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# Edexcel IGCSE 4MA1

4MA1/1HR (Higher Tier) – Thursday 16 May 2024

**100 marks · 2 hours · Calculator allowed**

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/1HR (Higher Tier), June 2024 series, sat Thursday 16 May 2024 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

# Practice Questions

*Try each question on paper first. Full worked solutions and mark schemes follow in the next section.*

**1.** Here are six number tiles from a puzzle set.
Five of the tiles already have a number painted on them. The sixth tile is still blank.

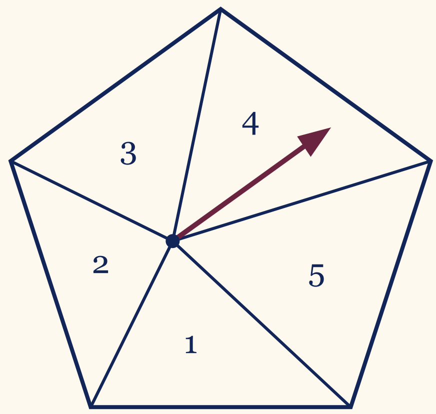
| | | | | | |
|---|---|---|---|---|---|
| 16 | 15 | 3 | 2 | 9 | |

Work out the number that must be painted on the blank tile so that the six numbers have a mean of $11$ *[3 marks]*

..............................

*[Total 3 marks]*

**2.** The five-sided spinner shown below is biased.




The table gives the probability that the spinner lands on each number when it is spun once.

| Number | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Probability** | $2x$ | 0.27 | 0.04 | $x$ | 0.12 |

Bethany is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on an odd number. *[4 marks]*

.......................................

*[Total 4 marks]*

**3.** Matteo sells plain plant pots and painted plant pots on his market stall.

He sells a total of $200$ pots such that

the number of plain pots sold :
the number of painted pots sold $= 3 : 2$

Matteo sells the plain pots for $£1.50$ each.
He sells the painted pots for $£1.75$ each.

$40\%$ of the price of a plain pot is profit.
$60\%$ of the price of a painted pot is profit.

Work out the total profit Matteo makes when he sells all $200$ pots. *[5 marks]*

.................................... £

*[Total 5 marks]*

**4.** Show that

$$\frac{2\frac{1}{3}}{5\frac{1}{4}} = \frac{4}{9}$$

You must show every stage of your working. *[3 marks]*

*[Total 3 marks]*

**5.** Radomir pays $5200$ euros into a savings bond.
The bond pays $2.5\%$ per year compound interest.
Radomir leaves his money in the bond for $4$ years.

Work out how much money Radomir will have in the savings bond at the end of the $4$ years.
Give your answer correct to the nearest euro. *[3 marks]*

.................................... euros

*[Total 3 marks]*

**6.** The diagram shows a solid cylinder cut from a length of hardwood.

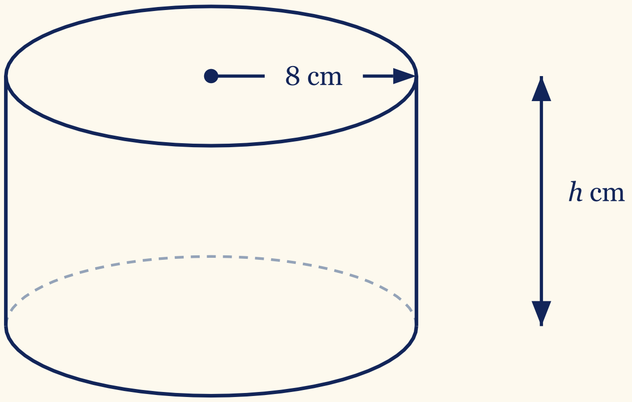



Diagram NOT accurately drawn

The cylinder has radius 8 cm and height $h$ cm.
The volume of the cylinder is 1208 $cm^3$

(a) Work out the value of $h$.
Give your answer correct to the nearest whole number. *[2 marks]*

The density of the hardwood is 1.25 $g/cm^3$

(b) Work out the mass of the cylinder.
Give your answer in kilograms. *[2 marks]*

(a) h = ..............................

(b) kilograms ..............................

*[Total 4 marks]*

**7.**

(a) Simplify $\frac{g^9}{g^2}$ *[1 mark]*

(b) Expand $5k^2(k^3+4)$ *[2 marks]*

(c) (i) Factorise $x^2-2x-63$
[2 marks]
(ii) Hence, solve $x^2-2x-63=0$ *[1 mark]*

(d) Solve the inequality $7-2y<3y-12$ *[3 marks]*

(a) ..............................

(b) ..............................

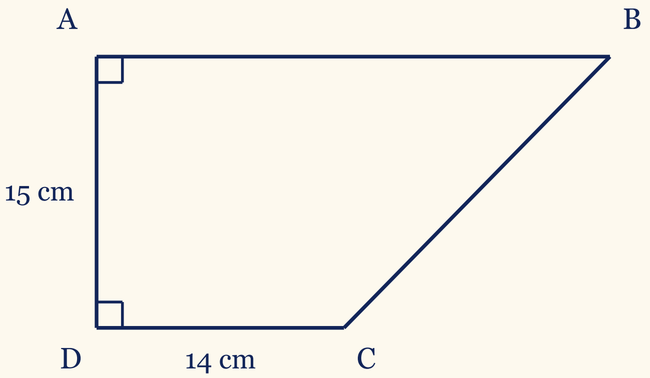
(c)(i) ..............................

(c)(ii) ..............................

(d) ..............................

*[Total 9 marks]*

**8.** $ABCD$ is a trapezium.




Angle $DAB$ and angle $ADC$ are right angles.

$AD = 15$ cm

$DC = 14$ cm

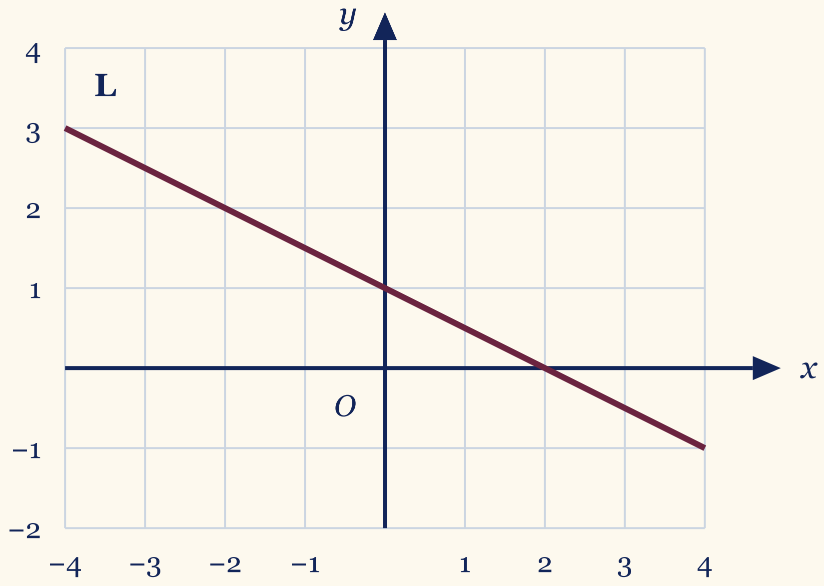
The area of the trapezium is $360$ cm²

Work out the perimeter of the trapezium. *[6 marks]*

.................................... cm

*[Total 6 marks]*

**9.** The straight line $L$ is drawn on the grid below.




Work out an equation of $L$.
Give your answer in the form $y = mx + c$ *[3 marks]*

..............................

*[Total 3 marks]*

**10.** Here are the numbers of loaves of bread left unsold by a bakery at the end of each of its last 11 days.

0 1 2 2 3 4 4 6 7 9 11

Work out the interquartile range of the numbers of loaves. *[2 marks]*

..............................

*[Total 2 marks]*

**11.** Two numbers, $A$ and $B$, are written below as products of their prime factors.

$A = 2^5 \times 5 \times 7^2$
$B = 2^3 \times 5^3 \times 7^4$

(a) Write down the highest common factor (HCF) of $5A$ and $2B$
Write your answer as a product of prime factors. *[2 marks]*

$A = 2^5 \times 5 \times 7^2$
$B = 2^3 \times 5^3 \times 7^4$

(b) Work out the value of $(AB)^2$
Write your answer as a product of prime factors. *[2 marks]*

(a) ..............................

(b) ..............................

*[Total 4 marks]*

**12.** Solve the simultaneous equations
$4x + 3y = 9.6$
$6x + 5y = 16.8$
You must show clear algebraic working. *[4 marks]*

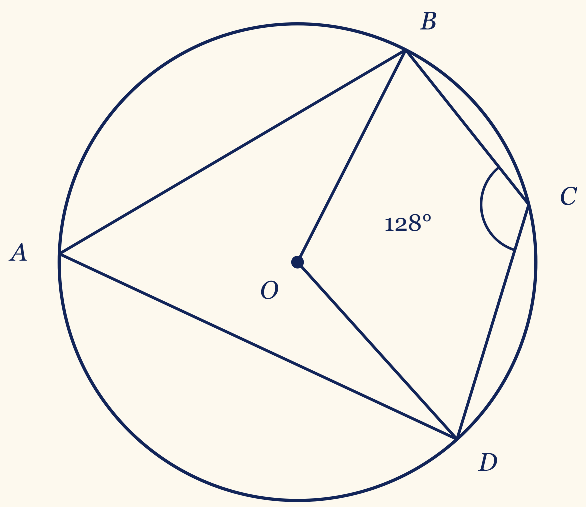
x = ..............................

y = ..............................

*[Total 4 marks]*

**13.** $A$, $B$, $C$ and $D$ are four points on a circle whose centre is $O$.
Angle $BCD$ is $128°$.

Diagram NOT accurately drawn




Work out the size of angle $OBD$.
Give a reason for each stage of your working. *[5 marks]*

angle OBD = ..............................

*[Total 5 marks]*

**14.** (a) Expand the brackets and simplify
$(3x+1)(2-x)(4+x)$ *[3 marks]*

(b) Simplify fully

$$\left(\frac{a^3b}{a^9b^5}\right)^{-\frac{1}{2}}$$ *[3 marks]*

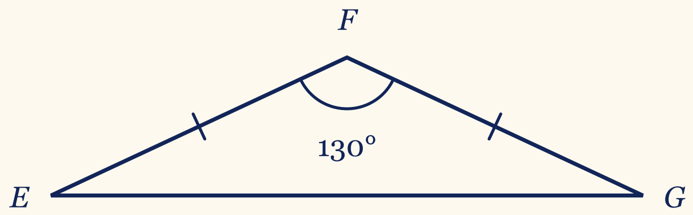
(a) ..............................

(b) ..............................

*[Total 6 marks]*

**15.** The diagram shows an isosceles triangle $EFG$.

Diagram NOT accurately drawn




$EF = GF$

Angle $EFG = 130°$

The area of triangle $EFG$ is $74$ cm²

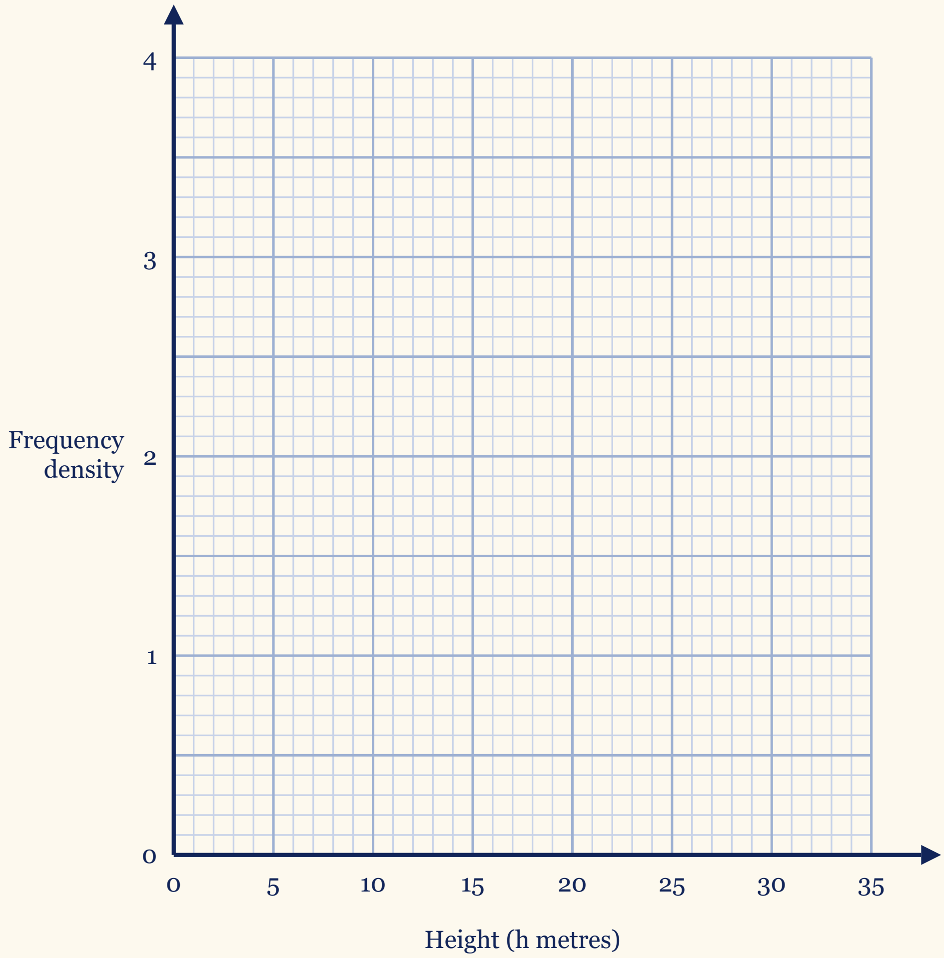
Calculate the length of $EF$.

Give your answer correct to $3$ significant figures. *[3 marks]*

.................................... cm

*[Total 3 marks]*

**16.** The table gives information about the heights, in metres, of the pine trees in a nature reserve.




| **Height** ($h$ **metres**) | **Frequency** |
|---|---|
| $0 < h \leq 2$ | 5 |
| $2 < h \leq 5$ | 12 |
| $5 < h \leq 10$ | 18 |
| $10 < h \leq 20$ | 14 |
| $20 < h \leq 35$ | 9 |

On the grid, draw a histogram for this information. *[3 marks]*

*[Total 3 marks]*

**17.** (a) $\left(\sqrt[4]{k^{12}}\right)^5 = k^n$

Work out the value of $n$. *[1 mark]*

(b) Write $\dfrac{7}{2-\sqrt{3}}$ in the form $\sqrt{c}+d$, where $c$ and $d$ are integers.

You must show your working clearly. *[3 marks]*

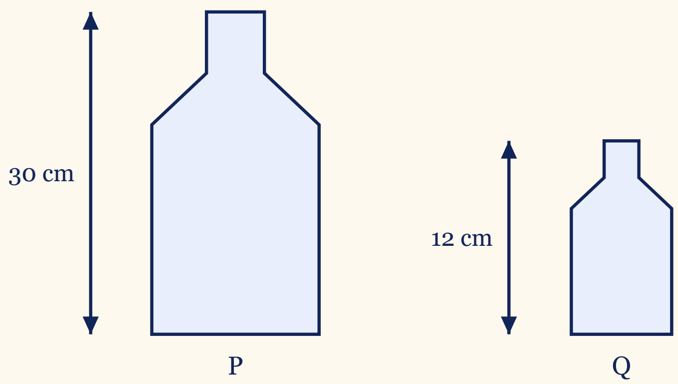
n = ..........................................

(b) ..........................................

*[Total 4 marks]*

**18.** The diagram shows two mathematically similar glass storage jars, $P$ and $Q$

Diagram NOT accurately drawn




The height of jar $P$ is 30 cm
The height of jar $Q$ is 12 cm

Given that

surface area of $P$ − surface area of $Q = 178.5\text{ cm}^2$

find the surface area of jar $P$ *[4 marks]*

.......................................... $\text{cm}^2$

*[Total 4 marks]*

**19.** The curve $C$ has equation $y = x^3 - 8x^2 - 12x + 5$

Curve $C$ has exactly two stationary points, one at the point $P$ and one at the point $Q$, such that

$x$ coordinate of point $P > x$ coordinate of point $Q$

Work out the coordinates of the point $P$.

You must show clear algebraic working. *[5 marks]*

..............................

*[Total 5 marks]*

**20.** (a) Write $2x^2 - 11x + 9$ in the form $a(x-b)^2 - c$, where $a$, $b$ and $c$ are numbers to be found. *[3 marks]*

A curve $C$ has equation $y = 2(x-3)^2 - 11(x-3) + 9$

The minimum point on $C$ is $P$

(b) Work out the coordinates of $P$ *[2 marks]*

(a) ..............................

(b) ..............................

*[Total 5 marks]*

**21.** There are $25$ beads in a jar such that

$6$ beads are green
$x$ beads are amber, where $x > 9$
the rest of the beads are cream

Idris takes at random two of the beads from the jar.

