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Circle Theorems

GCSE & IGCSE Practice Questions with Worked Solutions

GRADES 8-9 TARGETED

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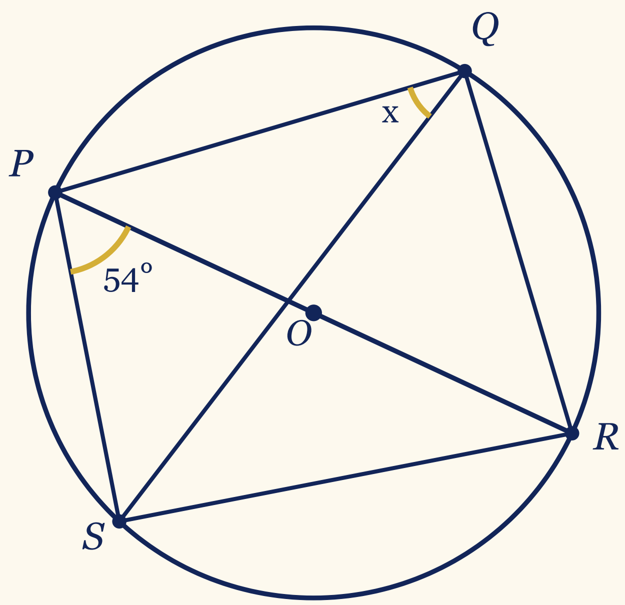
14 exam-style questions with full worked solutions and mark schemes.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1.

GRADE 8



Not drawn accurately

P, Q, R and S are points on a circle, centre O. POR is a straight line, so PR is a diameter. $\angle SPR = 54°$. Work out the size of the angle marked x ($\angle PQS$).

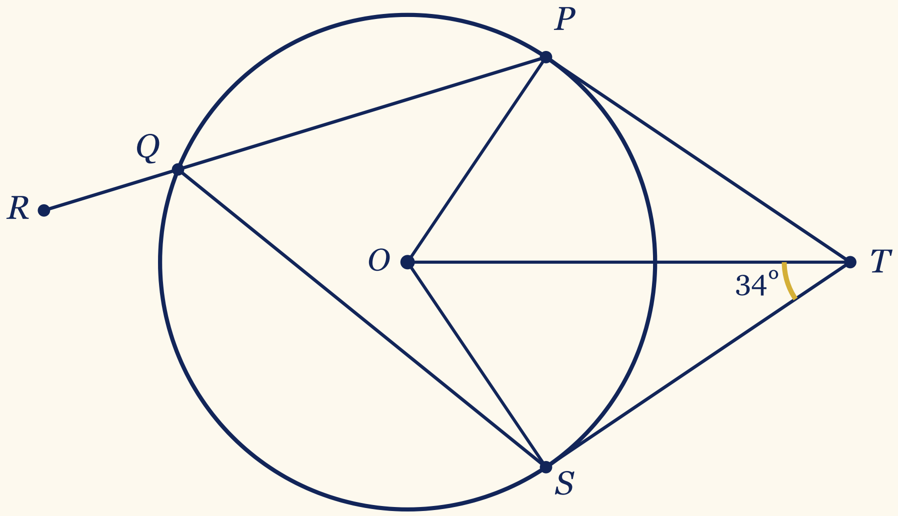
$x =$ .... °

[Total 3 marks]

2.

GRADE 8

Problem solving



Not drawn accurately

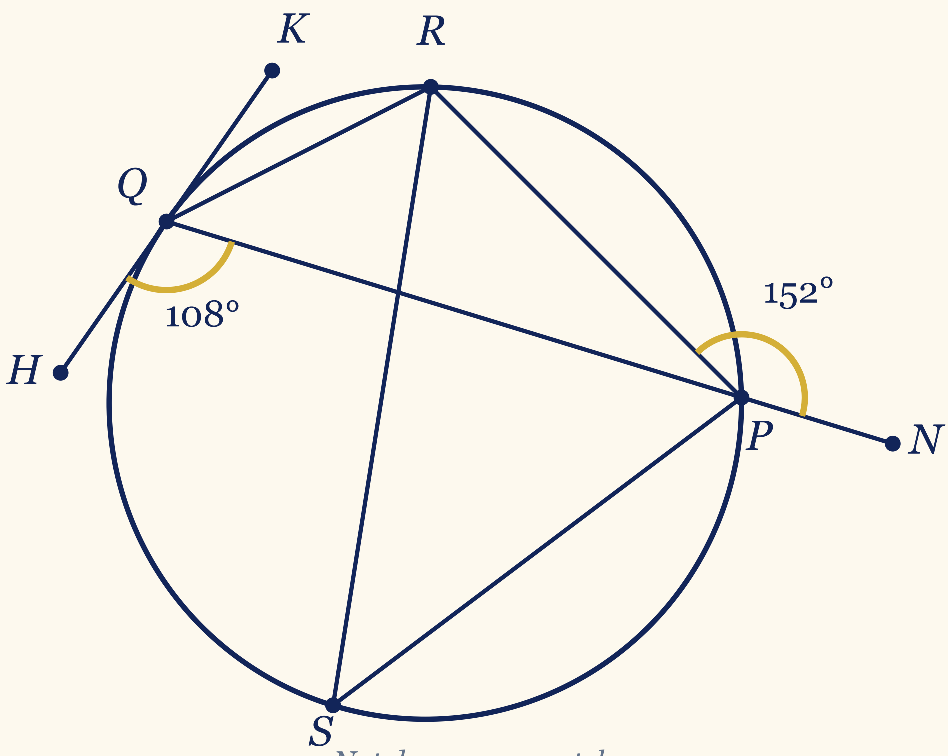
The diagram shows a circle, centre O. P, Q and S are points on the circle. TP and TS are tangents to the circle, and PQR is a straight line. $\angle STO = 34^\circ$. Work out the size of $\angle SQR$, giving reasons.

$\angle SQR =$ .... °

[Total 5 marks]

3.

GRADE 9



Not drawn accurately

P, Q, R and S lie on a circle. NPQ is a straight line and HK is the tangent to the circle at Q. $\angle HQP = 108^\circ$ and $\angle NPR = 152^\circ$. Find the size of $\angle RSP$, giving a reason for each step.

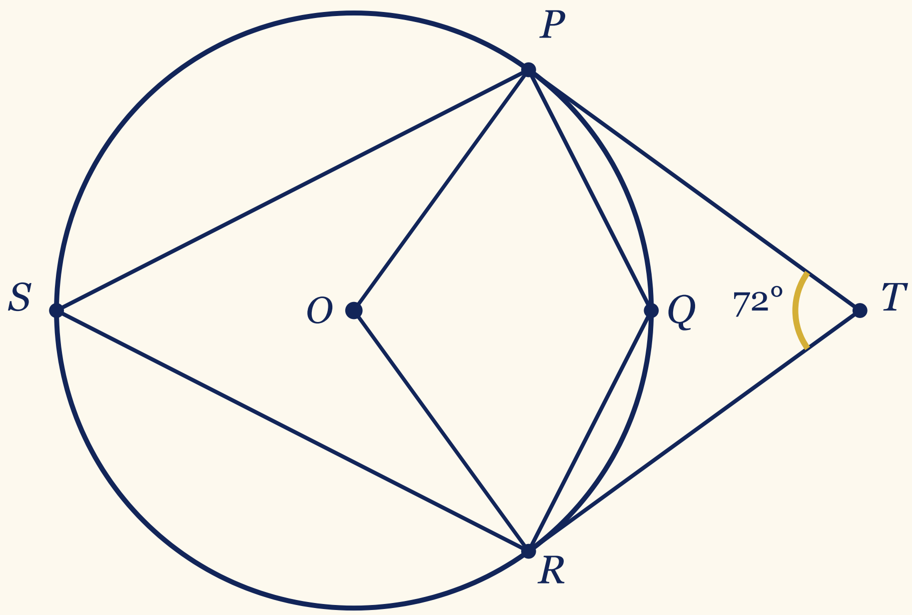
$\angle RSP =$ .... °

[Total 4 marks]

GRADE 8

Problem solving

4.



