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Edexcel IGCSE 4MA1

4MA1/2H (Higher Tier) – Monday 3 June 2024

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/2H (Higher Tier), June 2024 series, sat Monday 3 June 2024 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Eight numbers are written below in order of size.

$h \quad 6 \quad 7 \quad 8 \quad j \quad 16 \quad k \quad k$

where h, j and k are integers.

For these eight numbers:
the median is 10
the mode is 18
the range is 13

Find the value of h, the value of j and the value of k. *[3 marks]*

h = ...

j = ...

k = ...

[Total 3 marks]

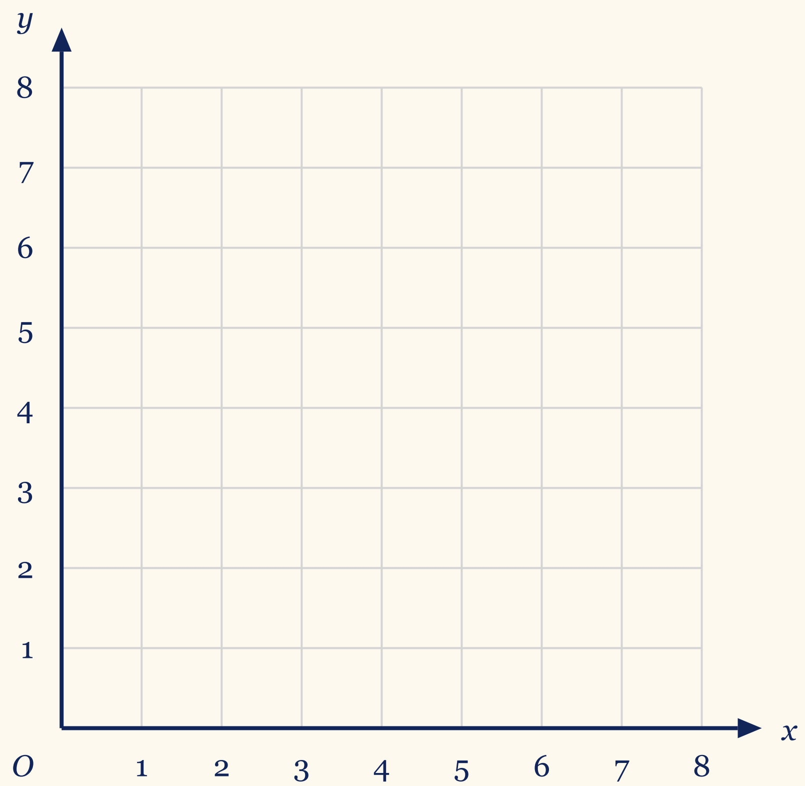
2. (a) On the grid below, draw the straight line whose equation is

(i) $y = 2$

(ii) $x = 6$

(iii) $y = x + 1$

Write the equation of each line beside the line you have drawn. *[3 marks]*



(b) On the same grid, shade the one region that satisfies all three of these inequalities at once

$y \geq 2$ $\qquad x \leq 6$ $\qquad y \leq x + 1$

Write the letter R inside the region you have shaded. *[1 mark]*

[Total 4 marks]

3. A cargo aircraft flies from Dubai to Tokyo.
The flight takes 9 hours 36 minutes.
The average speed of the aircraft for the whole flight is 820 km/h.
Work out the total distance the aircraft flies. *[3 marks]*

.... km

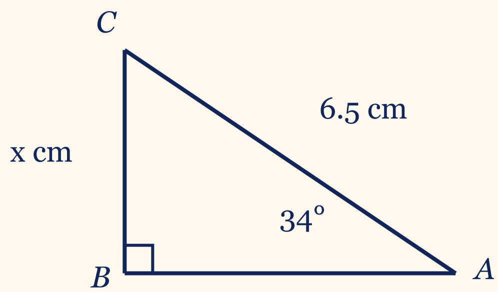
[Total 3 marks]

4. Show that $2\frac{4}{7} \times 3\frac{1}{9} = 8$
You must show all your working. *[3 marks]*

[Total 3 marks]

5. Triangle ABC is shown in the diagram below.

Diagram NOT
accurately drawn



Work out the value of x.
Give your answer correct to one decimal place. *[3 marks]*

x = ...

[Total 3 marks]

6. A cable car moves at a steady speed of w metres per second.
Work out the speed of the cable car in kilometres per hour.
Give your answer in terms of w in its simplest form. *[3 marks]*

.... kilometres per hour

[Total 3 marks]

7. The diagram shows the six-sided shape $ABCDEF$.

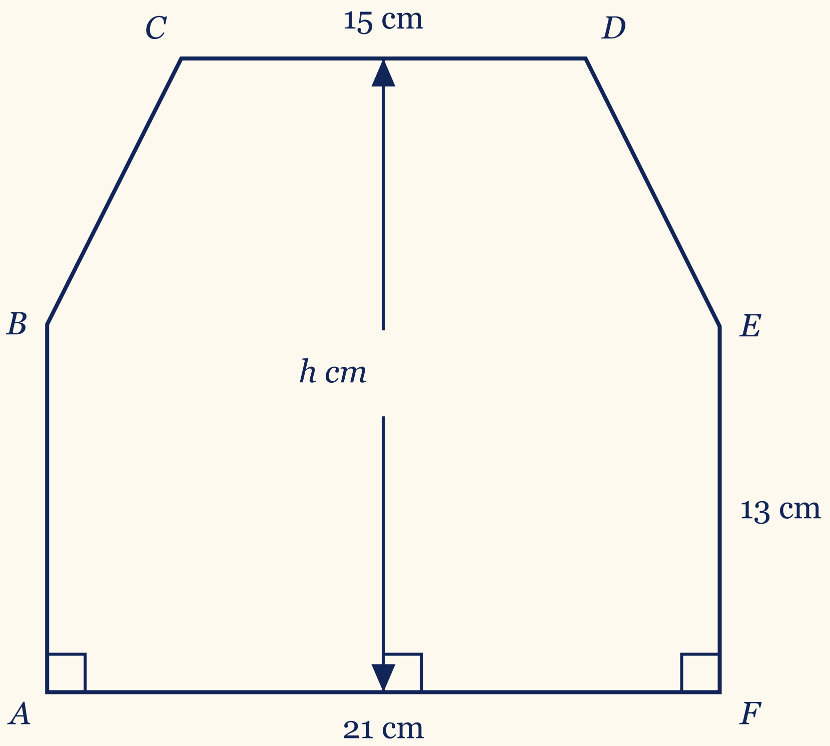


Diagram NOT accurately drawn

$AF = 21$ cm
$CD = 15$ cm
$AB = FE = 13$ cm

CD is parallel to AF.
The perpendicular height of the shape is h cm.
The shape has an area of 390 cm^2.

Find the value of h. *[4 marks]*

h = ...

[Total 4 marks]

8. Nikhil orders 600 seedlings for a plant nursery.
He orders marigold seedlings, aster seedlings and zinnia seedlings so that the number of each kind is in the ratio

$$\text{marigold} : \text{aster} : \text{zinnia} = 9 : 4 : 2$$

45% of the marigold seedlings are for orange flowers.

$\frac{5}{8}$ of the aster seedlings are for orange flowers.

All of the zinnia seedlings are for orange flowers.

Work out the number of seedlings that are for orange flowers. *[5 marks]*

....

[Total 5 marks]

9. Marek pays $4\,500$ koruna into a savings bond.
The bond runs for 4 years and pays 2.4% per year compound interest.

Work out how much money Marek will have in the bond at the end of the 4 years.
Give your answer correct to the nearest koruna. *[3 marks]*

.... koruna

[Total 3 marks]

10. Solve the simultaneous equations

$$6x + 4y = 1$$
$$3x + 5y = 8$$

You must show clear algebraic working. *[3 marks]*

x = ...

y = ...

[Total 3 marks]

11. (i) Write $x^2 + 9x - 22$ as a product of two brackets.
[2 marks]
(ii) Hence solve the equation $x^2 + 9x - 22 = 0$ *[1 mark]*

(i) ....

(ii) ....

[Total 3 marks]

12. Bilal records the number of bottles a machine fills each day for 7 days.

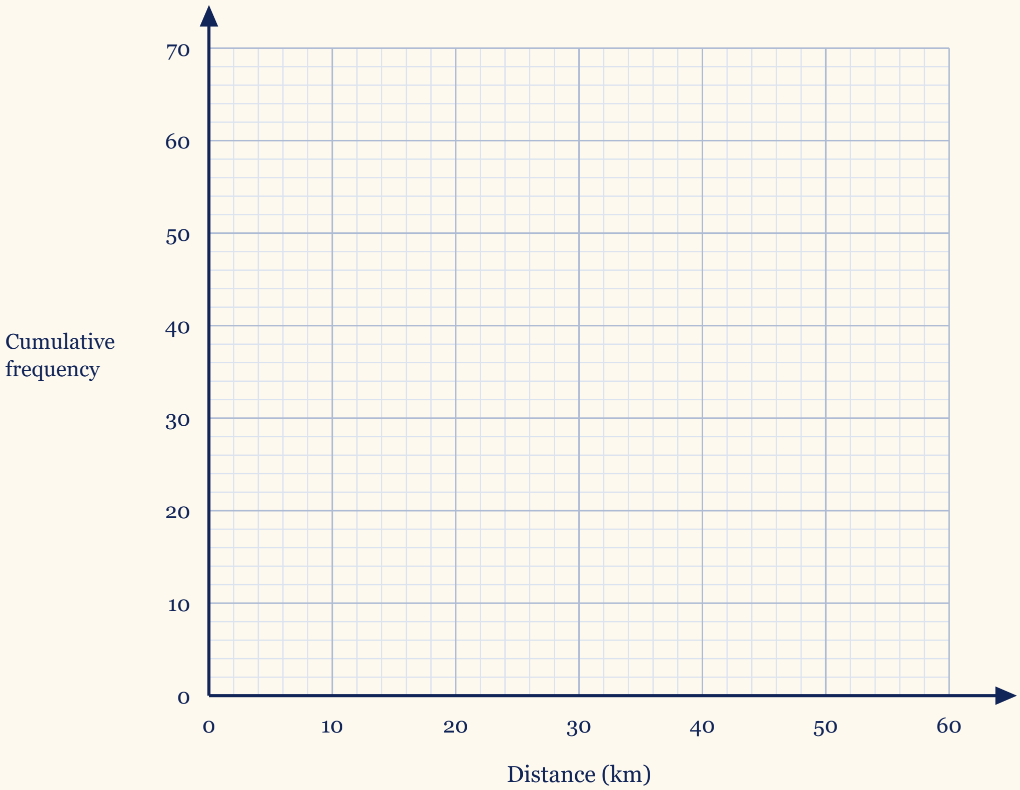
For the first 4 days, the mean number of bottles filled each day is $11\,800$
For the next 3 days, the mean number of bottles filled each day is $13\,207$

Work out the mean number of bottles filled each day for the 7 days. *[3 marks]*

....

[Total 3 marks]

13. A hospital manager records the distance, in km, that each of 70 nurses travels to work.
The table gives information about these distances.



Distance (d km)	Frequency
$0 < d \leq 10$	7
$10 < d \leq 20$	17
$20 < d \leq 30$	18
$30 < d \leq 40$	14
$40 < d \leq 50$	10
$50 < d \leq 60$	4

(a) Complete the cumulative frequency table.

Distance (d km)	Cumulative frequency
$0 < d \leq 10$	
$0 < d \leq 20$	
$0 < d \leq 30$	
$0 < d \leq 40$	
$0 < d \leq 50$	
$0 < d \leq 60$	

[1 mark]

(b) On the grid below, draw a cumulative frequency graph for your table. *[2 marks]*

(c) Use your graph to find an estimate for the interquartile range of the distances. *[2 marks]*

(d) Use your graph to find an estimate for the number of nurses who travel more than 46 km. *[2 marks]*

(c) km ....

(d) ....

[Total 7 marks]

14. (a) Show that the product $3y(2y+5)(y+7)$ can be written as ay^3+by^2+cy, where a, b and c are integers. *[3 marks]*

(b) Solve the equation $\dfrac{2x+3}{5}+\dfrac{6x-5}{4}=\dfrac{163}{100}$

You must show clear algebraic working. *[4 marks]*

x = ...

[Total 7 marks]

15. (a) Rearrange the formula

$$e=\sqrt{\frac{7g+5}{11+2g}}$$

to make g the subject. *[4 marks]*

(b) Solve the inequality $3y^2+4y-32>0$

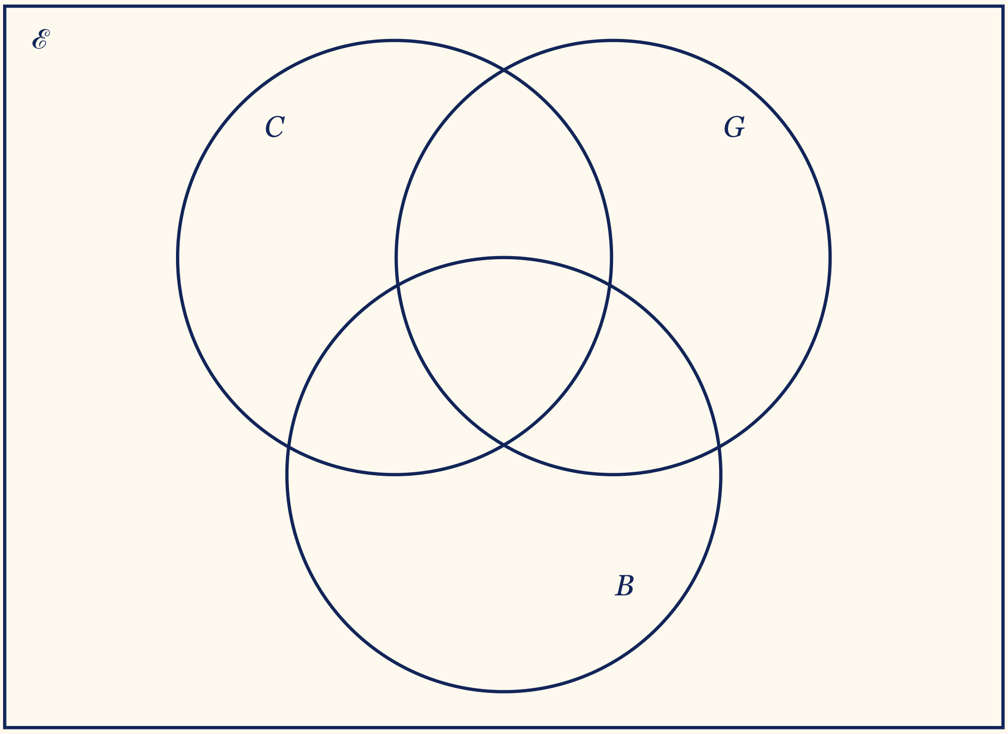
You must show your working clearly. *[3 marks]*

(a) ....

(b) ....

[Total 7 marks]

16. 60 members of a community centre were asked if they would like to join clubs for chess (C), for gardening (G) or for baking (B)



Of these members
9 chose chess, gardening and baking
17 chose chess and gardening
16 chose gardening and baking
20 chose chess and baking
28 chose gardening
39 chose baking
2 chose none of the clubs

(a) Using this information, complete the Venn diagram to show the numbers of members in each subset. *[3 marks]*

One of the members is chosen at random.
Given that this member chose gardening,

(b) find the probability that this member also chose chess. *[2 marks]*

(c) Find $\mathrm{n}(G \cap C')$ *[1 mark]*

(d) Find $\mathrm{n}([G \cup B] \cap C)$ *[1 mark]*

(b) ....

(c) ....

(d) ....

[Total 7 marks]

17. Q is directly proportional to the square root of d
When $d = 324$, $Q = 4.5$
Work out a formula for Q in terms of d *[3 marks]*

....

[Total 3 marks]

18. The straight line P has equation $5y + 2x = 7$

The straight line Q is perpendicular to P
Work out the gradient of Q *[2 marks]*

....

[Total 2 marks]

19. $$G = \frac{c}{2f - 3h}$$

Each of the three values below has been rounded.

$c = 8$ correct to the nearest whole number
$f = 6.62$ correct to 2 decimal places
$h = 1.2$ correct to 1 decimal place

Find the lower bound for the value of G
Give your answer correct to 3 decimal places.
You must show your working clearly. *[3 marks]*

....

[Total 3 marks]

20. Given that $k = x - y$ and $x = \dfrac{1}{4y}$

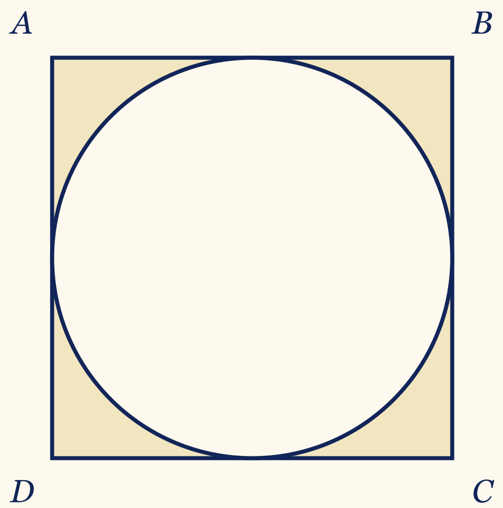
write $\dfrac{5k}{x+2}$ in the form $\dfrac{a - by^2}{c + dy}$, where a, b, c and d are integers. *[3 marks]*

....

[Total 3 marks]

21. The diagram shows a circle drawn inside a square $ABCD$.
Each side of the square is a tangent to the circle.

Not drawn accurately



The shaded regions have a total area of 80 cm².

Work out the length of AC.
Give your answer correct to 3 significant figures. *[5 marks]*

.... cm

[Total 5 marks]

22. The straight line L has equation $x + y = 5$
The curve C has equation $2x^2 + 3y^2 = 210$

Work out the coordinates of the points of intersection of L and C.
You must show clear algebraic working. *[5 marks]*

....

[Total 5 marks]

23. Simplify

$$\frac{30 \times 25^{2x+7}}{\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9}}$$

Write your answer in the form 5^w, where w is an expression in terms of x.
You must show each stage of your working clearly. *[3 marks]*

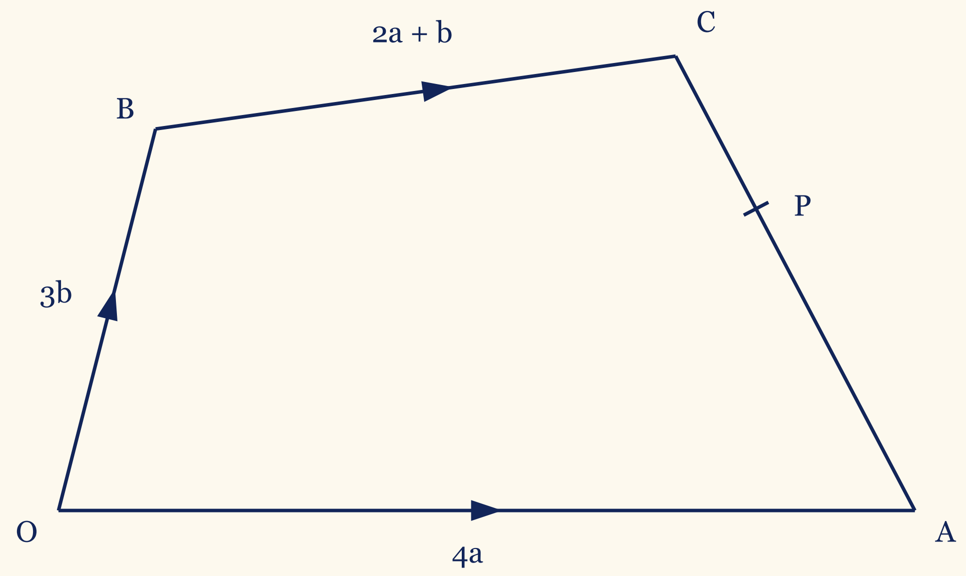
....

