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Edexcel IGCSE 4MA1

Paper 1F (Foundation Tier) – November 2024

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A (4MA1), Paper 1F (Foundation Tier), November 2024 – 100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. The table gives information about the total length, in kilometres, of the road network on each of seven islands.

Island	Length of road network (km)
Corvel	8947
Tarnholm	3808
Eskvale	13 600
Brindle	4763
Saltmere	6124
Northwick	2758
Greymoor	4361

(a) Which of these seven islands has the longest road network? *[1 mark]*

(b) Write the number 6124 in words. *[1 mark]*

The road network on Corvel is longer than the road network on Northwick.

(c) How much longer? *[1 mark]*

(d) Write down the value of the 6 in the number 4763 *[1 mark]*

Two numbers in the table round to 4000 when written correct to the nearest thousand.

(e) Write down these two numbers. *[1 mark]*

(a) .....

(b) .....

(c) km .....

(d) .....

(e) .....

and .....

[Total 5 marks]

2. (a) Write $10p - 4p + 9p$ in its simplest form. *[1 mark]*

(b) Simplify fully $9 \times 4q$. *[1 mark]*

(c) Solve $4r = 15$. *[1 mark]*

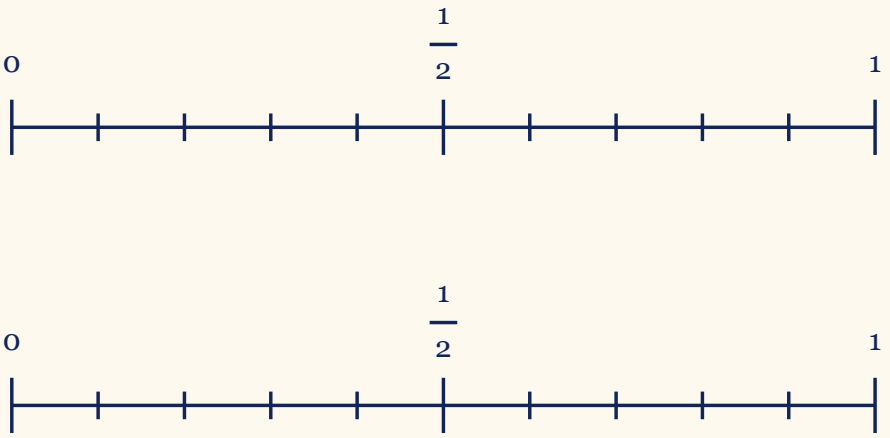
(a) .....

(b) .....

(c) r = ...

[Total 3 marks]

3. A tin contains 10 marbles.
5 marbles are black
3 marbles are green
2 marbles are red



Tub	A	B	C
Probability	0.7	0.45	1.2

Priya is going to take at random a marble from the tin.

(a) On the probability scale, mark with a cross the probability that the marble is black. *[1 mark]*

(b) On the probability scale, mark with a cross the probability that the marble is orange. *[1 mark]*

Lukas has three tubs of beads, A, B and C
He tries to find the probability of taking at random a white bead from each tub.
He writes his probabilities in a table.

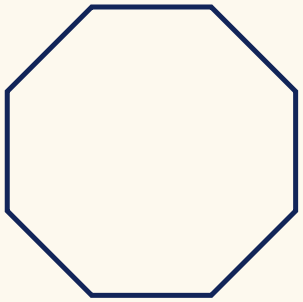
The probability that Lukas writes for tub C is incorrect.

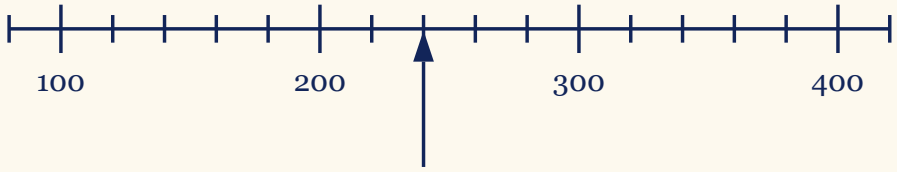
(c) Explain how you know that it is incorrect. *[1 mark]*

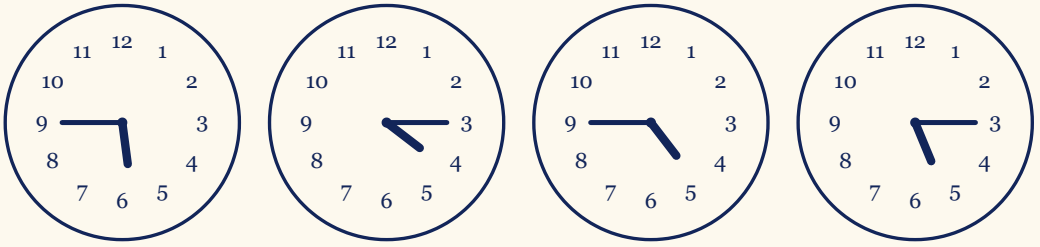
(c) .....

[Total 3 marks]

4. The diagram shows a polygon.







(a) Write down the mathematical name of this polygon. *[1 mark]*

(b) Write down the number that the arrow points to on the number line below. *[1 mark]*

Here are four clock faces, labelled A, B, C and D.

(c) Write down the letter of the clock face that shows quarter to five. *[1 mark]*

(d) Complete this sentence by writing a suitable metric unit on the answer line.
The length of a large cruise ship is 300 *[1 mark]*

(a) .....

(b) .....

(c) .....

(d) .....

[Total 4 marks]

5. Here is a list of six numbers.

2 8 9 18 24 28

(a) Using only the numbers in this list, write down

(i) an odd number

[1 mark]

(ii) a number that is a multiple of 4 and also a multiple of 6

[1 mark]

(iii) a cube number

[1 mark]

(iv) a prime number *[1 mark]*

(b) Work out the value of

$6^2 + 2^3 \times 5$ *[1 mark]*

(a)(i) .....

(a)(ii) .....

(a)(iii) .....

(a)(iv) .....

(b) .....

[Total 5 marks]

6. The first diagram shows two straight lines that meet at a point.
The second diagram shows the quadrilateral $ABDE$ together with the isosceles triangle BCD, in which $BD = DC$.
ABC is a straight line.

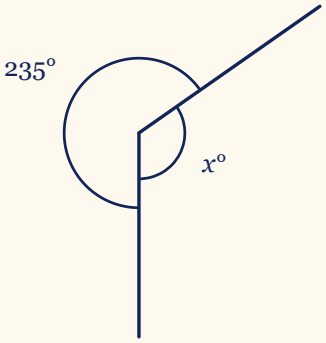
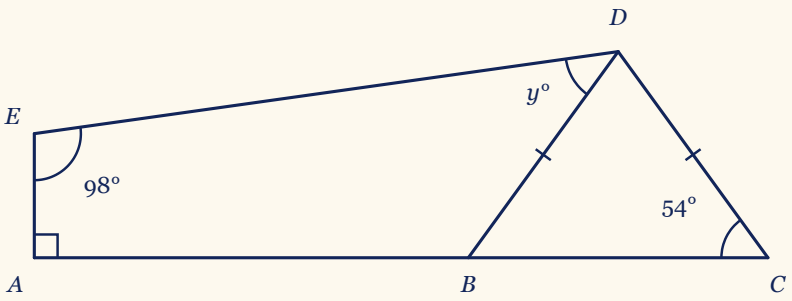


Diagram NOT accurately drawn

Diagram NOT accurately drawn



(a) (i) Find the value of x.
[1 mark]
(ii) Give the reason for your answer. *[1 mark]*

(b) Find the value of y. *[3 marks]*

(a)(i) x = ...

(a)(ii) .....

(b) y = ...

[Total 5 marks]

7. Douglas works at a bottling plant.

His normal hourly rate of pay is £14
His overtime hourly rate of pay is £21

Douglas is paid the normal hourly rate of pay for 35 hours in one week.
His total pay for this week is £679

Work out the number of hours of overtime he works in this week. *[4 marks]*

.....

[Total 4 marks]

8. (a) Simplify the expression $10x - 7y - 6x + 4y$ *[2 marks]*

You are given the formula $T = 4d - 6e$

(b) Work out the value of T when $d = 13$ and $e = 7$ *[2 marks]*

(c) Solve the equation $5p + 11 = 28$ *[2 marks]*

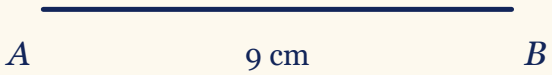
(a) .....

(b) T = ...

(c) p = ...

[Total 6 marks]

9. Triangle ABC is equilateral. Each of its three sides is 9 cm long.



Using only a ruler and a pair of compasses, construct triangle ABC
The side AB has already been drawn for you.
Every construction line you draw must be left showing. *[2 marks]*

[Total 2 marks]

10. There are 29 kayaks on the rack at a lake hire centre.

10 of the kayaks are yellow.
The rest of the kayaks are green or blue.

Erin takes one of these kayaks at random.

(a) Write down the probability that she takes a yellow kayak. *[1 mark]*

The probability that Erin takes a green kayak is $\frac{7}{29}$

(b) Work out the probability that she takes a blue kayak. *[2 marks]*

(a) .....

(b) .....

[Total 3 marks]

11. Oliver is going to bake some scones.

Here is a list of ingredients for making 24 scones.

Ingredients for 24 scones
120 g butter
60 g sugar
200 g flour

Oliver has
five 250 g packs of butter
750 g of sugar
1.4 kg of flour

Work out the maximum number of scones that Oliver can make.
Show your working clearly. *[4 marks]*

.....

[Total 4 marks]

12. The table gives information about the number of training sessions each of 30 members of a swimming club completed in one week.

Number of training sessions	Frequency
0	2
1	5
2	11
3	7
4	4
5	1

(a) Work out the mean number of training sessions. *[3 marks]*

Erin is a member of the swimming club.
The probability that Erin travels to the pool by bus is 0.79

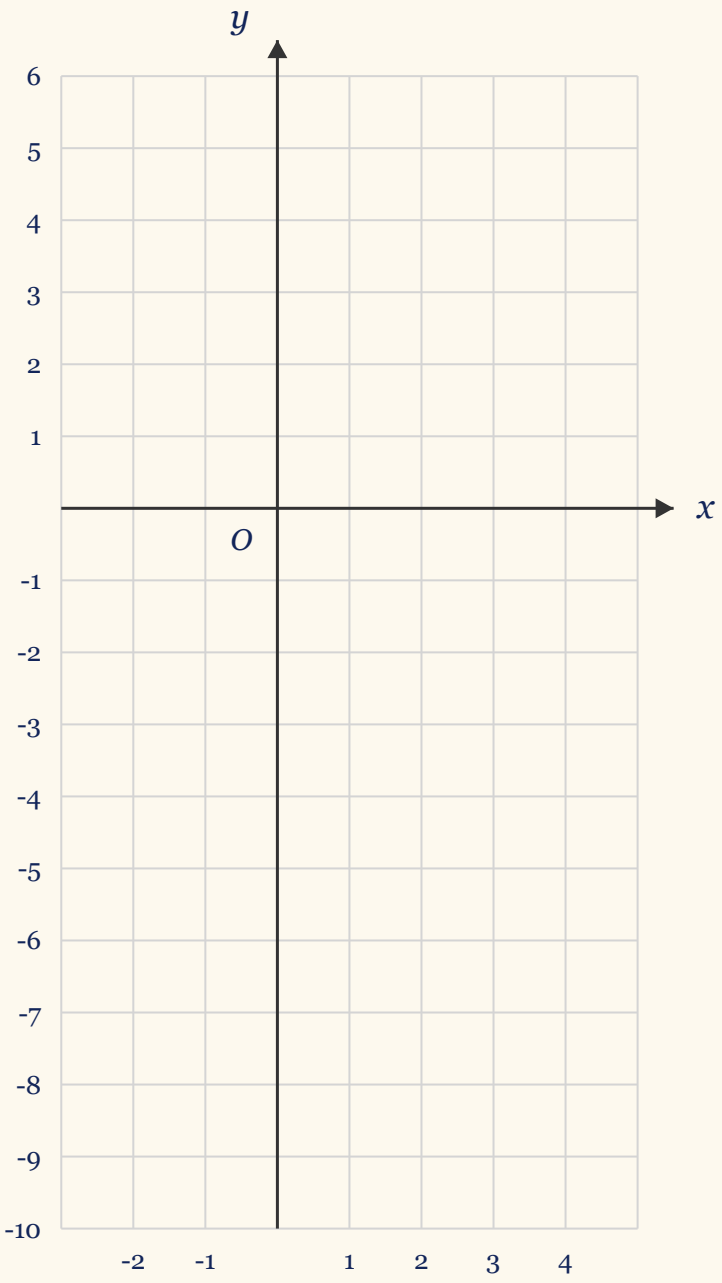
(b) Work out the probability that Erin does not travel to the pool by bus. *[1 mark]*

(a) .....

(b) .....

[Total 4 marks]

13. Draw the graph of $y = 2x - 3$ on the grid below.
Use values of x from -2 to 4 *[3 marks]*



[Total 3 marks]

14. Yaniv makes 490 clay pots each week for a pottery studio.
86 of the pots are glazed.

(a) Write 86 as a percentage of 490
Give your answer correct to one decimal place. *[2 marks]*

A glazed pot weighs 375 grams before it is fired.
The pot loses 12% of its weight when it is fired.

(b) Work out the weight of the pot after it is fired. *[3 marks]*

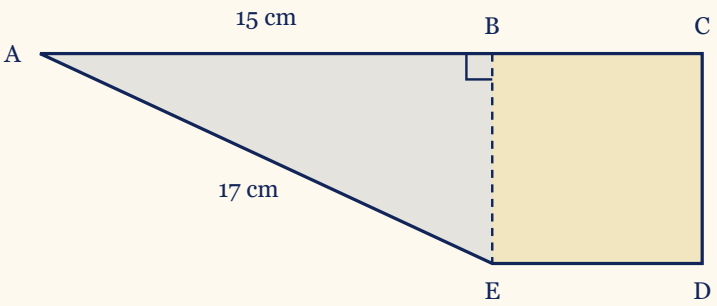
(a) % .....

(b) grams .....

[Total 5 marks]

15. The diagram shows the shape $ABCDE$.
It is formed by joining a right-angled triangle ABE to a square $BCDE$

Diagram NOT accurately drawn



ABC is a straight line.

$AB = 15$ cm and $AE = 17$ cm

Triangle ABE has a perimeter of 40 cm

Work out the area of shape $ABCDE$ *[4 marks]*

..... cm^2

[Total 4 marks]

16. Here are the first four terms of an arithmetic sequence.

1 4 7 10

(a) Work out an expression, in terms of n, for the nth term of this sequence. *[2 marks]*

A different arithmetic sequence has nth term $5n + 17$

(b) Work out the 12th term of this second sequence. *[1 mark]*

(a) .....

(b) .....

[Total 3 marks]

17. 450 employees at a city-centre office were asked how they travelled to work on Monday.
Each employee walked or travelled by bus or travelled by car or travelled by bicycle.
Each employee used just one method of travel.

One of these employees is chosen at random.
The table shows information about the probability of each method of travel.

Method of travel	walk	bus	car	bicycle
Probability	0.20	x	$2x$	0.26

Work out how many of the 450 employees travelled by car. *[4 marks]*

.....

[Total 4 marks]

18. Work out the highest common factor (HCF) of 72 and 108
You must show your working clearly. *[2 marks]*

.....

[Total 2 marks]

19. Elena is a driving instructor.
She keeps a record of the number of kilometres her car travels each month.

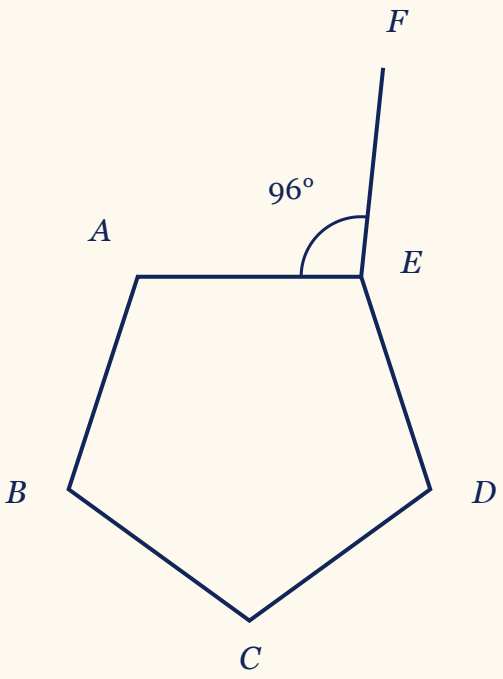
In April, the car travelled 943 kilometres.
This is 15% more than the number of kilometres the car travelled in March.

Work out the number of kilometres the car travelled in March. *[3 marks]*

..... kilometres

[Total 3 marks]

20. In the diagram, $ABCDE$ is a regular pentagon, and E is joined to a point F that lies outside the pentagon.
Not drawn accurately.