Not drawn accurately

The diagram shows a circle, centre O. P, Q, R and S are points on the circle. PT and RT are tangents from T. $\angle PTR = 72°$. Find the size of $\angle PQR$, giving reasons.

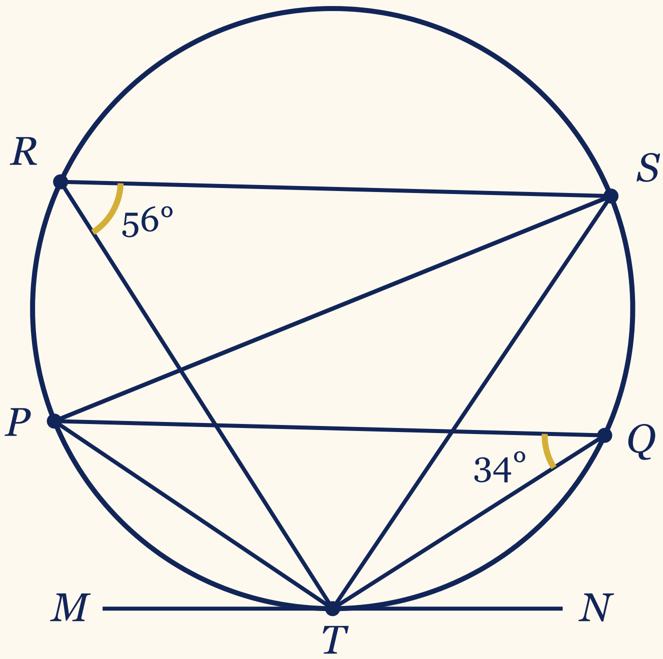
$\angle PQR =$ .... °

[Total 4 marks]

5.

GRADE 9

Problem solving



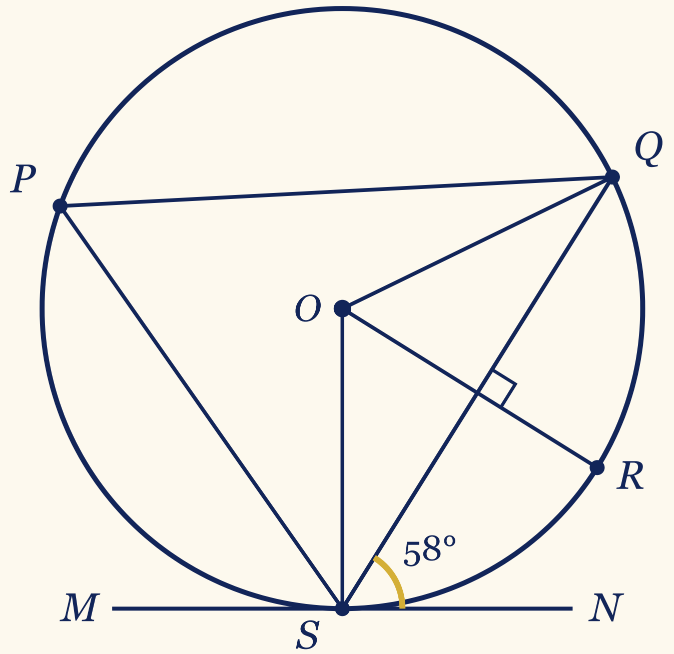
Not drawn accurately

P, Q, R, S and T lie on a circle. MN is the tangent to the circle at T. $\angle PQT = 34°$ and $\angle SRT = 56°$. Prove that the chord PS passes through the centre of the circle.

[Total 3 marks]

6.

GRADE 8



Not drawn accurately

P, Q, R and S lie on a circle, centre O. MN is the tangent at S.
$\angle QSN = 58°$.

(a) Work out the size of $\angle SOQ$. *[2 marks]*

(b) Explain why $\angle ROQ$ is half the size of $\angle SOQ$. *[2 marks]*

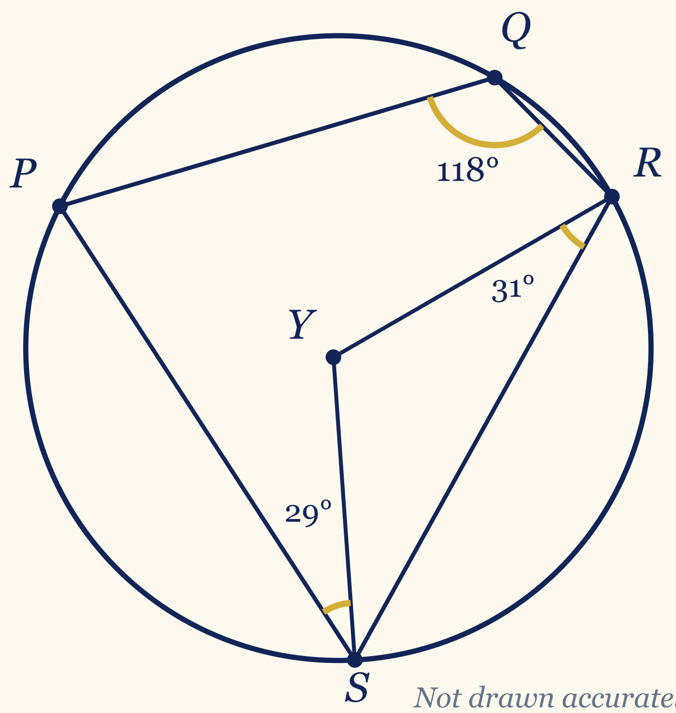
(a) $\angle SOQ =$ .... °

[Total 4 marks]

7.

GRADE 9

Problem solving



Not drawn accurately

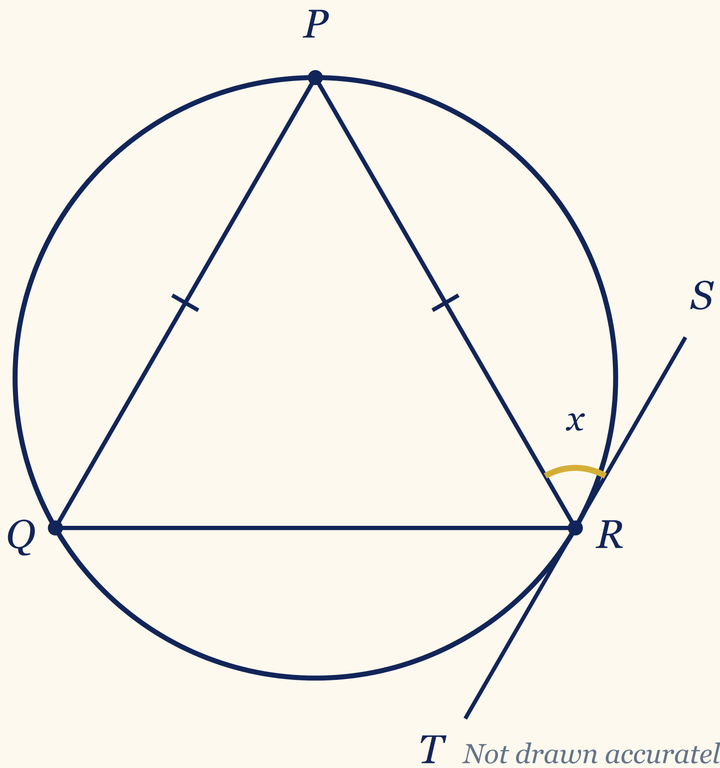
P, Q, R and S lie on a circle. Y is a point inside the circle, joined to S and R. $\angle PQR = 118°$, $\angle YSP = 29°$ and $\angle YRS = 31°$. Show that Y is not the centre of the circle.

[Total 3 marks]

8.

