IGCSEGCSEMATHS.CO.UK

Edexcel IGCSE 4MA1

Paper 1FR (Foundation Tier) – November 2024

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A (4MA1), Paper 1FR (Foundation Tier), November 2024 – 100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Seven numbers are printed in the box below.

6	15	19	28	38	44	48

Each of your answers must be one of these seven numbers.

Write down

(a) a number that is odd *[1 mark]*

(b) a number that is a multiple of 12 *[1 mark]*

(c) a number that is prime *[1 mark]*

(d) a number that is a factor of 24 *[1 mark]*

(a) .....

(b) .....

(c) .....

(d) .....

[Total 4 marks]

2. (a) Express the decimal 0.13 as a fraction. *[1 mark]*

(b) Fill in the empty box so that the statement below is correct.

$\frac{16}{20} = \frac{\square}{5}$ *[1 mark]*

(a) .....

[Total 2 marks]

3. (a) Complete this statement by writing a number in the box.

$$\frac{5073}{\square} = 19$$ *[1 mark]*

(b) Complete this statement by writing a number in the box.

The cube root of $\square$ is 14 *[1 mark]*

Here is a list of five numbers.

973 987 393 151 139

(c) Work out the difference between the largest number in the list and the smallest number in the list. *[2 marks]*

(c) .....

[Total 4 marks]

4. The pictogram shows information about the numbers of different types of vehicle that Emily counted passing the school gate.

Car	
Van	
Lorry	
Bus	
Motorbike	
Taxi	

Key:  represents 8 vehicles

(a) How many Vans did Emily count? *[1 mark]*

Emily counted 10 Motorbikes.

(b) Show this information on the pictogram. *[1 mark]*

Emily counted more Lorries than Buses.

(c) How many more? *[1 mark]*

(a) .....

(c) .....

[Total 3 marks]

5. A travel guide lists the average January temperature in five cities.

City	Temperature
Moscow	−8°C
Chennai	24°C
Riga	−3°C
Lisbon	12°C
Regina	−14°C

Here are the same five temperatures, in °C

−8 24 −3 12 −14

(a) Write these numbers in order of size.
Start with the smallest number. *[1 mark]*

(b) Work out the difference between the January temperature in Moscow and the January temperature in Lisbon. *[1 mark]*

The January temperature in Novosibirsk is 13°C lower than the January temperature in Riga.

(c) Work out the January temperature in Novosibirsk. *[1 mark]*

The January temperature in Marrakesh is 25°C higher than the January temperature in Regina.

(d) Work out the January temperature in Marrakesh. *[1 mark]*

(a) .....

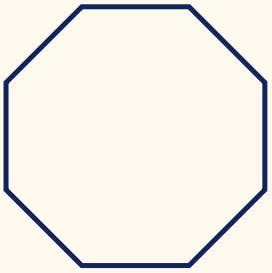
(b) °C .....

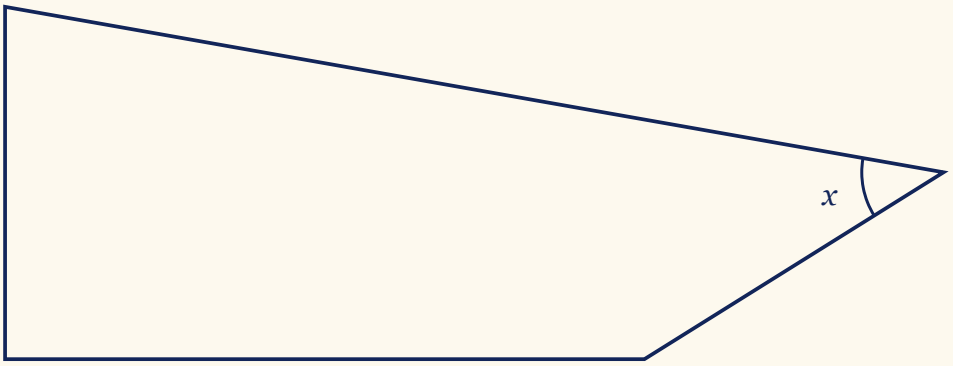
(c) °C .....

(d) °C .....

[Total 4 marks]

6. Here is a regular octagon.





(a) Write down how many lines of symmetry a regular octagon has. *[1 mark]*

The diagram below shows a different shape.

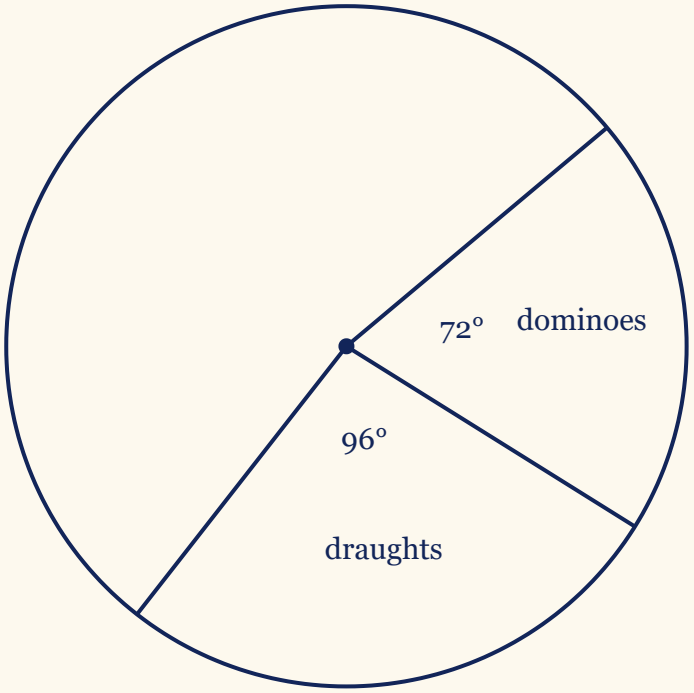
(b) On the shape, mark a right angle with the letter R.
[1 mark]
(c) On the shape, mark an obtuse angle with the letter O.
[1 mark]
(d) By measuring, work out the size of the angle marked x *[1 mark]*

(a) .....

(d) ° .....

[Total 4 marks]

7. At a community centre games evening, everyone played draughts or dominoes or backgammon or cribbage.
Each person played only one of these games.



Marina starts to draw a pie chart to show information about the games played.

24 people played dominoes.

(a) Work out the number of people who played draughts. *[2 marks]*

40 people played cribbage.

(b) Work out the size of the angle on the pie chart for the sector representing cribbage.
You do not need to complete the pie chart. *[2 marks]*

(a) .....

(b) .....

[Total 4 marks]

8. Elise buys

3 scented candles at $1.80 each

4 packets of flower seeds at $1.20 a packet

2 identical jars of honey

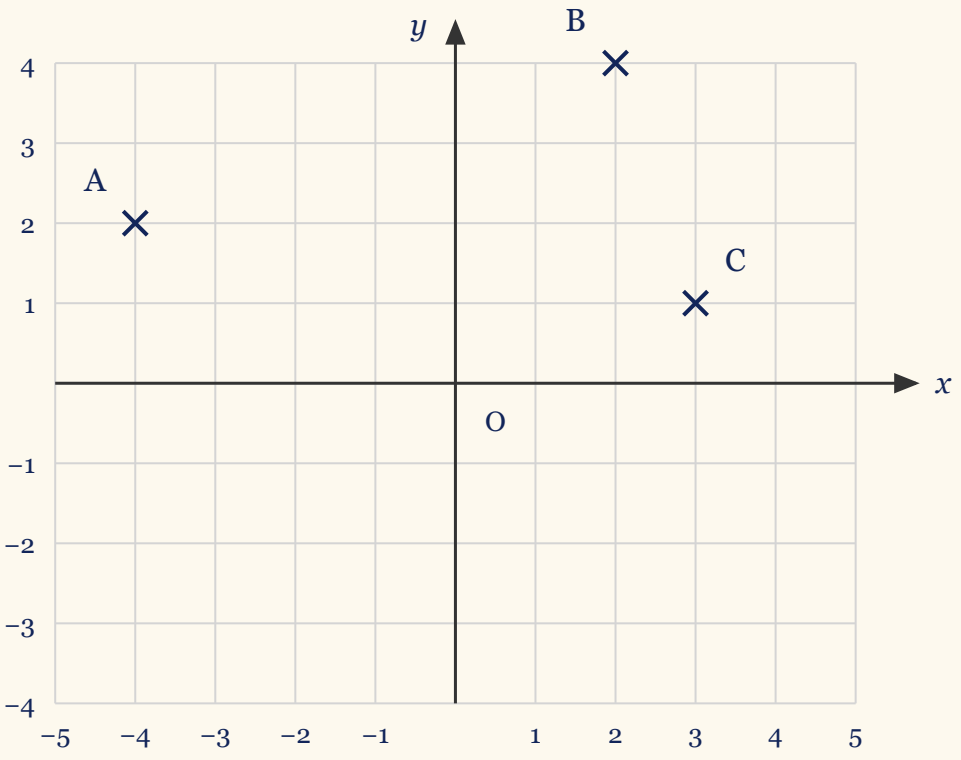
The total cost is $17.10

Work out the cost of one jar of honey. *[4 marks]*

$ .....

[Total 4 marks]

9. The grid shows three points, A, B and C.



(a) Write down the coordinates of A. *[1 mark]*

(b) Work out the coordinates of the midpoint of AB. *[2 marks]*

The point D is marked on the grid so that $ABCD$ is a rectangle.

(c) Work out the coordinates of D. *[2 marks]*

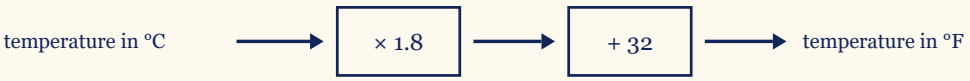
(a) .....

(b) .....

(c) .....

[Total 5 marks]

10. Ivy is baking from an American cookery book, so its oven temperatures are given in degrees Fahrenheit (°F) rather than in degrees Celsius (°C)
She uses this rule to change a temperature from °C to °F



(a) Change an oven temperature of $175°\text{C}$ to a temperature in °F *[2 marks]*

(b) Change an oven temperature of $482°\text{F}$ to a temperature in °C *[2 marks]*

(a) °F .....

(b) °C .....

[Total 4 marks]

11. (a) Simplify $c + c + c + c + c$ *[1 mark]*

(b) Simplify $7w + 10y - 9w + 2y$ *[2 marks]*

The nth term of a sequence is given by $7n - 4$

(c) Work out the 1st term and the 5th term of the sequence. *[2 marks]*

(d) Solve $7(x + 5) = 8 - 3x$
You must show clear algebraic working. *[3 marks]*

(a) .....

(b) .....

(c) 1st term .....

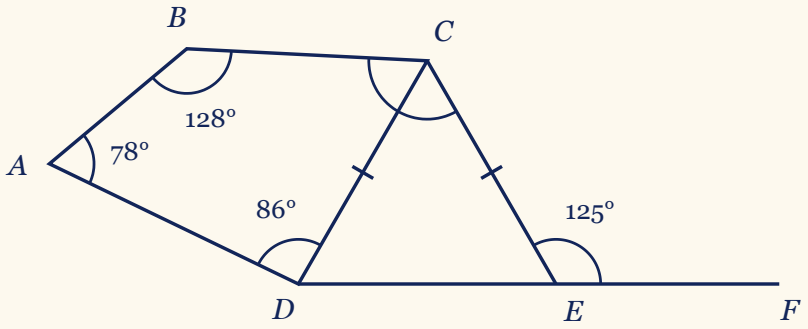
(c) 5th term .....

(d) x = ...

[Total 8 marks]

12. The diagram shows a quadrilateral $ABCD$ joined along CD to a triangle CDE.
In triangle CDE, $CD = CE$.
D, E and F lie on one straight line.

Diagram NOT accurately drawn



Work out the size of angle BCE.
Give a reason for each stage of your working. *[5 marks]*

angle BCE = ...

[Total 5 marks]

13. Rory has some marbles in a box.
He has n red marbles.
He has twice as many blue marbles as red marbles.
He has 7 more green marbles than red marbles.

(a) Write an expression, in terms of n, for the total number of red marbles, blue marbles and green marbles that Rory has.
Write your answer in its simplest form. *[2 marks]*

The total number of marbles that Rory has is T

(b) Write an expression, in terms of T and n, for the number of marbles that Rory has that are not red marbles, blue marbles or green marbles. *[1 mark]*

(a) .....

(b) .....

[Total 3 marks]

14. Use your calculator to work out the value of

$$\frac{\sqrt{17.8 \times 19.2}}{3.4^2 \times 0.23}$$

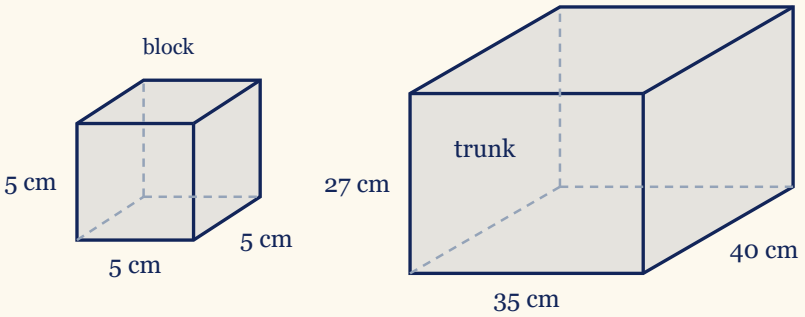
Write down all the figures on your calculator display. *[2 marks]*

.....

[Total 2 marks]

15. The diagram shows a wooden block and a storage trunk with a lid.

Diagram NOT accurately drawn



The block is a cube of side 5 cm.
Nathan has a large supply of these blocks.

The inside of the trunk is a cuboid measuring 27 cm by 35 cm by 40 cm.

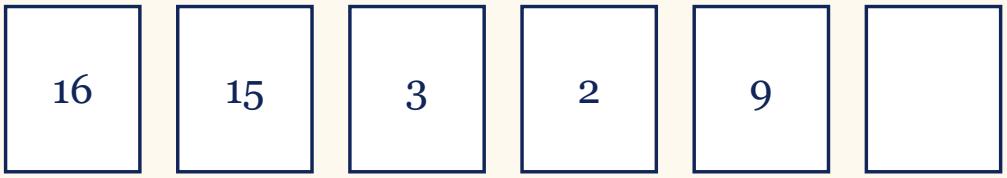
Nathan packs as many blocks as possible into the trunk so that the lid will still shut.

Work out the volume of the space inside the trunk that is not filled with blocks. *[4 marks]*

..... cm^3

[Total 4 marks]

16. Here are six tiles.
Five of the tiles have a number printed on them.

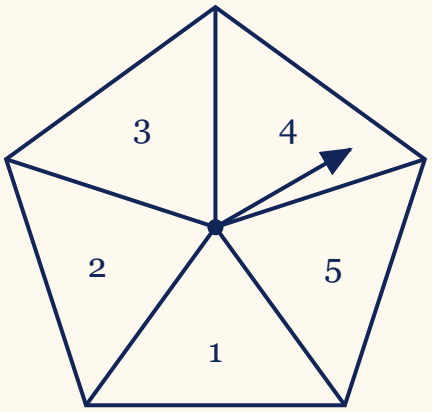


Work out the number that should be printed on the last tile so that the mean of the six numbers will be 11 *[3 marks]*

.....