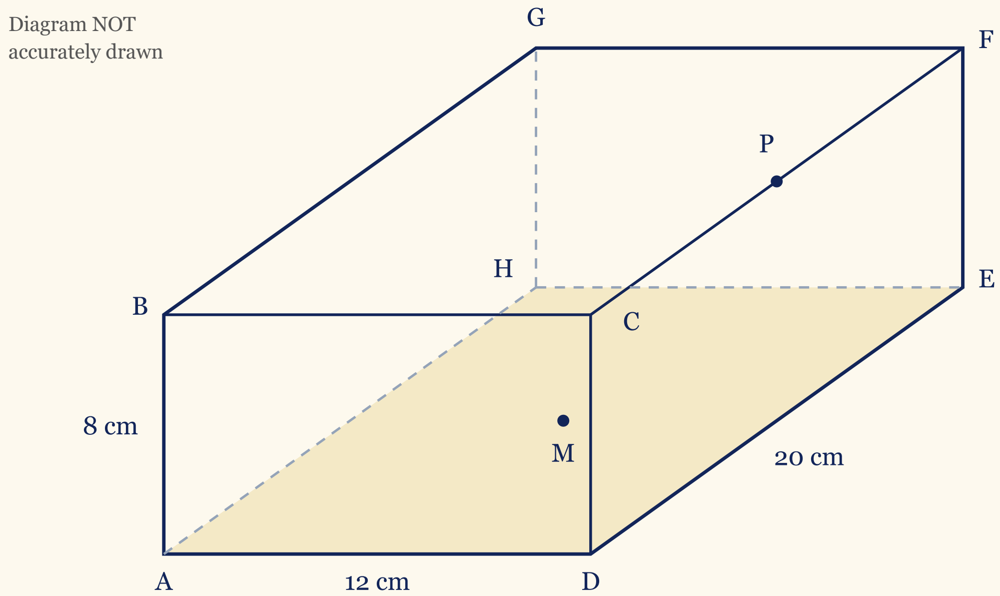
The probability that Idris takes one amber bead and one cream bead is $\dfrac{22}{75}$

Calculate the probability that Idris takes $2$ cream beads from the jar.
Show clear algebraic working. *[5 marks]*

.......................................

*[Total 5 marks]*

**22.** The diagram shows a cuboid $ABCDEFGH$.
The face $ADEH$ is the horizontal base.




$AB = 8$ cm, $AD = 12$ cm and $DE = 20$ cm

The point $M$ is the centre of the base $ADEH$ and the point $P$ is the midpoint of the edge $CF$

Find the size of angle $BMP$.
Give your answer correct to one decimal place. *[6 marks]*

.................................... °

*[Total 6 marks]*

**23.** The first three terms of an arithmetic sequence are shown below.

$(4x - 14), \quad (x + 2), \quad (7x - 9)$

Work out, as an integer, the sum of the first 40 terms of the sequence.
You must show clear algebraic working. *[4 marks]*

....................................

*[Total 4 marks]*

# Worked Solutions & Mark Schemes

*Every mark (M for method, A for accuracy) is shown, just like a real examiner.*

## Question 1 - Exam Solution

### Understanding the Question

**Given**

The five painted numbers are $16, 15, 3, 2$ and $9$.
There are $6$ tiles altogether, so six numbers go into the mean.
The mean of those six numbers must come out at $11$.

**Find**

The number painted on the blank tile.

### Plan the Solution

- A mean is a total shared out equally, so work backwards: the mean tells you what the total has to be.
- Multiply the mean by $6$ to get the total the six numbers must have.
- Add the five painted numbers, then take that sum away from the total. What is left is the missing number.

### Worked Solution [3 marks]

> Rule - Mean: the mean is the total of the numbers divided by how many numbers there are, so the total is $\text{mean} \times \text{how many}$.

**Step 1: work out the total the six numbers must have**

$11 \times 6 = 66$

*(Reason: A mean of $11$ means the six numbers share out to $11$ each, so their total is the mean multiplied by how many tiles there are. Every tile counts, blank or not.)*

**Step 2: add the five numbers that are already painted**

$16 + 15 + 3 + 2 + 9 = 45$

*(Reason: These five cannot change, so this much of the total is already accounted for.)*

**Step 3: subtract to find the missing number**

$66 - 45 = 21$

*(Reason: If the blank tile carries $x$, then $45 + x = 66$, so $x$ is what is left when 45 is taken from 66.)*

$21$

### Verification

✓ **Check 1:** Paint 21 on the blank tile and work the mean out forwards, the usual way round.

$$\frac{16 + 15 + 3 + 2 + 9 + 21}{6} = 11$$

✓ **Check 2:** Measure each painted number from the mean of 11 instead. The five sit $+5$, $+4$, $-8$, $-9$ and $-2$ away from it, and for the mean to be right the amounts above and below must cancel out. $5 + 4 - 8 - 9 - 2 = -10$ so the sixth number has to sit 10 above the mean, giving $11 + 10 = 21$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| The total six numbers need for a mean of 11 | M1 | A correct calculation for the total, $11 \times 6 = 66$, or a correct equation for the last tile using $x$. | ✓ |
| The five painted numbers totalled, then subtracted | M1 | A correct equation for $x$ with no fraction in it, or a correct calculation for the number on the last tile, $66 - 45$, where $16 + 15 + 3 + 2 + 9 = 45$. | ✓ |
| The number on the last tile | A1 | cao 21. A correct answer scores full marks unless it clearly comes from incorrect working, and if the answer line is left blank, check the tile itself. | ✓ |

**Full marks: 3/3**

## Question 2 - Exam Solution

### Understanding the Question

**Given**

**Find**

The spinner is biased and can land on $1, 2$, $3, 4$ or $5$.
Reading the table in order, the probabilities are $2x$, $0.27$, $0.04$, $x$ and $0.12$.
The spinner is spun $400$ times.

An estimate for the number of spins that land on an odd number, that is on $1$, $3$ or $5$.

### Plan the Solution

- The spinner must land on something, so the five probabilities total $1$. That one equation is enough to find $x$.
- Watch the two rows that hold the unknown: $1$ carries $2x$ and $4$ carries $x$, so there are three lots of $x$ altogether, not two.
- Add the probabilities of the three odd numbers, then multiply by $400$ to turn that probability into an expected number of spins.

### Worked Solution [4 marks]

> Rule - the probabilities of all the possible outcomes add up to $1$, and an estimate for a frequency is $\text{probability} \times \text{number of trials}$.

**Step 1: write down the total probability**

$$2x + 0.27 + 0.04 + x + 0.12 = 1$$
$$3x + 0.43 = 1$$

*(Reason: the spinner has to land on one of the five numbers, so the five probabilities account for every spin and must total $1$; collecting the $2x$ with the $x$ gives $3x$)*

**Step 2: solve for x**

$$3x = 1 - 0.43 = 0.57$$
$$x = \frac{0.57}{3} = 0.19$$

*(Reason: take the three known probabilities off $1$, then split what is left equally between the three lots of $x$)*

**Step 3: add the probabilities of the odd numbers**

$$P(\text{odd}) = P(1) + P(3) + P(5)$$
$$P(\text{odd}) = 2 \times 0.19 + 0.04 + 0.12 = 0.54$$

*(Reason: the odd numbers on this spinner are $1$, $3$ and $5$, and the probability of landing on one of them is the sum of the three separate probabilities because the spinner cannot land on two numbers at once)*

**Step 4: scale the probability up to 400 spins**

$400 \times 0.54 = 216$

*(Reason: an estimate for how often an event happens is its probability multiplied by the number of trials, and here there are 400 spins)*

**An estimate of 216 spins landing on an odd number**

### Verification

✓ **Check 1:** Put $x = 0.19$ back into the table and add all five probabilities, including the $2x$ row as $0.38$. $0.38 + 0.27 + 0.04 + 0.19 + 0.12 = 1$

✓ **Check 2:** Work in spins instead of probabilities, never finding $x$ at all: $0.27 \times 400 = 108$, $0.04 \times 400 = 16$ and $0.12 \times 400 = 48$, which leaves $400 - 108 - 16 - 48 = 228$ spins to share between $1$ and $4$ in the ratio $2 : 1$, so $152$ and $76$. $152 + 16 + 48 = 216$

✓ **Check 3:** Estimate the even numbers instead and take them off the $400$ spins. The even numbers are $2$ and $4$, with probability $0.27 + 0.19 = 0.46$, so $0.46 \times 400 = 184$. $400 - 184 = 216$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Uses the fact that the probabilities total 1: $2x + 0.27 + 0.04 + x + 0.12 = 1$ or $1 - (0.27 + 0.04 + 0.12) = 0.57$ | M1 | For a clear understanding that the total of the probabilities is 1. Also earned by finding estimates for the other three numbers: $0.27 \times 400 = 108$, $0.04 \times 400 = 16$ and $0.12 \times 400 = 48$ | ✓ |
| $x = \frac{0.57}{3} = 0.19$ or $2x = \frac{0.57}{3} \times 2 = 0.38$ | M1 | For a method to find the value of $x$ or of $2x$. By the frequency route, for $\frac{400 - 108 - 16 - 48}{3} = 76$ or $76 \times 2 = 152$ | ✓ |
| $(2 \times 0.19 + 0.04 + 0.12) \times 400$ or $2 \times 76 + 16 + 48$ | M1 | For a complete method: the three odd probabilities added and then scaled to the number of spins, or the three odd estimates added directly | ✓ |
| The estimate: 216 | A1 | For an answer of 216 | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Guidance on this question | **Note** | A correct answer scores full marks unless it comes from obviously incorrect working. An answer left as $\frac{216}{400}$ or equivalent is a probability rather than a number of spins and scores M3A0 | ✓ |

**Full marks: 4/4**

## Question 3 - Exam Solution

### Understanding the Question

**Given**

Matteo sells $200$ pots altogether, some plain and some painted.
The plain pots and the painted pots are sold in the ratio $3 : 2$.
A plain pot sells for $£1.50$ and a painted pot sells for $£1.75$.
$40\%$ of the price of a plain pot is profit, and $60\%$ of the price of a painted pot is profit.

**Find**

The total profit Matteo makes once all $200$ pots are sold.

### Plan the Solution

- Share the $200$ pots in the ratio $3 : 2$ to find how many of each kind are sold.
- Turn each number of pots into the money taken for that kind, by multiplying by the price of one pot.
- The two profit rates are different, so they can never be combined into a single percentage of the total takings. Apply each rate to the money taken for its own kind of pot.
- Add the two profits together.

### Worked Solution [5 marks]

> Rule - to share an amount in a given ratio, divide by the total number of shares to find one share; and a percentage of an amount is that percentage, written as a decimal, multiplied by the amount.

**Step 1: find one share of the ratio**

$3 + 2 = 5$

$\frac{200}{5} = 40$

*(Reason: the ratio $3 : 2$ splits the pots into $5$ equal shares, so dividing the $200$ pots by $5$ gives the size of one share)*

**Step 2: find how many pots of each kind are sold**

$3 \times 40 = 120$

$2 \times 40 = 80$

*(Reason: the plain pots take $3$ of the shares and the painted pots take $2$, and the two counts add back to $200$, which is the check that the sharing is right)*

**Step 3: work out the money taken for each kind**

$120 \times 1.50 = 180$

$80 \times 1.75 = 140$

*(Reason: multiply the number of pots by the price of one pot, so the plain pots take $£180$ and the painted pots take $£140$)*

**Step 4: take the profit rate of each kind out of its own takings**

$0.4 \times 180 = 72$

$0.6 \times 140 = 84$

*(Reason: each kind of pot has its own profit rate, so the plain rate is applied to the $£180$ and the painted rate is applied to the $£140$; a percentage is turned into a decimal before multiplying)*

**Step 5: add the two profits**

$72 + 84 = 156$

*(Reason: the total profit is the profit made on the plain pots plus the profit made on the painted pots)*

**A total profit of $£156$**

## Verification

✓ **Check 1:** Work out the profit on a single pot first, then multiply by how many of that kind are sold: $0.4 \times 1.50 = 0.60$ on a plain pot and $0.6 \times 1.75 = 1.05$ on a painted pot. $120 \times 0.60 + 80 \times 1.05 = 156$

✓ **Check 2:** Come at it from the other side. The part of each price that is not profit is $60\%$ of a plain pot and $40\%$ of a painted pot, so the total cost is $0.6 \times 180 + 0.4 \times 140 = 164$, and the total takings are $180 + 140 = 320$. $320 - 164 = 156$

✓ **Check 3:** Count the pots whose whole price is profit instead: $0.4 \times 120 = 48$ plain pots and $0.6 \times 80 = 48$ painted pots. $48 \times 1.50 + 48 \times 1.75 = 156$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Divides by the total number of shares: $\frac{200}{3+2}$ which gives $40$ | **M1** | For a method to find one share of the ratio, $\frac{200}{3+2} = 40$ | ✓ |
| $3 \times 40 = 120$ and $2 \times 40 = 80$ | **M1** | For a method to find the number of plain pots and the number of painted pots. Both are needed: $3 \times 40 = 120$ and $2 \times 40 = 80$ | ✓ |
| $120 \times 1.50 = 180$ and $80 \times 1.75 = 140$ | **M1** | For a method to find the money taken from the plain pots and the money taken from the painted pots, or the number of pots that are entirely profit, or the profit on a single pot of either kind. Examples of the last two routes: $0.4 \times 120 = 48$ and $0.6 \times 80 = 48$; $0.4 \times 1.50 = 0.60$ and $0.6 \times 1.75 = 1.05$. The printed scheme disagrees with itself on that last route - its condition allows the profit on a single pot of one kind or the other, while its worked example shows both - and it is the condition that carries the mark, so one is enough | ✓ |
| $0.4 \times 180 = 72$ and $0.6 \times 140 = 84$ | **M1** | For a complete method to find the total profit on the plain pots and the total profit on the painted pots: $0.4 \times 180 = 72$ and $0.6 \times 140 = 84$. By the other two routes, $48 \times 1.50 = 72$ and $48 \times 1.75 = 84$, or $120 \times 0.60 = 72$ and $80 \times 1.05 = 84$ | ✓ |
| The total profit: $156$ | **A1** | cao. For an answer of $156$. A correct answer scores full marks unless it comes from obviously incorrect working | ✓ |
| An answer of $164$ or $174$ | **SC** | Award SCB4 for either. $164$ is the two profit rates swapped, $0.6 \times 180 + 0.4 \times 140 = 164$ - which is why it is not a near miss but the total cost of the pots instead of the profit. | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| | | 174 is the ratio taken the other way round, giving 80 plain and 120 painted, so $0.4 \times 120 + 0.6 \times 210 = 174$ | |

**Full marks: 5/5**

## Question 4 - Exam Solution

### Understanding the Question

**Given**

One mixed number divided by another,

$$\frac{2\frac{1}{3}}{5\frac{1}{4}}.$$

The fraction it is claimed to come to, $\frac{4}{9}$, is printed for you.

**Find**

A complete piece of working that starts at $\frac{2\frac{1}{3}}{5\frac{1}{4}}$ and finishes at $\frac{4}{9}$. Because the answer is already printed, the working is the whole of the mark. An unsupported answer earns nothing here.

### Plan the Solution

- Turn each mixed number into a top-heavy (improper) fraction. The division rule only works on those, so $2\frac{1}{3}$ and $5\frac{1}{4}$ cannot be used as they stand.
- Turn the dividing fraction upside down and multiply by it instead.
- Multiply the numerators together and the denominators together.
- Cancel the result down to its lowest terms, and check that what is left is $\frac{4}{9}$.

### Worked Solution [3 marks]

> Rule - Dividing by a fraction: turn the second fraction upside down and multiply. In symbols, $\frac{a}{b}$ divided by $\frac{c}{d}$ is $\frac{a}{b} \times \frac{d}{c}$, and the first fraction is left exactly as it is.

**Step 1: write $2\frac{1}{3}$ and $5\frac{1}{4}$ as improper fractions**

$$2+\frac{1}{3}=\frac{6}{3}+\frac{1}{3}=\frac{7}{3}$$

$$5+\frac{1}{4}=\frac{20}{4}+\frac{1}{4}=\frac{21}{4}$$

*(Reason: A mixed number is a whole part plus a fraction part, so $2\frac{1}{3}$ means $2+\frac{1}{3}$. One whole is $\frac{3}{3}$, so two wholes are $\frac{6}{3}$. The same count of quarters turns $5\frac{1}{4}$ into $\frac{21}{4}$. The quicker version of the same thing is to multiply the whole number by the denominator and add the numerator.)*

**Step 2: replace the division with a multiplication**

$$\frac{\frac{7}{3}}{\frac{21}{4}}=\frac{7}{3}\times\frac{4}{21}$$

*(Reason: Dividing by $\frac{21}{4}$ asks how many lots of $\frac{21}{4}$ will fit, and that is the same as taking $\frac{4}{21}$ of the amount. Only the fraction being divided by is turned over; the first one is copied down unchanged.)*

**Step 3: multiply along the top and along the bottom**

$$\frac{7}{3}\times\frac{4}{21}=\frac{7\times 4}{3\times 21}=\frac{28}{63}$$

*(Reason: Multiplying fractions needs no common denominator: the two numerators multiply to give the new numerator, and the two denominators multiply to give the new denominator.)*

**Step 4: cancel down to lowest terms**

$$\frac{28}{63}=\frac{4\times 7}{9\times 7}=\frac{4}{9}$$

*(Reason: Both $28$ and $63$ are multiples of $7$, and $7$ is the highest number that divides into both, so cancelling it leaves the fraction in its lowest terms. Nothing above $1$ divides into both $4$ and $9$, so this is as far as it goes, and it is the fraction the question asked us to reach.)*

$$\frac{2\frac{1}{3}}{5\frac{1}{4}}=\frac{4}{9}\ \textbf{as required}$$

## Verification

✓ **Check 1:** Do the division the other way the mark scheme allows, with no reciprocal at all.

Put both mixed numbers over the common denominator $12$: $2\frac{1}{3}$ is $\frac{28}{12}$ and $5\frac{1}{4}$ is $\frac{63}{12}$.