GRADE 8

Problem solving



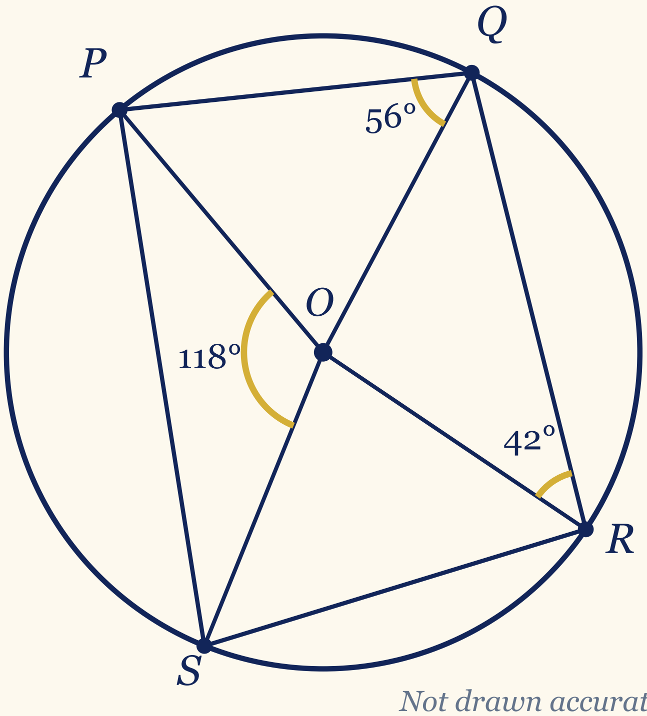
Not drawn accurately

P, Q and R lie on a circle with $PQ = PR$. ST is the tangent at R, and ST is parallel to PQ. $\angle SRP = x$. Prove that triangle PQR is an equilateral triangle.

[Total 4 marks]

9.

GRADE 8



Not drawn accurately

The diagram shows a circle, centre O. P, Q, R and S lie on the circle. $\angle POS = 118°$, $\angle PQO = 56°$ and $\angle ORQ = 42°$. Find the size of $\angle RSO$.

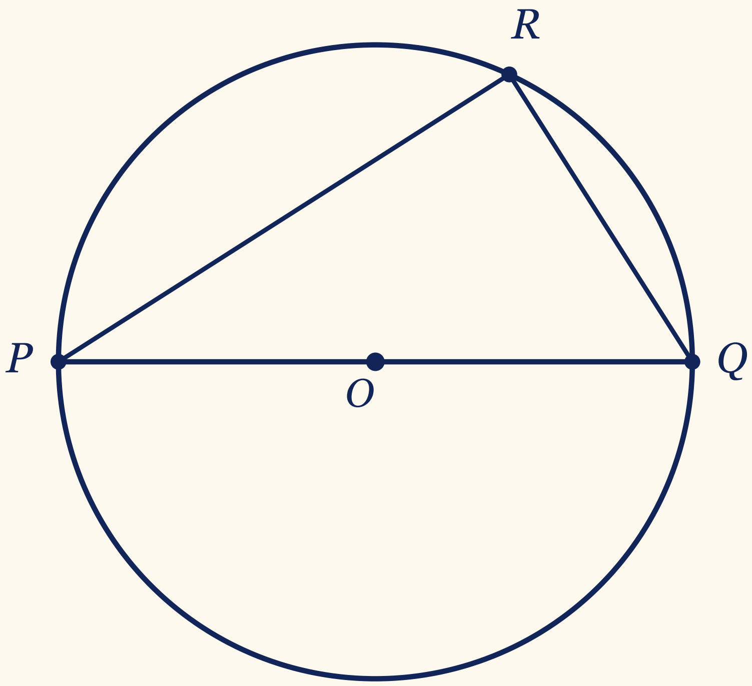
$\angle RSO =$ .... °

[Total 4 marks]

10.

GRADE 8

Problem solving



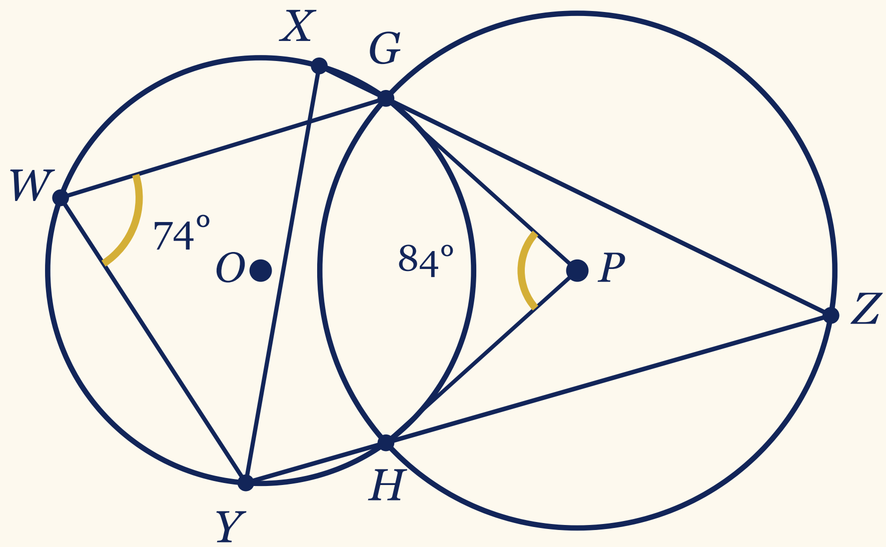
Not drawn accurately

The diagram shows a circle with centre O. PQ is a diameter and R is a point on the circumference. Prove that $\angle PRQ = 90°$.

[Total 3 marks]

11.

GRADE 9



Not drawn accurately

The diagram shows two intersecting circles with centres O and P. The circles intersect at G and H. W, X and Y are points on the circle with centre O, and Z is a point on the circle with centre P. XZ and ZY are straight lines. $\angle GWY = 74^\circ$ and $\angle GPH = 84^\circ$. Find the size of $\angle XYH$, giving reasons.

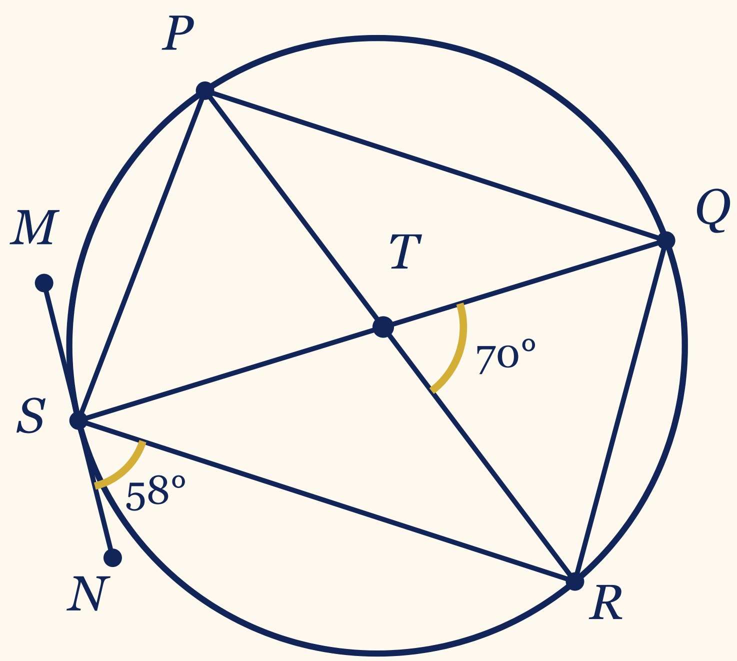
$\angle XYH =$ .... °

[Total 3 marks]

12.