[Total 3 marks]

17. A five-sided spinner is used in a board game.
The spinner is biased.



The table gives information about the probability that, when the spinner is spun once, it will land on each number.

Number	1	2	3	4	5
Probability	$2x$	0.27	0.04	x	0.12

Rosalind is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on an odd number. *[4 marks]*

.....

[Total 4 marks]

18. Alessandro sells plain bagels and sesame bagels on a market stall.

He sells a total of 200 bagels such that

the number of plain bagels sold : the number of sesame bagels sold = 3 : 2

Alessandro sells the plain bagels for £1.50 each.
He sells the sesame bagels for £1.75 each.

40% of the price of a plain bagel is profit.
60% of the price of a sesame bagel is profit.

Work out Alessandro's total profit when he sells all 200 bagels. *[5 marks]*

£ .....

[Total 5 marks]

19. Show that $2\frac{1}{3}$ divided by $5\frac{1}{4}$ gives $\frac{4}{9}$.
You must show your working. *[3 marks]*

[Total 3 marks]

20. Miroslav puts 5200 euros into a savings bond at his local credit union.
The bond runs for 4 years and it pays 2.5% per year compound interest.

Work out how much money Miroslav will have in the savings bond at the end of 4 years.
Give your answer correct to the nearest euro. *[3 marks]*

..... euros

[Total 3 marks]

21. The diagram shows a solid cylinder turned from a single block of beech wood.

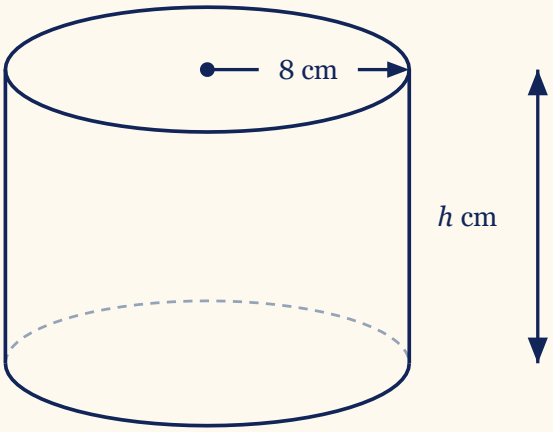


Diagram NOT accurately drawn

The cylinder has radius 8 cm and height h cm.
The volume of the cylinder is 1208 cm³

(a) Work out the value of h.
Give your answer correct to the nearest whole number. *[2 marks]*

The density of the beech wood is 1.25 g/cm³

(b) Work out the mass of the cylinder.
Give your answer in kilograms. *[2 marks]*

(a) h = ...

(b) kilograms .....

[Total 4 marks]

22. (a) Write $\frac{g^9}{g^2}$ as a single power of g. *[1 mark]*

(b) Multiply out the brackets $5k^2(k^3 + 4)$. *[2 marks]*

(c) (i) Write $x^2 - 2x - 63$ as a product of two brackets. *[2 marks]*

(ii) Hence solve the equation $x^2 - 2x - 63 = 0$. *[1 mark]*

(d) Solve the inequality $7 - 2y < 3y - 12$. *[3 marks]*

(a) .....

(b) .....

(c) (i) .....

(c) (ii) .....

(d) .....

[Total 9 marks]

23. The diagram shows a trapezium $ABCD$.

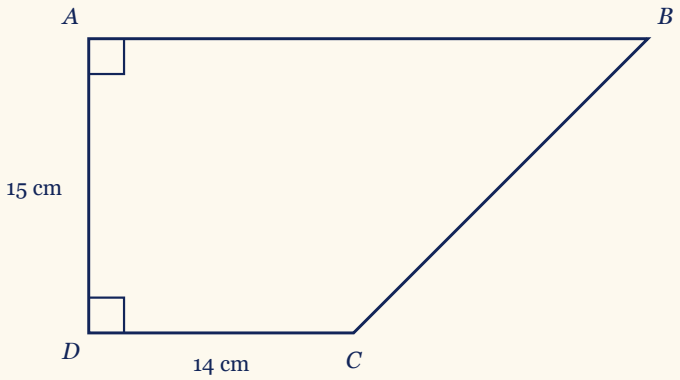


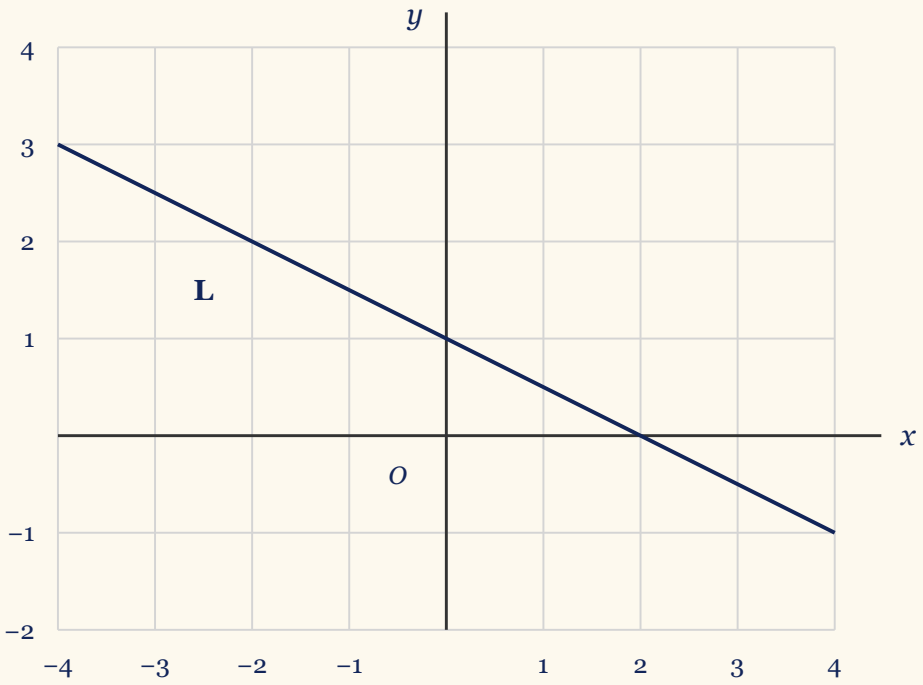
Diagram NOT accurately drawn

Angle DAB and angle ADC are right angles.
$AD = 15$ cm and $DC = 14$ cm
The area of the trapezium is 360 cm²
Work out the perimeter of the trapezium. *[6 marks]*

..... cm

[Total 6 marks]

24. The straight line L has been drawn on the grid below.



Work out an equation of the line L.
Write your answer in the form $y = mx + c$ *[3 marks]*

.....

[Total 3 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The seven numbers in the box: 6, 15, 19, 28, 38, 44, 48
Every answer has to be taken from that list, so nothing is worked out from scratch.

Find

(a) one of them that is odd (b) one of them that is a multiple of 12 (c) one of them that is prime (d) one of them that is a factor of 24

Plan the Solution

- Take one part at a time and read the whole list against that one property.
- Odd or even is settled by the last digit: a number ending in 0, 2, 4, 6 or 8 is even.
- A multiple of 12 sits in the 12 times table, so look for the number that 12 divides exactly.
- A prime has exactly two factors, itself and 1, so any number that splits into a product of two smaller numbers is out.
- A factor of 24 divides into 24 with nothing left over. Watch the direction here: 48 is a multiple of 24, not a factor of it.

Worked Solution [4 marks]

> Rule: a multiple of 12 is 12 times a whole number, and a factor of 24 is a whole number that divides 24 exactly. A prime has exactly two factors, and an odd number is one that 2 does not divide.

Step 1: pick out an odd number

$15 = 2 \times 7 + 1$

$19 = 2 \times 9 + 1$

(Reason: The other five numbers end in 6, 8, 8, 4 and 8, so halving each one leaves nothing over and they are all even. Only 15 and 19 leave 1 over, and either of them earns the mark.)

Step 2: pick out a multiple of 12

$48 = 12 \times 4$

$44 = 12 \times 3 + 8$

(Reason: The 12 times table runs 12, 24, 36, 48 and only the last of those is in the box. The second line shows the closest miss: 44 is 8 past a multiple of 12.)

Step 3: pick out a prime number

$6 = 2 \times 3$

$15 = 3 \times 5$

$28 = 2 \times 14$

$38 = 2 \times 19$

$44 = 2 \times 22$

$48 = 2 \times 24$

(Reason: Every number in the box except 19 splits into two smaller whole numbers, so each has more than two factors. Nothing from 2 to 4 divides 19, and $5 \times 5 = 25$ is already past it, so the testing stops there and 19 is prime.)

Step 4: pick out a factor of 24

$24 = 6 \times 4$

$48 = 24 \times 2$

(Reason: The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24, and the only one of those in the box is 6. The second line is the trap: 48 divides BY 24, which makes it a multiple, not a factor.)

(a) 15 (or 19)

(b) 48

(c) 19

(d) 6

Verification

✓ **Check 1:** Read the four answers back against the box: 15, 48, 19 and 6. Each one is a number that is actually printed in the box, which is what the question demands.

✓ **Check 2:** Test parts (b) and (d) the other way round, by dividing instead of multiplying. $\frac{48}{12} = 4$ and $\frac{24}{6} = 4$, so both divisions come out as whole numbers.

✓ **Check 3:** Count the factors of 19 directly, testing 2, 3 and 4, and stopping there because $5 \times 5 = 25$ is bigger than 19. None of them divides it, so 19 has only the factors 1 and 19,

which is exactly two.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) an odd number written down	**B1**	Accept 15 or 19. Both values given together, and no other value, also scores.	✓
(b) a multiple of 12 written down	**B1**	48, and no other number from the box. Any extra value written alongside it loses the mark.	✓
(c) a prime number written down	**B1**	19, and no other number from the box.	✓
(d) a factor of 24 written down	**B1**	6, and no other number from the box. 48 is a common wrong answer here because it is a multiple of 24 rather than a factor of it.	✓

Full marks: 4/4

Question 2 - Exam Solution

Understanding the Question

Given

The decimal 0.13.

The statement $\frac{16}{20} = \frac{\square}{5}$, with the numerator on the right missing.

Find

(a) 0.13 written as a fraction. (b) The number that belongs in the empty box.

Plan the Solution

- (a) Count the digits after the decimal point. Two decimal places means the number is a count of hundredths, so the denominator is 100.
- (b) Look at what has happened to the denominator: 20 has become 5. Do exactly the same thing to the numerator.

Worked Solution [2 marks]

Place value: a decimal with two decimal places is a number of hundredths, so it is written over 100. Equivalent fractions: whatever you divide the denominator by, you must divide the numerator by as well, or the fraction changes value.

Step 1: part (a), count the decimal places

$$0.13 = \frac{13}{100}$$

(Reason: There are two digits after the decimal point, so the last digit sits in the hundredths column. Thirteen hundredths is written as $\frac{13}{100}$*.)*

Step 2: part (a), check whether the fraction cancels

$$\frac{13}{100} = \frac{130}{1000}$$

(Reason: 13 *is a prime number and* 100 *is not a multiple of* 13*, so nothing cancels and* $\frac{13}{100}$ *is already in its simplest form. An equivalent fraction such as* $\frac{130}{1000}$ *is accepted as well.)*

Step 3: part (b), see what has happened to the denominator

$$\frac{20}{5} = 4$$

(Reason: The denominator has gone from 20 *down to* 5*, and* 20 *shared into groups of* 5 *gives* 4*, so the denominator has been divided by* 4*.)*

Step 4: part (b), do the same to the numerator

$$\frac{16}{4} = 4$$

(Reason: Whatever the denominator is divided by, the numerator must be divided by as well, otherwise the two fractions are not equal.)

Step 5: part (b), write out the completed statement

$$\frac{16}{20} = \frac{4}{5}$$

(Reason: The numerator on the right is 4*, so* 4 *is the number that goes in the box.)*

(a) $\frac{13}{100}$

(b) 4

Verification

✓ **Check 1:** Turn the part (a) answer back into a decimal. Dividing 13 by 100 moves every digit two places to the right. $\frac{13}{100} = 0.13$

✓ **Check 2:** Cross-multiply the two fractions in part (b). They are equivalent only if the two products are equal. $16 \times 5 = 80$ and $20 \times 4 = 80$

✓ **Check 3:** Write both fractions in part (b) as decimals on the calculator and compare them. $\frac{16}{20} = 0.8$ and $\frac{4}{5} = 0.8$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\frac{13}{100}$	**B1**	Or any equivalent fraction, for example $\frac{130}{1000}$. The fraction does not have to be given in its simplest form.	✓
(b) 4	**B1**	For 4 written in the box.	✓

Full marks: 2/2

Question 3 - Exam Solution

Understanding the Question

Given

(a) The statement $\frac{5073}{\square} = 19$, with one number missing.
(b) A number whose cube root is 14.
(c) The list
$973 \quad 987 \quad 393 \quad 151 \quad 139.$

Find

(a) The number that belongs in the box.
(b) The number that belongs in the box.
(c) The difference between the largest and the smallest number in the list.

Plan the Solution

- (a) Read the statement backwards. If 5073 divided by the box gives 19, then 19 lots of the box make 5073, so the box is $\frac{5073}{19}$.

- (b) Cubing undoes a cube root, so cube the 14.
- (c) Write the five numbers in order of size, read off the two ends, then subtract the smaller from the larger.

Worked Solution [4 marks]

> Inverse operations - if $\frac{a}{b} = c$ then $b = \frac{a}{c}$, and if $\sqrt[3]{n} = k$ then $n = k^3$. A difference is largest $-$ smallest.

(a) Turn the division round

$$\frac{5073}{\square} = 19 \implies \square = \frac{5073}{19}$$

$$\frac{5073}{19} = 267$$

(Reason: The box is what 5073 is divided by, so the box is 5073 shared into 19 equal parts. A calculator is allowed on this paper, and the division comes out exactly, with nothing left over.)

(b) Undo the cube root

$$\sqrt[3]{\square} = 14 \implies \square = 14^3$$

$$14^3 = 14 \times 14 \times 14 = 2744$$

(Reason: Cubing is the inverse of taking a cube root, so cube the 14. Working in two stages, $14 \times 14 = 196$ and then $196 \times 14 = 2744$.)

(c) Put the list in order and pick out the two ends

$$139 \quad 151 \quad 393 \quad 973 \quad 987$$

$$\text{largest} = 987, \quad \text{smallest} = 139$$

(Reason: Ordering the numbers makes the two ends of the list obvious, and this is where the method mark is earned - the mark scheme awards it for identifying 987 and 139, or simply for writing the numbers in order of size. Take care here: 973 and 987 differ only in the middle digit.)

(c) Subtract the smaller from the larger

$$987 - 139 = 848$$

(Reason: The difference between two numbers is what is left when the smaller is taken away from the larger.)

(a) 267

(b) 2744

(c) 848

Verification

✓ **Check 1:** Multiply the answer to (a) back into the statement: 19 lots of 267. $19 \times 267 = 5073$, the number the statement starts with.

