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Edexcel IGCSE 4MA1

4MA1/1H (Higher Tier) – Thursday 16 May 2024

100 marks · 2 hours · Calculator allowed

by Sir Faraz Hassan · IGCSE / GCSE Maths Specialist

Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/1H (Higher Tier), June 2024 series, sat Thursday 16 May 2024 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. The first four terms of an arithmetic sequence are shown below.

$1 \qquad 4 \qquad 7 \qquad 10$

(a) Work out an expression, in terms of n, for the nth term of this sequence. *[2 marks]*

A different arithmetic sequence has nth term $5n + 17$

(b) Work out the 12th term of this sequence. *[1 mark]*

(a) ....

(b) ....

[Total 3 marks]

2. 450 workers at a shipping depot were asked how they travelled to work on Tuesday.
Each worker walked or travelled by tram or travelled by taxi or travelled by bicycle.
Each worker used just one method of travel.

One of these workers is chosen at random.
The table shows information about the probability of each method of travel.

Method of travel	walk	tram	taxi	bicycle
Probability	0.20	x	$2x$	0.26

Work out how many of the 450 workers travelled by taxi. *[4 marks]*

....

[Total 4 marks]

3. Work out the highest common factor (HCF) of 72 and 108
You must show your working clearly. *[2 marks]*

....

[Total 2 marks]

4. Rosa records the number of kilometres her delivery van travels each month.

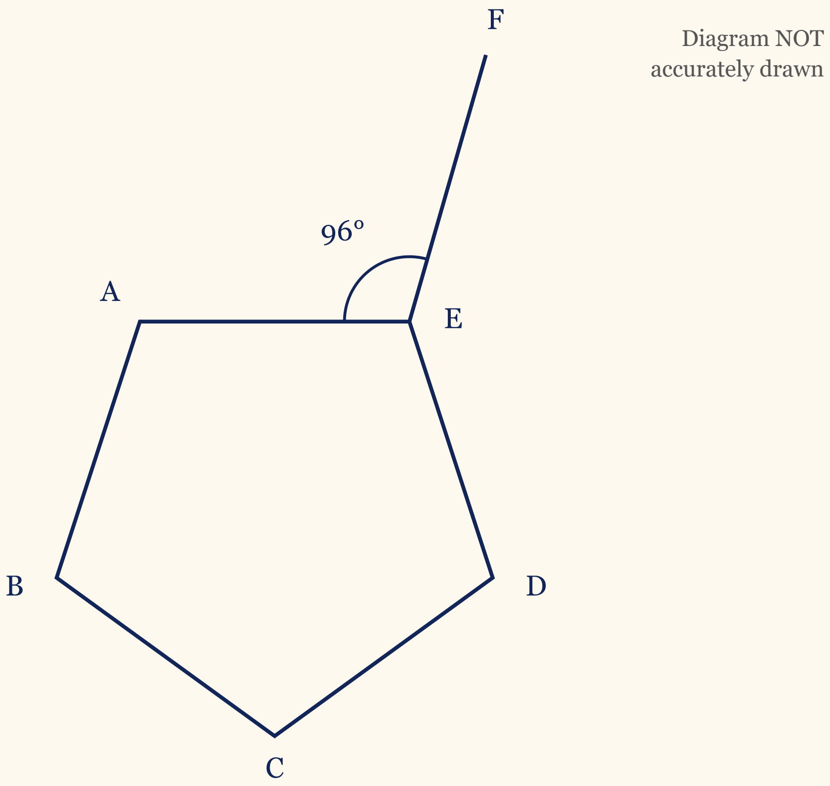
In April, the van travelled 943 kilometres.
This is 15% more than the number of kilometres the van travelled in March.

Work out the number of kilometres the van travelled in March. *[3 marks]*

.... kilometres

[Total 3 marks]

5. The diagram shows a regular pentagon $ABCDE$.
A straight line is drawn from the vertex E to the point F, as shown.



Angle $AEF = 96°$

Work out the size of the obtuse angle FED.
Show your working clearly. *[4 marks]*

.... °

[Total 4 marks]

6. (a) Multiply out the brackets and simplify
$(m + 5)(m - 8)$ *[2 marks]*

(b) Solve the equation

$$3n - 4 = \frac{5n + 6}{3}$$

You must show clear algebraic working. *[3 marks]*

(a) ....

(b) n = ...

[Total 5 marks]

7. $\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$
$A = \{\text{even numbers}\}$
$B = \{23, 29, 31\}$
$C = \{\text{multiples of } 3\}$

Yes	No
☐	☐

(a) Write down all the members of the set
(i) $B \cup C$
[1 mark]
(ii) $A' \cap C$ *[1 mark]*

(b) Is it true that $B \cap C = \varnothing$?
Tick one box below.
Then write down a reason for your answer. *[1 mark]*

The set D has 4 members, and $D \cap (A \cup C) = \varnothing$

(c) Write down all the members of set D *[2 marks]*

(a)(i) ....

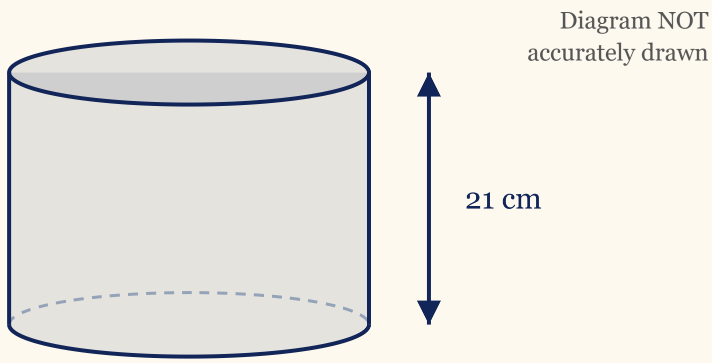
(a)(ii) ....

(b) ....

(c) ....

[Total 5 marks]

8. A cylindrical metal drum is stood upright on a workbench.



The volume of the drum is 1575 cm³.
The force the drum exerts on the workbench is 84 newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure on the workbench due to the drum. *[3 marks]*

.... newtons/cm²

[Total 3 marks]

9. The table gives the amount of maize produced by each of two countries in 2020

Country	Amount of maize (tonnes)
Ukraine	3.5×10^7
Portugal	8.2×10^5

(a) Write 3.5×10^7 as an ordinary number. *[1 mark]*

In 2020, Romania produced 6 780 000 more tonnes of maize than Portugal.

(b) Work out the amount of maize Romania produced in 2020
Give your answer in standard form. *[2 marks]*

(a) ....

(b) tonnes ....

[Total 3 marks]

10. (a) Simplify $(2p)^0$, given that $p > 0$. *[1 mark]*

$y^9 \times y^{-3} = y^n$

(b) Work out the value of n. *[1 mark]*

(c) Simplify fully $(5a^4c^2)^3$. *[2 marks]*

(a) ....

(b) n = ...

(c) ....

[Total 4 marks]

11. The diagram shows a wooden frame that supports a large advertising sign.

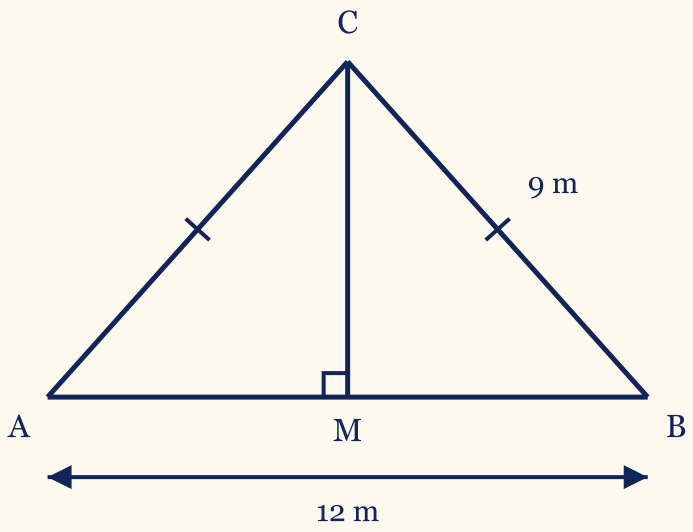


Diagram NOT accurately drawn

The frame is made from four lengths of timber, AB, AC, BC and MC
$AC = BC = 9$ m $AB = 12$ m
angle $AMC = 90°$

Martin is going to buy lengths of timber to make the frame.

The timber costs 21.50 euros per metre.
Each length of timber he buys has to be a whole number of metres.

Work out the total cost of the timber Martin needs to buy.
Show your working clearly. *[4 marks]*

.... euros

[Total 4 marks]

12. (a) Factorise fully $6y^2 - 5y - 4$ *[2 marks]*

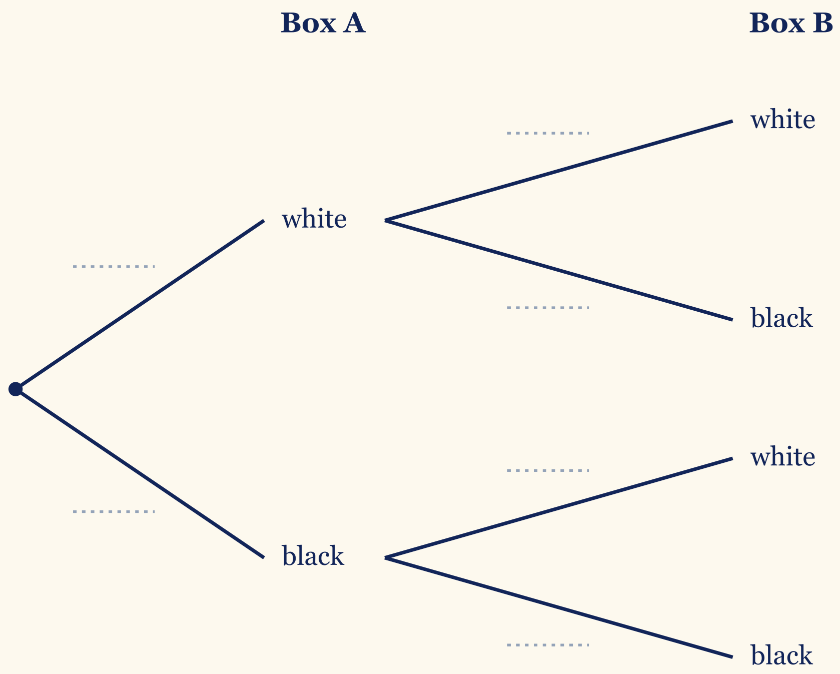
(b) Express $\dfrac{2x+1}{4x} + \dfrac{7-5x}{3x}$ as a single fraction in its simplest form. *[3 marks]*

(a) ....

(b) ....

[Total 5 marks]

13. Ravinder has two boxes of buttons.



In box A, there are 3 white buttons and 7 black buttons.
In box B, there are 5 white buttons and 4 black buttons.

Ravinder takes at random a button from box A and a button from box B

(a) Complete the probability tree diagram. *[2 marks]*

(b) Work out the probability that Ravinder takes two buttons of the same colour. *[3 marks]*

(b) ....

[Total 5 marks]

14. Owen, Bilal and Devika each raised money for a village library appeal.

The combined amount raised by Owen and Bilal is $15\,435$ dirhams.

The amount raised by Bilal is 45% more than the amount raised by Owen.

The amount raised by Bilal is $\frac{3}{2}$ times the amount raised by Devika.

Work out the amount raised by Devika. *[5 marks]*

.... dirhams

[Total 5 marks]

15. The function f is defined as

$$\mathrm{f} : x \mapsto \frac{3x+1}{x-2}$$

(a) Write down the value of x that cannot belong to any domain of the function f
[1 mark]
(b) Find the inverse function f^{-1}, giving your answer in the form $\mathrm{f}^{-1}(x) = \ldots$ *[3 marks]*

(a) ....

(b) ....

[Total 4 marks]

16. There are 20 beads in a jar.
15 of the beads are green
5 of the beads are white

Owen takes at random 3 beads from the jar.

Work out the probability that Owen takes at least one bead of each colour from the jar. *[4 marks]*

....

[Total 4 marks]

17. Show that $\dfrac{1+\sqrt{5}}{3-\sqrt{5}}$ can be written in the form $a+\sqrt{b}$, where a and b are integers.
You must show each stage of your working clearly. *[3 marks]*

[Total 3 marks]

18. The curve C has equation $y = x^3 - 40x + 1$
Work out the coordinates of the two points on C at which the gradient of the curve is 8 *[5 marks]*

First point ....

Second point ....

[Total 5 marks]

19. $ABCD$ is a quadrilateral.

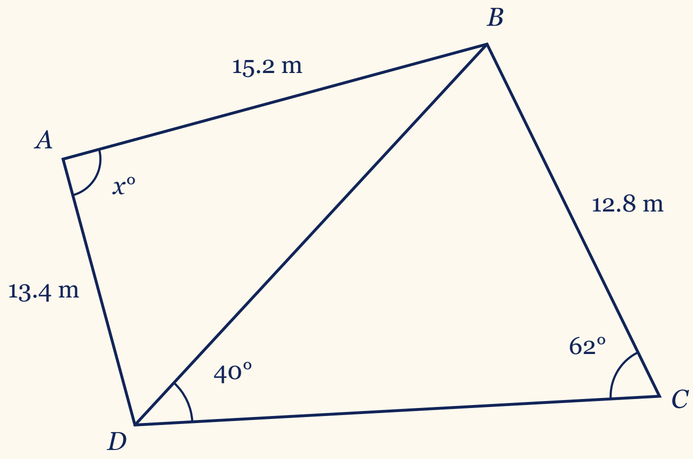


Diagram NOT accurately drawn

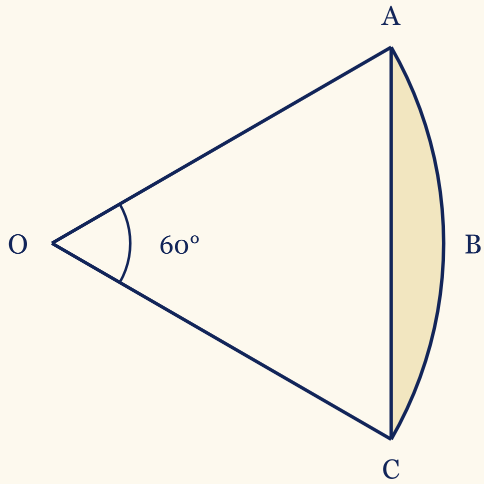
Work out the value of x.

Give your answer correct to 3 significant figures. *[5 marks]*

x = ...

[Total 5 marks]

20. In the diagram, $OABC$ is a sector of a circle with centre O.



Angle $AOC = 60°$

The shaded segment ABC has an area of 38 cm²

Work out the perimeter of the shaded segment ABC.

Give your answer correct to one decimal place. *[4 marks]*

.... cm

[Total 4 marks]

21. A curve has equation $y = \mathrm{f}(x)$.

The curve has exactly one minimum point, and the coordinates of that minimum point are $(5, -4)$.

Write down the coordinates of the minimum point on the curve with equation

(i) $y = \mathrm{f}(x + 7)$ *[1 mark]*

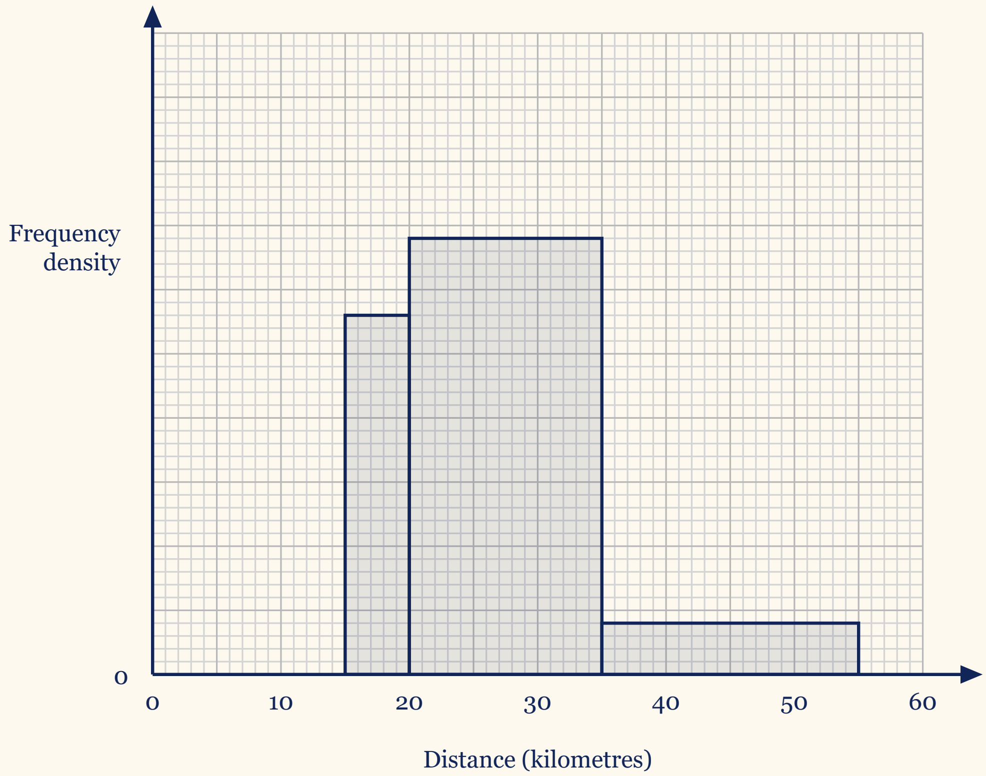
(ii) $y = \mathrm{f}(x) - 6$ *[1 mark]*

(i) ....