Twelfths divided by twelfths leaves a plain ratio of the two counts. $\dfrac{\frac{28}{12}}{\frac{63}{12}} = \dfrac{28}{63} = \dfrac{4}{9}$

✓ **Check 2:** Run the division backwards. If the answer is right, multiplying it by the divisor $5\frac{1}{4}$ must return the dividend $2\frac{1}{3}$. $\frac{4}{9} \times \frac{21}{4} = \frac{84}{36} = \frac{7}{3}$, and $\frac{7}{3}$ is $2\frac{1}{3}$

✓ **Check 3:** Test the cancelling on its own, without repeating it. Two fractions are equal exactly when their cross products match, so compare $\frac{28}{63}$ with $\frac{4}{9}$ that way.

$28 \times 9 = 252 = 63 \times 4$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Both mixed numbers written as improper fractions | M1 | For $\frac{7}{3}$ and $\frac{21}{4}$. A candidate who has already inverted the second fraction and written $\frac{4}{21}$ rather than $\frac{21}{4}$ still earns this mark. | ✓ |
| The division turned into a multiplication, or both fractions put over one denominator | M1 | For the intention to multiply the correct improper fraction by the inverted fraction, $\frac{7}{3} \times \frac{4}{21}$ or equivalent, for example $\frac{49}{21} \times \frac{4}{21}$. Also earned for writing the two fractions over the same common denominator, $\frac{28}{12}$ and $\frac{63}{12}$. | ✓ |
| The working completed to the printed fraction | A1 | For correctly completing to reach the required answer, for example $\frac{7}{3} \times \frac{4}{21} = \frac{28}{63} = \frac{4}{9}$, or $\frac{49}{21} \times \frac{4}{21} = \frac{196}{441} = \frac{4}{9}$, or the common-denominator route finishing at $\frac{28}{63} = \frac{4}{9}$. Cancelling the sevens inside the multiplication before multiplying is equally acceptable. | ✓ |
| Guidance carried on the mark scheme | Note | Working is required. The mark scheme header names question 4 as one of the questions where a correct answer is not taken to imply a correct | ✓ |

| Step | Mark | Description | Got it? |
| --- | --- | --- | --- |
| | | method, so the printed fraction copied out on its own scores nothing. Decimals written only as a check are ignored, neither credited nor penalised, provided the fraction working is there. | |

**Full marks: 3/3**

## Question 5 - Exam Solution

### Understanding the Question

**Given**

Paid into the bond: $5200$ euros
Compound interest: $2.5\%$ per year
Time left in the bond: $4$ years

**Find**

The amount in the bond after $4$ years, correct to the nearest euro

### Plan the Solution

- Turn the rate into a multiplier for one year: $100\% + 2.5\% = 102.5\%$, which is $1.025$.
- Compound interest means each year's interest is worked out on the new balance, so that same multiplier is used once for every year.
- Four years is therefore four multiplications by $1.025$, which is the same as multiplying by $1.025^4$.
- Keep the full decimal all the way through and round only at the very end, or the rounding error grows with every year.

### Worked Solution [3 marks]

Rule - Compound interest: $\text{final amount} = P \times \left(1 + \frac{r}{100}\right)^n$, where $P$ is the amount paid in, $r$ is the percentage rate per year and $n$ is the number of years.

**Step 1: Write the yearly rate as a multiplier**

$$1 + \frac{2.5}{100} = 1.025$$

*(Reason: The money is not replaced by the interest, it is added to it, so the bond is worth $102.5\%$ of what it was worth a year earlier. Writing that percentage as the decimal $1.025$ turns a year in the*

*bond into a single multiplication.)*

**Step 2: Multiply once for each year, to see what compound interest is doing**

$5200 \times 1.025 = 5330$

$5330 \times 1.025 = 5463.25$

$5463.25 \times 1.025 = 5599.83125$

$5599.83125 \times 1.025 = 5739.82703125$

*(Reason: Each line starts from the balance the line above finished with, which is exactly what makes the interest compound: the second year earns $133.25$ euros where the first earned only $130$ euros, because the second year's $2.5\%$ is taken on a bigger amount.)*

**Step 3: Do the same thing in one line with a power**

$5200 \times 1.025^4$

$1.025^4 = 1.103812890625$

$5200 \times 1.103812890625 = 5739.82703125$

*(Reason: Multiplying by $1.025$ four times in a row is multiplying by $1.025^4$ once, so the whole of Step 2 collapses into one calculation. In the exam this is the line to write, and the power key on the calculator does the rest.)*

**Step 4: Round to the nearest euro**

$5739.82703125 \text{ euros} \implies 5740 \text{ euros}$

*(Reason: The question asks for the nearest euro. The first digit after the decimal point is $8$, so the amount rounds up. Rounding here, at the end, and not earlier, is what keeps the answer inside the accepted $5739$ to $5740$.)*

**$5740$ euros**

## Verification

✓ **Check 1:** Undo the four years. Dividing the final amount by $1.025^4$ must give back exactly the amount that was paid in. $\dfrac{5739.82703125}{1.103812890625} = 5200$, the original amount

✓ **Check 2:** Look at the interest on its own. Four years of $2.5\%$ simple interest would be $4 \times 130 = 520$ euros, so compound interest must come to a little more than that, and only a little. $5739.82703125 - 5200 = 539.82703125$, which beats $520$ by about $20$ euros

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| $5200 \times 1.025 = 5330$ or $5200 \times 0.025 = 130$ | **M1** | For a method to find $2.5\%$ of $5200$, or $102.5\%$ of $5200$. Any equivalent working earns this. | ✓ |
| $5330 \times 1.025 = 5463.25$ and $5463.25 \times 1.025 = 5599.83\ldots$ and $5599.83\ldots \times 1.025 = 5739.8\ldots$ | **M1** | For a complete method: the multiplier used once for each of the $4$ years. The mark scheme quotes the candidate's own values, so this follows through from a wrong balance in an earlier year. | ✓ |
| $5200 \times 1.025^4$ | **M2** | The power method written in one line earns both method marks at once. The mark scheme also allows M2 for $5200 \times 1.025^5 = 5883\ldots$, which is a complete method spoiled only by using $5$ years instead of $4$. | ✓ |
| The value written on the answer line | **A1** | Accept anything from $5739$ to $5740$, which covers a candidate who truncated rather than rounded. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |
| If no other mark has been earned | **SC B1** | For any one of $5200 \times 0.1 (= 520)$ oe (reading the rate as $10\%$), $5200 \times 1.1 (= 5720)$ oe (one year at $10\%$), $5200 \times 0.9 (= 4680)$ oe (a $10\%$ decrease), $5200 \times 0.975 (= 5070)$ oe (one year of decrease instead of increase) or $5200 \times 0.975^4 (= 4699\ldots)$ oe (four years of decrease). Accept (1 + 0.025) as equivalent to 1.025 throughout, but not (1 + 2.5%). | ✓ |
| Equivalent forms of the multiplier | **Note** | Accept $(1 + 0.025)$ as equivalent to $1.025$ throughout, but do not accept $(1 + 2.5\%)$. | ✓ |

**Full marks: 3/3**

## Question 6 - Exam Solution

### Understanding the Question

**Given**

A solid hardwood cylinder of radius 8 cm and height $h$ cm.
Its volume is $1208$ cm³, so on this solid the volume is known and the height is not.
The hardwood has density $1.25$ g/cm³, so each cm³ of it has mass $1.25$ g.

**Find**

(a) The height $h$, correct to the nearest whole number. (b) The mass of the whole cylinder, in kilograms.

### Plan the Solution

- A cylinder is a circular face swept through its height, so $V = \pi r^2 h$. Putting the radius and the volume in leaves $h$ as the only unknown.
- Work out the area of the circular face first, then divide the volume by it. Keep the calculator's own $\pi$ all the way through and round once, at the very end of part (a).
- Density is the mass of one cm³, so $\text{mass} = \text{density} \times \text{volume}$ gives the mass in grams. The question wants kilograms, and $1000$ g make 1 kg.
- Part (b) does not need the answer to part (a). The volume is already given as $1208$ cm³, so use that rather than one rebuilt from a rounded height.

### Worked Solution [4 marks]

> Rule - Cylinder and density: $V = \pi r^2 h$, and $\text{mass} = \text{density} \times \text{volume}$.

**Step 1: Put the radius and the volume into the cylinder formula**

$\pi \times 8^2 \times h = 1208$

*(Reason: The circular face has radius $8$ cm, so its area is $\pi \times 8^2$ cm², and the volume of a cylinder is that area multiplied by the height. Writing the given volume on the other side turns the formula into an equation in $h$.)*

**Step 2: Work out the area of the circular face**

$8^2 = 64$

$\pi \times 64 = 201.0619\ldots$

*(Reason: Squaring the radius first keeps the working in one place: the face is $64\pi$ cm², a little over $201$ cm². Squaring the diameter here instead of the radius is the slip this question catches most often.)*

**Step 3: Make $h$ the subject**

$$h = \frac{1208}{\pi \times 8^2}$$

*(Reason: The face area multiplies the height, so dividing both sides by that area leaves the height on its own.)*

**Step 4: Work the height out on the calculator**

$$h = \frac{1208}{201.0619\ldots} = 6.0081\ldots$$

*(Reason: Use the $\pi$ key rather than a rounded value. Rounding the $201.0619\ldots$ before dividing moves the last figures of the height.)*

**Step 5: Round to the nearest whole number**

$$6.0081\ldots \approx 6$$

*(Reason: The first figure after the decimal point is $0$, which is below $5$, so the whole-number part stays at $6$.)*

**Step 6: Work out the mass of the cylinder in grams**

$$1.25 \times 1208 = 1510$$

*(Reason: Each cm³ of the wood has mass $1.25$ g and the solid holds $1208$ cm³ of it, so the mass is $1.25$ g taken $1208$ times. The volume used is the one the question states, not one rebuilt from the rounded height.)*

**Step 7: Change the grams into kilograms**

$$\frac{1510}{1000} = 1.51$$

*(Reason: There are $1000$ g in $1$ kg, so the number of kilograms is the number of grams divided by $1000$. The question asks for kilograms, so an answer left in grams is not finished.)*

**(a)** $h = 6$

**(b)** $1.51$ **kilograms**

## Verification

✓ **Check 1:** Put the unrounded height back into the volume formula. The volume the question states must come back. $\pi \times 8^2 \times 6.0081\ldots = 1208$ cm³

✓ **Check 2:** Is the size sensible, with no calculator? The face is a little over $200$ cm², and a height of $6$ cm on a face of $200$ cm² falls just short of the stated volume. $200 \times 6 = 1200$, just under $1208$, so the true height sits a little above $6$ and still rounds to $6$.

✓ **Check 3:** Divide the mass in grams by the volume. Density is the mass of one cm³, so the density given in the question must return. $\dfrac{1510}{1208} = 1.25$ g/cm³

✓ **Check 4:** Turn the kilograms back into grams, which undoes the last step. $1.51 \times 1000 = 1510$ g

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) An equation in $h$ built from the volume of the cylinder, $\pi \times 8^2 \times h = 1208$, or a correct calculation for $h$, $\dfrac{1208}{\pi \times 8^2}$ | **M1** | The method may be seen in stages: $\pi \times 8^2 = 201.06\ldots$ followed by a division by that figure earns the mark exactly as the single line does. Squaring the diameter, $16$, in place of the radius does not. | ✓ |
| $6$ | **A1** | Accept anything from $6$ to $6.02$. The band is stated with no reason on the paper, and this is the reason: a candidate who types a rounded $\pi$ lands a little high, at $\dfrac{1208}{3.14 \times 64} = 6.0111\ldots$, and the band is exactly wide enough for that and no wider. A correct answer scores both marks unless it comes from obviously incorrect working. | ✓ |
| (b) An equation built from density as mass per unit volume, $\dfrac{m}{1208} = 1.25$, or a calculation for the mass, $1208 \times 1.25$ $(= 1510)$ | **M1** | Any correct method for the mass earns this, including changing the density to $0.00125$ kg/cm³ first and multiplying by $1208$, which reaches kilograms in a single line. | ✓ |
| $1.51$ | **A1** | The answer must be in kilograms. $1510$ left in grams shows the method but not the change of unit, so it scores the method mark alone. A correct answer scores both marks unless it comes from obviously incorrect working. | ✓ |
| Part (b) is marked from the volume the question | **Note** | Rebuilding it from $h = 6$ gives $1206.37\ldots$ cm³ and a mass of $1507.96\ldots$ g. That still rounds to | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| states, 1208 cm³, not from a volume rebuilt out of the rounded height. | | 1.51 kg, so it is not penalised here, but the given volume is the one the mark scheme's own calculation uses. | |

**Full marks: 4/4**

## Question 7 - Exam Solution

### Understanding the Question

**Given**

$\frac{g^9}{g^2}$ - one power of $g$ divided by another.

$5k^2(k^3+4)$ - a single term multiplying a bracket.

$x^2-2x-63$ - a quadratic whose $x^2$ coefficient is $1$.

$7-2y<3y-12$ - a linear inequality with $y$ on both sides.

**Find**

(a) the quotient written as a single power of $g$. (b) the bracket expanded, one term at a time. (c)(i) the quadratic as a product of two brackets, then (ii) the two values of $x$ that follow from them. (d) the range of values of $y$ that satisfy the inequality.

### Plan the Solution

- (a) Dividing powers of the same base subtracts the indices, so this is $9-2$.
- (b) Multiply each of the two terms inside the bracket by $5k^2$, adding indices as you go.
- (c)(i) Look for two integers whose product is $-63$ and whose sum is $-2$.
- (c)(ii) A product is zero only when one of its factors is zero, so read one solution off each bracket. This is why the question says "Hence" - the factors are already done.
- (d) Rearrange it exactly as you would an equation, but reverse the inequality sign if you divide by a negative number.

### Worked Solution [9 marks]

Rule - Dividing powers: $\frac{a^m}{a^n}=a^{m-n}$. Expanding: multiply every term inside the bracket by the term outside. Factorising $x^2+bx+c$: find two integers with product $c$ and sum $b$

. Inequalities: solve as an equation, but reverse the sign if you multiply or divide by a negative number.

**(a) Subtract the indices**

$$\frac{g^9}{g^2} = g^{9-2} = g^7$$

*(Reason: The numerator is $9$ copies of $g$ and the denominator is $2$ of them, so $2$ cancel and $7$ are left.)*

**(b) Multiply each term in the bracket by $5k^2$**

$$5k^2 \times k^3 = 5k^5$$
$$5k^2 \times 4 = 20k^2$$

*(Reason: Multiplying powers of the same base adds the indices, so $k^2 \times k^3 = k^{2+3} = k^5$. The second term has no $k$ in the bracket, so its power of $k$ is unchanged.)*

**(b) Write the two terms as one expression**

$$5k^2(k^3 + 4) = 5k^5 + 20k^2$$

*(Reason: The powers $k^5$ and $k^2$ are different, so the two terms will not collect together. This is the final answer.)*

**(c)(i) Find two integers with product $-63$ and sum $-2$**

$$7 \times (-9) = -63$$
$$7 + (-9) = -2$$

*(Reason: The product is negative, so one integer is positive and the other is negative. The pairs that multiply to $-63$ are $1$ and $-63$, $3$ and $-21$, $7$ and $-9$ and their opposites, and only $7$ and $-9$ add to $-2$.)*

**(c)(i) Write the quadratic as two brackets**

$$x^2 - 2x - 63 = (x + 7)(x - 9)$$

*(Reason: The two integers go straight into the brackets. Expanding gives $x^2 - 9x + 7x - 63$, and the middle terms collect to $-2x$.)*

**(c)(ii) Set each bracket equal to zero**

$$(x + 7)(x - 9) = 0$$
$$x + 7 = 0 \implies x = -7$$
$$x - 9 = 0 \implies x = 9$$

*(Reason: A product of two numbers is zero only when at least one of them is zero, so each bracket gives one solution.)*

**(d) Collect the $y$ terms on the left and the numbers on the right**

$$7 - 2y < 3y - 12$$

$-2y - 3y < -12 - 7$

*(Reason: Subtract $3y$ from both sides and subtract $7$ from both sides. Adding or subtracting never changes the direction of an inequality.)*

**(d) Simplify each side**

$-5y < -19$

*(Reason: $-2y - 3y = -5y$ and $-12 - 7 = -19$.)*

**(d) Divide both sides by $-5$ and reverse the inequality**

$$y > \frac{19}{5}$$

*(Reason: Dividing by a negative number reverses the direction of an inequality, so the "less than" sign turns into a "greater than" sign. As a decimal the boundary is $3.8$, and the boundary itself is not included.)*

**(a)** $g^7$

**(b)** $5k^5 + 20k^2$

**(c)(i)** $(x+7)(x-9)$

**(c)(ii)** $x = -7$ **and** $x = 9$

**(d)** $y > \dfrac{19}{5}$

## Verification

✓ **Check 1:** Part (a) with $g = 2$. The numerator is $2^9 = 512$ and the denominator is $2^2 = 4$. $\frac{512}{4} = 128$ and $2^7 = 128$

✓ **Check 2:** Part (b) with $k = 2$. The bracket is $2^3 + 4 = 12$ and the term outside is $5 \times 4 = 20$. $20 \times 12 = 240$ and $5 \times 32 + 20 \times 4 = 240$

✓ **Check 3:** Expand the brackets from part (c)(i) and compare with the quadratic the question printed. $(x+7)(x-9) = x^2 - 9x + 7x - 63 = x^2 - 2x - 63$

✓ **Check 4:** Put both solutions from part (c)(ii) back into $x^2 - 2x - 63$. $81 - 18 - 63 = 0$ and $49 + 14 - 63 = 0$

✓ **Check 5:** Test part (d) either side of $3.8$, using $y = 4$ and then $y = 3$. At $y = 4$ the two sides are $-1$ and $0$, and $-1$ is less than $0$, so it works. At $y = 3$ the two sides are $1$ and $-3$, and $1$ is not less than $-3$, so it does not.