✓ **Check 2:** Test the answer to (b) by prime factors instead of by cubing: $2744 = 8 \times 343$, where $8 = 2^3$ and $343 = 7^3$. $\sqrt[3]{2744} = 2 \times 7 = 14$, so 2744 really is the number whose cube root is 14.

✓ **Check 3:** Add the difference found in (c) back on to the smallest number in the list. $139 + 848 = 987$, the largest number in the list.

✓ **Check 4:** Do the subtraction in (c) a second way, with no borrowing: take off 140 and then put the extra 1 back. $987 - 140 = 847$ and $847 + 1 = 848$, which agrees.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The number in the box	**B1**	For 267 written in the box.	✓
(b) The number in the box	**B1**	For 2744 written in the box.	✓
(c) Identify the largest and the smallest number	**M1**	For identifying 987 and 139 - they may be shown or circled - or for $987 - n$ where n is a number from the list, or for $m - 139$ where m is a number from the list, or for writing the numbers in order of size.	✓
(c) The difference	**A1**	For 848. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 4 - Exam Solution

Understanding the Question

Given

The key: one whole symbol represents 8 vehicles.
Each whole symbol is drawn as 4 small squares of equal size.
Car: 3 whole symbols. Van: 3 whole symbols and a half symbol.
Lorry: 4 whole symbols and one small square. Bus: one whole symbol and 3 small squares.
Taxi: 4 whole symbols. The Motorbike row is empty.
Emily counted 10 Motorbikes.

Find

(a) The number of Vans shown in the pictogram. (b) What must be drawn in the Motorbike row to show 10 Motorbikes.
(c) How many more Lorries than Buses Emily counted.

Plan the Solution

- Start with the key. It fixes the value of a whole symbol, and the way the symbol is divided fixes the value of one small square.
- For (a), count the whole symbols in the Van row, work out the value of the part symbol, and add.
- For (b), change 10 vehicles into small squares first, then group the small squares back into a symbol.
- For (c), total the Lorry row and the Bus row separately, then subtract.

Worked Solution [3 marks]

> Rule - Pictogram: a part of a symbol is worth the same part of the key. The symbol here is split into 4 equal small squares and the whole symbol is 8 vehicles, so one small square is $\frac{8}{4} = 2$ vehicles.

Step 1: work out what one small square is worth

$$\frac{8}{4} = 2$$

Car	
Van	
Lorry	
Bus	
Motorbike	
Taxi	

Key:  represents 8 vehicles

(Reason: The key gives the value of a whole symbol only. Every reading in this question uses part symbols as well, so the value of one small square is the number to have ready first.)

Step 2: read the Van row for part (a)

$3 \times 8 = 24$

$\frac{1}{2} \times 8 = 4$

$24 + 4 = 28$

(Reason: The Van row shows 3 whole symbols and a half symbol. Half a symbol is half of 8, which is the same as the two small squares that are drawn.)

Step 3: turn 10 into small squares for part (b)

$\frac{10}{2} = 5$

$5 = 4 + 1$

(Reason: Working in small squares avoids guessing how much of a symbol to draw. 5 small squares are needed, and 4 of them make one whole symbol, so one small square is left over.)

Step 4: total the Lorry row and the Bus row

$4 \times 8 + 2 = 34$

$8 + 3 \times 2 = 14$

(Reason: The Lorry row is 4 whole symbols and one small square. The Bus row is one whole symbol and 3 small squares, so the part symbol there is worth three small squares, not half a symbol.)

Step 5: subtract for part (c)

$34 - 14 = 20$

(Reason: The question asks how many more, so take the smaller row total away from the larger one.)

(a) 28 Vans

(b) One whole symbol and one small square drawn in the Motorbike row

(c) 20 more Lorries

Verification

✓ **Check 1:** Count the whole pictogram two different ways once the Motorbike row is complete. There are 71 small squares altogether, each worth 2 vehicles. $71 \times 2 = 142$, and adding the six row totals gives $24 + 28 + 34 + 14 + 10 + 32 = 142$

✓ **Check 2:** Do part (c) without totalling either row. The Lorry row has 17 small squares and the Bus row has 7, so subtract the small squares first and convert once. $17 - 7 = 10$ small squares, and $10 \times 2 = 20$

✓ **Check 3:** Work part (a) backwards. Turn 28 back into symbols and see whether that is what the Van row actually shows. $\frac{28}{8} = 3.5$ symbols, which is the three whole symbols and the half symbol that are drawn

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) 28	**B1**	cao. From $3.5 \times 8 = 28$, or from 14 small squares worth 2 each.	✓
(b) One whole symbol and one small square drawn in the Motorbike row	**B1**	oe, eg 5 small squares from the symbol shown. Whatever is drawn must come to 10.	✓
(c) 20	**B1**	cao. From $34 - 14 = 20$.	✓

Full marks: 3/3

Question 5 - Exam Solution

Understanding the Question

Given

Five January temperatures, in °C: -8 (Moscow), 24 (Chennai), -3 (Riga), 12 (Lisbon), -14 (Regina)
Novosibirsk is 13°C lower than Riga
Marrakesh is 25°C higher than Regina

Find

(a) the five temperatures written out in order, smallest first (b) the difference between -8°C and 12°C (c) the January temperature in Novosibirsk (d) the January temperature in Marrakesh

Plan the Solution

- Picture a number line. Temperatures below zero sit to the left of zero, and the further left a number is, the smaller it is.
- For (a), the below-zero temperatures come first, coldest first, then the above-zero ones.
- For (b), the difference is the size of the gap between the two temperatures on that line.
- For (c), lower means move to the left, so subtract. For (d), higher means move to the right, so add.

Worked Solution [4 marks]

> Rule - Numbers increase from left to right on a number line, so a temperature further below zero is the smaller one. A difference is the size of the gap between two temperatures, and subtracting a negative is the same as adding.

Step 1: put the five temperatures in order

$$-14 < -8 < -3 < 12 < 24$$

(Reason: Three of the temperatures are below zero, so those three come first. Of those, -14 is furthest below zero, so it is the smallest, and -3 is the closest to zero, so it is the largest of the three.)

Step 2: work out the difference for part (b)

$$12 - (-8) = 12 + 8 = 20$$

(Reason: Moscow is 8 degrees below zero and Lisbon is 12 degrees above zero, so the gap is the climb up to zero followed by the climb above it.)

Step 3: work out the temperature in Novosibirsk for part (c)

$$-3 - 13 = -16$$

(Reason: Lower means subtract. Riga is already 3 degrees below zero, and dropping a further 13 degrees puts Novosibirsk 16 degrees below zero.)

Step 4: work out the temperature in Marrakesh for part (d)

$-14 + 25 = 11$

(Reason: Higher means add. The first 14 degrees of the rise only bring Regina up to zero, so the remaining 11 degrees are above zero.)

(a) $-14, -8, -3, 12, 24$

(b) $20°C$

(c) $-16°C$

(d) $11°C$

Verification

✓ **Check 1:** Read the ordered list back: every temperature must be warmer than the one before it, and all five must appear once. The three below-zero temperatures come first, coldest first, then the two above zero, and no temperature is repeated or missing.

✓ **Check 2:** Count part (b) along the number line instead: 8 degrees from Moscow up to zero, then 12 degrees from zero up to Lisbon. $8 + 12 = 20$, which is the difference found in part (b).

✓ **Check 3:** Reverse part (c): put the 13 degrees back on the Novosibirsk temperature and Riga should come back. $-16 + 13 = -3$, the January temperature in Riga.

✓ **Check 4:** Reverse part (d): take the 25 degrees back off the Marrakesh temperature and Regina should come back. $11 - 25 = -14$, the January temperature in Regina.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $-14, -8, -3, 12, 24$	**B1**	All five temperatures written out in order, smallest first. No working is needed - the list alone earns the mark.	✓
(b) 20	**B1**	The answer alone earns the mark. A difference is a size, so it is not negative.	✓
(c) -16	**B1**	The answer alone earns the mark. The minus sign is part of the answer.	✓
(d) 11	**B1**	The answer alone earns the mark.	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

A regular octagon.
A four-sided shape whose left-hand edge is vertical and whose bottom edge is horizontal, with one angle marked x.

Find

(a) how many lines of symmetry a regular octagon has (b) a right angle on the shape, marked with the letter R (c) an obtuse angle on the shape, marked with the letter O (d) the size of x, found by measuring

Plan the Solution

- Part (a) is a rule, not a drawing job: a regular polygon with n sides has n lines of symmetry.
- Parts (b) and (c) are the two definitions. A right angle is exactly $90°$; an obtuse angle is bigger than $90°$ but smaller than a straight angle.
- Part (d) says by measuring, so the method is a protractor: centre on the point, base line along one arm, then read the scale that starts at zero on that arm.
- Finish by checking the reading against the angle sum of a quadrilateral, which must come to $360°$.

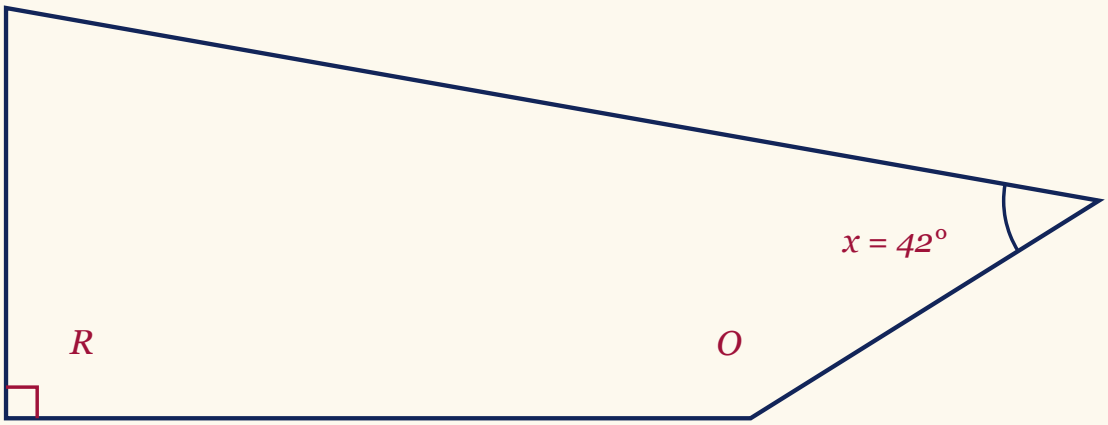
Worked Solution [4 marks]

> Regular polygons and quadrilaterals: a regular n-gon has n lines of symmetry and rotational symmetry of order n, and the four angles of any quadrilateral add up to $360°$.

Step 1: count the octagon's lines of symmetry

regular polygon with n sides $\rightarrow$ n lines of symmetry

$n = 8$



(Reason: An octagon has 8 sides, so it has 8 mirror lines: 4 joining opposite vertices, and 4 joining the midpoints of opposite sides. Nothing needs to be drawn or measured.)

Step 2: mark the right angle with R

vertical edge $\perp$ horizontal edge

angle at the bottom left vertex $= 90°$

(Reason: The left-hand edge runs straight up and the bottom edge runs straight across, so the corner where they meet is square. It is the only square corner on the shape, so the letter R goes at the bottom left vertex. A clear right-angle symbol drawn in that corner earns the mark just as well.)

Step 3: mark an obtuse angle with O

obtuse: bigger than $90°$ and smaller than a straight angle

angle at the bottom right vertex $\approx 148°$

(Reason: At the bottom right vertex the base stops and the edge turns back upwards, opening that corner out well past square but not as far as a straight line. That is the obtuse angle, so the letter O goes there.)

Step 4: measure the angle marked x

$x = 42°$

check: $360° - 90° - 148° - 80° = 42°$

(Reason: Put the centre of the protractor on the point and lay its base line along one of the two arms, then read up from zero on that arm. The reading is $42°$. Taking the other three angles from $360°$ gives the same value, which is why a hand measurement anywhere from $40°$ to $44°$ is accepted.)

(a) 8 lines of symmetry

(b) R at the bottom left vertex

(c) O at the bottom right vertex

(d) $x = 42°$

Verification

✓ **Check 1:** Add all four angles of the shape, in degrees. Any quadrilateral must total $360°$, so a measurement that breaks this is a measurement to take again.
$90 + 80 + 148 + 42 = 360$

✓ **Check 2:** Count the octagon's mirror lines in two families instead of using the rule: those through pairs of opposite vertices, and those through the midpoints of opposite sides.
$4 + 4 = 8$, which also matches its rotational symmetry of order 8.

✓ **Check 3:** Sanity-check the size before writing it down. The marked corner is the sharpest on the shape, so x has to come out acute. $42°$ is less than $90°$, and it sits inside the accepted $40°$ to $44°$ band.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) 8	**B1**	Award for 8 only. No working is required and none is expected.	✓
(b) R marked at the bottom left vertex	**B1**	Accept a clear, unambiguous right-angle symbol drawn in that corner instead of the letter R.	✓
(c) O marked at the bottom right vertex	**B1**	Award for a mark that clearly identifies that vertex and no other.	✓
(d) 42	**B1**	Allow 40 to 44, which is the tolerance for reading a protractor by hand.	✓

Full marks: 4/4

Question 7 - Exam Solution

Understanding the Question

Given

The sector for dominoes is $72°$, and 24 people played dominoes.
The sector for draughts is $96°$.
40 people played cribbage.
Each person played one game only, so the whole circle stands for everybody at the games evening.

Find

(a) the number of people who played draughts (b) the angle of the sector for cribbage

Plan the Solution

- A pie chart shares one full turn between the groups, so the same number of degrees stands for one person in every sector.
- The dominoes sector is the one we know completely: $72°$ for 24 people. Use it to find the angle for one person.
- Then (a) turns an angle into people, and (b) turns people into an angle. Both use that one figure.

Worked Solution [4 marks]

> Rule - Pie chart: $\text{angle for one person} = \dfrac{\text{angle of a known sector}}{\text{number of people in that sector}}$, and every sector on the chart uses that same figure.