(ii) ....

[Total 2 marks]

22. The incomplete histogram gives information about the distances, in kilometres, cycled by 100 members of a cycling club last Saturday.



Every member cycled at least 5 kilometres.
No member cycled more than 55 kilometres.
14 members cycled between 15 kilometres and 20 kilometres.

Complete the histogram. *[3 marks]*

[Total 3 marks]

23. An ornament is made by removing a hemisphere, shown shaded, from a solid cone, as shown in the diagram.

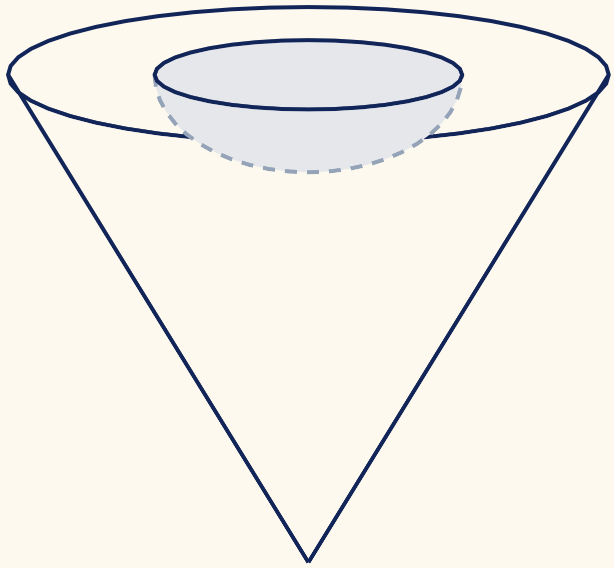


Diagram NOT accurately drawn

The radius of the hemisphere is $2x$ cm
The radius of the base of the cone is $5x$ cm
The vertical height of the cone is $6x$ cm

The volume of the ornament is 6948π cm³

Work out the total surface area of the solid hemisphere that has been removed from the cone.
Give your answer correct to the nearest integer. *[5 marks]*

.... cm²

[Total 5 marks]

24. A polygon has n sides, where $n > 5$
Written in order of size, the interior angles of this polygon form an arithmetic sequence.
The first term of the sequence is $84°$
The common difference of the sequence is $4°$

Find the total of all the interior angles of the polygon.
You must show clear algebraic working. *[6 marks]*

.... °

[Total 6 marks]

25. The function f is given by
$f(x) = 17 - 3x^2 + 12x$

Express $f(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants.
[4 marks]

f(x) = ...

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

An arithmetic sequence whose first four terms are $1, 4, 7, 10$.
A second, different arithmetic sequence, given by its nth term $5n + 17$.

Find

(a) An expression in terms of n for the n th term of the first sequence. (b) The 12th term of the second sequence.

Plan the Solution

- Subtract each term from the next to find the common difference d of the first sequence.
- Put a and d into the standard formula and simplify it to the form $dn + c$.
- Part (b) already gives a formula, so nothing has to be built there: substitute the position number into it.

Worked Solution [3 marks]

> Rule - nth term of an arithmetic sequence: $a + (n-1)d$, where a is the first term and d is the common difference. Expanded, this is always $dn + (a - d)$.

Step 1: find the common difference

$4 - 1 = 3$
$7 - 4 = 3$
$10 - 7 = 3$

(Reason: (Reason: the same gap of 3 appears every time, so the sequence really is arithmetic with $d = 3$.))

Step 2: put $a = 1$ and $d = 3$ into the formula

$a + (n-1)d = 1 + (n-1) \times 3$
$= 1 + 3n - 3$
$= 3n - 2$

(Reason: (Reason: the $3n$ comes from the common difference, and the -2 is the adjustment that pulls $3n$ back onto the first term.))

Step 3 (part b): substitute $n = 12$ into $5n + 17$

$5 \times 12 + 17 = 60 + 17 = 77$

(Reason: (Reason: the 12th term is the term in position 12, so the position number is what goes in place of n in the formula the question hands you.))

(a) $3n - 2$

(b) 77

Verification

✓ **Check 1:** Put $n = 1$ and $n = 4$ into $3n - 2$ and compare with the first and fourth terms printed in the question. $3 \times 1 - 2 = 1$ and $3 \times 4 - 2 = 10$, which are exactly the first and fourth terms printed.

✓ **Check 2:** Reach part (b) a different way. The first term of that sequence is $5 \times 1 + 17$, and it goes up in 5s, so the 12th term is 11 steps of 5 after it.
$22 + 11 \times 5 = 22 + 55 = 77$

✓ **Check 3:** Test the shape of the part (a) answer rather than its values: consecutive terms of $3n - 2$ must differ by the common difference. $(3(n+1) - 2) - (3n - 2) = 3$, the common difference found in step 1.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A correct method for the nth term: $3n + k$ where $k \neq -2$, or $3 \times n + k$, or $n \times 3 + k$.	M1	The multiplier of n must be 3; the constant k may be zero or absent.	✓
(a) $3n - 2$	A1	oe, eg $1 + (n-1)3$ or $3 \times n - 2$ or $n \times 3 - 2$ or $3x - 2$. Working is not required, so a correct answer scores full marks unless it clearly comes from incorrect working. Allow T_n, U_n or a_n for the nth term, but only M1 for $n = 3n - 2$ oe or $x = 3x - 2$.	✓

Step	Mark	Description	Got it?
(b) 77	**B1**	cao - correct answer only. The formula is given, so the single mark is for the value 77 and no method is required.	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

450 workers, each using exactly one of four methods of travel.
The probabilities are walk 0.20, tram x, taxi $2x$, bicycle 0.26.
The taxi probability is exactly twice the tram probability.

Find

How many of the 450 workers travelled by taxi.

Plan the Solution

- Add the four probabilities and set the total equal to 1.
- Subtract the two known probabilities to see what $x + 2x$ is worth.
- Divide by 3 to get x, then double it for the taxi probability.
- Multiply the taxi probability by 450.

Worked Solution [4 marks]

> Rule - Probability: for one set of outcomes that covers every case with no overlap, the probabilities add to 1, and the expected number in a group is $\text{probability} \times \text{total}$.

Step 1: Set the four probabilities equal to 1

$$0.20 + x + 2x + 0.26 = 1$$

(Reason: Every worker used just one of the four methods, so the four outcomes cover all 450 workers exactly once and no worker twice. Probabilities like that always add to 1.)

Step 2: Take out the two probabilities that are known

$$0.20 + 0.26 = 0.46$$

$3x = 1 - 0.46 = 0.54$

(Reason: Walking and cycling account for 0.46 of the workers, so the tram and taxi groups together must account for the remaining 0.54.)

Step 3: Find x, then the taxi probability

$$\frac{0.54}{3} = 0.18$$

$2 \times 0.18 = 0.36$

(Reason: The two unknown cells are x and $2x$, which is $3x$ altogether, so $x = 0.18$. The taxi cell is twice that, so the taxi probability is 0.36.)

Step 4: Turn the probability into a number of workers

$0.36 \times 450 = 162$

(Reason: Multiplying a probability by the total number of workers gives how many of the 450 fall into that group.)

162 workers travelled by taxi

Verification

✓ **Check 1:** Put $x = 0.18$ back into the table and add all four probabilities.
$0.20 + 0.18 + 0.36 + 0.26 = 1$

✓ **Check 2:** Work in people instead of probabilities. Walking gives $0.20 \times 450 = 90$ and cycling gives $0.26 \times 450 = 117$, so take those away and split what is left in the ratio $1 : 2$. $450 - 207 = 243$, and $\frac{2}{3} \times 243 = 162$

✓ **Check 3:** Add the four group sizes and see whether they come back to 450.
$90 + 81 + 162 + 117 = 450$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1 - (0.20 + 0.26)$ or 0.54 oe, or $0.20 + x + 2x + 0.26 = 1$ oe, or $x + 2x = 0.54$ oe	M1	Showing clear understanding that the total of the probabilities is 1. If the probabilities are given as percentages then the per cent sign must be seen.	✓

Step	Mark	Description	Got it?
$\frac{0.54}{3} = 0.18$ or $\frac{2}{3} \times 0.54 = 0.36$ or $0.54 \times 450 = 243$	**M1**	For a correct method to find x or $2x$.	✓
$(2\times)\ 0.18 \times 450$ oe, or 81, or 0.36×450 oe	**M1**	Or for $\frac{81}{450}$ or $\frac{162}{450}$.	✓
162	**A1**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Alternative method, carrying the same four marks	**Note**	The same marks are available for working in people throughout: $(0.2 \times 450) + (0.26 \times 450) = 207$ for the first mark, $450 - 207 = 243$ for the second, and $\frac{2}{3} \times 243 = 162$ for the third and the answer.	✓

Full marks: 4/4

Question 3 - Exam Solution

Understanding the Question

Given

The two numbers 72 and 108
Working must be shown, so the method itself carries a mark.

Find

The highest common factor (HCF) of 72 and 108, the largest number that divides into both of them exactly

Plan the Solution

- Break 72 down into a product of prime factors.
- Do the same for 108.
- Compare the two products and keep only the primes that appear in both, each taken to the lower power.
- Multiply those shared prime factors together to give the HCF.

Worked Solution [2 marks]

> Rule - HCF from prime factors: write each number as a product of primes, then multiply the primes the two numbers share, each one taken to the LOWER power it appears with. Taking the higher power would give the LCM instead.

Step 1: Write 72 as a product of prime factors

$72 = 8 \times 9$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$72 = 2^3 \times 3^2$

(Reason: halving repeatedly strips out every factor of 2, and what is left, 9, is a power of 3)

Step 2: Write 108 as a product of prime factors

$108 = 4 \times 27$
$108 = 2 \times 2 \times 3 \times 3 \times 3$
$108 = 2^2 \times 3^3$

(Reason: the same method on the second number, which stops at two twos because 27 is odd)

Step 3: Take the lower power of each shared prime

$2^2 = 4$
$3^2 = 9$

(Reason: the HCF has to divide BOTH numbers, so each prime may be used only as often as it appears in the number that has fewer of it, and here the twos are limited by 108 while the threes are limited by 72)

Step 4: Multiply the shared prime factors together

$2^2 \times 3^2 = 4 \times 9 = 36$

(Reason: every prime factor in this product sits inside both numbers, so the product divides both, and nothing larger can)

36

Verification

✓ **Check 1:** Divide each of the original numbers by 36 and confirm that both divisions come out exactly. $\frac{72}{36} = 2$ and $\frac{108}{36} = 3$, both whole numbers, so 36 really is a common factor.

✓ **Check 2:** List every factor of each number and compare the two lists. This is the mark scheme's alternative method and it uses no prime factors at all. The factors shared by both numbers are $1, 2, 3, 4, 6, 9, 12, 18$ and 36, and the largest of them is 36.

✓ **Check 3:** For any two numbers, the HCF multiplied by the LCM equals the product of the numbers. The LCM here takes the HIGHER power of each prime, $2^3 \times 3^3 = 216$. $36 \times 216 = 7776$ and $72 \times 108 = 7776$, so the two sides agree.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Any correct valid method with no errors, eg $72 = 2 \times 2 \times 2 \times 3 \times 3$ and $108 = 2 \times 2 \times 3 \times 3 \times 3$	M1	Also earned by starting to list at least four different factors of each number with no errors, by a factor tree or a ladder diagram, by a fully correct Venn diagram, or by any other clear method such as a table. Ignore 1 in a list of prime factors.	✓
36	A1	Dependent on the method mark. Accept $2^2 \times 3^2$ or equivalent. Working is required, so an answer written down with no method earns nothing.	✓

Full marks: 2/2

Question 4 - Exam Solution

Understanding the Question

Given

April's total is 943 kilometres.
April is 15% more than March, so the 15% is an increase ON THE MARCH FIGURE, not on April's.

Find

The number of kilometres the van travelled in March.

Plan the Solution

- This is a reverse percentage: the 943 is the answer to an increase, not the starting point.
- Turn the 15% increase into a single multiplier.
- Write March as an unknown, multiply it by that multiplier to make 943, then divide to undo the increase.

- Finish by increasing the answer by 15% again - it must come back to 943.

Worked Solution [3 marks]

> Rule - Reverse percentage: an increase multiplies, so undoing it divides. If $\text{new} = \text{original} \times \text{multiplier}$, then $\text{original} = \dfrac{\text{new}}{\text{multiplier}}$.

Step 1: Turn the 15% increase into a multiplier

$$100\% + 15\% = 115\%$$

$$115\% = \frac{115}{100} = 1.15$$

(Reason: An increase keeps the original 100% and adds 15% on top, so the whole of April is 115% of March.)

Step 2: Write the equation for April

$$1.15m = 943$$

(Reason: Let m be the number of kilometres in March. Multiplying m by 1.15 is what produced April's 943 kilometres.)

Step 3: Divide by the multiplier to undo the increase

$$m = \frac{943}{1.15} = 820$$

(Reason: Dividing by 1.15 reverses the increase and takes April back to March. Taking 15% off April is a different calculation, because that 15% would be measured against the wrong month.)

820 kilometres

Verification

✓ **Check 1:** Put the answer back into the question: increase 820 by 15% and see whether April's figure returns. $0.15 \times 820 = 123$ and $820 + 123 = 943$

✓ **Check 2:** Reach the same answer without the multiplier. If 115% is 943 kilometres, find 1% and then 100%. This is the mark scheme's second route. $\dfrac{943}{115} = 8.2$ so $8.2 \times 100 = 820$

✓ **Check 3:** Test the direction. March must be the smaller month, and the gap between the two months must be 15% of the MARCH figure. $943 - 820 = 123$ and $\dfrac{123}{820} = 0.15 = 15\%$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct first step: $1 + 0.15 = 1.15$ or $x + 0.15x = 943$ or $100\% + 15\% = 115\%$ or $\frac{943}{115} = 8.2$ oe	**M1**	Any one of these earns the first mark: the multiplier, an equation in the March value, the percentage total, or one per cent of April. Recognising that April is 115% of March is the whole of this mark.	✓
A complete method to reverse the increase: $\frac{943}{1.15}$ or $\frac{943}{115} \times 100$ or $943 \times \frac{100}{115}$ oe or 8.2×100	**M1**	Awarded on the candidate's own multiplier or own value of 1% from the first row, so the figures in quotation marks on the official scheme need not be correct. This mark is dependent on the first.	✓
820	**A1**	Working not required, so the correct answer scores full marks (unless it comes from obvious incorrect working). An answer of 801.55 earns nothing here: it comes from taking 15% off April instead of dividing by 1.15.	✓

Full marks: 3/3

Question 5 - Exam Solution

Understanding the Question

Given

A regular pentagon $ABCDE$.
A straight line EF drawn from the vertex E, with F outside the pentagon.
Angle $AEF = 96°$.

Find

The size of the obtuse angle FED.

Plan the Solution

- Three angles meet at the point E: the given angle AEF, the pentagon's own angle AED, and the angle FED that is wanted.

- So the only missing piece is the interior angle of a regular pentagon. Get it from the angle sum $(n-2) \times 180°$.
- Then take both known angles away from the full turn of $360°$.

Worked Solution [4 marks]

Angles at a point add up to $360°$. Each interior angle of a regular n-sided polygon is $\dfrac{(n-2) \times 180°}{n}$.

Step 1: Find the angle sum of the pentagon

$(5-2) \times 180 = 540$

(Reason: A pentagon splits into 3 triangles from one vertex, and each triangle holds $180°$, so its five interior angles add up to $540°$.)