GRADE 9

Problem solving



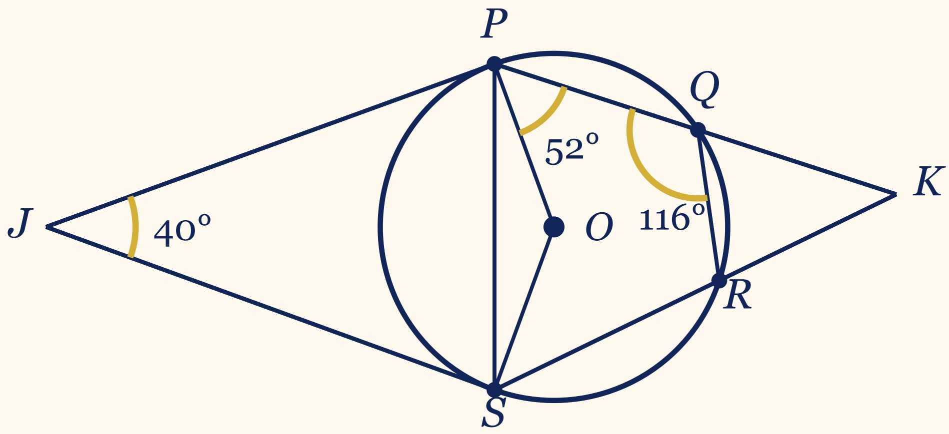
Not drawn accurately

P, Q, R and S lie on a circle. MN is a tangent at S, and PR and QS are straight lines meeting at T. $\angle QTR = 70°$ and $\angle NSR = 58°$. Show that T is not the centre of the circle.

[Total 4 marks]

13.

GRADE 9



Not drawn accurately

The diagram shows a circle, centre O. P, Q, R and S lie on the circle. JP and JS are tangents from J. PK and SK are straight lines. $\angle PJS = 40°$, $\angle OPQ = 52°$ and $\angle PQR = 116°$. Find the size of $\angle QKR$.

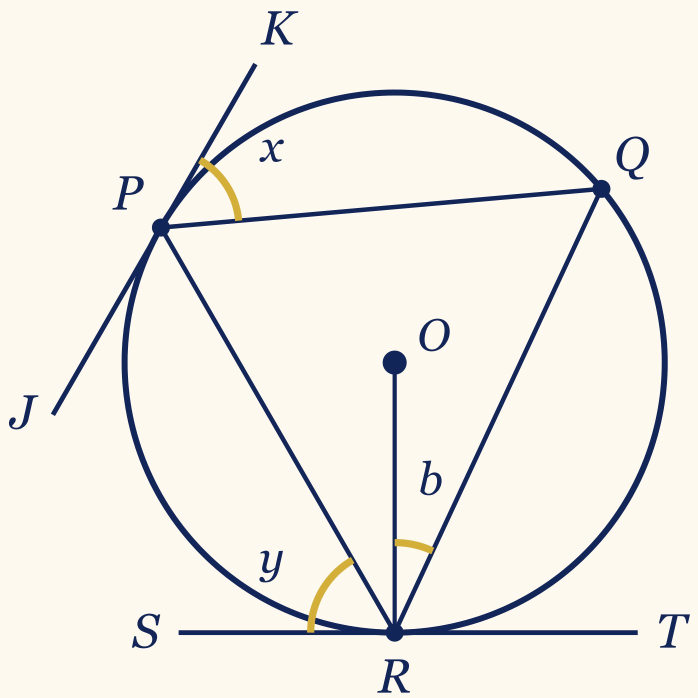
$\angle QKR =$ .... °

[Total 5 marks]

14.

GRADE 9

Problem solving



Not drawn accurately

The diagram shows a circle, centre O. JK is a tangent at P and ST is a tangent at R. Here x is the tangent-chord angle at P, y is the tangent-chord angle at R, and $b = \angle ORQ$. Prove that $b = x + y - 90°$. State any circle theorems that you use.

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

P, Q, R, S lie on a circle, centre O;
POR is a straight line, so PR is a diameter.
$\angle SPR = 54^\circ$.

Find

The size of the angle marked x ($\angle PQS$).

Plan the Solution

- PR is a diameter, so $\angle PQR$ is the angle in a semicircle and equals 90°.
- Use angles in the same segment to find $\angle SQR$.
- Subtract to find x.

Worked Solution [3 marks]

> **Rule - Angle chasing:** apply the angle in a semicircle and angles in the same segment, then subtract to reach the required angle.

Step 1: Use the diameter (angle in a semicircle).

$\angle PQR = 90^\circ$

(Reason: PR is a diameter, and the angle in a semicircle is 90°.)

Step 2: Angles in the same segment.

$\angle SQR = \angle SPR = 54^\circ$

(Reason: $\angle SQR$ and $\angle SPR$ both stand on the chord SR, so angles in the same segment are equal.)

Step 3: Subtract to find x.

$x = \angle PQR - \angle SQR = 90^\circ - 54^\circ = 36^\circ$

(Reason: x is the part of $\angle PQR$ that remains after $\angle SQR$ is removed.)

$$x = 36^\circ$$

Verification

✓ **Check 1:** $\angle PQS + \angle SQR = 36^\circ + 54^\circ = 90^\circ = \angle PQR$

✓ **Check 2:** $x = 36^\circ$ is acute and smaller than the right angle at Q, which fits the diagram.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle SQR = 54°$	**M1**	Method - angles in the same segment	✓
$\angle PQR = 90°$	**M1**	Method - the angle in a semicircle	✓
$x = 36°$	**A1**	Accuracy - correct final answer	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

Circle, centre O; P, Q, S on the circle. TP and TS are tangents from T; PQR is a straight line. $\angle STO = 34^\circ$.

Find

The size of $\angle SQR$.

Plan the Solution

- Tangent meets radius: $\angle TSO = 90^\circ$, then find $\angle TOS$ in triangle TSO.
- The two equal tangents double the angle at the centre: $\angle POS = 2 \times \angle TOS$.
- Angle at the centre is twice the angle at the circumference: find $\angle PQS$.
- Angles on a straight line: find $\angle SQR$.

Worked Solution [5 marks]

> **Rule - Work in from the tangent:** use the tangent-radius right angle, the symmetry of the two equal tangents, and the centre-to-circumference link, then finish on the straight line.

Step 1: Tangent meets radius.

$\angle TSO = 90^\circ$

(Reason: OS is a radius and TS is a tangent; a tangent meets a radius at 90°.)

Step 2: Angles in triangle TSO.

$\angle TOS = 180^\circ - 90^\circ - 34^\circ = 56^\circ$

(Reason: The angles in triangle TSO add up to 180°.)

Step 3: Two equal tangents (angle at the centre).

$\angle POS = 2 \times 56^\circ = 112^\circ$

(Reason: TP and TS are equal tangents from T, so triangles TPO and TSO are congruent; this doubles the angle at O.)

Step 4: Angle at the centre is twice the angle at the circumference.

$\angle PQS = \frac{1}{2} \times 112^\circ = 56^\circ$

(Reason: $\angle POS$ is at the centre and $\angle PQS$ is at the circumference, both standing on arc PS.)

Step 5: Angles on a straight line.

$\angle SQR = 180^\circ - 56^\circ = 124^\circ$

(Reason: PQR is a straight line, so the angles at Q add up to 180°.)

$$\angle SQR = 124^\circ$$

Verification

✓ **Check 1:** In triangle TSO: $90^\circ + 34^\circ + 56^\circ = 180^\circ$.

✓ **Check 2:** Quadrilateral $OPTS$: $90^\circ + 68^\circ + 90^\circ + 112^\circ = 360^\circ$ (with $\angle PTS = 2 \times 34^\circ = 68^\circ$).

✓ **Check 3:** At Q: $\angle PQS + \angle SQR = 56^\circ + 124^\circ = 180^\circ$, matching the straight line.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle TSO = 90^\circ$	**M1**	Method - tangent meets a radius at 90 degrees	✓
$\angle TOS = 56^\circ$	**M1**	Method - angles in a triangle	✓
$\angle POS = 112^\circ$	**M1**	Method - equal tangents give the angle at the centre	✓
$\angle PQS = 56^\circ$	**M1**	Method - angle at the centre is twice the circumference	✓
$\angle SQR = 124^\circ$	**A1**	Accuracy - correct final answer	✓

Full marks: 5/5

Question 3 - Exam Solution

Understanding the Question

Given

P, Q, R, S lie on a circle. NPQ is a straight line and HK is the tangent at Q. $\angle HQP = 108^\circ$ and $\angle NPR = 152^\circ$.

Find

The size of $\angle RSP$, giving a reason for each step.