Step 1: find the angle that stands for one person

$$\frac{72}{24} = 3$$

(Reason: the dominoes sector is $72°$ and it stands for 24 people, so one person is worth $3°$ of the pie chart)

Step 2: turn the draughts angle into people

$$\frac{96}{3} = 32$$

(Reason: the draughts sector is $96°$, and every $3°$ of it is one more person)

Step 3: turn the cribbage people into an angle

$$40 \times 3 = 120$$

(Reason: there are 40 people, and each one takes $3°$ of the pie chart)

(a) 32 people

(b) $120°$

Verification

✓ **Check 1:** Turn the answer to (a) back into an angle. If 32 people played draughts, their sector must be 32 lots of $3°$. $32 \times 3 = 96$, which is the draughts angle printed on the pie chart

✓ **Check 2:** Count the whole games evening two ways. One person is $3°$, so the full turn holds $\frac{360}{3} = 120$ people. The three known sectors use $72 + 96 + 120 = 288$ degrees, leaving $360 - 288 = 72$ degrees for backgammon, which is 24 people.
$24 + 32 + 40 + 24 = 120$, the same total, so the four sectors close the circle

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A method that fixes the size of one person or one degree, for example $\frac{72}{24} = 3$, $\frac{24}{72} = \frac{1}{3}$, $\frac{96}{72} = \frac{4}{3}$ or $24 \times \frac{360}{72} = 120$	**M1**	Any method that finds the degrees per person, the people per degree, the total number of people represented by the pie chart, or the scale factor between the two sectors.	✓
(a) 32	**A1**	Correct answer only. A correct answer scores both marks unless it comes from obviously incorrect working.	✓
(b) 3×40, or an equivalent such as $\frac{40}{24} \times 72$	**M1ft**	Follow through their own degrees-per-person figure. The working may already have been seen in part (a), for example $\frac{72}{24} = 3$.	✓
(b) 120	**A1**	Accept anything from 119.5 to 120.3.	✓

Full marks: 4/4

Question 8 - Exam Solution

Understanding the Question

Given

3 scented candles at $1.80 each, and 4 packets of flower seeds at $1.20 a packet
2 jars of honey, and the word identical tells us the two jars cost the same as each other
The whole order comes to $17.10
Three different items, and only one of the three has an unknown price

Find

The cost of ONE jar of honey The order pays for 2 jars, so the last move will be to split what the jars cost between them

Plan the Solution

- Price the two groups we can price: 3 candles at a known price each, and 4 packets at a known price each
- Add those two amounts to find how much of the $17.10 is already accounted for
- Take that away from the total, which leaves the cost of the two jars together
- Halve what is left, because the two jars are identical

Worked Solution [4 marks]

> Rule - when a total is made up of several items and only one item's price is unknown, take off everything you CAN price and whatever is left belongs to the rest:
> $\text{total} - \text{priced items} = \text{unpriced items}$

Step 1: price the two groups whose prices are given

$1.80 \times 3 = 5.40$

$1.20 \times 4 = 4.80$

$5.40 + 4.80 = 10.20$

(Reason: every candle costs the same and every packet costs the same, so each group is a multiplication rather than an addition, and the two group costs added together show that $10.20 of the order is already accounted for)

Step 2: take the priced items off the total

$17.10 - 10.20 = 6.90$

(Reason: the candles and the seed packets are paid for out of the same $17.10, so whatever is left of the total after they are removed has to be the cost of the two jars of honey together)

Step 3: split what is left between the two jars

$\frac{6.90}{2} = 3.45$

(Reason: the jars are identical, so the $6.90 they cost between them divides equally in two)

$3.45

Verification

✓ **Check 1:** Put the answer back into the shopping list and rebuild the whole order: $3 \times 1.80 = 5.40$ for the candles, $4 \times 1.20 = 4.80$ for the seeds and $2 \times 3.45 = 6.90$ for the honey. $5.40 + 4.80 + 6.90 = 17.10$, which is exactly the total the question states

✓ **Check 2:** Run the question backwards. If one jar really costs $3.45, then two jars cost $2 \times 3.45 = 6.90$, so taking that off the total should leave precisely the candles and the seeds. $17.10 - 6.90 = 10.20$, and the candles and seeds do come to $5.40 + 4.80 = 10.20$

✓ **Check 3:** Do the whole question again in cents, where every value is a whole number and no decimal point can slip: the candles cost $3 \times 180 = 540$ cents and the seeds cost $4 \times 120 = 480$ cents. $1710 - 540 - 480 = 690$ cents for two jars, so one jar is $\frac{690}{2} = 345$ cents, which is $3.45

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $3 \times 1.80 = 5.40$ or $4 \times 1.20 = 4.80$ or $3 \times 1.80 + 4 \times 1.20 = 10.20$	**M1**	for a method to find the cost of the candles, or the cost of the seed packets, or the total cost of the candles and the seed packets together	✓
eg $17.10 - (5.40 + 4.80) = 6.90$ or $17.10 - 10.20 = 6.90$	**M1**	for a method to find the cost of the 2 jars of honey. Follow through on the candidate's own group costs from the first mark	✓
eg $\frac{6.90}{2}$	**M1**	for a complete method: the cost of the two jars, halved. Follow through on the candidate's own value for the two jars	✓
3.45	**A1**	cao. A correct answer scores full marks on its own, unless it comes from obviously incorrect	✓

Step	Mark	Description	Got it?
		working	
If no other marks are awarded, an answer of 7.05 scores SCB2, and an answer of 14.10 scores SCB1	SC	both come from one nameable slip: taking off the price of a single candle and a single packet instead of all 3 candles and all 4 packets. That leaves 14.10 for the SCB1, and halving it gives the 7.05 for the SCB2	✓

Full marks: 4/4

Question 9 - Exam Solution

Understanding the Question

Given

Three points marked on a numbered grid: A, B and C.
$ABCD$ is a rectangle, so D is its fourth vertex - the corner opposite B.

Find

(a) The coordinates of A. (b) The coordinates of the midpoint of AB. (c) The coordinates of D.

Plan the Solution

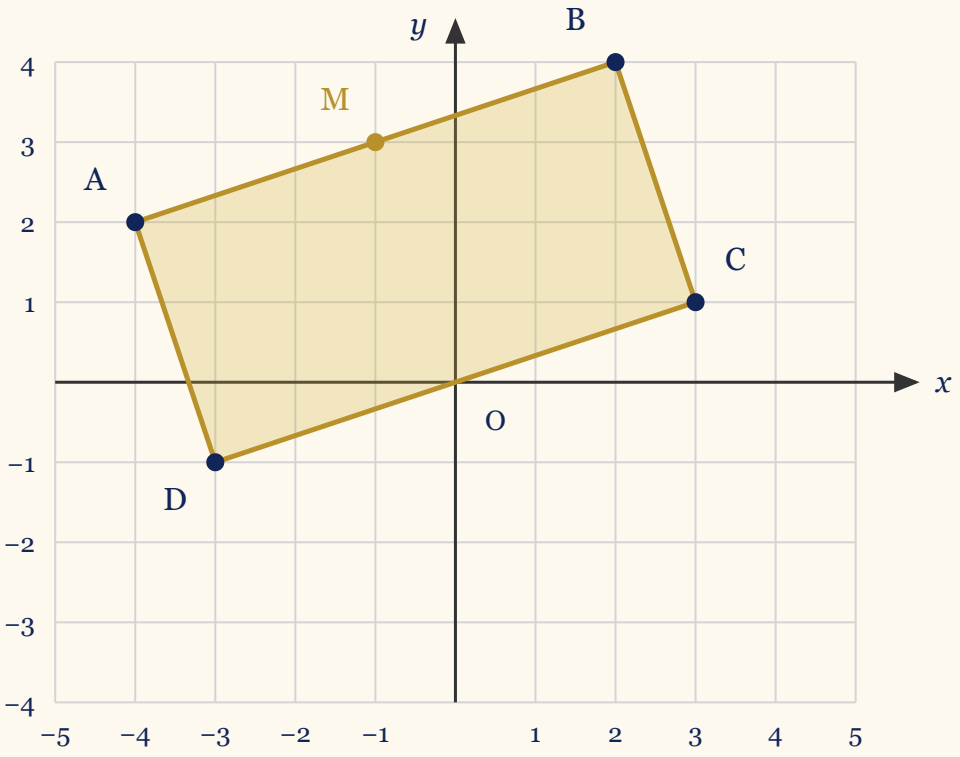
- Read A straight off the grid: count across from the y-axis first, then up from the x-axis, and write the across value first.
- For the midpoint, average the two x-coordinates and average the two y-coordinates. Averaging keeps the sign, so a negative coordinate is added, not ignored.
- For D, use the shape rather than a formula: in a rectangle AD and BC are opposite sides, so the move from A to D is the same as the move from B to C.
- The letters $ABCD$ go round the rectangle in order, so D is next to A and C, never next to B.

Worked Solution [5 marks]

> Rule - Midpoint: $x_M = \dfrac{x_1 + x_2}{2}$ and $y_M = \dfrac{y_1 + y_2}{2}$. Rule - Rectangle: opposite sides are equal and parallel, so the step from B to C is also the step from A to D.

Step 1: Read the coordinates of A off the grid

$A = (-4,\ 2)$



(Reason: From O, the cross at A is 4 squares to the left and 2 squares up. Left of the y-axis makes the x-coordinate negative, and the across value is always written first.)

Step 2: Average the coordinates of A and B

$$x_M = \frac{-4 + 2}{2} = -1$$

$$y_M = \frac{2 + 4}{2} = 3$$

$$M = (-1,\ 3)$$

(Reason: The midpoint sits halfway, so each of its coordinates is the mean of the two end values: add the pair and halve it. The x-coordinates are -4 and 2, and the y-coordinates are 2 and 4.)

Step 3: Step from A by the same move that takes B to C

$$(3 - 2,\ 1 - 4) = (1,\ -3)$$

$$D = (-4 + 1,\ 2 - 3) = (-3,\ -1)$$

(Reason: BC and AD are opposite sides of the rectangle, so they are the same length and point the same way. The move from B to C is 1 right and 3 down, so the same move from A lands on D.)

(a) $(-4,\ 2)$

(b) $(-1,\ 3)$

(c) $(-3,\ -1)$

Verification

✓ **Check 1:** Step from A to M, then from M to B. If M really is the midpoint, the two steps are identical. $(3,\ 1)$ both times, so M lies halfway along AB.

✓ **Check 2:** A rectangle needs a right angle at B. Multiply the two side steps $AB = (6,\ 2)$ and $BC = (1,\ -3)$ across and add: $6 \times 1 + 2 \times (-3)$. $6 - 6 = 0$, so AB and BC are perpendicular.

✓ **Check 3:** The diagonals of a rectangle bisect each other, so the midpoint of AC must come out the same as the midpoint of BD. Both give $(-0.5,\ 1.5)$, which also confirms $D = (-3,\ -1)$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The coordinates of A written down	**B1**	$(-4,\ 2)$, correct answer only. The x-coordinate must come first.	✓
(b) The coordinates of the midpoint of AB	**B2**	$(-1,\ 3)$. B1 for $(-1,\ y)$ or $(x,\ 3)$ or $(3,\ -1)$, or for the midpoint of AB shown unambiguously on the diagram.	✓
(c) The coordinates of D	**B2**	$(-3,\ -1)$. B1 for $(-3,\ y)$ or $(x,\ -1)$ or $(-1,\ -3)$, or for D marked correctly on the diagram.	✓

Full marks: 5/5

Question 10 - Exam Solution

Understanding the Question

Given

Find

A function machine that changes a temperature in degrees Celsius into a temperature in degrees Fahrenheit: multiply by 1.8, then add 32.
Part (a) starts with $175°\text{C}$, and part (b) starts with $482°\text{F}$.

(a) the same temperature written in degrees Fahrenheit. (b) the same temperature written in degrees Celsius. One machine, used forwards in (a) and backwards in (b).

Plan the Solution

- (a) Feed the temperature in at the left and work through the two boxes in the order they are drawn.
- (b) The temperature is coming out of the right-hand end, so travel the other way and undo each box: subtract 32 first, then divide by 1.8.
- Undoing a machine reverses the ORDER of the steps as well as the operations themselves, so the subtraction comes before the division.
- A calculator is allowed, so type each stage separately rather than one long expression - that is where marks are lost here.

Worked Solution [4 marks]

> Rule - forwards, follow the boxes as drawn: multiply by 1.8, then add 32.
> Backwards, use the inverse operations in the opposite order: subtract 32, then divide by 1.8.

Step 1 (a) - multiply by 1.8

$$175 \times 1.8 = 315$$

(Reason: The temperature going in is 175, and the first box it meets multiplies by 1.8. This product on its own earns the method mark.)

Step 2 (a) - then add 32

$$315 + 32 = 347$$

(Reason: What comes out of the first box goes straight into the second, so the 32 is added to 315 and never to 175.)

Step 3 (b) - undo the $+32$ first

$$482 - 32 = 450$$

(Reason: Part (b) walks back through the machine, and the last thing the machine did was add 32, so the first thing to undo is that addition.)

Step 4 (b) - then undo the $\times 1.8$

$\frac{450}{1.8} = 250$

(Reason: The opposite of multiplying by 1.8 is dividing by 1.8, and it is the 450 from step 3 that is divided, not the 482 the part started with.)

(a) $347°\text{F}$

(b) $250°\text{C}$

Verification

✓ **Check 1:** Put part (a)'s answer back through the machine in reverse: $347 - 32 = 315$, then $\frac{315}{1.8} = 175$. It lands on $175°\text{C}$, the temperature part (a) began with.

✓ **Check 2:** Send part (b)'s answer forwards through the machine as drawn: $250 \times 1.8 = 450$, then $450 + 32 = 482$. It lands on $482°\text{F}$, the temperature part (b) began with.

✓ **Check 3:** Is the machine itself being used properly? Water boils at $100°\text{C}$, and the machine gives $100 \times 1.8 + 32 = 212$. The familiar boiling point $212°\text{F}$ comes out, so the rule is being applied the right way round.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $175 \times 1.8 = 315$	**M1**	for a correct first step	✓
(a) $315 + 32 = 347$	**A1**	cao. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
(b) $482 - 32 = 450$	**M1**	for -32, or for dividing by 1.8	✓
(b) $\frac{450}{1.8} = 250$	**A1**	cao. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
(b) special case - an	**SC B1**	This row carries no mark of its own; it records what one particular wrong answer is worth. It comes from typing the	✓

Step	Mark	Description	Got it?
answer of 464.2 and so on		reverse machine into a calculator without brackets: the calculator works out $\frac{32}{1.8}$ first and subtracts only that from 482, instead of subtracting 32 and then dividing.	

Full marks: 4/4

Question 11 - Exam Solution

Understanding the Question

Given

(a) the sum $c + c + c + c + c$
(b) the expression $7w + 10y - 9w + 2y$
(c) a sequence whose nth term is $7n - 4$
(d) the equation $7(x + 5) = 8 - 3x$

Find

(a) the sum written as a single term in c
(b) the expression with its like terms collected (c) the 1st term and the 5th term of the sequence (d) the value of x, with algebraic working shown

Plan the Solution

- (a) count how many c terms are being added.
- (b) handle the w terms and the y terms separately, because they are not like terms.
- (c) substitute the position into the rule: $n = 1$ for the first term, then $n = 5$.
- (d) expand the bracket, gather the x terms on one side and the numbers on the other, then divide.

Worked Solution [8 marks]

> Like terms collect by adding their number parts only, and the letter is unchanged; a term of a sequence comes from substituting its position n into the nth term rule; and a linear equation is solved by expanding, collecting like terms, then dividing by the number in front of x.

Step 1: part (a), add the five c terms

$$c + c + c + c + c = 5c$$

(Reason: Adding c five times is the same as 5 lots of c. Only how many there are changes; the letter itself is unchanged.)

Step 2: part (b), collect the terms in w

$$7w - 9w = -2w$$

(Reason: The terms in w are like terms, so their number parts combine into one number part and the letter w is carried through unchanged.)

Step 3: part (b), collect the terms in y and write the answer

$$10y + 2y = 12y$$

$$7w + 10y - 9w + 2y = -2w + 12y$$

(Reason: A term in w and a term in y are not like terms, so they cannot be combined with each other and both are kept in the answer.)