Step 2: Find one interior angle of the regular pentagon

$\dfrac{540}{5} = 108$

$\angle AED = 108°$

(Reason: Regular means all five interior angles are equal, so share the $540°$ between the 5 vertices. The pentagon's own angle at E is the angle AED, because A and D are the vertices either side of E.)

Step 3: Use the angles at the point E

$\angle AEF + \angle AED + \angle FED = 360°$

$360 - 96 - 108 = 156$

$\angle FED = 156°$

(Reason: The three angles at E make one complete turn, so subtract the given $96°$ and the pentagon's $108°$ from $360°$.)

$$\angle FED = 156°$$

Verification

✓ **Check 1:** Work with exterior angles instead, so the interior angle is never used. Extend AE beyond E: the angle between that extension and ED is the exterior angle $\dfrac{360}{5} = 72$, and the angle between the extension and EF is $180 - 96 = 84$. $72 + 84 = 156$

✓ **Check 2:** Add the three angles that meet at E and see whether they close one full turn. $96 + 108 + 156 = 360$

✓ **Check 3:** The question asks for the obtuse angle, so a correct answer has to sit between 90° and 180°. Going round the point the other way gives $360 - 156 = 204$, which is reflex, not obtuse. 156° is obtuse, so it is the angle the question wants

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(5-2) \times 180 = 540$ or $\frac{360}{5} = 72$	M1	A correct first step towards an angle of the regular pentagon: either the angle sum or the exterior angle. If angles are written on the diagram they must come from correct working and be correctly assigned.	✓
$\frac{540}{5} = 108$ or $180 - 72 = 108$ or $180 - 96 = 84$	M1	Any one of these: the interior angle of the pentagon, or the angle between EF and AE extended. Follow through on the candidate's own value from the first mark.	✓
$72 + 84$ or $360 - (96 + 108)$ or $180 - (108 - 84)$	M1	A complete method that reaches the required angle, again on the candidate's own earlier values.	✓
156	A1	Working is required on this question, so an answer given with no working scores nothing.	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

Part (a): the product $(m+5)(m-8)$, two brackets to be multiplied out.

Part (b): the equation $3n - 4 = \frac{5n+6}{3}$, which has a fraction on one side only.

Find

Part (a): the expansion, written as a simplified quadratic in m. Part (b): the value of n that makes both sides equal. The equation is linear, so expect exactly one value.

Plan the Solution

- Part (a): multiply each term in the first bracket by each term in the second. That gives 4 terms, and only the two m terms can be combined.
- Part (b): the fraction is the obstacle, so multiply every term on both sides by 3 to clear it.
- Then collect the n terms on one side and the number terms on the other, and divide by the coefficient of n.

Worked Solution [5 marks]

> Rule - Expanding a pair of brackets: $(x+a)(x+b) = x^2 + (a+b)x + ab$. Rule - Clearing a fraction from an equation: multiply every term on both sides by the denominator, so the equation stays balanced.

Part (a), Step 1: multiply every term in the first bracket by every term in the second

$(m+5)(m-8) = m \times m + m \times (-8) + 5 \times m + 5 \times (-8)$

$= m^2 - 8m + 5m - 40$

(Reason: Each of the two terms in $(m+5)$ meets each of the two terms in $(m-8)$, so a correct expansion always has 4 terms before anything is simplified. Keeping the -8 with its sign is what stops the middle term coming out wrong.)

Part (a), Step 2: collect the like terms

$m^2 - 8m + 5m - 40 = m^2 - 3m - 40$

(Reason: The two middle terms are like terms, because both are a number of m, so they combine into a single m term. The m^2 term and the number term have nothing to pair with, so they are unchanged.)

Part (b), Step 1: clear the fraction

$$3 \times (3n-4) = 3 \times \frac{5n+6}{3}$$

$9n - 12 = 5n + 6$

(Reason: Multiplying by 3 cancels the denominator on the right. On the left the bracket must be multiplied out, so both $3n$ and -4 are tripled, not just the first term.)

Part (b), Step 2: collect the n terms on one side and the numbers on the other

$9n - 5n = 6 + 12$

$4n = 18$

(Reason: Subtracting $5n$ from both sides and adding 12 to both sides keeps the equation balanced, and leaves one n term against one number.)

Part (b), Step 3: divide by the coefficient of n

$$n = \frac{18}{4} = \frac{9}{2}$$

(Reason: Dividing both sides by 4 leaves n on its own. $\frac{18}{4}$ cancels by 2 to $\frac{9}{2}$, which is 4.5. A fraction is a perfectly acceptable final form here.)

(a) $m^2 - 3m - 40$

(b) $n = \frac{9}{2}$**, that is** 4.5

Verification

✓ **Check 1:** Part (a): put $m = 2$ into the original product and into the simplified answer. They must give the same number. $(2+5)(2-8) = 7 \times (-6) = -42$ and $2^2 - 3 \times 2 - 40 = 4 - 6 - 40 = -42$

✓ **Check 2:** Part (a): a second value, $m = -1$, in case $m = 2$ agreed by luck. A sign slip in the middle term shows up here. $(-1+5)(-1-8) = 4 \times (-9) = -36$ and $(-1)^2 - 3 \times (-1) - 40 = 1 + 3 - 40 = -36$

✓ **Check 3:** Part (b): substitute $n = \frac{9}{2}$ back into both sides of the original equation. Left side $3 \times 4.5 - 4 = 9.5$ and right side $\frac{5 \times 4.5 + 6}{3} = \frac{28.5}{3} = 9.5$

✓ **Check 4:** Part (b): solve a second way, by splitting the fraction instead of clearing it, so the two methods are independent. $3n - 4 = \frac{5}{3}n + 2$ gives $\frac{4}{3}n = 6$, so $n = 6 \times \frac{3}{4} = \frac{9}{2}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $m^2 - 8m + 5m - 40$	M1	For any 3 correct terms out of the 4, or for all 4 terms correct ignoring signs, or for $m^2 - 3m$, or for $-3m - 40$.	✓
(a) $m^2 - 3m - 40$	A1	Working is not required in this part, so a correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Step	Mark	Description	Got it?
(b) $9n - 12 = 5n + 6$	**M1**	For removal of the fraction and multiplying out the left-hand side. Or for separating the fraction on the right, as in $3n - 4 = \frac{5}{3}n + \frac{6}{3}$ or equivalent.	✓
(b) $9n - 5n = 6 + 12$ or $4n = 18$	**M1ft**	Follow through, dependent on a 4 term equation, for correctly rearranging their 4 term equation so that the n terms are on one side and the number terms on the other. Equivalents such as $-12 - 6 = 5n - 9n$ or $n = \frac{-18}{-4}$ also score.	✓
(b) $n = \frac{9}{2}$	**A1**	Dependent on M2. Or equivalent, for example $\frac{18}{4}$ or 4.5 or $4\frac{1}{2}$. Working is required in this part.	✓

Full marks: 5/5

Question 7 - Exam Solution

Understanding the Question

Given

The universal set $\mathcal{E} = \{23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$ $A = \{\text{even numbers}\}$, $B = \{23, 29, 31\}$ and $C = \{\text{multiples of } 3\}$, each taken from inside $\mathcal{E}$

For part (c): the set D has 4 members and $D \cap (A \cup C) = \varnothing$

Find

(a)(i) the members of $B \cup C$ (a)(ii) the members of $A' \cap C$ (b) whether $B \cap C = \varnothing$ is true, with a reason (c) the members of D

Plan the Solution

- Write A and C out in full first. Both are described in words, and only members of $\mathcal{E}$ count.
- Then read each symbol as an instruction: $\cup$ collects everything in either set, $\cap$ keeps only what is in both, and A' is everything in $\mathcal{E}$ that is not in A.

- For (c), a set that meets $A \cup C$ in nothing at all can only be made from what is left over, so list $A \cup C$ and take the rest of $\mathcal{E}$.

Worked Solution [5 marks]

> Rule - Set notation: $\cup$ means in one set or the other, $\cap$ means in both sets at once, A' means inside $\mathcal{E}$ but outside A, and $\varnothing$ is the set with no members.

Step 1: write A and C out as lists

$$A = \{24,\ 26,\ 28,\ 30,\ 32,\ 34\}$$
$$C = \{24,\ 27,\ 30,\ 33\}$$

(Reason: (Reason: both sets are described in words, so they must be turned into lists before any symbol can be used. Only the numbers 23 to 34 are available, so the multiples of 3 stop at 33.))

Step 2: part (a)(i), collect everything in B or in C

$$B \cup C = \{23,\ 29,\ 31\} \cup \{24,\ 27,\ 30,\ 33\}$$
$$B \cup C = \{23,\ 24,\ 27,\ 29,\ 30,\ 31,\ 33\}$$

(Reason: (Reason: a union keeps every member of either set, written once each and in order. Nothing appears in both lists here, so all 7 members survive.))

Step 3: part (a)(ii), write down A' and then intersect it with C

$$A' = \{23,\ 25,\ 27,\ 29,\ 31,\ 33\}$$
$$A' \cap C = \{23,\ 25,\ 27,\ 29,\ 31,\ 33\} \cap \{24,\ 27,\ 30,\ 33\}$$
$$A' \cap C = \{27,\ 33\}$$

(Reason: (Reason: A' is everything in $\mathcal{E}$ that is not even, which leaves the odd numbers. The intersection then keeps only those members that appear in both lists.))

Step 4: part (b), test B against C member by member

$$23 = 3 \times 7 + 2$$
$$29 = 3 \times 9 + 2$$
$$31 = 3 \times 10 + 1$$
$$B \cap C = \varnothing$$

(Reason: (Reason: every member of B leaves a remainder when divided by 3, so not one of them is a multiple of 3. The two sets therefore share no member, and an intersection with no members is the empty set, so the statement is true. Tick Yes.))

Step 5: part (c), list $A \cup C$

$$A \cup C = \{24,\ 26,\ 28,\ 30,\ 32,\ 34\} \cup \{24,\ 27,\ 30,\ 33\}$$
$$A \cup C = \{24,\ 26,\ 27,\ 28,\ 30,\ 32,\ 33,\ 34\}$$

(Reason: (Reason: 24 and 30 are in both lists but are written once each, so this union has 8 members.))

Step 6: take what is left of $\mathcal{E}$ to get D

$(A \cup C)' = \{23,\ 25,\ 29,\ 31\}$
$D = \{23,\ 25,\ 29,\ 31\}$

(Reason: (Reason: D shares nothing with $A \cup C$, so every member of D must come from the 4 numbers of $\mathcal{E}$ that the union left behind. That is exactly the 4 members the question asks for, so there is only one possible set.))

(a)(i) $B \cup C = \{23,\ 24,\ 27,\ 29,\ 30,\ 31,\ 33\}$

(a)(ii) $A' \cap C = \{27,\ 33\}$

(b) Yes - no member of B is a multiple of 3, so $B \cap C$ has no members

(c) $D = \{23,\ 25,\ 29,\ 31\}$

Verification

✓ **Check 1:** Count the union in (a)(i) instead of reading it. B has 3 members and C has 4, and no number is in both, so the union must hold $3 + 4 = 7$ members. The listed answer has exactly 7 members

✓ **Check 2:** Test the two conditions in (a)(ii) separately on every multiple of 3. Of 24, 27, 30 and 33, the even ones are barred by A'. 24 and 30 are even, so only 27 and 33 pass both tests

✓ **Check 3:** Count (c) a second way, and test each member. $\mathcal{E}$ has 12 members and $A \cup C$ has 8, so $12 - 8 = 4$ are left, and each should be odd and not a multiple of 3. 23, 25, 29 and 31 are all odd, none is a multiple of 3, and there are 4 of them, as the question states

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $B \cup C = \{23,\ 24,\ 27,\ 29,\ 30,\ 31,\ 33\}$	B1	All seven members listed, in any order, with no repeats.	✓
(a)(ii) $A' \cap C = \{27,\ 33\}$	B1	Both members listed, in any order, with no repeats.	✓

Step	Mark	Description	Got it?
(b) Yes, together with a reason such as: no member of B is a multiple of 3, so the sets have nothing in common	**B1**	B1 for Yes and a statement showing the correct meanings of intersection and of the empty set. If no box is ticked, the word Yes must appear in the written answer. Members, numbers, values or elements are all accepted wording.	✓
(c) $D = \{23,\ 25,\ 29,\ 31\}$	**B2**	B2 for the four correct numbers and no additions. B1 for three correct values with no more than one incorrect, or for four correct values with no more than one incorrect.	✓

Full marks: 5/5

Question 8 - Exam Solution

Understanding the Question

Given

A cylindrical drum standing upright, of volume 1575 cm³.
Its height is 21 cm, marked on the figure.
The force it exerts on the workbench is 84 newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Find

The pressure on the workbench, in newtons/cm². The area in that formula is the area of the face touching the workbench, so the circular end of the drum is what has to be found first.

Plan the Solution

- A cylinder is a prism, so its volume is the area of its circular end multiplied by its height.
- So the area of that end comes from dividing 1575 by 21, and the radius is never needed.
- Then put the force and that area into the pressure formula and divide.

Worked Solution [3 marks]

Rule - Cylinder and pressure: $\text{volume} = \text{cross-sectional area} \times \text{height}$, so $\text{area} = \dfrac{\text{volume}}{\text{height}}$, and then $\text{pressure} = \dfrac{\text{force}}{\text{area}}$.

Step 1: Find the area of the circular end

$1575 = \text{area} \times 21$

$\text{area} = \dfrac{1575}{21} = 75$

(Reason: The drum is a prism, so its volume is the area of the end multiplied by the height. Reversing that gives an end of area 75 cm², and no radius is needed to get there.)

Step 2: Put the force and the area into the formula

$\text{pressure} = \dfrac{\text{force}}{\text{area}} = \dfrac{84}{75}$

(Reason: The whole 84 newtons is carried by the 75 cm² of circle that touches the workbench.)

Step 3: Work out the division

$\dfrac{84}{75} = 1.12$

(Reason: This is the share of the force carried by each square centimetre of the workbench, which is what pressure measures.)

1.12 newtons/cm²

Verification

✓ **Check 1:** Multiply back. If the pressure is right, multiplying it by the area must return the force the question gives. $1.12 \times 75 = 84$ newtons, which is the force in the question.

✓ **Check 2:** Take the long way round, through the radius: $r = \sqrt{\dfrac{1575}{21\pi}} \approx 4.886$ cm.

$\pi \times 4.886^2 \approx 75$ cm², the same area, so the pressure is again $\dfrac{84}{75} = 1.12$.

✓ **Check 3:** A size check. 84 newtons spread over 75 cm² is a little more than one newton for each square centimetre, because 84 is a little more than 75. An answer of 1.12 sits just above 1, which is the size expected.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1575 = \text{area} \times 21$, or $\text{area} = 75$, or $1575 = \pi \times r^2 \times 21$, or $r^2 = \dfrac{1575}{21\pi} \approx 23.87$, or $r = \sqrt{\dfrac{1575}{21\pi}} \approx 4.886$	**M1**	for finding the area using $\text{volume} = \text{cross-sectional area} \times \text{height}$, or for finding r or r^2 using $\text{volume} = \pi r^2 h$. Note that r^2 and r may be rounded or truncated.	✓
$\dfrac{84}{75}$, or $\dfrac{84}{\pi \times 4.88^2}$, or $\dfrac{84}{\pi \times 23.8}$	**M1**	for $\dfrac{84}{\text{area of circle}}$, worked on their own area from the first mark.	✓
1.12	**A1**	accept 1.06 to 1.121. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given

A table of the maize produced by two countries in 2020: Ukraine 3.5×10^7 tonnes, Portugal 8.2×10^5 tonnes. Romania produced $6\,780\,000$ more tonnes than Portugal, so Romania's amount is not in the table.

Find

(a) The value 3.5×10^7 written as an ordinary number. (b) Romania's amount, written in standard form.

Plan the Solution

- Part (a) is a conversion, not a calculation: $\times 10^7$ moves every digit seven places to the left, so write the digits down and fill the gap with zeros.
- Part (b) mixes a standard form number with an ordinary one, so put both into the same form before adding anything.

- Add, then convert the total back to standard form, checking the front number really does sit between 1 and 10.

Worked Solution [3 marks]

> Rule - Standard form: $a \times 10^n$ means the digits of a shifted n places to the left, and a number is only in standard form when $1 \leq a < 10$.

Step 1: read what the power of ten is worth

$10^7 = 10\,000\,000$

(Reason: (Reason: the index 7 counts the zeros, so 10^7 is ten million.))

Step 2: multiply the front number by that power

$3.5 \times 10\,000\,000 = 35\,000\,000$

(Reason: (Reason: multiplying by 10^7 shifts 3.5 seven places to the left, which turns the 3.5 into 35 followed by six zeros.))