Angle $AEF = 96°$
Work out the size of the obtuse angle FED
You must show your working clearly. *[4 marks]*

..... °

[Total 4 marks]

21. (a) Expand and simplify $(m + 5)(m - 8)$ *[2 marks]*

(b) Solve $3n - 4 = \dfrac{5n + 6}{3}$

You must show clear algebraic working. *[3 marks]*

(a) .....

(b) n = ...

[Total 5 marks]

22. $\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$
$A = \{\text{even numbers}\}$
$B = \{23, 29, 31\}$
$C = \{\text{multiples of } 3\}$

(a) Write down all the members of the set

(i) $B \cup C$

[1 mark]

(ii) $A' \cap C$ *[1 mark]*

(b) Is it true that $B \cap C = \varnothing$?

Write Yes or No, and give a reason for your answer. *[1 mark]*

The set D has 4 members and is such that $D \cap (A \cup C) = \varnothing$

(c) Write down all the members of set D *[2 marks]*

(a)(i) .....

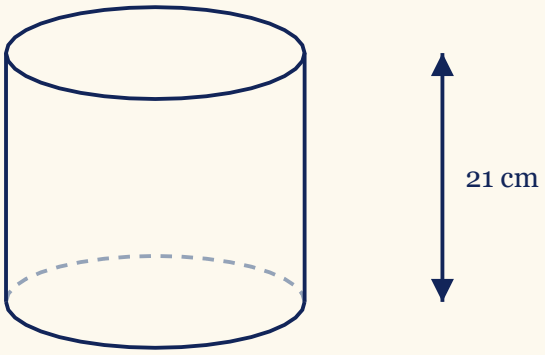
(a)(ii) .....

(b) .....

(c) .....

[Total 5 marks]

23. A solid metal cylinder is standing on a workbench.



Not drawn accurately

The volume of the cylinder is $1575\ \text{cm}^3$
The force exerted by the cylinder on the workbench is 84 newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the workbench due to the cylinder. *[3 marks]*

..... newtons/cm²

[Total 3 marks]

24. The table gives the amount of wheat harvested in each of two farming regions in 2020

Region	Amount of wheat (tonnes)
Amberdown	3.5×10^7
Fenwold	8.2×10^5

(a) Write 3.5×10^7 as an ordinary number. *[1 mark]*

In 2020, the Highmarsh region harvested 6 780 000 more tonnes of wheat than the Fenwold region.

(b) Work out the amount of wheat harvested in the Highmarsh region in 2020
Give your answer in standard form. *[2 marks]*

(a) .....

(b) tonnes .....

[Total 3 marks]

25. (a) Simplify $(2p)^0$ given that $p > 0$ *[1 mark]*

$y^9 \times y^{-3} = y^n$

(b) Work out the value of n *[1 mark]*

(c) Write $(5a^4c^2)^3$ in its simplest form. *[2 marks]*

(a) .....

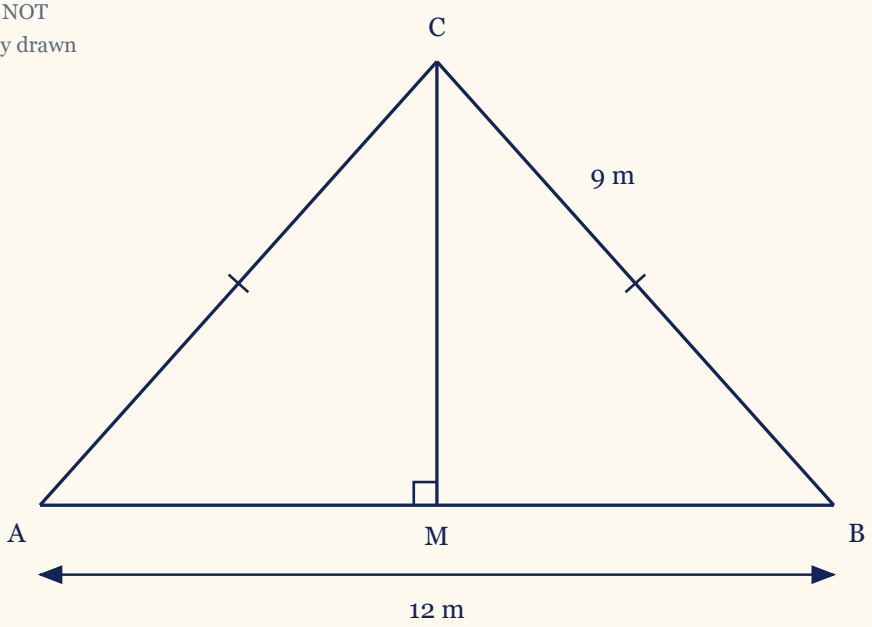
(b) n = ...

(c) .....

[Total 4 marks]

26. The diagram shows the timber frame for the end of a garden shelter.

Diagram NOT accurately drawn



The frame is made from four lengths of timber, AB, AC, BC and MC

$AC = BC = 9$ m and $AB = 12$ m
angle $AMC = 90°$

Nathan is going to buy lengths of timber to make the frame.

The timber costs 21.50 euros per metre.
Each length of timber he buys has to be a whole number of metres.

Work out the total cost of the timber Nathan needs to buy.
Show your working clearly. *[4 marks]*

..... euros

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Seven islands, and the length in kilometres of the road network on each: 8947, 3808, 13 600, 4763, 6124, 2758 and 4361.
Corvel holds the 8947 km network and Northwick the 2758 km one.

Find

(a) The island with the longest road network. (b) The number 6124 written out in words. (c) How much longer Corvel's network is than Northwick's. (d) What the 6 in 4763 is worth. (e) The two lengths that round to 4000 to the nearest thousand.

Plan the Solution

- (a) Compare by place value. A five-digit number beats every four-digit one, so count digits first.
- (b) Split 6124 into thousands, hundreds, tens and ones, then read the columns from the left.
- (c) How much longer is a difference, so subtract the smaller length from the larger.
- (d) Find which column the 6 stands in and give it that column's value.
- (e) Round each length to the nearest thousand and keep the ones that land on four thousand.

Worked Solution [5 marks]

> Every digit is worth its face value times the column it stands in, so $4763 = 4000 + 700 + 60 + 3$. Comparing, subtracting and rounding whole numbers all work column by column, starting from the largest column.

(a) Pick out the longest network

2758, 3808, 4361, 4763, 6124, 8947, 13 600

(Reason: Put the seven lengths in order. Six of them have four digits and only $13\,600$ has five, so it is the largest. That row is Eskvale.)

(b) Read 6124 out of its columns

$6124 = 6000 + 100 + 20 + 4$

(Reason: Six thousands, one hundred, two tens and four ones. Reading the columns from the left gives six thousand one hundred and twenty-four.)

(c) Subtract to find how much longer

$8947 - 2758 = 6189$

(Reason: Corvel's network is 8947 km and Northwick's is 2758 km, so take the smaller from the larger. Both the units column and the tens column need a borrow.)

(d) Give the 6 the value of its column

$4763 = 4000 + 700 + 60 + 3$

(Reason: The 6 stands in the tens column, so it is worth six tens. That is 60, not 6.)

(e) Round every length to the nearest thousand

$3808 \to 4000$

$4361 \to 4000$

$4763 \to 5000$

$2758 \to 3000$

(Reason: The hundreds digit decides the rounding, so a length rounds to 4000 exactly when it lies from 3500 up to 4500. Only 3808 and 4361 do; 4763 is past 4500, so it rounds to 5000.)

(a) Eskvale

(b) six thousand one hundred and twenty-four

(c) 6189 km

(d) 60

(e) 3808 and 4361

Verification

✓ **Check 1:** Add the difference back on to Northwick's length. It should come back to Corvel's. $2758 + 6189 = 8947$

✓ **Check 2:** Rebuild 4763 from its columns with the 6 put back in as 60. $4000 + 700 + 60 + 3 = 4763$

✓ **Check 3:** Read the words of part (b) back as a number: six thousand, one hundred, twenty-four. $6000 + 100 + 24 = 6124$

✓ **Check 4:** Count how many of the seven lengths lie from 3500 up to 4500. Exactly two do, 3808 and 4361, which is the number of answers part (e) asks for.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	Eskvale. Accept 13 600.	✓
(b)	**B1**	Six thousand one hundred and twenty four. A hyphen in twenty-four is accepted.	✓
(c)	**B1**	6189 and nothing else. Units are not required.	✓
(d)	**B1**	60. Accept 6 tens, or sixty.	✓
(e)	**B1**	3808 and 4361, in either order. Accept Tarnholm and Greymoor.	✓

Full marks: 5/5

Question 2 - Exam Solution

Understanding the Question

Given

$10p - 4p + 9p$ - three terms, every one of them a multiple of p.
$9 \times 4q$ - a number multiplied by a term in q.
$4r = 15$ - a one-step equation, with r multiplied by 4.

Find

The simplest form of $10p - 4p + 9p$. The simplest form of $9 \times 4q$. The value of r.

Plan the Solution

- Part (a) is a collecting exercise: the terms are like terms, so combine the coefficients and keep the p.
- Part (b) is a multiplication: multiply the two numbers and keep the q. There is no adding to do here.

- Part (c) is a one-step equation: apply the inverse of multiplying by 4 to both sides.
- Each part is worth one mark, so each answer is the whole of the work - no method marks are on offer.

Worked Solution [3 marks]

> Rule - Like terms are collected by combining their coefficients and keeping the letter; a term is multiplied by a number by multiplying its coefficient; and an equation is undone by the inverse operation, applied to both sides.

Step 1: (a) Combine the coefficients

$$10p - 4p + 9p = (10 - 4 + 9)p$$
$$(10 - 4 + 9)p = 15p$$

(Reason: Every term is a multiple of p, so they are like terms. Only the coefficients 10, -4 and 9 are combined; the letter is carried through unchanged, so the answer is a term in p and not a number.)

Step 2: (b) Multiply the numbers, keep the letter

$$9 \times 4q = 9 \times 4 \times q$$
$$9 \times 4 \times q = 36q$$

(Reason: Multiplication can be done in any order, so the two numbers 9 and 4 are multiplied first. Nothing is added here: $9 \times 4q$ is nine lots of $4q$, which is $36q$.)

Step 3: (c) Undo the multiplication

$$4r = 15$$
$$r = \frac{15}{4} = 3.75$$

(Reason: The 4 and the r are multiplied together, so the inverse of that multiplication is applied to both sides to leave r on its own. The answer does not have to be a whole number.)

(a) $15p$

(b) $36q$

(c) $r = 3.75$

Verification

✓ **Check 1:** Put $p = 2$ into the original expression and into the answer. The original gives $10(2) - 4(2) + 9(2) = 20 - 8 + 18$, and the answer gives $15(2)$. $30 = 30$, so the two expressions agree.

✓ **Check 2:** Put $q = 5$ into the original expression and into the answer. The original gives $9 \times 4(5) = 9 \times 20$, and the answer gives $36(5)$. $180 = 180$, so the two expressions agree.

✓ **Check 3:** Substitute the solution back into the equation: work out 4×3.75 and compare it with 15. $4 \times 3.75 = 15$, which is the right-hand side.

✓ **Check 4:** Read the answer as a division with a remainder instead: 15 divided by 4 is 3 remainder 3, so $r = 3\frac{3}{4}$. $3\frac{3}{4} = \frac{15}{4} = 3.75$, the same value in all three forms the mark scheme accepts.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Collect the like terms in p	**B1**	$15p$ with no further working needed. The letter must be present: 15 on its own is not the simplified expression.	✓
(b) Multiply the coefficient by 9	**B1**	$36q$. Adding instead of multiplying gives $13q$, which earns nothing.	✓
(c) Divide both sides by 4	**B1**	3.75, or any equivalent form: $\frac{15}{4}$ and $3\frac{3}{4}$ are both accepted.	✓

Full marks: 3/3

Question 3 - Exam Solution

Understanding the Question

Given

A tin of 10 marbles: 5 black, 3 green and 2 red. One marble is taken at random, so every marble is equally likely.
Two probability scales, each running from 0 to 1 in ten equal steps, to be marked with a cross.
Three probabilities written for three different tubs of beads: 0.7, 0.45 and 1.2.

Find

The probability that the marble is black, shown as a cross on the first scale. The probability that the marble is orange, shown as a cross on the second scale. A reason why 1.2 cannot be a probability at all.

Plan the Solution

- Parts (a) and (b) are answered ON the scale, so each answer is a cross rather than a written number. Work the probability out first, then find where it sits between 0 and 1.
- For (a), count the black marbles out of the total and write the fraction in its simplest form.
- For (b), check whether the tin holds any orange marbles at all. The three colours already account for all 10 marbles, so the event is impossible.
- For (c), compare 1.2 with the two ends of the probability scale. One sentence earns the mark, and it must say what is wrong with the value itself.

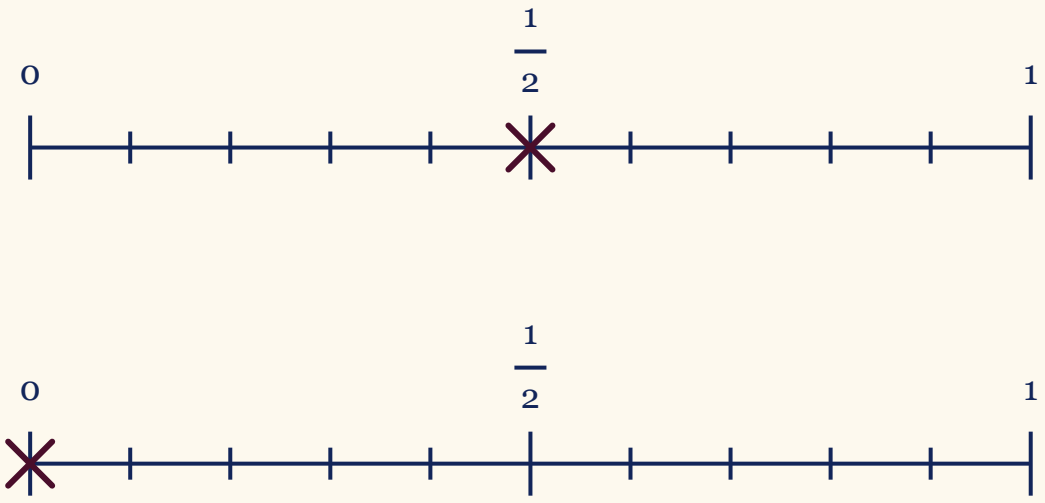
Worked Solution [3 marks]

> Rule - When the outcomes are equally likely, the probability of an event is the number of ways it can happen divided by the total number of outcomes, and the answer is always a number from 0, which means impossible, to 1, which means certain.

Step 1: (a) Write the probability of black as a fraction

$$P(\text{black}) = \frac{5}{10}$$

$$\frac{5}{10} = \frac{1}{2} = 0.5$$



(Reason: The tin holds 10 marbles and one is taken at random, so each marble is equally likely. The probability of black is therefore the number of black marbles over the total number of marbles, and that fraction is then written in its simplest form.)

Step 2: (a) Mark the cross halfway along the scale

$$\frac{1}{2} = 0.5$$

(Reason: A probability of $\frac{1}{2}$ sits exactly halfway between 0 and 1, which is the tick already labelled $\frac{1}{2}$ in the middle of the scale. The cross goes on the scale line itself, not above it or below it.)

Step 3: (b) Count the orange marbles

$$5 + 3 + 2 = 10$$

$$P(\text{orange}) = \frac{0}{10}$$

(Reason: The black, green and red marbles already account for every one of the 10 marbles in the tin, so there is no orange marble to take. The number of ways the event can happen is 0.)

Step 4: (b) Mark the cross at the left-hand end

$$\frac{0}{10} = 0$$

(Reason: An event that cannot happen has probability 0, and 0 is the left-hand end of the scale. An impossible event is still marked on the scale, not off it.)

Step 5: (c) Test the value written for tub C against the scale

$$1.2 - 1 = 0.2$$

$$1.2 \times 100 = 120$$

(Reason: The largest a probability can be is 1, which means certain, so 1.2 lies 0.2 beyond the right-hand end of the scale and no event can have it. Read as a percentage it is 120 per cent, and nothing is more than 100 per cent certain. The values 0.7 and 0.45 both sit on the scale, so those two are not the problem.)

(a) a cross on the scale at $\frac{1}{2}$

(b) a cross on the scale at 0

(c) 1.2 is greater than 1, and a probability can never be greater than 1

Verification

✓ **Check 1:** Add the probabilities of the three colours that are in the tin. If every marble has been counted once, they must total 1. $\frac{5}{10} + \frac{3}{10} + \frac{2}{10} = 1$, so nothing has been missed and there is no room left for an orange marble.