Step 4: part (c), the 1st term, where $n = 1$

$$7 \times 1 - 4 = 7 - 4 = 3$$

(Reason: The 1st term sits in position $n = 1$, so 1 is substituted for n in $7n - 4$.)

Step 5: part (c), the 5th term, where $n = 5$

$$7 \times 5 - 4 = 35 - 4 = 31$$

(Reason: The same rule is used with $n = 5$. Multiplying comes before subtracting, so the 35 is found first and the 4 is taken off afterwards.)

Step 6: part (d), expand the bracket

$$7(x + 5) = 7x + 35$$

$$7x + 35 = 8 - 3x$$

(Reason: Everything inside the bracket is multiplied by 7: $7 \times x = 7x$ and $7 \times 5 = 35$. The right-hand side is untouched.)

Step 7: part (d), gather the x terms and the numbers

$$7x + 3x = 8 - 35$$

$$10x = -27$$

(Reason: Adding $3x$ to both sides removes the $-3x$ from the right, and subtracting 35 from both sides removes it from the left, leaving the letters on one side and the numbers on the other.)

Step 8: part (d), divide to find x

$$x = \frac{-27}{10}$$

$$x = -2.7$$

(Reason: Both sides are divided by 10, the number multiplying x. The answer is negative because -27 is negative, and a decimal answer is perfectly acceptable here.)

(a) $5c$

(b) $-2w + 12y$

(c) 1st **term** 3, 5th **term** 31

(d) $x = -2.7$

Verification

✓ **Check 1:** Part (a): put $c = 2$ into both forms. The sum gives $2 + 2 + 2 + 2 + 2$ and the single term gives $5 \times 2.\ 10 = 10$, so the two forms agree

✓ **Check 2:** Part (b): put $w = 1$ and $y = 1$ into both forms. The original gives $7 + 10 - 9 + 2$ and the collected form gives $-2 + 12.\ 10 = 10$, so nothing was lost or gained in collecting

✓ **Check 3:** Part (c): list the sequence from $7n - 4$. The terms should rise by 7 each time, so the 5th term should be the 1st term plus four steps of $7.\ 3, 10, 17, 24, 31$ and $3 + 4 \times 7 = 31$

✓ **Check 4:** Part (d): substitute $x = -2.7$ into each side of the original equation. The left gives $7(-2.7 + 5) = 7 \times 2.3$ and the right gives $8 - 3 \times (-2.7) = 8 + 8.1$. $16.1 = 16.1$, so both sides balance

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $5c$	**B1**	cao	✓
(b) $-2w + 12y$	**B2**	oe (B1 for $-2w$ or $12y$)	✓
(c) 3 for the 1st term	**B1**	cao	✓
(c) 31 for the 5th term	**B1**	cao	✓
(d) $7x + 35$	**M1**	a correct expansion of the bracket (need not be in an equation)	✓
(d) $7x + 3x = 8 - 35$ oe, eg $10x = 8 - 35$ or $10x = -27$	**M1**	for isolating terms in x and number terms; ft from an incorrect expansion in the form $7x + c$ or $x + 35$	✓
(d) $x = -2.7$	**A1**	dep on M1, oe. Working required.	✓

Full marks: 8/8

Question 12 - Exam Solution

Understanding the Question

Given

$ABCD$ is a quadrilateral, and triangle CDE is joined to it along CD.
$CD = CE$, so triangle CDE is isosceles.
D, E and F lie on one straight line.
Marked on the figure: $\angle DAB = 78°$, $\angle ABC = 128°$, $\angle ADC = 86°$ and $\angle CEF = 125°$.

Find

The size of $\angle BCE$, the whole angle at C between CB and CE. A reason is wanted at every stage, so each line of working names the angle fact it uses.

Plan the Solution

- Start at E. The straight line DEF turns the $125°$ into $\angle CED$.
- $CD = CE$ copies that angle across to $\angle CDE$, and the triangle then gives $\angle DCE$.
- The four angles of $ABCD$ give the other piece, $\angle BCD$.
- CD lies inside the angle being asked for, so the two pieces at C add.

Worked Solution [5 marks]

> Angles on a straight line add to $180°$, angles in a triangle add to $180°$, and angles in a quadrilateral add to $360°$.

Step 1: the angle of the triangle at E

$$\angle CED = 180° - 125° = 55°$$

(Reason: Angles on a straight line add to $180°$, and DEF is a straight line, so $\angle CED$ and the marked $125°$ together make $180°$.)

Step 2: the other base angle of the triangle

$$\angle CDE = \angle CED = 55°$$

(Reason: $CD = CE$, so triangle CDE is isosceles and the two base angles, at D and at E, are equal.)

Step 3: the angle of the triangle at C

$\angle DCE = 180° - 55° - 55° = 70°$

(Reason: Angles in a triangle add to $180°$, so take both base angles of triangle CDE away from $180°$.)

Step 4: the fourth angle of the quadrilateral

$\angle BCD = 360° - (78° + 128° + 86°)$

$\angle BCD = 360° - 292° = 68°$

(Reason: Angles in a quadrilateral add to $360°$, and the other three angles of $ABCD$ are $78°$, $128°$ and $86°$.)

Step 5: put the two angles at C together

$\angle BCE = \angle BCD + \angle DCE$

$\angle BCE = 68° + 70° = 138°$

(Reason: CD lies inside $\angle BCE$, so $\angle BCD$ and $\angle DCE$ together make the whole angle at C.)

$$\angle BCE = 138°$$

Verification

- ✓ **Check 1:** Put the $68°$ back into the quadrilateral and add all four of its angles. $78° + 128° + 68° + 86° = 360°$
- ✓ **Check 2:** Add the three angles of triangle CDE, then check the straight line at E as well. $55° + 55° + 70° = 180°$ and $55° + 125° = 180°$
- ✓ **Check 3:** A different route to the same angle. $ABCED$ is a pentagon, so its five angles add to $540°$. Its angle at D is the whole of $\angle ADE = 86° + 55° = 141°$, and its angle at C is the angle being found. $540° - 78° - 128° - 55° - 141° = 138°$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\angle CED = 180° - 125° = 55°$ or $\angle BCD = 360° - (86° + 78° + 128°) = 68°$	M1	Either one of these two starting angles earns the mark. Angles may be marked on the diagram or labelled unambiguously in the working.	✓

Step	Mark	Description	Got it?
$\angle DCE = 180° - (2 \times 55°) = 70°$	**M1**	The apex angle of the isosceles triangle, from the candidate value for $\angle CED$. Angles may be marked on the diagram or labelled unambiguously in the working.	✓
$\angle BCE = 68° + 70° = 138°$	**A1**	For 138. A correct answer of 138 scores these 3 marks unless it comes from obvious incorrect working.	✓
Reasons: angles on a straight line add to $180°$; isosceles triangle; angles in a triangle add to $180°$; angles in a quadrilateral add to $360°$.	**B2**	Dependent on both method marks, for all correct reasons for the method used. Award B1 instead, dependent on one method mark, for one correct reason for the method used.	✓

Full marks: 5/5

Question 13 - Exam Solution

Understanding the Question

Given

Red marbles: n
Blue marbles: twice as many as red, so $2n$
Green marbles: 7 more than red, so $n + 7$
The total number of marbles in the box is T

Find

(a) An expression in terms of n for the number of red, blue and green marbles altogether, in its simplest form (b) An expression in terms of T and n for how many of the marbles are not red, blue or green

Plan the Solution

- Write each colour as an expression in n before adding anything.
- Read "twice as many" as $2 \times n$ and "7 more than" as $n + 7$, so the green marbles carry an n as well as the 7.
- Add the three expressions, then collect the n terms together and the number terms together.
- For part (b), the marbles of any other colour are what is left when the three colours are taken away from T, so subtract the whole of the part (a) expression.

Worked Solution [3 marks]

> Rule - Collecting like terms: write every quantity in terms of the same letter, add them, then add the n terms together and the number terms together. Only like terms may be combined.

Step 1: Write each colour in terms of n

$\text{red} = n$

$\text{blue} = 2 \times n = 2n$

$\text{green} = n + 7$

(Reason: (Reason: "twice as many blue as red" doubles the red amount, and "7 more green than red" adds 7 to it. Notice that green is $n + 7$, not just 7 - the n is still there.))

Step 2: Add the three amounts

$\text{total} = n + 2n + (n + 7)$

(Reason: (Reason: the total of the three colours is red plus blue plus green. The brackets keep the green expression together while it is written down.))

Step 3: Collect the like terms

$n + 2n + n = 4n$

$\text{total} = 4n + 7$

(Reason: (Reason: three lots of n come from red, blue and green, so their coefficients 1, 2 and 1 add up. The 7 has no n with it, so it cannot be combined with them and stays as it is.))

Step 4: (b) Take the three colours away from T

$\text{other colours} = T - (4n + 7)$

(Reason: (Reason: every marble is either one of the three colours or it is not, so the rest are T minus the number that are red, blue or green. The brackets matter: the whole of $4n + 7$ is subtracted, which is the same as $T - 4n - 7$.))

(a) $4n + 7$

(b) $T - (4n + 7)$

Verification

✓ **Check 1:** Put $n = 5$ into the words of the question: 5 red, twice as many blue so 10, and 7 more green than red so 12. Add them, then compare with the expression.
$5 + 10 + 12 = 27$ and $4 \times 5 + 7 = 27$

✓ **Check 2:** Repeat with a different value, $n = 10$, so that a rule which only happens to work once cannot pass: 10 red, 20 blue, 17 green. $10 + 20 + 17 = 47$ and $4 \times 10 + 7 = 47$

✓ **Check 3:** Test part (b) on real numbers. Take $n = 5$ and a box holding $T = 60$ marbles. Count the three colours, then count what is left. $60 - 27 = 33$ marbles of other colours, and $T - (4n + 7) = 60 - (20 + 7) = 33$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Write the blue marbles as $2n$, or the green marbles as $n + 7$	**M1**	for $2n$ oe or $n + 7$ oe	✓
(a) Add and collect the like terms	**A1**	for $4n + 7$ oe, eg $7 + 4n$. A correct answer scores full marks unless it comes from obviously incorrect working	✓
(b) Subtract the total of the three colours from T	**B1ft**	for $T - (4n + 7)$ oe, eg $T - 4n - 7$. Follow through T minus the candidate's own answer to part (a)	✓

Full marks: 3/3

Question 14 - Exam Solution

Understanding the Question

Given

The calculation $\dfrac{\sqrt{17.8 \times 19.2}}{3.4^2 \times 0.23}$, to be done on a calculator.
A square root on the top of the fraction, and a square followed by a multiplication on the bottom.

Find

The value of the expression. Every figure the calculator display shows, so the answer is not rounded.

Plan the Solution

- Work out the top and the bottom separately, then divide one by the other.
- Top: multiply 17.8 by 19.2 first, then take the square root of the product.

- Bottom: square 3.4 first, then multiply by 0.23.
- Keep the unrounded value on the display all the way through, and copy the final display in full.

Worked Solution [2 marks]

> Rule - a fraction bar is a division that acts on the whole of the top and the whole of the bottom, so each one is completed before dividing; and inside the bottom, 3.4^2 is squared before it is multiplied by 0.23.

Step 1: multiply the two numbers under the square root

$$17.8 \times 19.2 = 341.76$$

(Reason: The root sign covers the whole product, so the multiplication is done first and the root is taken of the single number it gives.)

Step 2: take the square root to get the numerator

$$\sqrt{341.76} = 18.48675201\ldots$$

(Reason: Leave this on the display or store it in the calculator's memory. Rounding here would change the later figures, and the question asks for all of them.)

Step 3: work out the denominator

$$3.4^2 = 11.56$$

$$11.56 \times 0.23 = 2.6588$$

(Reason: The square is worked out before the multiplication, following the order of operations. Squaring the whole product instead would give a different bottom line.)

Step 4: divide the numerator by the denominator

$$\frac{18.48675201\ldots}{2.6588} = 6.953043483\ldots$$

(Reason: The division is done last, using the unrounded numerator from Step 2.)

Step 5: write down every figure on the display

$$6.953043483$$

(Reason: The question asks for all the figures shown, so nothing is rounded off. A display with room for more figures shows 6.95304348325, and any answer that starts 6.95304 and continues with the display's own figures is the same answer.)

$$\boxed{6.953043483}$$

Verification

✓ **Check 1:** Reverse the division. Multiplying the answer by the denominator must give the numerator back. $6.953043483 \times 2.6588 = 18.48675201\ldots$, which is the numerator from Step 2

✓ **Check 2:** Estimate with easy numbers: $\sqrt{18 \times 19} = \sqrt{342} \approx 18.5$ on the top, and $3.4^2 \times 0.23 \approx 11.6 \times 0.23 \approx 2.7$ on the bottom. $\dfrac{18.5}{2.7} \approx 6.9$, which is close to 6.95, so the answer is the right size

✓ **Check 3:** Undo both operations at once. Squaring the answer and multiplying by 2.6588^2 must return the number that was under the root. $6.953043483^2 \times 2.6588^2 \approx 341.76$, which is 17.8×19.2

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$18.4(867\ldots)$ or $2.65(88)$ or 6.95 or 6.953 or 6.9530	**M1**	For a correct numerator $\sqrt{17.8 \times 19.2} = 18.4(867\ldots)$, or a correct denominator $3.4^2 \times 0.23 = 2.65(88)$, or the correct answer rounded or truncated to 3, 4 or 5 significant figures. The figures in brackets need not be shown.	✓
$6.95304(3483\ldots)$	**A1**	Correct answer scores full marks (unless from obvious incorrect working). The bracketed figures are the ones a display may or may not show, so 6.953043483 and any longer reading of the same display are accepted.	✓

Full marks: 2/2

Question 15 - Exam Solution

Understanding the Question

Given

A block that is a cube of side 5 cm, and a supply of identical blocks.
A trunk whose inside is a cuboid measuring 27 cm by 35 cm by 40 cm.

Find

The volume of the space inside the trunk that no block fills. The answer is a volume, so it is measured in cm^3.

As many blocks as possible are packed in, and the lid must still shut, so no block may stick out.

Plan the Solution

- Divide each inside length by the length of a block to see how many blocks fit along it, and round each answer DOWN, because part of a block cannot be packed.
- Multiply the three counts together for the number of blocks that fit.
- Work out the volume of one block, then the volume of all of them.
- Take the volume of the blocks away from the volume of the trunk.

Worked Solution [4 marks]

> Rule - blocks only fit a whole number of times along an edge, so every count is rounded down; then the volume of space left is the volume of the trunk take away the total volume of the blocks.

Step 1: how many blocks fit along each inside length

$$\frac{27}{5} = 5.4$$

$$\frac{35}{5} = 7$$

$$\frac{40}{5} = 8$$

(Reason: Only whole blocks can be packed, so each count is rounded down: 5 blocks along the 27 cm side, 7 along the 35 cm side and 8 along the 40 cm side.)

Step 2: the number of blocks that fit in the trunk

$$5 \times 7 \times 8 = 280$$

(Reason: The blocks fill complete rows, complete columns and complete layers, so the three counts multiply together.)

Step 3: the volume the blocks take up

$$5 \times 5 \times 5 = 125 \text{ cm}^3$$

$$280 \times 125 = 35\,000 \text{ cm}^3$$

(Reason: Each block is a cube, so its volume is 5 cubed, and every packed block has that same volume.)