[Total 3 marks]

24. The diagram shows the quadrilateral $OACB$.

In this quadrilateral $\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OB} = 3\mathbf{b}$ and $\overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

Diagram NOT accurately drawn



(a) Work out $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Write your answer as simply as possible. *[2 marks]*

The point P lies on AC so that $AP : PC = 3 : 2$

The point Q is placed so that OPQ is a straight line and BCQ is a straight line.

(b) Use a vector method to find $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Write your answer as simply as possible.

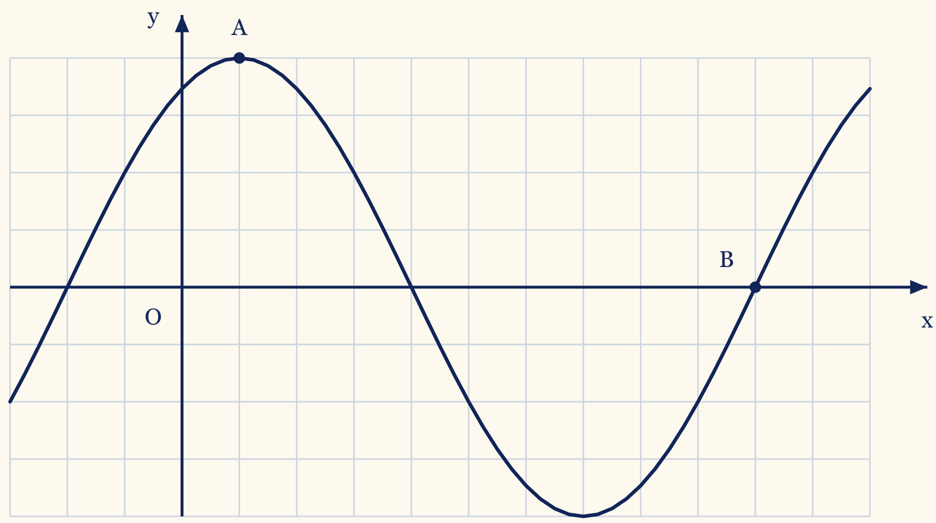
You must show your working clearly. *[4 marks]*

(a) AC = ...

(b) OQ = ...

[Total 6 marks]

25. The sketch shows the curve with equation $y = 2\sin(x + 60)^\circ$
The points A and B are marked on the curve.



(i) Find the coordinates of the point A *[1 mark]*

(ii) Find the coordinates of the point B *[1 mark]*

(i) ....

(ii) ....

[Total 2 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Eight numbers, in order of size:

$h \quad 6 \quad 7 \quad 8 \quad j \quad 16 \quad k \quad k$

h, j and k are integers, and the list is already ordered, so h is the smallest number and k is the largest.

The median is 10, the mode is 18 and the range is 13.

Find

The value of h. The value of j. The value of k.

Plan the Solution

- Take the three averages one at a time, and start with the one that gives a value on its own. Only k is written twice, so the repeated value in the list is k and the mode names it immediately.
- Then the median. There are eight numbers, an even amount, so the median sits halfway between the 4th and the 5th, which are 8 and j.
- Finish with the range, which links the two ends of an ordered list, so it ties h to the k already found.
- Write the completed list out at the end and read the three averages back off it, because the values must also leave the list in order of size.

Worked Solution [3 marks]

> Rule - For a list already written in order of size: the median of an even amount of numbers is the mean of the two middle ones, the mode is the value that occurs most often, and the range is $\text{largest} - \text{smallest}$.

Step 1: use the mode to find k

$k = 18$

(Reason: (Reason: the mode is the value that occurs most often. Every entry in the list is written once except k, which is written twice, so the mode has to be k itself. The mode is given as 18.))

Step 2: use the median to find j

$$\frac{8+j}{2} = 10$$

$$8 + j = 20$$

$$j = 12$$

(Reason: (Reason: with eight numbers in order the median is the mean of the 4th and the 5th, which are 8 and j. Multiplying the median by 2 gives 20 for the two of them together, and taking the 8 away leaves 12.))

Step 3: use the range to find h

$$k - h = 13$$

$$18 - h = 13$$

$$h = 18 - 13 = 5$$

(Reason: (Reason: the list is in order of size, so the largest number is k, now known to be 18, and the smallest is h. The range is the largest take away the smallest, so h sits 13 below 18.))

Step 4: write the completed list out

$$5 \quad 6 \quad 7 \quad 8 \quad 12 \quad 16 \quad 18 \quad 18$$

(Reason: (Reason: the question says the eight numbers are in order of size, so that has to still be true once the three values are put back. Reading left to right, the list never goes down, so the answers are consistent with the way the question presents them.))

$$h = 5$$

$$j = 12$$

$$k = 18$$

Verification

✓ **Check 1:** Put the three values back and find the middle pair of the completed list. The 4th and 5th numbers are 8 and 12, so take their mean. $\dfrac{8+12}{2} = 10$

✓ **Check 2:** The completed list runs from 5 up to 18, so subtract the smallest from the largest. $18 - 5 = 13$

✓ **Check 3:** Count how often each value occurs in the completed list, then read it from left to right to confirm it never goes down. Only 18 occurs twice, so the mode is 18, and $5 \quad 6 \quad 7 \quad 8 \quad 12 \quad 16 \quad 18 \quad 18$ is in order of size.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One of the three values found, or a correct statement leading to one of them	M1	For a correct value for h, j or k, or for a correct statement for one of them, e.g. $k = 18$, or $\frac{8+j}{2} = 10$, or $8 + j = 2 \times 10$, or $10 \times 2 - 8$, or $j = 12$, or $k - h = 13$, or $18 - h = 13$, or $h = 5$. The scheme quotes the 18 in that last statement, so the candidate's own value of k may stand in its place.	✓
Two of the three values found, or correct statements for two of them	M1	For 2 correct values from h, j or k, or for 2 correct statements for them.	✓
All three values correct	A1	$h = 5$, $j = 12$ and $k = 18$. All three are needed for this mark.	✓
A correct answer written down with little or no working	Note	A correct answer scores full marks, unless it clearly comes from obviously incorrect working.	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

A square grid whose x-axis and y-axis are both numbered from 0 to 8.
Three equations to draw: $y = 2$, $x = 6$ and $y = x + 1$.
Three inequalities to satisfy together: $y \geq 2$, $x \leq 6$ and $y \leq x + 1$.

Find

Part (a): each of the three lines drawn on the grid, with its equation written beside it. Part (b): the one region where all three inequalities hold at once, shaded and marked R.

Plan the Solution

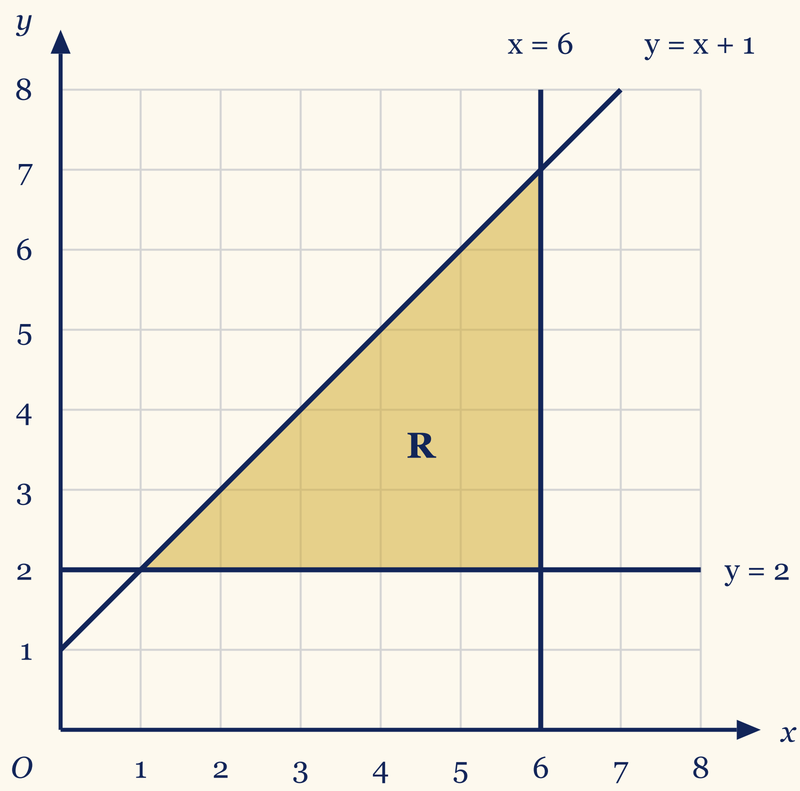
- Sort the three equations by shape before drawing anything. $y = 2$ has no x in it, so every point on it sits at the same height and the line is horizontal. $x = 6$ has no y in it, so it is vertical. $y = x + 1$ has both letters, so it slopes.
- A straight line needs only two points. Take the two edges of the grid for the first two lines, and substitute two values of x for the third.
- Each inequality keeps one side of its own line. Settle which side by testing a single point that is not on any of the lines, rather than by reading the direction off the sign.
- The region is where all three sides overlap. Work out the three points where the boundaries cross, so the shading can be drawn exactly, then shade it and mark it R.

Worked Solution [4 marks]

> Rule - A line $y = a$ is horizontal, a line $x = a$ is vertical, and a line $y = mx + c$ slopes. An inequality keeps one whole side of its boundary line, and which side is settled by testing one point that is not on the line.

Step 1: draw $y = 2$, the horizontal line

$$y = 2$$
$$(0, 2) \quad \text{and} \quad (8, 2)$$



(Reason: (Reason: the equation fixes the height and says nothing at all about x, so every point 2 units above the x-axis lies on it. Joining $(0, 2)$ to $(8, 2)$ carries the line right across the grid, which is well over the 2 cm the mark scheme asks for.))

Step 2: draw $x = 6$, the vertical line

$$x = 6$$
$$(6, 0) \quad \text{and} \quad (6, 8)$$

(Reason: (Reason: this time the equation fixes how far across the point is and leaves the height free, so the line runs straight up from $(6, 0)$ to $(6, 8)$. It is the mirror image of Step 1: no y in the equation means vertical, no x means horizontal.))

Step 3: draw $y = x + 1$, the sloping line

$$x = 0: \quad y = 0 + 1 = 1$$
$$x = 7: \quad y = 7 + 1 = 8$$
$$(0, 1) \quad \text{and} \quad (7, 8)$$

(Reason: (Reason: both letters appear, so pick two values of x and work out the matching y. $x = 8$ is no use here because it would give $y = 9$, which is off the top of the grid, so $x = 7$ is the last value that still lands on the paper.))

Step 4: test the point $(4, 3)$ to fix which side of each line to keep

$$y \geq 2 \text{ gives } 3 \geq 2$$
$$x \leq 6 \text{ gives } 4 \leq 6$$
$$y \leq x + 1 \text{ gives } 3 \leq 4 + 1$$

(Reason: (Reason: $(4, 3)$ is not on any of the three lines, so it is strictly on one side of each of them. All three inequalities are true there, so the side to keep is above $y = 2$, to the left of $x = 6$ and below $y = x + 1$. Testing a point is safer than reading the direction off the sign, because below and above only mean anything once the line is on the paper.))

Step 5: find the corners, then shade and label the region

$$y = 2 \text{ meets } y = x + 1: \quad 2 = x + 1 \implies x = 1$$
$$y = 2 \text{ meets } x = 6: \quad (6, 2)$$
$$x = 6 \text{ meets } y = x + 1: \quad y = 6 + 1 = 7$$
$$(1, 2), \quad (6, 2), \quad (6, 7)$$

(Reason: (Reason: two boundaries cross at every corner, so taking the lines in pairs gives the three corners exactly rather than by eye. The first pair gives the corner on the left, the second the corner on the right, and the third the corner at the top. Shade the triangle those three points enclose and write R inside it, well clear of the edges.))

(a)(i) $y = 2$ drawn from $(0, 2)$ to $(8, 2)$

(a)(ii) $x = 6$ drawn from $(6, 0)$ to $(6, 8)$

(a)(iii) $y = x + 1$ drawn from $(0, 1)$ to $(7, 8)$

(b) R is the triangle with corners $(1, 2)$, $(6, 2)$ and $(6, 7)$

Verification

✓ **Check 1:** Take a point well inside the shaded triangle, $(4, 3)$, and put it into all three inequalities. $3 \geq 2$, $4 \leq 6$ and $3 \leq 4 + 1$, so $(4, 3)$ really is in the region.

✓ **Check 2:** Step just across each boundary in turn and confirm the point leaves the region: $(4, 1)$ below $y = 2$, $(7, 3)$ to the right of $x = 6$, and $(2, 5)$ above $y = x + 1$. $1 < 2$, $7 > 6$ and $5 > 3$, so each point breaks exactly one inequality and all three lines are genuine edges of the region.

✓ **Check 3:** Count the whole-number points of the region column by column. The column at x runs from $y = 2$ up to $y = x + 1$, which is x points, and the columns run from $x = 1$ to $x = 6$. $1 + 2 + 3 + 4 + 5 + 6 = 21$ points, the highest of them at $(6, 7)$, which is the top corner found in Step 5.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) the line $y = 2$ drawn	**B1**	For the horizontal line $y = 2$ drawn on the grid.	✓
(a)(ii) the line $x = 6$ drawn	**B1**	For the vertical line $x = 6$ drawn on the grid.	✓
(a)(iii) the line $y = x + 1$ drawn	**B1**	For the sloping line $y = x + 1$ drawn on the grid, through $(0, 1)$ and $(7, 8)$.	✓
Guidance on the three lines of part (a)	**Note**	A line may be solid, dotted or dashed. Each one must be at least 2 cm long, and the lines need not be labelled to earn the marks in part (a).	✓
(b) the correct region shaded and marked R	**B1ft**	For the correct region indicated. The follow-through is dependent on at least 2 of the 3 marks in part (a), and on a vertical line, a horizontal line and a sloping line of positive gradient all having been drawn.	✓

Step	Mark	Description	Got it?
(b) special case: the two letters read the wrong way round	**SC B1**	For $y = x + 1$, $y = 6$ and $x = 2$ drawn, with the region those three enclose shaded. That is what comes out when $y \geq 2$ and $x \leq 6$ are read as $x \geq 2$ and $y \leq 6$, swapping the two letters and leaving the third inequality alone.	✓

Full marks: 4/4

Question 3 - Exam Solution

Understanding the Question

Given

Time for the flight: 9 hours 36 minutes
Average speed for the whole flight: 820 km/h

Find

The total distance the aircraft flies, in kilometres

Plan the Solution

- The speed is given in kilometres per hour, so the time has to be in hours before the two can be multiplied together.
- Write the 36 minutes as a fraction of an hour, then add it to the 9 whole hours.
- Multiply the speed by that time in hours.

Worked Solution [3 marks]

> Rule - Speed, distance and time: $\text{distance} = \text{speed} \times \text{time}$, with the time measured in hours whenever the speed is in kilometres per hour.

Step 1: Write the 36 minutes as a fraction of an hour

$$\frac{36}{60} = 0.6$$

(Reason: An hour is 60 minutes, so 36 minutes is $\frac{36}{60}$ of an hour, and that fraction cancels to 0.6 of an hour.)

Step 2: Write the whole flight time in hours

$9 + 0.6 = 9.6$

(Reason: The time is now measured in the same unit as the speed. Writing 9.36 here is the classic trap: that reads the 36 as hundredths of an hour rather than as minutes.)

Step 3: Multiply the speed by the time

$820 \times 9.6 = 7\,872$ km

(Reason: The aircraft covers 820 kilometres in each hour of the flight, so multiplying the speed by the number of hours gives the whole distance.)

$7\,872$ km

Verification

- ✓ **Check 1 - divide the distance back by the speed:** Dividing the distance by the speed has to give the time back. If it does not come out as 9.6 hours, the multiplication was wrong. $\frac{7\,872}{820} = 9.6$ hours, which is 9 hours 36 minutes
- ✓ **Check 2 - split the flight into the whole hours and the last part hour:** In 9 hours the aircraft covers $820 \times 9 = 7\,380$ km, and in the final 0.6 of an hour it covers $820 \times 0.6 = 492$ km. $7\,380 + 492 = 7\,872$ km, the same total reached by a different split
- ✓ **Check 3 - is the size sensible:** Round the speed down to 800 km/h. The distance should then come out a little below the answer, and a flight of nearly ten hours should cover a few thousand kilometres. $800 \times 9.6 = 7\,680$ km, just below $7\,872$ km, so the answer is the right size

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Convert the time into hours (or into minutes)	M1	For a correct conversion of the time, e.g. 9.6 hours, or $9\frac{36}{60}$ hours, or $9\frac{3}{5}$ hours oe, or 576 minutes	✓
Use speed $\times$ time with the time in hours	M1	e.g. 820×9.6, or $820 \times \frac{576}{60}$, or $576 \times \frac{820}{60}$, or $576 \times \frac{41}{3}$ (allow 13.7 for $\frac{41}{3}$) oe. Allow the use of 9.36 for this mark. Award M2 for the split method	✓

Step	Mark	Description	Got it?
		$820 \times 9 + \frac{820}{60} \times 36 = 7\,380 + 492$, or for $\frac{34\,560}{60 \times 60} \times 820$ oe	
Answer	**A1**	$7\,872$. A correct answer scores full marks, unless it comes from obviously incorrect working	✓
Special case	**SC B1**	Award one mark for $7\,675.2$ if no other marks are awarded. That value is 820×9.36, the distance produced by reading 9 hours 36 minutes as 9.36 hours instead of 9.6 hours	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

The first mixed number: $2\frac{4}{7}$

The second mixed number: $3\frac{1}{9}$

The value the product has to be shown to equal: 8

Find

Working that reaches exactly 8, not a decimal that rounds to it. On a show-that question the working IS the answer, so every stage has to be written down.

Plan the Solution

- Mixed numbers cannot be multiplied a piece at a time. Multiplying the whole numbers together and the fractions together would give only two of the four products that a multiplication of two sums produces, and it comes to a value nowhere near the target.
- So turn each mixed number into a single improper fraction first.
- Then cancel before multiplying: 18 and 9 share a factor of 9, and 28 and 7 share a factor of 7, which leaves a multiplication of two whole numbers.

Worked Solution [3 marks]

Rule - Multiplying mixed numbers: rewrite each mixed number as an improper fraction using $\dfrac{\text{whole} \times \text{denominator} + \text{numerator}}{\text{denominator}}$, cancel any factor common to a numerator and a denominator, then multiply the numerators together and the denominators together.