✓ **Check 2:** Work part (a) the other way round. The marbles that are not black are the 3 green and the 2 red, so subtract that probability from 1. $1 - \frac{1}{2} = \frac{1}{2}$, the same probability, so black and not-black are equally likely.

✓ **Check 3:** Change part (a) into a decimal and a percentage, then read the position off the scale: halfway along ought to be 50 per cent. $\frac{5}{10} = 0.5$, which is 50 per cent, and the middle tick of a scale from 0 to 1 is exactly where the cross was put.

✓ **Check 4:** Try to mark all three of Lukas's values on a probability scale and see which one has nowhere to go. 0.7 and 0.45 both land on the scale, while $1.2 - 1 = 0.2$ shows that the third lands past the end of it.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A cross on the probability scale at $\frac{1}{2}$	**B1**	The cross must be at $\frac{1}{2}$, the middle tick, and on the scale line. Correct answer only, so a cross anywhere else scores nothing and no working can earn the mark.	✓
(b) A cross on the probability scale at 0	**B1**	The cross must be at 0, the left-hand end. Correct answer only. Leaving the scale blank because the event cannot happen scores nothing: 0 is itself a probability.	✓
(c) A correct reason	**B1**	Any statement that a probability cannot be more than 1 or equivalent: it is over 1; it is more than 100 per cent; probability runs from 0 to 1; 1.2 is impossible; it has to be 1 or less. A contradictory answer scores nothing, and so does saying only that the value is too high. Arguing that the probabilities should add up to 1 also scores nothing - these three are for three different tubs, so their total means nothing at all.	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

A polygon drawn from eight straight sides of equal length.
A number line whose labelled marks are 100, 200, 300 and 400, with five equal small divisions between one labelled mark and the next, and an arrow pointing at a mark between 200 and 300.
Four clock faces, A, B, C and D, each showing a different time.
A sentence giving the length of a large cruise ship as 300 of a metric unit that has been left out.

Find

The mathematical name of the polygon. The number the arrow points to. The letter of the clock face that shows quarter to five. The metric unit that completes the sentence.

Plan the Solution

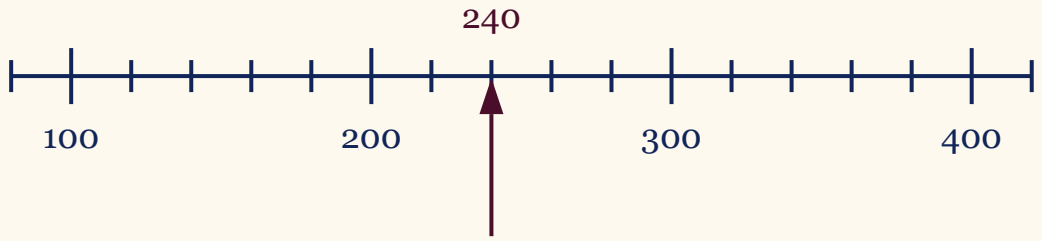
- Name the polygon by counting its sides. The name comes from the number of sides, so count them one at a time round the outline rather than judging by the look of the shape.
- On a number line, work out what ONE small division is worth before reading anything off it: divide the gap between two labelled marks by the number of divisions inside that gap, then count on from the nearer label.
- Read a clock in two halves. The long hand gives the minutes and the short hand gives the hour. Quarter to means the long hand is at 9, and the hour is the one the short hand is heading towards, not the one it has just left.
- For the unit, test each candidate in turn: turn 300 of it into metres and ask whether a ship could really be that long.

Worked Solution [4 marks]

> Rule - On a number line one small division is worth the gap between two labelled marks divided by the number of divisions in it; on a clock face the long hand counts 5 minutes for every number it passes; and a polygon is named by how many sides it has.

Step 1: (a) Count the sides of the polygon

$4 + 4 = 8$



(Reason: Four of the edges run straight across or straight up and down - the top, the bottom and the two ends - and four more cut across the corners, so the outline is made of 8 straight sides. A polygon with 8 sides is an octagon.)

Step 2: (b) Work out what one small division is worth

$$\frac{100}{5} = 20$$

(Reason: From 100 to 200 the line climbs by 100, and that gap is split into 5 equal divisions. Count the GAPS and not the ticks: between the two labels there are 5 gaps but only 4 ticks, so each division is worth 20.)

Step 3: (b) Count on from 200 to the arrow

$$200 + 2 \times 20 = 240$$

(Reason: The arrow stands 2 small divisions to the right of 200, and Step 2 found each division to be worth 20, so add 2 lots of 20 on to 200.)

Step 4: (c) Read the four clock faces

$$9 \times 5 = 45$$

$$60 - 45 = 15$$

(Reason: Every number on the dial stands for 5 minutes, so a long hand at 9 means 45 minutes past the hour, which leaves 15 minutes - a quarter of an hour - still to run. Quarter to five therefore needs the long hand at 9 AND the short hand between 4 and 5, on its way to 5. Only clock C has both. Clock A has its long hand at 9 but its short hand between 5 and 6, so that face is quarter to six, and clocks B and D have the long hand at 3, which is quarter PAST.)

Step 5: (d) Test each metric unit for length

$$\frac{300}{1000} = 0.3$$

$$\frac{300}{100} = 3$$

$$300 \times 1000 = 300\,000$$

(Reason: Turn 300 of each unit into metres. 300 millimetres is only 0.3 of a metre and 300 centimetres is only 3 metres, both far too short for a ship, while 300 kilometres is 300 000 metres, which is longer than many countries are wide. Metres is the one unit left, and 300 metres is about right for a large cruise ship.)

(a) octagon

(b) 240

(c) C

(d) metres

Verification

✓ **Check 1:** Test the count of sides a different way. The angles inside a polygon with n sides add up to $(n-2)\times 180$ degrees, so work that out for 8 sides and share it equally between the corners. $(8-2)\times 180 = 1080$ and $\dfrac{1080}{8} = 135$, so each corner of the figure is a 135 degree angle, which is exactly what the corners of a regular octagon measure.

✓ **Check 2:** Read the number line from the other end. Counting back, the arrow stands 3 small divisions short of 300. $300 - 3\times 20 = 240$, the same number, so the reading does not depend on which label it was counted from.

✓ **Check 3:** Come at the time from the words instead of from the dial. A quarter of an hour is 60 minutes shared into 4, and quarter TO means that many minutes before the next hour, so work out which number on the dial the long hand must reach. $\dfrac{60}{4} = 15$ minutes before the hour, which is $60 - 15 = 45$ minutes past it, and $\dfrac{45}{5} = 9$ puts the long hand on the 9 - which is where clock C has it, with its short hand not yet at 5.

✓ **Check 4:** Measure the ship in people rather than in units. A grown adult is about 2 metres tall, so divide the length by 2 for each candidate unit and see which count is believable. $\dfrac{300}{2} = 150$ adults lying end to end if the unit is metres, which is a believable ship, against $\dfrac{300\,000}{2} = 150\,000$ adults if the unit is kilometres, which is not.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) octagon	**B1**	Correct answer only. The word octagon on its own earns the mark, and regular octagon earns it too. Naming any other polygon scores nothing, and so does describing the shape without naming it.	✓

Step	Mark	Description	Got it?
(b) 240	**B1**	Correct answer only. 240 with no working still earns the mark. The two wrong readings this question is built to catch are 220, from counting one division instead of two, and 250, from sharing the 100 between 4 divisions instead of 5. Neither earns anything.	✓
(c) C	**B1**	A lower case c is accepted as well as a capital C. The question asks for a letter, so it is the letter that must be written down. Clock A is the near miss, since it also has its long hand at 9.	✓
(d) metres	**B1**	The abbreviation m is accepted as well as the word metres. Any other unit scores nothing: the sentence measures a length, so a unit of mass or of capacity is wrong outright, and centimetres or kilometres would make the ship 3 metres or 300 000 metres long.	✓

Full marks: 4/4

Question 5 - Exam Solution

Understanding the Question

Given

A list of six numbers:
$2,\ 8,\ 9,\ 18,\ 24,\ 28.$
For part (b), the calculation $6^2 + 2^3 \times 5$, which mixes powers, a multiplication and an addition.

Find

(a) One number from the list for each of four properties: odd, a multiple of both numbers, a cube, and prime. Each answer must be taken from the list, not invented.
(b) The value of $6^2 + 2^3 \times 5$.

Plan the Solution

- Part (a) is answered by testing the list, not by recalling a fact. Take one property at a time and run every number in the list past it.
- Odd means it will not divide by 2. A multiple of both 4 and 6 is a multiple of 12, their lowest common multiple. A cube is a whole number times itself times itself. A prime has exactly two factors.
- Each property is matched by exactly one number in this list, so each part of (a) has a single right answer.
- Part (b) is order of operations: powers first, then the multiplication, then the addition last.

Worked Solution [5 marks]

> Rule - Number properties, then order of operations: a number is odd if it leaves a remainder of 1 on division by 2; it is a cube if it equals $k \times k \times k$ for some whole number k; it is prime if it has exactly two factors. In a calculation, work out powers first, then multiplication, and addition last.

Step 1: (a)(i) test the list for an odd number

$2, 8, 18, 24, 28$ all divide by 2

$9 = 2 \times 4 + 1$

(Reason: An odd number leaves a remainder of 1 when it is divided by 2. Five of the six numbers are even, so 9 is the only odd number available.)

Step 2: (a)(ii) test the list for a multiple of both 4 and 6

LCM of 4 and $6 = 12$

$24 = 12 \times 2$

$24 = 4 \times 6 = 6 \times 4$

(Reason: A number in both times tables is a multiple of the lowest common multiple, 12. Only 24 is a multiple of 12 here. The near misses are worth naming: 8 and 28 are multiples of 4 but not of 6, and 18 is a multiple of 6 but not of 4.)

Step 3: (a)(iii) test the list for a cube number

$8 = 2 \times 2 \times 2 = 2^3$

$1^3 = 1$

$3^3 = 27$

(Reason: A cube number is a whole number multiplied by itself and then by itself again. The cubes around this list are 1, 8 and 27, and only 8 appears in the list.)

Step 4: (a)(iv) test the list for a prime number

2 has factors 1 and 2 only

$9 = 3 \times 3$

$8, 18, 24, 28$ are even and larger than 2

(Reason: A prime number has exactly two factors, itself and 1. Any even number larger than 2 has 2 as a third factor, and 9 has the extra factor 3, so 2 is the only prime in the list. It is the one even prime, which is why it is easy to miss.)

Step 5: (b) work out $6^2 + 2^3 \times 5$

$6^2 = 36$

$2^3 \times 5 = 8 \times 5 = 40$

$6^2 + 2^3 \times 5 = 36 + 40 = 76$

(Reason: Powers are worked out first, then the multiplication, and the addition comes last. Adding first would give $44 \times 5 = 220$, a completely different value, which is why the order matters.)

(a)(i) 9

(a)(ii) 24

(a)(iii) 8

(a)(iv) 2

(b) 76

Verification

✓ **Check 1:** Go back to the list and test all six numbers against all four properties, rather than only checking the four answers chosen. Odd: only 9. Multiple of 12: only 24. Cube: only 8. Prime: only 2. Each property is matched once, so every answer in part (a) is forced.

✓ **Check 2:** Redo part (b) a different way. Both terms are multiples of 4, since $36 = 4 \times 9$ and $40 = 4 \times 10$, so the 4 can be taken out. $4 \times (9 + 10) = 4 \times 19 = 76$, the same value as before.

✓ **Check 3:** Replace the multiplication with repeated addition: $2^3 \times 5$ means five lots of 8. $8 + 8 + 8 + 8 + 8 = 40$ and $36 + 40 = 76$, so the answer to (b) is confirmed without multiplying.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) writes down 9	**B1**	cao. 9 is the only odd number in the list. Working is not required, so a correct answer alone scores the mark.	✓
(a)(ii) writes down 24	**B1**	cao. Accept 24 only. 8 or 28 (a multiple of 4 alone) and 18 (a multiple of 6 alone) score no mark.	✓
(a)(iii) writes down 8	**B1**	cao. $8 = 2^3$. A square number such as 9 scores no mark.	✓
(a)(iv) writes down 2	**B1**	cao. 2 is the only prime in the list; 9 has the factor 3 and the rest are even and larger than 2.	✓

Step	Mark	Description	Got it?
(b) writes down 76	**B1**	cao. Working is not required, so a correct answer scores full marks. 220 (adding before multiplying) and 66 (reading 2^3 as 2×3) score no mark.	✓

Full marks: 5/5

Question 6 - Exam Solution

Understanding the Question

Given

Two straight lines meet at a point. One angle at that point is 235° and the other is x°.
$ABDE$ is a quadrilateral, BCD is an isosceles triangle with $BD = DC$, and ABC is a straight line.
In the second figure, angle $EAB = 90^\circ$, angle $AED = 98^\circ$ and angle $DCB = 54^\circ$.

Find

The value of x, the angle fact that gives it, and the value of y.

Plan the Solution

- Part (a): the two marked angles are the only angles at that point, so together they must make one complete turn of 360°.
- Part (b): the isosceles triangle gives the angle at B inside the triangle; the straight line ABC turns that into $\angle ABD$, which is the quadrilateral's angle at B.
- Then use the four angles of $ABDE$. Keep to that order: y cannot be found until $\angle ABD$ is known.

Worked Solution [5 marks]

> Angles around a point add up to 360°. Angles on a straight line add up to 180°. The base angles of an isosceles triangle are equal, and the four angles of a quadrilateral add up to 360°.

Step 1: one complete turn at the point

$x + 235 = 360$

$360 - 235 = 125$

(Reason: the two marked angles are the only angles at that point, and a complete turn is $360°$)

Step 2: say which angle fact was used

$235 + 125 = 360$

(Reason: part (a)(ii) wants the fact, not the arithmetic: angles around a point add up to $360°$)

Step 3: the angles of isosceles triangle BCD

$\angle DBC = \angle DCB = 54°$

$180 - 54 - 54 = 72$

(Reason: $BD = DC$, so the angles opposite those two equal sides are equal, and the apex angle $\angle BDC$ is whatever is left of $180°$)

Step 4: the quadrilateral's angle at B

$\angle ABD = 180 - \angle DBC$

$180 - 54 = 126$

(Reason: ABC is a straight line, so $\angle ABD$ and $\angle DBC$ together make $180°$. This obtuse angle is the one inside $ABDE$, not the $54°$ inside the triangle)

Step 5: the four angles of quadrilateral ABDE

$\angle EAB + \angle ABD + \angle BDE + \angle DEA = 360$

$90 + 126 + 98 = 314$

$360 - 314 = 46$

(Reason: the four angles of any quadrilateral add up to $360°$, so y is what the other three leave)

(a)(i) $x = 125$

(a)(ii) Angles around a point add up to $360°$

(b) $y = 46$

Verification

- ✓ **Check 1:** Put the answer to part (a) back beside the given angle and add them $235 + 125 = 360$, exactly one complete turn
- ✓ **Check 2:** Add all four angles of $ABDE$ with $y = 46$ in place $90 + 126 + 46 + 98 = 360$
- ✓ **Check 3:** Take a different route entirely. $AEDC$ is also a quadrilateral, since its fourth side lies along the straight line ABC. Its angle at D is y plus the apex angle $72°$, and

$46 + 72 = 118$, so its four angles are $90°$, $98°$, $118°$ and $54°$
$90 + 98 + 118 + 54 = 360$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $x = 125$	**B1**	cao. No working is required, so 125 on its own scores the mark.	✓
(a)(ii) correct reason	**B1**	Angles around a point add up to $360°$. The words 'around a point' (or 'at a point') and the value 360 must both appear.	✓
(b) $\angle ABD = 180 - 54 = 126$ or $\angle BDC = 180 - 2 \times 54 = 72$ or $\angle BDC = 180 - 108 = 72$	**M1**	Any one of these three. If angles are written on the diagram they must be correctly assigned, and if angle notation is used it must be correctly assigned.	✓
(b) $360 - (98 + 90 + 126)$ or $360 - (98 + 90 + 54 + 72)$ or $360 - 314$	**M1**	For a complete method that reaches y. The quotation marks the mark scheme puts round 126 and 72 mean the candidate's own value from the previous mark may be used here.	✓
(b) $y = 46$	**A1**	cao	✓
Note	**(no mark)**	Working is not required in part (b), so a correct answer scores full marks unless it comes from obviously incorrect working. A common wrong answer is 100, which is what putting the apex angle $72°$ into the quadrilateral in place of $\angle ABD = 126°$ gives.	✓

Full marks: 5/5

Question 7 - Exam Solution

Understanding the Question

Given

Normal rate of pay: £14 for each hour
Overtime rate of pay: £21 for each hour
35 hours of the week are paid at the normal rate
Total pay for the week: £679

Find

The number of hours of overtime worked in this week

Plan the Solution

- Work out the pay earned at the normal rate for the 35 hours.
- Take that away from the total pay, so that what is left is the pay earned at the overtime rate.
- Divide the overtime pay by the overtime rate to turn it back into a number of hours.

