB is twice OP , because P is its midpoint, so $\overrightarrow{OB} = 4\mathbf{m}$. Going from O to A and then along AB , which is the same vector as $\overrightarrow{OC}$, also reaches B , so the two answers must add up to $4\mathbf{m}$.

$$(2\mathbf{m} - \mathbf{n}) + (2\mathbf{m} + \mathbf{n}) = 4\mathbf{m}$$

✓ **Check 2 - (b) by a route through P:** Go from C to D through P and A instead. C to P is back along $\mathbf{n}$, P to A is back along $\mathbf{n}$ again, and then A to D is given. This route uses only the vectors in the question, not the answers to (a).

$$\overrightarrow{CD} = -\mathbf{n} - \mathbf{n} + 10\mathbf{m} - 3\mathbf{n} = 10\mathbf{m} - 5\mathbf{n}, \text{ the same vector.}$$

✓ **Check 3 - with numbers:** Take a real rhombus: $O(0, 0)$, $A(3, 4)$, $B(8, 4)$ and $C(5, 0)$, with every side 5 long. Its diagonals cross at their midpoint $P(4, 2)$, so $2\mathbf{m} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$,

$$\mathbf{m} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ and } \mathbf{n} = \overrightarrow{AP} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}. \text{ Then } 2\mathbf{m} - \mathbf{n} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \text{ which is } \overrightarrow{OA}, \text{ and}$$

$$\overrightarrow{AD} = 10\mathbf{m} - 3\mathbf{n} = \begin{pmatrix} 17 \\ 16 \end{pmatrix}, \text{ which puts } D \text{ at } (20, 20).$$

$$\overrightarrow{CD} = \begin{pmatrix} 15 \\ 20 \end{pmatrix} = 5 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 5\overrightarrow{OA}$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The vector $\overrightarrow{OA}$	Note	The Answer column gives $2\mathbf{m} - \mathbf{n}$, and the Marks column gives 1 for it.	✓
(a)(ii) The vector $\overrightarrow{OC}$	Note	The Answer column gives $2\mathbf{m} + \mathbf{n}$, and the Marks column gives 1 for it.	✓
(b) The vector $\overrightarrow{CD}$	M2	For $\overrightarrow{CD} = 10\mathbf{m} - 5\mathbf{n}$. Allow M2 for equivalents: $\overrightarrow{CD} = -2\mathbf{n} + 10\mathbf{m} - 3\mathbf{n}$ or $\overrightarrow{CD} = -(2\mathbf{m} + \mathbf{n}) + 2\mathbf{m} - \mathbf{n} + 10\mathbf{m} - 3\mathbf{n}$. For M2, FT their (a), e.g. $\overrightarrow{CD} = \text{their } \overrightarrow{CO} + \text{their } \overrightarrow{OA} + 10\mathbf{m} - 3\mathbf{n}$.	✓
(b) In place of the M2: a correct route for $\overrightarrow{CD}$	M1	Or for correct route for $\overrightarrow{CD}$ using the lines of the diagram with $\overrightarrow{AD}$, e.g. $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$ oe.	✓

Step	Mark	Description	Got it?
(b) $\overrightarrow{CD}$ as a multiple of $\overrightarrow{OA}$	A1	For $\overrightarrow{CD} = 5\overrightarrow{OA}$ leading to CD is parallel to OA [$\therefore OACD$ is a trapezium]. Dependent on M2.	✓

Full marks: 5/5

Question 22 - Exam Solution

Understanding the Question

Given

The curve $y = x^n + qx^2 + 9x$, with two unknown constants, n and q

Its gradient function,

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

There is no calculator, so every step is done by hand, and every value stays exact.

Find

(a) The values of n and q (b) The coordinates of the turning points. The gradient function is a quadratic, so expect two points

Plan the Solution

- (a) Differentiate y term by term, keeping n and q as letters, then match the result term by term with $3x^2 - 12x + 9$.
- (b) At a turning point the gradient is zero, so solve $3x^2 - 12x + 9 = 0$ for x . Then put each value of x into the equation of the curve, using the values from (a), to find y .

Worked Solution [6 marks]

Rule - Differentiation and turning points: the derivative of ax^k is kax^{k-1} , and at a turning point the gradient is zero, so $\frac{dy}{dx} = 0$.

