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Edexcel IGCSE 4MA1

4MA1/2HR (Higher Tier) – Monday 3 June 2024

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/2HR (Higher Tier), June 2024 series, sat Monday 3 June 2024 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

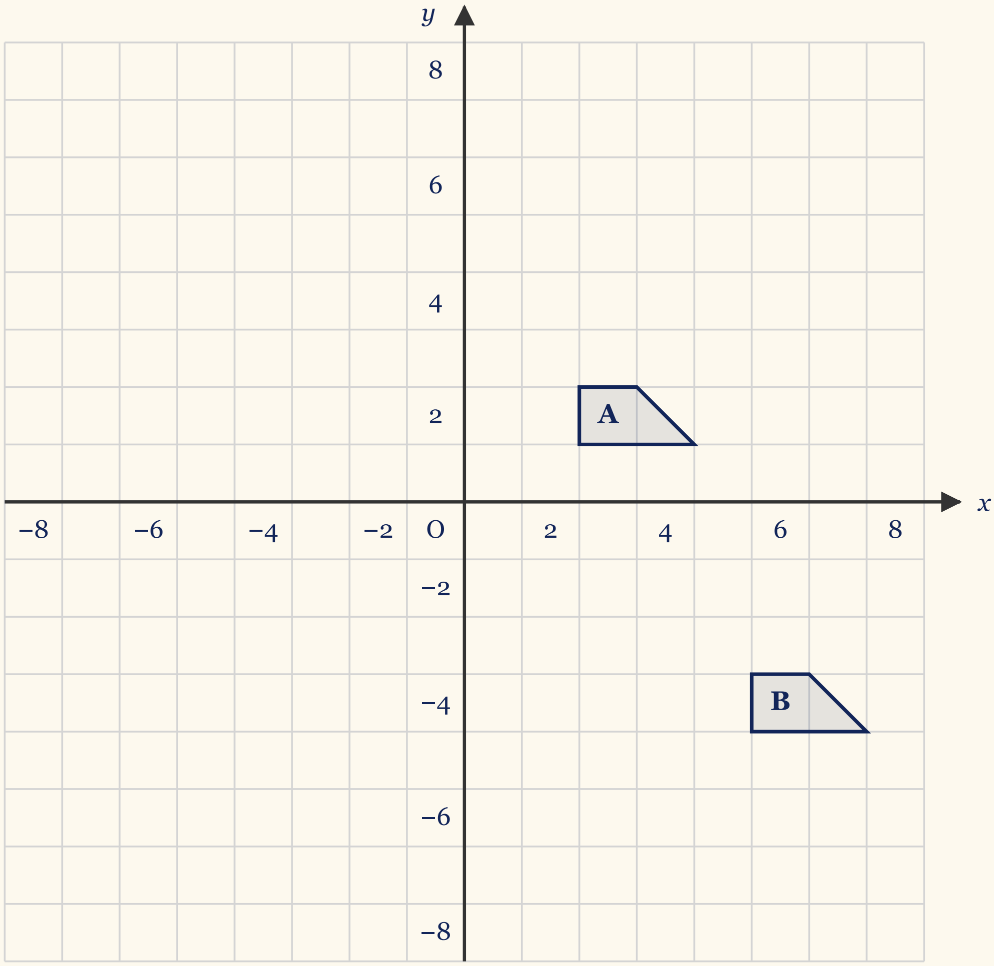
Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Express 1 400 as a product of powers of its prime factors.
You must show your working clearly. *[3 marks]*

....

[Total 3 marks]

2. The grid shows shape A and shape B.



(a) Describe fully the single transformation that maps shape A onto shape B. *[2 marks]*

(b) On the grid above, rotate shape A through 180° about $(-1, 0)$. Label your shape C. *[2 marks]*

(a) ....

[Total 4 marks]

3. These four numbers are written in order of size, smallest first.

$x \qquad x \qquad y \qquad 15$

Here x and y are integers.

For these four numbers,
the median is 12.5
the range is 4

Work out the value of x and the value of y. *[2 marks]*

x = ...

y = ...

[Total 2 marks]

4. $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{factors of } 6\}$
$B = \{\text{prime numbers}\}$

(a) List all the members of the set
(i) $A \cup B$
[1 mark]
(ii) A' *[1 mark]*

Ravinder claims that $A \cap B = \varnothing$
Ravinder is wrong.
(b) Explain why. *[1 mark]*

C is a set with 4 members, and
$A \cap C$ has 2 members
$B \cap C$ has 2 members
No member belongs to both $A \cap C$ and $B \cap C$.
(c) List all 4 members of set C *[2 marks]*

(a)(i) ....

(a)(ii) ....

(b) ....

(c) ....