Plan the Solution

- Alternate segment theorem: the tangent-chord angle $\angle HQP$ equals $\angle QRP$.
- Angles on a straight line at P: find $\angle RPQ$.
- Angles in triangle PQR: find $\angle PQR$.
- Angles in the same segment: $\angle RSP = \angle PQR$.

Worked Solution [4 marks]

> **Rule - Transfer angles round the circle:** use the alternate segment theorem, a straight line and a triangle, then move the angle to the required point with the same-segment rule.

Step 1: Alternate segment theorem.

$\angle QRP = 108^\circ$

(Reason: $\angle HQP$ is the angle between the tangent HK and the chord QP; by the alternate segment theorem it equals $\angle QRP$ in the alternate segment.)

Step 2: Angles on a straight line at P.

$\angle RPQ = 180^\circ - 152^\circ = 28^\circ$

(Reason: NPQ is a straight line, so $\angle NPR$ and $\angle RPQ$ add up to 180°.)

Step 3: Angles in triangle PQR.

$\angle PQR = 180^\circ - 108^\circ - 28^\circ = 44^\circ$

(Reason: The angles in triangle PQR add up to 180°.)

Step 4: Angles in the same segment.

$\angle RSP = \angle PQR = 44^\circ$

(Reason: $\angle RSP$ and $\angle PQR$ both stand on the chord RP, so angles in the same segment are equal.)

$$\angle RSP = 44^\circ$$

Verification

✓ **Check 1:** Triangle PQR: $108^\circ + 28^\circ + 44^\circ = 180^\circ$.

✓ **Check 2:** Straight line NPQ at P: $\angle NPR + \angle RPQ = 152^\circ + 28^\circ = 180^\circ$.

✓ **Check 3:** The same-segment angles agree: $\angle RSP = \angle PQR = 44^\circ$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle QRP = 108^\circ$	M1	Method - alternate segment theorem	✓
$\angle RPQ = 28^\circ$	M1	Method - angles on a straight line	✓
$\angle PQR = 44^\circ$	M1	Method - angles in a triangle	✓
$\angle RSP = 44^\circ$	A1	Accuracy - correct final answer	✓

Full marks: 4/4

Question 4 - Exam Solution

Understanding the Question

Given

Circle, centre O; P, Q, R, S on the circle.

PT and RT are tangents from T.

$\angle PTR = 72^\circ$.

Find

The size of $\angle PQR$, giving reasons.

Plan the Solution

- Tangent meets radius: both $\angle OPT$ and $\angle ORT$ are 90°.
- Angles in quadrilateral $OPTR$ add to 360°: find the central angle $\angle POR$.
- Angle at the centre is twice the angle at the circumference: find $\angle PSR$.
- Opposite angles of a cyclic quadrilateral: find $\angle PQR$.

Worked Solution [4 marks]

> **Rule - From the tangents to the far angle:** use the two tangent-radius right angles inside the quadrilateral, then the centre-to-circumference link, and finish with the cyclic-quadrilateral rule.

Step 1: Tangent meets radius (two right angles).

$\angle OPT = \angle ORT = 90^\circ$

(Reason: OP and OR are radii and PT, RT are tangents; a tangent meets a radius at 90°.)

Step 2: Angles in quadrilateral OPTR.

$\angle POR = 360^\circ - 90^\circ - 90^\circ - 72^\circ = 108^\circ$

(Reason: $OPTR$ is a quadrilateral, and its angles add up to 360°.)

Step 3: Angle at the centre is twice the angle at the circumference.

$\angle PSR = \frac{1}{2} \times 108^\circ = 54^\circ$

(Reason: $\angle POR$ is at the centre and $\angle PSR$ is at the circumference, both standing on arc PR.)

Step 4: Opposite angles of a cyclic quadrilateral.

$\angle PQR = 180^\circ - 54^\circ = 126^\circ$

(Reason: $PQRS$ is a cyclic quadrilateral, so the opposite angles $\angle PQR$ and $\angle PSR$ add up to 180°.)

$$\angle PQR = 126^\circ$$

Verification

✓ **Check 1:** Quadrilateral $OPTR$: $90^\circ + 90^\circ + 72^\circ + 108^\circ = 360^\circ$.

✓ **Check 2:** Opposite angles: $\angle PQR + \angle PSR = 126^\circ + 54^\circ = 180^\circ$.

✓ **Check 3:** The angle at the centre (108°) is exactly twice the angle at the circumference (54°).

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle OPT = \angle ORT = 90^\circ$	**M1**	Method - tangent meets a radius at 90 degrees	✓
$\angle POR = 108^\circ$	**M1**	Method - angles in a quadrilateral add to 360 degrees	✓
$\angle PSR = 54^\circ$	**M1**	Method - angle at the centre is twice the circumference	✓
$\angle PQR = 126^\circ$	**A1**	Accuracy - correct final answer	✓

Full marks: 4/4

Question 5 - Exam Solution

Understanding the Question

Given

P, Q, R, S, T lie on a circle. MN is the tangent at T.

$\angle PQT = 34°$ and $\angle SRT = 56°$.

Prove

That the chord PS passes through the centre of the circle.

Plan the Solution

- Alternate segment theorem: find the two tangent-chord angles $\angle PTM$ and $\angle STN$.
- Angles on the straight line MTN: find $\angle PTS$.
- If $\angle PTS = 90°$, then PS is a diameter (angle in a semicircle), so it passes through the centre.

Worked Solution [3 marks]

> **Rule - A right angle means a diameter:** show that the angle at T is $90°$; the converse of the angle in a semicircle then forces PS to be a diameter through the centre.

Step 1: Alternate segment theorem.

$$\angle PTM = \angle PQT = 34° \quad \text{and} \quad \angle STN = \angle SRT = 56°$$

(Reason: $\angle PTM$ is the tangent-chord angle for chord TP, so it equals $\angle PQT$ in the alternate segment; similarly $\angle STN = \angle SRT$.)

Step 2: Angles on a straight line.

$$\angle PTS = 180° - 34° - 56° = 90°$$

(Reason: MTN is a straight line, so $\angle PTM + \angle PTS + \angle STN = 180°$.)

Step 3: Angle in a semicircle (converse).

$$\angle PTS = 90° \Rightarrow PS \text{ is a diameter}$$

(Reason: The angle in a semicircle is $90°$; conversely, if an inscribed angle is $90°$ then the chord opposite it is a diameter, which passes through the centre.)

Therefore PS is a diameter, so it passes through the centre of the circle.

Verification

✓ **Check 1:** Angles on the straight line at T: $34^\circ + 90^\circ + 56^\circ = 180^\circ$.

✓ **Check 2:** The two given angles satisfy $34^\circ + 56^\circ = 90^\circ$, which is why $\angle PTS = 180^\circ - 90^\circ = 90^\circ$.

✓ **Check 3:** $\angle PTS = 90^\circ$ is the angle in a semicircle, confirming PS is a diameter.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle PTM = 34^\circ$ and $\angle STN = 56^\circ$	M1	Method - alternate segment theorem	✓
$\angle PTS = 90^\circ$	M1	Method - angles on a straight line	✓
PS is a diameter through the centre	A1	Accuracy - angle in a semicircle, so PS is a diameter	✓

Full marks: 3/3

Question 6 - Exam Solution

Understanding the Question

Given

P, Q, R, S lie on a circle, centre O;
MN is the tangent at S.
$\angle QSN = 58^\circ$.

Find

(a) The size of $\angle SOQ$.
(b) Why $\angle ROQ$ is half the size of $\angle SOQ$.

Plan the Solution

- Part (a): alternate segment theorem, then the centre-to-circumference link.
- Part (b): the radius perpendicular to a chord bisects both the chord and the angle at the centre.

Part (a): Work out $\angle SOQ$ [2 marks]

Rule - Alternate segment, then double at the centre: transfer the tangent-chord angle to the circumference, then use the centre-to-circumference link.

Step 1: Alternate segment theorem.

$\angle QPS = \angle QSN = 58^\circ$

(Reason: $\angle QSN$ is the tangent-chord angle for chord SQ, equal to $\angle QPS$ in the alternate segment.)