Step 1: (a) Differentiate $y = x^n + qx^2 + 9x$ term by term

$$\frac{dy}{dx} = nx^{n-1} + 2qx + 9$$

(Reason: Every term follows the same rule: multiply by the power, then take 1 off the power. So x^n becomes nx^{n-1} . In qx^2 the constant q simply stays as a multiplier, so that term becomes $2qx$, and $9x$

becomes 9.)

Step 2: (a) Match the terms with the given gradient function

$$nx^{n-1} + 2qx + 9 = 3x^2 - 12x + 9$$

$$n - 1 = 2 \implies n = 3$$

$$2q = -12 \implies q = -6$$

(Reason: The two expressions are equal for every value of x , so their terms must match one by one. The only term that can match $3x^2$ is nx^{n-1} : its power $n - 1$ must be 2, which gives $n = 3$, and its coefficient n must be 3, which agrees. The x terms give $2q = -12$, so $q = -6$, and the constant 9 is already the same on both sides.)

Step 3: (b) Set $\frac{dy}{dx}$ equal to zero

$$3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

(Reason: At a turning point the curve is momentarily flat, so its gradient is 0. Every coefficient is a multiple of 3, so dividing the whole equation by 3 gives a simpler quadratic with exactly the same solutions.)

Step 4: (b) Factorise and solve

$$(x - 1)(x - 3) = 0$$

$$x - 1 = 0 \implies x = 1$$

$$x - 3 = 0 \implies x = 3$$

(Reason: The two numbers that multiply to 3 and add to -4 are -1 and -3 . A product is zero only when one of its factors is zero, which gives the two values of x . The quadratic formula on

$$3x^2 - 12x + 9 = 0 \text{ gives the same pair: } x = \frac{12 \pm \sqrt{144 - 108}}{6} = \frac{12 \pm 6}{6}, \text{ which is 3 or 1.})$$

Step 5: (b) Find each y -coordinate from the equation of the curve

$$y = x^3 - 6x^2 + 9x$$

$$1^3 - 6 \times 1^2 + 9 \times 1 = 1 - 6 + 9 = 4$$

$$3^3 - 6 \times 3^2 + 9 \times 3 = 27 - 54 + 27 = 0$$

(Reason: With $n = 3$ and $q = -6$ from (a), the curve is $y = x^3 - 6x^2 + 9x$. The y -coordinates come from this equation, not from $\frac{dy}{dx}$, which is 0 at both points. Powers are worked out before multiplying: $3^3 = 27$ and $6 \times 3^2 = 6 \times 9 = 54$. So the turning points are $(1, 4)$ and $(3, 0)$.)

$$\text{(a) } n = 3, q = -6$$

$$\text{(b) } (1, 4) \text{ and } (3, 0)$$

Verification

- ✓ **Check 1 - (a) differentiate the finished curve:** With $n = 3$ and $q = -6$ the curve is $y = x^3 - 6x^2 + 9x$. Differentiating it term by term should give back the gradient function the question states. $\frac{dy}{dx} = 3x^2 - 2 \times 6x + 9 = 3x^2 - 12x + 9$
- ✓ **Check 2 - (b) the gradient is zero at $x = 1$ and $x = 3$:** Put each value of x into $3x^2 - 12x + 9$. Both should give 0. $3 \times 1^2 - 12 \times 1 + 9 = 0$ and $3 \times 3^2 - 12 \times 3 + 9 = 0$
- ✓ **Check 3 - (b) the curve in factorised form:** Taking out the common factor x gives $y = x(x^2 - 6x + 9) = x(x - 3)^2$. This form finds each y -coordinate by a different calculation. $1 \times (1 - 3)^2 = 4$ and $3 \times (3 - 3)^2 = 0$
- ✓ **Check 4 - (b) both points really are turning points:** The gradient should change sign at each turning point. At $x = 0$ and $x = 4$, outside the two points, $3x^2 - 12x + 9$ is 9. At $x = 2$, between them, it should be negative. $3 \times 2^2 - 12 \times 2 + 9 = -3$: positive, negative, positive, so the curve rises to $(1, 4)$, falls to $(3, 0)$ and rises again

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The values of n and q	Note	The Answer column gives $[n =] 3$, $[q =] -6$, and the Marks column gives 2 for it.	✓
(a) Each value, n or q	B1	For each correct value.	✓
(b) The two turning points	Note	The Answer column gives $(1, 4)$ and $(3, 0)$, and the Marks column gives 4 for it.	✓
(b) One correct point, or both values of x	B3	For $(1, 4)$ or $(3, 0)$, or for two correct values of x .	✓
(b) In place of the B3: the equation factorised, or the quadratic formula	M2	Or for $[3](x - 1)(x - 3) [= 0]$ oe, or $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 3 \times 9}}{2 \times 3}$ oe.	✓
(b) In place of the B3 or the M2: the gradient set to zero	M1	Or for writing $\frac{dy}{dx} = 0$, or for $3x^2 - 12x + 9 = 0$.	✓