Step 3: write Portugal's amount as an ordinary number

$8.2 \times 10^5 = 820\,000$

(Reason: (Reason: the extra tonnes are given as an ordinary number, so Portugal's amount has to be an ordinary number too before the two can be added.))

Step 4: add the extra tonnes Romania produced

$820\,000 + 6\,780\,000 = 7\,600\,000$

(Reason: (Reason: Romania produced $6\,780\,000$ tonnes more than Portugal, and more means add.))

Step 5: put the total back into standard form

$7\,600\,000 = 7.6 \times 10^6$

(Reason: (Reason: the point moves six places to sit after the 7, and 7.6 is between 1 and 10, so this is standard form. Writing 76×10^5 would be the same number but not standard form.))

(a) $35\,000\,000$

(b) 7.6×10^6 **tonnes**

Verification

✓ **Check 1:** Work part (b) backwards. Take the extra $6\,780\,000$ tonnes off the answer and the table's figure for Portugal should come back.
$7\,600\,000 - 6\,780\,000 = 820\,000 = 8.2 \times 10^5$

✓ **Check 2:** Undo part (a) by dividing the ordinary number by 10^7. A correct conversion gives back the 3.5 the table prints. $\dfrac{35\,000\,000}{10^7} = 3.5$

✓ **Check 3:** Add in hundred-thousands instead, so neither number is ever written out in full: $6\,780\,000$ is 67.8 lots of 10^5. $(8.2 + 67.8) \times 10^5 = 76 \times 10^5 = 7.6 \times 10^6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Write 3.5×10^7 as an ordinary number	**B1**	$35\,000\,000$ cao	✓
(b) Add the extra tonnes to Portugal's amount	**M1**	$8.2 \times 10^5 + 6\,780\,000$ oe, or $820\,000 + 6\,780\,000$ oe, or $7\,600\,000$ or 76×10^5 oe, or 7.6×10^n where $n \neq 6$. Allow a correct mixture of ordinary numbers and standard form numbers.	✓
(b) Give the total in standard form	**A1**	7.6×10^6. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 10 - Exam Solution

Understanding the Question

Given

(a) the expression $(2p)^0$, with $p > 0$, so the bracket is never zero
(b) $y^9 \times y^{-3} = y^n$, a power of y multiplied by another power of y
(c) the expression $(5a^4c^2)^3$, a product of three factors inside one bracket

Find

(a) $(2p)^0$ in its simplest form (b) the value of the index n (c) $(5a^4c^2)^3$ written as one simplified product

Plan the Solution

- Each part is one index law, so name the law first and then apply it.

- (a) uses the zero index. The condition $p > 0$ is there to guarantee the bracket is not zero, which is the one case the law excludes.
- (b) is a multiplication of powers of the same letter, so the indices are ADDED. The second index is negative, so adding it makes the result smaller.
- (c) is a bracket raised to a power, so every factor inside is raised to that power: the number 5 and both letters. The indices on the letters are MULTIPLIED, not added.
- Finish (c) by writing the number first, then a, then c, as a single product.

Worked Solution [4 marks]

> Index laws: $x^0 = 1$ for any $x \neq 0$, $x^m \times x^n = x^{m+n}$, $(x^m)^n = x^{mn}$ and $(kx)^n = k^n x^n$.

Step 1: part (a), use the zero index

$(2p)^0 = 1$

(Reason: (Reason: anything that is not zero raised to the power 0 equals 1. Here $p > 0$, so $2p$ is a positive number and the law applies. The 2 does not survive: the whole bracket carries the index, not just the p.))

Step 2: part (b), add the indices

$y^9 \times y^{-3} = y^{9+(-3)} = y^6$

$n = 6$

(Reason: (Reason: multiplying powers of the same letter adds the indices. Adding -3 is the same as subtracting 3, so $9 + (-3) = 6$. Comparing y^6 with y^n gives the index n directly.))

Step 3: part (c), give every factor inside the bracket the power 3

$(5a^4c^2)^3 = 5^3 \times (a^4)^3 \times (c^2)^3$

(Reason: (Reason: the bracket is a product of three things, 5, a^4 and c^2, and cubing a product cubes each of them. Forgetting to cube the 5 is the commonest way to lose a mark here.))

Step 4: work out the three factors

$5^3 = 5 \times 5 \times 5 = 125$

$(a^4)^3 = a^{4\times3} = a^{12}$

$(c^2)^3 = c^{2\times3} = c^6$

(Reason: (Reason: a power raised to another power multiplies the indices, because a^4 is being written down 3 times. The number is different: 5 has no index of its own to combine, so it is simply cubed to 125.))

Step 5: write it as one product

$(5a^4c^2)^3 = 125a^{12}c^6$

(Reason: (Reason: the number goes first, then the letters in alphabetical order. Nothing can be added or cancelled here because 125, a^{12} and c^6 are multiplied, not added, so this is fully simplified.))

(a) 1

(b) $n = 6$

(c) $125a^{12}c^6$

Verification

✓ **Check 1:** Part (a) with a number. Put $p = 5$, so the bracket is $2 \times 5 = 10$, which is not zero. $10^0 = 1$, and the same happens for every positive p.

✓ **Check 2:** Part (b) with a number. Put $y = 2$. Then $y^9 = 512$ and $y^{-3} = \frac{1}{8}$.

$512 \times \frac{1}{8} = 64 = 2^6$, so $n = 6$.

✓ **Check 3:** Part (c) with numbers. Put $a = 2$ and $c = 3$. The bracket is $5 \times 16 \times 9 = 720$, so the left-hand side is 720^3. $720^3 = 373\,248\,000$ and $125 \times 2^{12} \times 3^6 = 373\,248\,000$, so the two forms agree.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) 1	**B1**	cao. The whole bracket carries the index, so 2 on its own scores nothing.	✓
(b) $n = 6$	**B1**	Accept y^6.	✓
(c) $125a^{12}c^6$	**B2**	Fully simplified. Multiplication signs between the terms are accepted, so $125 \times a^{12} \times c^6$ also scores B2.	✓
(c) a product in the form ka^pc^q with 2 of k, p, q correct	**B1**	Partial credit, eg $5a^{12}c^6$ where the coefficient was not cubed. Allow $125a^{12}$ or $125c^6$ or $a^{12}c^6$, as long as they are not added to any other terms.	✓

Full marks: 4/4

Question 11 - Exam Solution

Understanding the Question

Given

A triangular frame ABC with $AC = BC = 9$ m and $AB = 12$ m. A strut MC from the apex to the point M on AB, with angle $AMC = 90°$. Timber costs 21.50 euros per metre, and every length bought is a whole number of metres.

Find

The total cost, in euros, of the four lengths AB, AC, BC and MC.

Plan the Solution

- The two sloping sides are equal and the strut is perpendicular to the base, so M is the midpoint of AB and triangle AMC is right-angled.
- Use Pythagoras' theorem in triangle AMC to find the length of the strut MC.
- Round that length UP to a whole number of metres, because a shorter length would not reach.
- Add the four lengths, then multiply the total by the price of one metre.

Worked Solution [4 marks]

> Rule - Pythagoras' theorem: in a right-angled triangle the square on the hypotenuse equals the sum of the squares on the other two sides, so a shorter side is $\sqrt{\text{hypotenuse}^2 - \text{other short side}^2}$.

Step 1: Find AM

$$AM = MB = \frac{12}{2} = 6 \text{ m}$$

(Reason: $AC = BC$ and angle $AMC = 90°$, so triangles AMC and BMC are congruent and M is the midpoint of AB.)

Step 2: Use Pythagoras' theorem to find MC

$$MC^2 = 9^2 - 6^2 = 81 - 36 = 45$$

$$MC = \sqrt{45} = 3\sqrt{5} = 6.7082\ldots \text{ m}$$

(Reason: Triangle AMC is right-angled at M, and the 9 m side AC is its hypotenuse, so the strut is the square root of $81 - 36$.)

Step 3: Round the strut up to a whole number of metres

$6 < 6.7082... < 7$

(Reason: Every length bought is a whole number of metres. A 6 m length would be too short to reach, so Martin has to buy a 7 m length for the strut.)

Step 4: Add the four lengths

$7 + 9 + 9 + 12 = 37$ m

(Reason: The four lengths he buys are the strut MC, the two sloping sides AC and BC, and the base AB.)

Step 5: Work out the cost

$37 \times 21.50 = 795.50$

(Reason: Each metre of timber costs 21.50 euros, so multiply the total length by 21.50.)

795.50 euros

Verification

✓ **Check 1:** Put the strut back into Pythagoras' theorem. Its square is 45, and half the base squared is 36, so $6^2 + 45 = 36 + 45 = 81$ and $81 = 9^2$. The hypotenuse comes back to $AC = 9$ m, so $MC = \sqrt{45}$ is right.

✓ **Check 2:** Reach the strut by trigonometry instead. The base angle is $\cos^{-1}\left(\frac{6}{9}\right) = 48.1896...°$, and then $MC = 9\sin(48.1896...°)$. $MC = 6.7082...$ m, the same length the Pythagoras route gave, so it still rounds up to 7 m.

✓ **Check 3:** Split the final multiplication. $37 \times 21 = 777$ and $37 \times 0.50 = 18.50$. $777 + 18.50 = 795.50$, which matches the cost worked out in one go.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Set up Pythagoras' theorem in triangle AMC	M1	for $9^2 - \left(\frac{12}{2}\right)^2 (= 81 - 36 = 45)$ or $(MC)^2 + \left(\frac{12}{2}\right)^2 = 9^2$ oe	✓

Step	Mark	Description	Got it?
Find the length of the strut MC	**M1**	for $\sqrt{9^2 - \left(\frac{12}{2}\right)^2}$ oe, that is $\sqrt{81-36} = \sqrt{45} = 3\sqrt{5} = 6.7(08\ldots)$	✓
Cost the four whole-number lengths	**M1**	for their whole-number strut used as $(7+9+9+12) \times 21.5(0)$ or $37 \times 21.5(0)$	✓
State the total cost	**A1**	for $795.5(0)$. Working required.	✓
Special case - the strut never rounded up	**SC**	SC B3 for awrt 789 if no other marks are earned. That is the total costed from $6.7\ldots$ m of strut instead of the 7 m length that has to be bought.	✓
Alternative method for the first two marks	**Note**	M2 for $\cos^{-1}\left(\frac{6}{9}\right) = 48.1(896\ldots)°$ and $MC = 6\tan(48.1\ldots°)$ or $MC = 9\sin(48.1\ldots°)$, each $= 6.7\ldots$	✓

Full marks: 4/4

Question 12 - Exam Solution

Understanding the Question

Given

The quadratic expression $6y^2 - 5y - 4$

The sum of two algebraic fractions

$$\frac{2x+1}{4x} + \frac{7-5x}{3x}$$

Find

(a) $6y^2 - 5y - 4$ written as a product of two brackets, with nothing left to take out

(b) The two fractions written as one fraction, in its simplest form

Plan the Solution

- (a) Multiply the coefficient of y^2 by the constant term. Look for two numbers with that product whose sum is the coefficient of y, use them to split the middle term, then factorise in pairs.
- (b) The denominators are $4x$ and $3x$, so both fractions go over $12x$. Add the numerators, expand the brackets, collect like terms, then look for anything that cancels.

Worked Solution [5 marks]

> To factorise $ay^2 + by + c$, find two numbers whose product is ac and whose sum is b. To add algebraic fractions, write each one over the lowest common denominator, then add the numerators.

Step 1: the two numbers for part (a)

$6 \times (-4) = -24$

$-8 \times 3 = -24$

$-8 + 3 = -5$

(Reason: The coefficient of y^2 times the constant term is -24. Of all the pairs multiplying to -24, only -8 and 3 add to -5, which is the coefficient of y.)

Step 2: split the middle term

$6y^2 - 5y - 4 = 6y^2 - 8y + 3y - 4$

(Reason: $-5y$ is written as $-8y + 3y$. That is the same quantity in two pieces, so the expression has not been changed, but it can now be grouped in pairs.)

Step 3: factorise in pairs

$6y^2 - 8y + 3y - 4 = 2y(3y - 4) + 1(3y - 4)$

$2y(3y - 4) + 1(3y - 4) = (2y + 1)(3y - 4)$

(Reason: The first two terms have $2y$ in common and the last two have 1. Both pairs leave $(3y - 4)$, so that bracket is a common factor of the whole expression and comes out. Neither bracket has a factor left inside it, so this is the full factorisation.)

Step 4: a common denominator for part (b)

$$\frac{2x+1}{4x} + \frac{7-5x}{3x} = \frac{3(2x+1)}{12x} + \frac{4(7-5x)}{12x}$$

(Reason: The lowest common multiple of $4x$ and $3x$ is $12x$. The first fraction is multiplied top and bottom by 3, the second top and bottom by 4, so neither fraction changes in value.)

Step 5: add the numerators and expand

$$\frac{3(2x+1) + 4(7-5x)}{12x} = \frac{6x + 3 + 28 - 20x}{12x}$$

(Reason: Two fractions over the same denominator add by adding their numerators. Every term inside a bracket is multiplied by the number in front of it: 3 multiplies both $2x$ and 1, and 4 multiplies both 7 and $-5x$.)

Step 6: collect like terms

$3 + 28 = 31$

$$\frac{6x+3+28-20x}{12x}=\frac{31-14x}{12x}$$

(Reason: The number terms give 31 and the x terms give $6x-20x=-14x$. Since 31 and 14 share no factor, and the numerator has a number term so no x can be cancelled, the fraction is already in its simplest form.)

(a) $(2y+1)(3y-4)$

(b) $\dfrac{31-14x}{12x}$

Verification

✓ **Check 1:** Multiply the brackets in part (a) back out and collect the y terms.
$6y^2-8y+3y-4=6y^2-5y-4$

✓ **Check 2:** Put $y=2$ into the original expression, and into the factorised form. Both must give the same number. $6\times2^2-5\times2-4=10$ and
$(2\times2+1)\times(3\times2-4)=5\times2=10$

✓ **Check 3:** Put $x=1$ into the two fractions in part (b), and into the single fraction.
$\frac{3}{4}+\frac{2}{3}=\frac{17}{12}$ and $\frac{31-14}{12}=\frac{17}{12}$

✓ **Check 4:** Repeat with $x=2$. One value agreeing could be luck, two is evidence.
$\frac{5}{8}-\frac{3}{6}=\frac{1}{8}$ and $\frac{31-28}{24}=\frac{3}{24}=\frac{1}{8}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A pair of brackets of the right form, or the middle term split and factorised in pairs	M1	For $(2y\pm1)(3y\pm4)$ or $(2y\pm4)(3y\pm1)$, or for $2y(3y-4)+1(3y-4)$ or $3y(2y+1)-4(2y+1)$. The factors must be of the form $(ay+b)$ where a and b are integers. Condone the use of a different letter in place of y.	✓
(a) The full factorisation	A1	For $(2y+1)(3y-4)$, or $(3y-4)(2y+1)$. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working. Ignore further working if the quadratic is then solved to find roots.	✓

Step	Mark	Description	Got it?
(b) Both fractions over a common denominator, with the intention to add	M1	For $\frac{3(2x+1)}{12x}+\frac{4(7-5x)}{12x}$ or $\frac{3x(2x+1)}{12x^2}+\frac{4x(7-5x)}{12x^2}$, or for a single correct fraction such as $\frac{3(2x+1)+4(7-5x)}{12x}$ or $\frac{3x(2x+1)+4x(7-5x)}{12x^2}$ oe. For this mark $12x$ may be written as $(3)(4x)$ or $(4)(3x)$, and $12x^2$ as $(3x)(4x)$.	✓
(b) A correct single fraction with every bracket expanded	M1	For $\frac{6x+3+28-20x}{12x}$ oe, or $\frac{6x^2+3x+28x-20x^2}{12x^2}$ oe, or $\frac{31x-14x^2}{12x^2}$ oe.	✓
(b) The single fraction in its simplest form	A1	For $\frac{31-14x}{12x}$, or $\frac{14x-31}{-12x}$. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 5/5

Question 13 - Exam Solution

Understanding the Question

Given

Box A holds 3 white buttons and 7 black buttons, so 10 buttons altogether.
Box B holds 5 white buttons and 4 black buttons, so 9 buttons altogether.
One button is taken from each box, so what happens in one box cannot change the other.

Find

(a) The six probabilities that belong on the branches of the tree. (b) The probability that the two buttons match in colour.

Plan the Solution

- Write each probability as the number of buttons of that colour over the total in that box.