✓ **Check 6:** The boundary itself. Put $y = \frac{19}{5}$ into both sides. Both sides come to $-\frac{3}{5}$, so at the boundary the two sides are equal, which confirms the inequality is strict.

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) $\frac{g^9}{g^2} = g^7$ | **B1** | For $g^7$. | ✓ |
| (b) $5k^5 + 20k^2$ | **B2** | For $5k^5 + 20k^2$. Award 1 mark only, B1, for either $5k^5$ or $20k^2$ alone. | ✓ |
| (c)(i) $(x \pm 7)(x \pm 9)$ | **M1** | For $(x \pm 7)(x \pm 9)$, or for $(x + a)(x + b)$ where $ab = -63$ or $a + b = -2$, with $a$ and $b$ integers. | ✓ |
| (c)(i) $(x + 7)(x - 9)$ | **A1** | For the correct factors. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |
| (c)(ii) $-7, 9$ | **B1** | Must follow through from part (c)(i), and is dependent on factorising in the form $(x + p)(x + q)$ where $p$ and $q$ are integers. | ✓ |
| (d) $-2y - 3y < -12 - 7$ or $7 + 12 < 3y + 2y$ | **M1** | For a rearrangement with the $y$ terms on one side and the numerical terms on the other in a correct inequality, or for the correct simplification of the $y$ terms or of the numbers on one side in a correct inequality. The sign may be $=$ or the incorrect inequality sign. | ✓ |
| (d) $-5y < -19$ or $19 < 5y$ | **M1** | For the correct simplification of the $y$ terms on one side and the numbers on the other side in a correct inequality, or for a correct inequality with the wrong sign. The sign may be $=$ or the incorrect inequality sign. | ✓ |
| (d) $y > \frac{19}{5}$ | **A1** | Or equivalent, for example $y > 3.8$ or $3.8 < y$. It must be given as the correct inequality on the answer line. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |

**Full marks: 9/9**

## Question 8 - Exam Solution

### Understanding the Question

**Given**

$ABCD$ is a trapezium, with angle $DAB$ and angle $ADC$ both right angles
$AD = 15$ cm and $DC = 14$ cm
Area of $ABCD$ is 360 cm²

**Find**

The perimeter of $ABCD$, in cm - so all four sides are needed, and two of them are not given

### Plan the Solution

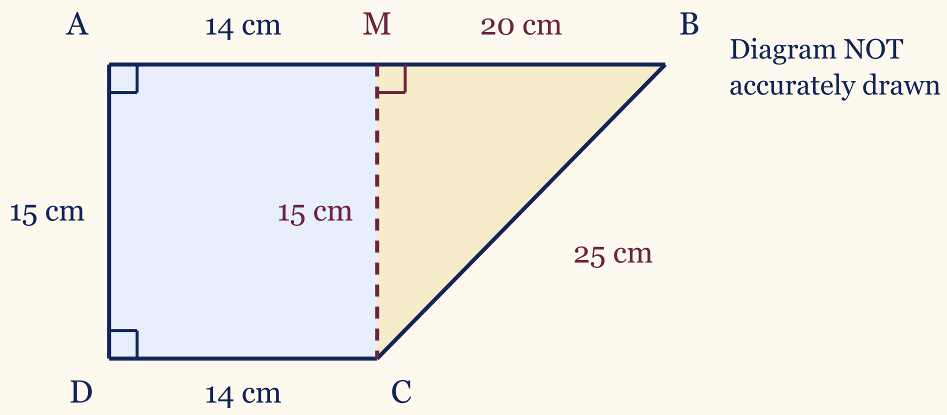
- Both $AB$ and $DC$ are perpendicular to $AD$, so they are the parallel sides and $AD$ is the distance between them.
- Put the given area into the trapezium formula and solve for the missing parallel side $AB$.
- Drop a perpendicular from $C$ to $AB$, meeting it at $M$. That cuts the trapezium into rectangle $AMCD$ and right-angled triangle $MBC$.
- Use Pythagoras in triangle $MBC$ to find the slant side $CB$, then add the four sides.

### Worked Solution [6 marks]

Rule - Trapezium: $\text{Area} = \dfrac{a+b}{2} \times h$, where $a$ and $b$ are the parallel sides and $h$ is the perpendicular distance between them.

**Step 1: name the parallel sides and the height**

$a = DC = 14$
$h = AD = 15$




*(Reason: Angle $DAB$ and angle $ADC$ are both right angles, so $AB$ and $DC$ are both perpendicular to $AD$. That makes them the parallel pair, with $AD$ as the distance between them and $AB$ as the unknown side b.)*

**Step 2: put the area into the formula**

$$\frac{14 + AB}{2} \times 15 = 360$$

*(Reason: Everything in the formula is now known except $AB$, so the area gives one equation in one unknown.)*

**Step 3: solve for $AB$**

$$14 + AB = \frac{2 \times 360}{15}$$

$$14 + AB = 48$$

$$AB = 48 - 14 = 34$$

*(Reason: Multiply both sides by 2 and divide by 15 to undo the formula. That leaves the two parallel sides adding to 48, so take the known one, $DC = 14$, off the total.)*

**Step 4: split the trapezium at $M$**

$$AM = DC = 14$$

$$MC = AD = 15$$

$$MB = 34 - 14 = 20$$

*(Reason: $M$ is the point on $AB$ with $MC$ perpendicular to $AB$. Then $AMCD$ has four right angles, so it is a rectangle and its opposite sides are equal. What is left of $AB$ is the base of the triangle.)*

**Step 5: Pythagoras in triangle $MBC$**

$$CB^2 = 15^2 + 20^2$$

$$CB^2 = 225 + 400 = 625$$

$CB = \sqrt{625} = 25$

*(Reason: Angle $BMC$ is a right angle, so the square on the slant side $CB$ is the sum of the squares on $MC$ and $MB$.)*

**Step 6: add the four sides**

Perimeter $= AB + BC + CD + DA$

Perimeter $= 34 + 25 + 14 + 15 = 88$

*(Reason: The perimeter is the distance all the way round the outside, so each of the four sides is counted exactly once. The dashed line $MC$ is a construction line and is not part of it.)*

**88 cm**

## Verification

✓ **Check 1:** Put $AB = 34$ back into the trapezium formula and see whether the given area returns. $\dfrac{34+14}{2} \times 15 = 24 \times 15 = 360$

✓ **Check 2:** Work the area out a second way, as rectangle $AMCD$ plus triangle $MBC$, which never uses the trapezium formula at all.
$14 \times 15 + \dfrac{20 \times 15}{2} = 210 + 150 = 360$

✓ **Check 3:** Test $CB$ without Pythagoras. The legs $15$ and $20$ are $5$ times $3$ and $4$, so triangle $MBC$ is a scaled $3$, $4$, $5$ triangle and its hypotenuse must be $5$ times $5$.
$5 \times 5 = 25$

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| $\dfrac{14+AB}{2} \times 15 = 360$ oe, or $360 - 14 \times 15 (= 150)$ oe | M1 | for setting up an equation using the area of the trapezium, or for a method to find the area of the triangle | ✓ |
| $AB = 34$ or $MB = 20$ | A1 | could be seen on the diagram, where $M$ is the point on $AB$ with $MC$ perpendicular to $AB$ | ✓ |
| $(CB^2 =) 15^2 + 20^2 (= 625)$ or $(CB^2 =) 15^2 + MB^2$ | M1 | allow use of their $MB$ | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| $(CB =)\sqrt{15^2+20^2}(= 25)$ or $(CB =)\sqrt{15^2+MB^2}$ | **M1** | allow use of their $MB$ | ✓ |
| $14+15+34+25$ oe, or $14+15+14+MB+CB$ oe | **M1ft** | dep on the previous two M marks, for a method to find the perimeter of the trapezium; allow use of their $MB$ and their $CB$ | ✓ |
| 88 | **A1** | cao. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |

**Full marks: 6/6**

## Question 9 - Exam Solution

### Understanding the Question

**Given**

A straight line $L$ drawn on a grid, with both axes numbered.
One square on the grid stands for $1$ unit across and $1$ unit up.
The line runs right across the grid, from the left edge to the right edge.

**Find**

An equation of $L$, written in the form $y = mx + c$. That is two numbers: the gradient $m$, and the value $c$ where the line crosses the $y$-axis.

### Plan the Solution

- Pick two points where the line passes exactly through a crossing of the grid lines, so both coordinates can be read with nothing estimated.
- Divide the change in $y$ by the change in $x$ between those two points. That is the gradient.
- Read the value of $y$ where the line cuts the $y$-axis. That is $c$.
- Put both numbers into $y = mx + c$, then test the equation on a third point of the line.

### Worked Solution [3 marks]

Rule - Straight line: $y = mx + c$, where $m$ is the gradient $\dfrac{\text{change in } y}{\text{change in } x}$ and $c$ is the value of $y$ where the line crosses the $y$-axis.

**Step 1: read two exact points off the line**

$(-4, 3)$ and $(4, -1)$

*(Reason: The line meets each edge of the grid at a crossing of the grid lines, so both readings are exact. A point taken from the middle of a square would only be an estimate.)*

**Step 2: work out the gradient $m$**

$$\frac{-1-3}{4-(-4)} = \frac{-4}{8} = -\frac{1}{2}$$

$$m = -\frac{1}{2}$$

*(Reason: Going from $(-4, 3)$ to $(4, -1)$, the $y$ value falls by $4$ while the $x$ value rises by $8$. A line that falls from left to right has a negative gradient.)*

**Step 3: read the value of $c$**

$c = 1$

*(Reason: The line crosses the $y$-axis one square above the origin, at $(0, 1)$, and that value of $y$ is $c$.)*

**Step 4: write the equation in the form the question asks for**

$$y = -\frac{1}{2}x + 1$$

*(Reason: The gradient goes in front of the $x$ and the intercept goes on the end. Writing $y = -0.5x + 1$ says exactly the same thing and is equally acceptable.)*

$$y = -\frac{1}{2}x + 1$$

## Verification

✓ **Check 1:** Put $x = 4$ into the equation and compare it with the point the line reaches at the right-hand edge of the grid. $-\frac{1}{2} \times 4 + 1 = -1$, and the grid shows the line at $(4, -1)$.

✓ **Check 2:** Test a point that was not used to build the equation: the line cuts the $x$-axis at $(2, 0)$. $-\frac{1}{2} \times 2 + 1 = 0$, so $(2, 0)$ lies on the line, as the grid shows.

✓ **Check 3:** Check the shape of the answer rather than a number: a gradient of $-\frac{1}{2}$ means the line should drop one square for every two squares it moves to the right. Across the whole grid the line moves $8$ squares right and drops $4$ squares, which is one down for every two across.

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| One correct piece of the equation: the gradient $-\frac{1}{2}$, or $-\frac{1}{2}x + c$, or $mx + 1$ with $m \neq 0$ | B1 | The gradient alone scores, however it is written - $-0.5$, $\frac{-1}{2}$, or the working $\frac{3-(-1)}{-4-4}$. The commonest loss is dividing the wrong way round, change in $x$ over change in $y$, which gives $-2$ and scores nothing. | ✓ |
| Both parts correct, but not yet written as $y = mx + c$ | B2 | Award for $-\frac{1}{2}x + 1$ with no $y =$ in front, or for a correct equation in another form such as $2y + x = 2$. Also award for $y = -\frac{1}{2}x + c$ or for $y = mx + 1$ with $m \neq 0$. | ✓ |
| The equation in the form asked for: $y = -\frac{1}{2}x + 1$ | B3 | Accept any equivalent, for example $y = -0.5x + 1$, provided it is in the form $y = mx + c$. A correct equation left as $2y + x = 2$ scores $2$ of the $3$ marks, because the question names the form it wants. | ✓ |

**Full marks: 3/3**

## Question 10 - Exam Solution

### Understanding the Question

**Given**

The numbers of unsold loaves on $11$ days, already written in ascending order:
$0, 1, 2, 2, 3, 4, 4, 6, 7, 9, 11$

**Find**

The interquartile range, $IQR = Q_3 - Q_1$, where $Q_1$ is the lower quartile and $Q_3$ is the upper quartile. One number, and it is a spread, A list, not a table, so every value counts once - including the repeated $2$ and the repeated $4$.

not an average - expect it to be smaller than the range.

### Plan the Solution

- Check the list really is in order, then count how many values there are: $n = 11$.
- Use the position rule. With $n = 11$ every quartile position comes out a whole number, so no averaging of two values is needed.
- Find $Q_1$ and $Q_3$ by counting along the list to those positions - the mark scheme gives the method mark for these two values alone.
- Subtract: $Q_3 - Q_1$.

### Worked Solution [2 marks]

> Rule - Quartiles by position: for $n$ values written in order, $Q_1$ is the $\frac{n+1}{4}$th value, the median is the $\frac{n+1}{2}$th value, $Q_3$ is the $\frac{3(n+1)}{4}$th value, and $IQR = Q_3 - Q_1$.

**Step 1: Put the values in order and count them**

$0, 1, 2, 2, 3, 4, 4, 6, 7, 9, 11$

$n = 11$

*(Reason: (Reason: quartiles are positions in an ORDERED list. This list is already ascending, so nothing has to be moved - but the count $n = 11$ is what every position below is worked out from.))*

**Step 2: Find the middle value**

$$\frac{11+1}{2} = 6$$

$$\text{median} = \text{6th value} = 4$$

*(Reason: (Reason: the median splits the list into $5$ values below it and $5$ above it, which is what makes the quartile positions land on whole numbers. The mark scheme accepts the median $4$ as evidence of the method.))*

**Step 3: Find the lower quartile**

$$\frac{11+1}{4} = 3$$

$$Q_1 = \text{3rd value} = 2$$

*(Reason: (Reason: $\frac{n+1}{4}$ gives a POSITION, not a value. Counting along the list to that position is what turns it into the lower quartile.))*

**Step 4: Find the upper quartile**

$$\frac{3(11+1)}{4} = 9$$

$$Q_3 = 9\text{th value} = 7$$

*(Reason: (Reason: three quarters of the way along, so the $9$th of the $11$ values. Counting from the other end it is the $3$rd from the top, which is a quick way to check it.))*

**Step 5: Subtract the quartiles**

$$IQR = Q_3 - Q_1 = 7 - 2 = 5$$

*(Reason: (Reason: the interquartile range is the width of the middle half of the data, so it is always the upper quartile MINUS the lower quartile - never the two added, and never the median subtracted from anything.))*

**Interquartile range = $5$ loaves**

### Verification

- ✓ **Check 1:** Use the other standard method: delete the median $4$ and take the middle of each half. Lower half $0, 1, 2, 2, 3$ has middle $2$; upper half $4, 6, 7, 9, 11$ has middle $7$. $7 - 2 = 5$
- ✓ **Check 2:** Count in from both ends independently, so neither quartile is found from the other. The $3$rd value up from the bottom is $2$, and the $3$rd value down from the top is $7$. $7 - 2 = 5$
- ✓ **Check 3:** Sanity check the size. The range is $11 - 0 = 11$, and the interquartile range measures only the middle half, so it must come out smaller than the range. $5 < 11$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Both quartiles correctly identified: $2$ and $7$ | **M1** | Method mark for reading off both quartiles. Working that identifies the median $(4)$ may also be seen and is accepted as part of the method. | ✓ |
| $7 - 2 = 5$ | **A1** | cao. A correct answer of $5$ scores full marks, unless it comes from obviously incorrect working. | ✓ |

**Full marks: 2/2**

## Question 11 - Exam Solution

### Understanding the Question

**Given**

$A = 2^5 \times 5 \times 7^2$

$B = 2^3 \times 5^3 \times 7^4$

Both numbers are already broken into their prime factors, so no factor tree is needed.

**Find**

(a) The highest common factor of $5A$ and $2B$, written as a product of prime factors

(b) The value of $(AB)^2$, written as a product of prime factors

### Plan the Solution

- Multiply each given number by its extra factor first: the $5$ in $5A$ raises the index of $5$ by one, and the $2$ in $2B$ raises the index of $2$ by one.
- For the HCF, compare the two lists prime by prime and keep the lower power each time.
- For part (b), build $AB$ by adding the indices of each prime, then square the result by doubling every index.
- Never multiply the numbers out: the answer is wanted as a product of prime factors, and the indices carry all the information.

### Worked Solution [4 marks]

> Rule - Index laws on prime factors: to multiply two numbers, add the indices of each prime; to square a number, double every index; and for the HCF, keep the LOWEST power of each prime that appears in both numbers.