Step 4: the volume inside the trunk

$$27 \times 35 \times 40 = 37\,800 \text{ cm}^3$$

(Reason: The INSIDE lengths are the ones to use, because it is the space inside the trunk that is being filled.)

Step 5: take the blocks away from the trunk

$37\,800 - 35\,000 = 2800\ \text{cm}^3$

(Reason: What is left is the space no block reaches: a gap of 2 cm right across the top, because five layers of blocks stand only 25 cm tall.)

$$2800\ \text{cm}^3$$

Verification

✓ **Check 1:** Look at the gap at the top instead. Five layers of blocks stand 25 cm tall, so $27 - 25 = 2$ cm of height is empty right across the floor area of the trunk. $2 \times 35 \times 40 = 2800\ \text{cm}^3$, the same answer from a completely different starting point

✓ **Check 2:** Count in block-sized spaces. The trunk has room for $\dfrac{37\,800}{125} = 302.4$ block-sized spaces, but only 280 of them actually hold a block. $302.4 - 280 = 22.4$ spaces are empty, and $22.4 \times 125 = 2800$

✓ **Check 3:** Put the two volumes back together. The blocks and the empty space must fill the inside of the trunk exactly. $35\,000 + 2800 = 37\,800\ \text{cm}^3$, which is the volume worked out in Step 4

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\dfrac{35}{5} = 7$ or $\dfrac{27}{5} = 5.4$ or $\dfrac{40}{5} = 8$ or $5 \times 5 \times 5 = 125$ or $27 \times 35 \times 40 = 37\,800$ or $25 \times 35 \times 40 = 35\,000$	M1	For finding the number of blocks that fit along one inside length of the trunk, or the volume of one block, or the volume taken up by the blocks.	✓
$5 \times 7 \times 8 = 280$ and $5 \times 5 \times 5 = 125$, or $27 - 5 \times 5 = 2$, or	M1	For the total number of blocks that will fit together with the volume of one block; or the number of centimetres of height left with no block in it; or the volume of the trunk together	✓

Step	Mark	Description	Got it?
$27 \times 35 \times 40 = 37\,800$ and $25 \times 35 \times 40 = 35\,000$, or $\frac{37\,800}{125} - 5 \times 7 \times 8 = 22.4$		with the total volume of the blocks; or the space left at the top written as a number of blocks. Note that a count of 6 layers along the 27 cm side is not a count of blocks that fit: those layers would stand 30 cm tall and the lid would not shut.	
$37\,800 - 280 \times 125$ or equivalent, e.g. $37\,800 - 35\,000$ or $2 \times 35 \times 40$ or 22.4×125	**M1**	For a fully correct method to find the volume of the space left. The official scheme prints the values of this row in quotation marks, which is its way of saying that the candidate's own earlier values may be used, provided the method itself is complete.	✓
2800	**A1**	cao. A correct answer scores full marks unless it comes from obviously incorrect working. The unit is given on the answer line, so it need not be repeated.	✓

Full marks: 4/4

Question 16 - Exam Solution

Understanding the Question

Given

Six tiles in a row. Five of them are printed with 16, 15, 3, 2 and 9.
The mean of all six numbers has to be 11.
The sixth tile is blank, so its number is the only thing that can be chosen.

Find

The number that belongs on the blank tile.

Plan the Solution

- A mean of 11 spread over 6 numbers fixes what those numbers must add up to, so start with that total.

- Add the five numbers that are already printed.
- Whatever is left over from the total is the number for the blank tile.

Worked Solution [3 marks]

> Mean: total $=$ mean $\times$ number of values, so a known mean can always be turned back into a total.

Step 1: turn the mean into the total the six numbers must make

$$11 \times 6 = 66$$

(Reason: The mean is the total shared equally between the tiles, so multiplying the mean by how many numbers there are gives that total back.)

Step 2: add the five numbers that are already printed

$$16 + 15 + 3 + 2 + 9 = 45$$

(Reason: These five cannot be changed, so their sum is the part of the 66 that is already on the tiles.)

Step 3: take the five away from the total

$$45 + x = 66$$

$$66 - 45 = 21$$

(Reason: The blank tile has to make up the difference between what the five printed numbers give and the total the mean demands.)

21

Verification

✓ **Check 1:** Print 21 on the last tile, add all six numbers, then divide by 6.

$16 + 15 + 3 + 2 + 9 + 21 = 66$ and $\dfrac{66}{6} = 11$, which is the mean asked for.

✓ **Check 2:** Measure each printed number against 11 instead of totalling: 16 is 5 above, 15 is 4 above, 3 is 8 below, 2 is 9 below and 9 is 2 below. Those five sit 10 below the mean altogether, so the last number must sit 10 above it: $11 + 10 = 21$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Work out the total the six numbers need, or	M1	$11 \times 6 = 66$ or $\dfrac{16 + 15 + 3 + 2 + 9 + x}{6} = 11$ - a correct	✓

Step	Mark	Description	Got it?
write the mean as an equation		calculation for the total, or a correct equation for the last tile using x.	
Reach an equation with no fraction in it, or the subtraction that gives the last number	M1	$16 + 15 + 3 + 2 + 9 + x = 66$ oe, eg $45 + x = 66$ or $66 - (16 + 15 + 3 + 2 + 9)$.	✓
The number printed on the last tile	A1	21 - a correct answer scores full marks unless it comes from obviously incorrect working. If the answer line is blank, check the tile.	✓

Full marks: 3/3

Question 17 - Exam Solution

Understanding the Question

Given

The spinner is biased, so the five numbers are not equally likely.
The probabilities of landing on 1, 2, 3, 4 and 5 are $2x$, 0.27, 0.04, x and 0.12.
The spinner is to be spun 400 times.

Find

An estimate for how many of the 400 spins land on an odd number. The odd numbers on this spinner are 1, 3 and 5.

Plan the Solution

- Use the fact that the five probabilities have a total of 1 to form an equation in x.
- Solve that equation, then double the answer, because the probability of landing on 1 is $2x$.
- Add the three odd probabilities, then multiply that total by 400.

Worked Solution [4 marks]

> Rule - Expected frequency: the probabilities of all the possible outcomes have a total of 1, and an estimate for the number of times an event happens is its probability multiplied by the number of trials.

Step 1: Write down what the five probabilities must total

$$2x + 0.27 + 0.04 + x + 0.12 = 1$$

(Reason: Every spin lands on exactly one of the five numbers, so the five probabilities have a total of 1 *.)*

Step 2: Collect the like terms

$$0.27 + 0.04 + 0.12 = 0.43$$

$$3x + 0.43 = 1$$

(Reason: The three known probabilities come to 0.43*, and* $2x$ *and* x *combine to give* $3x$*.)*

Step 3: Solve for x

$$3x = 1 - 0.43 = 0.57$$

$$x = \frac{0.57}{3} = 0.19$$

$$2x = 2 \times 0.19 = 0.38$$

(Reason: Take 0.43 *from both sides, then divide by* 3*. The table gives the probability of landing on* 1 *as* $2x$*, which is double this.)*

Step 4: Add the probabilities of the odd numbers

$$0.38 + 0.04 + 0.12 = 0.54$$

(Reason: The odd numbers are 1*,* 3 *and* 5*, so the three probabilities to add are* $2x$*,* 0.04 *and* 0.12*.)*

Step 5: Multiply the probability by the number of spins

$$0.54 \times 400 = 216$$

(Reason: An estimate for how often an event happens is its probability multiplied by the number of trials, and the spinner is spun 400 *times.)*

216

Verification

- ✓ **Check 1 - do the five probabilities total 1:** With $x = 0.19$ the two unknown entries in the table are 0.38 and 0.19, so the even probabilities come to $0.27 + 0.19 = 0.46$. Odd and even together give $0.54 + 0.46 = 1$, so nothing has been left out and nothing counted twice.
- ✓ **Check 2 - count the spins instead of the probabilities:** Estimate the even outcomes directly: $0.27 \times 400 = 108$ for the 2 and $0.19 \times 400 = 76$ for the 4. The even spins come to $108 + 76 = 184$, so $400 - 184 = 216$ spins are left for the odd numbers.
- ✓ **Check 3 - is the size sensible?** A little over half of the probability sits on the odd numbers, so a little over half of the 400 spins should land on one of them. Half of 400 is 200, and 216 is a little more than that.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Use the total probability: $1-(0.27+0.04+0.12)=0.57$ or $2x+0.27+0.04+x+0.12=1$	**M1**	for showing a clear understanding that the total of the probabilities is 1, or for finding estimates for the number of times the spinner will land on 2, 3 and 5	✓
Find the unknown entry: $\frac{0.57}{3}=0.19$, or $2x=0.38$, or $\frac{400-108-16-48}{3}=76$	**M1**	for a method to find the value of x or of $2x$, or an estimate for the number of times the spinner will land on 4 or on 1	✓
A complete method: $(2\times 0.19+0.04+0.12)\times 400$ or $2\times 76+16+48$	**M1**	for a complete method	✓
The answer 216	**A1**	for an answer of 216. An answer of $\frac{216}{400}$ scores M3A0.	✓
Note	**note**	A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 18 - Exam Solution

Understanding the Question

Given

A total of 200 bagels are sold
plain : sesame = $3:2$
A plain bagel costs £1.50 and a sesame bagel costs £1.75
Profit is 40% of a plain bagel's price, and 60% of a sesame bagel's price

Find

The total profit, in pounds, on all 200 bagels

Plan the Solution

- Share the 200 bagels in the ratio $3 : 2$ to find how many of each kind are sold.
- Multiply each count by its own price, to get the money taken on the plain bagels and the money taken on the sesame bagels.
- Take 40% of the plain money and 60% of the sesame money. The two percentages are different, so the two kinds must be kept apart.
- Add the two profits. Taking one single percentage of the whole £320 taken would not work, because neither 40% nor 60% applies to all of it.

Worked Solution [5 marks]

> Rule - Ratio then percentage: divide by the total number of parts to get one share, multiply up to get each quantity, then take each percentage of its own amount and add.

Step 1: Find one share of the ratio

$$3 + 2 = 5$$

$$\frac{200}{5} = 40$$

(Reason: The ratio $3 : 2$ splits the 200 bagels into 5 equal shares, so one share is 40 bagels.)

Step 2: Find how many of each kind are sold

$$3 \times 40 = 120 \text{ plain}$$

$$2 \times 40 = 80 \text{ sesame}$$

(Reason: Plain takes 3 of the shares and sesame takes 2. Check that they rebuild the total: $120 + 80 = 200$.)

Step 3: Work out the money taken on each kind

$$120 \times 1.50 = 180$$

$$80 \times 1.75 = 140$$

(Reason: The two kinds are priced differently, so their takings are worked out separately: £180 from the plain bagels and £140 from the sesame bagels.)

Step 4: Take each percentage of its own amount

$$0.4 \times 180 = 72$$

$$0.6 \times 140 = 84$$

(Reason: Write each percentage as a decimal first: $40\% = 0.4$ and $60\% = 0.6$. The 40% belongs to the plain takings only, and the 60% to the sesame takings only.)

Step 5: Add the two profits

$$72 + 84 = 156$$

(Reason: The profit on the plain bagels and the profit on the sesame bagels together give Alessandro's total profit on all 200 bagels.)

£156

Verification

✓ **Check 1:** Work out the profit on ONE bagel of each kind first. A plain bagel gives $0.4 \times 1.50 = 0.60$ and a sesame bagel gives $0.6 \times 1.75 = 1.05$. Then multiply by how many are sold: $120 \times 0.60 = 72$ and $80 \times 1.05 = 84$. $72 + 84 = 156$, the same total from a completely different order of working

✓ **Check 2:** Count how many bagels' worth of money is profit. 40% of 120 plain bagels is 48 bagels' worth, and 60% of 80 sesame bagels is also 48 bagels' worth. Now price those: $48 \times 1.50 = 72$ and $48 \times 1.75 = 84$. 156 again, reached without ever finding the takings

✓ **Check 3:** A size check. Altogether Alessandro takes $180 + 140 = 320$ pounds. Part of that earns 40% profit and part earns 60%, so the answer must sit strictly between those two percentages of £320. $\dfrac{156}{320} = 0.4875$, which lies between 0.4 and 0.6 as it must

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\dfrac{200}{3+2} = 40$	M1	for a method to find one 'share' of the ratio	✓
$3 \times 40 = 120$ and $2 \times 40 = 80$	M1	for a method to find the number of plain bagels and the number of sesame bagels	✓
$120 \times 1.50 = 180$ and $80 \times 1.75 = 140$	M1	for a method to find the money taken from the plain bagels and from the sesame bagels, or the number of bagels that are entirely profit ($0.4 \times 120 = 48$ and $0.6 \times 80 = 48$), or the profit on a single plain bagel or a single sesame bagel ($0.4 \times 1.50 = 0.60$ or $0.6 \times 1.75 = 1.05$)	✓

Step	Mark	Description	Got it?
$0.4 \times 180 = 72$ and $0.6 \times 140 = 84$	**M1**	for a complete method to find the total profit on the plain bagels and the total profit on the sesame bagels	✓
$72 + 84 = 156$	**A1**	cao. A correct answer scores full marks unless it follows obviously incorrect working. The answer is £156.	✓
Special case	**SC**	award SC B4 for an answer of 164 or 174. An answer of 164 comes from swapping the two percentages over, $0.6 \times 180 + 0.4 \times 140$; an answer of 174 comes from reading the ratio the wrong way round, so 80 plain bagels and 120 sesame ones. This row carries no mark of its own.	✓

Full marks: 5/5

Question 19 - Exam Solution

Understanding the Question

Given

Two mixed numbers, $2\frac{1}{3}$ and $5\frac{1}{4}$, and the first is to be divided by the second.

The result that has to be reached: $\frac{4}{9}$.

Find

A complete line of working that starts at $2\frac{1}{3}$ and $5\frac{1}{4}$ and finishes at $\frac{4}{9}$. This is a show that question, so the working is the answer. Quoting the result on its own, or reaching it only in decimals, earns nothing.

Plan the Solution

- Write each mixed number as an improper fraction first. A mixed number cannot be divided as it stands, because the whole number and the fraction would have to be handled separately.

- Replace the division by a multiplication. Dividing by a fraction is the same as multiplying by its reciprocal, so $\frac{21}{4}$ is turned upside down to give $\frac{4}{21}$, and the first fraction is left alone.
- Multiply the numerators together and the denominators together, then cancel the result down to its lowest terms and confirm it is $\frac{4}{9}$.

Worked Solution [3 marks]

> Dividing by a fraction means multiplying by its reciprocal, so $\frac{a}{b}$ divided by $\frac{c}{d}$ becomes $\frac{a}{b} \times \frac{d}{c}$. Only the second fraction is turned upside down.

Step 1: Write each mixed number as an improper fraction

$$2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$$
$$5\frac{1}{4} = \frac{5 \times 4 + 1}{4} = \frac{21}{4}$$

(Reason: Multiply the whole number by the denominator and add the numerator, keeping the same denominator. There are $2 \times 3 + 1 = 7$ thirds in $2\frac{1}{3}$, and $5 \times 4 + 1 = 21$ quarters in $5\frac{1}{4}$.)