- Check that each fork adds to 1 before going any further.
- Two buttons match in only two ways: white then white, or black then black.
- Multiply along each of those two routes, then add the two results.

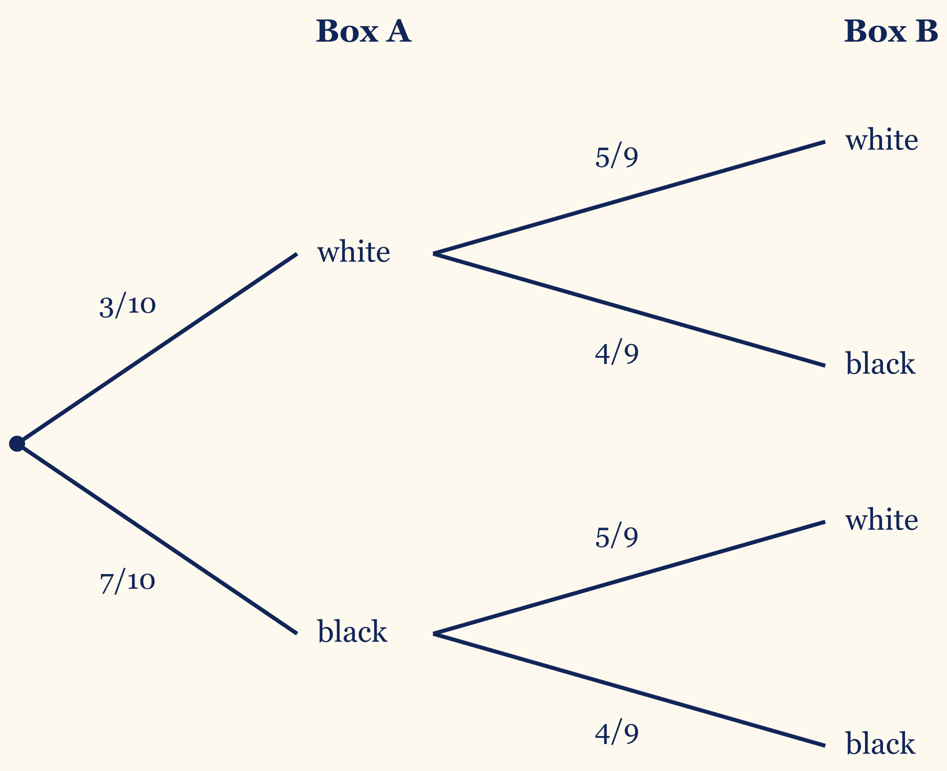
Worked Solution [5 marks]

> Rule - Tree diagrams: multiply ALONG a pair of branches, add BETWEEN separate routes. For two picks that do not affect each other, $P(\text{both}) = P(\text{first}) \times P(\text{second})$.

Step 1: the box A probabilities

$$P(\text{white}) = \frac{3}{10}$$

$$P(\text{black}) = \frac{7}{10}$$



(Reason: Box A holds 10 buttons and 3 of them are white, so 3 of the 10 equally likely picks give white. The pair adds to 1, as every fork must.)

Step 2: the box B probabilities

$$P(\text{white}) = \frac{5}{9}$$

$P(\text{black}) = \frac{4}{9}$

(Reason: Box B holds 9 buttons. This same pair goes on BOTH box B forks, because taking a button out of box A leaves box B exactly as it was.)

Step 3: multiply along the two same-colour routes

$$P(\text{white, white}) = \frac{3}{10} \times \frac{5}{9} = \frac{15}{90}$$

$$P(\text{black, black}) = \frac{7}{10} \times \frac{4}{9} = \frac{28}{90}$$

(Reason: Multiplying along a pair of branches gives the probability of that whole route through the tree. Leaving both products over 90 means they can be added straight away.)

Step 4: add the two same-colour routes

$$P(\text{same colour}) = \frac{15}{90} + \frac{28}{90}$$

$$P(\text{same colour}) = \frac{43}{90}$$

(Reason: The two routes cannot both happen, so their probabilities add. $\frac{43}{90}$ will not cancel, because 43 is prime, and as a decimal it is $0.477\ldots$, a little under a half.)

(a) $\frac{3}{10}$ and $\frac{7}{10}$ on the box A branches; $\frac{5}{9}$ and $\frac{4}{9}$ on both box B forks

(b) $\frac{43}{90}$

Verification

✓ **Check 1:** Every fork of a tree must add to 1. Add each pair that was written on the diagram. $\frac{3}{10} + \frac{7}{10} = 1$ and $\frac{5}{9} + \frac{4}{9} = 1$

✓ **Check 2:** Work out the opposite outcome, two different colours, and see whether the two probabilities add to 1. $\frac{3}{10} \times \frac{4}{9} + \frac{7}{10} \times \frac{5}{9} = \frac{47}{90}$ and $\frac{43}{90} + \frac{47}{90} = 1$

✓ **Check 3:** Count instead of multiplying. There are $10 \times 9 = 90$ equally likely pairs of buttons, of which $3 \times 5 = 15$ are both white and $7 \times 4 = 28$ are both black. $\frac{15+28}{90} = \frac{43}{90}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) All three pairs on the correct branches: $\frac{3}{10}$, $\frac{7}{10}$ on box A; $\frac{5}{9}$, $\frac{4}{9}$ on each box B fork	**B2**	B2 for all 3 correct pairs of probabilities on the correct branches. If not B2, then award B1 for 1 correct pair of probabilities on a correct branch. Allow equivalent fractions or decimals, to 2 dp truncated or rounded, ie 0.55(...) and/or 0.44(...)	✓
(b) One correct product, eg $\frac{3}{10} \times \frac{5}{9}$ or $\frac{7}{10} \times \frac{4}{9}$	**M1ft**	Follow through on the candidate's own tree, provided every probability used is less than 1. Any one of the four products earns this mark, including a different-colour one such as $\frac{3}{10} \times \frac{4}{9}$.	✓
(b) A complete method: $\frac{3}{10} \times \frac{5}{9} + \frac{7}{10} \times \frac{4}{9}$ or $1 - \left(\frac{3}{10} \times \frac{4}{9} + \frac{7}{10} \times \frac{5}{9}\right)$	**M1ft**	Both same-colour routes added, or both different-colour routes subtracted from 1. Follow through on the candidate's own tree, again with every probability less than 1.	✓
(b) $\frac{43}{90}$	**A1ft**	Or equivalent: 0.47(77...) to 2 dp truncated or rounded, or 47.(77) per cent to 2 sf truncated or rounded.	✓
Note	**Note**	Working is not required in part (b), so a correct answer scores full marks unless it follows obviously incorrect working. This row carries no mark of its own.	✓

Full marks: 5/5

Question 14 - Exam Solution

Understanding the Question

Given

Find

The amount Devika raised, in dirhams.

Owen and Bilal together raised $15\,435$ dirhams.
Bilal raised 45% more than Owen.
Bilal raised $\frac{3}{2}$ times as much as Devika.
Devika's amount is not linked to the total directly, only through Bilal.

Plan the Solution

- Call the amount Owen raised x dirhams, so the amount Bilal raised is $1.45x$ dirhams.
- Add the two amounts, put the sum equal to $15\,435$, and solve for x.
- Scale Owen's amount by 1.45 to get Bilal's amount.
- Bilal's amount is $\frac{3}{2}$ of Devika's, so undo that by multiplying Bilal's amount by $\frac{2}{3}$.

Worked Solution [5 marks]

> Rule - a rise of 45% is a multiplier of $1 + 0.45 = 1.45$, and a statement of the form $P = \frac{3}{2} \times Q$ is undone by $Q = \frac{2}{3} \times P$.

Step 1: write both amounts using one letter

$$\text{Owen} = x$$
$$\text{Bilal} = x + 0.45x = 1.45x$$

(Reason: Bilal's amount is Owen's amount plus another 45% of it, and $1 + 0.45 = 1.45$, so one letter now carries both people.)

Step 2: use the combined total to find Owen's amount

$$x + 1.45x = 15\,435$$
$$2.45x = 15\,435$$
$$x = \frac{15\,435}{2.45} = 6300$$

(Reason: Adding the two expressions gives 2.45 lots of x, so dividing the total by 2.45 leaves one lot. Owen raised 6300 dirhams.)

Step 3: work out Bilal's amount

$$\text{Bilal} = 15\,435 - 6300 = 9135$$

(Reason: The pair raised $15\,435$ between them, so taking Owen's share off the total leaves Bilal's. Multiplying instead gives the same value, since $1.45 \times 6300 = 9135$.)

Step 4: work back from Bilal to Devika

$$\text{Bilal} = \frac{3}{2} \times \text{Devika}$$

$$\text{Devika} = 9135 \times \frac{2}{3} = 6090$$

(Reason: Bilal's amount is the larger one here, so Devika's amount is found by reversing the $\frac{3}{2}$ and multiplying by $\frac{2}{3}$. Devika raised 6090 dirhams.)

6090 dirhams

Verification

✓ **Check 1:** Add Owen's 6300 to Bilal's 9135 and compare the sum with the combined total in the question. $6300 + 9135 = 15\,435$, which is the combined amount given.

✓ **Check 2:** Work out how much more Bilal raised than Owen, as a fraction of Owen's amount. $\dfrac{9135 - 6300}{6300} = 0.45$, so Bilal did raise 45% more.

✓ **Check 3:** Scale the answer back up by $\frac{3}{2}$ and see whether Bilal's amount reappears. $6090 \times \frac{3}{2} = 9135$, which is Bilal's amount.

✓ **Check 4:** Take the whole journey in one fraction instead: Bilal is $\frac{29}{49}$ of the pair's total, and Devika is $\frac{2}{3}$ of Bilal. $15\,435 \times \dfrac{58}{147} = 6090$, the same answer by a route that never names Owen.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Set up the link between two of the amounts	**B1**	Any one of $1 + 1.45 = 2.45$, $B = 1.45A$, $A : B = 100 : 145$, $100 + 145 = 245$ or $B : C = 3 : 2$. Any letters may be used, provided the ratio is identified with the right pair of people.	✓
A method to find Owen's amount	**M1**	$x + 1.45x = 15\,435$, $\dfrac{15\,435}{2.45}$, $\dfrac{15\,435}{245} \times 100$ or 63×100 seen, or the value 6300 seen anywhere.	✓

Step	Mark	Description	Got it?
A method to find Bilal's amount	**M1**	$15\,435 - 6300$, 1.45×6300 or 145×63, each using the candidate's own value for Owen, or the value 9135 seen anywhere.	✓
A method to find Devika's amount	**M1**	$9135 \times \frac{2}{3}$ or $\frac{9135}{\frac{3}{2}}$ or equivalent, using the candidate's own value for Bilal.	✓
Correct answer	**A1**	6090, correct answer only.	✓
One-step alternative to the first two method marks	**Note**	A candidate who goes straight to Bilal's amount with $\frac{15\,435}{\frac{1}{1.45}+1} = 9135$ or $15\,435 \times \frac{29}{49} = 9135$ earns both method marks at once, so award M2 for either.	✓
Answer with no working	**Note**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 5/5

Question 15 - Exam Solution

Understanding the Question

Given

The function f sends an input x to $\frac{3x+1}{x-2}$.
The rule is one fraction, so its denominator decides which inputs the function can handle.
No domain is stated, no figure is given and there is no data to read off.

Find

(a) The one value of x that no domain of f may contain. (b) A formula for $f^{-1}(x)$, written in terms of x.

Plan the Solution

- For part (a), set the denominator equal to 0 and solve it. That input is the only one the rule cannot handle.
- For part (b), write y for the output, so that $y = \mathrm{f}(x)$, and then rearrange to make x the subject.
- Multiply both sides by $x - 2$ straight away, so no fraction is left to carry through the algebra.
- Gather every term containing x on one side, factorise, then divide by the bracket.
- Finish by renaming y as x, because the answer is asked for in terms of x.

Worked Solution [4 marks]

> Rule - a fraction has no value when its denominator is 0, and a function is inverted by putting $y = \mathrm{f}(x)$, rearranging until x is the subject, then writing x in place of y.

Step 1: (a) find where the rule breaks down

$$x - 2 = 0$$
$$x = 2$$

(Reason: The rule divides by $x - 2$, and division by 0 gives no number at all. Every other input leaves a denominator that is not 0, so 2 is the single value that must be kept out of every domain.)

Step 2: (b) write y for the output and clear the fraction

$$y = \frac{3x + 1}{x - 2}$$
$$y(x - 2) = 3x + 1$$

(Reason: Calling the output y turns the mapping into an equation linking x and y. Multiplying both sides by $x - 2$ clears the fraction, which is what earns the first method mark.)

Step 3: (b) expand, then collect the x terms on one side

$$yx - 2y = 3x + 1$$
$$yx - 3x = 1 + 2y$$

(Reason: Expanding the bracket puts x in two separate places. Moving $3x$ to the left and the term in y to the right leaves every x on one side and nothing else there.)

Step 4: (b) factorise and make x the subject

$$x(y - 3) = 1 + 2y$$
$$x = \frac{1 + 2y}{y - 3}$$

(Reason: Both terms on the left share a factor of x, so taking it outside a bracket leaves a single x multiplied by $y - 3$. Dividing by that bracket makes x the subject, and this factorising step is the second method mark.)

Step 5: (b) write the inverse in terms of x

$$f^{-1}(x) = \frac{1+2x}{x-3}$$

(Reason: The rearranged equation says what input f needed in order to produce y, which is exactly what f^{-1} does. The letter is only a label, so y is renamed x to match the form the question asks for.)

(a) $x = 2$

(b) $\mathrm{f}^{-1}(x) = \dfrac{1+2x}{x-3}$

Verification

✓ **Check 1:** Put $x = 2$ into the top and the bottom of the rule separately, and see what the fraction would ask for. $3 \times 2 + 1 = 7$ and $2 - 2 = 0$, so $\mathrm{f}(2)$ asks for 7 divided by 0, which is not a number.

✓ **Check 2:** Send a number through f and then feed the output back through f^{-1}. $\mathrm{f}(3) = \dfrac{3 \times 3 + 1}{3 - 2} = 10$ and $\mathrm{f}^{-1}(10) = \dfrac{1 + 2 \times 10}{10 - 3} = 3$, the number we started with.

✓ **Check 3:** Do it the other way round, starting with f^{-1} and finishing with f. $\mathrm{f}^{-1}(4) = \dfrac{1 + 2 \times 4}{4 - 3} = 9$ and $\mathrm{f}(9) = \dfrac{3 \times 9 + 1}{9 - 2} = 4$, so the two rules really do undo each other.

✓ **Check 4:** Split the original rule to see which outputs f can never produce, and compare that with the inverse. $\dfrac{3x+1}{x-2} = 3 + \dfrac{7}{x-2}$, and $\dfrac{7}{x-2}$ is never 0, so f never outputs 3. The inverse must leave that value out, and its denominator $x - 3$ does exactly that.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The value left out of every domain	B1	$(x =)2$. Accept $x = 2$ and $x \neq 2$, and accept it stated in words as the value x cannot take. Any response that contains 2 is also acceptable.	✓
(a) Forms that are not accepted	Note	Do not accept the answer written with an inequality sign: $x > 2$, $x < 2$, $x \geq 2$ or $x \leq 2$. Do not accept 2 given alongside another number, for example 2 and 3.	✓

Step	Mark	Description	Got it?
(b) Clear the fraction	**M1**	$y(x-2)=3x+1$ or equivalent, or $yx-2y=3x+1$ or equivalent. The same working with the two letters interchanged, $x(y-2)=3y+1$ or $yx-2x=3y+1$, earns the mark just as well.	✓
(b) Factorise correctly	**M1**	$x(y-3)=1+2y$ or equivalent, or $y(x-3)=1+2x$ or equivalent from the interchanged working. The mark is for the factorising, not for the tidying that follows.	✓
(b) The inverse function	**A1**	$\dfrac{1+2x}{x-3}$ or equivalent, for example $\dfrac{-1-2x}{3-x}$. The answer must be in terms of x.	✓
Answer with no working	**Note**	Working is not required in part (b), so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 16 - Exam Solution

Understanding the Question

Given

A jar holding 20 beads: 15 green and 5 white.
3 beads are taken at random, and none of them is put back.

Find

The probability that the 3 beads include at least one green bead and at least one white bead.

Plan the Solution

- At least one of each colour means anything except all one colour, and there are only two ways to get all one colour, so the complement is much quicker than listing every mixed order.
- Work out the probability of 3 greens, work out the probability of 3 whites, add them, then take the total away from 1.

- Nothing is put back, so the count of that colour and the total both drop by one at every pick.