**Step 1: Write $5A$ and $2B$ as products of prime factors**

$5A = 5 \times 2^5 \times 5 \times 7^2 = 2^5 \times 5^2 \times 7^2$

$2B = 2 \times 2^3 \times 5^3 \times 7^4 = 2^4 \times 5^3 \times 7^4$

*(Reason: The extra $5$ joins the one $5$ already in $A$, giving $5^2$; the extra $2$ joins the $2^3$ in $B$, giving $2^4$.)*

**Step 2: Keep the lowest power of each prime**

lower of $2^5$ and $2^4$ is $2^4$

lower of $5^2$ and $5^3$ is $5^2$

lower of $7^2$ and $7^4$ is $7^2$

$2^4 \times 5^2 \times 7^2 = 19\,600$

*(Reason: A common factor cannot use more copies of a prime than the smaller supply allows, so $2^4$ is as far as the twos go. The product is the HCF; the numerical value $19\,600$ is shown only as a check, since the answer is wanted in index form.)*

### Step 3: Write $AB$ as a product of prime factors

$$AB = 2^{5+3} \times 5^{1+3} \times 7^{2+4} = 2^8 \times 5^4 \times 7^6$$

*(Reason: Every prime in $A$ meets the same prime in $B$, and when two powers of one prime are multiplied together the indices are combined by ADDING them, never by multiplying them.)*

### Step 4: Square $AB$ by doubling every index

$$(AB)^2 = 2^{8\times2} \times 5^{4\times2} \times 7^{6\times2} = 2^{16} \times 5^8 \times 7^{12}$$

*(Reason: Squaring is multiplying the number by itself, so each index is added to a copy of itself, which doubles it: $8$ becomes $16$, $4$ becomes $8$ and $6$ becomes $12$.)*

**(a)** $2^4 \times 5^2 \times 7^2$

**(b)** $2^{16} \times 5^8 \times 7^{12}$

## Verification

- ✓ **Check 1:** Divide both numbers by the HCF and see whether anything is left in common. $5A = 39\,200$ and $2B = 4\,802\,000$. $\dfrac{39\,200}{19\,600} = 2$ and $\dfrac{4\,802\,000}{19\,600} = 245$, and $2$ and $245$ share no factor, so nothing more can be taken out.
- ✓ **Check 2:** Use the HCF and LCM together. The LCM takes the HIGHEST power of each prime, giving $2^5 \times 5^3 \times 7^4 = 9\,604\,000$, and HCF times LCM must equal the product of the two numbers. $19\,600 \times 9\,604\,000 = 39\,200 \times 4\,802\,000$, so the pair of answers is consistent.
- ✓ **Check 3:** Square each number first instead of multiplying first: $A^2 = 2^{10} \times 5^2 \times 7^4$ and $B^2 = 2^6 \times 5^6 \times 7^8$. $2^{10} \times 5^2 \times 7^4 \times 2^6 \times 5^6 \times 7^8 = 2^{16} \times 5^8 \times 7^{12}$, the same answer by a different order of working.
- ✓ **Check 4:** A square number has an even index on every prime, because each index has been doubled. $16$, $8$ and $12$ are all even, so the answer really is a perfect square.

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) Correct HCF as a product of prime factors | **B2** | $2^4 \times 5^2 \times 7^2$ or equivalent, for example $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7$ | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) Partially correct HCF | **B1** | For $2^m \times 5^n \times 7^p$ with two of $m = 4$, $n = 2$, $p = 2$, or for $19\,600$ without sight of the correct factorisation, or a fully correct Venn diagram for $5A$ and $2B$, or an answer of $2^3 \times 5 \times 7^2$ or equivalent | ✓ |
| (b) A correct product of prime factors on the way to the answer | **M1** | For any correct product of prime factors, for example $AB = 2^8 \times 5^4 \times 7^6$, or $A^2 = 2^{10} \times 5^2 \times 7^4$, or $B^2 = 2^6 \times 5^6 \times 7^8$, or for $(AB)^2$ evaluated as $3.5\ldots \times 10^{20}$, oe | ✓ |
| (b) Correct answer as a product of prime factors | **A1** | $2^{16} \times 5^8 \times 7^{12}$ or equivalent. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |
| (b) Special case, if no other marks are earned | **SC B1** | For $2^c \times 5^d \times 7^f$ with two of $c = 16$, $d = 8$, $f = 12$. The error behind it is doubling only some of the indices. | ✓ |

**Full marks: 4/4**

## Question 12 - Exam Solution

### Understanding the Question

**Given**

$4x + 3y = 9.6$

$6x + 5y = 16.8$

Two linear equations in the same two unknowns, so one solution pair is expected.

**Find**

The value of $x$ and the value of $y$ that satisfy both equations at once.

### Plan the Solution

- Both equations are linear, so use elimination: scale them until one unknown has the same coefficient in each.
- The $x$ terms are $4x$ and $6x$, and the lowest common multiple of $4$ and $6$ is $12$, so multiply the first equation by $3$ and the second by $2$.
- Both scaled equations then have $+12x$, so subtracting removes $x$ and leaves $y$ on its own.

- Substitute that value back into an original equation to get $x$, then test the pair in both original equations.

### Worked Solution [4 marks]

> Rule - Elimination: multiply each equation so that one unknown has the same coefficient in both, subtract to remove that unknown, then substitute the value found back into an original equation.

**Step 1: Match the $x$ terms**

$$4x + 3y = 9.6 \implies 12x + 9y = 28.8$$
$$6x + 5y = 16.8 \implies 12x + 10y = 33.6$$

*(Reason: (Reason: multiplying every term of the first equation by $3$ and every term of the second by $2$ makes both $x$ coefficients $12$. Multiplying an equation through by a number does not change its solutions.))*

**Step 2: Subtract to remove $x$**

$$(12x + 10y) - (12x + 9y) = 33.6 - 28.8$$
$$y = 4.8$$

*(Reason: (Reason: the $12x$ terms are identical and cancel, and $10y - 9y = y$, so one subtraction leaves y alone.))*

**Step 3: Substitute $y = 4.8$ into the first equation**

$$4x + 3 \times 4.8 = 9.6$$
$$4x + 14.4 = 9.6$$
$$4x = 9.6 - 14.4 = -4.8$$
$$x = \frac{-4.8}{4} = -1.2$$

*(Reason: (Reason: replacing y with $4.8$ leaves an equation in $x$ alone. The $14.4$ moves across the equals sign, so it changes sign, and dividing by $4$ finishes it.))*

$$x = -1.2$$
$$y = 4.8$$

### Verification

✓ **Check 1:** Put $x = -1.2$ and $y = 4.8$ back into the first equation.
$4 \times (-1.2) + 3 \times 4.8 = -4.8 + 14.4 = 9.6$

✓ **Check 2:** The same pair must also satisfy the second equation, which was never used to find $y$. $6 \times (-1.2) + 5 \times 4.8 = -7.2 + 24 = 16.8$

✓ **Check 3:** Eliminate the other way round: multiply the first equation by $5$ and the second by $3$ to remove $y$ instead, giving $20x + 15y = 48$ and $18x + 15y = 50.4$. $2x = 48 - 50.4 = -2.4$, so $x = -1.2$ as before.

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Multiply one or both equations so that one unknown has matching coefficients, with the correct operation selected, for example $12x + 9y = 28.8$ with $12x + 10y = 33.6$, or $20x + 15y = 48$ with $18x + 15y = 50.4$. | M1 | For multiplication of one or both equations with the correct operation selected (allow one arithmetic error). If the operation is not shown, assume it is the one that at least $2$ of the $3$ terms have been calculated for. A correct rearrangement of one equation with substitution into the other earns this mark instead. | ✓ |
| First unknown found: $x = -1.2$ or $y = 4.8$. | A1 | Either value, or equivalent. dep on M1 | ✓ |
| Substitute the value found into one of the original equations, or repeat the scaling and subtraction for the second unknown. | dM1 | For substitution of the found variable, or for repeating the steps of the first M1 for the second variable. dep on M1 | ✓ |
| Both values stated, with the algebraic working shown: $x = -1.2$ and $y = 4.8$. | A1 | Both values, or equivalent. Working required. dep on M1 | ✓ |

**Full marks: 4/4**

## Question 13 - Exam Solution

### Understanding the Question

**Given**

**Find**

$A$, $B$, $C$ and $D$ lie on a circle with centre $O$
Angle $BCD$ is $128^{\circ}$, marked at $C$ on the diagram
$OB$ and $OD$ are radii of the same circle, so they are equal in length

The size of angle $OBD$, in degrees A reason for every stage of the working, not just the arithmetic

### Plan the Solution

- The four vertices of $ABCD$ all sit on the circle, so it is a cyclic quadrilateral. Its opposite angles add up to 180 degrees, and that turns the marked angle at $C$ into angle $BAD$.
- Angle $BAD$ and angle $BOD$ stand on the same arc $BD$, one at the circumference and one at the centre, so the angle at the centre is double the other one.
- Triangle $OBD$ has two sides that are radii, so it is isosceles. Sharing what is left of 180 degrees between its two equal base angles gives the answer.

### Worked Solution [5 marks]

> Rule - three circle facts in a chain: opposite angles of a cyclic quadrilateral add to 180 degrees; the angle at the centre is twice the angle at the circumference standing on the same arc; and the base angles of an isosceles triangle are equal.

**Step 1: find angle $BAD$**

$$180 - 128 = 52$$

*(Reason: $ABCD$ is a cyclic quadrilateral, so angle $BAD$ and angle $BCD$ are opposite angles and add up to 180 degrees)*

**Step 2: find the obtuse angle $BOD$ at the centre**

$$2 \times 52 = 104$$

*(Reason: angle $BAD$ and angle $BOD$ both stand on arc $BD$, one at the circumference and one at the centre, and the angle at the centre is twice the angle at the circumference)*

**Step 3: share the rest of triangle $OBD$**

$$\frac{180 - 104}{2} = 38$$

*(Reason: $OB$ and $OD$ are radii, so triangle $OBD$ is isosceles; angle $OBD$ and angle $ODB$ are equal base angles, so they share what is left of 180 degrees equally)*

**angle $OBD$ = $38^{\circ}$**

### Verification

- ✓ **Check 1 - do the three angles of triangle $OBD$add up:** The triangle has the apex angle at $O$ and the two equal base angles at $B$ and $D$. Add all three. $104 + 38 + 38 = 180$
- ✓ **Check 2 - go round by the reflex angle instead:** Double angle $BCD$ to get the reflex angle at the centre, take that from $360$ to get the obtuse angle $BOD$, then halve what is left of the triangle. $2 \times 128 = 256$, then $360 - 256 = 104$, and $\dfrac{180 - 104}{2} = 38$
- ✓ **Check 3 - run the chain backwards from the answer:** Start at $38$ degrees, rebuild the angle at the centre, halve it for the angle at the circumference, and see whether the marked angle at $C$ comes back. $180 - 2 \times 38 = 104$, then $\dfrac{104}{2} = 52$, and $180 - 52 = 128$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Angle $BAD$ : $180 - 128 = 52$ | **M1** | Award for $180 - 128$, or for the reflex angle route $2 \times 128 = 256$. Angles must be identified either by notation or by being correctly positioned on the diagram. | ✓ |
| Obtuse angle $BOD$: $2 \times 52 (= 104)$ | **M1 dep** | Dependent on the first M1. Award for $2 \times 52$, or for $360 - 256 (= 104)$ on the reflex route. The scheme prints both of those values in quotation marks, so a candidate's own earlier angle follows through. Angles must again be identified by notation or by position on the diagram. | ✓ |
| Angle $OBD$ = 38 | **A1** | A correct answer of $38$ scores the first three marks on its own, unless it comes from obviously incorrect working. | ✓ |
| All three reasons stated correctly for the method used | **B2** | Dependent on a fully correct method for finding angle $OBD$. The reasons are: opposite angles of a cyclic quadrilateral sum to $180$ degrees; the angle at the centre is twice the angle at the circumference, or equally the angle at the circumference is half the angle at the centre; the base angles of an isosceles triangle are equal and the angles in a triangle add to $180$ degrees. On the reflex route, angles around a point adding to $360$ degrees replaces the cyclic quadrilateral reason. | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Only one correct circle theorem given for their method | (B1) | Dependent on the first M1. One correct circle theorem for their own method earns one of the two reasoning marks. | ✓ |
| The reflex route is worth full marks | Note | A candidate who writes reflex $BOD$ first is on the mark scheme's second route: $2 \times 128 = 256$ then $360 - 256 = 104$, which reaches the same obtuse angle at the centre. | ✓ |

**Full marks: 5/5**

## Question 14 - Exam Solution

### Understanding the Question

**Given**

(a) a product of three brackets, $(3x+1)(2-x)(4+x)$
(b) a fraction in $a$ and $b$ raised to the power $-\frac{1}{2}$, $\left(\frac{a^3b}{a^9b^5}\right)^{-\frac{1}{2}}$

Two separate pieces of algebra: one expansion, one index question.

**Find**

(a) the expansion written as a simplified cubic in $x$ (b) one simplified term in $a$ and $b$, with positive indices

### Plan the Solution

- (a) Expand two of the three brackets first, then multiply that quadratic by the bracket that is left over.
- (a) Collect like terms and write the cubic in descending powers.
- (b) Simplify inside the bracket first, using $\frac{a^m}{a^n} = a^{m-n}$ on each letter separately.
- (b) Then apply the power outside the bracket, using $(a^m)^n = a^{mn}$.

### Worked Solution [6 marks]

Rule - expand a triple product two brackets at a time; for indices, $\dfrac{a^m}{a^n} = a^{m-n}$ and $(a^m)^n = a^{mn}$.

**Step 1: expand the first two brackets**

$$(3x+1)(2-x) = 6x - 3x^2 + 2 - x$$
$$= -3x^2 + 5x + 2$$

*(Reason: Each term of the first bracket multiplies each term of the second, and then $6x - x = 5x$. Any two of the three brackets may be chosen here; the other two pairings give $-x^2 - 2x + 8$ and $3x^2 + 13x + 4$, and both finish at the same cubic.)*

**Step 2: multiply that quadratic by the third bracket**

$$(-3x^2 + 5x + 2)(4 + x) = -12x^2 - 3x^3 + 20x + 5x^2 + 8 + 2x$$

*(Reason: Each of the three terms of $-3x^2 + 5x + 2$ multiplies each of the two terms of $4 + x$, so six terms appear before any tidying up.)*

**Step 3: collect like terms**

$$-3x^3 + (5-12)x^2 + (20+2)x + 8$$
$$= -3x^3 - 7x^2 + 22x + 8$$

*(Reason: The cubic term and the constant stand alone. The $x^2$ column is the one that mixes signs, so it is written out in full before it is added, and the $x$ column adds two positive terms.)*

**Step 4: simplify inside the bracket in part (b)**

$$\frac{a^3b}{a^9b^5} = a^{3-9}b^{1-5} = a^{-6}b^{-4}$$

*(Reason: Dividing powers of the same letter subtracts the indices, letter by letter: $3 - 9 = -6$ and $1 - 5 = -4$. The single $b$ on the top counts as $b^1$.)*

**Step 5: apply the power outside the bracket**

$$\left(a^{-6}b^{-4}\right)^{-\frac{1}{2}} = a^{-6\times-\frac{1}{2}}\,b^{-4\times-\frac{1}{2}}$$
$$= a^3b^2$$

*(Reason: Raising a power to a power multiplies the indices. Both products are of two negatives, so both indices come out positive and no negative index is left in the answer.)*

**(a)** $-3x^3 - 7x^2 + 22x + 8$

**(b)** $a^3b^2$

## Verification

✓ **Check 1:** Put $x = 1$ into the original product and into the expansion. The brackets become $(3+1)(2-1)(4+1)$. $4 \times 1 \times 5 = 20$ and $-3 - 7 + 22 + 8 = 20$

✓ **Check 2:** Put $x = -1$, which tests the signs that $x = 1$ cannot. The brackets become $(-3+1)(2+1)(4-1)$. $-2 \times 3 \times 3 = -18$ and $3 - 7 - 22 + 8 = -18$

✓ **Check 3:** Put $a = 2$ and $b = 2$ into part (b). The fraction becomes $\dfrac{16}{16\,384} = \dfrac{1}{1\,024}$, and a power of $-\dfrac{1}{2}$ means the square root of the reciprocal. $\sqrt{1\,024} = 32$ and the answer gives $2^3 \times 2^2 = 32$

✓ **Check 4:** Do part (b) the other way round: deal with the negative sign in the index first by turning the fraction upside down, $\left(\dfrac{a^3b}{a^9b^5}\right)^{-\frac{1}{2}} = \left(\dfrac{a^9b^5}{a^3b}\right)^{\frac{1}{2}}.\ \left(a^6b^4\right)^{\frac{1}{2}} = a^3b^2$

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) Expand two of the three brackets | **M1** | for a correct method to expand two brackets, with at least 3 of the 4 terms correct (or 2 of 3 terms correct), eg $6x - 3x^2 + 2 - x$. Do not award for two separate pair-expansions written side by side, nor for one pair-expansion with the remaining bracket simply added on rather than multiplied. | ✓ |
| (a) Multiply that quadratic by the remaining bracket | **M1ft** | follow through, dependent on the first M1 and on a quadratic having been reached, for a correct method to multiply by the third bracket - allow one further error, eg $-12x^2 - 3x^3 + 20x + 5x^2 + 8 + 2x$ | ✓ |
| (a) Collect like terms | **A1** | for $-3x^3 - 7x^2 + 22x + 8$ or equivalent, but it must be simplified, eg $22x - 3x^3 - 7x^2 + 8$. A correct answer scores full marks unless it follows obviously incorrect working. | ✓ |
| (a) Alternative - expand all three brackets in one go | **Note** | M2 for a complete expansion showing 8 terms of which at least 4 are correct (M1 only, for at least 4 correct terms from any number of terms), then the same A1 for the simplified answer. | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) No working shown | **B2** | if no working is shown, award B2 for 3 correct terms out of a maximum of 4. | ✓ |
| (b) One correct simplification | **M1** | for simplifying the $a$ and the $b$ terms in the fraction, or for applying the power $\frac{1}{2}$ to at least 3 of the 4 indices in $a^3$, $b$, $a^9$, $b^5$, or for applying the negative power to at least 3 of those 4, eg $\left(a^{-6}b^{-4}\right)^{-\frac{1}{2}}$ or $\left(\frac{a^9b^5}{a^3b}\right)^{\frac{1}{2}}$ | ✓ |
| (b) A second correct simplification | **M1** | for two of: simplifying the $a$ and the $b$ terms in the fraction; applying the power $\frac{1}{2}$ to at least 3 of the 4 indices; applying the negative power to at least 3 of the 4 indices, eg $\left(a^6b^4\right)^{\frac{1}{2}}$ or $\left(\frac{1}{a^3b^2}\right)^{-1}$ | ✓ |
| (b) Final answer | **A1** | for $a^3b^2$, accepting $\frac{1}{a^{-3}b^{-2}}$. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |

**Full marks: 6/6**

## Question 15 - Exam Solution

### Understanding the Question

**Given**

Triangle $EFG$ with $EF = GF$ - the two equal sides meet at $F$

Angle $EFG = 130°$, the angle between those two sides

Area of triangle $EFG$ = 74 cm²

**Find**

The length of $EF$, correct to $3$ significant figures

### Plan the Solution

- The marked angle sits between the two equal sides, which is exactly the arrangement $\frac{1}{2}ab\sin C$ needs - no right angle and no perpendicular height required.
- Call each equal side $x$. Both $a$ and $b$ are then $x$, so the formula gives one equation in $x^2$.
- Rearrange for $x^2$, square root, and round only at the very end.