Step 1: Write each mixed number as an improper fraction

$$2\frac{4}{7} = \frac{2 \times 7 + 4}{7} = \frac{18}{7}$$
$$3\frac{1}{9} = \frac{3 \times 9 + 1}{9} = \frac{28}{9}$$

(Reason: One whole is $\frac{7}{7}$ in the first number and $\frac{9}{9}$ in the second, so the whole number is multiplied by the denominator and the numerator is added to it. The denominator itself never changes.)

Step 2: Cancel the common factors before multiplying

$$\frac{18}{7} \times \frac{28}{9} = \frac{2}{1} \times \frac{4}{1}$$

(Reason: The two numerators are multiplied together and the two denominators are multiplied together, so any factor above a line may be cancelled with any factor below either line. Here 18 and 9 both divide by 9, leaving 2 and 1; and 28 and 7 both divide by 7, leaving 4 and 1.)

Step 3: Multiply what is left

$$\frac{2}{1} \times \frac{4}{1} = \frac{8}{1} = 8$$

(Reason: Both denominators are now 1, so the product is a whole number, and it is the 8 the question asked to be shown.)

$$2\frac{4}{7} \times 3\frac{1}{9} = \frac{18}{7} \times \frac{28}{9} = 8 \textbf{ as required}$$

Verification

✓ **Check 1 - multiply straight out, with no cancelling:** Cancelling is a shortcut, so the long way must give the same thing. Multiply the numerators and the denominators as they stand: $18 \times 28 = 504$ and $7 \times 9 = 63$. $\dfrac{504}{63} = 8$, because $8 \times 63 = 504$

✓ **Check 2 - work backwards from the answer:** If the product really is 8, then dividing 8 by the second mixed number must give the first one back. Dividing by $\dfrac{28}{9}$ is the same as multiplying by $\frac{9}{28}$. $8 \times \frac{9}{28} = \frac{72}{28} = \frac{18}{7}$, which is $2\frac{4}{7}$ again

✓ **Check 3 - expand instead of converting:** Treat each mixed number as a sum and multiply out all four products of $\left(2 + \frac{4}{7}\right)\left(3 + \frac{1}{9}\right)$. This route never forms an improper fraction at all, so it tests the conversions rather than repeating them. $6 + \frac{2}{9} + \frac{12}{7} + \frac{4}{63} = 8$, and the two middle terms are exactly what is lost by multiplying only the whole numbers and only the fractions

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write both mixed numbers as improper fractions	M1	For correct improper fractions $\frac{18}{7}$ and $\frac{28}{9}$	✓
Cancel, or show a clear intention to multiply	M1dep	For cancelling the fractions fully, or for cancelling partially with a clear intention to multiply, or for not cancelling at all with a clear intention to multiply. An arithmetic error in the multiplication is allowed. For example $\frac{18}{7} \times \frac{28}{9} = \frac{504}{63}$ oe, or a partial cancel such as $\frac{2 \times 28}{7} = \frac{56}{7}$, or both fractions written over 63 as $\frac{162}{63} \times \frac{196}{63} = \frac{31\,752}{3\,969}$ oe	✓
Reach the given value from fully correct working	A1	For a correct answer of 8 from fully correct working. Working is required: the value is printed in the question, so an unsupported 8 earns nothing	✓
Note - the alternative presentation	Note	A candidate may write $8 = \frac{8}{1}$, possibly under the given 8, and then need only show that their fraction comes to $\frac{8}{1}$	✓

Full marks: 3/3

Question 5 - Exam Solution

Understanding the Question

Given

Triangle ABC, with the right angle at B
$AC = 6.5$ cm, the side facing the right angle
The angle at A is $34°$
$BC = x$ cm

Find

The value of x, correct to one decimal place

Plan the Solution

- Stand at the $34°$ angle and name the two sides that matter from there: AC is the hypotenuse and BC is the side opposite the angle.
- Opposite together with hypotenuse is the sine ratio, so that is the ratio to write down.
- Rearrange to make x the subject, evaluate it with the calculator in degree mode, then round to one decimal place.

Worked Solution [3 marks]

Sine ratio in a right-angled triangle: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, which rearranges to $\text{opposite} = \text{hypotenuse} \times \sin\theta$.

Step 1: Name the sides from the angle you know

$AC = 6.5$ cm

$BC = x$ cm

(Reason: The right angle sits at B, so the side facing it, AC, is the hypotenuse. Standing at A, the side BC faces the $34°$ angle, so it is the opposite side.)

Step 2: Write the sine ratio for the $34°$ angle

$$\sin 34° = \frac{x}{6.5}$$

(Reason: Opposite over hypotenuse is the sine ratio, and here that is x over 6.5. Only the two lengths named in Step 1 appear, so no other side is needed.)

Step 3: Make x the subject and evaluate

$$x = 6.5 \times \sin 34° = 3.63475\ldots$$

(Reason: Multiplying both sides of the ratio by 6.5 leaves x on its own. Keep the full display value for now, with the calculator in degree mode, and round only at the last step.)

Step 4: Round to one decimal place

$x = 3.6$

(Reason: The digit in the second decimal place is 3, which is below 5, so the digit in the first decimal place is left alone.)

$$x = 3.6$$

Verification

✓ **Check 1:** Work backwards. Put the unrounded answer back into the ratio and take the inverse sine: the angle at A has to come back as $34°$. $\sin^{-1}\left(\frac{3.63475}{6.5}\right) = 34.0°$

✓ **Check 2:** Test the triangle a different way, with no trigonometry in the test itself. The other short side is $AB = 5.38874\ldots$, so the squares of the two short sides must add to the square of the hypotenuse, and $6.5^2 = 42.25$. $5.38874^2 + 3.63475^2 = 42.25$

✓ **Check 3:** A size check. The angle is under $45°$, so the side facing it must be the shorter of the two short sides, and both must be shorter than the hypotenuse. $3.6 < 5.4 < 6.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write a correct trigonometric statement for x	**M1**	$\sin 34° = \frac{x}{6.5}$, or $\frac{x}{\sin 34°} = \frac{6.5}{\sin 90°}$, or $\cos 56° = \frac{x}{6.5}$, or $6.5^2 - (6.5 \times \cos 34°)^2$, or equivalent	✓
A fully correct method to find x	**M1**	$x = 6.5 \times \sin 34°$, or $x = \frac{6.5 \times \sin 34°}{\sin 90°}$, or $x = \sqrt{6.5^2 - (6.5 \times \cos 34°)^2}$, or $x = 6.5 \times \cos 56°$, or equivalent	✓
The value of x, correct to one decimal place	**A1**	awrt 3.6	✓
Correct answer seen with no working	**Note**	A correct answer scores full marks, unless it comes from working that is obviously incorrect.	✓

Full marks: 3/3

Question 6 - Exam Solution

Understanding the Question

Given

The speed of the cable car is w metres per second.
There are 60 seconds in a minute and 60 minutes in an hour.
There are 1000 metres in 1 kilometre.

Find

The same speed written in kilometres per hour, in terms of w, in its simplest form.

Plan the Solution

- Change the time unit first: multiply by 3600, because one hour is 60×60 seconds.
- Change the distance unit next: divide by 1000, because one kilometre is 1000 metres.
- Simplify the single multiplier $\dfrac{3600}{1000}$ and write the answer as a multiple of w.

Worked Solution [3 marks]

> Rule - Unit conversion: there are 3600 seconds in one hour and 1000 metres in one kilometre, so a speed in metres per second becomes a speed in kilometres per hour by multiplying by 3600 and then dividing by 1000.

Step 1: Change the seconds into hours

$60 \times 60 = 3600$

$w \times 3600 = 3600w$

(Reason: There are 60 seconds in a minute and 60 minutes in an hour, so one hour is 3600 seconds. In one hour the cable car covers 3600 lots of w metres.)

Step 2: Change the metres into kilometres

$\dfrac{3600w}{1000}$ kilometres per hour

(Reason: There are 1000 metres in 1 kilometre, so the number of kilometres is the number of metres over 1000.)

Step 3: Simplify the multiplier

$$\frac{3600}{1000} = \frac{18}{5} = 3.6$$

$$\frac{3600w}{1000} = 3.6w$$

(Reason: Dividing the top and the bottom by 200 turns $\frac{3600}{1000}$ into $\frac{18}{5}$, which is 3.6. The w is carried through untouched, so the speed is 3.6 times w.)

$3.6w$ kilometres per hour

Verification

✓ **Check 1:** Try a real speed. Put $w = 10$: in one hour the cable car covers $10 \times 3600 = 36000$ metres, and 36000 metres is 36 kilometres. $3.6 \times 10 = 36$

✓ **Check 2:** Convert the answer back the other way. At $3.6w$ kilometres per hour the cable car covers $3600w$ metres in 3600 seconds. $\frac{3600w}{3600} = w$

✓ **Check 3:** Compare with the fraction form the mark scheme also accepts. $\frac{18}{5}$ and 3.6 are the same number, so $\frac{18}{5}w$ and $3.6w$ are the same answer. $\frac{18}{5} = 3.6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One correct conversion, or the combined multiplier on its own	M1	For one of $\frac{w}{1000}$, $\frac{w}{10^3}$, $w \times 10^{-3}$, $0.001w$ oe, or $(w \times 60 \times 60)$ oe, or $w \times 3600$ or $\frac{w}{\frac{1}{3600}}$ oe. Also for $\frac{3600}{1000}$ or $\frac{18}{5}$ or 3.6 oe, without a link to w.	✓
A fully correct method, including w	M1	For $\frac{w \times 60 \times 60}{1000}$ oe, for example $w \times \frac{3600}{1000}$.	✓

Step	Mark	Description	Got it?
The simplified answer	**A1**	For $3.6w$, or $\frac{18}{5}w$, or $3\frac{3}{5}w$; allow $3.6 \times w$. A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

$AF = 21$ cm, $CD = 15$ cm,
$AB = FE = 13$ cm
CD is parallel to AF, and the figure marks a right angle at A and at F
The area of $ABCDEF$ is 390 cm^2, and its perpendicular height is h cm

Find

The value of h, the perpendicular distance from AF up to CD

Plan the Solution

- The right angles at A and F make $ABEF$ a rectangle, so cut the shape along BE into a rectangle and a trapezium.
- Work out the rectangle's area and take it off 390. What is left is the trapezium $BCDE$.
- The trapezium's parallel sides are known, so its area formula gives its height.
- Add that height to the 13 cm below BE to reach h.

Worked Solution [4 marks]

> Rule - split a composite shape into parts whose areas you can write down. Rectangle: area is length times width. Trapezium: area is $\frac{1}{2}(a+b) \times$ height, where a and b are the two parallel sides.

Step 1: the area of rectangle $ABEF$

$21 \times 13 = 273$

(Reason: The right angles at A and F make AB and FE both perpendicular to AF, and the question says they are equal, so $ABEF$ is a rectangle 21 cm by 13 cm.)

Step 2: the area left for trapezium $BCDE$

$390 - 273 = 117$

(Reason: The rectangle and the trapezium together make the whole shape, so whatever is left of the 390 cm^2 belongs to $BCDE$.)

Step 3: the trapezium's parallel sides

$$\frac{1}{2} \times (21 + 15) = 18$$

(Reason: BE is the same length as AF, so the two parallel sides of the trapezium are 21 cm and 15 cm. Halving their sum gives the number that multiplies the trapezium's height.)

Step 4: the height of the trapezium

$$\frac{117}{18} = 6.5$$

(Reason: Dividing the trapezium's area by that 18 leaves its height, in centimetres.)

Step 5: the perpendicular height of the whole shape

$13 + 6.5 = 19.5$

(Reason: h runs the whole way from AF up to CD, so it is the rectangle's 13 cm plus the trapezium's 6.5 cm.)

$$h = 19.5$$

Verification

- ✓ **Check 1:** Put the two parts back together: the rectangle is 273 and the trapezium is 18×6.5. $273 + 117 = 390$
- ✓ **Check 2:** Find the area a completely different way. Draw the rectangle that encloses the shape, $21 \times 19.5 = 409.5$, then cut off the two corner triangles: their bases add to $21 - 15 = 6$ and each has height $19.5 - 13 = 6.5$. $409.5 - \frac{1}{2} \times 6 \times 6.5 = 390$
- ✓ **Check 3:** That second method turns the question into the equation $18h + 39 = 390$. Substitute the answer back into it. $18 \times 19.5 + 39 = 390$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct area calculation linked to the shape, for example $13 \times 21 = 273$ or $\frac{1}{2}(15+21) \times y$	**M1**	Any one correct area expression for a part of the shape, or for the whole of it. The trapezium's height may appear as $h - 13$, or as x or y or any other letter, and even as h itself; all are acceptable. Brackets need not be used for this mark.	✓
The areas of all the parts considered, for example $390 - 273 = 117$, or $21h$ together with $2 \times \frac{1}{2} \times 3(h-13)$	**M1**	The parts need not be added or subtracted to give the whole shape for this mark. The scheme quotes the 273, so the candidate's own value from the first mark may stand there. The same freedom over the letter used for the trapezium's height applies, and correct use of brackets is required.	✓
A correct calculation for a height, for example $\frac{117}{18} = 6.5$, or a correct equation such as $273 + 18(h-13) = 390$ or $18h = 351$	**M1**	Award for reaching the height of the trapezium, or the height of the whole shape, or a correct equation in which that height is the unknown. The scheme quotes the 117, so the candidate's own value from the previous mark may stand there. The same freedom over the letter used for that height applies, and correct use of brackets is required here too.	✓
$h = 19.5$	**A1**	Accept any equivalent form, for example $\frac{39}{2}$.	✓
A correct answer with no working scores all four marks, unless it clearly follows incorrect working.	**Note**	The mark scheme prints 6.5 once, in the third M1 row above, as one of the ways that mark can be earned; it says nothing about what a script ending there scores.	✓

Full marks: 4/4

Question 8 - Exam Solution

Understanding the Question

Given

Find

Total order: 600 seedlings Ratio $\text{marigold} : \text{aster} : \text{zinnia} = 9 : 4 : 2$ 45% of the marigold seedlings are for orange flowers $\frac{5}{8}$ of the aster seedlings are for orange flowers Every zinnia seedling is for orange flowers	The number of the 600 seedlings that are for orange flowers

Plan the Solution

- Add the ratio numbers, then divide 600 by that total to find one part.
- Multiply one part by each ratio number to get the size of the three groups.
- Take 45% of the marigolds, $\frac{5}{8}$ of the asters, and every one of the zinnias.
- Add the three orange counts. Each group has its own fraction, so the fractions can never simply be added first.

Worked Solution [5 marks]

> Rule - Sharing in a ratio: add the ratio numbers, divide the total by that sum to find one part, then multiply one part by each ratio number to get the group sizes.

Step 1: Add the ratio numbers

$$9 + 4 + 2 = 15$$

(Reason: the ratio splits the order into 15 equal parts, not into 3 equal groups)

Step 2: Work out one part

$$\frac{600}{15} = 40$$

(Reason: dividing the whole order by the number of parts gives the number of seedlings in a single part)

Step 3: Work out the size of each group

$\text{marigold: } 9 \times 40 = 360$

$\text{aster: } 4 \times 40 = 160$

$\text{zinnia: } 2 \times 40 = 80$

(Reason: each group is its own ratio number of parts, and the three come back to $360 + 160 + 80 = 600$, which is the first sign the shares are right)

Step 4: Count the orange marigolds

$0.45 \times 360 = 162$

(Reason: 45% is 0.45, and the percentage is of the marigold group only, never of the whole order)

Step 5: Count the orange asters

$$\frac{5}{8} \times 160 = 100$$

(Reason: five of every eight asters are for orange flowers, so multiply the aster group by that fraction and not by the part left over)

Step 6: Add the three orange counts

$162 + 100 + 80 = 342$

(Reason: all 80 zinnias are for orange flowers, so the whole zinnia group is counted)

342 seedlings

Verification

✓ **Check 1 - count in ratio parts instead:** Work out how many of the 15 parts are orange: $0.45 \times 9 = 4.05$ parts of marigold, $\frac{5}{8} \times 4 = 2.5$ parts of aster, and both zinnia parts. $4.05 + 2.5 + 2 = 8.55$ parts, and $\frac{8.55}{15} \times 600 = 342$

✓ **Check 2 - one fraction of the whole order:** Collapse the three shares into a single fraction of the order: $\frac{45}{100} \times \frac{9}{15} + \frac{5}{8} \times \frac{4}{15} + \frac{2}{15} = \frac{57}{100}$. $\frac{57}{100} \times 600 = 342$, so 57% of the order is orange

✓ **Check 3 - count what is not orange:** The seedlings that are not for orange flowers are $0.55 \times 360 = 198$ marigolds, $\left(1 - \frac{5}{8}\right) \times 160 = 60$ asters and no zinnias at all. $600 - 198 - 60 = 342$, which agrees, and 342 sits between 80 and 600 as it must

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{600}{9+4+2} = 40$	M1	A correct method to find one share. Also allow $0.45 \times 600 = 270$ or $\frac{5}{8} \times 600 = 375$, or the fraction of a share that is orange, $0.45 \times 9 = 4.05$.	✓

Step	Mark	Description	Got it?
$2 \times 40 = 80$	**M1**	A correct method to find the number of zinnia seedlings. $\frac{2}{15} \times 600 = 80$ also scores it and implies the first M1, as does the fraction of a share that is orange aster, $\frac{5}{8} \times \frac{4}{15} = \frac{1}{6}$.	✓
$0.45 \times 360 = 162$	**M1**	A correct method to find the number of orange marigold seedlings, or the total of the orange parts, $4.05 + 2.5 + 2 = 8.55$, which implies the first two M marks.	✓
$\frac{5}{8} \times 160 = 100$	**M1**	A correct method to find the number of orange aster seedlings, or multiplying the total of the correct orange shares by the order, $\frac{8.55}{9+4+2} \times 600 = 342$, which implies all the previous M marks.	✓
$162 + 100 + 80 = 342$	**A1**	cao. 342 seedlings are for orange flowers.	✓
A correct answer with no working shown	**Note**	A correct answer scores full marks, unless it comes from obviously incorrect working. This row earns nothing on its own.	✓

Full marks: 5/5

Question 9 - Exam Solution

Understanding the Question

Given

Amount paid in: $4\,500$ koruna
Interest: 2.4% per year, compound
Time: 4 years

Find

The value of the bond at the end of the 4 years, to the nearest koruna

Plan the Solution

- Write 2.4% as a decimal and add it to 1 to get the multiplier for one year.

- Multiply by that number once for each year, always starting from the balance the year opened with.
- Collect the four multiplications into a single power, 1.024^4, which is the same calculation in one line.
- Round at the very end only, so no part of a koruna is lost part way through.

Worked Solution [3 marks]

> Rule - Compound interest: the interest for each year is worked out on the balance at the START of that year, so the balance is multiplied by the same number once per year. After n years an amount P growing at a rate r (as a decimal) is worth $P \times (1+r)^n$.