Worked Solution [4 marks]

> Rule - Pay in two parts:
> $\text{total pay} = \text{normal hours} \times \text{normal rate} + \text{overtime hours} \times \text{overtime rate}$

Step 1: The pay earned at the normal rate

$$35 \times 14 = 490$$

(Reason: (Reason: Douglas is paid £14 for each of the 35 hours at the normal rate, so £490 of his pay has nothing to do with overtime.))

Step 2: The pay that is left for overtime

$$679 - 490 = 189$$

(Reason: (Reason: the week's pay is only these two parts, so everything above the £490 must have been earned at the overtime rate. That leaves £189 of overtime pay.))

Step 3: Turn the overtime pay back into hours

$$\frac{189}{21} = 9$$

(Reason: (Reason: each hour of overtime is worth £21, so dividing the overtime pay by 21 counts how many overtime hours it took to earn it.))

9 hours of overtime

Verification

- ✓ **Check 1:** Build the week's pay forwards from the answer: 35 hours at £14 gives 490, and 9 hours at £21 gives 189. $490 + 189 = 679$, the total pay the question gives
- ✓ **Check 2:** Count the week a different way. Douglas works $35 + 9 = 44$ hours altogether, and every hour earns at least £14, while each overtime hour earns an extra $21 - 14 = 7$ pounds on top: $44 \times 14 = 616$ and $9 \times 7 = 63$. $616 + 63 = 679$, the same total by a route that never uses the £189
- ✓ **Check 3:** Check the answer is sensible. Overtime is paid in whole hours here, and 189 is a multiple of 21, so the division has to come out exactly. 9 is a whole number of hours, and $9 \times 21 = 189$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$35 \times 14 = 490$	M1	Method for the pay earned at the normal hourly rate.	✓
$679 - 490 = 189$	M1	Method for the part of the total pay earned at the overtime rate, allowing their own value in place of 490. The official scheme also awards this mark and the next one together, as M2, for $679 - 9 \times 21$ or $490 + 9 \times 21$ or equivalent.	✓
$\frac{189}{21}$	M1	Method for turning the overtime pay into hours, allowing their own value in place of 189, divided by the overtime rate.	✓
9	A1	Working not required, so a correct answer scores full marks, unless it comes from obvious incorrect working.	✓

Full marks: 4/4

Question 8 - Exam Solution

Understanding the Question

Given	Find

(a) The expression $10x - 7y - 6x + 4y$, which has two x terms and two y terms. (b) The formula $T = 4d - 6e$, together with $d = 13$ and $e = 7$. (c) The equation $5p + 11 = 28$, which has one unknown letter.

(a) The expression written with as few terms as possible. (b) The value of T. (c) The value of p.

Plan the Solution

- (a) Group the terms that carry the same letter, then combine their number parts, keeping the sign that sits in front of each term.
- (b) Replace d and e by their values, work out the two products, then subtract.
- (c) Undo the $+11$ first, then undo the multiplication by 5, doing the same to both sides each time.

Worked Solution [6 marks]

> Rule - Collect, substitute, then undo: like terms are terms carrying exactly the same letter, and only those may be combined; a letter is replaced by its value before any arithmetic is done; and an equation is solved by undoing its operations in reverse order, doing the same to both sides.

Step 1: Sort part (a) into like terms

$$10x - 7y - 6x + 4y$$

$$(10x - 6x) + (-7y + 4y)$$

(Reason: Like terms carry exactly the same letter, so the two x terms go together and the two y terms go together. The sign written in front of a term belongs to that term, so $-7y$ keeps its minus sign when it is moved.)

Step 2: Combine each pair

$$10x - 6x = 4x$$

$$-7y + 4y = -3y$$

$$10x - 7y - 6x + 4y = 4x - 3y$$

(Reason: 10 of something with 6 of it taken away leaves 4 of it. For the y terms, start at -7 and add 4, which moves up the number line to -3, so that term is still negative.)

Step 3: Substitute the given values in part (b)

$$T = 4d - 6e$$

$$T = 4 \times 13 - 6 \times 7$$

(Reason: Each letter is replaced by its own value, so d becomes 13 and e becomes 7. Writing $4d$ means 4 multiplied by d, so the multiplication signs are put back in.)

Step 4: Work out the two products, then subtract

$$4 \times 13 = 52$$

$$6 \times 7 = 42$$

$$T = 52 - 42 = 10$$

(Reason: Multiplication is done before subtraction, so both products are worked out first. The formula then takes 42 away from 52.)

Step 5: Undo the addition in part (c)

$$5p + 11 = 28$$

$$5p + 11 - 11 = 28 - 11$$

$$5p = 17$$

(Reason: On the left, 11 has been added, so 11 is taken off both sides. Doing the same to each side keeps the equation balanced and leaves $5p$ on its own.)

Step 6: Undo the multiplication

$$\frac{5p}{5} = \frac{17}{5}$$

$$p = \frac{17}{5} = 3.4$$

(Reason: $5p$ means 5 multiplied by p, so both sides are divided by 5. Since 17 is not a multiple of 5, the answer is a fraction, and $\frac{17}{5}$ and 3.4 are the same number.)

(a) $4x - 3y$

(b) $T = 10$

(c) $p = \frac{17}{5} = 3.4$

Verification

- ✓ **Check 1:** Part (a): put $x = 2$ and $y = 1$ into both forms. The original gives $20 - 7 - 12 + 4$ and the simplified form gives $8 - 3$, so the two expressions must agree. $5 = 5$
- ✓ **Check 2:** Part (b): work backwards. Adding back the 42 that was subtracted must return the first product, 4×13. $10 + 42 = 52 = 4 \times 13$

✓ **Check 3:** Part (c): put $p = 3.4$ back into the left-hand side of the original equation and compare it with the right-hand side. $5 \times 3.4 + 11 = 17 + 11 = 28$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $4x - 3y$	**B2**	Both terms correct, in either order, so $-3y + 4x$ also scores B2. If not B2, award B1 for $4x$ alone or for $-3y$ alone.	✓
(b) 4×13 and 6×7, or 52 and 42	**M1**	Method mark for substituting both values and forming the two products. The sign attached to 6×7 may be either way round.	✓
(b) 10	**A1**	Working not required, so a correct answer scores full marks (unless it comes from obviously incorrect working).	✓
(b) -50	**SC B1**	Special case, awarded only when no other marks are earned. It names one specific error: substituting the two values the wrong way round gives $4e - 6d = 28 - 78 = -50$.	✓
(c) $5p = 28 - 11$ or $5p = 17$ or $p + \frac{11}{5} = \frac{28}{5}$ oe	**M1**	Method mark for a correct first step towards getting p on its own. Also allow $11 - 28 = -5p$ or $\frac{28 - 11}{5}$.	✓
(c) $\frac{17}{5}$	**A1**	Or equivalent, for example 3.4 or the mixed number $3\frac{2}{5}$. Working not required, so a correct answer scores full marks.	✓

Full marks: 6/6

Question 9 - Exam Solution

Understanding the Question

Given

Find

Triangle ABC is equilateral, so $AB = BC = CA = 9$ cm

The side AB is already drawn, and it is exactly 9 cm long

A ruler and a pair of compasses only, with every construction line left showing

The position of the third vertex C, and then the completed triangle

Plan the Solution

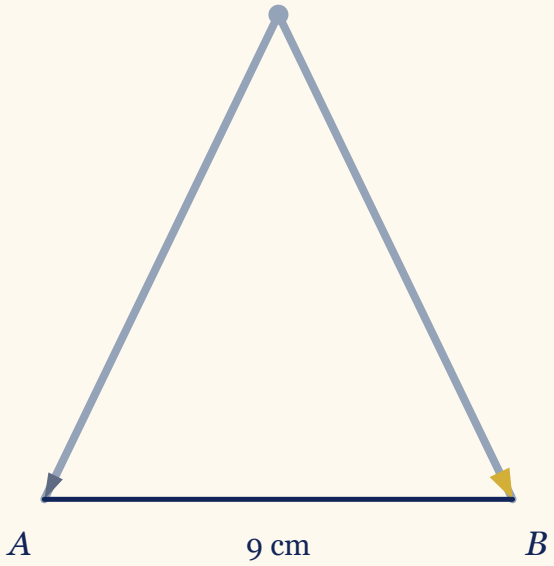
- The third vertex has to be 9 cm from A and 9 cm from B at the same time.
- Every point 9 cm from A lies on one arc, and every point 9 cm from B lies on another.
- The two arcs cross at only one point above AB, so that crossing point must be C.
- Draw both arcs first, then join the crossing point to each end of the given side. The arcs stay on the page: they are the working, and the marks are for them.

Worked Solution [2 marks]

> Rule - Constructing an equilateral triangle: the third vertex is the point that is the same distance from both ends of the given side. An arc of radius 9 cm centred on A and an arc of radius 9 cm centred on B cross at exactly that point.

Step 1: Open the compasses to the length of a side

compass radius $= AB = 9$ cm



(Reason: (Reason: every side of the triangle is 9 cm, and AB is already drawn 9 cm long. Resting the compass point on A and the pencil on B therefore sets the radius without measuring anything. Do not change that setting again.))

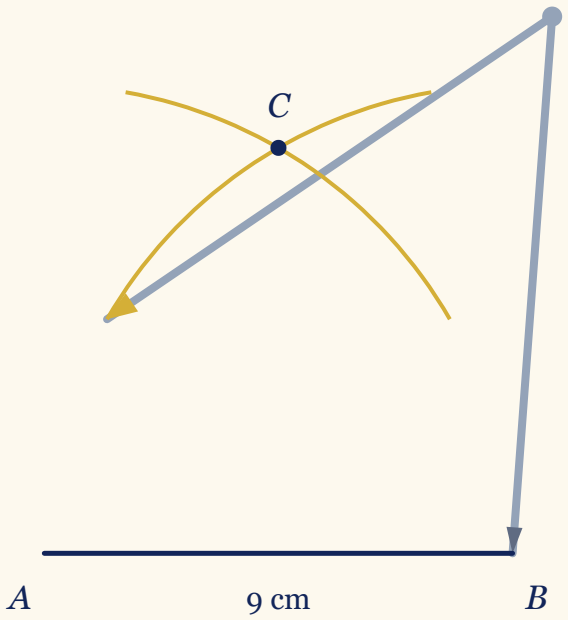
Step 2: Draw an arc centred on A

$AC = 9$ cm

(Reason: (Reason: with the point on A, every place the pencil can reach is 9 cm from A. Sweep a generous arc right across the space above AB, so there is plenty of it for the second arc to cross.))

Step 3: Draw an arc centred on B

$BC = 9$ cm

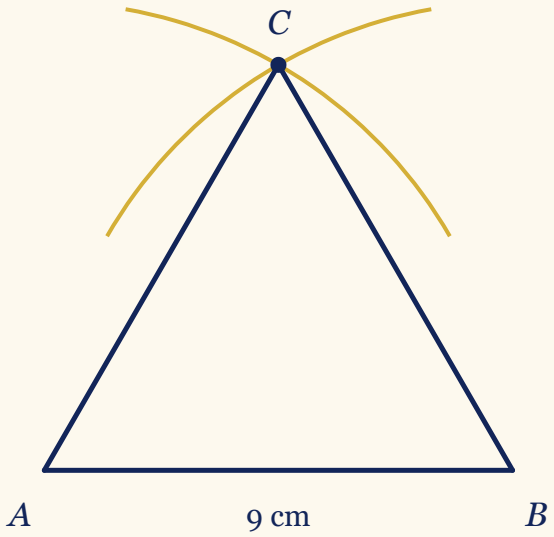


(Reason: (Reason: keeping the same setting, lift the compasses, put the point on B and sweep a second arc. Where the two arcs cross, the point is 9 cm from A and 9 cm from B at once, so that crossing is C .))

Step 4: Join the triangle and leave the arcs showing

$AB = BC = CA = 9$ cm

$\angle A = \angle B = \angle C = 60^\circ$



(Reason: (Reason: use the ruler to join C to A and C to B. All three sides now measure 9 cm, so the triangle is equilateral and every angle is $60°$. Do not rub the arcs out: without them the answer scores only one of the two marks.))

Triangle ABC constructed with two arcs of radius 9 cm, one centred on A and one centred on B, crossing at C, with all construction arcs left showing

Verification

- ✓ **Check 1:** Measure AC and BC with the ruler. Both were drawn with the compasses fixed at one setting, so both must come out the same as AB. $AC = BC = AB = 9$ cm
- ✓ **Check 2:** Measure straight up from AB to C. An equilateral triangle of side 9 cm stands $9 \sin 60°$ high, so a badly set pair of compasses shows up at once.
  $9 \sin 60° = 7.79$ cm $(2$ d.p.$)$
- ✓ **Check 3:** Put a protractor on each corner in turn. All three sides are equal, so all three angles are equal, and the three of them add up to $180°$. $\dfrac{180°}{3} = 60°$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two arcs of radius 9 cm drawn, one centred on A and one centred on B, crossing above AB	**B1**	Awarded for two intersecting arcs within or on the guidelines of the overlay. Also awarded for an accurate triangle drawn with no arcs.	✓
Triangle ABC completed from the crossing point, with the correct intersecting arcs 9 cm from A and 9 cm from B still showing	**B2**	Full marks. The arcs must lie within or on the guidelines of the overlay.	✓
Note	**note**	Working required. Here the arcs are the working: a triangle measured out with a ruler and a protractor, with no arcs left on the page, is worth B1 only.	✓

Full marks: 2/2

Question 10 - Exam Solution

Understanding the Question

Given

There are 29 kayaks on the rack, and one is taken at random, so every kayak is equally likely
10 of the kayaks are yellow
The rest are green or blue, and
$P(\text{green}) = \frac{7}{29}$

Find

(a) $P(\text{yellow})$ (b) $P(\text{blue})$

Plan the Solution

- Part (a) is a straight count: the number of yellow kayaks over the 29 kayaks on the rack.
- The green probability is already written over 29, so its numerator, 7, is the number of green kayaks.
- Every kayak is yellow, green or blue, so taking the yellow ones and the green ones away from 29 leaves the blue ones.

- Put that count over the same total. There is a second route for part (b) - take both known probabilities away from 1 - and it is used as a check.

Worked Solution [3 marks]

> Rule - For equally likely outcomes, $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$, and the probabilities of all the separate colours add up to 1.

Step 1: (a) Write the yellow probability straight from the counts

$$P(\text{yellow}) = \frac{10}{29}$$

(Reason: (Reason: 10 of the 29 kayaks are yellow, and each one is equally likely to be taken. There is nothing to cancel here: 29 is a prime number, so $\frac{10}{29}$ is already in its simplest form.))

Step 2: Turn the green probability back into a number of kayaks

$$\frac{7}{29} \times 29 = 7$$

(Reason: (Reason: the denominator of $\frac{7}{29}$ is the total number of kayaks, so the numerator is already a count. 7 of the kayaks on the rack are green.))

Step 3: (b) Subtract to find how many kayaks are blue

$$29 - 10 - 7 = 12$$

(Reason: (Reason: every kayak on the rack is yellow, green or blue. Take away the 10 yellow ones and the 7 green ones, and only the blue ones are left.))

Step 4: Write the blue probability over the same total

$$P(\text{blue}) = \frac{12}{29}$$

(Reason: (Reason: the rack still holds 29 kayaks, so the total underneath does not change - only the count on top does. As a decimal this is $0.4137\ldots$, which the mark scheme accepts anywhere from 0.41 to 0.42.))

(a) $\dfrac{10}{29}$

(b) $\dfrac{12}{29}$

Verification

✓ **Check 1:** The kayak taken is certainly yellow, green or blue, so the three probabilities must add to 1. $\frac{10}{29} + \frac{7}{29} + \frac{12}{29} = \frac{29}{29} = 1$

✓ **Check 2:** Work part (b) the other way round: instead of counting kayaks, take both known probabilities away from 1. $1 - \frac{10}{29} - \frac{7}{29} = \frac{12}{29}$

✓ **Check 3:** Count the kayaks instead of the probabilities: the three colours must account for the whole rack. $10 + 7 + 12 = 29$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $P(\text{yellow}) = \frac{10}{29}$	**B1**	Or any equivalent. Accept the decimal 0.34(48275...) or the percentage 34.(48275...)%, truncated or rounded.	✓
(b) A complete method for the blue kayaks: $29 - 10 - 7$, or $\frac{29 - 10 - 7}{29}$, or $1 - \frac{10 + 7}{29}$	**M1**	Also awarded for the blue count 12 on its own, or for the decimal method 1 - 0.34(482...) - 0.24(137...).	✓
(b) $P(\text{blue}) = \frac{12}{29}$	**A1**	Or any equivalent. Accept 0.41(37931...) to 0.42, or 41.(37931...)% to 42%.	✓
Note	**note**	Working is not required in part (b), so a correct answer scores full marks on its own, unless it comes from obvious incorrect working. Incorrect probability notation is penalised only once across the question.	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

A recipe that makes 24 scones uses 120 g of butter, 60 g of sugar and 200 g of flour. Oliver has five 250 g packs of butter, 750 g of sugar and 1.4 kg of flour.
One of the three amounts is written in kilograms while the recipe is written in grams.