Worked Solution [4 marks]

> For picks without replacement, multiply the probabilities of the successive picks, taking one off the count of that colour and one off the total each time. Then take the probability that all 3 beads are the same colour away from 1.

Step 1: the probability that all three beads are green

$$\frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} = \frac{2730}{6840} = \frac{91}{228}$$

(Reason: There are 15 green beads out of 20. With one green gone there are 14 green out of 19, and then 13 green out of 18.)

Step 2: the probability that all three beads are white

$$\frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{60}{6840} = \frac{1}{114}$$

(Reason: The same reducing pattern, starting from the 5 white beads: 5 out of 20, then 4 out of 19, then 3 out of 18. Only one white bead is used up at each pick, and the jar holds just 5 of them, so all three whites can still be drawn.)

Step 3: add the two all-one-colour probabilities

$$\frac{91}{228} + \frac{1}{114} = \frac{91}{228} + \frac{2}{228} = \frac{93}{228}$$

(Reason: All green and all white cannot both happen, so the two probabilities simply add. Doubling 114 gives the common denominator 228.)

Step 4: take the all-one-colour probability away from 1

$$1 - \frac{93}{228} = \frac{135}{228} = \frac{45}{76}$$

(Reason: Every draw that is not all green and not all white has at least one bead of each colour, so the complement is the answer. Dividing top and bottom by 3 turns $\frac{135}{228}$ into $\frac{45}{76}$.)

$$\frac{45}{76}$$

Verification

✓ **Check 1 - count the favourable orders instead:** Two greens and a white can come out in 3 orders, each worth $\frac{15}{20} \times \frac{14}{19} \times \frac{5}{18}$, and two whites and a green in 3 orders, each worth $\frac{5}{20} \times \frac{4}{19} \times \frac{15}{18}$. $\frac{3150}{6840} + \frac{900}{6840} = \frac{4050}{6840} = \frac{45}{76}$

✓ **Check 2 - count selections rather than orders:** There are 1140 ways of choosing 3 beads from 20 when the order is ignored. Of those, 455 are all green and 10 are all white, so the rest are mixed. $\frac{1140 - 455 - 10}{1140} = \frac{675}{1140} = \frac{45}{76}$

✓ **Check 3 - the three outcomes must account for everything:** All green, all white and a mixture are the only possibilities, so their probabilities have to total 1. The all-one-colour probability was $\frac{93}{228}$, which cancels to $\frac{31}{76}$. $\frac{45}{76} + \frac{31}{76} = \frac{76}{76} = 1$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One correct product of three fractions	**M1**	For one mixed order, $\frac{15}{20} \times \frac{14}{19} \times \frac{5}{18}$ or $\frac{5}{20} \times \frac{4}{19} \times \frac{15}{18}$ in any order, or for one all-one-colour product, $\frac{15}{20} \times \frac{14}{19} \times \frac{13}{18}$ or $\frac{5}{20} \times \frac{4}{19} \times \frac{3}{18}$. The products must be correct but need not be evaluated, and equivalent decimals to 2 decimal places, truncated or rounded, are accepted.	✓
All the products one route needs	**M1**	For $3 \times \frac{15}{20} \times \frac{14}{19} \times \frac{5}{18}$ or $3 \times \frac{5}{20} \times \frac{4}{19} \times \frac{15}{18}$, or for both mixed products in any order, or for both all-one-colour products.	✓
A complete method using correct products	**M1**	For adding all six mixed orders, or for the complement $1 - \left(\frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} + \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18}\right)$.	✓
The probability	**A1**	$\frac{45}{76}$ or an equivalent fraction. Accept 0.59 to 2 decimal places, truncated or rounded, or 59.2% to 2 significant figures. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Alternative route, using	**Note**	A candidate who works with a green and a white among the first two picks earns the first two method marks together,	✓

Step	Mark	Description	Got it?
the first two beads only		for $\frac{15}{20} \times \frac{5}{19} = \frac{15}{76}$ and $\frac{5}{20} \times \frac{15}{19} = \frac{15}{76}$. This row awards nothing on its own.	

Full marks: 4/4

Question 17 - Exam Solution

Understanding the Question

Given

One fraction with a surd in its denominator: $\frac{1+\sqrt{5}}{3-\sqrt{5}}$.

The useful fact behind the whole method: $\sqrt{5} \times \sqrt{5} = 5$, a whole number.

Find

Integers a and b with $\frac{1+\sqrt{5}}{3-\sqrt{5}} = a + \sqrt{b}$. This is a show-that question, so the marks are for the stages of working, not for the final line on its own.

Plan the Solution

- Multiply the numerator and the denominator by the conjugate of the denominator, $3+\sqrt{5}$. That is multiplying by 1, so the value of the fraction does not change.
- Expand the numerator with all four products, then collect the two $\sqrt{5}$ terms.
- Expand the denominator. The two middle terms cancel, so a whole number is left.
- Divide every term of the numerator by that whole number, then read off a and b.

Worked Solution [3 marks]

> Rule - Rationalising a denominator: multiply the top and the bottom by the conjugate, using $(p-\sqrt{q})(p+\sqrt{q}) = p^2 - q$, which is a difference of two squares and always leaves a whole number underneath.

Step 1: Multiply top and bottom by the conjugate $3+\sqrt{5}$

$$\frac{1+\sqrt{5}}{3-\sqrt{5}} = \frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

(Reason: (Reason: $\dfrac{3+\sqrt{5}}{3+\sqrt{5}}$ is equal to 1, so the value is untouched. The conjugate is chosen because it is the one multiplier that clears the surd from the bottom.))

Step 2: Expand the numerator

$$(1+\sqrt{5})(3+\sqrt{5}) = 3+\sqrt{5}+3\sqrt{5}+5$$

$$3+\sqrt{5}+3\sqrt{5}+5 = 8+4\sqrt{5}$$

(Reason: (Reason: all four products are written out. The last one is $\sqrt{5}\times\sqrt{5}=5$, so $3+5=8$ and the two surd terms collect as $1\sqrt{5}+3\sqrt{5}=4\sqrt{5}$.))

Step 3: Expand the denominator

$$(3-\sqrt{5})(3+\sqrt{5}) = 9+3\sqrt{5}-3\sqrt{5}-5$$

$$(3-\sqrt{5})(3+\sqrt{5}) = 9-5 = 4$$

(Reason: (Reason: the two middle terms are equal and opposite, so they cancel. What is left is 3^2 with $\sqrt{5}\times\sqrt{5}$ taken away from it, and no surd survives.))

Step 4: Divide every term by the denominator

$$\frac{8+4\sqrt{5}}{4} = \frac{8}{4}+\frac{4\sqrt{5}}{4}$$

$$\frac{8}{4}+\frac{4\sqrt{5}}{4} = 2+\sqrt{5}$$

(Reason: (Reason: the fraction bar applies to the whole numerator, so both terms are divided. $\dfrac{4\sqrt{5}}{4}$ leaves $1\sqrt{5}$, which is written $\sqrt{5}$.))

Step 5: Read off a and b

$2+\sqrt{5} = a+\sqrt{b}$ gives $a=2$ and $b=5$

(Reason: (Reason: the whole-number part is a and the number under the root is b. Both are integers, which is exactly what the question asked to be shown.))

$$\frac{1+\sqrt{5}}{3-\sqrt{5}} = 2+\sqrt{5}$$

so $a=2$ and $b=5$, both integers

Verification

✓ **Check 1:** Multiply the answer back by the original denominator. If $2+\sqrt{5}$ is right, $(2+\sqrt{5})(3-\sqrt{5})$ must return the original numerator $1+\sqrt{5}$.

$(2+\sqrt{5})(3-\sqrt{5}) = 6-2\sqrt{5}+3\sqrt{5}-5 = 1+\sqrt{5}$

✓ **Check 2:** A calculator is allowed here, so work out the original fraction and the answer as decimals and compare them. $\dfrac{1+\sqrt{5}}{3-\sqrt{5}} \approx 4.236$ and $2+\sqrt{5} \approx 4.236$

✓ **Check 3:** Test the form itself. The question demands integers, so 4 must divide both terms of $8+4\sqrt{5}$ exactly, and the surd must be left with a coefficient of 1. $\dfrac{8}{4}=2$ and $\dfrac{4}{4}=1$, so $a=2$ and $b=5$ are integers and the answer is genuinely in the form $a+\sqrt{b}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Multiply the numerator and the denominator by $3+\sqrt{5}$	**M1**	For rationalising the denominator by multiplying the numerator and the denominator by $3+\sqrt{5}$, or by $-3-\sqrt{5}$, or equivalent.	✓
Expand both brackets	**M1**	Numerator correctly expanded, and may be simplified to at least 2 terms, such as $8+4\sqrt{5}$; and denominator correctly expanded, and may be simplified to 1 term, such as 4.	✓
Simplify to the required form	**A1 dep on M2**	For $2+\sqrt{5}$ from correct working. Dependent on both method marks.	✓
Working required	**Note**	This row earns nothing. The mark scheme states that working is required, so the answer $2+\sqrt{5}$ written down with no rationalising stage shown scores no marks at all.	✓

Full marks: 3/3

Question 18 - Exam Solution

Understanding the Question

Given

The curve C has equation
$y = x^3 - 40x + 1$
The gradient at each of the points wanted is 8

Find

The coordinates of both points on C where the gradient is 8 The gradient function of a cubic is a quadratic, so expect 2 points, one with a positive x and one with a negative x

Plan the Solution

- Differentiate the equation of the curve to get the gradient function $\frac{dy}{dx}$
- Set that gradient function equal to 8 and solve the quadratic for x
- Substitute each value of x back into the equation of the curve to get its y, then pair the two coordinates up

Worked Solution [5 marks]

> Rule - Gradient of a curve: the gradient at any point is $\frac{dy}{dx}$, so a question that gives you a gradient is telling you to solve $\frac{dy}{dx} = 8$. Differentiate term by term, using $\frac{d}{dx}(ax^n) = anx^{n-1}$

Step 1: differentiate the equation of the curve

$y = x^3 - 40x + 1$

$\frac{dy}{dx} = 3x^2 - 40$

(Reason: Each term drops one power and is multiplied by the old power: x^3 becomes $3x^2$, the $-40x$ becomes -40 because x has power 1, and the constant 1 differentiates to nothing at all.)

Step 2: set the gradient function equal to 8 and solve

$3x^2 - 40 = 8$

$3x^2 = 48$

$x^2 = 16$

$x = 4$ or $x = -4$

(Reason: The gradient function is a quadratic, so $x^2 = 16$ has two square roots and both of them count. Losing the negative root here is what turns a five-mark answer into a three-mark one.)

Step 3: find the y-coordinate at x = 4

$y = 4^3 - 40 \times 4 + 1 = 64 - 160 + 1 = -95$

(Reason: The x values come from the gradient function, but the y values must come from the curve itself, so substitute back into $y = x^3 - 40x + 1$ and not into the derivative.)

Step 4: find the y-coordinate at x = -4

$y = (-4)^3 - 40 \times (-4) + 1 = -64 + 160 + 1 = 97$

(Reason: Watch both signs: cubing a negative keeps it negative, so $(-4)^3 = -64$, while $-40 \times (-4)$ is a minus times a minus and so comes out positive.)

First point $(4, -95)$

Second point $(-4, 97)$

Verification

✓ **Check 1:** Put both answers back into the gradient function. Squaring either 4 or -4 gives 16, so both points must give the same gradient. $3 \times 16 - 40 = 8$ at each point, as required

✓ **Check 2:** Use symmetry. The part $x^3 - 40x$ is an odd function, so the two points sit symmetrically about the point of the curve where $x = 0$. The midpoint of the two answers should therefore have height 1, which is what the curve gives at $x = 0$. $\dfrac{-95 + 97}{2} = 1$, matching the curve at $x = 0$

✓ **Check 3:** Measure the gradient at $x = 4$ from the curve alone, with no differentiation: take the chord from $x = 3.999$ to $x = 4.001$ and divide the change in y by the change in x. the chord gradient is 8.000001, which is 8 to every figure a calculator would show

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$3x^2$ or -40	**M1**	for differentiating one of the first two terms correctly	✓
$3x^2 - 40$	**A1**	for both terms correct and no additions	✓
$3x^2 - 40 = 8$	**M1ft**	dep on M1, for equating their quadratic derivative with 8. The derivative must be of the form	✓

Step	Mark	Description	Got it?
		$ax^2 - 40$ or $3x^2 - b$ with $a \neq 0$ and $b \neq 0$	
$y = 4^3 - 40 \times 4 + 1$ or $y = (-4)^3 - 40 \times (-4) + 1$	**M1ft**	dep on the previous M1, for substituting at least one x value into y	✓
$(4, -95)$ and $(-4, 97)$	**A1**	both coordinates must be paired correctly	✓
Working not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	**Note**	Following through from $ax^2 - 40 = 8$ or $3x^2 - b = 8$, their x values must be correct.	✓

Full marks: 5/5

Question 19 - Exam Solution

Understanding the Question

Given

Quadrilateral $ABCD$, with $AB = 15.2$ m, $AD = 13.4$ m and $BC = 12.8$ m. Angle $BDC = 40°$ and angle $BCD = 62°$.
The diagonal BD splits the shape into triangle ABD and triangle BCD. It is the only length that belongs to both.

Find

The value of x, which is the size of angle DAB in degrees, correct to 3 significant figures.

Plan the Solution

- Triangle BCD is the only one with enough information to start: it has two angles and the side $BC = 12.8$ m opposite one of them, which is the sine rule situation.
- Use the sine rule there to find the diagonal BD.
- Triangle ABD then has all three sides and no angle, which is the cosine rule situation. Rearrange it for $\cos x$.
- Keep the unrounded BD in the calculator throughout and round only at the very end.

Worked Solution [5 marks]

Rule - Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$, each side paired with the angle opposite it. Cosine rule, rearranged for an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Step 1: pair each side with its opposite angle in triangle BCD

$$\frac{BD}{\sin 62^\circ} = \frac{12.8}{\sin 40^\circ}$$

(Reason: BD is opposite the 62° angle at C, and the 12.8 m side is opposite the 40° angle at D, so those are the two pairs the sine rule needs)

Step 2: work out the diagonal BD

$$BD = \frac{12.8 \times \sin 62^\circ}{\sin 40^\circ} = 17.58\ldots \text{ m}$$

(Reason: multiplying both sides by $\sin 62^\circ$ leaves BD on its own; the full unrounded value stays in the calculator for the next step)

Step 3: put the cosine rule into triangle ABD

$$BD^2 = 13.4^2 + 15.2^2 - 2 \times 13.4 \times 15.2 \times \cos x$$

$$309.13\ldots = 179.56 + 231.04 - 407.36 \cos x$$

(Reason: x sits between the two known sides AD and AB, and $BD = 17.58\ldots$ m is the side opposite it, so all three sides are known and only $\cos x$ is not)

Step 4: rearrange for the cosine

$$\cos x = \frac{13.4^2 + 15.2^2 - 17.58\ldots^2}{2 \times 13.4 \times 15.2}$$

$$\cos x = \frac{410.6 - 309.13\ldots}{407.36} = 0.2490\ldots$$

(Reason: the cosine rule is one equation in one unknown here, so the two squares go to one side and the product $2 \times 13.4 \times 15.2 = 407.36$ divides through)

Step 5: undo the cosine

$$x = \cos^{-1}(0.2490\ldots) = 75.577\ldots$$

$$x = 75.6 \text{ (3 s.f.)}$$

(Reason: the cosine came out positive, so x is acute; rounding only at this last line keeps every figure of BD in play)

$$x = 75.6$$

Verification

✓ **Check 1 - rebuild the diagonal from the answer:** Run the cosine rule forwards instead of backwards: put $x = 75.6$ into $BD = \sqrt{410.6 - 407.36 \cos x}$. $\sqrt{410.6 - 407.36 \times \cos 75.6^\circ} = 17.586\ldots$, which is 17.6 m to 3 significant figures - the same as the $17.58\ldots$ m the sine rule gave.