Step 1: Turn the rate into a one-year multiplier

$$\frac{2.4}{100} = 0.024$$

$$1 + 0.024 = 1.024$$

(Reason: the balance keeps its whole self and gains 2.4% on top, so one year of growth is a single multiplication by 1.024 rather than by 0.024)

Step 2: Multiply once for each of the four years

$$4\,500 \times 1.024 = 4\,608$$

$$4\,608 \times 1.024 = 4\,718.592$$

$$4\,718.592 \times 1.024 = 4\,831.838208$$

$$4\,831.838208 \times 1.024 = 4\,947.802324992$$

(Reason: each year's interest is worked out on the balance that year started with, not on the original $4\,500$, which is exactly what makes the interest compound rather than simple)

Step 3: Do the same four multiplications in one line

$$4\,500 \times 1.024^4 = 4\,947.802324992$$

(Reason: multiplying by 1.024 four times is multiplying by 1.024 to the power 4, so a calculator reaches the same balance in a single press)

Step 4: Round to the nearest koruna

$$4\,947.802324992 \approx 4\,948$$

(Reason: the first digit after the decimal point is 8, so the amount rounds up to the next whole koruna)

$4\,948$ koruna

Verification

✓ **Check 1 - undo the four years:** Divide the final balance by the multiplier once for each year. If the four years were applied correctly, the amount paid in comes back exactly.

$$\frac{4\,947.802324992}{1.024^4} = 4\,500$$

✓ **Check 2 - compare it with simple interest:** Simple interest would pay $4\,500 \times 0.024 = 108$ koruna every year, so 4 years of it would be 432 koruna and the bond would hold $4\,932$. Compound interest must pay a little more than that, and never a lot more over 4 years. $4\,947.802324992 - 4\,500 = 447.802324992$ of interest, which is more than 432 and only a little more

✓ **Check 3 - read the growth as a single percentage:** Work out the four-year growth factor on its own, then apply the part above 1 to the amount paid in. This never uses the year-by-year balances, so it is an independent route to the interest. $1.024^4 = 1.099511627776$, and $0.099511627776 \times 4\,500 = 447.802324992$, the same interest as Check 2

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$4\,500 \times 1.024 = 4\,608$	**M1**	A correct start to a compound method: $4\,500 \times 1.024$ or equivalent, or the first year's interest on its own, $4\,500 \times 0.024 = 108$.	✓
$4\,831.838208 \times 1.024 = 4\,947.802324992$	**M1**	Carrying the multiplication through all four years: $4\,608 \times 1.024 = 4\,718.592$, then $4\,718.592 \times 1.024 = 4\,831.838208$, then $4\,831.838208 \times 1.024 = 4\,947.802324992$. The mark scheme prints these figures in quotation marks, which means the candidate's own running totals are followed through. The power form scores M2 on its own, and the mark scheme allows $4\,500 \times 1.024^4$ or $4\,500 \times 1.024^5$ for it.	✓
$4\,948$	**A1**	Anything from $4\,947$ to $4\,948$ is accepted, which covers a candidate who truncates the unrounded $4\,947.802324992$ instead of rounding it.	✓
One of the recognised wrong methods, and no other mark earned	**SC B1**	Simple interest in place of compound: $4\,500 \times 0.024 \times 4 = 432$ or $0.096 \times 4\,500 = 432$; the simple total $4\,500 + 4\,500 \times 0.024 \times 4 = 4\,932$ or $4\,500 \times 1.096 = 4\,932$; a depreciation multiplier, $0.976 \times 4\,500 = 4\,392$ or	✓

Step	Mark	Description	Got it?
		$0.904 \times 4\,500 = 4\,068$ or $0.976^4 \times 4\,500 = 4\,083.304660992$; or three years instead of four, $4\,500 \times 1.024^3 = 4\,831.838208$.	
A correct answer with no working shown	Note	A correct answer scores full marks, unless it comes from obviously incorrect working. This row earns nothing on its own.	✓

Full marks: 3/3

Question 10 - Exam Solution

Understanding the Question

Given

The first equation: $6x + 4y = 1$
The second equation: $3x + 5y = 8$
Two linear equations in the same two unknowns, so one pair of values has to satisfy both at once.

Find

The value of x and the value of y that fit both equations together. Neither equation is quadratic, so expect exactly one solution pair.

Plan the Solution

- Compare the x terms first: $6x$ and $3x$. Doubling the second equation makes them match, so the first equation needs no work at all.
- Subtract one equation from the other to cancel x, leaving a single equation in y.
- Solve that for y, then substitute the value back into one of the original equations to reach x.
- Test the pair in both original equations at the end, because a slip in either value shows up immediately there.

Worked Solution [3 marks]

> Rule - Elimination: multiply one or both equations so that one letter has the same number in front of it in each, then add or subtract the equations to remove that letter. What is left

is a single equation in one unknown, and putting its value back into either original equation gives the other unknown.

Step 1: Match the number of x in the two equations

$3x + 5y = 8$

$6x + 10y = 16$

(Reason: the first equation already carries $6x$, so doubling every term of the second one gives it $6x$ as well; multiplying a whole equation by 2 keeps it true, because both sides are doubled together)

Step 2: Subtract to remove x, and solve for y

$(6x + 10y) - (6x + 4y) = 16 - 1$

$6y = 15$

$y = \frac{15}{6} = \frac{5}{2} = 2.5$

(Reason: both equations now carry $6x$, so taking one from the other cancels x and leaves an equation in y on its own)

Step 3: Put $y = 2.5$ back into an original equation

$6x + 4 \times 2.5 = 1$

$6x + 10 = 1$

$6x = -9$

$x = \frac{-9}{6} = -\frac{3}{2} = -1.5$

(Reason: with y known, the first equation has a single unknown left in it, and choosing an original equation rather than the doubled one means any slip made in the doubling cannot travel into x)

$$x = -1.5$$
$$y = 2.5$$

Verification

✓ **Check 1 - the first equation:** Put $x = -1.5$ and $y = 2.5$ into $6x + 4y = 1$. The left-hand side has to come out as the right-hand side.
$6 \times (-1.5) + 4 \times 2.5 = -9 + 10 = 1$

✓ **Check 2 - the second equation:** The second equation was the one that got doubled, so it is the one a multiplying slip would spoil. Put the same pair into $3x + 5y = 8$ in its original form. $3 \times (-1.5) + 5 \times 2.5 = -4.5 + 12.5 = 8$

✓ **Check 3 - solve again, removing the other letter:** Multiply the first equation by 5 and the second by 4, so both carry $20y$, then subtract. This route never uses the value of y found above, so it reaches x independently. $30x + 20y = 5$ and $12x + 20y = 32$, so $18x = -27$ and $x = -1.5$ again

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$6x + 10y = 16$ with $6x + 4y = 1$, giving $6y = 15$	**M1**	A correct method to eliminate one letter: multiplying one or both equations so that x or y carries the same number in each, together with the correct operation to remove it, which can be shown by 2 of the 3 terms being correct for the subtraction or the addition. One arithmetic error in the multiplying is allowed. The other route scores the same mark: a correct substitution of one letter into the other equation, for example $6x + 4 \times \dfrac{8 - 3x}{5} = 1$ or $3 \times \dfrac{1 - 4y}{6} + 5y = 8$. The mark is for the method and not for what the method produces, although a correct result seen earns it.	✓
$6x + 4 \times 2.5 = 1$	**M1dep**	A correct method to reach the value of the other letter, dependent on the first M1: substituting the letter already found into either equation, which does not then have to be solved, or starting again with a fresh elimination or substitution.	✓
$x = -1.5$, $y = 2.5$	**A1**	Both values, and dependent on the first M1. Any equivalent exact form is accepted, so $-\dfrac{3}{2}$ or $-1\dfrac{1}{2}$ is as good as -1.5, and $\dfrac{5}{2}$ or $2\dfrac{1}{2}$ as good as 2.5. It must be a vulgar fraction, a mixed number or a decimal, so a fraction left unfinished, such as $y = \dfrac{12.5}{5}$, is not accepted.	✓
Working required	**Note**	The mark scheme prints the words working required beside the answer, and the question asks for clear algebraic working, so a correct pair of values written down with no algebra behind it earns nothing at all. This row carries no mark of its own.	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

$x^2 + 9x - 22$, a quadratic expression in which the coefficient of x^2 is 1.
Part (ii) sets that same expression equal to 0, and the word "Hence" says to use the brackets from part (i) rather than start again.

Find

(i) the expression written as a product of two brackets. (ii) the values of x that satisfy $x^2 + 9x - 22 = 0$. A quadratic, so expect two solutions.

Plan the Solution

- Look for two numbers whose product is -22 and whose sum is 9.
- List the factor pairs of -22 and test the sum of each one, so the search is complete rather than lucky.
- Write the brackets, then expand them to confirm they give $x^2 + 9x - 22$ back.
- For (ii), a product is 0 only when one of the factors is 0, so set each bracket equal to 0 in turn.

Worked Solution [3 marks]

> Factorising $x^2 + bx + c$: find two numbers p and q with $pq = c$ and $p + q = b$. Then $x^2 + bx + c = (x + p)(x + q)$.

Step 1: Name the two conditions the numbers must meet

$x^2 + 9x - 22$

$pq = -22$

$p + q = 9$

(Reason: Expanding $(x + p)(x + q)$ gives $x^2 + (p + q)x + pq$, so the constant term is the product of the two numbers and the coefficient of x is their sum. The product is negative, so one number is positive and the other is negative.)

Step 2: Test every factor pair of the constant

$-22 = 1 \times (-22) = (-1) \times 22 = 2 \times (-11) = (-2) \times 11$

$1 + (-22) = -21$

$(-1) + 22 = 21$
$2 + (-11) = -9$
$(-2) + 11 = 9$

(Reason: Only four pairs of whole numbers multiply to -22, and only the last of them adds to 9. Listing them all is what makes this a search rather than a guess, so $p = -2$ and $q = 11$.)

Step 3: Write the brackets, then expand to check

$x^2 + 9x - 22 = (x - 2)(x + 11)$
$(x - 2)(x + 11) = x^2 + 11x - 2x - 22$
$x^2 + 11x - 2x - 22 = x^2 + 9x - 22$

(Reason: Each number goes inside its own bracket with its own sign, so -2 gives $(x - 2)$ and 11 gives $(x + 11)$. Multiplying out returns the original expression, which is the answer to part (i).)

Step 4: Solve by setting each bracket equal to zero

$(x - 2)(x + 11) = 0$
$x - 2 = 0 \implies x = 2$
$x + 11 = 0 \implies x = -11$

(Reason: Two numbers multiply to 0 only if at least one of them is 0, so each bracket supplies one solution. Notice the sign flips: a bracket that reads $+11$ gives the negative solution, and a bracket that reads -2 gives the positive one. This is why the question says "Hence": part (i) has already done the work.)

(i) $(x - 2)(x + 11)$

(ii) $x = 2$ or $x = -11$

Verification

✓ **Check 1:** Multiply the brackets out term by term: $x \times x$, then $x \times 11$, then $-2 \times x$, then -2×11. $(x - 2)(x + 11) = x^2 + 9x - 22$, the expression the question printed.

✓ **Check 2:** Substitute each solution back into $x^2 + 9x - 22$. A solution must make the whole expression 0. $2^2 + 9 \times 2 - 22 = 0$ and $(-11)^2 + 9 \times (-11) - 22 = 0$

✓ **Check 3:** Use the sum and product of the roots, which never touches the brackets. For $x^2 + bx + c$ the two solutions add to $-b$ and multiply to c, so here they must add to -9 and multiply to -22. $2 + (-11) = -9$ and $2 \times (-11) = -22$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) $(x \pm 2)(x \pm 11)$	**M1**	Or $(x+a)(x+b)$ where $ab=-22$ or $a+b=9$. The method mark is for the right pair of numbers, whatever the signs.	✓
(i) $(x-2)(x+11)$	**A1**	A correct answer scores full marks, unless it comes from obviously incorrect working.	✓
(ii) $2, -11$	**B1ft**	Follow through from their factors in part (i): the two solutions must be the ones their own brackets give. A candidate who wrote $(x+2)(x-11)$ in (i) and then $-2, 11$ here still earns this mark.	✓

Full marks: 3/3

Question 12 - Exam Solution

Understanding the Question

Given

The number of bottles filled is recorded on 7 days in a row.
The mean for the first 4 days is $11\,800$ bottles a day.
The mean for the next 3 days is $13\,207$ bottles a day.

Find

The mean number of bottles filled each day across all 7 days.

Plan the Solution

- Two means cannot simply be averaged with each other here, because the two blocks are different sizes: 4 days against 3 days.
- Turn each mean back into a total instead. Multiply each mean by the number of days it covers.
- Add the two totals to get the total for the whole week, then divide that by 7.

Worked Solution [3 marks]

Rule - Mean: mean $= \dfrac{\text{total}}{\text{number of values}}$, so rearranged, total $=$ mean $\times$ number of values.

Step 1: Total for the first 4 days

$4 \times 11\,800 = 47\,200$

(Reason: Each of those 4 days contributes the mean of $11\,800$ to the total, so the block total is 4 lots of $11\,800$.)

Step 2: Total for the next 3 days

$3 \times 13\,207 = 39\,621$

(Reason: The same idea for the second block: multiply its mean by the number of days that block covers.)

Step 3: Total for all 7 days

$47\,200 + 39\,621 = 86\,821$

(Reason: The two blocks cover every one of the 7 days once and do not overlap, so the two totals add to the whole week's total.)

Step 4: Divide the total by the number of days

$$\frac{86\,821}{7} = 12\,403$$

(Reason: The mean for the week is the total for the week divided by the number of days in it, which is 7.)

$12\,403$ bottles

Verification

✓ **Check 1:** Work backwards. If the mean over the 7 days really is $12\,403$, then multiplying it by 7 must rebuild the total found in Step 3. $7 \times 12\,403 = 86\,821$, which is the same total

✓ **Check 2:** A different route: start every day at the lower mean $11\,800$. Each of the last 3 days is $13\,207 - 11\,800 = 1\,407$ above that, and the extra is shared over 7 days.
$$11\,800 + \frac{3 \times 1\,407}{7} = 11\,800 + 603 = 12\,403$$

✓ **Check 3:** Sanity check the size. The answer must sit between the two means, and nearer the lower one, because 4 of the 7 days come from the block with the lower mean. $11\,800 < 12\,403 < 13\,207$, and it is 603 above the lower mean but 804 below the upper one

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$4 \times 11\,800 = 47\,200$ or $3 \times 13\,207 = 39\,621$ or $86\,821$	M1	for one correct product, or for the sum of the two products	✓
$\dfrac{47\,200 + 39\,621}{7}$ or $\dfrac{86\,821}{7}$	M1	for a fully correct method to find the mean for the 7 days, using their own two block totals	✓
$12\,403$	A1	cao. A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 13 - Exam Solution

Understanding the Question

Given

The distances, in km, travelled to work by 70 nurses, grouped into six classes of width 10 km.
Frequencies $7, 17, 18, 14, 10, 4$.
A grid running from 0 to 60 km across and 0 to 70 up.

Find

(a) The six cumulative frequencies. (b) The cumulative frequency graph, drawn on the grid. (c) An estimate for the interquartile range of the distances. (d) An estimate for the number of nurses who travel more than 46 km.

Plan the Solution

- Build a running total down the frequency column; the last one must be the whole group.
- Plot each running total at the upper end of its class, then join the points up.
- Read across from $\dfrac{70}{4}$ and from $\dfrac{3 \times 70}{4}$ on the cumulative frequency axis, down to the distance axis, and subtract the two distances.
- For part (d), read up from 46 km to the graph, then take that reading away from 70.

Worked Solution [7 marks]

Rule - Cumulative frequency: plot the running total at the upper end of each class and join the points. The lower quartile sits $\frac{n}{4}$ of the way up the cumulative frequency axis and the upper quartile $\frac{3n}{4}$ of the way up. The interquartile range is the gap between the two distances they lead to, never the gap between the two heights.

Step 1: Add the frequencies down the table

7

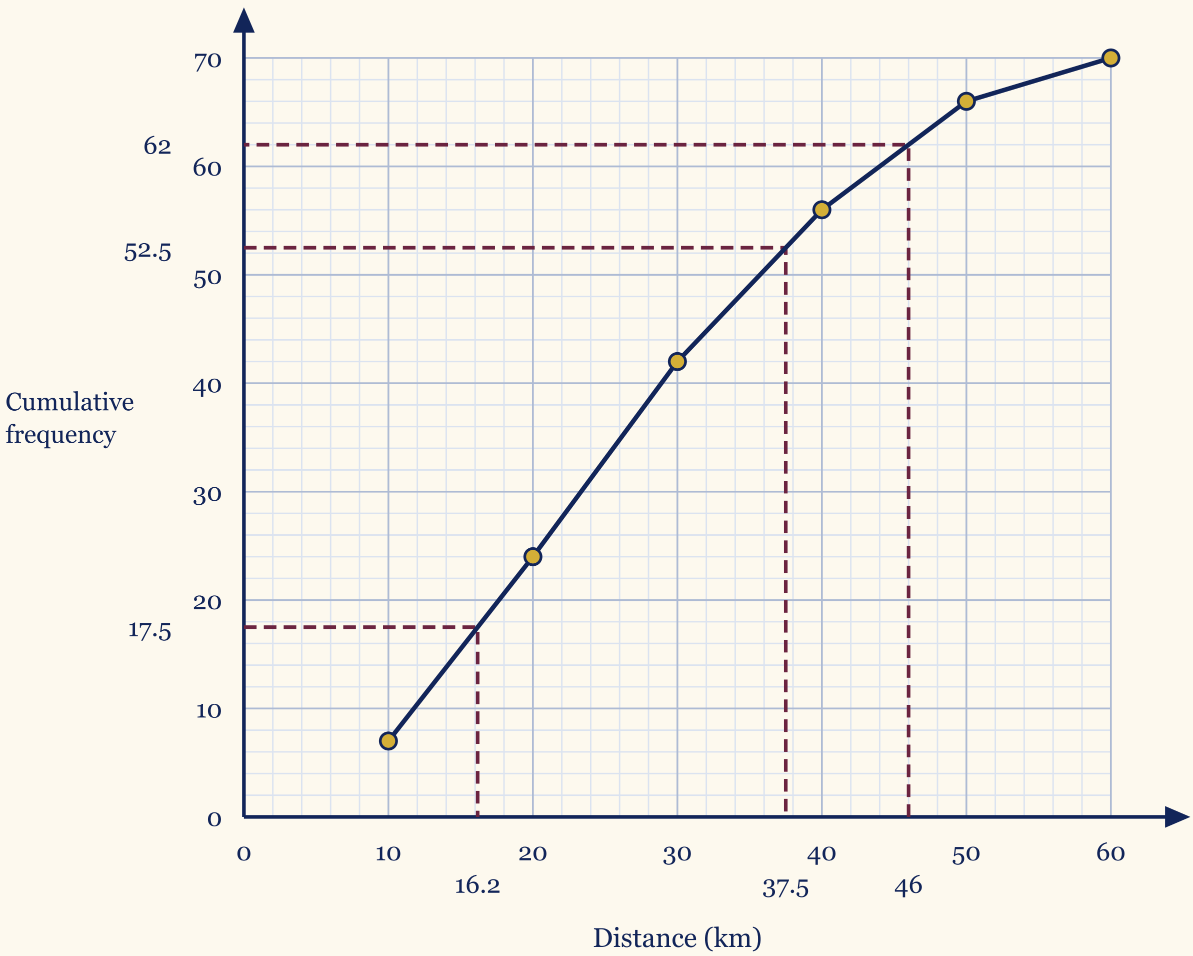
$7 + 17 = 24$

$24 + 18 = 42$

$42 + 14 = 56$

$56 + 10 = 66$

$66 + 4 = 70$



(Reason: Each cumulative frequency counts everyone so far, so the previous total is carried forward and the next class added on. The last total has to come to 70*, the whole group of nurses.)*

Step 2: Plot each total at the upper end of its class

$(10, 7) \quad (20, 24) \quad (30, 42)$
$(40, 56) \quad (50, 66) \quad (60, 70)$

(Reason: A cumulative total counts everyone at or below the top of the class, so 24 *belongs at a distance of* 20 *km and never at the midpoint* 15 *km. Joining the points with a smooth curve or with straight line segments both earn the marks.)*

Step 3: Read the two quartiles from the graph

$$\frac{70}{4} = 17.5$$

$$\frac{3 \times 70}{4} = 52.5$$

$$Q_1 \approx 16.2 \text{ km}$$

$$Q_3 \approx 37.5 \text{ km}$$

(Reason: Go up the cumulative frequency axis to 17.5 *for the lower quartile and to* 52.5 *for the upper quartile, across to the graph, then straight down to the distance axis. The mark scheme accepts a lower quartile anywhere from* 16 *to* 18 *and an upper quartile from* 36 *to* 38*.)*

Step 4: Subtract for the interquartile range

$$37.5 - 16.2 = 21.3 \text{ km}$$

(Reason: The interquartile range is the width of the middle half of the data, so it is a distance. Subtract the two readings taken off the distance axis, not the two heights read up the cumulative frequency axis.)