Find

The largest number of scones Oliver can bake before one of the ingredients runs out.

Plan the Solution

- Put every amount into grams first, so the store cupboard and the recipe can be compared.
- Take each ingredient in turn and work out how many batches of 24 that ingredient alone would allow.
- The smallest of the three answers is the limit, because that ingredient runs out first. The other two are not the answer, however large they are.
- Turn batches back into scones by multiplying the limiting number of batches by 24.

Worked Solution [4 marks]

> Rule - The ingredient that runs out first sets the limit: divide the amount held by the amount one batch needs, do this for every ingredient, take the smallest result, then multiply by 24.

Step 1: Write every amount in grams

$1.4 \times 1000 = 1400$ g of flour

$5 \times 250 = 1250$ g of butter

(Reason: (Reason: the recipe is written in grams, so the 1.4 kg of flour has to become 1400 g and the five packs have to be added together before anything can be compared. The sugar is already in grams at 750 g.))

Step 2: How many batches does the butter allow?

$$\frac{1250}{120} = 10.4\ldots \text{ batches}$$

(Reason: (Reason: one batch takes 120 g of butter, so dividing the butter held by 120 counts how many whole batches the butter could supply.))

Step 3: How many batches does the sugar allow?

$$\frac{750}{60} = 12.5 \text{ batches}$$

(Reason: (Reason: one batch takes 60 g of sugar, so the sugar he has is divided by 60 in exactly the same way as the butter.))

Step 4: How many batches does the flour allow?

$$\frac{1400}{200} = 7 \text{ batches}$$

(Reason: (Reason: one batch takes 200 g of flour, and 1400 divides by 200 exactly, so the flour allows precisely 7 batches with nothing left over.))

Step 5: Take the smallest, then count the scones

smallest $= 7$ batches, from the flour

$7 \times 24 = 168$

(Reason: (Reason: the flour is the first ingredient to run out, so it decides the total. Only 7 batches can be baked, and each batch makes 24 scones.))

168 scones

Verification

- ✓ **Check 1:** Rebuild the shopping list. 168 scones is 7 full batches, so multiply each recipe amount by 7 and compare with what Oliver actually has. $7 \times 200 = 1400$ g of flour, which is all of it; $7 \times 120 = 840$ g of butter out of 1250 g; $7 \times 60 = 420$ g of sugar out of 750 g - every amount is available
- ✓ **Check 2:** Test that 168 really is the maximum by trying one more batch. An eighth batch would need 8×200 g of flour. $1600 > 1400$, so an eighth batch is impossible and 192 scones cannot be made
- ✓ **Check 3:** Reach the answer a different way, working per single scone instead of per batch. One scone takes $\frac{200}{24}$ g of flour, so the flour allows $1400 \times \frac{24}{200}$ scones.

  $1400 \times \frac{24}{200} = 168$, the same limit found by a completely different route

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Convert the flour into grams	M1	for $1.4 \times 1000 = 1400$ oe	✓

Step	Mark	Description	Got it?
Work out how many batches ONE ingredient allows	M1	for $\frac{5 \times 250}{120} = 10.4\ldots$ or $\frac{750}{60} = 12.5$ or $\frac{1400}{200} = 7$ oe; or for the grams needed by one scone, $\frac{120}{24} = 5$ or $\frac{60}{24} = 2.5$ or $\frac{200}{24} = 8.3\ldots$ oe. A correct list of multiples of 120, 60 or 200 scores this mark	✓
Work out how many scones ALL THREE ingredients allow	M1	for 250 and 300 and 168 oe; or for the three batch counts $10.4\ldots$ and 12.5 and 7 each multiplied by 24	✓
Choose the smallest and state the number of scones	A1	for 168. Working is required, and this mark is not awarded unless all three method marks are earned	✓

Full marks: 4/4

Question 12 - Exam Solution

Understanding the Question

Given

A frequency table for the 30 members: 0, 1, 2, 3, 4, 5 training sessions, with frequencies 2, 5, 11, 7, 4, 1

The frequencies total 30, which is the number of members the question states

The probability that Erin travels to the pool by bus is 0.79

Find

(a) The mean number of training sessions for the members of the club (b) The probability that Erin does not travel to the pool by bus

Plan the Solution

- Part (a): a frequency table is a shorthand for a long list. Turn it back into a total by multiplying each number of sessions by how many members did that number.
- Add those products to get the total number of sessions swum by the whole club, then divide by how many members there are.

- Part (b): travelling by bus and not travelling by bus are the only two outcomes, so their probabilities add to 1. Subtract to get the one that is missing.

Worked Solution [4 marks]

> Rule - Mean from a frequency table: $\text{mean} = \dfrac{\text{total of all the values}}{\text{total frequency}}$. Rule - Complementary events: an event and its opposite have probabilities that add to 1.

Step 1: Multiply each number of sessions by its frequency

$0 \times 2 = 0$

$1 \times 5 = 5$

$2 \times 11 = 22$

$3 \times 7 = 21$

$4 \times 4 = 16$

$5 \times 1 = 5$

(Reason: each row stands for a whole group of members, so the 7 members who trained 3 times contribute 21 sessions between them, not 3)

Step 2: Add the products to get the total number of sessions

$0 + 5 + 22 + 21 + 16 + 5 = 69$

(Reason: adding the products puts the whole club back together as one running total, which is the top of the mean)

Step 3: Add the frequencies to get the total number of members

$2 + 5 + 11 + 7 + 4 + 1 = 30$

(Reason: this is the bottom of the mean, and it agrees with the 30 members the question states, so no row has been missed)

Step 4: Divide the total number of sessions by the total number of members

$\dfrac{69}{30} = 2.3$

(Reason: the mean shares the 69 sessions equally between the 30 members)

Step 5: Part (b) - take the probability of travelling by bus away from 1

$1 - 0.79 = 0.21$

(Reason: Erin either travels by bus or does not, and nothing else is possible, so the two probabilities must add to 1)

(a) 2.3 training sessions

(b) 0.21

Verification

✓ **Check 1:** Multiply the mean back by the number of members. If 2.3 really is the fair share, 30 shares of it must rebuild the total. $2.3 \times 30 = 69$, the total found in step 2

✓ **Check 2:** A mean must lie between the smallest and the largest value in the table, and it should sit near the tallest bar. Most members, 11 of them, trained 2 times. 0 is the least and 5 is the most, and 2.3 lies between them, just above 2

✓ **Check 3:** Add the two probabilities in part (b). Two outcomes that cover every possibility must account for the whole of the probability between them. $0.79 + 0.21 = 1$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Multiply each value by its frequency and add the products	**M1**	for at least 4 products added, which need not be evaluated, or for 71 (the total reached when the top row 0×2 is copied down as 2)	✓
(a) Divide the total by the total frequency	**M1**	for their 69 divided by 30, that is $\frac{69}{30}$	✓
(a) Answer	**A1**	cao 2.3. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working	✓
(b) Answer	**B1**	0.21 or an equivalent, for example $\frac{21}{100}$ or 21 per cent	✓

Full marks: 4/4

Question 13 - Exam Solution

Understanding the Question

Given	Find

The equation $y = 2x - 3$
Values of x from -2 to 4
A printed grid to draw the graph on

The graph of $y = 2x - 3$, drawn across the whole of that interval

Plan the Solution

- $y = 2x - 3$ is of the form $y = mx + c$, so its graph is a straight line and a short table of values is enough.
- Put each whole value of x from -2 to 4 into the equation and work out y.
- Plot each pair on the grid and rule one line through them.
- Take the line all the way from $x = -2$ to $x = 4$: the marks are for a line across the whole interval, not for the points on their own.

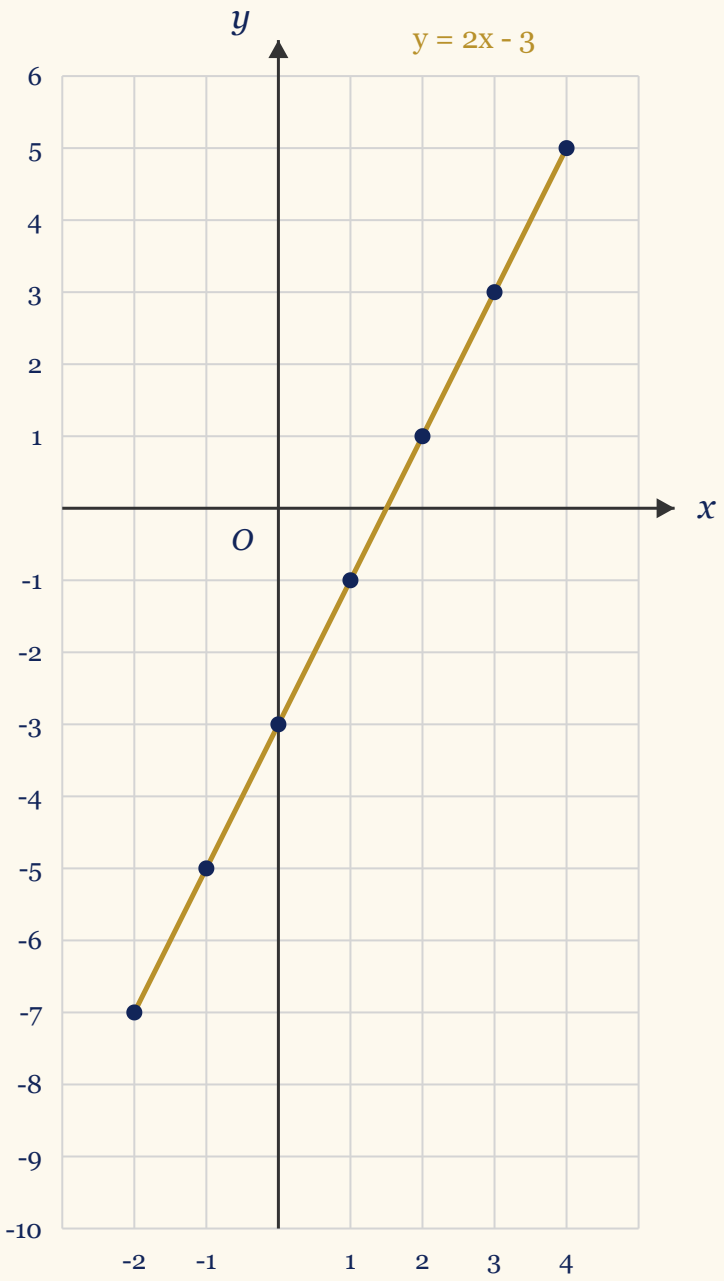
Worked Solution [3 marks]

> Rule - A straight line $y = mx + c$: m is the gradient, the steps up for each step across, and c is where the line crosses the y-axis. Two points fix the line and a third is a check, so a table of values is the safest way to draw one.

Step 1: Work out y at the two ends, $x = -2$ and $x = 4$

$2 \times (-2) - 3 = -4 - 3 = -7$

$2 \times 4 - 3 = 8 - 3 = 5$



(Reason: (Reason: the ends of the interval are the two points the question names, and between them they fix the whole line. Multiply by 2 first and take the 3 off afterwards; a negative value of x still has to go through that multiplication before the subtraction.))

Step 2: Fill in the values in between

$$2 \times (-1) - 3 = -5$$

$$2 \times 0 - 3 = -3$$

$$2 \times 1 - 3 = -1$$

$$2 \times 2 - 3 = 1$$

$$2 \times 3 - 3 = 3$$

(Reason: (Reason: only two points are strictly needed for a straight line, but a full table catches a slip. Each y should be 2 more than the one before it, and here they run $-7, -5, -3, -1, 1, 3, 5$ - a

constant step, which is exactly what a straight line means.))

Step 3: Plot the seven points and rule one line through them

$(-2, -7),\ (-1, -5),\ (0, -3),\ (1, -1),\ (2, 1),\ (3, 3),\ (4, 5)$

(Reason: (Reason: read each pair as across first, then up or down. Every point should land on one straight line; if one is off, that value of y is worth checking again before the line is ruled. Use a ruler and take the line right from $x = -2$ to $x = 4$.))

Step 4: Read the gradient back off the drawing as a check

$$\frac{5 - (-7)}{4 - (-2)} = \frac{12}{6} = 2$$

(Reason: (Reason: from one end of the drawn line to the other is 12 squares up for 6 squares across, so the drawing has gradient 2 - the number in front of the x. A line drawn with the right gradient through the right point on the y-axis cannot be anything else.))

The straight line through $(-2, -7)$ and $(4, 5)$, ruled right across the grid from $x = -2$ to $x = 4$

Verification

- ✓ **Check 1:** Take a point off the middle of the drawn line, $(3, 3)$, and put $x = 3$ back into the equation. It has to give the same y. $2 \times 3 - 3 = 3$
- ✓ **Check 2:** Count the squares between the two ends of the drawn line: 12 up for 6 across. The gradient of the drawing must equal the 2 in the equation. $\frac{12}{6} = 2$
- ✓ **Check 3:** The line should cut the y-axis at -3, the constant term, and the middle point of the seven should sit halfway between the two ends. $\frac{-7 + 5}{2} = -1$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
At least 2 correct points stated (a table counts) or plotted, or a line with a positive gradient through $(0, -3)$, or a line with a gradient of 2	**B1**	The first band. Enough correct work to show the equation has been used, but no	✓

Step	Mark	Description	Got it?
		correct line yet.	
A correct straight line segment through at least 3 of $(-2,-7), (-1,-5), (0,-3), (1,-1), (2,1), (3,3), (4,5)$, or all seven of them plotted but not joined	**B2**	The second band. A correct short line, or seven correct points with no line ruled through them.	✓
A correct line drawn between $x = -2$ and $x = 4$	**B3**	Full marks. The line is correct and it covers the whole interval the question asks for.	✓
Note	**note**	The three marks are one B3 award, not three marks collected in turn: B1 and B2 say what a partly correct answer is worth. A correct line that stops short of either end drops to the B2 band, so the last thing to check is that the line	✓

Step	Mark	Description	Got it?
		reaches both ends.	

Full marks: 3/3

Question 14 - Exam Solution

Understanding the Question

Given

490 pots are made each week.
86 of those pots are glazed.
A glazed pot weighs 375 grams before it is fired.
Firing takes away 12% of that weight.

Find

(a) 86 written as a percentage of 490, correct to one decimal place. (b) The weight of one pot after firing, in grams.

Plan the Solution

- Part (a): put the glazed pots over the total made, $\frac{86}{490}$, then multiply by 100.
- Keep the full decimal on the calculator and round only at the very end, or the last digit can come out wrong.
- Part (b): find 12% of 375 grams, then take that loss away from 375 grams.
- A one-step alternative for part (b): losing 12% leaves 88%, so multiply by 0.88.

Worked Solution [5 marks]

Rule - Percentage of a total: $\frac{\text{part}}{\text{whole}} \times 100$. Rule - Percentage decrease: take the loss off the original, or multiply the original by $\frac{100 - 12}{100}$.

Step 1: Write the glazed pots as a fraction of all the pots

$$\frac{86}{490} = 0.175510\ldots$$

(Reason: A percentage compares a part with a whole, so the 86 glazed pots go on top and the 490 pots made altogether go underneath.)

Step 2: Turn the decimal into a percentage

$0.175510\ldots \times 100 = 17.5510\ldots\%$

(Reason: Multiplying by 100 converts a decimal to a percentage, in the same way that 0.5 becomes 50%.)

Step 3: Round to one decimal place

$17.5510\ldots \approx 17.6$

(Reason: The digit in the second decimal place is 5, so the first decimal place rounds up from 5 to 6. Rounding the answer, not the working, is what keeps the last digit safe.)

Step 4: Work out the weight the pot loses in the kiln

$$\frac{12}{100} = 0.12$$

$$0.12 \times 375 = 45$$

(Reason: The pot loses 12% of the weight it had before firing, so the multiplier goes on the weight of one pot, 375 grams.)

Step 5: Take the loss away from the starting weight

$375 - 45 = 330$

(Reason: What comes out of the kiln is what went in minus what was lost, so the baked pot weighs 330 grams.)