Step 2: Angle at the centre is twice the angle at the circumference.

$\angle SOQ = 2 \times 58^\circ = 116^\circ$

(Reason: $\angle SOQ$ is at the centre and $\angle QPS$ is at the circumference, both standing on arc SQ.)

$$\angle SOQ = 116^\circ$$

MARK SCHEME

Step	Mark	Description	Got it?
$\angle QPS = 58^\circ$	M1	Method - alternate segment theorem	✓
$\angle SOQ = 116^\circ$	A1	Accuracy - angle at centre is twice the circumference	✓

Full marks: 2/2

Part (b): Explain why $\angle ROQ = \frac{1}{2}\angle SOQ$ [2 marks]

Rule - Isosceles symmetry: the radius that is perpendicular to a chord bisects both the chord and the angle at the centre.

Step 1: Isosceles triangle and perpendicular bisector.

$OQ = OS$ (radii), so triangle OSQ is isosceles. OR meets chord SQ at right angles, so it bisects SQ.

(Reason: All radii are equal, and the perpendicular from the centre of a circle to a chord bisects the chord.)

Step 2: Bisecting the base bisects the angle at O.

Bisecting the base of the isosceles triangle also bisects the apex angle, so $\angle ROQ = \frac{1}{2} \times \angle SOQ$.

(Reason: In an isosceles triangle, the line from the apex perpendicular to the base bisects the apex angle.)

Therefore $\angle ROQ = \frac{1}{2} \times \angle SOQ$.

MARK SCHEME

Step	Mark	Description	Got it?
Triangle OSQ isosceles and OR bisects SQ	M1	Reason - perpendicular from the centre bisects a chord	✓
$\angle ROQ = \frac{1}{2}\angle SOQ$	A1	Reason - the bisector halves the apex angle	✓

Full marks: 2/2

Verification

✓ **Check 1 (a):** $\angle SOQ = 2 \times \angle QPS = 2 \times 58^\circ = 116^\circ$

✓ **Check 2 (b):** OR splits triangle OSQ symmetrically, so $\angle ROQ = \angle SOR = \frac{1}{2} \times 116^\circ = 58^\circ$, which is half of $\angle SOQ$.

Question 7 - Exam Solution

Understanding the Question

Given

P, Q, R, S lie on a circle. Y is inside, joined to S and R.

$\angle PQR = 118°$, $\angle YSP = 29°$, $\angle YRS = 31°$.

Show

That Y is not the centre of the circle.

Plan the Solution

- Cyclic quadrilateral $PQRS$: find $\angle PSR$.
- Subtract to find $\angle RSY$.
- If Y were the centre, YS and YR would be radii, giving equal base angles. Check whether $\angle RSY = \angle SRY$.

Worked Solution [3 marks]

> **Rule - Assume and contradict:** suppose Y is the centre; two radii would force equal base angles, so if the angles differ, Y cannot be the centre.

Step 1: Opposite angles of a cyclic quadrilateral.

$\angle PSR = 180° - 118° = 62°$

(Reason: $PQRS$ is a cyclic quadrilateral, so the opposite angles $\angle PQR$ and $\angle PSR$ add up to $180°$.)

Step 2: Subtract to find angle RSY.

$\angle RSY = \angle PSR - \angle PSY = 62° - 29° = 33°$

(Reason: $\angle PSY$ is part of $\angle PSR$, so subtracting it leaves $\angle RSY$.)

Step 3: Test the "centre" assumption.

If Y were the centre, YS and YR would be radii, so triangle RSY would be isosceles with $\angle RSY = \angle SRY$. But $\angle RSY = 33°$ and $\angle SRY = 31°$.

(Reason: A triangle made from two radii is isosceles, so its base angles would be equal - but here they are not.)

Since $\angle RSY = 33° \neq 31° = \angle SRY$, Y is not the centre of the circle.

Verification

✓ **Check 1:** Cyclic quadrilateral: $\angle PQR + \angle PSR = 118^\circ + 62^\circ = 180^\circ$.

✓ **Check 2:** The parts add up: $\angle PSY + \angle RSY = 29^\circ + 33^\circ = 62^\circ = \angle PSR$.

✓ **Check 3:** Two radii would force $\angle RSY = \angle SRY$, but $33^\circ \neq 31^\circ$, confirming Y is not the centre.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle PSR = 62^\circ$	**M1**	Reason - opposite angles in a cyclic quadrilateral	✓
$\angle RSY = 33^\circ$	**M1**	Method - angle subtraction	✓
$33^\circ \neq 31^\circ$, so Y is not the centre	**A1**	Reason - two radii would give equal base angles	✓

Full marks: 3/3

Question 8 - Exam Solution

Understanding the Question

Given

Triangle PQR with P, Q, R on a circle; $PQ = PR$.

ST is a tangent at R, parallel to PQ; $\angle SRP = x$.

Prove

That triangle PQR is an equilateral triangle.

Plan the Solution

- Alternate segment theorem: $\angle PQR = x$.
- Isosceles triangle ($PQ = PR$): $\angle PRQ = x$.
- Alternate angles (ST parallel to PQ): $\angle QPR = x$.
- All three angles equal, so the triangle is equilateral.

Worked Solution [4 marks]

> **Rule - Show all three angles are equal:** a triangle is equilateral when its three angles are equal; find each angle in turn and show they are all x.

Step 1: Alternate segment theorem.

$$\angle PQR = x$$

(Reason: $\angle SRP$ is the angle between the tangent ST and the chord RP, so by the alternate segment theorem it equals $\angle PQR$ in the alternate segment.)

Step 2: Isosceles triangle.

$$\angle PRQ = x$$

(Reason: $PQ = PR$, so triangle PQR is isosceles; its base angles are equal, so $\angle PRQ = \angle PQR = x$.)

Step 3: Alternate angles.

$$\angle QPR = x$$

(Reason: ST is parallel to PQ, so $\angle SRP$ and $\angle QPR$ are alternate angles and are equal.)

Step 4: All three angles are equal.

$$\angle PQR = \angle PRQ = \angle QPR = x$$

(Reason: A triangle with three equal angles is equilateral, so each angle is $60°$.)

All three angles equal x, so triangle PQR is equilateral (each angle is $60°$).

Verification

✓ **Check 1:** The three equal angles sum to $180°$: $x + x + x = 3x = 180°$, so $x = 60°$.

✓ **Check 2:** Equal angles give equal sides, so $PQ = QR = RP$, which is exactly what equilateral means.

✓ **Check 3:** With $x = 60°$, the tangent-chord angle is $60°$, consistent with an equilateral triangle inscribed in the circle.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle PQR = x$	M1	Reason - alternate segment theorem	✓
$\angle PRQ = x$	M1	Reason - isosceles triangle (PQ = PR)	✓
$\angle QPR = x$	M1	Reason - alternate angles (ST parallel to PQ)	✓
All three angles equal, so equilateral	A1	Reason - three equal angles mean equilateral	✓

Full marks: 4/4

Question 9 - Exam Solution

Understanding the Question

Given

Circle, centre O; P, Q, R, S on the circle.

$\angle POS = 118^\circ$, $\angle PQO = 56^\circ$, $\angle ORQ = 42^\circ$.

Find

The size of $\angle RSO$.

Plan the Solution

- Triangle POS is isosceles (two radii): find $\angle OSP$.
- Triangle QOR is isosceles (two radii): find $\angle OQR$.
- Combine at Q to get $\angle PQR$, then use the cyclic quadrilateral for $\angle RSP$.
- Subtract to find $\angle RSO$.

Worked Solution [4 marks]

> **Rule - Radii make isosceles triangles:** every pair of radii forms an isosceles triangle with equal base angles; use these, then the cyclic-quadrilateral rule, and subtract.

Step 1: Isosceles triangle POS.

$$\angle OSP = \frac{1}{2}(180^\circ - 118^\circ) = \frac{1}{2} \times 62^\circ = 31^\circ$$

(Reason: $OP = OS$ are radii, so triangle POS is isosceles; the base angles are equal, each being half of what is left after the apex angle.)