Step 5: Read the graph at 46 km

$$56 + \frac{6}{10} \times 10 = 62$$

(Reason: The graph climbs from 56 *at* 40 *km to* 66 *at* 50 *km, and* 46 *km is* $\frac{6}{10}$ *of the way along that step. So about* 62 *nurses travel* 46 *km or less.)*

Step 6: Take that reading away from the total

$$70 - 62 = 8$$

(Reason: A cumulative frequency graph gives the number at or below a distance, so the number above it is the whole group minus that reading. The answer has to be a whole number of nurses.)

(a) $7, 24, 42, 56, 66, 70$

(b) the six points plotted and joined, as shown above

(c) 21.3 **km**

(d) 8 **nurses**

Verification

✓ **Check 1:** Add every frequency on its own: $7 + 17 + 18 + 14 + 10 + 4$. The running total finishes on 70, the number of nurses, so nobody has been lost or counted twice.

✓ **Check 2:** The lower quartile has to land in the class holding the 17.5th value, and the upper quartile in the class holding the 52.5th value. 16.2 lies inside $10 < d \leq 20$ and 37.5 lies inside $30 < d \leq 40$, so both readings sit in the right class.

✓ **Check 3:** Count part (d) from the other end instead. From 46 km to 50 km is $\frac{4}{10}$ of a class holding 10 nurses, and the last class holds 4 more. $4 + 4 = 8$, which matches the number read off the graph.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Cumulative frequency table	**B1**	All six values correct: $7, 24, 42, 56, 66, 70$.	✓
(b) Cumulative frequency graph	**B2**	Fully correct graph: six points plotted at the upper end of each class and joined with a curve or with line segments. Award B1 for five correct points plotted and joined, or for five or six points plotted but not joined, or for five or six points plotted consistently within each interval rather than at its upper end, at their correct heights and joined, for example at $5, 15, 25, 35, 45$ and 55. Any of the B1 options may follow through from a part (a) table with one error in it, provided its values are ascending. A bar-chart type graph scores zero marks. Ignore any part of the graph before $(10, 7)$.	✓
(c) Quartile readings	**M1ft**	A correct method allowing readings to be taken on the distance axis from cumulative frequency 52.5 (or 53.25) and from 17.5 (or 17.75), or equivalent. Follow through from their own graph. The readings themselves are 16 to 18 and 36 to 38, but for this mark they need not be correct provided correct working is shown, such as lines or marks at those two cumulative frequencies with the matching points indicated on the distance axis.	✓
(c) Interquartile	**A1ft**	A single value from 18 to 22, or follow through from their own cumulative frequency graph, unless it comes from	✓

Step	Mark	Description	Got it?
range		obviously incorrect working.	
(d) Reading at 46 km	**M1ft**	A line up from 46 to the graph and a reading across, or a reading of 61 to 64 which need not be a whole number, from their own graph.	✓
(d) Number of nurses	**A1ft**	6, 7, 8 or 9, follow through from their own graph. It must be a whole number, and again the mark is not awarded where the answer comes from obviously incorrect working.	✓

Full marks: 7/7

Question 14 - Exam Solution

Understanding the Question

Given

Part (a): the product of three factors, $3y(2y+5)(y+7)$, and the target form $ay^3 + by^2 + cy$ with a, b and c integers.
Part (b): the equation $\frac{2x+3}{5} + \frac{6x-5}{4} = \frac{163}{100}$, with denominators 5, 4 and 100.

Find

(a) The expanded and simplified cubic, with every line of the expansion shown.
(b) The value of x, with clear algebraic working.

Plan the Solution

- (a) Multiply two of the three factors first, simplify, then multiply that result by the third factor. Any order gives the same cubic, so choose the pair that keeps the arithmetic smallest.
- (a) Collect the two middle terms before the last multiplication, so only three terms have to be multiplied by $3y$ at the end.
- (b) Write the left-hand side over the lowest common denominator 20, then multiply both sides by 20 to clear the fractions.
- (b) Gather the x terms on one side and the numbers on the other, then divide.

Worked Solution [7 marks]

Rule - Expanding a triple product: multiply any two factors first, simplify, then multiply by the third. Multiplication is associative, so the order cannot change the answer. Rule - Clearing fractions: multiplying every term of an equation by the lowest common denominator leaves an equivalent equation with no fractions in it, so $\dfrac{p}{20} = \dfrac{q}{100}$ becomes $p = \dfrac{20q}{100}$.

Step 1: Expand the pair of brackets

$$(2y+5)(y+7) = 2y^2 + 14y + 5y + 35$$

(Reason: Every term in the first bracket multiplies every term in the second, so there are four products to write down: $2y \times y$, then $2y \times 7$, then $5 \times y$, then 5×7. Leaving the $3y$ out until later keeps these numbers small.)

Step 2: Collect the two middle terms

$$14y + 5y = 19y$$
$$(2y+5)(y+7) = 2y^2 + 19y + 35$$

(Reason: The two middle products are both multiples of y, so they are like terms and add together. The quadratic is now in three terms, which is what the final multiplication needs.)

Step 3: Multiply every term by $3y$

$$3y \times 2y^2 = 6y^3$$
$$3y \times 19y = 57y^2$$
$$3y \times 35 = 105y$$
$$3y(2y^2 + 19y + 35) = 6y^3 + 57y^2 + 105y$$

(Reason: Multiply the numbers and add the indices: $y \times y^2 = y^3$ and $y \times y = y^2$. Every term of the quadratic must be multiplied, not just the first one.)

Step 4: Compare with the form asked for

$$ay^3 + by^2 + cy = 6y^3 + 57y^2 + 105y$$
$$a = 6, \quad b = 57, \quad c = 105$$

(Reason: Matching term by term gives the three coefficients. All three are whole numbers, and there is no constant term, so the product really does have the form $ay^3 + by^2 + cy$ with a, b and c integers, which is what part (a) asked to be shown.)

Step 5: Put the left-hand side over one denominator

$$\frac{2x+3}{5} + \frac{6x-5}{4} = \frac{4(2x+3)}{20} + \frac{5(6x-5)}{20}$$
$$\frac{4(2x+3) + 5(6x-5)}{20} = \frac{163}{100}$$

(Reason: The lowest common denominator of 5 and 4 is 20. The first fraction is multiplied top and bottom by 4 and the second by 5, which changes how each fraction is written but not its value.)

Step 6: Expand and simplify the numerator

$$4(2x + 3) + 5(6x - 5) = 8x + 12 + 30x - 25$$
$$8x + 12 + 30x - 25 = 38x - 13$$
$$\frac{38x - 13}{20} = \frac{163}{100}$$

(Reason: Take the minus sign with the 5: $5 \times (-5) = -25$, not $+25$. Then $8x + 30x = 38x$ and $12 - 25 = -13$.)

Step 7: Clear the fractions

$$38x - 13 = \frac{163 \times 20}{100}$$
$$38x - 13 = 32.6$$

(Reason: Multiplying both sides by 20 removes the denominator on the left, and the right-hand side must be multiplied by 20 as well. Since $163 \times 20 = 3260$, the right-hand side is 32.6.)

Step 8: Collect the terms and divide

$$38x = 32.6 + 13$$
$$38x = 45.6$$
$$x = \frac{45.6}{38} = 1.2$$

(Reason: Adding 13 to both sides puts the number terms on the right, leaving one term in x on the left. Dividing by 38 gives the solution, and the decimal is exact rather than rounded.)

(a) $3y(2y + 5)(y + 7) = 6y^3 + 57y^2 + 105y$, **so** $a = 6$, $b = 57$, $c = 105$

(b) $x = 1.2$

Verification

✓ **Check 1:** Put $y = 2$ into the original product and into the cubic. If the two forms are the same expression, they must give the same number. $3 \times 2 \times 9 \times 9 = 486$ and $6 \times 8 + 57 \times 4 + 105 \times 2 = 486$, so the two agree.

✓ **Check 2:** Expand in a different order: multiply $3y$ by the first bracket instead, then by the second. $3y(2y + 5) = 6y^2 + 15y$, and $(6y^2 + 15y)(y + 7) = 6y^3 + 42y^2 + 15y^2 + 105y$, which simplifies to $6y^3 + 57y^2 + 105y$.

✓ **Check 3:** Substitute $x = 1.2$ into the left-hand side of the original equation.
$\dfrac{5.4}{5} + \dfrac{2.2}{4} = 1.08 + 0.55 = 1.63$, and $\dfrac{163}{100} = 1.63$, so both sides match.

✓ **Check 4:** Solve part (b) a second way: multiply every term of the original equation by 100 instead of 20. $20(2x+3) + 25(6x-5) = 163$ gives $190x - 65 = 163$, so $190x = 228$ and $\dfrac{228}{190} = 1.2$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) First expansion	**M1**	An expansion of one pair of the three factors with only one error, for example $(2y+5)(y+7) = 2y^2 + 14y + 5y + 35$, or $3y(2y+5) = 6y^2 + 15y$, or $3y(y+7) = 3y^2 + 21y$. Do not award this mark for $6y^2 + 15y + 3y^2 + 21y$.	✓
(a) Second expansion	**M1**	Follow through, dependent on the first M1, allowing one further error: $(6y^2 + 15y)(y+7) = 6y^3 + 42y^2 + 15y^2 + 105y$, or $(3y^2 + 21y)(2y+5) = 6y^3 + 15y^2 + 42y^2 + 105y$, or $3y(2y^2 + 19y + 35) = 6y^3 + 57y^2 + 105y$. Alternatively M2 for 3 correct terms out of a maximum of 4 of $6y^3 + 42y^2 + 15y^2 + 105y$, and M1 for 2 correct out of a maximum of 4.	✓
(a) Simplified cubic	**A1**	cao, dependent on M1. The terms may be in any order but must be simplified: $6y^3 + 57y^2 + 105y$. Accept $a = 6$, $b = 57$, $c = 105$. Working is required.	✓
(b) Common denominator	**M1**	Writing the fractions over a common denominator (two fractions are enough), or a method to remove the denominator by multiplying each term by, for example, 20 or 100. If the numerator is expanded, allow one error. For example $\dfrac{4(2x+3) + 5(6x-5)}{20} = 1.63$, or $20(2x+3) + 25(6x-5) = 163$, which may all be written over 100.	✓
(b) Brackets and fractions removed	**M1**	Removing the brackets and the fractions on the left-hand side, in an equation with no more than one error from expanding the numerator, or an equation with the terms on the numerator simplified with no more than one such	✓

Step	Mark	Description	Got it?
		error. For example $8x + 12 + 30x - 25 = 32.6$, or $40x + 60 + 150x - 125 = 163$, or $\dfrac{38x - 13}{20} = \dfrac{163}{100}$.	
(b) Terms collected	M1	Terms in x on one side and number terms on the other, in a correct equation. For example $8x + 30x = 32.6 - 12 + 25$, or $38x = 45.6$, or $190x = 228$.	✓
(b) Solution	A1	1.2 or equivalent, dependent on M1. Working is required.	✓

Full marks: 7/7

Question 15 - Exam Solution

Understanding the Question

Given

The formula $e = \sqrt{\dfrac{7g+5}{11+2g}}$, in which g sits under a square root and in both parts of a fraction.
The inequality $3y^2 + 4y - 32 > 0$, a quadratic whose y^2 coefficient is positive.

Find

(a) g written in terms of e. (b) every value of y that makes the quadratic greater than 0. Expect two separate regions, not one.

Plan the Solution

- (a) Square both sides to remove the root, then multiply by the denominator so that nothing is left underneath a fraction.
- (a) Two terms will contain g. Collect them on one side, factorise g out and divide by the bracket that is left.
- (b) Factorise the quadratic, then set each bracket equal to 0 to get the two critical values.
- (b) A positive quadratic dips below the axis between its roots and sits above it outside them, so the solution is the two outer regions.

Worked Solution [7 marks]

> Rule - Changing the subject: clear roots and fractions first, gather every term containing the new subject on one side, then factorise it out and divide by the bracket. For a quadratic inequality: factorise, find the critical values, then choose the regions from the shape of the curve.

Step 1: square both sides

$$e^2 = \frac{7g+5}{11+2g}$$

(Reason: Squaring is the inverse of the square root, and it is applied to the whole of each side, so the left becomes e^2 *and the right loses its root sign completely.)*

Step 2: multiply by the denominator and expand

$$e^2(11+2g) = 7g+5$$
$$11e^2 + 2e^2g = 7g+5$$

(Reason: While g *is still inside the denominator nothing can be collected, so the fraction has to go first. Multiplying both sides by* $11+2g$ *clears it, and expanding the bracket puts every term on one level.)*

Step 3: gather the terms in g on one side

$$2e^2g - 7g = 5 - 11e^2$$

(Reason: There are now two terms containing g*, one on each side. Both must end up together before* g *can be taken out as a factor, and everything without a* g *in it moves the other way.)*

Step 4: factorise out g, then divide

$$g(2e^2-7) = 5 - 11e^2$$
$$g = \frac{5-11e^2}{2e^2-7}$$

(Reason: As far as g *is concerned,* $2e^2 - 7$ *is just the number multiplying it, so dividing both sides by that bracket leaves* g *on its own. That bracket is never zero:* $2e^2 = 7$ *would need* $\frac{7g+5}{11+2g}$ *to equal* $\frac{7}{2}$*, and no value of* g *makes that true.)*

Step 5: factorise the quadratic in part (b)

$$3y^2 + 4y - 32 = 3y^2 + 12y - 8y - 32$$
$$3y^2 + 12y - 8y - 32 = 3y(y+4) - 8(y+4)$$
$$3y(y+4) - 8(y+4) = (3y-8)(y+4)$$

(Reason: The middle term is split using two numbers whose product is $3 \times (-32) = -96$ *and whose sum is* 4*. Those numbers are* 12 *and* -8*, and grouping in pairs then gives the common bracket.)*

Step 6: find the critical values

$$3y - 8 = 0 \implies y = \frac{8}{3}$$

$$y + 4 = 0 \implies y = -4$$

(Reason: A product is zero exactly when one of its factors is zero, so the quadratic takes the value 0 at these two values of y and nowhere else. They are the boundaries between the positive and the negative parts of the curve, and neither is itself a solution, because the inequality is strict.)

Step 7: choose the regions

$$y < -4 \text{ or } y > \frac{8}{3}$$

(Reason: The coefficient of y^2 is 3, which is positive, so the curve is a U shape: below the axis between the critical values and above it outside them. Testing $y = 0$, which lies between them, gives -32, confirming that the middle region is not wanted.)

$$\textbf{(a) } g = \frac{5 - 11e^2}{2e^2 - 7}$$

$$\textbf{(b) } y < -4 \text{ or } y > \frac{8}{3}$$

Verification

✓ **Check 1 - put a number through both formulas:** Take $g = 3$ in the original: $7 \times 3 + 5 = 26$ and $11 + 2 \times 3 = 17$, so $e^2 = \frac{26}{17}$. Then $11e^2 = \frac{286}{17}$ and $2e^2 = \frac{52}{17}$.

The rearranged formula gives $\dfrac{5 - \frac{286}{17}}{\frac{52}{17} - 7} = \dfrac{-201}{-67} = 3$, the value g started at.

✓ **Check 2 - the other accepted form:** Multiply the numerator and the denominator of $\dfrac{5 - 11e^2}{2e^2 - 7}$ by -1. Every sign flips and the value is unchanged, giving $\dfrac{11e^2 - 5}{7 - 2e^2}$, which is the alternative form the mark scheme accepts, so the two answers are one expression written two ways.

✓ **Check 3 - expand the factorisation:** Multiply $(3y - 8)(y + 4)$ out again and collect the middle terms. $3y^2 + 12y - 8y - 32 = 3y^2 + 4y - 32$, which is exactly the quadratic the question gives.

✓ **Check 4 - one value from each region:** Substitute $y = -5$, $y = 0$ and $y = 3$ into $3y^2 + 4y - 32$, one from each side of the critical values and one from between them.