Step 2: Turn the division into a multiplication

$$\frac{7}{3} \times \frac{4}{21}$$

(Reason: The reciprocal of $\frac{21}{4}$ is $\frac{4}{21}$, so dividing by $\frac{21}{4}$ is the same as multiplying by $\frac{4}{21}$. Only the second fraction is inverted; the first one stays exactly as it is.)

Step 3: Multiply the numerators, and multiply the denominators

$$\frac{7}{3} \times \frac{4}{21} = \frac{7 \times 4}{3 \times 21} = \frac{28}{63}$$

(Reason: When two fractions are multiplied, the two numerators give the new numerator and the two denominators give the new denominator. There is no need for a common denominator here, because this is a multiplication and not an addition.)

Step 4: Cancel down to lowest terms

$$\frac{28}{63} = \frac{4}{9}$$

(Reason: Both numbers are multiples of 7, since $28 = 7 \times 4$ and $63 = 7 \times 9$. Dividing the top and the bottom by 7 leaves $\frac{4}{9}$, which is the result the question asked for.)

$$\frac{7}{3} \times \frac{4}{21} = \frac{28}{63} = \frac{4}{9}\text{, as required}$$

Verification

✓ **Check 1:** Multiplying undoes dividing, so multiplying the result by $5\frac{1}{4}$ must give back $2\frac{1}{3}$.
$\frac{4}{9} \times \frac{21}{4} = \frac{84}{36} = \frac{7}{3} = 2\frac{1}{3}$

✓ **Check 2:** The second route in the mark scheme: put both improper fractions over the common denominator 12, after which the division is just one numerator over the other.
$\frac{7}{3} = \frac{28}{12}$ and $\frac{21}{4} = \frac{63}{12}$, giving $\frac{28}{63} = \frac{4}{9}$ again.

✓ **Check 3:** A calculator check in decimals: $2\frac{1}{3}$ is 2.3333 and $5\frac{1}{4}$ is 5.25, each to 4 decimal places. The division gives 0.4444, and $\frac{4}{9}$ is 0.4444 to 4 decimal places, so the two agree.
The mark scheme allows decimals as a check only, never in place of the fraction working.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{7}{3}$ and $\frac{21}{4}$	M1	For both mixed numbers written as improper fractions. A candidate who writes $\frac{4}{21}$ rather than $\frac{21}{4}$ has still earned this mark, because inverting the second fraction is the next step anyway.	✓
$\frac{7}{3} \times \frac{4}{21}$ or equivalent, for example $\frac{49}{21} \times \frac{4}{21}$, or $\frac{28}{12}$ and $\frac{63}{12}$	M1	For the intention to multiply the correct improper fraction by the inverted fraction, or for writing the two fractions over the same common denominator.	✓

Step	Mark	Description	Got it?
$\frac{7}{3} \times \frac{4}{21} = \frac{28}{63} = \frac{4}{9}$ or equivalent, correctly shown	A1	For completing the working correctly to reach the required answer. Working is required, so the printed result copied out on its own scores nothing.	✓
Decimal working	Note	Ignore any decimals used as checking. A decimal answer may sit alongside the fraction working, but it cannot replace it.	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

Amount put in at the start: 5200 euros
Time in the bond: 4 years
Compound interest: 2.5% per year, so each year's interest is worked out on the balance at the start of that year

Find

The amount in the bond after 4 years, correct to the nearest euro

Plan the Solution

- Turn the 2.5% increase into a decimal multiplier.
- Compound interest applies that multiplier once for every year, so raise it to the power 4 rather than multiplying the interest by 4.
- Round to the nearest euro only at the very end, so no accuracy is lost partway through.

Worked Solution [3 marks]

> Rule - Compound growth:
> final amount $=$ starting amount $\times$ (yearly multiplier)^n, where n is the number of years.

Step 1: Write the 2.5% increase as a multiplier

$100\% + 2.5\% = 102.5\%$

$$102.5\% = \frac{102.5}{100} = 1.025$$

(Reason: Adding 2.5% to a balance leaves 102.5% of it, and 102.5% as a decimal is 1.025, so one year of interest is one multiplication.)

Step 2: Apply the multiplier once for each of the 4 years

$$\text{amount} = 5200 \times 1.025 \times 1.025 \times 1.025 \times 1.025$$
$$\text{amount} = 5200 \times 1.025^4$$

(Reason: With compound interest each year's interest is paid on the new balance, so the multiplier acts on the previous year's amount. Four years means four multiplications, which is written 1.025^4.)

Step 3: Work out the value

$$5200 \times 1.025^4 = 5200 \times 1.103812890625 = 5739.82703125$$

(Reason: Keep the full figure the calculator gives here. Rounding partway through a compound interest calculation moves the final amount, because every later year is worked out from it.)

Step 4: Round to the nearest euro

$$5739.82703125 \approx 5740$$

(Reason: The first digit after the decimal point is 8, which is 5 or more, so the whole-euro part rounds up from 5739 to 5740.)

5740 euros

Verification

- ✓ **Check 1:** Build the balance one year at a time instead of using a power, multiplying by 1.025 four times. $5200 \to 5330 \to 5463.25 \to 5599.83125 \to 5739.82703125$
- ✓ **Check 2:** Add the four separate interest payments to the 5200 euros put in. They grow a little each year because the balance grows.
  $130 + 133.25 + 136.58125 + 139.99578125 = 539.82703125$, and
  $5200 + 539.82703125 = 5739.82703125$
- ✓ **Check 3:** Compare with simple interest, where the same 130 euros would be paid every year. Compound interest must come out a little higher, but not wildly higher.
  $5200 + 4 \times 130 = 5720$, and $5720 < 5739.83$, so the compound amount is above it by under 20 euros.
- ✓ **Check 4:** Work backwards: undo the four years of growth and the starting amount should reappear. $\dfrac{5739.82703125}{1.025^4} = 5200$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A method to find 2.5% of 5200, or 102.5% of 5200: $5200 \times 1.025\ (= 5330)$ or $5200 \times 0.025\ (= 130)$	**M1**	One year's growth, by either the multiplier or the interest itself. $(1 + 0.025)$ counts as 1.025.	✓
A complete method for all four years: $5330 \times 1.025\ (= 5463.25)$ and $5463.25 \times 1.025\ (= 5599.83\ldots)$ and $5599.83\ldots \times 1.025\ (= 5739.8\ldots)$	**M1**	The scheme quotes each yearly amount, so this follows through on the candidate's own earlier figures provided the method is a complete four-year one.	✓
Alternative to the two method marks above, in one line: 5200×1.025^4, or $5200 \times 1.025^5\ (= 5883\ldots)$	**M2**	The single-power method earns both method marks at once. The 1.025^5 version still shows compound growth, so it earns the method marks even though counting 5 years loses the accuracy mark.	✓
5740	**A1**	Accept anything from 5739 to 5740, which covers a candidate who truncates $5739.82\ldots$ instead of rounding it. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Special case, if no other marks are awarded: $5200 \times 0.1\ (= 520)$, $5200 \times 1.1\ (= 5720)$, $5200 \times 0.9\ (= 4680)$, $5200 \times 0.975\ (= 5070)$ or $5200 \times 0.975^4\ (= 4699\ldots)$	**B1**	Each names one specific error. The 0.1 and 1.1 lines come from treating 4 years at 2.5% as a single 10%; the 0.9, 0.975 and 0.975^4 lines come from taking the interest off instead of adding it on.	✓
Accept $(1 + 0.025)$ as equivalent to 1.025 throughout, but do not accept $(1 + 2.5\%)$.	**note**	$(1 + 2.5\%)$ mixes a number with a percentage, so it is not a multiplier and earns nothing on its own.	✓

Full marks: 3/3

Question 21 - Exam Solution

Understanding the Question

Given

A solid cylinder of radius 8 cm and height h cm.
Its volume is 1208 cm³, so the volume is known and the height is not.
The wood has density 1.25 g/cm³, which means every cm³ of it has mass 1.25 g.

Find

(a) The height h, correct to the nearest whole number. (b) The mass of the cylinder, in kilograms.

Plan the Solution

- Put the radius and the volume into $V = \pi r^2 h$ and solve the equation for h.
- Keep the unrounded height on the calculator and round only at the very end of part (a), so no accuracy is lost partway through.
- For part (b) use $\text{mass} = \text{density} \times \text{volume}$. The density is in grams per cm³, so the mass arrives in grams and is changed to kilograms last.
- Part (b) does not need the answer to part (a). The volume is already given as 1208 cm³, so use that rather than a volume rebuilt from a rounded height.

Worked Solution [4 marks]

> Rule - Cylinder and density: $V = \pi r^2 h$, and $\text{mass} = \text{density} \times \text{volume}$.

Step 1: Put the radius and the volume into the cylinder formula

$$\pi \times 8^2 \times h = 1208$$

(Reason: The radius is 8 cm, so the circular face has area $\pi \times 8^2$ cm², and the volume of a cylinder is that area multiplied by the height. Writing the given volume on the other side turns the formula into an equation in h.)

Step 2: Work out the area of the circular face

$$8^2 = 64$$

$$\pi \times 64 = 201.0619\ldots$$

(Reason: Squaring the radius first keeps the working in one place: the face has area 64π cm², which is about 201 cm².)

Step 3: Rearrange to make h the subject

$$h = \frac{1208}{\pi \times 8^2}$$

(Reason: The area of the circular face multiplies the height, so dividing both sides by that area leaves the height on its own.)

Step 4: Work out the height

$$h = \frac{1208}{201.0619\ldots} = 6.0081\ldots$$

(Reason: Use the calculator's own π key rather than a rounded value. Rounding the $201.0619\ldots$ before dividing moves the last figures of the height.)

Step 5: Round to the nearest whole number

$$6.0081\ldots \approx 6$$

(Reason: The first digit after the decimal point is 0, which is less than 5, so the whole-number part stays at 6.)

Step 6: Work out the mass of the cylinder in grams

$$1.25 \times 1208 = 1510$$

(Reason: Every cm³ of the wood has mass 1.25 g and there are 1208 cm³ of it, so the mass is 1.25 g taken 1208 times. The volume used is the one the question gives, not one rebuilt from the rounded height.)

Step 7: Change the mass from grams to kilograms

$$\frac{1510}{1000} = 1.51$$

(Reason: There are 1000 g in 1 kg, so the number of kilograms is the number of grams divided by 1000. The question asks for kilograms, so the grams answer on its own is not finished.)

(a) $h = 6$

(b) 1.51 kilograms

Verification

- ✓ **Check 1:** Put the unrounded height back into the volume formula. The volume the question gives must reappear. $\pi \times 8^2 \times 6.0081\ldots = 1208$ cm³
- ✓ **Check 2:** Divide the mass in grams by the volume. Density is mass per unit volume, so the density given in the question must come back. $\frac{1510}{1208} = 1.25$ g/cm³
- ✓ **Check 3:** Convert the kilograms back to grams, which undoes the last step. $1.51 \times 1000 = 1510$ g

✓ **Check 4:** Is the size sensible? A height of exactly 6 cm on a face of about 201 cm² would give a volume a little below the one stated, so the true height must be a little above 6. $6 \times 201 = 1206$, just under 1208, so a height slightly above 6 is right and it still rounds to 6.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) An equation in h built from the volume of the cylinder, $\pi \times 8^2 \times h = 1208$, or a correct calculation for h, $\dfrac{1208}{\pi \times 8^2}$	**M1**	The method may be seen in stages: $\pi \times 8^2 = 201.06\ldots$ followed by a division by that figure earns it just as the single line does. Using the diameter, 16, instead of the radius does not.	✓
6	**A1**	Accept anything from 6 to 6.02. The band exists because a candidate working with a rounded π lands a little above the true height: $\dfrac{1208}{3.14 \times 64} = 6.011\ldots$. A correct answer scores both marks unless it comes from obviously incorrect working.	✓
(b) An equation built from density as mass per unit volume, $\dfrac{m}{1208} = 1.25$, or a calculation for the mass, $1208 \times 1.25\ (= 1510)$	**M1**	Any correct method for the mass earns this, including converting the density to 0.00125 kg/cm³ first and multiplying by 1208, which reaches the kilograms in one line.	✓
1.51	**A1**	The answer must be in kilograms. 1510 left in grams shows the method but not the conversion, so it scores the method mark alone. A correct answer scores both marks unless it comes from obviously incorrect working.	✓
Part (b) is marked from the volume the question gives, 1208 cm³, not from a volume rebuilt out of a rounded height.	**note**	Rebuilding it from $h = 6$ gives 1206.37… cm³ and a mass of 1507.96… g. That still rounds to 1.51 kg here, so it is not penalised, but the given volume is the one the mark scheme's own calculation uses.	✓

Full marks: 4/4

Question 22 - Exam Solution

Understanding the Question

Given

(a) $\frac{g^9}{g^2}$ - one power of g divided by another power of the same letter.
(b) $5k^2(k^3+4)$ - a single term standing outside a bracket.
(c) $x^2-2x-63$ - a quadratic whose x^2 coefficient is 1.
(d) $7-2y<3y-12$ - a linear inequality with y on both sides.

Find

(a) The quotient written as one power of g. (b) The expansion, as two terms. (c) (i) The two brackets, and (ii) the solutions that follow from them. A quadratic has two, so expect two values of x. (d) The values of y that satisfy the inequality, given as an inequality.

Plan the Solution

- (a) Divide powers of one base by subtracting the indices.
- (b) Multiply each term inside the bracket by $5k^2$, adding indices wherever two powers of k meet.
- (c) (i) Hunt for two integers with product -63 and sum -2. (ii) A product is zero only when one factor is zero, so read a solution off each bracket.
- (d) Move the y terms to the side that leaves a positive coefficient, then divide. Doing it that way means the inequality sign never has to be reversed.

Worked Solution [9 marks]

> Index laws: $\frac{a^m}{a^n}=a^{m-n}$ and $a^m\times a^n=a^{m+n}$. For x^2+bx+c, two integers with product c and sum b give the brackets. An inequality is solved exactly like an equation, except that multiplying or dividing both sides by a negative number reverses the sign.

Step 1: subtract the indices in $\frac{g^9}{g^2}$

$$\frac{g^9}{g^2}=g^{9-2}$$

$= g^7$

(Reason: Dividing powers of the same base subtracts the indices, $\frac{a^m}{a^n} = a^{m-n}$, because 2 of the 9 factors of g cancel and 7 survive.)

Step 2: multiply k^3 by $5k^2$

$5k^2 \times k^3 = 5k^5$

(Reason: Multiplying powers of the same base combines the two indices 2 and 3 into a single index, and the 5 is simply carried along.)

Step 3: multiply 4 by $5k^2$, then write both terms

$5k^2 \times 4 = 20k^2$

$5k^2(k^3 + 4) = 5k^5 + 20k^2$

(Reason: The 4 carries no k, so only the numbers multiply, $5 \times 4 = 20$, and the k^2 is unchanged. Each term inside the bracket gets multiplied, so there are two terms in the answer.)