✓ **Check 2 - close the angles of triangle ABD:** The cosine rule also gives the other two angles of triangle ABD: angle $ADB = 56.85^\circ$ and angle $ABD = 47.57^\circ$. A triangle that has been solved correctly must close. $75.58^\circ + 56.85^\circ + 47.57^\circ = 180^\circ$

✓ **Check 3 - is the size sensible?** In triangle ABD the longest side is $BD = 17.58\ldots$ m, so the largest angle must be the one facing it, which is x. And $13.4^2 + 15.2^2 = 410.6$ is bigger than $BD^2 = 309.13\ldots$, so that angle is acute. 75.6° is the largest of the three angles and is under 90°, as both tests require.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Sine rule set up in triangle BCD: $\dfrac{BD}{\sin 62^\circ} = \dfrac{12.8}{\sin 40^\circ}$ oe	**M1**	for correct use of the sine rule for BD	✓
$BD = \dfrac{12.8}{\sin 40^\circ} \times \sin 62^\circ = 17.5(82\ldots)$	**M1**	for finding BD, truncated or rounded	✓
$17.5(82\ldots)^2 = 13.4^2 + 15.2^2 - 2 \times 13.4 \times 15.2 \times \cos x$	**M1**	for correct use of the cosine rule	✓
$\cos x = \dfrac{13.4^2 + 15.2^2 - 17.5(82\ldots)^2}{2 \times 13.4 \times 15.2}$ oe, or $\cos x = 0.247$ to 0.256 oe	**M1**	for a correct rearrangement of $\cos x$	✓
$x = 75.6$	**A1**	accept 75.1 to 75.7. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Step	Mark	Description	Got it?
Alternative route to BD, through CD: $CD = \dfrac{12.8}{\sin 40°} \times \sin 78° = 19.4(781\ldots)$ and $BD = \dfrac{19.4(781\ldots)}{\sin 78°} \times \sin 62° = 17.5(82\ldots)$	M2	this pair of lines replaces the first two rows and is worth the same two marks; the third angle of triangle BCD is $180° - 40° - 62° = 78°$	✓

Full marks: 5/5

Question 20 - Exam Solution

Understanding the Question

Given

A sector $OABC$ of a circle with centre O
Angle $AOC = 60°$
The area of the shaded segment ABC is 38 cm²
The radius is not given, and every length in the question depends on it.

Find

The perimeter of the shaded segment ABC, correct to one decimal place The perimeter of a segment is its arc plus its chord, so the radius has to be found first.

Plan the Solution

- The segment is what is left when triangle OAC is cut away from the sector, so its area is the sector's area minus the triangle's area.
- Both of those are a multiple of r^2, so the given area becomes one equation in r^2 alone. One division and one square root then give the radius.
- The boundary of the segment is its curved edge plus its straight edge: the arc ABC plus the chord AC. The two radii lie inside the sector and are not part of that boundary.
- Because $OA = OC$ and the angle between them is $60°$, triangle OAC is equilateral, so the chord is the same length as the radius and needs no separate calculation.

Worked Solution [4 marks]

Rule - Segment of a circle: its area is $\frac{60}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin 60^\circ$ and its perimeter is the arc $\frac{60}{360} \times 2\pi r$ plus the chord AC.

Step 1: write the area of the segment in terms of r

$$\frac{60}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \times \sin 60^\circ = 38$$

(Reason: the sector is $\frac{60}{360}$ of the whole circle, and triangle OAC has two sides of length r with 60° between them, so its area is $\frac{1}{2} r^2 \sin 60^\circ$. Taking the triangle away from the sector leaves exactly the shaded segment.)

Step 2: collect the two terms into a single number

$$\frac{\pi}{6} - \frac{\sqrt{3}}{4} = 0.5235988 - 0.4330127$$

$$0.5235988 - 0.4330127 = 0.0905861$$

(Reason: both terms carry a factor of r^2, so the equation becomes $r^2 \times 0.0905861 = 38$. Keeping seven decimal places here is deliberate: this number is a small difference between two much larger ones, so rounding it early would move the final answer.)

Step 3: divide to find r^2, then take the square root

$$\frac{38}{0.0905861} = 419.49$$

$$r = \sqrt{419.49} = 20.4815$$

(Reason: dividing both sides by 0.0905861 leaves r^2 on its own, and the radius is its positive square root, because a length cannot be negative.)

Step 4: the length of the arc ABC

$$\frac{60}{360} \times 2 \times \pi \times 20.4815 = 21.4481$$

(Reason: the arc is the same fraction of the circumference, $\frac{60}{360}$, that the sector is of the whole circle.)

Step 5: the length of the chord AC

$$OA = OC = 20.4815$$

$$AC = 20.4815$$

(Reason: two sides of triangle OAC are radii, so they are equal, and the angle between them is 60°. The other two angles are equal to each other and together make 120°, so each of them is 60° as well and the triangle is equilateral. The chord is therefore the radius itself.)

Step 6: add the two edges of the segment

$21.4481 + 20.4815 = 41.9296$

(Reason: the boundary of the shaded segment is made of exactly two pieces, the curved arc ABC and the straight chord AC. Nothing else lies on its edge.)

Step 7: round to one decimal place

$41.9296 \to 41.9$ cm

(Reason: the digit after the first decimal place is a 2, which is less than 5, so the first decimal place is left as it is.)

41.9 cm

Verification

✓ **Check 1:** Put the unrounded $r^2 = 419.4905$ back into the two areas the segment is built from: the sector, $\frac{\pi}{6} \times 419.4905$, and the triangle, $\frac{\sqrt{3}}{4} \times 419.4905$. Their difference must return the area the question gives. $219.6447 - 181.6447 = 38$

✓ **Check 2:** Divide the perimeter by the radius. Every 60° segment is the same shape, so this ratio is fixed at $\frac{\pi}{3} + 1 = 2.0472$ whatever the radius happens to be. $\frac{41.9296}{20.4815} = 2.0472$

✓ **Check 3:** Scale a segment up instead. A 60° segment of a circle of radius 1 has perimeter 2.0472, and multiplying every length by 20.4815 must give the perimeter found above. $2.0472 \times 20.4815 = 41.93$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct expression for the area of the segment, e.g. $\frac{60}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin 60^\circ = 38$	M1	for a correct expression for the area of the segment, or equivalent. The expression may be embedded in an equation, e.g. $\frac{60}{360} \times \pi r^2 = 38 + \frac{1}{2} r^2 \sin 60^\circ$, or written with the two coefficients already simplified as $\frac{\pi r^2}{6} - \frac{\sqrt{3}}{4} r^2$.	✓
A correct expression for r^2 or r, e.g.	M1	dependent on the first M1, for a correct expression for r^2 or r. The values these come	✓

Step	Mark	Description	Got it?
$r^2 = \dfrac{38}{\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}} =$ 419.49		to are 419.490 and 20.4815, and a value rounded or truncated from either is accepted.	
Using the radius to find the arc, e.g. $\dfrac{60}{360} \times 2 \times \pi \times 20.4815 = 21.4481$	M1	for using their value of r to find the arc length, e.g. $\dfrac{\pi}{6} \times 20.4815 \times 2$, or equivalent. Follow through on their radius.	✓
41.9	A1	for 41.9 cm. Allow anything from 41 to 42, which covers a radius that was rounded or truncated on the way. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working.	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

A curve with equation $y = \mathrm{f}(x)$, and nothing else about f at all.
Its one and only minimum point is at $(5, -4)$.

Find

(i) the minimum point of $y = \mathrm{f}(x + 7)$
(ii) the minimum point of $y = \mathrm{f}(x) - 6$

Plan the Solution

- Decide, for each equation, whether the change is INSIDE the bracket or OUTSIDE it. Inside means a horizontal translation; outside means a vertical one.
- A translation slides the whole curve without turning or stretching it, so the minimum point stays a minimum point. Only its coordinates move.
- Apply the translation to the single point $(5, -4)$. A horizontal translation changes only the x-coordinate; a vertical one changes only the y-coordinate.

- Watch the sign inside the bracket: $+7$ inside moves the curve in the negative x direction, not the positive one.

Worked Solution [2 marks]

> Rule - Translating a curve: $y = \mathrm{f}(x + a)$ moves every point a units in the negative x direction, and $y = \mathrm{f}(x) + b$ moves every point b units in the positive y direction.

Step 1: read what $y = \mathrm{f}(x + 7)$ does

$$y = \mathrm{f}(x + 7)$$

(Reason: The $+7$ is inside the bracket, so it acts on x before f does anything: the translation is horizontal. The curve reaches its lowest point when the input to f is 5, which now happens when $x + 7 = 5$, so the whole curve has slid 7 units in the negative x direction.)

Step 2: translate the minimum point for part (i)

$$(5 - 7, -4) = (-2, -4)$$

(Reason: Only the x-coordinate is affected. A horizontal translation cannot change how high a point sits, so the y-coordinate is still -4.)

Step 3: read what $y = \mathrm{f}(x) - 6$ does

$$y = \mathrm{f}(x) - 6$$

(Reason: The -6 is outside the bracket, so it is applied after f has done its work: every output is 6 smaller, so the whole curve drops 6 units.)

Step 4: translate the minimum point for part (ii)

$$(5, -4 - 6) = (5, -10)$$

(Reason: Only the y-coordinate is affected. A vertical translation cannot move a point sideways, so the x-coordinate is still 5.)

(i) $(-2, -4)$

(ii) $(5, -10)$

Verification

✓ **Check 1:** Ask where the new curve repeats the old low point. $y = \mathrm{f}(x + 7)$ takes the value $\mathrm{f}(5)$ when $x + 7 = 5$, that is when $x = -2$, and the height there is the same $\mathrm{f}(5) = -4$. the minimum is at $(-2, -4)$

✓ **Check 2:** Test both parts on a curve that really does have its minimum at $(5, -4)$. Take $\mathrm{f}(x) = (x - 5)^2 - 4$. Then $\mathrm{f}(x + 7) = (x + 2)^2 - 4$, whose minimum is where

$x + 2 = 0$, and $f(x) - 6 = (x-5)^2 - 10$. $(-2, -4)$ for (i) and $(5, -10)$ for (ii)

✓ **Check 3:** Count which coordinate is allowed to move. Part (i) is a horizontal translation, so the y-coordinate must survive unchanged as -4; part (ii) is a vertical translation, so the x-coordinate must survive unchanged as 5. each answer changes exactly one coordinate, and it is the expected one

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) minimum point of $y = f(x+7)$	**B1**	cao $(-2, -4)$. Accept $x = -2$ and $y = -4$ written separately, or the pair labelled on a sketch.	✓
(ii) minimum point of $y = f(x) - 6$	**B1**	cao $(5, -10)$. Accept $x = 5$ and $y = -10$ written separately.	✓
Guidance on the two common slips	**Note**	A translation applied the wrong way round - moving the curve to the right for $y = f(x+7)$, or upwards for $y = f(x) - 6$ - scores nothing. There is no method mark in either part: the answer is written down, so each mark is all or nothing.	✓

Full marks: 2/2

Question 22 - Exam Solution

Understanding the Question

Given

A histogram of the distances cycled by 100 members, with three bars already drawn.
Every distance is at least 5 kilometres and at most 55 kilometres, so the bars must cover 5 to 55 with no gaps.
14 members are in the 15 to 20 class - the only frequency the question states.

Find

The one bar that is missing: the distances it covers, and how tall to draw it. Expect a frequency density, not a frequency - the height of a histogram bar is $\frac{\text{frequency}}{\text{class width}}$.

The vertical axis carries no numbers, so the scale has to be worked out first.

Plan the Solution

- Use the one stated frequency, 14 over the 15 to 20 class, to fix what one small square is worth.
- Read the other two bars off that scale and turn each height back into a frequency.
- Subtract the three frequencies from 100 to get the missing frequency.
- The missing class runs from 5 to 15, so divide by that width to get the height to draw.

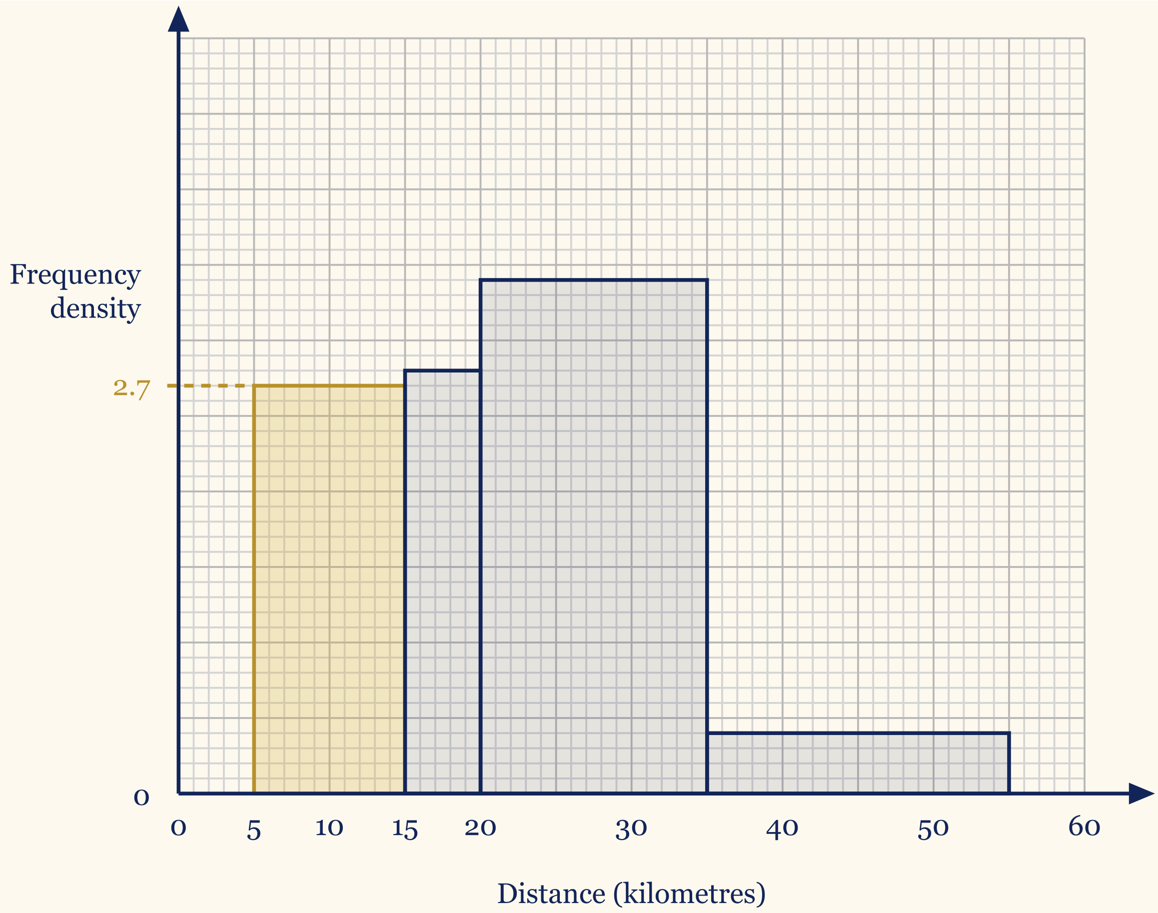
Worked Solution [3 marks]

> Rule - Histograms: the AREA of a bar is its frequency, so
> $\text{frequency density} = \dfrac{\text{frequency}}{\text{class width}}$.

Step 1: use the stated frequency to fix the scale

$$\frac{14}{20-15} = \frac{14}{5} = 2.8$$

$$\frac{2.8}{28} = 0.1$$



(Reason: (Reason: the 15 to 20 bar is 28 small squares tall and stands for a frequency density of 2.8, so one small square is 0.1 on the vertical axis. Nothing else on the graph can be read until this is known.))

Step 2: read the other two bars off that scale

$$34 \times 0.1 = 3.4$$
$$3.4 \times (35 - 20) = 51$$
$$4 \times 0.1 = 0.4$$
$$0.4 \times (55 - 35) = 8$$

(Reason: (Reason: the middle bar is 34 small squares tall and the flat one is 4. Multiplying each frequency density by its class width turns the area of the bar back into a number of members.))

Step 3: take the known frequencies away from the total

$$14 + 51 + 8 = 73$$
$$100 - 73 = 27$$

(Reason: (Reason: every member is somewhere on the histogram, so the four bars must account for all 100. The three drawn bars hold 73 of them, leaving 27 for the missing one.))

Step 4: turn that frequency into a height

$$15 - 5 = 10$$

$$\frac{27}{10} = 2.7$$

(Reason: (Reason: the shortest distance is 5 kilometres and the next bar starts at 15, so the missing class is 5 to 15. Dividing the frequency by that class width gives the frequency density, which is what the vertical axis measures.))

Step 5: draw the bar

$2.7 \times 10 = 27$

(Reason: (Reason: a bar from 5 to 15 with its top at 2.7 has area 27, which is exactly the 27 members it has to represent. On the printed grid that top edge is 27 small squares above the horizontal axis.))