They give 23, -32 and 7, so the expression is positive on the two outer regions only, which is what the answer claims.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$e^2 = \dfrac{7g+5}{11+2g}$	**M1**	For removing the square root.	✓
$11e^2 + 2e^2g = 7g + 5$	**M1**	For multiplying by the denominator and expanding, in a correct equation.	✓
eg $2e^2g - 7g = 5 - 11e^2$ or $11e^2 - 5 = 7g - 2e^2g$ oe	**M1**	For gathering the terms in g on one side and the other terms on the other side, in a correct equation.	✓
$g = \dfrac{5-11e^2}{2e^2-7}$	**A1**	oe. Accept for example $g = \dfrac{11e^2-5}{7-2e^2}$ or $g = \dfrac{\frac{5}{e^2}-11}{2-\frac{7}{e^2}}$ or $g = \dfrac{\frac{5-11e^2}{e^2-3.5}}{2}$.	✓
Correct answer scores full marks	**Note**	unless it comes from obviously incorrect working.	✓
$(3y-8)(y+4)$	**M1**	For a correct factorisation, or correct use of the quadratic formula $\dfrac{-4 \pm \sqrt{4^2 - 4\times 3\times(-32)}}{2\times 3}$, or as far as $\dfrac{-4\pm\sqrt{400}}{6}$.	✓
$\left(y - \dfrac{8}{3}\right)(y+4)$	**Note**	is not a valid factorisation of the given quadratic, so it scores no marks unless it is preceded by dividing the quadratic by 3.	✓
$y = \dfrac{8}{3}, y = -4$	**A1**	dep on M1, for both correct critical values. Allow 2.6 or better, or 2.7.	✓

Step	Mark	Description	Got it?
$y < -4, y > \frac{8}{3}$	A1	oe dep on M1, and working is required. Allow x in place of y, and accept the interval form $(-\infty, -4)$ with $\left(\frac{8}{3}, \infty\right)$, or the union of the two.	✓

Full marks: 7/7

Question 16 - Exam Solution

Understanding the Question

Given

60 members were asked about three clubs: chess C, gardening G and baking B
$\mathrm{n}(C \cap G \cap B) = 9$
$\mathrm{n}(C \cap G) = 17$, $\mathrm{n}(G \cap B) = 16$, $\mathrm{n}(C \cap B) = 20$
$\mathrm{n}(G) = 28$ and $\mathrm{n}(B) = 39$, and 2 members joined no club at all

Find

(a) the number of members in each of the eight regions of the Venn diagram (b) the probability that a member chose chess, given that the member chose gardening (c) $\mathrm{n}(G \cap C')$ (d) $\mathrm{n}([G \cup B] \cap C)$

Plan the Solution

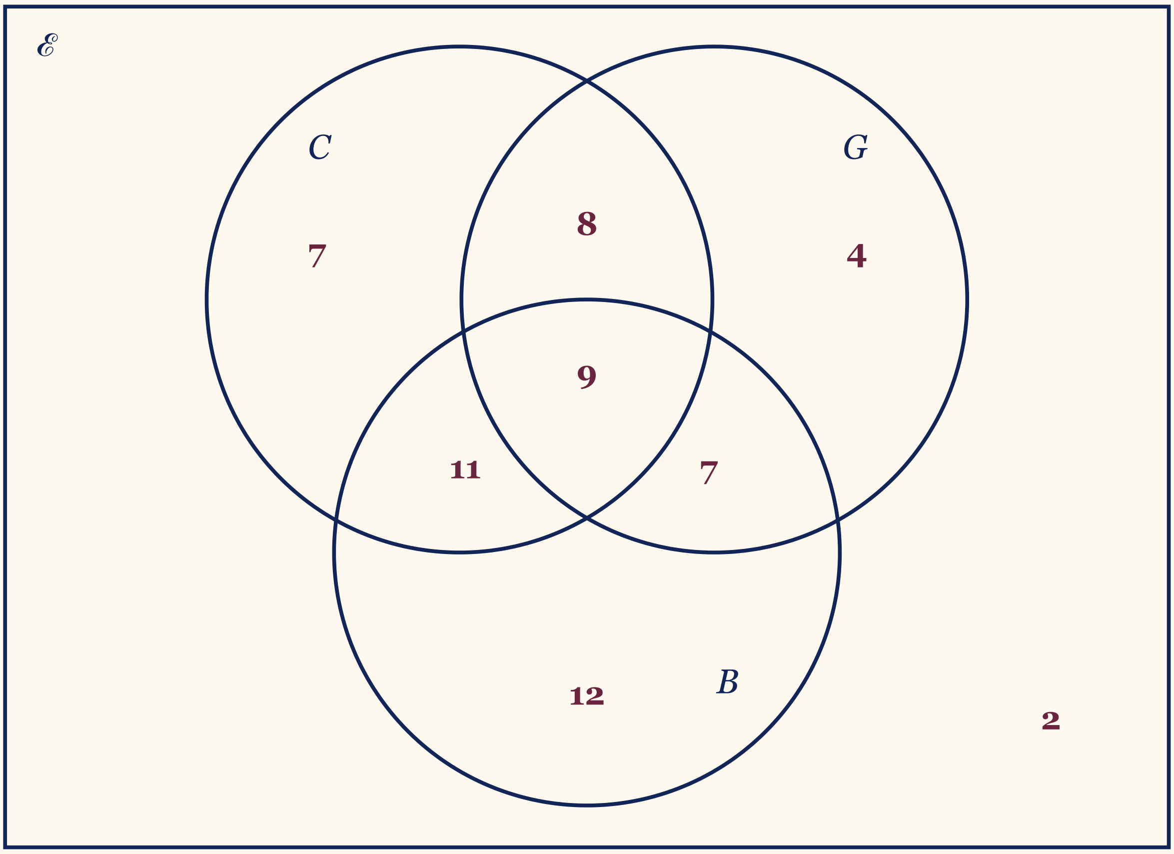
- Start in the middle. The 9 who joined all three clubs go in the centre region, and every other total in the question already includes them.
- Work outwards. Take the centre off each pair total to get the members in exactly two clubs, then take the filled regions off 28 and 39 to get the members in exactly one.
- Finish with the 60. The last region is whatever is left once the 2 outside the circles are set aside.
- Then read parts (b), (c) and (d) straight off the completed diagram - no new working is needed.

Worked Solution [7 marks]

Rule - overlapping sets: a pair total counts everyone in both circles, including everyone in all three. So $\mathrm{n}(C \cap G) = 17$ leaves $17 - 9 = 8$ members in chess and gardening but not baking.

Step 1: put the 9 who joined all three in the centre

$$\mathrm{n}(C \cap G \cap B) = 9$$



(Reason: The centre region lies inside all three circles, so it holds exactly the members who chose every club. Filling it first is what makes every later subtraction possible.)

Step 2: take the centre off each pair total

$$17 - 9 = 8$$
$$16 - 9 = 7$$
$$20 - 9 = 11$$

(Reason: Each pair total already contains the 9 in the centre, so removing them leaves the members in exactly two of the clubs: 8 in chess and gardening only, 7 in gardening and baking only, and 11 in chess and baking only.)

Step 3: fill the gardening-only and baking-only regions

$$28 - 8 - 9 - 7 = 4$$
$$39 - 11 - 9 - 7 = 12$$

(Reason: The 28 who chose gardening are spread over four regions, so take away the three that are already filled. The 39 who chose baking work the same way.)

Step 4: use the 60 to finish the chess-only region

$8 + 9 + 11 + 4 + 7 + 12 = 51$

$60 - 2 - 51 = 7$

(Reason: Every region except chess-only is now known. The 2 who joined nothing sit outside the circles, so they come off the total before the rest is shared out. The diagram is complete.)

Step 5, part (b): restrict to the gardening circle

$4 + 8 + 9 + 7 = 28$

$8 + 9 = 17$

$\mathrm{P}(C \mid G) = \dfrac{17}{28}$

(Reason: Being told the member chose gardening narrows the choice from 60 members to the 28 inside that circle. Of those, the 8 and the 9 also chose chess.)

Step 6, part (c): $\mathrm{n}(G \cap C')$

$4 + 7 = 11$

(Reason: C' is everything outside the chess circle, so keep only the parts of the gardening circle that lie outside C: gardening only and gardening with baking.)

Step 7, part (d): $\mathrm{n}([G \cup B] \cap C)$

$8 + 9 + 11 = 28$

(Reason: $G \cup B$ is everything in the gardening or baking circles, and intersecting with C keeps the parts of the chess circle that overlap them - so every region of C except chess only.)

(a) chess only 7; chess and gardening only 8; gardening only 4; chess and baking only 11; all three 9; gardening and baking only 7; baking only 12; outside 2

(b) $\dfrac{17}{28}$

(c) 11

(d) 28

Verification

✓ **Check 1 - every member is somewhere:** Add all eight regions of the completed diagram, including the 2 outside the circles. The result must be the 60 members who were asked. $7 + 8 + 4 + 11 + 9 + 7 + 12 + 2 = 60$

✓ **Check 2 - the three pair totals come back:** For each pair of clubs, add the exactly-two region to the centre and compare with the 17, 16 and 20 the question gives. $8+9=17$, $7+9=16$, $11+9=20$

✓ **Check 3 - part (d) a second way:** Everyone in the chess circle who is not in chess alone must have chosen gardening or baking, so part (d) is the whole chess circle less the chess-only region. $7+8+9+11=35$ and $35-7=28$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) the chess-only region	**B1**	For 7 in chess only.	✓
(a) the other seven regions	**B2**	For all 7 of the other regions correct: 8, 4, 11, 9, 7, 12 and the 2 outside the circles.	✓
(a) most of the other regions	**(B1)**	For 4, 5 or 6 of those other seven regions correct.	✓
(b) the conditional probability	**B2**	$\frac{17}{28}$ or equivalent: 0.61 or 61% or 0.607 or 60.7% or better. Follow through from the candidate's own Venn diagram, or work from the values given in the question text.	✓
(b) a partly correct fraction	**(B1)**	For 17 as the numerator, or 28 as the denominator, of a fraction between 0 and 1.	✓
(c) the gardening members outside chess	**B1ft**	11, follow through from the candidate's own Venn diagram, or from the values given in the question text.	✓
(d) the chess members who also chose gardening or baking	**B1ft**	28, follow through from the candidate's own Venn diagram, or from the values given in the question text.	✓
Follow through in (b), (c) and (d)	**Note**	Follow through is only available where the candidate has actually written numbers into the regions of the Venn diagram.	✓

Full marks: 7/7

Question 17 - Exam Solution

Understanding the Question

Given

$Q \propto \sqrt{d}$ - a direct proportion to the square root, not to d itself

One pair of matching values: $Q = 4.5$ when $d = 324$

Find

A formula for Q in terms of d The answer must be written as $Q = \ldots$, so the constant has to be worked out first

Plan the Solution

- Replace the proportion sign by a constant of proportionality k to get an equation.
- Substitute the pair $Q = 4.5$, $d = 324$ into that equation. 324 is a square number, so the root is exact.
- Solve for k, then put its value back into the equation and leave d as a letter.

Worked Solution [3 marks]

> Rule - Direct proportion: $Q \propto \sqrt{d}$ means $Q = k\sqrt{d}$ for one fixed constant k. A single pair of values is enough to fix k, and the finished formula must show d as a letter.

Step 1: write the proportion as an equation

$$Q = k\sqrt{d}$$

(Reason: Directly proportional means one quantity is a fixed multiple of the other, so a single constant k links them. The multiplier is applied to $\sqrt{d}$, not to d, because that is what the question says is proportional to Q.)

Step 2: substitute the pair of values

$$4.5 = k\sqrt{324}$$

$$\sqrt{324} = 18$$

$$4.5 = 18k$$

(Reason: The given pair obeys the same rule, so putting it in turns the equation into one with only k unknown. Here $18^2 = 324$, so the square root is exact and nothing is rounded.)

Step 3: solve for the constant

$k = \dfrac{4.5}{18} = 0.25$

(Reason: Divide both sides by the number that is multiplying k. The constant is a pure number: it carries no Q and no d.)

Step 4: write the formula

$Q = 0.25\sqrt{d}$

(Reason: Putting the value of k back into $Q = k\sqrt{d}$ gives what was asked for. The equivalent forms $Q = \dfrac{\sqrt{d}}{4}$ and $Q = \sqrt{\dfrac{d}{16}}$ are also accepted, but an answer that does not start $Q =$ is not.)

$$Q = 0.25\sqrt{d}$$

Verification

✓ **Check 1:** Put $d = 324$ back into the formula. The square root of 324 is 18. $0.25 \times 18 = 4.5$, which is the value of Q the question gives.

✓ **Check 2:** Test one of the equivalent forms the mark scheme accepts, $Q = \sqrt{\dfrac{d}{16}}$, at the same value of d. It reaches the answer without dividing by a root at all. $\sqrt{\dfrac{324}{16}} = \sqrt{20.25} = 4.5$, the same value again.

✓ **Check 3:** A structure check rather than an arithmetic one. Proportion to $\sqrt{d}$ means d must be multiplied by 4 for Q to double. Take $d = 4 \times 324 = 1296$. $0.25\sqrt{1296} = 0.25 \times 36 = 9$, which is exactly twice 4.5. A formula using d instead of $\sqrt{d}$ would have given four times as much.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Writes the proportion as an equation, eg $Q = k\sqrt{d}$ or $kQ = \sqrt{d}$ or $Q = \sqrt{kd}$	M1	Any correct algebraic form with a constant of proportionality, or equivalent, provided $k \neq 1$. Use of $\propto$ in place of $=$ is condoned for the method marks.	✓

Step	Mark	Description	Got it?
Substitutes the given pair, eg $4.5 = k \times \sqrt{324}$, or states $k = 0.25$	M1	Or equivalent value of the constant, eg $k = \frac{1}{4}$. Award both method marks for this line if $Q = k\sqrt{d}$ is never written down.	✓
Gives the formula $Q = 0.25\sqrt{d}$	A1	Or equivalent, but it must be in the form $Q = \ldots$ eg $Q = \frac{\sqrt{d}}{4}$ or $Q = \sqrt{\frac{d}{16}}$. A correct answer scores full marks unless it comes from obviously incorrect working. A value of k alone, with no formula, does not earn this mark.	✓

Full marks: 3/3

Question 18 - Exam Solution

Understanding the Question

Given

The straight line P has equation $5y + 2x = 7$ - written with the x and y terms on the same side, not as $y = mx + c$

The straight line Q is perpendicular to P

Find

The gradient of Q A gradient is a number, so the answer carries no unit. Nothing is asked about where either line sits, so the 7 will not appear in the answer

Plan the Solution

- Rearrange $5y + 2x = 7$ into the form $y = mx + c$, because the gradient is only readable once y is the subject.
- Read the gradient of P as the number multiplying x, sign included.
- Apply the perpendicular rule to that gradient to get the gradient of Q. Then check the two gradients multiply to -1.

Worked Solution [2 marks]

> Rule - Perpendicular gradients: if two lines are perpendicular then $m_1 \times m_2 = -1$, so each gradient is the negative reciprocal of the other - turn the fraction upside down and

change its sign. A horizontal line and a vertical line are the one exception, because a vertical line has no gradient.

Step 1: make y the subject

$$5y + 2x = 7$$
$$5y = 7 - 2x$$
$$y = \frac{7 - 2x}{5}$$
$$y = \frac{7}{5} - \frac{2}{5}x$$

(Reason: Subtracting $2x$ from both sides and then dividing every term by 5 leaves y on its own. Splitting the single fraction into two is what makes the coefficient of x visible; the mark scheme also accepts the unsplit form $y = \frac{7-2x}{5}$ for the method mark.)

Step 2: read the gradient of P

$$m_1 = -\frac{2}{5} = -0.4$$

(Reason: Comparing $y = \frac{7}{5} - \frac{2}{5}x$ with $y = mx + c$ gives $c = \frac{7}{5}$ and a gradient of $-\frac{2}{5}$. The minus sign is part of the gradient, not a stray subtraction: P falls from left to right.)

Step 3: turn that gradient into the gradient of Q

$$m_1 \times m_2 = -1$$
$$-\frac{2}{5} \times m_2 = -1$$
$$m_2 = \frac{5}{2} = 2.5$$

(Reason: Dividing both sides by $-\frac{2}{5}$ is the same as multiplying by its reciprocal, so the gradient of Q is the gradient of P turned upside down with its sign changed. One line falls and the other rises, so the two gradients must have opposite signs.)

$$\frac{5}{2} = 2.5$$

Verification

✓ **Check 1:** Multiply the two gradients together. Perpendicular lines must give -1, and this is the definition the answer has to satisfy. $-\dfrac{2}{5} \times \dfrac{5}{2} = -\dfrac{10}{10} = -1$, so the two gradients really are perpendicular. Keeping the sign would have given $+1$ instead.

✓ **Check 2:** Get the gradient of P from two points instead of from $y = mx + c$, so the rearranging is never used. Both $(1, 1)$ and $(6, -1)$ satisfy $5y + 2x = 7$. $\dfrac{-1-1}{6-1} = -\dfrac{2}{5}$, the same gradient for P that Step 2 read off, so Step 1 was rearranged correctly.

✓ **Check 3:** A geometry check, using no gradient rule at all. From $(1, 1)$, a step of 5 right and 2 down stays on P and lands on $(6, -1)$. Turning that step a quarter turn gives 2 right and 5 up, landing on $(3, 6)$. The three squared lengths are $5^2 + 2^2 = 29$, $2^2 + 5^2 = 29$ and $3^2 + 7^2 = 58$. $29 + 29 = 58$, so by the converse of Pythagoras the corner at $(1, 1)$ is a right angle. The turned step rises 5 for every 2 across, a gradient of $\dfrac{5}{2}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Rearranges $5y + 2x = 7$ so the gradient can be seen, eg $y = \dfrac{7-2x}{5}$ or $y = -0.4x + \ldots$, or writes an equation whose gradient is 2.5 oe, eg $y = \dfrac{5}{2}x + \ldots$	**M1**	Oe. Writing the gradient of P as $-\dfrac{2}{5}$ on its own earns this mark. Otherwise the rearrangement need not be simplified, provided the gradient of P can be read from it. The term $\dfrac{5}{2}x$ written on its own also earns this mark, so a candidate who goes straight to the perpendicular line's equation is not penalised for skipping the gradient of P.	✓
States the gradient of the perpendicular line, $\dfrac{5}{2}$	**A1**	Oe, eg 2.5. The gradient must be stated, but isw if it is stated and then used inside an equation. A correct answer scores full marks unless it comes from obviously incorrect working, so no working is needed for the 2 marks.	✓
Special case: an equation of a line with gradient $\dfrac{5}{2}$ is given, but $\dfrac{5}{2}$ is never written on its own	**SC B1**	Awarded only if no other mark is earned. The named error is answering with a whole equation when the question asked for a number: the perpendicular gradient has in fact been found, so one mark is recovered, but the answer as written does not state it.	✓

Full marks: 2/2

Question 19 - Exam Solution

Understanding the Question

Given

$G = \dfrac{c}{2f - 3h}$

$c = 8$, correct to the nearest whole number

$f = 6.62$, correct to 2 decimal places

$h = 1.2$, correct to 1 decimal place

Find

The lower bound for the value of G, correct to 3 decimal places

Plan the Solution

- Turn each rounded value into a pair of bounds by going half a rounding unit either side.
- Decide which bound of each letter makes G as small as it can be.
- Substitute those bounds, work the denominator out on its own, then divide.
- Round the result to 3 decimal places.

Worked Solution [3 marks]

Rule - Bounds: a rounded value lies within half a rounding unit of the value stated, so 8 to the nearest whole number means $7.5 \leq c < 8.5$. A fraction is smallest when its numerator is smallest and its denominator is largest, so for $G = \dfrac{c}{2f - 3h}$ take the lower bound of c, the upper bound of f and the lower bound of h.

Step 1: Write down the bounds for each letter

$7.5 \leq c < 8.5$

$6.615 \leq f < 6.625$

$1.15 \leq h < 1.25$

(Reason: Each value can be out by half of the unit it was rounded to. Half of 1 is 0.5, half of 0.01 is 0.005, and half of 0.1 is 0.05.)

Step 2: Choose the bound of each letter that makes G smallest

$$\frac{7.5}{2 \times 6.625 - 3 \times 1.15}$$

(Reason: The numerator must be as small as possible, so c takes its lower bound 7.5. The denominator must be as large as possible, so f takes its upper bound 6.625 while h takes its lower bound 1.15, because $3h$ is taken away. This is the trap in the question: the lower bound of G needs the upper bound of one letter and the lower bound of the other.)