Step 2: Isosceles triangle QOR.

$$\angle OQR = \angle ORQ = 42^\circ$$

(Reason: $OQ = OR$ are radii, so triangle QOR is isosceles; its base angles are equal.)

Step 3: Cyclic quadrilateral PQRS.

$$\angle PQR = \angle PQO + \angle OQR = 56^\circ + 42^\circ = 98^\circ$$

$$\angle RSP = 180^\circ - 98^\circ = 82^\circ$$

(Reason: Adding the two parts at Q gives $\angle PQR$; then $PQRS$ is a cyclic quadrilateral, so the opposite angles $\angle PQR$ and $\angle RSP$ add up to 180°.)

Step 4: Subtract to find angle RSO.

$$\angle RSO = \angle RSP - \angle OSP = 82^\circ - 31^\circ = 51^\circ$$

(Reason: $\angle OSP$ is part of $\angle RSP$, so subtracting it leaves $\angle RSO$.)

$$\angle RSO = 51^\circ$$

Verification

✓ **Check 1:** Triangle POS: $118^\circ + 31^\circ + 31^\circ = 180^\circ$.

✓ **Check 2:** Cyclic quadrilateral: $\angle PQR + \angle RSP = 98^\circ + 82^\circ = 180^\circ$.

✓ **Check 3:** The parts add up: $\angle RSO + \angle OSP = 51^\circ + 31^\circ = 82^\circ = \angle RSP$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle OSP = 31^\circ$	M1	Method - isosceles triangle (two radii)	✓
$\angle OQR = 42^\circ$	M1	Method - isosceles triangle (two radii)	✓
$\angle RSP = 82^\circ$	M1	Method - cyclic quadrilateral opposite angles	✓
$\angle RSO = 51^\circ$	A1	Accuracy - correct final answer	✓

Full marks: 4/4

Question 10 - Exam Solution

Understanding the Question

Given

A circle with centre O. PQ is a diameter and R is a point on the circumference.

Prove

That $\angle PRQ = 90^\circ$.

Plan the Solution

- Draw the radius OR, splitting triangle PQR into two isosceles triangles.
- Label the base angles x and y.
- Use the angle sum of triangle PQR to show $2x + 2y = 180^\circ$, hence $x + y = 90^\circ$.

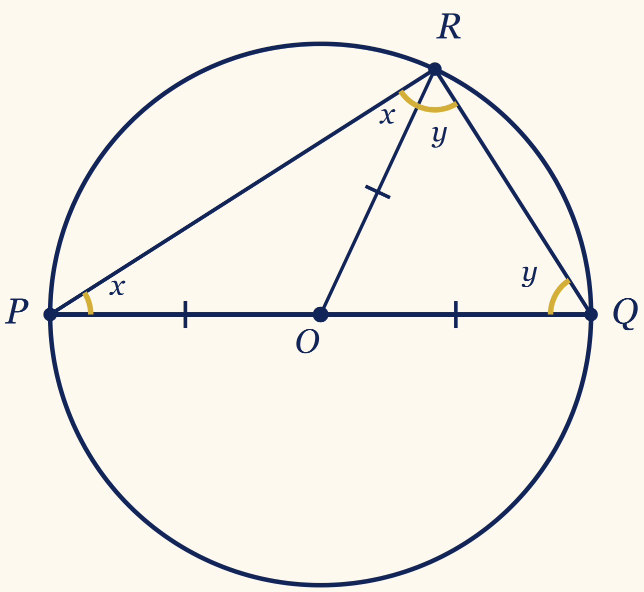
Worked Solution [3 marks]

> **Rule - Two radii, two isosceles triangles:** the radius to R creates two isosceles triangles; adding their base angles and using the triangle angle sum forces the angle at R to be 90°.

Step 1: Split into two isosceles triangles.

Draw OR. Since $OP = OR$ and $OQ = OR$ (radii):

$$\angle OPR = \angle ORP = x \quad \text{and} \quad \angle OQR = \angle ORQ = y$$



Not drawn accurately

(Reason: OP, OQ and OR are all radii, so triangles OPR and OQR are isosceles, with equal base angles.)

Step 2: Angle sum of triangle PQR.

The angle at R is $\angle PRQ = x + y$, so:

$x + x + y + y = 180°$

(Reason: The angles in triangle PQR - namely x, y and $x + y$ - add up to $180°$.)

Step 3: Solve for the angle at R.

$2x + 2y = 180°$, so $x + y = 90°$

$\angle PRQ = x + y = 90°$

(Reason: Dividing by 2 gives $x + y = 90°$, and $\angle PRQ$ equals $x + y$.)

Therefore $\angle PRQ = 90°$ (the angle in a semicircle is a right angle).

Verification

✓ **Check 1:** $x + x + y + y = 2(x + y) = 2 \times 90° = 180°$, matching the triangle angle sum.

✓ **Check 2:** The proof uses no fixed value for x or y, so $\angle PRQ = 90°$ for any point R on the semicircle.

✓ **Check 3:** A right angle at R means the diameter PQ subtends $90°$, which is exactly the angle in a semicircle.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two isosceles triangles, base angles x and y	M1	Method - split using the radius OR	✓
$x + x + y + y = 180°$	M1	Method - angles in a triangle	✓
$x + y = 90°$, so $\angle PRQ = 90°$	A1	Reason - the angle in a semicircle is 90 degrees	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

Two intersecting circles, centres O and P, meeting at G and H. W, X, Y on circle O; Z on circle P. XZ and ZY are straight lines.

$\angle GWY = 74^\circ$ and $\angle GPH = 84^\circ$.

Find

The size of $\angle XYH$, giving reasons.

Plan the Solution

- Circle P: $\angle GPH$ is at the centre, so the inscribed angle $\angle GZH$ is half of it.
- Circle O: $\angle YXG$ equals $\angle GWY$ (same segment).
- These are two angles of triangle XZY (as G lies on XZ and H lies on ZY). Use the triangle angle sum for $\angle XYH$.

Worked Solution [3 marks]

Rule - Work through each circle: use the centre-to-circumference rule in circle P and the same-segment rule in circle O to fill in two angles of triangle XZY, then use the angle sum.

Step 1: Circle P - angle at the centre.

$\angle GZH = \frac{1}{2} \times \angle GPH = \frac{1}{2} \times 84^\circ = 42^\circ$

(Reason: In circle P, $\angle GPH$ is at the centre and $\angle GZH$ is at the circumference, both standing on arc GH.)

Step 2: Circle O - angles in the same segment.

$\angle YXG = \angle GWY = 74^\circ$

(Reason: In circle O, $\angle YXG$ and $\angle GWY$ both stand on the chord GY, so angles in the same segment are equal.)

Step 3: Angles in triangle XZY.

G lies on XZ and H lies on ZY, so $\angle XZY = \angle GZH = 42^\circ$ and $\angle XYH = \angle XYZ$.

$\angle XYH = 180^\circ - 74^\circ - 42^\circ = 64^\circ$

(Reason: The angles in triangle XZY add up to 180°; $\angle XYH$ is the angle at Y because H lies on ZY.)

$$\angle XYH = 64°$$

Verification

- ✓ **Check 1:** Triangle XZY: $74° + 42° + 64° = 180°$.
- ✓ **Check 2:** $\angle GZH = 42°$ is exactly half of $\angle GPH = 84°$ (centre is twice the circumference).
- ✓ **Check 3:** $\angle YXG = \angle GWY = 74°$, confirming the same-segment step.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle GZH = 42°$	**M1**	Method - angle at the centre is twice the circumference (circle P)	✓
$\angle YXG = 74°$	**M1**	Method - angles in the same segment (circle O)	✓
$\angle XYH = 64°$	**A1**	Accuracy - angles in a triangle	✓

Full marks: 3/3

Question 12 - Exam Solution

Understanding the Question

Given

P, Q, R, S lie on a circle. MN is a tangent at S; PR and QS are straight lines meeting at T.
$\angle QTR = 70°$ and $\angle NSR = 58°$.

Show

That T is not the centre of the circle.