Step 4: find two integers with the right product and sum

$7 \times (-9) = -63$

$7 + (-9) = -2$

(Reason: For $x^2 + bx + c$ with $b = -2$ and $c = -63$, the pair must multiply to c and add to b. A negative product means one integer is negative, and the negative one is the larger in size because the sum is negative.)

Step 5: write the factorised form

$x^2 - 2x - 63 = (x + 7)(x - 9)$

(Reason: The coefficient of x^2 is 1, so each bracket starts with x, and one integer of the pair goes into each bracket.)

Step 6: set each bracket equal to zero

$(x + 7)(x - 9) = 0$

$x + 7 = 0 \quad \text{or} \quad x - 9 = 0$

$x = -7 \quad \text{or} \quad x = 9$

(Reason: A product of two numbers is zero only when at least one of them is zero, so each bracket gives one solution. This is why part (i) makes part (ii) short.)

Step 7: collect the y terms on the right

$7 - 2y < 3y - 12$

$7 + 12 < 3y + 2y$

$19 < 5y$

(Reason: Add $2y$ to both sides and add 12 to both sides. Adding never changes the direction of the sign, and gathering the y terms on the right leaves $5y$, a positive coefficient.)

Step 8: divide both sides by 5

$$\frac{19}{5} < y$$

$$y > \frac{19}{5} = 3.8$$

(Reason: Dividing by the positive number 5 leaves the sign as it is. Reading the statement from the other end turns $\frac{19}{5} < y$ into $y > \frac{19}{5}$, which is the form the answer line asks for.)

(a) g^7

(b) $5k^5 + 20k^2$

(c) (i) $(x + 7)(x - 9)$

(c) (ii) $x = -7$ or $x = 9$

(d) $y > \frac{19}{5}$, that is $y > 3.8$

Verification

✓ **Check 1:** Part (a) with a number in place of the letter. Put $g = 2$, so that $g^9 = 512$ and $g^2 = 4$. $\frac{512}{4} = 128$ and $2^7 = 128$

✓ **Check 2:** Part (b) evaluated both ways. Put $k = 2$ into the bracketed form and into the expansion. $5 \times 4 \times (8 + 4) = 240$ and $5 \times 32 + 20 \times 4 = 240$

✓ **Check 3:** Part (c) both ways round. Multiply the brackets back out, then put each solution into the original quadratic. $(x + 7)(x - 9) = x^2 - 9x + 7x - 63 = x^2 - 2x - 63$, and $9^2 - 2(9) - 63 = 0$, $(-7)^2 - 2(-7) - 63 = 0$

✓ **Check 4:** Part (d) tested on either side of 3.8. Substitute $y = 4$ and then $y = 3$ into the original inequality. At $y = 4$ it reads $-1 < 0$, which is true. At $y = 3$ it reads $1 < -3$, which is false.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) g^7	**B1**	The quotient written as one power. No working is needed.	✓

Step	Mark	Description	Got it?
(b) $5k^5 + 20k^2$	**B2**	Both terms correct. Award B1 only for $5k^5$ or for $20k^2$ alone.	✓
(c) (i) $(x \pm 7)(x \pm 9)$	**M1**	For $(x \pm 7)(x \pm 9)$, or for $(x + a)(x + b)$ where $ab = -63$ or $a + b = -2$, with a and b integers.	✓
(c) (i) $(x + 7)(x - 9)$	**A1**	For the correct factors. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
(c) (ii) $-7,\ 9$	**B1**	Follow through from (c)(i), dependent on factorising in the form $(x + p)(x + q)$ where p and q are integers.	✓
(d) $7 + 12 < 3y + 2y$	**M1**	For a rearrangement with the y terms on one side and the numerical terms on the other in a correct inequality, or for the correct simplification of the y terms or of the numbers on one side in a correct inequality. The sign may be $=$ or the incorrect inequality sign.	✓
(d) $19 < 5y$ or $-5y < -19$	**M1**	For the correct simplification of the y terms on one side and the numbers on the other in a correct inequality, or a correct inequality with the wrong sign. The sign may be $=$ or the incorrect inequality sign. Accept $y = \frac{19}{5}$ or equivalent here.	✓
(d) $y > \frac{19}{5}$	**A1**	Or equivalent, for example $y > 3.8$ or $3.8 < y$. It must be given as the correct inequality on the answer line. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 9/9

Question 23 - Exam Solution

Understanding the Question

Given

Find

Trapezium $ABCD$, with AB parallel to DC.
$AD = 15$ cm and $DC = 14$ cm, with right angles at A and at D.
The area of the trapezium is 360 cm².

The perimeter of the trapezium. The top side AB and the sloping side CB are both unknown, so the area has to give one before the other can be found.

Plan the Solution

- Only AB is missing from the area formula, so put the area of 360 into it and solve for AB.
- Drop a perpendicular from C onto AB, meeting it at M. That splits the trapezium into a rectangle and a right-angled triangle.
- The triangle has short sides 15 and MB, so Pythagoras' theorem gives the sloping side CB.
- Add the four sides for the perimeter.

Worked Solution [6 marks]

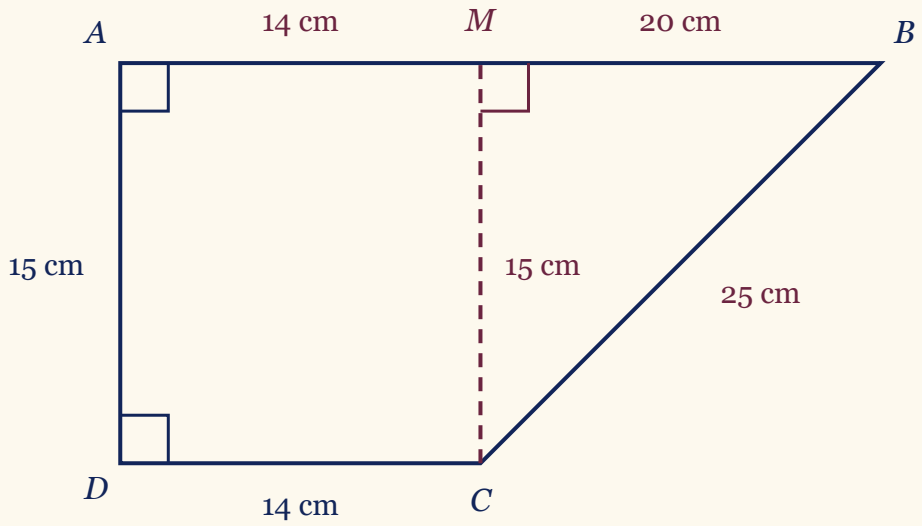
> Area of a trapezium: half the sum of the two parallel sides, times the distance between them, $\frac{a+b}{2} \times h$. Pythagoras' theorem in a right-angled triangle: $c^2 = a^2 + b^2$, where c is the hypotenuse.

Step 1: use the area to find AB

$$\frac{AB + 14}{2} \times 15 = 360$$

$$AB + 14 = \frac{2 \times 360}{15} = 48$$

$$AB = 48 - 14 = 34$$



(Reason: The parallel sides are AB and DC, and AD is the distance between them because it is perpendicular to both. Multiplying by 2 and dividing by 15 undoes the formula.)

Step 2: drop a perpendicular from C onto AB

$$AM = DC = 14$$
$$CM = AD = 15$$
$$MB = AB - AM$$
$$34 - 14 = 20$$

(Reason: $AMCD$ has four right angles, so it is a rectangle and its opposite sides are equal. What is left of the top side is the base of the right-angled triangle MCB.)

Step 3: Pythagoras' theorem in triangle MCB

$$CB^2 = CM^2 + MB^2$$
$$15^2 + 20^2 = 225 + 400 = 625$$
$$CB = \sqrt{625} = 25$$

(Reason: Angle CMB is a right angle, so CB is the hypotenuse. Squaring the two short sides and adding gives 625, and the square root of that is the length itself.)

Step 4: add the four sides

$$AD + DC + CB + BA$$
$$15 + 14 + 25 + 34 = 88$$

(Reason: The perimeter is the distance all the way round: the two lengths given in the question, the sloping side from Step 3, and the whole top side from Step 1.)

88 cm

Verification

✓ **Check 1:** Put $AB = 34$ back into the area formula: half of $34 + 14$, times the height 15. $\frac{48}{2} \times 15 = 360$, which is the area the question gives.

✓ **Check 2:** The right-angled triangle has short sides 15 and 20, and those are 5 times 3 and 5 times 4, so it is a scaled up $3, 4, 5$ triangle. $5 \times 5 = 25$, the same sloping side that Pythagoras' theorem gave.

✓ **Check 3:** Rebuild the area a different way: rectangle $AMCD$ is 15×14 and triangle MCB is half of 15×20. $210 + 150 = 360$, so the split of the top side into 14 and 20 is right.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{14 + AB}{2} \times 15 = 360$ oe, or $360 - 14 \times 15 = 150$ oe	**M1**	For setting up an equation using the area of the trapezium, or for a method to find the area of the triangle.	✓
$AB = 34$ or $MB = 20$, where M is the point on AB for which MC is perpendicular to AB	**A1**	Could be seen on the diagram.	✓
$(CB^2 =)\ 15^2 + 20^2\ (= 625)$, or $(CB^2 =)\ 15^2 + MB^2$	**M1**	Allow use of their MB.	✓
$(CB =)\ \sqrt{15^2 + 20^2}\ (= 25)$, or $(CB =)\ \sqrt{15^2 + MB^2}$	**M1**	Allow use of their MB.	✓
$14 + 15 + 34 + 25$ oe, or $14 + 15 + 14 + MB + CB$ oe	**M1ft**	Dependent on the previous two method marks. For a method to find the perimeter of the trapezium, allowing use of their MB and their CB.	✓
88	**A1**	cao. A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 6/6

Question 24 - Exam Solution

Understanding the Question

Given

A straight line L drawn on a square grid
The grid runs from -4 to 4 across and from -2 to 4 up, one square to each unit

Find

An equation of L written in the form $y = mx + c$

Plan the Solution

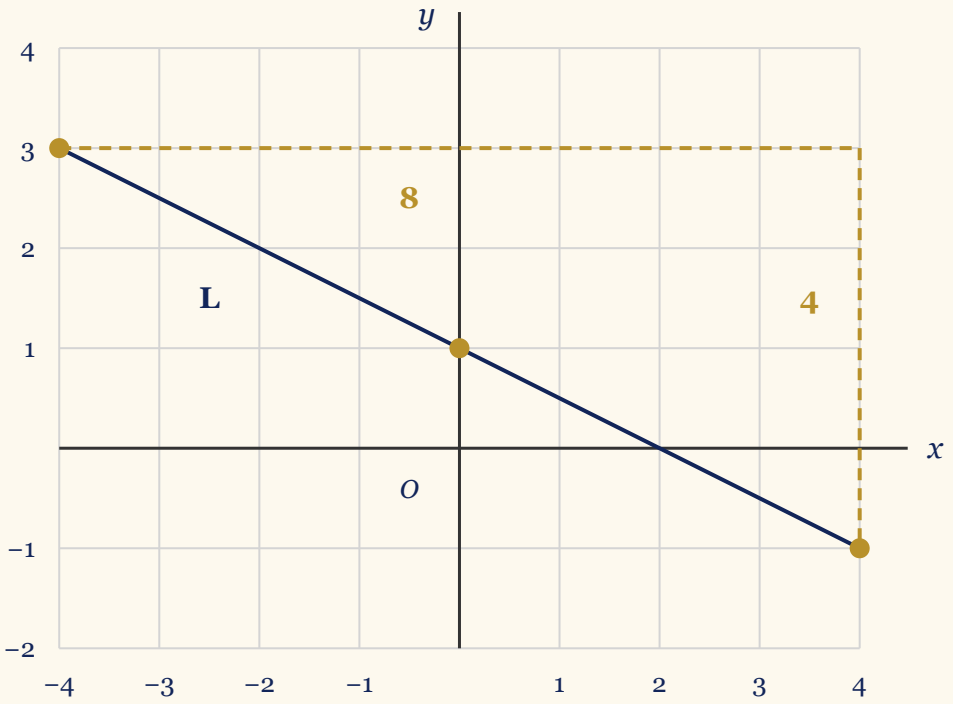
- Pick two points where L passes exactly through the corner of a grid square, so both coordinates can be read without estimating.
- Work out the gradient m from those two points.
- Read c off the graph where the line crosses the y-axis.
- Put m and c into $y = mx + c$.

Worked Solution [3 marks]

> Gradient and intercept: for a straight line, $m = \dfrac{\text{change in } y}{\text{change in } x}$, and c is the y-value where the line crosses the y-axis.

Step 1: read two points off the line

$(-4, 3)$ and $(4, -1)$



(Reason: The line passes exactly through the corner of a grid square at each of these two points, so both coordinates can be read off the grid instead of estimated. Any two exact points on the line will do.)

Step 2: work out the gradient m

$$\frac{-1-3}{4-(-4)} = \frac{-4}{8} = -\frac{1}{2}$$

(Reason: Going from $(-4, 3)$ to $(4, -1)$ the line moves 8 squares to the right and 4 squares down, so the change in y goes on top of the change in x and the gradient is negative.)

Step 3: read the value of c

$$c = 1$$

(Reason: The line crosses the y-axis at $(0, 1)$, and in $y = mx + c$ the number c is that crossing value.)

Step 4: write the equation in the form $y = mx + c$

$$y = -\frac{1}{2}x + 1$$

(Reason: Putting $m = -\frac{1}{2}$ and $c = 1$ into $y = mx + c$ gives the equation. The question asks for this exact form, so leave it written this way.)

$$y = -\frac{1}{2}x + 1$$

Verification

✓ **Check 1:** Put $x = 4$ into the answer. It should give back the second point that was read off the grid, $(4, -1)$. $y = -\frac{1}{2} \times 4 + 1 = -1$

✓ **Check 2:** A different point again: the line crosses the x-axis at $(2, 0)$, so putting $x = 2$ in must give zero. $y = -\frac{1}{2} \times 2 + 1 = 0$

✓ **Check 3:** Count squares on the grid. A gradient of $-\frac{1}{2}$ means the line falls one square for every two squares it travels to the right. From $(0, 1)$ going 2 right and 1 down reaches $(2, 0)$, which is on the line.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The gradient on its own, or one of the two numbers in the equation	**B1**	for $m = -\frac{1}{2}$ or gradient $= -\frac{1}{2}$ oe, eg $\frac{3-(-1)}{-4-4}$, or for $-\frac{1}{2}x + c$, or for $mx + 1$ where m is not zero	✓
Both numbers right, or a correct equation written the wrong way round	**B2**	for $-\frac{1}{2}x + 1$ without the $y =$, or for $y = -\frac{1}{2}x + c$, or for $y = mx + 1$ where m is not zero, or for a correct equation in the wrong form, eg $2y + x = 2$	✓
The equation of L in the form the question asks for	**B3**	for $y = -\frac{1}{2}x + 1$ oe, eg $y = -0.5x + 1$	✓
Note on the form of the answer	**note**	A correct equation written any other way, such as $2y + x = 2$, earns B2 and not B3, because the question asks for the $y = mx + c$ form. Reading the gradient upside down, as -2 instead of $-\frac{1}{2}$, scores no marks for the gradient.	✓

Full marks: 3/3

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