Step 3: Work out the largest possible denominator

$2 \times 6.625 = 13.25$

$3 \times 1.15 = 3.45$

$13.25 - 3.45 = 9.8$

(Reason: Deal with the denominator on its own before dividing. Doubling the upper bound of f and then subtracting three lots of the lower bound of h gives the largest value the denominator can take.)

Step 4: Divide, then round to 3 decimal places

$$\frac{7.5}{9.8} = \frac{75}{98} = 0.7653061224$$

$0.7653061224 \approx 0.765$

(Reason: Multiplying top and bottom by 10 turns $\frac{7.5}{9.8}$ into $\frac{75}{98}$, which is a tidier thing to key in. The fourth decimal digit is 3, so the third digit stays as it is.)

Lower bound $= 0.765$

Verification

✓ **Check 1:** Multiply the answer back by the largest denominator. It should return the smallest numerator, 7.5. $0.7653061224 \times 9.8 = 7.5$

✓ **Check 2:** Swap each chosen bound for the other one, one at a time. If the right three bounds were picked, every swap must make G bigger. $\frac{8.5}{9.8} = 0.8673469388$, $\frac{7.5}{9.78} = 0.7668711656$, $\frac{7.5}{9.5} = 0.7894736842$, all bigger than 0.765

✓ **Check 3:** Confirm the denominator can never turn negative, otherwise the largest denominator would not give the smallest G. The smallest it can be pairs the lower bound of f with the upper bound of h. $2 \times 6.615 - 3 \times 1.25 = 13.23 - 3.75 = 9.48$, which is positive

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write down any one correct bound, for example $7.5 \le c < 8.5$	B1	For a correct upper or lower bound: $7.5, 8.5, 6.615, 6.625, 1.15, 1.25$. Allow $8.4\dot{9}$ for 8.5, $6.624\dot{9}$ for 6.5 and $1.24\dot{9}$ for 1.25. That 6.5 appears to be a slip for 6.625, which is what $6.624\dot{9}$ equals.	✓
$\dfrac{7.5}{2 \times 6.625 - 3 \times 1.15}$	M1	For the lower bound of c over 2 times the upper bound of f minus 3 times the lower bound of h, where $7.5 \le \text{LB}_c < 8$, $6.62 < \text{UB}_f \le 6.625$ and $1.15 \le \text{LB}_h < 1.2$.	✓
A candidate who drops the 2 and the 3 from the denominator and works out $\dfrac{7.5}{6.625 - 1.15} = 1.369863$	SC B1	A special case, awarded in addition to the first B1. The error is reading $2f - 3h$ as $f - h$: the bounds themselves are chosen correctly, but the coefficients are dropped.	✓
0.765	A1	awrt 0.765 (the full value is 0.7653061224). Working is required. Dependent on completely correct bounds, and dependent on the M1.	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

$k = x - y$

$x = \dfrac{1}{4y}$

Two facts linking k, x and y. The required form contains only y, so k and x must both disappear.

Find

$\dfrac{5k}{x+2}$ written as $\dfrac{a - by^2}{c + dy}$. The four integers a, b, c and d.

Plan the Solution

- Replace x by $\dfrac{1}{4y}$ everywhere, so that y is the only letter left.
- Write the top, $5k$, as one fraction over $4y$.
- Write the bottom, $x+2$, as one fraction over the same $4y$.
- Divide the two fractions. The $4y$ cancels, which is what leaves integers behind.

Worked Solution [3 marks]

> Rule - Divide by a fraction by multiplying by its reciprocal: $\dfrac{P}{Q}$ divided by $\dfrac{R}{S}$ is $\dfrac{P}{Q} \times \dfrac{S}{R}$.
>
> Here Q and S are both $4y$, so they cancel each other.

Step 1: write k as a single fraction over $4y$

$$k = x - y = \frac{1}{4y} - y$$

$$y = \frac{4y^2}{4y}$$

$$k = \frac{1}{4y} - \frac{4y^2}{4y} = \frac{1-4y^2}{4y}$$

(Reason: Two fractions can only be subtracted over a common denominator. Multiplying y top and bottom by $4y$ turns it into $\dfrac{4y^2}{4y}$, which changes how it is written but not what it is worth.)

Step 2: write $x+2$ over the same denominator

$$x + 2 = \frac{1}{4y} + 2$$

$$2 = \frac{8y}{4y}$$

$$x + 2 = \frac{1}{4y} + \frac{8y}{4y} = \frac{1+8y}{4y}$$

(Reason: Since $2 \times 4y = 8y$, the whole number 2 is $\dfrac{8y}{4y}$. This numerator, $1+8y$, is the denominator the final answer keeps.)

Step 3: multiply the top by 5

$$5k = 5 \times \frac{1-4y^2}{4y} = \frac{5-20y^2}{4y}$$

(Reason: The 5 multiplies the numerator as a whole, so every term inside the bracket is multiplied by it. The denominator is untouched.)

Step 4: divide, and cancel the $4y$

$$\frac{5k}{x+2} = \frac{5-20y^2}{4y} \times \frac{4y}{1+8y}$$
$$= \frac{5-20y^2}{1+8y}$$

(Reason: Dividing by $\frac{1+8y}{4y}$ is the same as multiplying by $\frac{4y}{1+8y}$. The $4y$ now appears once above and once below, so it cancels and no fraction is left inside a fraction.)

Step 5: read off the four integers

$$\frac{a-by^2}{c+dy} = \frac{5-20y^2}{1+8y}$$
$$a=5,\ b=20,\ c=1,\ d=8$$

(Reason: Comparing with the required form, a is the constant on top, b is the number subtracted from it, and c and d are the two numbers underneath. All four are integers, as the question demands.)

$$\frac{5k}{x+2} = \frac{5-20y^2}{1+8y}$$
$$a=5, b=20, c=1, d=8$$

Verification

✓ **Check 1:** Put $y=1$ into the original. Then $x=\frac{1}{4}$, $k=\frac{1}{4}-1=-\frac{3}{4}$ and $x+2=\frac{9}{4}$, so the original is $-\frac{15}{4}\times\frac{4}{9}$. The original gives $-\frac{5}{3}$, and the answer gives $\frac{5-20}{1+8}=-\frac{15}{9}=-\frac{5}{3}$. They agree.

✓ **Check 2:** Now a negative value, which tests the signs. Put $y=-1$. Then $x=-\frac{1}{4}$, $k=-\frac{1}{4}+1=\frac{3}{4}$ and $x+2=\frac{7}{4}$. The original gives $\frac{15}{4}\times\frac{4}{7}=\frac{15}{7}$, and the answer gives $\frac{5-20}{1-8}=\frac{15}{7}$. They agree again, and this time both are positive.

✓ **Check 3:** Check the FORM, not just the value. The question asks for $\dfrac{a - by^2}{c + dy}$: a constant minus a multiple of y^2 on top, and a constant plus a multiple of y underneath, with all four numbers integers. $5 - 20y^2$ over $1 + 8y$ matches that pattern exactly, and neither k nor x survives anywhere in it.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Substitute $x = \dfrac{1}{4y}$ into $\dfrac{5k}{x+2}$, giving $5\left(\dfrac{1}{4y} - y\right)$ over $\dfrac{1}{4y} + 2$.	M1	A correct substitution leaving only values of y; or an expression containing xy (not just x); or a correct denominator of $1 + 8y$.	✓
Multiply every term by $4y$, or write top and bottom over $4y$: $\dfrac{5 - 20y^2}{4y}$ over $\dfrac{1 + 8y}{4y}$.	M1	Multiplying every term by $4y$ or a multiple of $4y$; or writing numerator and denominator over $4y$ or a multiple of it; or a correct expansion containing an xy term (xy may be replaced by 0.25); or three of a, b, c, d correct in the required form.	✓
$\dfrac{5 - 20y^2}{1 + 8y}$	A1	Or equivalent, e.g. $\dfrac{-5 + 20y^2}{-1 - 8y}$ or $\dfrac{20 - 80y^2}{4 + 32y}$, i.e. $a = 5$, $b = 20$, $c = 1$, $d = 8$ or any common multiple of them.	✓
A correct answer written down with no working	Note	A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 21 - Exam Solution

Understanding the Question

Given	Find

A square $ABCD$ with a circle inside it, and every side of the square is a tangent to that circle.
The shaded regions together have an area of 80 cm².
The figure is not drawn accurately, so nothing may be measured off it.

The length of AC, which is a diagonal of the square, correct to 3 significant figures.

Plan the Solution

- Turn the word tangent into a length. Each side touches the circle once, so the circle is squeezed between opposite sides and its diameter is the side of the square. Call the radius r; the side is then $2r$.
- Shaded means the square with the circle taken out of it, so 80 becomes one equation in r alone.
- $4 - \pi$ is just a number, so r^2 comes out by dividing. Nothing here needs r to be a tidy value.
- AC is the diagonal of a square, so Pythagoras across two sides finishes it. Keep the full accuracy and round once, at the end.

Worked Solution [5 marks]

> Rule - Circle inscribed in a square: the side of the square is $2r$, the shaded area is $(2r)^2 - \pi r^2$, and the diagonal follows from Pythagoras, $AC^2 = x^2 + x^2$, where x is the side.

Step 1: turn the tangents into a length

$x = 2r$

(Reason: A tangent meets a circle exactly once, so the circle is held tight between each pair of opposite sides. The distance across the circle is therefore the whole side of the square: if the radius is r, the side x is $2r$. This is the only place the word tangent is used, and it is what makes one letter enough.)

Step 2: write the shaded area in that one letter

$(2r)^2 - \pi r^2 = 80$
$4r^2 - \pi r^2 = 80$

(Reason: The shaded parts are what is left when the circle is cut out of the square, so their total area is the area of the square minus the area of the circle. The square has side $2r$ and the circle has radius r, so both areas are written with the same letter. Note that $(2r)^2$ expands to $4r^2$, not to $2r^2$ - the 2 is squared as well.)

Step 3: factorise, then divide

$r^2(4 - \pi) = 80$

$r^2 = \dfrac{80}{4 - \pi} = 93.1958\ldots$

(Reason: Both terms carry r^2, so it comes out as a common factor and leaves the bracket $4 - \pi$, which is only a number, about 0.8584. Dividing 80 by it gives r^2 in one step, and there is no need to find r itself.)

Step 4: square the side of the square

$x^2 = (2r)^2 = 4r^2$

$4 \times 93.1958\ldots = 372.7833\ldots$

(Reason: The diagonal is built from the side, not from the radius. Squaring $2r$ multiplies r^2 by 4, so the square of the side is four times the $93.1958\ldots$ found above. Keeping x^2 rather than x saves a root that Pythagoras would only square again.)

Step 5: Pythagoras across the square

$AC^2 = x^2 + x^2 = 2x^2$

$2 \times 372.7833\ldots = 745.5667\ldots$

$AC = \sqrt{745.5667\ldots} = 27.3050\ldots$

(Reason: The sides AB and BC meet at a right angle at B, and AC is the hypotenuse of that right-angled triangle. Both sides are the same length, so the square of the diagonal is simply twice the square of the side, and the square root of it is the length wanted.)

Step 6: round to 3 significant figures

$AC = 27.3050\ldots$

$AC = 27.3$ cm

(Reason: The first three significant figures of $27.3050\ldots$ are 2, 7 and 3, and the digit after them is 0, so nothing rounds up. The answer is a length, so it is measured in centimetres.)

$$AC = 27.3 \text{ cm}$$

Verification

✓ **Check 1:** Rebuild both areas from $r^2 = 93.1958\ldots$ and see whether the shading comes back. The square has area $4r^2$ and the circle has area πr^2, so subtracting them must return the area the question gives. $372.7833 - 292.7833 = 80$, which is the 80 cm² of shading the question started from

✓ **Check 2:** A test that does not depend on the size at all. For a circle inscribed in any square the shaded share is fixed at $1 - \dfrac{\pi}{4}$, so dividing the given 80 by the area of the square found above should reproduce it. $\dfrac{80}{372.7833} = 0.2146$ and $1 - \dfrac{\pi}{4} = 0.2146$, so the square has come out the right size

✓ **Check 3:** Finally test the diagonal itself. In every square, whatever its size, the diagonal divided by the side is $\sqrt{2}$. The side itself is the square root of the $372.7833\ldots$ found in step 4, which is $19.3075\ldots$, so the ratio can be tested against $1.4142\ldots$. $\dfrac{27.3050}{19.3075} = 1.4142$, which is $\sqrt{2}$ to four decimal places

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write the total shaded area in one variable, e.g. $(2r)^2 - \pi r^2$	M1	A correct expression for the area of the shaded parts in one variable only. For this mark only, accept it without the brackets, e.g. $2r^2 - \pi r^2$, and accept the same thing written in the side of the square, $x^2 - \pi \times (0.5x)^2$. Any letter may be used.	✓
$4r^2 - \pi r^2 = 80$	M1	A correct equation in one variable with the brackets expanded, or equivalent, e.g. $r^2 - 0.25\pi r^2 = 20$, $x^2 - 0.25\pi x^2 = 80$ or $4x^2 - \pi x^2 = 320$. It may be seen later in the working.	✓
$r^2 = \dfrac{80}{4 - \pi} = 93.19\ldots$	M1	A correct expression for the radius squared or the radius, or for the side of the square squared or the side of the square, e.g. $r = \sqrt{\dfrac{80}{4-\pi}} = 9.65\ldots$, $x^2 = \dfrac{80}{1 - 0.25\pi} = 372.78\ldots$ or $x = \sqrt{\dfrac{320}{4-\pi}} = 19.307\ldots$.	✓
A correct calculation for AC, e.g. $\sqrt{(2 \times 9.65)^2 + (2 \times 9.65)^2}$	M1	For a correct calculation to find the length of AC from the candidate's own radius or side, e.g. $2 \times \sqrt{9.65^2 + 9.65^2}$,	✓

Step	Mark	Description	Got it?
		$\sqrt{19.307^2 + 19.307^2}$, $\sqrt{8 \times \dfrac{80}{4-\pi}}$ or $\dfrac{2 \times 9.65}{\sin 45}$.	
The length of AC	A1	27.3. Accept any value from 27.3 to 27.5, which covers the different points at which a candidate may round.	✓
A correct answer written down with no working	Note	A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 5/5

Question 22 - Exam Solution

Understanding the Question

Given

The straight line L: $x + y = 5$
The curve C: $2x^2 + 3y^2 = 210$
One equation is linear; the other is quadratic in both variables.

Find

The coordinates of every point that lies on L and on C at the same time. A quadratic appears on the way, so expect two solution pairs.

Plan the Solution

- Make y the subject of the linear equation.
- Substitute that expression into the curve's equation, so one equation in x alone is left.
- Expand, collect like terms and write the result as $ax^2 + bx + c = 0$.
- Divide through by the common factor, then factorise to get both values of x.
- Put each value of x back into $y = 5 - x$ to find its partner.

Worked Solution [5 marks]

> Rule - Substitution (line into curve): make one variable the subject of the linear equation, substitute it into the quadratic equation, solve the quadratic that results, then substitute

each root back into the linear equation to find the other coordinate.

Step 1: Make y the subject of the line's equation

$x + y = 5$

$y = 5 - x$

(Reason: The curve's equation contains x^2 and y^2, so it is the straight line that gets substituted into the curve, never the other way round. Rearranging the curve instead would bring in a square root.)

Step 2: Substitute into the curve's equation

$2x^2 + 3y^2 = 210$

$2x^2 + 3(5 - x)^2 = 210$

(Reason: Every y in the curve's equation is replaced by $5 - x$, which leaves one equation in x alone. Keep the bracket: $(5 - x)^2$ is not $25 + x^2$.)

Step 3: Expand and collect into a three-term quadratic

$(5 - x)^2 = 25 - 10x + x^2$

$2x^2 + 3(25 - 10x + x^2) = 210$

$2x^2 + 75 - 30x + 3x^2 = 210$

$5x^2 - 30x + 75 = 210$

$5x^2 - 30x - 135 = 0$

(Reason: Square the bracket first, then multiply every term inside it by 3. The two x^2 terms collect to $5x^2$, and taking 210 from both sides sets the quadratic equal to zero.)

Step 4: Solve the quadratic

$x^2 - 6x - 27 = 0$

$(x - 9)(x + 3) = 0$

$x = 9$ or $x = -3$

(Reason: Every term has a factor of 5, so dividing through by 5 gives a simpler quadratic with exactly the same roots. Two numbers with product -27 and sum -6 are -9 and 3.)

Step 5: Find the y that goes with each x

$y = 5 - 9 = -4$

$y = 5 - (-3) = 8$

(Reason: Both points lie on the line, so each x takes its own y from $y = 5 - x$. Pairing them the wrong way round gives two points that are on neither curve, and it is the commonest way to lose the final mark here.)

$$(9, -4)$$
$$(-3, 8)$$

Verification

✓ **Check 1 - the point $(9, -4)$:** Put $x = 9$ and $y = -4$ into both original equations. A point of intersection must satisfy each of them. $9 + (-4) = 5$ and $2 \times 9^2 + 3 \times (-4)^2 = 162 + 48 = 210$

✓ **Check 2 - the point $(-3, 8)$:** Put $x = -3$ and $y = 8$ into both original equations, squaring the negative carefully. $-3 + 8 = 5$ and $2 \times (-3)^2 + 3 \times 8^2 = 18 + 192 = 210$

✓ **Check 3 - the two roots against the quadratic:** Add and multiply the roots without re-solving. For $x^2 - 6x - 27 = 0$ the sum of the roots must be 6 and their product must be -27. $9 + (-3) = 6$ and $9 \times (-3) = -27$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Substitute $y = 5 - x$ into the curve's equation	M1	Substitution of $x = \pm 5 \pm y$ or $y = \pm 5 \pm x$ into $2x^2 + 3y^2 = 210$, or a correct equation formed using $x = \pm 5 \pm y$ or $y = \pm 5 \pm x$, to obtain an equation in x only or in y only. For example $2x^2 + 3(5 - x)^2 = 210$.	✓
Multiply out and collect terms	M1	For multiplying out and collecting terms, forming a three-term quadratic in any form $ax^2 + bx + c = 0$ where at least 2 of the coefficients are correct. For example $5x^2 - 30x - 135 = 0$ or $x^2 - 6x - 27 = 0$, or $5y^2 - 20y - 160 = 0$ or $y^2 - 4y - 32 = 0$.	✓
Complete method to solve their three-term quadratic	M1	For a complete method to solve their three-term quadratic: correct factorisation, for example $(x - 9)(x + 3) = 0$; or substitution into the formula, allowing one sign error and some simplification, as far as $\dfrac{6 \pm \sqrt{36 + 108}}{2}$ or $\dfrac{4 \pm \sqrt{16 + 128}}{2}$; or completing the square, for example $(x - 3)^2 - 3^2 - 27 = 0$; or for seeing $x = 9, x = -3$ or $y = 8, y = -4$. Incorrect labelling of x and y is allowed at this stage.	✓
Substitute the two found	M1ft	For substituting their 2 found values of x (or of y) into a suitable equation, for example $y = 5 - 9$ and	✓

Step	Mark	Description	Got it?
values back to get the other variable		$y = 5 - (-3)$, with use of the quadratic equation allowed; or fully correct values for the other variable. Substitution must be seen for incorrect x or y values, and the labels for x and y must be correct here.	
Both pairs of coordinates	A1	$(9, -4)$ and $(-3, 8)$. Working is required, and the mark is dependent on both of the first two method marks, so an answer written down with no algebra behind it does not earn it.	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

The expression $\dfrac{30 \times 25^{2x+7}}{\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9}}$

Every base in it is built from 5: $25 = 5^2$, $30 = 6 \times 5$ and $180 = 36 \times 5$.