A bar from 5 to 15 kilometres drawn at a frequency density of 2.7

Verification

✓ **Check 1 - do the four bars hold 100 members?** Work out the area of every bar and add them, using frequency density times class width. $2.7 \times 10 + 14 + 51 + 8 = 100$

✓ **Check 2 - count small squares instead of using the scale:** One small square is 1 kilometre across and 0.1 up, so ten small squares stand for one member. The missing bar is 10 squares across and 27 up. $\frac{10 \times 27}{10} = 27$

✓ **Check 3 - count large squares:** The heavy grid lines fall every 5 small squares, so one large square is 25 small squares, which is 2.5 members. The completed histogram covers 40 large squares. $40 \times 2.5 = 100$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Fix the frequency density scale	M1	for finding the frequency density $\frac{14}{5} = 2.8$, or a correct value on the frequency density scale, or ten small squares standing for one member oe, or one large square standing for 2.5 members oe, or 51 and 8 assigned to the correct bars	✓
Find the missing frequency	M1	for a method to find the total frequency of the bars given, or a method to find the missing frequency: $14 + 51 + 8 = 73$ oe, or $100 - (14 + 51 + 8) = 27$ oe	✓

Step	Mark	Description	Got it?
Draw the bar	**A1**	for the correct bar, frequency 27: drawn from 5 to 15 with its top at 2.7	✓
Special case - right height, wrong left edge	**SC B2**	for a bar of height 2.7 drawn from 0 to 15: the frequency density is right, but the bar has been started at the axis instead of at 5	✓
Special case - divided by the wrong width	**SC B2**	for a bar of height 1.8 drawn from 0 to 15: the missing frequency of 27 has been spread over a width of 15 instead of 10	✓
Working is not required	**Note**	A correct histogram scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 23 - Exam Solution

Understanding the Question

Given

A cone of base radius $5x$ cm and vertical height $6x$ cm

A hemisphere of radius $2x$ cm removed from the cone's top face

The volume of what is left is 6948π cm³

Find

The total surface area of the solid hemisphere that was removed, to the nearest integer

Plan the Solution

- What is left is the cone minus the hemisphere, so write both volumes in terms of x.
- Every length is a multiple of x, so both volumes come out as a multiple of πx^3. Subtract, set the result equal to 6948π and solve for x^3.
- Take the cube root to get x, then the hemisphere's own radius $r = 2x$.
- The removed hemisphere is SOLID, so its total surface area is the curved half-sphere plus the flat circular face it was cut on.

Worked Solution [5 marks]

Cone: $V = \frac{1}{3}\pi r^2 h$. Hemisphere: $V = \frac{2}{3}\pi r^3$. Total surface area of a solid hemisphere: $2\pi r^2 + \pi r^2 = 3\pi r^2$.

Step 1: Write the volume of the cone in terms of x

$$V_{\text{cone}} = \frac{1}{3}\pi(5x)^2(6x)$$

$$V_{\text{cone}} = \frac{1}{3}\pi \times 25x^2 \times 6x = 50\pi x^3$$

(Reason: The base radius is $5x$ and the vertical height is $6x$. Squaring the radius gives $25x^2$, and $\frac{1}{3} \times 25 \times 6 = 50$.)

Step 2: Write the volume of the hemisphere that is removed

$$V_{\text{hemisphere}} = \frac{1}{2} \times \frac{4}{3}\pi(2x)^3$$

$$V_{\text{hemisphere}} = \frac{2}{3}\pi \times 8x^3 = \frac{16}{3}\pi x^3$$

(Reason: A hemisphere is half a sphere, and the sphere here has radius $2x$. Cubing the whole bracket gives $8x^3$. This is where a missing bracket costs the mark, since $2x^3$ is not $(2x)^3$.)

Step 3: Form an equation for the volume of the ornament and solve for x^3

$$50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$$

$$\frac{134}{3}\pi x^3 = 6948\pi$$

$$x^3 = \frac{6948 \times 3}{134} = 155.552\ldots$$

(Reason: What is left is the cone minus the hemisphere, and $50 - \frac{16}{3} = \frac{134}{3}$. Every term carries a factor of π, so the π cancels from both sides before x^3 is found.)

Step 4: Take the cube root to find x

$$x = \sqrt[3]{155.552\ldots} = 5.378\ldots$$

(Reason: Keep the unrounded value in the calculator. Rounding x to 5.4 here would drag the final area off by several square centimetres.)

Step 5: Work out the total surface area of the solid hemisphere

$$r = 2x = 2 \times 5.378\ldots = 10.756\ldots$$
$$S = 3\pi r^2 = 3\pi \times (10.756\ldots)^2$$

$S = 1090.39\ldots$

(Reason: The hemisphere is solid, so its surface is the curved half-sphere $2\pi r^2$ together with the flat circular face πr^2, which is $3\pi r^2$ altogether. To the nearest integer $1090.39\ldots$ is 1090.)

1090 cm²

Verification

✓ **Check 1:** Put the value back into the volume. The cone is $50\pi x^3$ and the hemisphere is $\frac{16}{3}\pi x^3$, so what is left is $\frac{134}{3}\pi x^3$. $\frac{134}{3} \times 155.552\ldots = 6948$, so the volume really is 6948π cm³

✓ **Check 2:** Add the two surfaces separately instead of using $3\pi r^2$. With $r = 10.756\ldots$ cm the curved part is $2\pi r^2$ and the flat circle is πr^2. $726.93\ldots + 363.46\ldots = 1090.39\ldots$, the same total

✓ **Check 3:** A whole sphere of radius $10.756\ldots$ cm has surface area $4\pi r^2 = 1453.85\ldots$. A solid hemisphere is three quarters of that: half the curved surface plus one flat circle. $\frac{3}{4} \times 1453.85\ldots = 1090.39\ldots$, which agrees

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Volume of the cone, or of the hemisphere, in terms of x	M1	For $\frac{1}{3}\pi \times (5x)^2 \times 6x$ oe or $50\pi x^3$ oe, or for $\frac{1}{2} \times \frac{4}{3} \times \pi \times (2x)^3$ oe or $\frac{16}{3}\pi x^3$ oe, or for the whole sphere $\frac{4}{3} \times \pi \times (2x)^3$ oe or $\frac{32}{3}\pi x^3$ oe. Ignore missing brackets around $5x$ and $2x$ for this mark.	✓
A correct equation for the volume of the shape	M1	For $\frac{1}{3}\pi \times (5x)^2 \times 6x - \frac{1}{2} \times \frac{4}{3} \times \pi \times (2x)^3 = 6948\pi$ oe, or $50\pi x^3 - \frac{16}{3}\pi x^3 = 6948\pi$ oe, or $\frac{134}{3}\pi x^3 = 6948\pi$ oe. If it is not expanded at this stage then the brackets must be seen.	✓

Step	Mark	Description	Got it?
Rearranging the correct equation to find x^3 or x	**M1**	For $x^3 = \dfrac{6948\pi \times 3}{134\pi} = 155.552\ldots$ oe, or $x = \sqrt[3]{155.552\ldots} = 5.37(8\ldots)$ oe. Accept 5.4 or better.	✓
Surface area of the hemisphere	**M1**	For $3 \times \pi \times (2 \times 5.37\ldots)^2$ oe or $12 \times \pi \times (5.37\ldots)^2$ oe.	✓
The answer	**A1**	1090, allow 1086 to 1100. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Special case: using 6948 without the π	**SC**	SC B3 for $x^3 = 49.5(138\ldots)$ or $x = 3.67(205\ldots)$, and SC B4 for a final answer of awrt 508. This is the one wrong route the scheme still rewards, and naming it is the point: the π must be cancelled from both sides, not divided out of one.	✓

Full marks: 5/5

Question 24 - Exam Solution

Understanding the Question

Given

A polygon with n sides, where $n > 5$
Its interior angles, in order of size, are an arithmetic sequence with first term $a = 84$ and common difference $d = 4$
Standard fact: the interior angles of any n -sided polygon add up to $(n-2) \times 180°$

Find

The total of all n interior angles, in degrees Expect a quadratic in n, so expect two roots and only one that satisfies $n > 5$

Plan the Solution

- Write that same total twice: once as an arithmetic series in n, once with the polygon angle rule.

- Set the two expressions equal, since they count the same angles, and tidy the result into a quadratic.
- Solve the quadratic, then use the condition $n > 5$ to decide which root is this polygon.
- Substitute that value of n back into $(n-2) \times 180$ to get the sum.

Worked Solution [6 marks]

> Rule: an arithmetic series with first term a and common difference d has $S_n = \frac{n}{2}[2a + (n-1)d]$, and the interior angles of an n-sided polygon sum to $(n-2) \times 180°$.

Step 1: write the angle total as an arithmetic series

$$S_n = \frac{n}{2}[2(84) + (n-1)(4)]$$

$$S_n = \frac{n}{2}[168 + 4n - 4] = \frac{n}{2}[164 + 4n]$$

$$S_n = 82n + 2n^2$$

(Reason: The n angles are an arithmetic sequence with $a = 84$ and $d = 4$, so their total is the sum of the first n terms. Halving the bracket now keeps the algebra tidy.)

Step 2: write the same total using the polygon rule

$$(n-2) \times 180 = 180n - 360$$

(Reason: Every polygon with n sides has interior angles adding to $(n-2) \times 180$ degrees, so this is the same total written a second way.)

Step 3: equate the two totals and form a quadratic

$$82n + 2n^2 = 180n - 360$$

$$2n^2 - 98n + 360 = 0$$

$$n^2 - 49n + 180 = 0$$

(Reason: The two expressions count the same angles, so they are equal. Collecting every term on one side gives a three term quadratic, and every coefficient has a factor of 2, so divide right through by 2.)

Step 4: solve the quadratic

$$(n-4)(n-45) = 0$$

$$n = 4 \text{ or } n = 45$$

(Reason: Two numbers multiplying to 180 and adding to 49 are 4 and 45, so it factorises. The quadratic formula gives the same pair: $\sqrt{49^2 - 4(180)} = \sqrt{1681} = 41$.)

Step 5: use the condition on n to choose the root

$n > 5 \implies n = 45$

(Reason: The root $n = 4$ really does solve the equation, since a quadrilateral with angles $84°$, $88°$, $92°$ and $96°$ has an angle sum of $360°$. The question rules it out with $n > 5$, which is exactly why that condition is given.)

Step 6: work out the sum of the interior angles

$(45 - 2) \times 180 = 43 \times 180 = 7740$

(Reason: With $n = 45$ the polygon angle rule gives the total straight away, in degrees.)

$7740°$

Verification

✓ **Check 1:** Total the 45 angles as a series instead of using the polygon rule:

$\frac{45}{2}[2(84) + 44(4)]$. $\frac{45}{2} \times 344 = 45 \times 172 = 7740$

✓ **Check 2:** The largest angle is $84 + 44(4) = 260$, and an arithmetic sequence has mean equal to the average of its first and last terms, so the mean angle is $\frac{84 + 260}{2}$.

$\frac{84 + 260}{2} = 172 \implies 45 \times 172 = 7740$

✓ **Check 3:** Each exterior angle is 180 minus its interior angle, and exterior angles always total 360, so 45×180 minus the answer must come to 360. $8100 - 7740 = 360$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Substitute $a = 84$ and $d = 4$ into $S_n = \frac{n}{2}[2a + (n-1)d]$	M1	For $\frac{n}{2}[2(84) + (n-1)(4)]$ or $\frac{n}{2}[168 + 4n - 4]$ or $\frac{n}{2}[164 + 4n]$ or $82n + 2n^2$ oe	✓
Equate that sum with $(n-2) \times 180$	M1	For $\frac{n}{2}[164 + 4n] = (n-2) \times 180$ or $82n + 2n^2 = (n-2) \times 180$ oe. The sum must come from a correct substitution of a and d into $\frac{n}{2}[2a + (n-1)d]$	✓
Multiply out and collect terms to form a three	M1	For $n^2 - 49n + 180 = 0$ oe, or any form of $an^2 + bn + c = 0$ with at least two of the	✓

Step	Mark	Description	Got it?
term quadratic		coefficients correct. Allow $n^2 - 49n = -180$	
Solve their three term quadratic	**M1ft**	Any correct method, allowing one sign error and some simplification, for example $(n-45)(n-4)=0$ or $n = \dfrac{49 \pm \sqrt{2401-720}}{2}$ or $\left(n - \dfrac{49}{2}\right)^2 - \left(\dfrac{49}{2}\right)^2 = -180$. If factorising, allow brackets which expand to give two of the three terms correct. Or the correct value $n = 45$ (ignore $n = 4$)	✓
Substitute their n to find the angle sum	**M1**	For $(45-2) \times 180$ or $\dfrac{45}{2}[2(84)+(45-1)(4)]$ oe, or the same with 44 in place of 45. Note $n > 5$	✓
State the sum of the interior angles	**A1**	7740. Working required. Accept 7560 or 7480	✓

Full marks: 6/6

Question 25 - Exam Solution

Understanding the Question

Given

$f(x) = 17 - 3x^2 + 12x$

The coefficient of x^2 is -3, so the curve is a parabola that opens downwards.

Find

The constants a, b and c that write $f(x)$ as $a - b(x-c)^2$. The target form has a minus in front of the square, so b is the positive number 3, not -3.

Plan the Solution

- Rewrite the terms in the usual order, then take the factor -3 out of the two x terms only, leaving the 17 outside.
- Complete the square on the simple quadratic left inside the bracket.

- Multiply the whole bracket by -3 again and collect the constants, so the answer ends up in the form the question asks for.

Worked Solution [4 marks]

Rule - Completing the square: take the x^2 coefficient out as a factor first, then use $x^2 + px = (x + \frac{p}{2})^2 - (\frac{p}{2})^2$ on what is left inside the bracket.

Step 1: take the factor -3 out of the x terms

$$f(x) = 17 - 3x^2 + 12x$$
$$f(x) = 17 - 3(x^2 - 4x)$$

(Reason: Only the two x terms go inside the bracket. Dividing $-3x^2$ by -3 gives x^2, and dividing $12x$ by -3 gives $-4x$.)

Step 2: complete the square inside the bracket

$$x^2 - 4x = (x-2)^2 - 4$$

(Reason: Half of -4 is -2, so the square is $(x-2)^2$. Expanding that gives $x^2 - 4x + 4$, which is 4 too big, so 4 is taken off again.)

Step 3: put that back in and multiply through by -3

$$f(x) = 17 - 3[(x-2)^2 - 4]$$
$$f(x) = 17 - 3(x-2)^2 + 12$$

(Reason: Every term inside the square bracket is multiplied by the factor -3, including the constant.)

Step 4: collect the two constants

$$f(x) = 17 + 12 - 3(x-2)^2$$
$$f(x) = 29 - 3(x-2)^2$$

(Reason: The question wants the constant written first, so reading off the form $a - b(x-c)^2$ gives $a = 29$, $b = 3$ and $c = 2$.)

$$f(x) = 29 - 3(x-2)^2$$

Verification

✓ **Check 1:** Expand the answer again and compare it, term by term, with the $f(x)$ the question prints. $29 - 3(x^2 - 4x + 4) = 29 - 3x^2 + 12x - 12 = 17 - 3x^2 + 12x$

✓ **Check 2:** Put $x = 5$ into both forms. They are the same function, so they must give the same value. $17 - 3(5)^2 + 12(5) = 2$ and $29 - 3(5-2)^2 = 2$

✓ **Check 3:** The coefficient of x^2 is negative, so f has a maximum, and the completed square says that maximum is a at $x = c$. Substituting $x = 2$ tests both constants at once. $f(2) = 17 - 3(2)^2 + 12(2) = 29$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Factorise the x terms, e.g. $-3(x^2 - 4x)$, or state $b = 3$.	**M1**	For factorising $-3x^2 + 12x$, or for the correct value of b (that is $b = 3$) embedded in an incorrect final answer of the form $a - 3(x - c)^2$.	✓
$-3[(x-2)^2 \dots]$ or $-3(x-2)^2 \dots$	**M1**	For a correct first step to complete the square.	✓
$-3[(x-2)^2 - (2)^2] \dots$ or $-3(x-2)^2 + 12 \dots$	**M1**	For a correct second step to complete the square, or equivalent.	✓
$f(x) = 29 - 3(x-2)^2$	**A1**	Or equivalent, e.g. $-3(x-2)^2 + 29$. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working.	✓
Alternative method: multiply out $a - b(x-c)^2$ to get $-bx^2 + 2bcx - bc^2 + a$, then equate coefficients.	**Note**	The mark scheme allows this route for the same 4 marks: M1 for multiplying out, M1 for equating coefficients ($2bc = 12$ or $a - bc^2 = 17$), M1 for finding at least two of a, b, c, then A1 for the answer. This row carries no mark of its own.	✓

Full marks: 4/4

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