[Total 5 marks]

5. A craft studio makes hanging ornaments from copper wire.
The diagram shows the design for one ornament.
The design is a circle inside a square $ABCD$

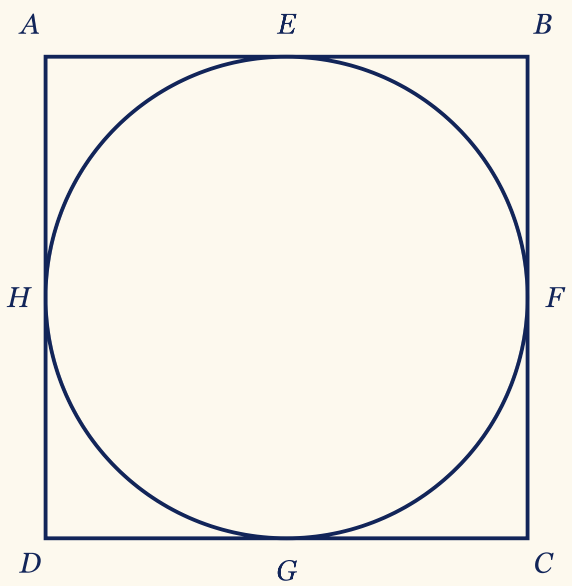


Diagram NOT accurately drawn

The circle touches the square at the points E, F, G and H

The area of the square is 81 cm²

Work out the total length of copper wire needed to make the square and the circle.
Give your answer correct to 3 significant figures. *[4 marks]*

.... cm

[Total 4 marks]

6. (a) Solve the equation $\frac{2f}{3} = 4f - 17$

You must show clear algebraic working. *[3 marks]*

(b) Simplify $(e + 12)^0$, given that $e > 0$ *[1 mark]*

(c) Simplify fully $\frac{12a^4h^6}{4ah^2}$ *[2 marks]*

(d) Factorise fully $20x^5y + 12x^3y^4$ *[2 marks]*

(a) f = ...

(b) ....

(c) ....

(d) ....

[Total 8 marks]

7. $\frac{3^{-2} \times 3^5}{3^{10}} = 3^n$

Work out the value of n *[2 marks]*

n = ...

[Total 2 marks]

8. In a clearance sale at an electrical goods shop, all normal prices are reduced by 17%

The sale price of a table fan is $6\,225$ rupees.

Work out the normal price of the table fan. *[3 marks]*

.... rupees

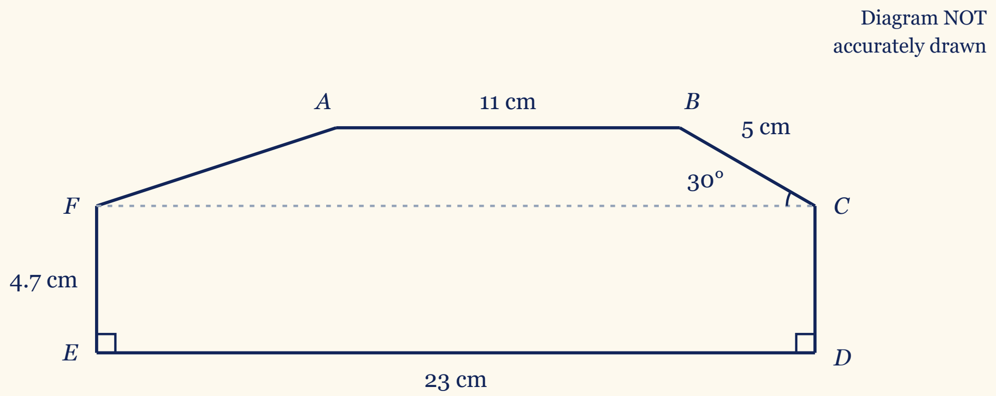
[Total 3 marks]

9. (a) Express 6.04×10^5 as an ordinary number.
[1 mark]
(b) Express $0.000\,07$ in standard form.
[1 mark]
(c) Work out $\dfrac{7.6 \times 10^{10}}{4 \times 10^5 - 2 \times 10^4}$
Give your answer in standard form. *[2 marks]*

(a) ....
(b) ....
(c) ....

[Total 4 marks]

10. $ABCDEF$ is a hexagon.
The measurements of the hexagon are shown on the diagram.