(a) 17.6%

(b) 330 grams

Verification

- ✓ **Check 1:** Put the rounded percentage back. 17.6% is 0.176, so take that fraction of the 490 pots. $0.176 \times 490 = 86.24$, within a quarter of a pot of 86, which is all a rounded percentage can give.
- ✓ **Check 2:** Test the rounding boundary. Half way between 17.5 and 17.6 is 17.55. $17.5510\ldots > 17.55$, so one decimal place gives 17.6 and not 17.5.
- ✓ **Check 3:** Do part (b) by the other method. Losing 12% means keeping 88%, so multiply by 0.88. $0.88 \times 375 = 330$ grams, the same weight reached a different way.
- ✓ **Check 4:** Compare the fired weight with the raw weight as a fraction. $\frac{330}{375} = 0.88 = 88\%$, so exactly 12% of the weight has gone.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\frac{86}{490} \times 100$ or $0.175510\ldots \times 100$	M1	A correct method for writing one number as a percentage of another. The multiplication by 100 may be implied.	✓
(a) 17.6	A1	awrt 17.6. Working is not required, so a correct answer scores both marks unless it comes from obviously incorrect working.	✓
(b) $\frac{12}{100} \times 375$ or $0.12 \times 375\ (= 45)$	M1	A calculation must be seen. Writing 12% of 375 in words is not enough unless 45 is seen. The split $37.5 + 3.75 + 3.75$ earns the same mark.	✓
(b) $375 - 45$	M1	Awarded on the candidate's own loss, so $375 - 37.5 - 3.75 - 3.75$ and $375 - 37.5 - 7.5$ both score it.	✓
(b) 330	A1	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Alternative for (b): $\frac{88}{100} \times 375$	M2	The one-step multiplier method earns both method marks at once, because $100\% - 12\% = 88\%$ does the subtraction inside the multiplier.	✓

Full marks: 5/5

Question 15 - Exam Solution

Understanding the Question

Given

Shape $ABCDE$ is a right-angled triangle ABE joined to a square $BCDE$ along the side BE.
ABC is a straight line, so the right angle of the triangle is at B.
$AB = 15$ cm and $AE = 17$ cm.
The perimeter of triangle ABE is 40 cm.

Find

The area of the whole shape $ABCDE$, in cm^2. Two shapes, so expect two areas and one addition at the end.

Plan the Solution

- Only two of the triangle's three sides are given, and the missing one is BE. The perimeter supplies it in a single subtraction.
- BE is also a side of the square, so finding it unlocks both areas at once. This is why the question gives a perimeter rather than the side itself.
- Work out the area of the square, then the area of the triangle, then add them.
- Nothing needs to be taken away: the triangle and the square meet along BE and do not overlap.

Worked Solution [4 marks]

> Rule - Composite shape: split it into parts, work out each area, then add. Area of a triangle = $\frac{1}{2} \times$ base $\times$ height, and area of a square = side $\times$ side.

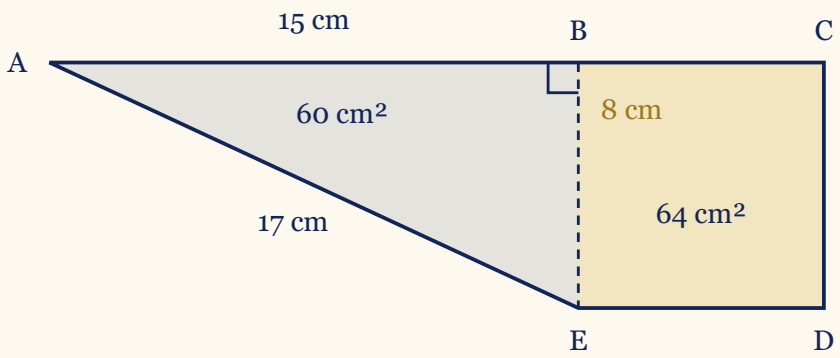
Step 1: Use the perimeter to find BE

Perimeter of $ABE = AB + AE + BE$

$40 = 15 + 17 + BE$

$BE = 40 - 32 = 8$ cm

Diagram NOT accurately drawn



(Reason: The perimeter is the total of all three sides, so the unknown side is whatever is left once the two known sides are taken off: $15 + 17 = 32$, and $40 - 32 = 8$.)

Step 2: Work out the area of the square $BCDE$

Area of $BCDE = BE \times BE$

$= 8 \times 8 = 64$ cm^2

(Reason: All four sides of a square are equal, so $BE = 8$ cm makes BC, CD and DE each 8 cm as well.)

Step 3: Work out the area of the triangle ABE

Area of $ABE = \frac{1}{2} \times AB \times BE$

$= \frac{1}{2} \times 15 \times 8 = 60$ cm^2

(Reason: The right angle is at B, so the two sides that meet there, AB and BE, are the base and the height. The sloping side 17 cm is the hypotenuse and is never the height.)

Step 4: Add the two areas

Area of $ABCDE = 60 + 64$

$= 124$ cm^2

(Reason: The shape is exactly the triangle and the square with nothing counted twice. The join BE is a side of each piece, not a piece of area, so the two totals simply add.)

124 cm^2

Verification

✓ **Check 1:** Test BE a completely different way. Triangle ABE is right-angled at B, so Pythagoras must hold with AE as the hypotenuse. $15^2 + 8^2 = 225 + 64 = 289 = 17^2$, so $BE = 8$ cm is right, and the perimeter really is $15 + 17 + 8 = 40$ cm.

✓ **Check 2:** Treat the shape as one trapezium instead of two pieces. $AC = AB + BC = 15 + 8 = 23$ cm, and $ACDE$ has parallel sides AC and ED with height BE. $\frac{1}{2} \times (23 + 8) \times 8 = 124$, the same total by a route that never splits the shape up.

✓ **Check 3:** A size check. The whole shape sits inside a rectangle 23 cm by 8 cm, of area 184 cm². What that rectangle has and the shape does not is the triangle above AE, whose legs are 15 cm and 8 cm. $184 - \frac{1}{2} \times 15 \times 8 = 184 - 60 = 124$, and 124 is comfortably less than 184, as it must be.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$40 - (17 + 15) = 8$ or $\sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8$	M1	For a correct method to find BE. May be seen on the diagram.	✓
$8 \times 8 = 64$	M1	For the area of the square. The scheme writes the side in quotes, so a candidate's own value	✓

Step	Mark	Description	Got it?
		for BE is used. May be seen on the diagram.	
$\frac{15 \times 8}{2} = 60$ oe	**M1**	For the area of the triangle. Again the scheme quotes the side, so a candidate's own value for BE is used. May be seen on the diagram.	✓
$60 + 64 = 124$	**A1**	Working not required, so a correct answer scores full marks (unless it comes from obviously incorrect working).	✓
Alternative to the two area marks: $\frac{1}{2} \times (15 + 8 + 8) \times 8$	**M2**	Use of the trapezium formula on $ACDE$ earns both area marks at once. It replaces the square and triangle rows above rather than adding to them, so the question is still out of 4 .	✓

Full marks: 4/4

Question 16 - Exam Solution

Understanding the Question

Given

The first four terms of an arithmetic sequence: $1,\ 4,\ 7,\ 10$

A second, different arithmetic sequence whose nth term is $5n + 17$

Find

(a) An expression, in terms of n, for the n th term of the first sequence. (b) The 12th term of the second sequence.

Plan the Solution

- Subtract each term from the one after it. Equal gaps confirm the sequence is arithmetic and give the common difference d.
- The nth term of an arithmetic sequence always starts with dn. So write it as $3n + k$ and pin down k using the first term.
- Part (b) needs no listing at all. The 12th term is just the value of $5n + 17$ when $n = 12$.

Worked Solution [3 marks]

> Rule - nth term of an arithmetic sequence: $n\text{th term} = a + (n-1)d$, where a is the first term and d is the common difference. Multiplied out, this is always $dn + (a - d)$.

Step 1: check that the sequence really is arithmetic

$4 - 1 = 3$

$7 - 4 = 3$

$10 - 7 = 3$

(Reason: All three gaps are equal, so the sequence is arithmetic with common difference $d = 3$.)

Step 2: start the expression with $3n$

$n\text{th term} = 3n + k$

(Reason: Going up in 3s means the terms track the 3 times table $3n$, shifted by some fixed number k. Reaching this line alone earns the method mark.)

Step 3: find k from the first term

$3 \times 1 + k = 1$

$3 + k = 1$

$k = 1 - 3 = -2$

(Reason: Putting $n = 1$ into the expression must give the first term, 1. Rearranging leaves k on its own.)

Step 4: write down the nth term

$n\text{th term} = 3n - 2$

(Reason: Test it on the fourth term: $3 \times 4 - 2 = 10$, which is the term printed in the question.)

Step 5: substitute $n = 12$ into $5n + 17$

$5 \times 12 + 17$

(Reason: The 12th term is the value of the expression at $n = 12$, so there is no need to write out all twelve terms.)

Step 6: work out the value

$= 60 + 17$

$= 77$

(Reason: Multiplication is done before addition.)

(a) $n\text{th term} = 3n - 2$

(b) 77

Verification

✓ **Check 1:** Put $n = 1$ and $n = 4$ into $3n - 2$ and compare with the terms printed in the question. $3 \times 1 - 2 = 1$ and $3 \times 4 - 2 = 10$, the first and fourth terms.

✓ **Check 2:** Reach part (a) a different way: use the standard formula $a + (n - 1)d$ with $a = 1$ and $d = 3$, then multiply out. $1 + (n - 1) \times 3 = 3n - 3 + 1 = 3n - 2$

✓ **Check 3:** Reach part (b) without substituting. The second sequence has first term $5 \times 1 + 17 = 22$ and rises in 5s, so its 12th term is 11 steps further on. $22 + 11 \times 5 = 22 + 55 = 77$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) An expression of the form $3n + k$, with $k \neq -2$	**M1**	Method mark for using the common difference as the coefficient of n. Accept $3 \times n + k$ or $n \times 3 + k$, and k may be zero or absent.	✓
(a) $3n - 2$	**A1**	Accept any equivalent form, eg $1 + (n - 1)3$, $3 \times n - 2$ or $n \times 3 - 2$, and allow $x = 3n - 2$, $3x - 2$ or $T_n = 3n - 2$. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working. An answer written as $n = 3n - 2$ scores M1 only.	✓
(b) 77	**B1**	Correct answer only, from $5 \times 12 + 17$. No method is required for the mark.	✓

Full marks: 3/3

Question 17 - Exam Solution

Understanding the Question

Given

450 employees, each of whom used exactly one of four methods of travel
The probability table: walk 0.20, bus x, car $2x$, bicycle 0.26

Find

How many of the 450 employees travelled by car.

The car probability is written as $2x$, so it is twice the bus probability

Plan the Solution

- Every employee used one method and no employee used two, so the four probabilities describe every possibility once and must add to 1.
- Add the two probabilities that are given as numbers, then subtract that total from 1 to see how much probability is left for the bus and the car together.
- That leftover is $x + 2x$, which is $3x$, so divide by 3 to find x and then double it for the car.
- Finally turn the car probability into a number of employees by multiplying by 450.

Worked Solution [4 marks]

> Rule - Probabilities of a complete set of outcomes: when every outcome is listed and no two of them can happen together, the probabilities add to 1. And the expected number in a group is probability $\times$ total.

Step 1: write down that the four probabilities add to 1

$$0.20 + x + 2x + 0.26 = 1$$

(Reason: Each employee used just one method, and the four methods are the only ones offered, so the four outcomes cover everything and cannot overlap. Reaching this line, in any equivalent form, earns the first method mark.)

Step 2: collect the two probabilities that are already numbers

$$0.20 + 0.26 = 0.46$$

$$1 - 0.46 = 0.54$$

$$3x = 0.54$$

(Reason: The walkers and the cyclists account for 0.46 of the probability, so 0.54 is left for the bus and the car. On the left, $x + 2x$ collects to $3x$.)

Step 3: find x, the probability for the bus

$$x = \frac{0.54}{3} = 0.18$$

(Reason: The leftover probability is shared out in equal lots of x, so dividing tells you what one lot is worth.)

Step 4: double it to get the probability for the car

$$2x = 2 \times 0.18 = 0.36$$

(Reason: The car cell of the table reads $2x$, which is twice the bus probability, so the car probability is 0.36.)

Step 5: turn the probability into a number of employees

$0.36 \times 450 = 162$

(Reason: A probability of 0.36 means 0.36 of the group, and the group is 450 employees.)

162 employees travelled by car

Verification

✓ **Check 1:** Work out all four group sizes and add them up. They must come back to 450: walkers $0.20 \times 450 = 90$, bus $0.18 \times 450 = 81$, car 162, cyclists $0.26 \times 450 = 117$. $90 + 81 + 162 + 117 = 450$, the number of employees the question states.

✓ **Check 2:** Reach the answer without ever finding x. Count the walkers and the cyclists first, take them off the 450, then split what is left so that the car group is twice the bus group, which makes the car group two thirds of it. $90 + 117 = 207$, then $450 - 207 = 243$, and $\frac{2}{3} \times 243 = 162$.

✓ **Check 3:** Put the value of x back into the table and add the four probabilities. $0.20 + 0.18 + 0.36 + 0.26 = 1$, so the completed table is a proper probability distribution.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1 - (0.20 + 0.26) = 0.54$ oe, or $x + 2x + 0.26 + 0.20 = 1$ oe, or $x + 2x = 0.54$ oe	M1	Method mark for showing clear understanding that the total of the probabilities is 1. If the probabilities are given as percentages then the $\%$ sign must be seen.	✓
$\frac{0.54}{3} = 0.18$, or $\frac{2}{3} \times 0.54 = 0.36$ oe, or $0.54 \times 450 = 243$	M1	For a correct method to find x or $2x$. The mark scheme writes the 0.54 in quotation marks, which means a candidate's own earlier value may be used here in its place.	✓

Step	Mark	Description	Got it?
$(2\times)\ 0.18 \times 450$ oe, or 81, or 0.36×450 oe	**M1**	Or for $\frac{81}{450}$ or $\frac{162}{450}$. Again the 0.18 may be the candidate's own value.	✓
162	**A1**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Alternative full method, worth the same four marks	**note**	0.2×450 and 0.26×450, or $90 + 117 = 207$, or $0.46 \times 450 = 207$, earns the first method mark; $450 - 207 = 243$ earns the second; $\frac{1}{3} \times 243 = 81$ or $\frac{2}{3} \times 243 = 162$ earns the third; and the answer is still 162.	✓

Full marks: 4/4

Question 18 - Exam Solution

Understanding the Question

Given

The two numbers 72 and 108
The working has to be shown, so a method must appear on the page and not just the answer.

Find

The highest common factor (HCF) of 72 and 108 - the largest number that divides into both of them exactly.

Plan the Solution

- Break each number down into a product of prime factors.
- Pick out the primes that appear in both lists, and take the lower power of each one.
- Multiply those shared prime factors together to get the HCF.
- Check by dividing 72 and 108 by the answer - both divisions must come out exactly.

Worked Solution [2 marks]

> Rule - Highest common factor: write each number as a product of prime factors, then multiply together only the primes the two numbers share, taking the lower power of each. Taking the higher power gives the lowest common multiple instead, which is a different number.

Step 1: write 72 as a product of prime factors

$72 = 2 \times 36 = 2 \times 2 \times 18 = 2 \times 2 \times 2 \times 9$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

(Reason: (Reason: halving as far as it will go strips out every factor of 2, and what is left, 9, is 3×3 .))

Step 2: write 108 as a product of prime factors

$108 = 2 \times 54 = 2 \times 2 \times 27$

$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

(Reason: (Reason: 27 is odd, so the halving stops after two steps, and $27 = 3 \times 3 \times 3$.))

Step 3: multiply the prime factors the two numbers share

$\text{HCF} = 2^2 \times 3^2 = 4 \times 9 = 36$

(Reason: (Reason: 72 brings three 2s and two 3s, while 108 brings two 2s and three 3s. Only what BOTH lists can supply may be used, so the lower power of each shared prime is the most that can divide into both numbers.))

36

Verification

- ✓ **Check 1:** Divide each of the two numbers by 36 and see whether the division is exact. $\frac{72}{36} = 2$ and $\frac{108}{36} = 3$, both whole numbers, so 36 really is a common factor.
- ✓ **Check 2:** Look at the two quotients 2 and 3. If a larger common factor existed, these two would still share a factor above 1. 2 and 3 share no factor above 1, so nothing bigger than 36 divides both numbers - it is the HIGHEST common factor, not just a common one.
- ✓ **Check 3:** List every factor of each number and compare the two lists. 72: $1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$. 108: $1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108$. The common ones are $1, 2, 3, 4, 6, 9, 12, 18, 36$, and the largest of them is 36.
- ✓ **Check 4:** Use the fact that the HCF multiplied by the lowest common multiple equals the product of the two numbers. Taking the HIGHER power of each shared prime gives the LCM, $2^3 \times 3^3 = 216$. $36 \times 216 = 7776$ and $72 \times 108 = 7776$, so the pair 36 and 216 is consistent - 36 is the HCF and 216 is the LCM.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Any correct valid method with no errors, for example: starting to list at least four different factors of each number; or both prime factorisations seen, $2 \times 2 \times 2 \times 3 \times 3$ and $2 \times 2 \times 3 \times 3 \times 3$ (a factor tree or a ladder diagram counts, and 1 is ignored); or a fully correct Venn diagram; or another clear method such as a division table.	**M1**	Method mark for a correct approach with no errors. Partial factorisations are accepted in the same spirit, for example $4 \times 2 \times 3 \times 3$ and $4 \times 3 \times 3 \times 3$, or 2×36 and 3×36.	✓
36	**A1**	Dependent on the method mark. Working is required. Accept $2^2 \times 3^2$ or equivalent as the final answer.	✓

Full marks: 2/2

Question 19 - Exam Solution

Understanding the Question

Given

The car travelled 943 kilometres in April. The April distance is 15% more than the March distance.