Find

The whole expression as one power of 5, written 5^w. w as an expression in terms of x, so the x terms must not cancel away.

Plan the Solution

- Take the square factor out of $\sqrt{180}$, and split 30 into a whole number times 5.
- Rewrite both powers in base 5, using $\sqrt{5} = 5^{\frac{1}{2}}$.
- Add the indices on the top, and add the indices on the bottom.
- Cancel the whole-number factor, then subtract the indices to divide.

Worked Solution [3 marks]

Rule - Index laws in a single base: $(a^m)^n = a^{mn}$, $a^m \times a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$ and $\sqrt{a} = a^{\frac{1}{2}}$.

Step 1: Write 30 and $\sqrt{180}$ in terms of 5

$30 = 6 \times 5$

$$\sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5} = 6 \times 5^{\frac{1}{2}}$$

(Reason: Pulling the square factor 36 out of 180 leaves $\sqrt{5}$ behind, and it puts the same factor of 6 on the top and on the bottom.)

Step 2: Rewrite both powers in base 5

$25^{2x+7} = (5^2)^{2x+7} = 5^{4x+14}$

$$\left(\sqrt{5}\right)^{4x+9} = \left(5^{\frac{1}{2}}\right)^{4x+9} = 5^{\frac{4x+9}{2}}$$

(Reason: Raising a power to a power multiplies the two indices, and the multiplier acts on the whole of the index inside the bracket, never on part of it.)

Step 3: Collect the numerator and the denominator

$30 \times 25^{2x+7} = 6 \times 5 \times 5^{4x+14} = 6 \times 5^{4x+15}$

$$\sqrt{180} \times \left(\sqrt{5}\right)^{4x+9} = 6 \times 5^{\frac{1}{2}} \times 5^{\frac{4x+9}{2}} = 6 \times 5^{2x+5}$$

(Reason: A lone 5 counts as 5^1, so the top index is $1 + (4x+14) = 4x+15$. Underneath, $\frac{1}{2} + \frac{4x+9}{2} = \frac{4x+10}{2} = 2x+5$.)

Step 4: Divide by subtracting the indices

$$\frac{6 \times 5^{4x+15}}{6 \times 5^{2x+5}} = 5^{(4x+15)-(2x+5)}$$

$5^{(4x+15)-(2x+5)} = 5^{2x+10}$

(Reason: The factor of 6 cancels, and subtracting the indices gives $4x - 2x = 2x$ and $15 - 5 = 10$.)

5^{2x+10}, so $w = 2x + 10$

Verification

✓ **Check 1:** Put $x = 0$ into the original expression. The top is $30 \times 25^7 = 183\,105\,468\,750$ and the bottom is $6\sqrt{5} \times \left(\sqrt{5}\right)^9 = 6 \times 5^5 = 18\,750$. $\frac{183\,105\,468\,750}{18\,750} = 9\,765\,625 = 5^{10}$, and the answer gives $2(0) + 10 = 10$.

✓ **Check 2:** Group the plain numbers first: $\dfrac{30}{\sqrt{180}} = \dfrac{30}{6\sqrt{5}} = \dfrac{5}{\sqrt{5}} = \sqrt{5}$. The two powers then leave $5^{(4x+14)-\dfrac{4x+9}{2}} = 5^{\dfrac{4x+19}{2}}.5^{\frac{1}{2}} \times 5^{\dfrac{4x+19}{2}} = 5^{\dfrac{4x+20}{2}} = 5^{2x+10}$, the same answer from a completely different grouping.

✓ **Check 3:** Count the x terms and the constants separately. The top index is $4x + 15$ and the bottom index is $2x + 5$. $4x - 2x = 2x$ and $15 - 5 = 10$, so $w = 2x + 10$ still contains x, as the question requires.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Any two of the conversions into base 5: $30 = 6 \times 5$, $\sqrt{180} = 6\sqrt{5}$, $25^{2x+7} = (5^2)^{2x+7}$, $\left(\sqrt{5}\right)^{4x+9} = \left(5^{\frac{1}{2}}\right)^{4x+9}$	M1	Or equivalent: $30 = 5 \times 2 \times 3$, $\sqrt{180} = 2 \times 3 \times \sqrt{5}$, $5^{2(2x+7)}$, $5^{\frac{1}{2}(4x+9)}$, $\dfrac{30}{\sqrt{180}} = \sqrt{5}$. Also award for 5^{4x+15} as the numerator, or 5^{2x+5} as the denominator.	✓
One correct combined expression in terms of 6 (or 2 and 3) and powers of 5, for example $\dfrac{6 \times 5 \times 5^{4x+14}}{6 \times 5^{0.5} \times 5^{2x+4.5}}$	M1	Some cancelling may already have taken place, for example $\dfrac{5^{4x+15}}{5^{2x+5}}$. Also award for a fully correct expression in terms of $\sqrt{5}$ and powers, $\dfrac{\left(\sqrt{5}\right)^{8x+30}}{\left(\sqrt{5}\right)^{4x+10}}$, or in terms of 25 and powers, $\dfrac{25^{2x+7.5}}{25^{x+2.5}}$.	✓
5^{2x+10}	A1	Dependent on both method marks. Allow $w = 2x + 10$. Working must be shown.	✓

Full marks: 3/3

Question 24 - Exam Solution

Understanding the Question

Given

A quadrilateral $OACB$ with $\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OB} = 3\mathbf{b}$ and $\overrightarrow{BC} = 2\mathbf{a} + \mathbf{b}$

P lies on AC with $AP : PC = 3 : 2$

OPQ and BCQ are both straight lines

$\mathbf{a}$ and $\mathbf{b}$ are not parallel, so an $\mathbf{a}$ part can never be turned into a $\mathbf{b}$ part

Find

(a) $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, simplified

(b) $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, with the vector working shown

Plan the Solution

- For (a), travel from A to C along sides whose vectors are given: back along OA, then up OB, then along BC.
- For (b), find $\overrightarrow{OP}$ first, because $AP : PC = 3 : 2$ fixes $\overrightarrow{AP}$ as a fraction of the $\overrightarrow{AC}$ just found.
- Write $\overrightarrow{OQ}$ twice, once along each straight line, each time with its own unknown multiplier.
- Match the $\mathbf{a}$ parts and match the $\mathbf{b}$ parts. That is two equations in two unknowns, so both multipliers can be found.

Worked Solution [6 marks]

> Rule - Vector journeys and straight lines: every route between two points gives the same vector, and travelling against an arrow reverses the sign. If X, Y and Z lie on one straight line then $\overrightarrow{XZ} = k\,\overrightarrow{XY}$ for some number k.

Step 1: travel from A to C the long way round

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BC}$$

$$\overrightarrow{AC} = -4\mathbf{a} + 3\mathbf{b} + (2\mathbf{a} + \mathbf{b})$$

$$\overrightarrow{AC} = 4\mathbf{b} - 2\mathbf{a}$$

(Reason: There is no vector given straight from A to C, so go round three sides that are given. Going from A to O is against the arrow, so $4\mathbf{a}$ becomes $-4\mathbf{a}$. Then $-4\mathbf{a} + 2\mathbf{a} = -2\mathbf{a}$ and $3\mathbf{b} + \mathbf{b} = 4\mathbf{b}$.)

Step 2: find $\overrightarrow{OP}$

$$AP : PC = 3 : 2 \implies \overrightarrow{AP} = \frac{3}{5}\overrightarrow{AC}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = 4\mathbf{a} + \frac{3}{5}(4\mathbf{b} - 2\mathbf{a}) = \frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

(Reason: The ratio $3:2$ cuts AC into 5 equal parts, and AP is 3 of those parts, measured from A.)

Step 3: use the straight line OPQ

$$\overrightarrow{OQ} = \lambda\,\overrightarrow{OP}$$

$$\overrightarrow{OQ} = \lambda\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$$

(Reason: O, P and Q lie on one straight line, so $\overrightarrow{OQ}$ is parallel to $\overrightarrow{OP}$, and parallel vectors are multiples of one another. The multiplier λ is not known yet.)

Step 4: use the straight line BCQ

$$\overrightarrow{OQ} = \overrightarrow{OB} + \mu\,\overrightarrow{BC}$$

$$\overrightarrow{OQ} = 3\mathbf{b} + \mu(2\mathbf{a} + \mathbf{b})$$

(Reason: B, C and Q lie on one straight line, so $\overrightarrow{BQ}$ is a multiple of $\overrightarrow{BC}$. This is a second, completely separate way of reaching Q.)

Step 5: match the two expressions

$$\frac{14}{5}\lambda = 2\mu$$

$$\frac{12}{5}\lambda = 3 + \mu$$

(Reason: Both expressions are the same vector $\overrightarrow{OQ}$. Since $\mathbf{a}$ and $\mathbf{b}$ are not parallel, they can only be equal if the $\mathbf{a}$ parts agree and the $\mathbf{b}$ parts agree.)

Step 6: solve for λ

$$\mu = \frac{7}{5}\lambda$$

$$\frac{12}{5}\lambda = 3 + \frac{7}{5}\lambda$$

$$\lambda = 3$$

(Reason: Halving the first equation gives μ in terms of λ. Substituting it into the second removes μ, and $\frac{12}{5} - \frac{7}{5} = 1$, which leaves $\lambda = 3$ directly.)

Step 7: put $\lambda = 3$ back in

$$\overrightarrow{OQ} = 3\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$$

$\overrightarrow{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$

(Reason: Multiply each part by 3*. Written as decimals this is* $8.4\mathbf{a} + 7.2\mathbf{b}$*, which is the same answer in a different form.)*

$$\textbf{(a) } \overrightarrow{AC} = 4\mathbf{b} - 2\mathbf{a}$$

$$\textbf{(b) } \overrightarrow{OQ} = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$$

Verification

✓ **Check 1:** Test part (a) on the other route round the quadrilateral. If $\overrightarrow{AC}$ is right then $\overrightarrow{OA} + \overrightarrow{AC}$ must equal $\overrightarrow{OB} + \overrightarrow{BC}$, because both are $\overrightarrow{OC}$. $4\mathbf{a} + (4\mathbf{b} - 2\mathbf{a}) = 2\mathbf{a} + 4\mathbf{b}$ and $3\mathbf{b} + (2\mathbf{a} + \mathbf{b}) = 2\mathbf{a} + 4\mathbf{b}$

✓ **Check 2:** Test part (b) on the line that was not used to find λ. From $\mu = \frac{7}{5}\lambda$ with $\lambda = 3$ comes $\mu = \frac{21}{5}$. Put that into the BCQ expression from Step 4.

$3\mathbf{b} + \frac{21}{5}(2\mathbf{a} + \mathbf{b}) = \frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$

✓ **Check 3:** Replace the letters with numbers and do the whole thing again as coordinates. Take $\mathbf{a} = (3, 1)$ and $\mathbf{b} = (-1, 2)$, which are not parallel. Then $A = (12, 4)$, $B = (-3, 6)$, $C = (2, 10)$ and $P = (6, 7.6)$. Find where the line through O and P crosses the line through B and C. The two lines cross at $(18, 22.8)$, and the answer rebuilds that point exactly: $8.4 \times 3 + 7.2 \times (-1) = 18$ and $8.4 \times 1 + 7.2 \times 2 = 22.8$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A complete vector route from A to C	M1	eg $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BC}$ or eg $-4\mathbf{a} + 3\mathbf{b} + 2\mathbf{a} + \mathbf{b}$	✓
(a) The simplified answer	A1	$4\mathbf{b} - 2\mathbf{a}$ or equivalent, but it must be simplified, eg $-2\mathbf{a} + 4\mathbf{b}$ or $2(2\mathbf{b} - \mathbf{a})$. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Step	Mark	Description	Got it?
(b) An expression for $\overrightarrow{OP}$	**M1ft**	Follow through their $\overrightarrow{AC}$. eg $\overrightarrow{OP} = 4\mathbf{a} + \frac{3}{5}(4\mathbf{b} - 2\mathbf{a})$ or eg $\overrightarrow{OP} = 3\mathbf{b} + 2\mathbf{a} + \mathbf{b} - \frac{2}{5}(4\mathbf{b} - 2\mathbf{a})$, either giving $\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$ or $2.8\mathbf{a} + 2.4\mathbf{b}$. It may appear as part of another vector equation.	✓
(b) One correct expression for $\overrightarrow{OQ}$ or $\overrightarrow{PQ}$	**M1ft**	Follow through their $\overrightarrow{AC}$, or equivalent. eg $\overrightarrow{OQ} = \lambda\left(\frac{14}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}\right)$ or $\overrightarrow{OQ} = 3\mathbf{b} + \mu(2\mathbf{a} + \mathbf{b})$ or $\overrightarrow{OQ} = 4\mathbf{b} + 2\mathbf{a} + \omega(2\mathbf{a} + \mathbf{b})$ or $\overrightarrow{PQ} = k(2.8\mathbf{a} + 2.4\mathbf{b})$. This mark can be awarded even if the previous mark was not.	✓
(b) Two correct expressions for $\overrightarrow{OQ}$ or $\overrightarrow{PQ}$	**M1ft**	Two of the expressions above, or equivalent, follow through their $\overrightarrow{AC}$. Two are needed because one alone still has an unknown multiplier in it.	✓
(b) The final vector	**A1**	$\frac{42}{5}\mathbf{a} + \frac{36}{5}\mathbf{b}$ or equivalent, eg $8.4\mathbf{a} + 7.2\mathbf{b}$. Working is required, and this mark depends on both method marks in part (b).	✓

Full marks: 6/6

Question 25 - Exam Solution

Understanding the Question

Given

The curve $y = 2\sin(x + 60)^\circ$, sketched on a grid with no numbers on either axis

Find

The coordinates of A The coordinates of B

The point A at the top of the first hill, and the point B where the curve crosses the x -axis after the lowest point

Plan the Solution

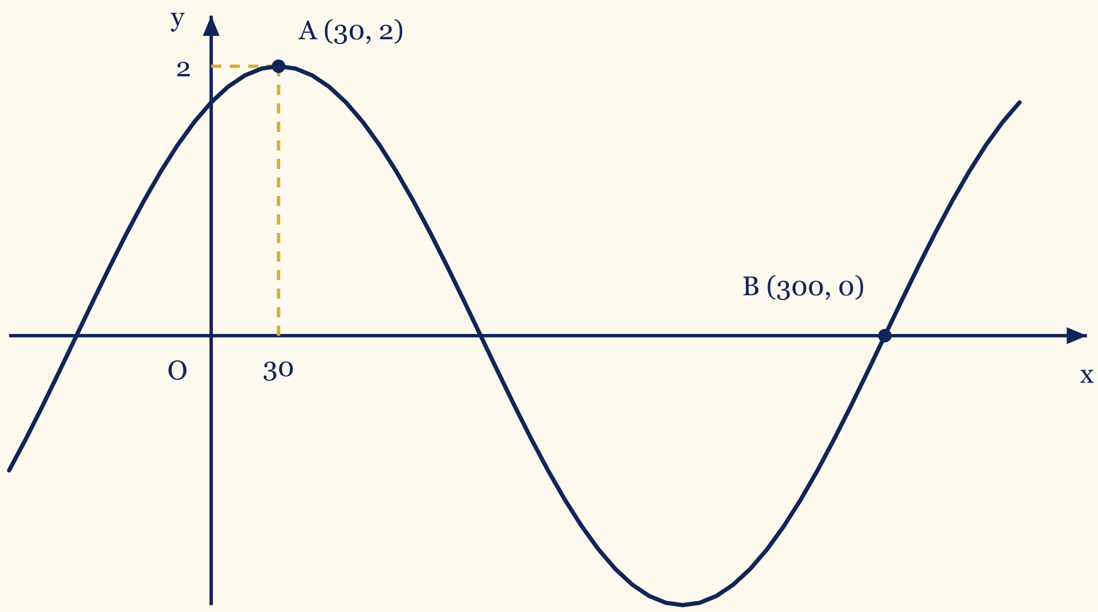
- Treat the whole bracket $x + 60$ as the angle. Whatever a plain sine does at some angle, this curve does when the bracket reaches that angle.
- A sine never rises above 1, so the greatest height here is 2×1. Find the x that puts the bracket at 90.
- For B, put $y = 0$, list every angle in view whose sine is 0, and pick the crossing the sketch marks.

Worked Solution [2 marks]

> For $y = a\sin(x + c)^\circ$ the greatest value is a, reached when $x + c = 90$. The curve meets the x-axis when $x + c = 0,\ 180,\ 360, \ldots$

Step 1: Read what the sketch is telling you

$y = 2\sin(x + 60)^\circ$



(Reason: The dot at A sits at the top of the curve, so A is where y is as large as it gets. The dot at B sits on the x-axis, so at B the value of y is 0.)

Step 2: The x-coordinate of A

$\sin(x + 60)^\circ = 1$

$x + 60 = 90$

$x = 90 - 60 = 30$

(Reason: A sine reaches its largest value, 1, when its own angle is 90. Here the angle is the whole bracket, so set the bracket equal to 90 and move the 60 across.)

Step 3: The y-coordinate of A

$y = 2 \times 1 = 2$

$A = (30, 2)$

(Reason: At the top of the hill the sine is exactly 1, so the height is just the 2 in front of it. That 2 is how far the curve reaches above and below the x-axis.)

Step 4: The coordinates of B

$2\sin(x + 60)^\circ = 0$

$x + 60 = 0,\ 180,\ 360$

$x = -60,\ 120,\ 300$

$B = (300, 0)$

(Reason: A sine is 0 at 0, 180 and 360, so those three angles give the three crossings in view. The sketch marks B on the way back up, after the lowest point, which is the third of them.)

(i) $A = (30, 2)$

(ii) $B = (300, 0)$

Verification

✓ **Check 1:** Put $x = 30$ back into the equation. The bracket becomes $30 + 60 = 90$, and the sine of 90 is 1. $2 \times 1 = 2$, which is the height the sketch draws A at.

✓ **Check 2:** Put $x = 300$ back in. The bracket becomes $300 + 60 = 360$, and the sine of 360 is 0. $2 \times 0 = 0$, so B really does sit on the x-axis.

✓ **Check 3:** Read the curve as $y = 2\sin x^\circ$ slid 60 to the left. A plain $2\sin x^\circ$ peaks at 90 and rises through zero at 360. $90 - 60 = 30$ and $360 - 60 = 300$, which is both answers over again from a different direction.

✓ **Check 4:** One full wave is 360, and a peak sits three quarters of a wave before the next rising crossing, which is 270. $300 - 30 = 270$, so the two answers are the right distance apart.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) The coordinates of A	**B1**	cao for $(30, 2)$. Both coordinates are needed, and in that order.	✓
(ii) The coordinates of B	**B1**	cao for $(300, 0)$. Correct answer only, so 300 on its own does not score.	✓

Full marks: 2/2

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