Plan the Solution

- Alternate segment theorem: find $\angle SQR$.
- Angles on a straight line at T: find $\angle STR$.
- If T were the centre, $\angle STR$ would equal $2 \times \angle SQR$. Check whether this holds.

Worked Solution [4 marks]

> **Rule - Assume and contradict:** suppose T is the centre; the angle at the centre would be twice the inscribed angle, so if that fails, T cannot be the centre.

Step 1: Alternate segment theorem.

$\angle SQR = \angle NSR = 58°$

(Reason: $\angle NSR$ is the tangent-chord angle for chord SR, equal to $\angle SQR$ in the alternate segment.)

Step 2: Angles on a straight line at T.

$\angle STR = 180° - 70° = 110°$

(Reason: QS is a straight line, so $\angle QTR$ and $\angle STR$ add up to $180°$.)

Step 3: Apply the "centre" assumption.

If T were the centre, TS and TR would be radii, and:

$\angle STR = 2 \times \angle SQR = 2 \times 58° = 116°$

(Reason: The angle at the centre is twice the angle at the circumference standing on the same arc SR.)

Step 4: Compare.

But $\angle STR = 110° \neq 116°$.

(Reason: The actual angle at T does not match the value it would have if T were the centre.)

Since $\angle STR = 110° \neq 116° = 2 \times \angle SQR$, T is not the centre of the

circle.

Verification

✓ **Check 1:** Straight line at T: $\angle QTR + \angle STR = 70^\circ + 110^\circ = 180^\circ$.

✓ **Check 2:** If T were the centre, $\angle STR = 2 \times 58^\circ = 116^\circ$, but the straight line gives 110° - the two disagree.

✓ **Check 3:** $110^\circ \neq 116^\circ$ confirms T cannot be the centre.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle SQR = 58^\circ$	M1	Reason - alternate segment theorem	✓
$\angle STR = 110^\circ$	M1	Method - angles on a straight line	✓
If centre, $\angle STR = 116^\circ$	M1	Reason - angle at centre is twice the circumference	✓
$116^\circ \neq 110^\circ$, so T is not the centre	A1	Reason - the values disagree	✓

Full marks: 4/4

Question 13 - Exam Solution

Understanding the Question

Given

Circle, centre O; P, Q, R, S on the circle. JP and JS are tangents from J; PK and SK are straight lines.

$\angle PJS = 40^\circ$, $\angle OPQ = 52^\circ$, $\angle PQR = 116^\circ$.

Find

The size of $\angle QKR$.

Plan the Solution

- Tangents from J are equal, so triangle JPS is isosceles: find the base angles.
- Tangent meets radius, then the isosceles triangle OPS: find $\angle SPO$.
- Add at P to get $\angle SPQ$, then use the cyclic quadrilateral for $\angle SRQ$.
- Angles on the straight lines PK and SK: find $\angle KQR$ and $\angle KRQ$.
- Angles in triangle QKR: find $\angle QKR$.

Worked Solution [5 marks]

> **Rule - Work from the tangents to the triangle:** build up the angles from the two tangents, use the centre and the cyclic quadrilateral, then finish with the triangle at K.

Step 1: Tangents from J are equal (isosceles triangle).

$$\angle JPS = \angle JSP = \frac{1}{2}(180^\circ - 40^\circ) = \frac{1}{2} \times 140^\circ = 70^\circ$$

(Reason: JP and JS are equal tangents from J, so triangle JPS is isosceles with equal base angles.)

Step 2: Tangent-radius, then isosceles triangle OPS.

$$\angle JSO = 90^\circ, \text{ so } \angle PSO = 90^\circ - 70^\circ = 20^\circ, \text{ and } \angle SPO = \angle PSO = 20^\circ$$

(Reason: OS is a radius and JS a tangent, so $\angle JSO = 90^\circ$; and $OP = OS$ are radii, so triangle OPS is isosceles with equal base angles.)

Step 3: Cyclic quadrilateral PQRS.

$$\angle SPQ = \angle SPO + \angle OPQ = 20^\circ + 52^\circ = 72^\circ$$

$$\angle SRQ = 180^\circ - 72^\circ = 108^\circ$$

(Reason: Adding the parts at P gives $\angle SPQ$; then $PQRS$ is a cyclic quadrilateral, so opposite angles $\angle SPQ$ and $\angle SRQ$ add up to 180°.)

Step 4: Angles on the straight lines.

$\angle KQR = 180° - 116° = 64°$ and $\angle KRQ = 180° - 108° = 72°$

(Reason: PQK and SRK are straight lines, so the angles at Q and at R each add up to $180°$.)

Step 5: Angles in triangle QKR.

$\angle QKR = 180° - 64° - 72° = 44°$

(Reason: The angles in triangle QKR add up to $180°$.)

$$\angle QKR = 44°$$

Verification

✓ **Check 1:** Triangle JPS: $40° + 70° + 70° = 180°$.

✓ **Check 2:** Cyclic quadrilateral: $\angle SPQ + \angle SRQ = 72° + 108° = 180°$.

✓ **Check 3:** Triangle QKR: $64° + 72° + 44° = 180°$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle JPS = \angle JSP = 70°$	**M1**	Method - tangents from a point are equal (isosceles)	✓
$\angle SPO = 20°$	**M1**	Method - tangent-radius and isosceles triangle	✓
$\angle SRQ = 108°$	**M1**	Method - cyclic quadrilateral opposite angles	✓
$\angle KQR = 64°, \angle KRQ = 72°$	**M1**	Method - angles on straight lines	✓
$\angle QKR = 44°$	**A1**	Accuracy - angles in a triangle	✓

Full marks: 5/5

Question 14 - Exam Solution

Understanding the Question

Given

Circle, centre O. JK is a tangent at P and ST is a tangent at R.

x is the tangent-chord angle at P, y the tangent-chord angle at R, and $b = \angle ORQ$.

Prove

That $b = x + y - 90^\circ$.

Plan the Solution

- Alternate segment theorem: $\angle PRQ = x$.
- Tangent meets radius at R: $\angle ORS = 90^\circ$.
- Subtract to find $\angle ORP = 90^\circ - y$.
- b is $\angle PRQ$ minus $\angle ORP$; simplify to get the result.

Worked Solution [4 marks]

> **Rule - Build the target from known angles:** write b as a difference of angles found with the alternate segment theorem and the tangent-radius right angle, then simplify.

Step 1: Alternate segment theorem.

$\angle PRQ = x$

(Reason: x is the angle between the tangent at P and the chord PQ, so by the alternate segment theorem it equals $\angle PRQ$ in the alternate segment.)

Step 2: Tangent meets radius.

$\angle ORS = 90^\circ$

(Reason: OR is a radius and RS is a tangent; a tangent meets a radius at 90°.)

Step 3: Subtract to find angle ORP.

$\angle ORP = \angle ORS - \angle SRP = 90^\circ - y$

(Reason: $y = \angle SRP$ is part of $\angle ORS$, so subtracting it gives $\angle ORP$.)

Step 4: Form b and simplify.

$b = \angle PRQ - \angle ORP = x - (90^\circ - y) = x + y - 90^\circ$

(Reason: b is the part of $\angle PRQ$ that remains after $\angle ORP$ is removed; expanding the bracket gives the result.)

Therefore $b = x + y - 90^\circ$, as required.

Verification

✓ **Check 1:** For example, if $x = 55^\circ$ and $y = 60^\circ$, the formula gives $b = 55^\circ + 60^\circ - 90^\circ = 25^\circ$, which agrees with the actual $\angle ORQ$.

✓ **Check 2:** Rearranging gives $x + y = b + 90^\circ$, so the two tangent-chord angles together are always 90° more than b.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle PRQ = x$	M1	Reason - alternate segment theorem	✓
$\angle ORS = 90^\circ$	M1	Reason - tangent meets a radius at 90 degrees	✓
$\angle ORP = 90^\circ - y$	M1	Method - angle subtraction	✓
$b = x + y - 90^\circ$	A1	Accuracy - correct algebraic result	✓

Full marks: 4/4

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