Find

The number of kilometres the car travelled in March. The March distance is the amount that was increased, so it is the 100% amount.

Plan the Solution

- March is the starting amount, so March counts as 100%.
- Adding 15% makes April 115% of March, which is a multiplier of 1.15.
- March was multiplied by 1.15 to give 943, so divide by 1.15 to get back to March.
- Do not take 15% off 943 - that would be a percentage of the wrong amount.

Worked Solution [3 marks]

Reversing a percentage increase: new $=$ original $\times$ multiplier, so original $= \dfrac{\text{new}}{\text{multiplier}}$.

Step 1: Turn the increase into a multiplier

$100\% + 15\% = 115\%$

$115\% = \dfrac{115}{100} = 1.15$

(Reason: March is the amount being increased, so March is 100%. April is 15% more, so April is 115% of March.)

Step 2: Write the relationship as an equation

$m \times 1.15 = 943$

(Reason: Let m be the number of kilometres travelled in March. Multiplying m by 1.15 gives the April distance, 943.)

Step 3: Divide by the multiplier

$m = \dfrac{943}{1.15} = 820$

(Reason: Dividing undoes the multiplication, so the April distance divided by 1.15 gives the March distance.)

820 kilometres

Verification

- ✓ **Check 1:** Increase the answer by 15% again and see whether April comes back. $820 \times 1.15 = 943$, the April distance.
- ✓ **Check 2:** Work out 15% of 820 on its own and add it on. $0.15 \times 820 = 123$ and $820 + 123 = 943$.
- ✓ **Check 3:** Use the unitary method instead: 115% is 943 kilometres, so find 1% and multiply by 100. $\dfrac{943}{115} = 8.2$ and $8.2 \times 100 = 820$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1 + 0.15 = 1.15$, or $100\% + 15\% = 115\%$, or	M1	A correct first step: the multiplier, the total percentage, an equation in m, or the	✓

Step	Mark	Description	Got it?
$m + 0.15m = 943$, or $\frac{943}{115} = 8.2$ oe		value of 1%.	
$\frac{943}{1.15}$, or $\frac{943}{115} \times 100$, or $943 \times \frac{100}{115}$, or 8.2×100	**M1 dep**	A complete method for the March distance, using their own multiplier or their own 1% value. Dependent on the first method mark.	✓
820	**A1**	Working is not required, so a correct answer scores full marks unless it clearly comes from incorrect working.	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

$ABCDE$ is a regular pentagon.
The point F lies outside the pentagon, and E is joined to F.
Angle $AEF = 96°$

Find

The size of the obtuse angle FED.

Plan the Solution

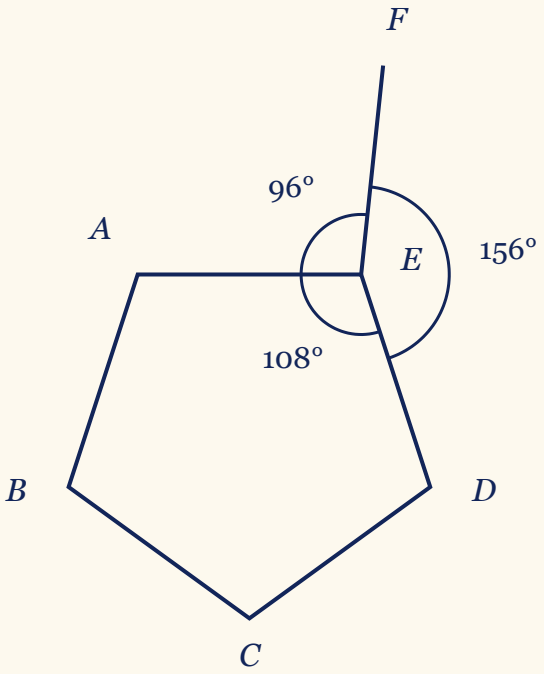
- A regular pentagon has 5 equal interior angles, so work one of them out from the angle sum $(n - 2) \times 180°$.
- That interior angle is the pentagon's own angle at E, which is angle AED.
- Angles AEF, AED and FED all meet at E and fill one full turn, so take the two known angles away from $360°$.

Worked Solution [4 marks]

Rule - Angles at a point add to $360°$. One interior angle of a regular n-sided polygon is $\frac{(n - 2) \times 180°}{n}$.

Step 1: the angle sum of a pentagon

$(5-2)\times 180^\circ = 540^\circ$



(Reason: A pentagon can be cut into 3 triangles from one vertex, and each triangle contributes 180°.)

Step 2: one interior angle of the regular pentagon

$$\frac{540^\circ}{5} = 108^\circ$$

$$\angle AED = 108^\circ$$

(Reason: All 5 interior angles of a regular pentagon are equal, so each one is a fifth of 540°. The pentagon's own angle at E is the angle between EA and ED.)

Step 3: the three angles at E fill one full turn

$$\angle AEF + \angle AED + \angle FED = 360^\circ$$

$$\angle FED = 360^\circ - 96^\circ - 108^\circ = 156^\circ$$

(Reason: The three angles round the point E leave no gap and do not overlap, so together they make one complete turn.)

Step 4: check it is the obtuse angle that has been found

$90^\circ < 156^\circ < 180^\circ$

(Reason: Turning the other way round from EF to ED gives the reflex angle $360^\circ - 156^\circ = 204^\circ$, so 156° is the obtuse one the question asks for.)

$$\angle FED = 156^\circ$$

Verification

✓ **Check 1:** Produce AE past E to a point G. Angles on the straight line AEG give $\angle FEG = 180° - 96° = 84°$, and the exterior angle of the pentagon at E is $\frac{360°}{5} = 72°$. EG lies inside angle FED, so the two pieces add. $84° + 72° = 156°$

✓ **Check 2:** Add the three angles that meet at E: the given $96°$, the pentagon's own $108°$ and the answer. They must come to one full turn. $96° + 108° + 156° = 360°$

✓ **Check 3:** Is the size sensible? The five interior angles come to $5 \times 108° = 540°$, which matches the angle sum, and the answer is bigger than a right angle but smaller than a straight line. $90° < 156° < 180°$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Start on the pentagon's angles	**M1**	$(5-2) \times 180° = 540°$ or $\frac{360°}{5} = 72°$	✓
One angle of the pentagon, or the piece on the straight line	**M1**	$\frac{540°}{5} = 108°$ or $180° - 72° = 108°$ or $180° - 96° = 84°$	✓
A complete method for angle FED	**M1**	$84° + 72°$ or $360° - (96° + 108°)$ or $180° - (108° - 84°)$	✓
The answer	**A1**	156, and the working must be shown	✓
Note	**note**	Angles written on the diagram earn these marks only if they come from correct working and are correctly assigned.	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

Part (a): the product of two brackets, $(m + 5)(m - 8)$.

Find

(a) The expansion of $(m + 5)(m - 8)$, simplified to a quadratic in m with the

Part (b): the equation $3n - 4 = \dfrac{5n + 6}{3}$ - a linear equation with n on both sides, and the right-hand side written over a denominator of 3.

like terms collected. (b) The value of n. The paper asks for clear algebraic working, so the answer alone earns nothing here.

Plan the Solution

- (a) Multiply each of the two terms in the first bracket by each of the two terms in the second. That gives four terms, two of which are terms in m.
- (a) Collect those two terms in m. The number term and the term in m^2 have nothing to collect with.
- (b) The right-hand side is divided by 3, so multiply BOTH sides by 3 to clear the fraction. The whole of the left-hand side is multiplied, not just its first term.
- (b) Then gather the n terms on one side and the number terms on the other, and divide to finish.

Worked Solution [5 marks]

> Rule - Expand then collect: $(a + b)(c + d) = ac + ad + bc + bd$, and only like terms may be added. Rule - Clear the fraction first: multiplying both sides of an equation by the denominator removes it, and every term on each side is multiplied.

Step 1: multiply each term in the first bracket by each term in the second

$$(m + 5)(m - 8) = m \times m + m \times (-8) + 5 \times m + 5 \times (-8)$$

$$(m + 5)(m - 8) = m^2 - 8m + 5m - 40$$

(Reason: Each of the two terms in the first bracket has to meet both terms in the second, so there are four products. Keeping the sign with the 8 is what makes the third and fourth products come out as $-8m$ and -40.)

Step 2: collect the two terms in m

$$-8m + 5m = -3m$$

$$(m + 5)(m - 8) = m^2 - 3m - 40$$

(Reason: Only $-8m$ and $5m$ are like terms. The m^2 term and the -40 have nothing to pair with, so they are already simplified.)

Step 3: clear the fraction in part (b) by multiplying both sides by 3

$$3 \times (3n - 4) = 3 \times \frac{5n + 6}{3}$$

$$9n - 12 = 5n + 6$$

(Reason: Multiplying by 3 undoes the division by 3 on the right, so the fraction disappears. On the left the 3 multiplies the whole bracket, which is exactly what the mark scheme means by removal of the fraction AND multiplying out the left-hand side.)

Step 4: gather the n terms on one side and the numbers on the other

$$9n - 5n = 6 + 12$$

$$4n = 18$$

(Reason: Subtracting $5n$ from both sides clears the n from the right; adding 12 to both sides clears the number from the left. Doing both leaves a single term in n equal to a single number.)

Step 5: divide both sides by 4

$$n = \frac{18}{4}$$

$$n = \frac{9}{2} = 4.5$$

(Reason: Dividing by the coefficient of n leaves n on its own. The fraction $\frac{18}{4}$ cancels by 2 to $\frac{9}{2}$, and the mark scheme accepts either form, or 4.5.)

(a) $m^2 - 3m - 40$

(b) $n = \frac{9}{2} = 4.5$

Verification

✓ **Check 1:** Part (a), by substitution. Put $m = 2$ into the original brackets and into the expansion. If the expansion is right, the two must give the same number.
$(2 + 5)(2 - 8) = 7 \times (-6) = -42$ and $2^2 - 3 \times 2 - 40 = 4 - 6 - 40 = -42$

✓ **Check 2:** Part (a), by the zeros. The brackets are zero when $m = -5$ and when $m = 8$, so the expansion must be zero at both of those values too.
$(-5)^2 - 3 \times (-5) - 40 = 25 + 15 - 40 = 0$ and
$8^2 - 3 \times 8 - 40 = 64 - 24 - 40 = 0$

✓ **Check 3:** Part (b), by substitution into the ORIGINAL equation, fraction and all. Put $n = 4.5$ into each side separately. Left-hand side $3 \times 4.5 - 4 = 13.5 - 4 = 9.5$; right-hand side $\frac{5 \times 4.5 + 6}{3} = \frac{28.5}{3} = 9.5$. The two sides agree.

✓ **Check 4:** Part (b), by trapping the solution. Both sides are straight lines, so they cross once. Test $n = 4$ and $n = 5$ and see the left-hand side overtake the right. At $n = 4$ the

sides are 8 and $\frac{26}{3} \approx 8.67$; at $n = 5$ they are 11 and $\frac{31}{3} \approx 10.33$. The crossing lies between 4 and 5, as 4.5 does.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $m^2 - 8m + 5m - 40$	M1	For any 3 correct terms out of the 4, or for all 4 terms correct ignoring signs, or for $m^2 - 3m$ with anything following, or for the $-3m - 40$ part alone.	✓
(a) $m^2 - 3m - 40$	A1	Working is not required in part (a), so a correct answer on its own scores both marks, unless it follows obviously incorrect working.	✓
(b) $9n - 12 = 5n + 6$ or equivalent	M1	For removal of the fraction AND multiplying out the left-hand side, or for separating the fraction on the right-hand side within an equation, eg $3n - 4 = \frac{5}{3}n + \frac{6}{3}$.	✓
(b) $9n - 5n = 6 + 12$ or $4n = 18$ or equivalent	M1ft	Dependent on a 4 term equation. For correctly rearranging their 4 term equation so that the terms in n are on one side and the number terms on the other. Follow through their own equation, eg $-12 - 6 = 5n - 9n$ or $-4n = -18$ scores this mark too.	✓
(b) $n = \frac{9}{2}$	A1	Dependent on M2. Working is required in part (b). Or equivalent, eg $\frac{18}{4}$ or 4.5 or $4\frac{1}{2}$.	✓

Full marks: 5/5

Question 22 - Exam Solution

Understanding the Question

Given

$\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$, the universal set, with 12 members

Find

The members of $B \cup C$ The members of $A' \cap C$ Whether

$A = \{\text{even numbers}\}$, $B = \{23, 29, 31\}$, $C = \{\text{multiples of } 3\}$
D has exactly 4 members, and
$D \cap (A \cup C) = \varnothing$

$B \cap C = \varnothing$ is true, and why The members of D

Plan the Solution

- A and C are described in words, so write both out in full first. Every part after that is read straight off the lists.
- Union $\cup$ collects, intersection $\cap$ filters: build $B \cup C$ by listing B and adding what C brings that is new.
- For $A' \cap C$, write A' (everything in $\mathcal{E}$ that is not even) and keep only what also appears in C.
- For D, read the condition backwards: sharing nothing with $A \cup C$ means D can only use the numbers left in $\mathcal{E}$ once $A \cup C$ is removed.

Worked Solution [5 marks]

> Rule - Set notation: $\cup$ keeps everything that is in either set, $\cap$ keeps only what is in both, A' is everything in $\mathcal{E}$ that is not in A, and $\varnothing$ is the set with no members at all.

Step 1: Write out A and C in full

$A = \{24, 26, 28, 30, 32, 34\}$
$C = \{24, 27, 30, 33\}$

(Reason: The even members of $\mathcal{E}$ are $24, 26, 28, 30, 32, 34$, and the multiples of 3 between 23 and 34 are $24, 27, 30, 33$. All four parts are answered from these two lists, so it is worth writing them once rather than sifting the numbers again each time.)

Step 2: (a)(i) Build the union B u C

$B \cup C = \{23, 29, 31\} \cup \{24, 27, 30, 33\}$
$B \cup C = \{23, 24, 27, 29, 30, 31, 33\}$

(Reason: The union keeps every number that is in B or in C or in both, and each member is written once only. Nothing appears in both sets here, so all 3 members of B and all 4 members of C survive, giving 7 numbers.)

Step 3: (a)(ii) Find A' first, then intersect with C

$A' = \{23, 25, 27, 29, 31, 33\}$
$A' \cap C = \{23, 25, 27, 29, 31, 33\} \cap \{24, 27, 30, 33\}$
$A' \cap C = \{27, 33\}$

(Reason: A' is everything in $\mathcal{E}$ that is not even, which is the odd numbers. The intersection then keeps only what is in both lists, so it is asking for the multiples of 3 that are odd: 24 and 30 are multiples of 3 but they are even, so they are ruled out.)

Step 4: (b) Test whether B and C share anything

$$B \cap C = \{23, 29, 31\} \cap \{24, 27, 30, 33\}$$
$$B \cap C = \varnothing$$

(Reason: Checking the members of B one at a time: 23, 29 and 31 are none of them multiples of 3, so no number lies in both sets. An intersection with no members is the empty set, so the statement is true and the answer is Yes.)

Step 5: (c) Remove A u C from the universal set

$$A \cup C = \{24, 26, 27, 28, 30, 32, 33, 34\}$$
$$n(A \cup C) = 6 + 4 - 2 = 8$$
$$D = \{23, 25, 29, 31\}$$

(Reason: $D \cap (A \cup C) = \varnothing$ says D shares no member with $A \cup C$, so every member of D has to come from what is left of $\mathcal{E}$ once $A \cup C$ is taken out. Only $23, 25, 29, 31$ are left, and that is exactly the 4 members D is stated to have, so there is no choice about it.)