### Worked Solution [3 marks]

> Rule - Area from two sides and the angle between them: $\text{Area} = \frac{1}{2}ab\sin C$, where $C$ is the angle between the sides $a$ and $b$.

**Step 1: call each equal side $x$ and build the area equation**

$$EF = GF = x$$

$$\frac{1}{2} \times x \times x \times \sin 130^\circ = 74$$

*(Reason: Both sides in the formula are the same length here, and $130^\circ$ is the angle between them, so the left-hand side is the area of this triangle.)*

**Step 2: rearrange to find $x^2$**

$$x^2 = \frac{2 \times 74}{\sin 130^\circ} = 193.200\ldots$$

*(Reason: Multiply both sides by $2$ to clear the half, then divide by $\sin 130^\circ$. Keep the whole decimal on the calculator - rounding at this point moves the third significant figure of the answer.)*

**Step 3: square root, then round**

$$x = \sqrt{193.200\ldots} = 13.8996\ldots$$

$$EF = 13.9 \text{ cm}$$

*(Reason: The square root undoes the square. Only now round: $13.8996\ldots$ to $3$ significant figures is $13.9$. The units are centimetres because the area was in square centimetres.)*

$$EF = 13.9 \text{ cm (correct to 3 significant figures)}$$

### Verification

✓ **Check 1:** Put the unrounded length back into the area formula: $\frac{1}{2} \times 13.8996\ldots \times 13.8996\ldots \times \sin 130^\circ$. It gives $74$ cm², the area the question states.

✓ **Check 2:** A different formula. The perpendicular from $F$ to $EG$ splits the $130^\circ$ into two halves of $65^\circ$, so the base is $2 \times 13.8996\ldots \times \sin 65^\circ = 25.194\ldots$ and the height is $13.8996\ldots \times \cos 65^\circ = 5.874\ldots$. Half of $25.194\ldots \times 5.874\ldots$ is $74$ again, from working that never uses $\sin C$.

✓ **Check 3:** Is the size sensible? $\sin 130^\circ$ is less than $1$, so the area has to be less than $\frac{1}{2}x^2$, which is $96.6\ldots$. The given $74$ is comfortably below that bound, so a side near $13.9$ cm is the right order of size.

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| e.g. $\frac{1}{2} \times EF \times GF \times \sin 130^\circ = 74$ oe, or $EF \times GF \times \sin 130^\circ = 2 \times 74$ oe | **M1** | for setting up an equation using the area of a triangle formula | ✓ |
| $(EF^2 =)\dfrac{2 \times 74}{\sin 130^\circ}$ $(= 193.2\ldots)$ oe, or $(EF =)\sqrt{\dfrac{2 \times 74}{\sin 130^\circ}}$ | **M1** | for a complete method to find $EF^2$ or $EF$ | ✓ |
| $13.9$ | **A1** | awrt 13.9 | ✓ |
| A correct answer scores full marks unless it comes from obviously incorrect working. | **Note** | no mark of its own - it records how the A1 is applied | ✓ |

**Full marks: 3/3**

## Question 16 - Exam Solution

### Understanding the Question

**Given**

A grouped frequency table for $58$ pine trees.
The class widths are $2$, $3$, $5$, $10$ and $15$ metres, so the classes are not equally wide.

**Find**

The histogram: how tall to draw each of the five bars, and how wide. Because the classes are unequal, the height of a bar is the frequency density, not the frequency.

| | |
|---|---|
| A grid with frequency density up the vertical axis. | Expect five different heights, one of which reaches $4$ at the top of the grid. |

## Plan the Solution

- Work out the width of every class first, by subtracting its two boundaries.
- Divide each frequency by its own class width to get that class's frequency density.
- Draw each bar from its lower boundary to its upper boundary at that height, with no gaps between bars.
- Check by reading the areas back: a bar's area must return the frequency it came from.

## Worked Solution [3 marks]

> Rule - Histogram with unequal class widths: $\text{frequency density} = \dfrac{\text{frequency}}{\text{class width}}$, so the AREA of a bar is its frequency and the HEIGHT is not.

**Step 1: Work out each class width**

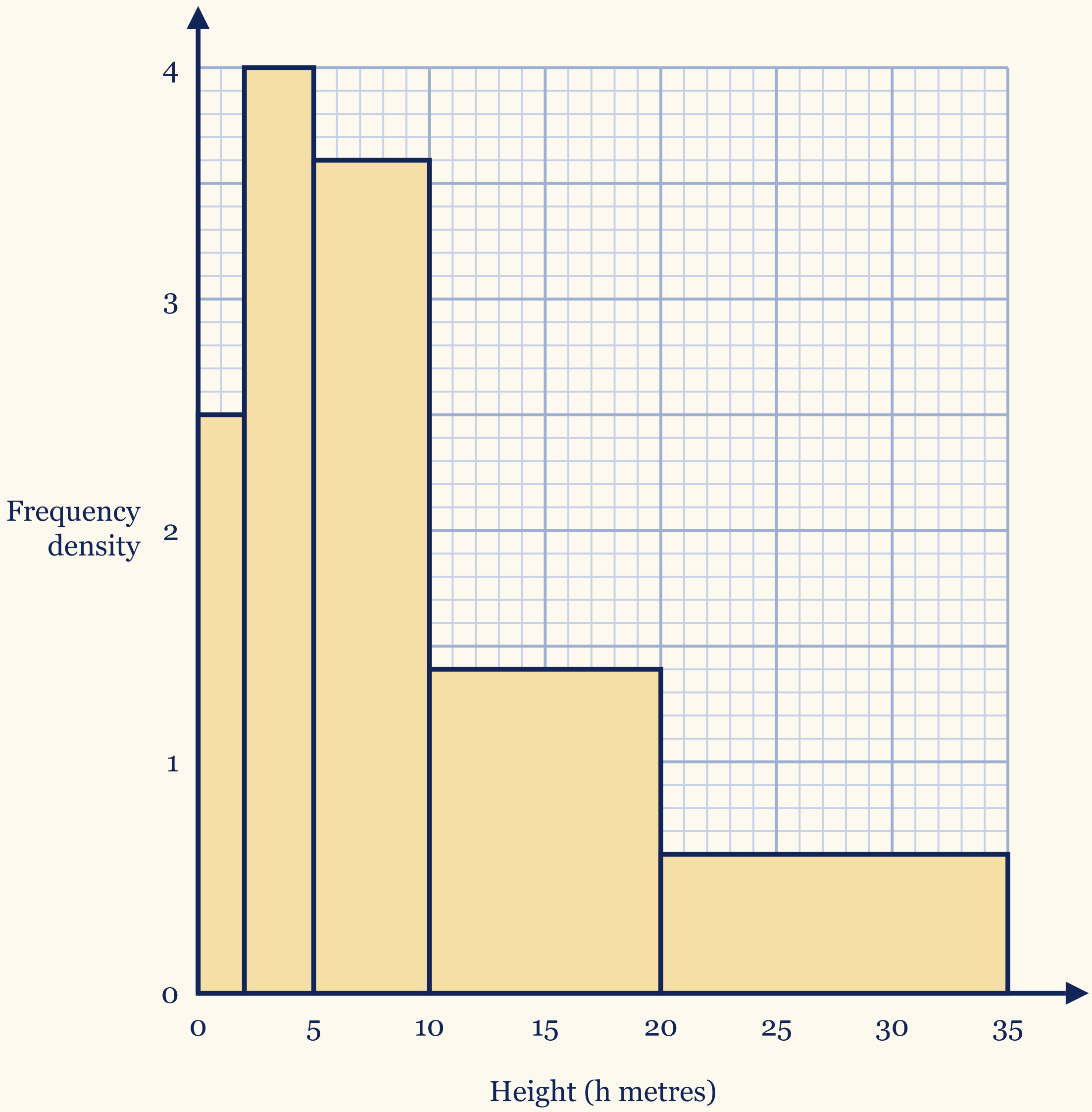
$2 - 0 = 2$

$5 - 2 = 3$

$10 - 5 = 5$

$20 - 10 = 10$

$35 - 20 = 15$




*(Reason: The width is the upper boundary minus the lower boundary. The five widths come out as* $2, 3$ *,* $5, 10$ *and* $15$ *- all different, which is exactly why the frequencies cannot be used as heights.)*

**Step 2: Divide each frequency by its own class width**

$$\frac{5}{2} = 2.5$$

$$\frac{12}{3} = 4$$

$$\frac{18}{5} = 3.6$$

$$\frac{14}{10} = 1.4$$

$$\frac{9}{15} = 0.6$$

*(Reason: Each division shares the frequency out over the metres its class covers, so a frequency density is trees per metre of height. Take the width from Step 1, not the upper boundary: the $5 < h \leq 10$ class is $5$ metres wide, not $10$.)*

### Step 3: Plot the heights and draw the bars

$2.5 = 25 \times 0.1$

$4 = 40 \times 0.1$

$3.6 = 36 \times 0.1$

$1.4 = 14 \times 0.1$

$0.6 = 6 \times 0.1$

*(Reason: One small square up the frequency density axis is $0.1$, so every height lands exactly on a ruled line and nothing has to be judged by eye. Each bar runs from its lower boundary to its upper boundary, so the five bars fill the axis from $0$ to $35$ with no gaps between them.)*

$0 < h \leq 2$ **frequency density** $2.5$

$2 < h \leq 5$ **frequency density** $4$

$5 < h \leq 10$ **frequency density** $3.6$

$10 < h \leq 20$ **frequency density** $1.4$

$20 < h \leq 35$ **frequency density** $0.6$

### Verification

- ✓ **Check 1:** Read the areas back. Width times height must return the frequency that bar was drawn from. $2 \times 2.5 = 5$, $3 \times 4 = 12$, $5 \times 3.6 = 18$, $10 \times 1.4 = 14$, $15 \times 0.6 = 9$
- ✓ **Check 2:** Add the five areas. The whole histogram has to hold every tree, and the table lists $58$ of them. $5 + 12 + 18 + 14 + 9 = 58$
- ✓ **Check 3:** A histogram drawn at half height everywhere is still in the correct ratio, and halving ours gives $1.25$, $2$, $1.8$, $0.7$ and $0.3$ - the five heights the mark scheme names in its special case. Doubling those must return our heights. $2 \times 1.25 = 2.5$, $2 \times 2 = 4$, $2 \times 1.8 = 3.6$, $2 \times 0.7 = 1.4$, $2 \times 0.3 = 0.6$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Any 3 correct frequency densities, or 3 correct bars | **M1** | Three of $\frac{5}{2} = 2.5$, $\frac{12}{3} = 4$, $\frac{18}{5} = 3.6$, $\frac{14}{10} = 1.4$, $\frac{9}{15} = 0.6$, or equivalent. The working alone earns this - the bars need not be drawn yet. | ✓ |
| Any 4 correct frequency densities, or 4 correct bars | **M1** | One more than the first method mark: 4 of the five, by working or by drawing. This is why a single slip, such as dividing by an upper boundary, still leaves 2 marks available. | ✓ |
| A completely correct histogram | **A1** | All five bars: correct widths, edge to edge from 0 to 35, and correct heights. Marked with the overlay. | ✓ |
| All five bars of correct width, with heights in the correct ratio | **SC B2** | Awarded if no other marks are earned - for example bars drawn at 1.25, 2, 1.8, 0.7 and 0.3. Those are every frequency density halved, which is what happens when the frequency is divided by twice the class width. | ✓ |
| Guidance | **Note** | A correct answer scores full marks, unless it comes from obviously incorrect working. | ✓ |

**Full marks: 3/3**

## Question 17 - Exam Solution

### Understanding the Question

**Given**

(a) $\left(\sqrt[4]{k^{12}}\right)^5 = k^n$ - a fourth root raised to a power, written as a single power of $k$.

(b) $\frac{7}{2-\sqrt{3}}$ - a fraction whose denominator contains a surd.

**Find**

(a) the value of $n$. (b) the integers $c$ and $d$ for which $\frac{7}{2-\sqrt{3}} = \sqrt{c} + d$. Note the order: the surd is written first, so the whole number $d$ is the second term.

### Plan the Solution

- Part (a): rewrite the fourth root as the power $\frac{1}{4}$, then multiply the indices twice - once for the root and once for the outer power $5$.
- Part (b): multiply the top and the bottom by the conjugate $2+\sqrt{3}$. The denominator becomes a difference of two squares, so the surd disappears from it.
- Finish part (b) by turning the $7\sqrt{3}$ term into a single square root, because the answer has to look like $\sqrt{c}+d$.

### Worked Solution [4 marks]

> Rule - Index laws: $\sqrt[n]{x}=x^{\frac{1}{n}}$ and $\left(x^a\right)^b=x^{ab}$. Rule - Rationalising a denominator: multiply above and below by the conjugate, and move a coefficient inside a root with $a\sqrt{b}=\sqrt{a^2b}$.

**Step 1: write the fourth root of $k^{12}$ as a power**

$$\sqrt[4]{k^{12}}=\left(k^{12}\right)^{\frac{1}{4}}$$
$$\left(k^{12}\right)^{\frac{1}{4}}=k^{12\times\frac{1}{4}}=k^3$$

*(Reason: An $n$th root is the power $\frac{1}{n}$, and a power raised to a power multiplies the two indices. Here $12\times\frac{1}{4}=3$, so the fourth root of $k^{12}$ is simply $k^3$.)*

**Step 2: raise $k^3$ to the power $5$**

$$\left(k^3\right)^5=k^{3\times5}=k^{15}$$
$$k^n=k^{15}\implies n=15$$

*(Reason: The same law applies again: $3\times5=15$. The two sides are powers of the same base, so the indices must be equal and $n=15$. The whole of part (a) is one $\frac{12}{4}\times5$ calculation.)*

**Step 3: multiply the top and the bottom by the conjugate $2+\sqrt{3}$**

$$\frac{7}{2-\sqrt{3}}=\frac{7\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$$

*(Reason: The conjugate of $2-\sqrt{3}$ is $2+\sqrt{3}$ - the same two terms with the sign between them reversed. Multiplying top and bottom by the same thing does not change the value of the fraction, only*

*how it is written.)*

**Step 4: expand the numerator and the denominator**

$7\left(2+\sqrt{3}\right)=14+7\sqrt{3}$

$\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)=4+2\sqrt{3}-2\sqrt{3}-3=1$

$\dfrac{14+7\sqrt{3}}{1}=14+7\sqrt{3}$

*(Reason: The denominator is a difference of two squares, $2^2-\left(\sqrt{3}\right)^2$: the two middle terms cancel and $4-3=1$ is left. That is the whole point of the conjugate - the surd has gone from the bottom.)*

**Step 5: write the $7\sqrt{3}$ term as a single square root**

$7\sqrt{3}=\sqrt{7^2\times 3}=\sqrt{147}$

*(Reason: A number in front of a square root can be taken inside it, but only after it has been squared: $a\sqrt{b}=\sqrt{a^2b}$. The answer must have the shape $\sqrt{c}+d$, so the coefficient has nowhere else to go.)*

**Step 6: put the answer in the form $\sqrt{c}+d$**

$14+7\sqrt{3}=\sqrt{147}+14$

$c=147$

$d=14$

*(Reason: The question asks for the surd first and the integer second, so the two terms are swapped round. Both $147$ and $14$ are integers, which is what the question demands of c and d.)*

**(a)** $n=15$

**(b)** $\sqrt{147}+14$

## Verification

✓ **Check 1:** Part (a) with a number in place of $k$. Put $k=2$ into the original expression, and into $k^{15}$ separately. $\left(\sqrt[4]{2^{12}}\right)^5=\left(\sqrt[4]{4096}\right)^5=8^5=32768$ and $2^{15}=32768$

✓ **Check 2:** Part (b) backwards. If the answer is right then multiplying it by the original denominator $2-\sqrt{3}$ must give back the numerator $7$.