Full marks: 6/6

Question 23 - Exam Solution

Understanding the Question

Given

$\frac{2x^2 + 10x}{x^2 - 25}$, an algebraic fraction with a

quadratic expression on the top and another on the bottom

No calculator is allowed, and none is needed: every step is factorising by hand.

Find

The fraction in its simplest form, where the top and the bottom have no factor in common.

Plan the Solution

- Factorise the top by taking out the highest common factor of $2x^2$ and $10x$.
- Factorise the bottom as a difference of two squares, because $25 = 5^2$.
- Divide the top and the bottom by the bracket they share.

Worked Solution [3 marks]

Rule - Simplifying an algebraic fraction: factorise the top and the bottom fully, then cancel any factor they have in common. Only a factor of the whole top and of the whole bottom may be cancelled, never a single term.

Step 1: Factorise the numerator

$$2x^2 + 10x = 2x \times x + 2x \times 5$$

$$2x^2 + 10x = 2x(x + 5)$$

(Reason: The highest common factor of $2x^2$ and $10x$ is $2x$: the highest common factor of the numbers 2 and 10 is 2, and both terms contain x . Dividing each term by $2x$ gives $\frac{2x^2}{2x} = x$ and $\frac{10x}{2x} = 5$, and those two go inside the bracket. This factorised top is what the first B1 on the mark scheme is for.)

Step 2: Factorise the denominator

$$x^2 - 25 = x^2 - 5^2$$

$$x^2 - 25 = (x - 5)(x + 5)$$

(Reason: 25 is 5^2 , so $x^2 - 25$ is a difference of two squares, and $a^2 - b^2 = (a - b)(a + b)$ with $a = x$ and $b = 5$. Multiplying the brackets back out, $-5x$ and $+5x$ cancel and leave $x^2 - 25$. This factorised bottom is what the second B1 is for.)

Step 3: Cancel the common factor

$$\frac{2x^2 + 10x}{x^2 - 25} = \frac{2x(x + 5)}{(x - 5)(x + 5)}$$

$$\frac{2x(x + 5)}{(x - 5)(x + 5)} = \frac{2x}{x - 5}$$

(Reason: The bracket $(x + 5)$ is a factor of the whole top and of the whole bottom, so both are divided by it. That is allowed: the fraction only has a value when $x^2 - 25$ is not 0, and then $x + 5$ is not 0 either. What is left has no factor in common, so it is fully simplified. The x on the top does not cancel with the x in $x - 5$: there it is a term of the bottom, not a factor of it.)

$$\frac{2x}{x - 5}$$

Verification

- ✓ **Check 1 - put $x = 10$ into both:** The simplified fraction must equal the original for every value of x where the original has a value. With $x = 10$ the original gives

$$\frac{2 \times 10^2 + 10 \times 10}{10^2 - 25} = \frac{300}{75} = 4, \quad \frac{2 \times 10}{10 - 5} = \frac{20}{5} = 4$$

- ✓ **Check 2 - put $x = 3$ into both:** A second value, this time one that makes the bottom negative. With $x = 3$ the original gives $\frac{2 \times 3^2 + 10 \times 3}{3^2 - 25} = \frac{48}{-16} = -3$.

$$\frac{2 \times 3}{3 - 5} = \frac{6}{-2} = -3$$

- ✓ **Check 3 - rebuild the original:** Multiplying the top and the bottom of the answer by $(x + 5)$ must give back the fraction the question started with: $2x(x + 5) = 2x^2 + 10x$ on the top and $(x - 5)(x + 5) = x^2 - 25$ on the bottom.

$$\frac{2x(x + 5)}{(x - 5)(x + 5)} = \frac{2x^2 + 10x}{x^2 - 25}$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The simplified fraction	Note	The Answer column gives $\frac{2x}{x - 5}$ final answer, and the Marks column gives 3 for it.	✓

Step	Mark	Description	Got it?
The top factorised	B1	For $2x(x + 5)$.	✓
The bottom factorised	B1	For $(x - 5)(x + 5)$.	✓

Full marks: 3/3

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