(a)(i) $B \cup C = \{23, 24, 27, 29, 30, 31, 33\}$

(a)(ii) $A' \cap C = \{27, 33\}$

(b) Yes, because there are no multiples of 3 in set B

(c) $D = \{23, 25, 29, 31\}$

Verification

- ✓ **Check 1:** Count the union a different way. $n(B \cup C) = n(B) + n(C) - n(B \cap C)$, and $B \cap C$ is empty, so nothing is double counted. $3 + 4 - 0 = 7$, and the list in (a)(i) has 7 members
- ✓ **Check 2:** Test the two answers to (a)(ii) against both conditions, and test the numbers that were rejected. A member of $A' \cap C$ must be odd and a multiple of 3. $27 = 3 \times 9$ and $33 = 3 \times 11$ are both odd; the only other multiples of 3 in $\mathcal{E}$ are 24 and 30, which are even and so belong to A, not A'
- ✓ **Check 3:** Count part (c) instead of listing it. $\mathcal{E}$ has 12 members and $A \cup C$ has 8, so count how many numbers are left over and compare with the 4 members D must have. $12 - 8 = 4$ numbers left over, matching the 4 members of D, and none of $23, 25, 29, 31$ is even or a multiple of 3

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $B \cup C = \{23, 24, 27, 29, 30, 31, 33\}$	**B1**	All seven values $23, 24, 27, 29, 30, 31, 33$, in any order, with no repeats.	✓
(a)(ii) $A' \cap C = \{27, 33\}$	**B1**	Both values $27, 33$, in any order, with no repeats.	✓
(b) Yes, with a correct reason	**B1**	Yes together with a statement showing correct meanings of intersection and empty set, for example there are no multiples of 3 in set B, or the two sets have no members in common, or $23, 29, 31$ are not in C. This is not an exhaustive list; allow element or value for member. If Yes is not written on the answer line, it must be stated in the reason.	✓
(c) $D = \{23, 25, 29, 31\}$	**B2**	B2 for the four correct numbers $23, 25, 29, 31$ and no additions. B1 for three correct values with no more than one incorrect, or for four correct values with no more than one incorrect.	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

Volume of the cylinder: 1575 cm³
Height of the cylinder: 21 cm, marked on the figure
Force on the workbench: 84 newtons
The formula is given in the question:

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Find

The pressure on the workbench, in newtons/cm² The formula needs an area, and no area is given, so this is a two-step question.

The area the cylinder presses on is not given.

Plan the Solution

- The area in the formula is the area actually being pressed on: the flat circular base of the cylinder.
- A cylinder is a prism, so its volume is the area of that base multiplied by the height. Dividing the volume by the height therefore undoes the multiplication and leaves the base area.
- Then put the force over that area.
- The radius is never needed. Working it out first reaches the same base area by a longer road, which is why the mark scheme allows either.

Worked Solution [3 marks]

> Rule - Pressure and prisms: $\text{pressure} = \dfrac{\text{force}}{\text{area}}$, and for any prism $\text{volume} = \text{cross-sectional area} \times \text{height}$.

Step 1: turn the volume and the height into the area of the base

$$\frac{1575}{21} = 75 \text{ cm}^2$$

(Reason: the cylinder is a prism, so 1575 is the base area multiplied by 21. Dividing by the height undoes that multiplication and leaves the area of the circular base.)

Step 2: divide the force by that area

$$\frac{84}{75} = 1.12 \text{ newtons/cm}^2$$

(Reason: pressure is the force spread over the area it presses on, so the 84 newtons goes over the base area worked out in Step 1, not over any other length in the question.)

1.12 newtons/cm²

Verification

✓ **Check 1:** Multiply the base area back by the height. The volume the question gives should come back. $75 \times 21 = 1575 \text{ cm}^3$

✓ **Check 2:** Multiply the pressure by the area. The force the question gives should come back. $1.12 \times 75 = 84$ newtons

✓ **Check 3:** Take a different road entirely. Substituting $\text{area} = \dfrac{\text{volume}}{\text{height}}$ into the pressure formula gives force times height over volume, with no area worked out at all.

$$\frac{84 \times 21}{1575} = 1.12$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Base area from the volume and the height: $\dfrac{1575}{21} = 75$	**M1**	for a correct method to find the cross-sectional area - either $\dfrac{1575}{21}$, or finding r^2 or r from $1575 = \pi r^2 \times 21$. Note that r^2 and r may be rounded or truncated.	✓
Force over that area: $\dfrac{84}{75}$ or equivalent	**M1**	for $\dfrac{84}{\text{their area of the circle}}$. Follow through on the area found in the first step, including one reached through the radius.	✓
1.12	**A1**	for 1.12 newtons/cm². Accept anything from 1.06 to 1.121, which covers a rounded or truncated radius. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working.	✓

Full marks: 3/3

Question 24 - Exam Solution

Understanding the Question

Given

The table gives two amounts, both in standard form: Amberdown 3.5×10^7 tonnes and Fenwold 8.2×10^5 tonnes
Highmarsh harvested $6\,780\,000$ more tonnes of wheat than Fenwold
The Highmarsh amount is not in the table. It is described in words, and only in terms of the Fenwold amount.

Find

(a) 3.5×10^7 written as an ordinary number (b) the amount Highmarsh harvested, given in standard form and not as an ordinary number

Plan the Solution

- (a) The index says how far the digits move. Multiplying by 10^7 makes every digit worth ten million times as much, so the point in 3.5 travels seven places and zeros fill the gaps.
- (b) A standard form number and an ordinary number cannot be added as they stand, so turn 8.2×10^5 into an ordinary number first.
- Then add the extra tonnes, and put the total back into standard form.
- That last conversion is the step that is easy to lose: an ordinary number is the right amount of wheat but the wrong form of answer, and the question asks for the form.

Worked Solution [3 marks]

> Rule - Standard form: a number is written $A \times 10^n$, where A is at least 1 and less than 10, and the index n counts the places every digit moves.

Step 1: (a) move the digits seven places

$$3.5 \times 10^7 = 35\,000\,000$$

(Reason: the index 7 says multiply by 10 seven times, so the point in 3.5 moves seven places to the right and the empty places are filled with zeros. The result has 8 digits, one more than the index.)

Step 2: (b) write the Fenwold amount as an ordinary number

$$8.2 \times 10^5 = 820\,000$$

(Reason: the two amounts that have to be added are written in different forms, and only numbers in the same form can be added. The index 5 moves the point in 8.2 five places to the right.)

Step 3: add the extra tonnes

$$820\,000 + 6\,780\,000 = 7\,600\,000$$

(Reason: harvesting $6\,780\,000$ more tonnes means that many tonnes on top of the Fenwold amount, so the two are added.)

Step 4: put the total back into standard form

$$7\,600\,000 = 7.6 \times 10^6$$

(Reason: standard form needs a number between 1 and 10 in front, so the digits are written as 7.6. Counting from 7.6 up to $7\,600\,000$ is six places, so the index is 6.)

(a) $35\,000\,000$

(b) 7.6×10^6 tonnes

Verification

✓ **Check 1:** Turn the part (b) answer back into an ordinary number and take the extra tonnes away again. The Fenwold amount must come back. $7\,600\,000 - 6\,780\,000 = 820\,000$

✓ **Check 2:** Work part (b) again without ever leaving standard form. Counted in multiples of 10^6, the Fenwold amount is 0.82 and the extra is 6.78, so the two mantissas add directly. $0.82 \times 10^6 + 6.78 \times 10^6 = 7.6 \times 10^6$

✓ **Check 3:** Undo part (a) instead of redoing it: divide the ordinary number by 10^7. The figure printed in the table must come back. $\dfrac{35\,000\,000}{10^7} = 3.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $35\,000\,000$	**B1**	for $35\,000\,000$. Working is not required, so the number on its own earns the mark, written with spaces or with no separator at all.	✓
(b) $820\,000 + 6\,780\,000$ or equivalent	**M1**	for a correct method: $8.2 \times 10^5 + 6\,780\,000$ or $820\,000 + 6\,780\,000$. A correct mixture of ordinary numbers and standard form numbers is allowed. Also award it for the digits reached but not converted, $7\,600\,000$ or 76×10^5, and for 7.6×10^n with any index other than 6, which is the right digits with the places miscounted.	✓
(b) 7.6×10^6	**A1**	for 7.6×10^6. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working. An ordinary number left as the final answer scores the method mark only, because the question asks for standard form.	✓

Full marks: 3/3

Question 25 - Exam Solution

Understanding the Question

Given

(a) $(2p)^0$, with $p > 0$
(b) $y^9 \times y^{-3} = y^n$
(c) $(5a^4c^2)^3$

Find

(a) the value of $(2p)^0$ (b) the value of n
(c) $(5a^4c^2)^3$ written as a single product of a number, a power of a and a power of c

Three separate index laws, one per part, and no calculator work is needed.

Plan the Solution

- Part (a): the whole bracket carries the power 0, and the condition $p > 0$ is there to guarantee the base is not zero.
- Part (b): the two powers share the base y, so multiplying them adds the indices; then match indices on both sides.
- Part (c): the power 3 is outside a bracket holding three factors, so every factor is cubed, and a power of a power multiplies the indices.
- Finish each part by checking the answer against a numerical substitution.

Worked Solution [4 marks]

> Rule - Index laws: $x^0 = 1$ for any non-zero x; $x^m \times x^n = x^{m+n}$; and $(x^m)^n = x^{mn}$, where every factor inside a bracket is raised to the outside power.

Part (a): use the zero-index law

$$(2p)^0 = \frac{(2p)^1}{(2p)^1}$$

$$(2p)^0 = 1$$

(Reason: (Reason: $p > 0$, so the base $2p$ is never zero, and any non-zero quantity divided by itself is 1. The 2 is inside the bracket, so it carries the power 0 as well and does not survive.))

Part (b): multiply the powers by adding the indices

$$y^9 \times y^{-3} = y^{9+(-3)}$$

$$9 + (-3) = 6$$

$$y^9 \times y^{-3} = y^6$$

(Reason: (Reason: the base is y on both factors, so the indices add. The negative index means a division, so of the nine y factors on the top, three are cancelled and 6 are left.))

Part (b): compare the two sides

$$y^6 = y^n$$

$$n = 6$$

(Reason: (Reason: the two sides are powers of the same base, so their indices must be equal.))

Part (c): raise every factor inside the bracket to the power 3

$$(5a^4c^2)^3 = 5^3 \times (a^4)^3 \times (c^2)^3$$

(Reason: (Reason: the bracket holds three factors multiplied together, and the outside power applies to each of them. The common slip is to cube the letters and leave the 5 untouched.))

Part (c): work out each factor

$5^3 = 5 \times 5 \times 5 = 125$

$(a^4)^3 = a^{4\times 3} = a^{12}$

$(c^2)^3 = c^{2\times 3} = c^6$

(Reason: (Reason: a power raised to a power multiplies the two indices, because a^4 is written down 3 times.))

Part (c): put the three factors back together

$(5a^4c^2)^3 = 125a^{12}c^6$

(Reason: (Reason: the coefficient and the two powers are multiplied, so the simplified form is one product with no bracket left.))

(a) 1

(b) $n = 6$

(c) $125a^{12}c^6$

Verification

✓ **Check 1:** Part (a): put $p = 3$ into the bracket first, then apply the power.
$(2 \times 3)^0 = 6^0 = 1$

✓ **Check 2:** Part (b): put $y = 2$ into both sides and work them out as numbers.
$512 \times \frac{1}{8} = 64$ and $2^6 = 64$

✓ **Check 3:** Part (c): put $a = 2$ and $c = 3$ into the bracket and into the answer.
$(5 \times 16 \times 9)^3 = 720^3 = 373\,248\,000$ and $125 \times 4096 \times 729 = 373\,248\,000$

✓ **Check 4:** Part (c): count the letters instead of using a law. Writing $5a^4c^2$ down three times gives $5 \times 5 \times 5$ at the front, $4 + 4 + 4$ factors of a and $2 + 2 + 2$ factors of c. $125a^{12}c^6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Simplify $(2p)^0$	**B1**	Answer 1, cao. The whole bracket carries the power 0, so $2p$ is not left behind.	✓
(b) Value of n in	**B1**	Answer 6, cao, from $9 + (-3)$.	✓

Step	Mark	Description	Got it?
$y^9 \times y^{-3} = y^n$			
(c) Simplify $(5a^4c^2)^3$ fully	**B2**	Answer $125a^{12}c^6$. Multiplication signs between the terms are accepted, and $125a^{12}$, $125c^6$ or $a^{12}c^6$ is allowed as long as it is not added to any other term.	✓
(c) Partial credit	**B1**	A product in the form ka^pc^q where 2 from k, p or q are correct, for example $5a^{12}c^6$ or $125a^{12}3c^6$.	✓
Note	**note**	The named error behind the partial credit row is a power of a power being ADDED instead of multiplied: $(a^4)^3$ read as a^7 rather than a^{12}. This row carries no mark of its own.	✓

Full marks: 4/4

Question 26 - Exam Solution

Understanding the Question

Given

Triangle ABC with $AC = BC = 9$ m and $AB = 12$ m, so the triangle is isosceles
M lies on AB with angle $AMC = 90°$, so MC is perpendicular to the base
The frame uses four lengths: AB, AC, BC and MC
Timber costs 21.50 euros per metre and is bought in whole metres only

Find

The total cost, in euros, of the four lengths of timber Nathan buys

Plan the Solution

- Because $AC = BC$, the perpendicular MC meets AB at its midpoint, so $AM = 6$ m
- Use Pythagoras in the right-angled triangle AMC to find MC
- Round MC UP to a whole number of metres, because a shorter length would not reach
- Add the four whole-metre lengths, then multiply the total by 21.50

Worked Solution [4 marks]

> Rule - Pythagoras in a right-angled triangle: $a^2 + b^2 = c^2$, so a shorter side is $\sqrt{c^2 - b^2}$.
> Then, when timber is sold in whole metres, every length is rounded UP before it is priced.

Step 1: find AM, half of the base

$$AM = MB = \frac{12}{2} = 6 \text{ m}$$

(Reason: $AC = BC$, so triangle ABC is isosceles and the perpendicular from C lands on the midpoint of AB)

Step 2: use Pythagoras in triangle AMC

$$MC^2 = 9^2 - 6^2 = 81 - 36 = 45$$

$$MC = \sqrt{45} = 3\sqrt{5} \approx 6.71 \text{ m}$$

(Reason: angle $AMC = 90°$, so AC is the hypotenuse of triangle AMC and the shorter side MC comes from subtracting the squares)

Step 3: round MC up to a whole number of metres

$$MC \approx 6.71 \implies \text{buy a } 7 \text{ m length}$$

(Reason: each length bought has to be a whole number of metres, and a 6 m length would be too short, so the next whole metre up is the one Nathan can use)

Step 4: add the four lengths that are bought

$$7 + 9 + 9 + 12 = 37 \text{ m}$$

(Reason: the frame needs MC, AC, BC and AB, and it is the 7 m length from Step 3 that is actually bought)

Step 5: multiply the total length by the price per metre

$$37 \times 21.50 = 795.50 \text{ euros}$$

(Reason: every metre bought costs 21.50 euros, so the cost is the total length multiplied by the price per metre)

795.50 euros

Verification

✓ **Check 1:** Rebuild the right angle at M: the two shorter sides squared are 45 and $6^2 = 36$, and $45 + 36 = 81$. $81 = 9^2$, which is AC^2, so triangle AMC closes exactly and $MC = \sqrt{45}$ is right.

✓ **Check 2:** Find MC by trigonometry instead: $\cos CAM = \frac{6}{9}$ gives angle $CAM = 48.19°$, and $MC = 6 \times \tan 48.19°$. This gives $MC \approx 6.71$ m again, so the same 7 m length is bought and the total is unchanged.

✓ **Check 3:** Price the lengths one at a time rather than as one total: $7 \times 21.50 = 150.50$, $9 \times 21.50 = 193.50$ twice, and $12 \times 21.50 = 258.00$.
$150.50 + 193.50 + 193.50 + 258.00 = 795.50$ euros, which agrees with the boxed answer.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(MC)^2 + \left(\frac{12}{2}\right)^2 = 9^2$ oe, or $9^2 - \left(\frac{12}{2}\right)^2$ $(= 81 - 36 = 45)$	**M1**	A correct Pythagoras statement for MC, using half of AB as the base of the right-angled triangle.	✓
$\sqrt{9^2 - \left(\frac{12}{2}\right)^2}$ oe $\left(= \sqrt{45} = 3\sqrt{5} = 6.7(08\ldots)\right)$	**M1**	The square root taken, so MC is reached as a length rather than left as 45.	✓
$(7 + 9 + 9 + 12) \times 21.5(0)$ oe	**M1**	Their MC rounded up to a whole number of metres, added to the other three lengths, and the total multiplied by the price per metre.	✓
$795.5(0)$	**A1**	Working required, so a bare correct answer with no method shown does not earn this mark.	✓
Alternative, by trigonometry, with $\frac{12}{2} = 6$ as half of AB: $\cos(CAM) = \frac{6}{9} \implies CAM = 48.1(896\ldots)°$ and $MC = 6 \times \tan 48.1\ldots°$ or $MC = 9 \times \sin 48.1\ldots°$ $(= 6.7\ldots)$	**M2**	The angle and the length together earn the first two method marks; this replaces the two Pythagoras rows above and is not additional to them.	✓

Step	Mark	Description	Got it?
Answer of awrt 789 from using $6.7\ldots$ metres of timber for MC	**SC**	The named error: pricing the exact length $6.7\ldots$ m instead of the 7 m length that has to be bought, giving $36.7 \times 21.50 = 789.05$.	✓

Full marks: 4/4

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