$\left(14+7\sqrt{3}\right)\left(2-\sqrt{3}\right)=28-14\sqrt{3}+14\sqrt{3}-21=7$

✓ **Check 3:** Part (b) as decimals, which is an entirely different test from the algebra. Work out the printed fraction on the calculator, then work out $\sqrt{147}+14$.

$$\frac{7}{2-\sqrt{3}} \approx 26.1244 \text{ and } \sqrt{147}+14 \approx 12.1244+14 \approx 26.1244$$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) Give the single index $n$ | **B1** | $15$. The scheme also accepts the answer written as $k^{15}$ rather than as a bare number. | ✓ |
| (b) Multiply the numerator and the denominator by the conjugate | **M1** | For multiplying the numerator and the denominator of the fraction by $2+\sqrt{3}$ or by $-2-\sqrt{3}$, for example $\frac{7\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$ or $\frac{7\left(-2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(-2-\sqrt{3}\right)}$. | ✓ |
| (b) Expand to reach a fraction with a rational denominator | **M1** | Dependent on the previous method mark. For example $\frac{14+7\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3}$ or $\frac{14+7\sqrt{3}}{4-3}$ or $\frac{14+7\sqrt{3}}{1}$, and equally the negative versions $\frac{-14-7\sqrt{3}}{-4-2\sqrt{3}+2\sqrt{3}+3}$ or $\frac{-14-7\sqrt{3}}{-4+3}$ or $\frac{-14-7\sqrt{3}}{-1}$. | ✓ |
| (b) The answer, with the working shown | **A1** | $\sqrt{147}+14$, dependent on both method marks. Working is required, so the answer alone does not earn this mark - which is why the question says to show the working clearly. | ✓ |
| (b) Special case - the right answer without the method | **SC** | SCB1 for $\sqrt{147}+14$ gained with no method marks awarded, and SCB2 for $\sqrt{147}+14$ gained with the first method mark awarded. The error being marked is not an arithmetic one: it is answering from a calculator display, or jumping to $14+7\sqrt{3}$ with no rationalising shown, on a question whose marks are mostly for the method. | ✓ |

**Full marks: 4/4**

## Question 18 - Exam Solution

### Understanding the Question

**Given**

Two mathematically similar jars, $P$ and $Q$

Height of $P$ is 30 cm, height of $Q$ is 12 cm

Surface area of $P$ minus surface area of $Q$ is $178.5$ cm²

**Find**

The surface area of jar $P$, in cm²

### Plan the Solution

- Divide the two heights to get the length scale factor.
- Square it, because areas scale by the square of the length scale factor.
- That turns the two surface areas into $25$ parts and $4$ parts of one size, so the given difference is $21$ of those parts.
- Divide $178.5$ by $21$ to get one part, then take $25$ of them.

### Worked Solution [4 marks]

> Rule - Similar solids: if every length is multiplied by $k$, then every area is multiplied by $k^2$ and every volume by $k^3$.

**Step 1: Work out the length scale factor**

$$\frac{30}{12} = \frac{5}{2} = 2.5$$

*(Reason: the jars are mathematically similar, so every length on $P$ is $2.5$ times the matching length on $Q$)*

**Step 2: Square it to get the area scale factor**

$$\frac{5^2}{2^2} = \frac{25}{4} = 6.25$$

*(Reason: an area is a length times a length, so it is multiplied by $2.5$ twice, not once; the surface areas are therefore in the ratio $25$ to $4$)*

**Step 3: Turn that ratio into parts**

$$25 - 4 = 21$$

*(Reason: the surface areas are $25$ parts and $4$ parts of one size, which is the same as calling them $25x$ and $4x$, so the difference is the number of parts between them)*

### Step 4: Work out one part

$$\frac{178.5}{21} = 8.5$$

*(Reason: the difference of $178.5$ cm² given in the question is exactly those $21$ parts)*

### Step 5: Scale up to jar P

$$25 \times 8.5 = 212.5$$

*(Reason: jar $P$ is $25$ of those parts, so multiply one part by $25$)*

**Surface area of jar $P$ = $212.5$ cm²**

## Verification

✓ **Check 1:** Work backwards from the answer. Divide it by the area scale factor $6.25$ to get the smaller surface area, then take the difference. $\frac{212.5}{6.25} = 34$ and $212.5 - 34 = 178.5$, the difference the question gives

✓ **Check 2:** Reach it from the other end. The difference is $5.25$ times the smaller surface area, so divide by $5.25$ first and scale up afterwards. $\frac{178.5}{5.25} = 34$ and $34 \times 6.25 = 212.5$

✓ **Check 3:** Is it sensible? The two surface areas must be in the ratio $25$ to $4$, and the larger one must be bigger than the difference it was built from. $\frac{212.5}{34} = 6.25$, and $212.5$ is bigger than $178.5$

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Length scale factor from the two heights, e.g. $\frac{30}{12} = 2.5$ or the ratio $5$ to $2$ | M1 | for a method to find the ratio for the lengths, or the linear scale factor | ✓ |
| An equation in the surface areas, e.g. $5^2x - 2^2x = 178.5$ | M1 | for setting up an equation using the surface areas | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| or<br>$6.25y - y = 178.5$ | | | |
| A complete method, e.g. $\dfrac{178.5}{5^2 - 2^2} \times 5^2$ or $\dfrac{178.5}{6.25 - 1} \times 6.25$ | **M1** | for a complete method | ✓ |
| The surface area of jar P | **A1** | for $212.5$ or an equivalent value | ✓ |
| A correct answer with no working | **Note** | a correct answer scores full marks, unless it comes from obviously incorrect working | ✓ |
| Using the length ratio instead of the area ratio, giving $1487.5$ | **Note** | this answer comes from dividing by 3 instead of 21, that is from using the ratio 5 to 2 without squaring it. The first method mark is still earned, because the length ratio itself has been found correctly; the second and the third need the area ratio, so the slip scores 1 of the 4 marks | ✓ |

**Full marks: 4/4**

## Question 19 - Exam Solution

### Understanding the Question

**Given**

The curve $C$ has equation $y = x^3 - 8x^2 - 12x + 5$, a cubic

There are exactly two stationary points, $P$ and $Q$

The $x$ coordinate of $P$ is the greater of the two

**Find**

The coordinates of $P$, so both an $x$ value and a $y$ value

### Plan the Solution

- Differentiate to get $\dfrac{dy}{dx}$, which is the gradient function.

- Set the gradient function to zero and solve the quadratic; that gives both stationary $x$ values.
- Take the greater of the two, because the question says $P$ is the point with the greater $x$ coordinate.
- Substitute that $x$ into the equation of $C$, not into the derivative, to get the $y$ coordinate.

### Worked Solution [5 marks]

> Stationary points: solve $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ for $x$, then put each $x$ back into the equation of the curve to find $y$.

**Step 1: differentiate the equation of $C$**

$y = x^3 - 8x^2 - 12x + 5$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 16x - 12$

*(Reason: Each term drops one power and is multiplied by the old power: $x^3$ gives $3x^2$, $-8x^2$ gives $-16x$, $-12x$ gives $-12$, and the constant $5$ disappears.)*

**Step 2: a stationary point has zero gradient**

$3x^2 - 16x - 12 = 0$

*(Reason: At a stationary point the tangent is horizontal, so the gradient function is zero there. This is a quadratic, which is why $C$ has two stationary points and not one.)*

**Step 3: solve the quadratic**

$(3x + 2)(x - 6) = 0$

$x = -\frac{2}{3}$ or $x = 6$

*(Reason: Expanding $(3x + 2)(x - 6)$ gives $3x^2 - 18x + 2x - 12 = 3x^2 - 16x - 12$, so the factorisation is right, and a product is zero only when one bracket is zero. The formula gives the same two values, since $\frac{16 \pm \sqrt{256 + 144}}{6} = \frac{16 \pm 20}{6}$.)*

**Step 4: pick the point with the greater $x$ coordinate**

$6 > -\frac{2}{3}$

*(Reason: The question defines $P$ as the stationary point whose $x$ coordinate is the greater one, so $P$ has $x = 6$ and $Q$ has $x = -\frac{2}{3}$. Only $P$ is wanted here.)*

**Step 5: substitute $x = 6$ into the equation of the curve**

$y = 6^3 - 8 \times 6^2 - 12 \times 6 + 5$

$y = 216 - 288 - 72 + 5 = -139$

*(Reason: The stationary point sits on $C$, so its $y$ coordinate comes from the curve's own equation, never from the derivative. Note that $8 \times 6^2$ squares first and then multiplies, giving $288$, not $48^2$.)*

$$P = (6, -139)$$

## Verification

✓ **Check 1:** Differentiate a second time, $\frac{d^2y}{dx^2} = 6x - 16$, and test both stationary values. At $x = 6$: $6 \times 6 - 16 = 20$, which is positive, so $P$ is a minimum. At $x = -\frac{2}{3}$: $6 \times \left(-\frac{2}{3}\right) - 16 = -20$, which is negative, so $Q$ is a maximum. Two stationary points, one of each kind, exactly as the question states.

✓ **Check 2:** Rebuild the $y$ coordinate by nested multiplication, $((x - 8)x - 12)x + 5$, which uses the coefficients in a different order and never squares or cubes anything. $((6 - 8) \times 6 - 12) \times 6 + 5 = -24 \times 6 + 5 = -144 + 5 = -139$

✓ **Check 3:** Test both roots at once against the quadratic's coefficients: for $ax^2 + bx + c = 0$ the roots sum to $-\frac{b}{a}$ and multiply to $\frac{c}{a}$. $6 + \left(-\frac{2}{3}\right) = \frac{16}{3}$ and $6 \times \left(-\frac{2}{3}\right) = -4$, matching $-\frac{-16}{3} = \frac{16}{3}$ and $\frac{-12}{3} = -4$, so neither root is wrong.

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Differentiate: $\frac{dy}{dx} = 3x^2 - 16x - 12$ | M1 | for differentiation with at least two terms correct | ✓ |
| Set the derivative equal to zero: $3x^2 - 16x - 12 = 0$ | M1ft | (dep on previous M1) for their $\frac{dy}{dx} = 0$ | ✓ |
| Solve the quadratic, e.g. $(3x + 2)(x - 6) = 0$, or | M1ft | (dep on 1st M1) for the correct $x$ value (of 6), ignore the other $x$ value, or for solving their three term quadratic equation using any | ✓ |

| Step | Mark | Description | Got it? |
| --- | --- | --- | --- |
| $x = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4 \times 3 \times (-12)}}{2 \times 3}$ | | correct method. If factorising, allow brackets which expanded give 2 out of 3 terms correct. If using the formula, allow one sign error and some simplification - as far as $\dfrac{16 \pm \sqrt{256+144}}{6}$. If completing the square, then as far as shown on the left-hand side. The award of this mark implies the previous M mark | |
| Substitute into the equation of the curve: $6^3 - 8 \times 6^2 - 12 \times 6 + 5$ | M1ft | (dep on 1st M1) for $x = 6$ substituted into the correct equation for curve $C$, or (dep on 1st M1 and two values for $x$) for their greatest $x$ value substituted into the correct equation for curve $C$, ignoring any attempt to substitute their least $x$ value | ✓ |
| Working required. Coordinates of $P$: $(6, -139)$ | A1 | (dep on M2) cao | ✓ |

**Full marks: 5/5**

## Question 20 - Exam Solution

### Understanding the Question

**Given**

the quadratic $2x^2 - 11x + 9$, to be written as $a(x-b)^2 - c$
the curve $C$ with equation
$y = 2(x-3)^2 - 11(x-3) + 9$
$P$ is the lowest point on $C$

**Find**

the three numbers $a$, $b$ and $c$ the coordinates of $P$, which will be a pair of fractions

### Plan the Solution

- Take the factor $2$ out of the $x^2$ and $x$ terms only, leaving the $+9$ outside the bracket.
- Complete the square inside the bracket, then multiply the bracket by $2$ and combine what is left with the $+9$.
- For part (b), notice that every $x$ of part (a) has become $x-3$, so the completed square of part (a) can be reused with $x-3$ written in place of $x$.
- A squared bracket is never negative, so $y$ is smallest when that bracket is zero. Solve that equation for $x$ and read the $y$ value straight off.

### Worked Solution [5 marks]

> Rule - Completing the square: take the coefficient of $x^2$ outside a bracket, halve the coefficient of $x$ inside that bracket, then subtract the square of that half. A squared bracket is never negative, so the whole expression is at its smallest when the bracket equals $0$.

**Step 1: (a) Take the factor of $2$ out of the $x$ terms**

$$2x^2 - 11x + 9 = 2\left(x^2 - \frac{11}{2}x\right) + 9$$

*(Reason: Only the $x^2$ and $x$ terms carry the factor of $2$. Dividing $-11$ by $2$ gives the $-\frac{11}{2}$ inside, and the $+9$ is left outside so nothing has been changed.)*

**Step 2: Complete the square inside the bracket**

$$x^2 - \frac{11}{2}x = \left(x - \frac{11}{4}\right)^2 - \frac{121}{16}$$

*(Reason: Half of $\frac{11}{2}$ is $\frac{11}{4}$, so the bracket is $\left(x - \frac{11}{4}\right)^2$. Squaring $\frac{11}{4}$ gives $\frac{121}{16}$, and that has to be taken away again because the square put it there.)*

**Step 3: Multiply the bracket back by $2$ and tidy the constant**

$$2\left[\left(x - \frac{11}{4}\right)^2 - \frac{121}{16}\right] + 9$$

$$2 \times \frac{121}{16} = \frac{121}{8}$$

$$\frac{121}{8} - 9 = \frac{49}{8}$$

$$2x^2 - 11x + 9 = 2\left(x - \frac{11}{4}\right)^2 - \frac{49}{8}$$

*(Reason: The $\frac{121}{16}$ is inside the bracket, so it is doubled as well and becomes $\frac{121}{8}$. Taking that away and adding the 9 leaves $\frac{49}{8}$ to be taken away.)*

**Step 4: Read off $a$, $b$ and $c$**

$a = 2$

$b = \frac{11}{4}$

$c = \frac{49}{8}$

*(Reason: Comparing with $a(x-b)^2 - c$ term by term. In decimals these are 2, 2.75 and 6.125, which the mark scheme also accepts.)*

**Step 5: (b) Spot that $C$ is part (a) with $x - 3$ written in place of $x$**

$y = 2(x-3)^2 - 11(x-3) + 9$

$$y = 2\left((x-3) - \frac{11}{4}\right)^2 - \frac{49}{8}$$

*(Reason: Part (a) is true for whatever is written in place of $x$, and here that is $x - 3$ in all three places. So no fresh completing of the square is needed.)*

**Step 6: The square is at its smallest when it is zero**

$$(x-3) - \frac{11}{4} = 0$$

*(Reason: A square is never negative, and the number in front of it, 2, is positive. So $y$ is least exactly when the squared bracket is 0.)*

**Step 7: Add the 3 back to get the $x$ coordinate**

$x - 3 = \frac{11}{4}$

$x = \frac{11}{4} + 3$

$\frac{11}{4} + \frac{12}{4} = \frac{23}{4}$

*(Reason: The bracket is zero when $x - 3 = \frac{11}{4}$, so add 3 to both sides. Writing 3 as $\frac{12}{4}$ keeps both fractions over the same denominator.)*

**Step 8: Read off the $y$ coordinate**

$y = -\dfrac{49}{8}$

*(Reason: With the squared bracket equal to $0$, all that is left of $y$ is the $-\dfrac{49}{8}$. In decimals $P$ is $(5.75,\ -6.125)$.)*

**(a)** $2\left(x - \dfrac{11}{4}\right)^2 - \dfrac{49}{8}$

**(b)** $P$ **is at** $\left(\dfrac{23}{4},\ -\dfrac{49}{8}\right)$

## Verification

✓ **Check 1:** Expand part (a) again. The two constants must recombine to the original $+9$: doubling $\dfrac{121}{16}$ gives $\dfrac{121}{8}$, and the answer takes $\dfrac{49}{8}$ away from it. $\dfrac{121}{8} - \dfrac{49}{8} = 9$

✓ **Check 2:** Put $x = \dfrac{23}{4}$ straight into the equation of $C$. Then $x - 3 = \dfrac{11}{4}$, so the three terms are $2 \times \dfrac{121}{16}$, $-11 \times \dfrac{11}{4}$ and $+9$, written over $8$. $\dfrac{121}{8} - \dfrac{242}{8} + \dfrac{72}{8} = -\dfrac{49}{8}$

✓ **Check 3:** A different route to the same $x$. The roots of $2x^2 - 11x + 9 = 0$ are $x = 1$ and $x = 4.5$, and a parabola is symmetric about the midpoint of its roots. That midpoint is the $b$ of part (a), and adding the shift of $3$ gives the $x$ coordinate of $P$. $\dfrac{1 + 4.5}{2} = 2.75$ and $2.75 + 3 = 5.75$

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) The factor of $2$ taken out of the $x$ terms: $2\left(x^2 - \dfrac{11}{2}x\right)$ | M1 | for taking out a factor of $2$ | ✓ |
| (a) The square completed inside the bracket: $2\left[\left(x - \dfrac{11}{4}\right)^2 - \dfrac{121}{16}\right]$ | M1 | for correctly completing the square | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| (a) $2\left(x-\frac{11}{4}\right)^2-\frac{49}{8}$ | A1 | or equivalent, for example $2(x-2.75)^2-6.125$. Allow $a=2$ , $b=\frac{11}{4}$, $c=\frac{49}{8}$ stated separately. A correct answer scores full marks unless it follows obviously incorrect working. | ✓ |
| (a) Alternative method: expand $a(x-b)^2-c$ to $ax^2-2abx+ab^2-c$ and compare coefficients. | Note | The alternative scheme carries the same three marks: M1 for the correct expansion, M1 for setting up $-2ab=-11$ and $ab^2-c=9$, then A1 for the same answer. | ✓ |
| (a) Answer left as $2\left(x-\frac{11}{4}\right)^2$ with the constant term never dealt with. | SC B1 | a special case: if no other marks are awarded, this earns 1 mark. The error is stopping before the $\frac{121}{16}$ has been doubled and combined with the $+9$. | ✓ |
| (b) $\left(\frac{23}{4}, -\frac{49}{8}\right)$ | B2 | or equivalent, for example $(5.75,\ -6.125)$. Follow through is allowed from the candidate's own $b$ and $c$ in part (a). | ✓ |
| (b) Only one coordinate right, for example $x=\frac{23}{4}$ with no $y$ value. | (B1) | for one correct coordinate, follow through allowed. Shown in brackets because it is the partial award inside the B2 above, not a mark on top of it. | ✓ |

**Full marks: 5/5**

## Question 21 - Exam Solution

### Understanding the Question

**Given**

A jar holds $25$ beads: $6$ green, $x$ amber and the rest cream, with $x>9$.

**Find**

The probability that both beads are cream, with clear algebraic working.

Two beads are taken at random, one after the other and not replaced.
The probability of one amber and one cream bead is $\frac{22}{75}$.

### Plan the Solution

- Write the number of cream beads in terms of $x$: the three colours use up all $25$ beads.
- Multiply along one branch for amber then cream, then double it, because one of each colour happens in two orders.
- Set that equal to $\frac{22}{75}$ and clear the fractions to get a quadratic in $x$.
- Factorise, then use $x > 9$ to choose between the two roots.
- Finish with two cream beads, remembering that the second denominator drops to $24$.

### Worked Solution [5 marks]

> Rule - Taking two objects without replacement: the second fraction has a denominator one smaller, and one of each colour happens in two orders, so
> $P(\text{one of each}) = 2 \times P(\text{first colour, then second})$.

**Step 1: Count the cream beads in terms of $x$**

$$\text{cream} = 25 - 6 - x = 19 - x$$

*(Reason: (Reason: green, amber and cream are the only colours, so the three counts add to $25$ and the cream count is whatever is left.))*

**Step 2: Write the probability of one amber and one cream bead**

$$P(\text{amber, then cream}) = \frac{x}{25} \times \frac{19-x}{24}$$
$$P(\text{cream, then amber}) = \frac{19-x}{25} \times \frac{x}{24}$$
$$P(\text{one of each}) = 2 \times \frac{x}{25} \times \frac{19-x}{24}$$

*(Reason: (Reason: the first bead is not replaced, so the second denominator is $24$. The two orders give the same product, so one of them is doubled.))*

**Step 3: Clear the fractions to get a quadratic in $x$**

$$2 \times \frac{x}{25} \times \frac{19-x}{24} = \frac{22}{75}$$

$$\frac{x(19-x)}{300} = \frac{22}{75}$$

$$x(19-x) = 300 \times \frac{22}{75} = 88$$

$$x^2 - 19x + 88 = 0$$

*(Reason: (Reason: the factor $2$ has already been folded into the left-hand side, so multiplying both sides by the common denominator clears every fraction at once and collecting the terms on one side gives a quadratic.))*

**Step 4: Solve the quadratic, then use $x > 9$**

$$x^2 - 19x + 88 = 0$$

$$(x-8)(x-11) = 0$$

$$x = 8 \text{ or } x = 11$$

$$x > 9 \implies x = 11$$

*(Reason: (Reason: $8 \times 11 = 88$ and $8 + 11 = 19$, so the brackets are correct. Both roots satisfy the given probability, which is exactly why the question states $x > 9$ - it is the condition that decides between them.))*

**Step 5: Work out the probability of two cream beads**

$$\text{cream} = 19 - 11 = 8$$

$$P(2 \text{ cream}) = \frac{8}{25} \times \frac{7}{24}$$

$$= \frac{56}{600} = \frac{7}{75}$$

*(Reason: (Reason: once one cream bead has been taken there are $7$ cream beads left out of $24$, and $\frac{56}{600}$ cancels by $8$.))*

$$\frac{7}{75}$$

### Verification

✓ **Check 1:** Put $x = 11$ back into the probability the question gives. The jar then holds $6$ green, $11$ amber and $8$ cream beads, which is $25$ altogether. $2 \times \frac{11}{25} \times \frac{8}{24} = \frac{88}{300} = \frac{22}{75}$

✓ **Check 2:** Count pairs instead of using order. There are $\frac{8 \times 7}{2} = 28$ ways to choose $2$ cream beads from $8$, and $\frac{25 \times 24}{2} = 300$ ways to choose any $2$ beads from $25$. Order never enters this method, so it is independent of the branch working. $\frac{28}{300} = \frac{7}{75}$

✓ **Check 3:** Test the rejected root. If $x = 8$ there would be $11$ cream beads, and the answer would be different, so the condition $x > 9$ really is doing the work of choosing the root. $\frac{11}{25} \times \frac{10}{24} = \frac{11}{60}$, which is not $\frac{7}{75}$

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| A correct product for one amber and one cream bead in one order: $\frac{x}{25} \times \frac{25-(x+6)}{24}$ or $\frac{x}{25} \times \frac{19-x}{24}$ | M1 | for a correct product for P(amber, cream), oe | ✓ |
| Double it and set it equal to the given probability: $2 \times \left(\frac{x}{25} \times \frac{19-x}{24}\right) = \frac{22}{75}$ | M1 | for setting up a correct equation in $x$, oe | ✓ |
| Clear the fractions: $2x^2 - 38x + 176 (= 0)$ oe eg $x^2 - 19x + 88 (= 0)$ | M1 | for dealing with the fractions to set up a correct quadratic equation | ✓ |
| $x = 11$ or cream $= 25 - 6 - 11 (= 8)$ | M1 | for $x = 11$ or cream $= 25 - 6 - 11 (= 8)$. | ✓ |
| Working required. Final answer $\frac{7}{75}$ | A1 | oe eg $0.093...$ or $9.3...\%$. Working is required, so an answer with no algebraic working scores no marks. | ✓ |

**Full marks: 5/5**

## Question 22 - Exam Solution

### Understanding the Question

**Given**

**Find**

$ABCDEFGH$ is a cuboid whose horizontal base is the rectangle $ADEH$. $AB = 8$ cm, $AD = 12$ cm and $DE = 20$ cm.
$M$ is the centre of the base, so it is the midpoint of each diagonal of $ADEH$.
$P$ is the midpoint of $CF$, a top edge running from the front of the solid to the back.

The size of angle $BMP$, correct to one decimal place. Triangle $BMP$ is a slanting slice through the solid with no right angle in it, so expect the cosine rule rather than ordinary trigonometry.

### Plan the Solution

- The cosine rule needs all three sides, so find $MP$, $BM$ and $BP$ first. Each one is the hypotenuse of a right-angled triangle hidden inside the cuboid.
- Every horizontal distance measured from $M$ is a half, because $M$ is the centre of the base and not a corner of it: half of $AD$ is 6 cm and half of $DE$ is 10 cm.
- $B$ and $P$ are both 8 cm above the base, so $BP$ is purely horizontal - the easiest of the three.
- Finish with the cosine rule rearranged for the angle, and round only on the very last line.

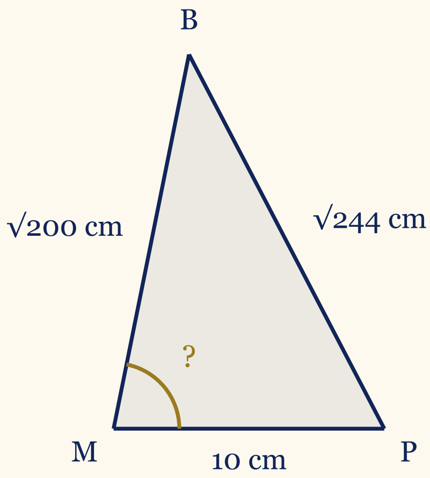
### Worked Solution [6 marks]

> Rule - Cosine rule for an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where $a$ is the side facing the angle. Every length inside a cuboid comes from Pythagoras.

**Step 1: Work out $MP$**

$$MP = \sqrt{(\text{half of } AD)^2 + AB^2}$$

$$\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$$




*(Reason: $M$ and $P$ are both $10$ cm from the front face, so nothing changes in the front-to-back direction. Going from $M$ to $P$ you move across by half of $AD$, which is $6$ cm, and straight up by $AB = 8$ cm.)*

**Step 2: Work out $BM$**

$$AM = \sqrt{(\text{half of } AD)^2 + (\text{half of } DE)^2}$$

$$\sqrt{6^2 + 10^2} = \sqrt{36 + 100} = \sqrt{136}$$

$$BM = \sqrt{AM^2 + AB^2}$$

$$\sqrt{136 + 64} = \sqrt{200} = 14.14 \text{ cm}$$

*(Reason: $AM$ is half of the base diagonal $AE$: across $6$ cm and back $10$ cm. $B$ is straight above $A$, so $AM$ and $AB$ meet at a right angle and Pythagoras applies again. Keep the $200$ rather than the decimal, because it goes straight into the cosine rule.)*

**Step 3: Work out $BP$**

$$BP = \sqrt{AD^2 + (\text{half of } DE)^2}$$

$$\sqrt{12^2 + 10^2} = \sqrt{144 + 100} = \sqrt{244} = 15.62 \text{ cm}$$

*(Reason: $B$ is directly above $A$ and $P$ is level with it, both $8$ cm up, so $BP$ has no vertical part at all. Flatten it onto the top face: across the full $12$ cm, then back half of $20$ cm.)*

**Step 4: Put the three sides into the cosine rule**

$$\cos BMP = \frac{BM^2 + MP^2 - BP^2}{2 \times BM \times MP}$$

$$\frac{200 + 100 - 244}{2 \times \sqrt{200} \times 10} = 0.19799$$

*(Reason: The side facing angle $BMP$ is $BP$, so $BP^2 = 244$ is the one that gets subtracted. The squares $200$ and $100$ go in exactly as they came out of Pythagoras, with no rounding on the way.)*

**Step 5: Take the inverse cosine, then round**

$BMP = \cos^{-1}(0.19799) = 78.58^\circ$

$BMP = 78.6^\circ$ to 1 decimal place

*(Reason: A positive cosine means the angle is acute, which is what the picture shows. Rounding is left to this last line so that nothing is lost on the way, and the mark scheme accepts anything from 78.5 to 78.7.)*

**Angle $BMP = 78.6^\circ$**

### Verification

✓ **Check 1:** Drop the cuboid onto axes, with $A$ at the origin, $D$ at $(12, 0, 0)$, $H$ at $(0, 20, 0)$ and $B$ at $(0, 0, 8)$. Then $M$ is $(6, 10, 0)$ and $P$ is $(12, 10, 8)$. The dot product of the two vectors leaving $M$ gives the cosine without the cosine rule being used at all. The vectors are $(-6, -10, 8)$ and $(6, 0, 8)$, so the dot product is $-36 + 0 + 64 = 28$. Dividing by $10\sqrt{200}$ gives $0.19799$ once more, and the same $78.6^\circ$.

✓ **Check 2:** Work out the other two angles of triangle $BMP$ with the same cosine rule and add all three. A triangle that closes must give $180^\circ$. The angle at $P$ is $62.55^\circ$ and the angle at $B$ is $38.87^\circ$, and $78.58 + 62.55 + 38.87 = 180.00$.

✓ **Check 3:** A shape check that needs almost no arithmetic. In any triangle the longest side lies opposite the largest angle, and the three sides here are $MP = 10$ cm, $BM = 14.14$ cm and $BP = 15.62$ cm. $BP$ is the longest, and it is the side facing $M$, so the angle at $M$ has to be the biggest of the three - and $78.6^\circ$ beats $62.6^\circ$ and $38.9^\circ$.

### Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| $MP = \sqrt{8^2 + 6^2} \quad (= 10)$ | **M1** | For a method to find $MP$. It may be seen in later working, for example in the correct place in the cosine rule. A bare 10 with no correct method, or not identified as $MP$, scores M0. | ✓ |
| $BM = \sqrt{\left(\sqrt{10^2 + 6^2}\right)^2 + 8^2} \quad (= \sqrt{200} = 10\sqrt{2} = 14.1\ldots)$ | **M1** | For a method to find $BM$. | ✓ |

| Step | Mark | Description | Got it? |
|---|---|---|---|
| $BP = \sqrt{12^2 + 10^2} \quad (= \sqrt{244} = 2\sqrt{61} = 15.6\ldots)$ | M1 | For a method to find $BP$. | ✓ |
| For example $244 = 10^2 + 200 - 2 \times 10 \times \sqrt{200} \cos BMP$ , or $\cos BMP = \dfrac{10^2 + 200 - 244}{2 \times 10 \times \sqrt{200}}$ oe | M1 | For correct substitution into the cosine rule. The scheme prints each length in quotation marks, so a candidate's own values for $MP$, $BM$ and $BP$ follow through. | ✓ |
| $BMP = \cos^{-1}\left(\dfrac{10^2 + 200 - 244}{2 \times 10 \times \sqrt{200}}\right)$ oe | M1 | For a complete correct method to find angle $BMP$. | ✓ |
| 78.6 | A1 | Accept anything from $78.5$ to $78.7$. A correct answer scores full marks unless it comes from obviously incorrect working. | ✓ |

**Full marks: 6/6**

## Question 23 - Exam Solution

### Understanding the Question

**Given**

The first three terms of an arithmetic sequence are $(4x - 14)$, $(x + 2)$ and $(7x - 9)$.

In an arithmetic sequence consecutive terms differ by the same amount, the common difference $d$.

**Find**

The sum of the first $40$ terms of the sequence, written as an integer.

### Plan the Solution

- The gap from the first term to the second equals the gap from the second to the third. Write that as one equation in $x$ and solve it.

- Put the value of $x$ back into the three expressions to get the first term $a$ and the common difference $d$.
- Substitute $a$, $d$ and $n = 40$ into the sum formula for an arithmetic series.

### Worked Solution [4 marks]

> Rule - Arithmetic series: $S_n = \frac{n}{2}[2a + (n-1)d]$, where $a$ is the first term, $d$ the common difference and $n$ the number of terms.

**Step 1: the two gaps are equal**

$(x+2) - (4x-14) = (7x-9) - (x+2)$

$-3x + 16 = 6x - 11$

*(Reason: The sequence is arithmetic, so the second term minus the first equals the third minus the second. Removing the brackets is where marks are lost: $-(4x-14)$ gives $-4x+14$, not $-4x-14$.)*

**Step 2: solve for $x$**

$16 + 11 = 6x + 3x$

$27 = 9x$

$x = 3$

*(Reason: Collecting the $x$ terms on the side where they stay positive avoids a sign slip. Dividing $27$ by $9$ gives $x = 3$.)*

**Step 3: the first term and the common difference**

$4 \times 3 - 14 = -2$

$3 + 2 = 5$

$7 \times 3 - 9 = 12$

$5 - (-2) = 7$

$12 - 5 = 7$

*(Reason: Substituting $x = 3$ turns the three expressions into the numbers $-2$, $5$ and $12$. Both gaps come to $7$, which confirms the sequence really is arithmetic, so $a = -2$ and $d = 7$.)*

**Step 4: sum the first $40$ terms**

$$S_{40} = \frac{40}{2}[2 \times (-2) + 39 \times 7]$$

$20 \times (-4 + 273) = 20 \times 269 = 5380$

*(Reason: The formula uses $(n-1)$ lots of the common difference, not $n$ lots, because the first term already sits at the start of the sequence. With $n = 40$ that is $39$ lots of $7$, giving $-4 + 273 = 269$ inside the bracket.)*

5380

## Verification

✓ **Check 1:** Put $x = 3$ back into the three printed expressions and compare the two gaps. The terms are $-2, 5, 12$, with gaps $5 - (-2) = 7$ and $12 - 5 = 7$, so the sequence is arithmetic.

✓ **Check 2:** Pair the terms instead of using the formula. The $40$th term is $-2 + 39 \times 7 = 271$, and the $40$ terms fall into $20$ pairs, each pair worth $-2 + 271$. $20 \times 269 = 5380$, the same total.

✓ **Check 3:** Use the mean of the first and last terms. That mean is $\dfrac{-2 + 271}{2} = 134.5$, and there are $40$ terms. $134.5 \times 40 = 5380$, so all three methods agree.

## Mark Scheme Breakdown

| Step | Mark | Description | Got it? |
|---|---|---|---|
| Set up an equation in $x$ | M1 | For $(x + 2) - (4x - 14) = (7x - 9) - (x + 2)$ oe, e.g. $6x - 11 = 16 - 3x$, or two simultaneous equations in $x$ and $d$. | ✓ |
| Correct values, or correct substitution | M1 | For $x = 3$ and $a = -2$ and $d = 7$, or $x = 3$ with $S_{40}$ expressed in terms of $x$. Allow $(40 - 1)$ for $39$. | ✓ |
| Substitute into the sum formula | M1 | For $\frac{40}{2}(2 \times (-2) + 39 \times 7)$ oe. Allow their own $a$ and their own $d$, or their own $x$ as long as it is clearly stated. Allow $(40 - 1)$ for $39$. | ✓ |
| The integer sum | A1 | For $5380$, dependent on at least one method mark. | ✓ |
| Working required | Note | The paper asks for clear algebraic working and the accuracy mark is dependent, so a correct answer written down with no algebraic working earns nothing. The common slip is taking $40$ lots of the common difference instead of $39$, which gives $5520$. The third method mark needs the 39 itself - the scheme allows only (40 - 1) as another way of writing it - so that slip scores 2 of the 4 marks. | ✓ |

**Full marks: 4/4**

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