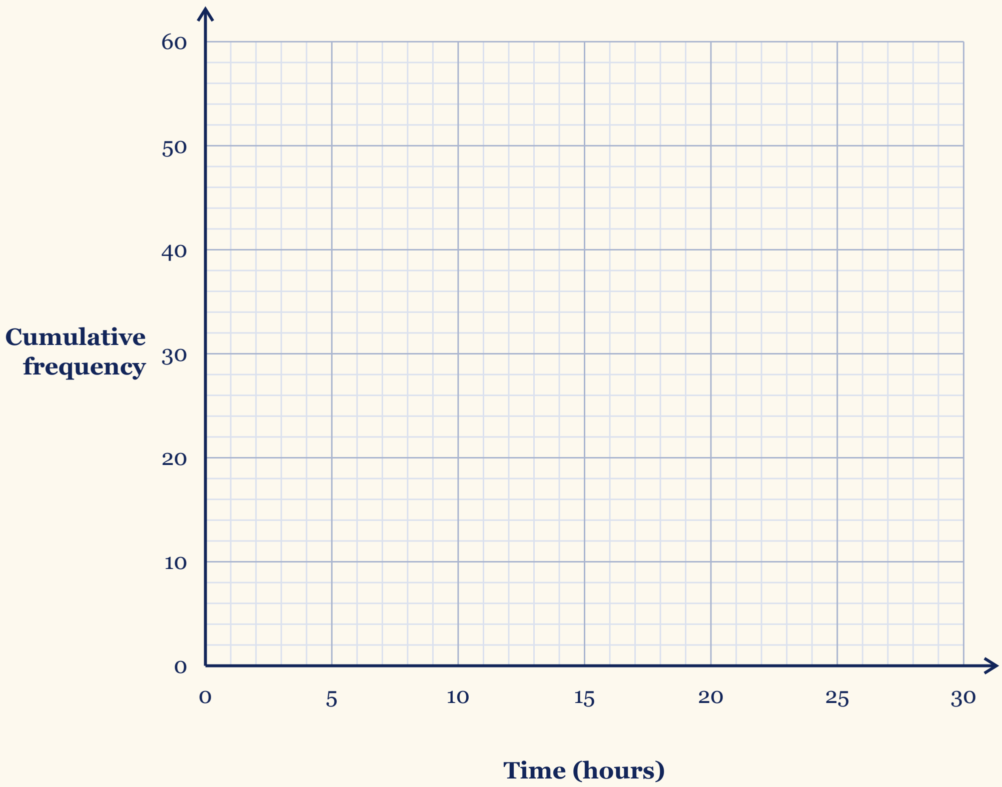
The lines AB, FC and ED are parallel, and angle BCF is 30°.

Work out the area of $ABCDEF$.
You must show all your working. *[5 marks]*

.... cm²

[Total 5 marks]

11. The cumulative frequency table gives information about the time, in hours, that each of 60 members of a swimming club spent training in one week.



Time (t hours)	Cumulative frequency
$0 < t \leq 5$	6
$0 < t \leq 10$	17
$0 < t \leq 15$	27
$0 < t \leq 20$	42
$0 < t \leq 25$	53
$0 < t \leq 30$	60

(a) On the grid below, draw a cumulative frequency graph for the information in the table. *[2 marks]*

(b) Use your graph to find an estimate for the interquartile range of the times. *[2 marks]*

25 members spent more than W hours training.

(c) Use your graph to find an estimate for the value of W *[2 marks]*

One of the 60 members is chosen at random.
This member spent H hours training.

(d) Find the probability that $5 < H \leq 10$ *[1 mark]*

(b) hours ....

(c) W = ...

(d) ....

[Total 7 marks]

12. The diagram shows triangle ACD.
B is a point on AC and E is a point on AD, so that ABC and AED are straight lines.
BE is parallel to CD

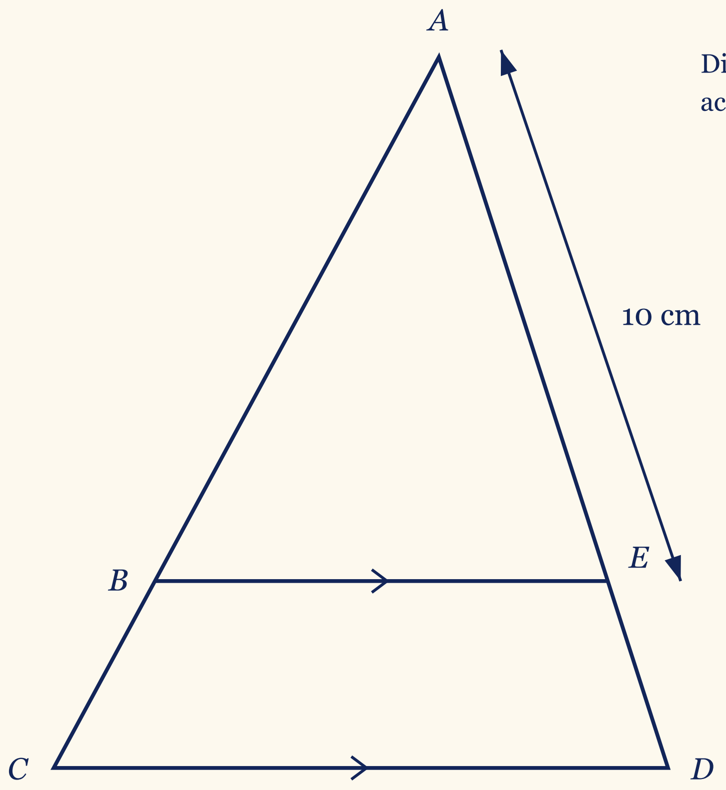


Diagram NOT accurately drawn

$AE = 10$ cm and $CD = 1.5 \times BE$

(a) Work out the length of ED *[2 marks]*

$AB = (2x + 5)$ cm and $BC = (3x - 5)$ cm

(b) Work out the value of x *[2 marks]*

(a) cm ....

(b) x = ...

[Total 4 marks]

13. OAB is a sector of a circle. The centre of the circle is O and its radius is r cm.

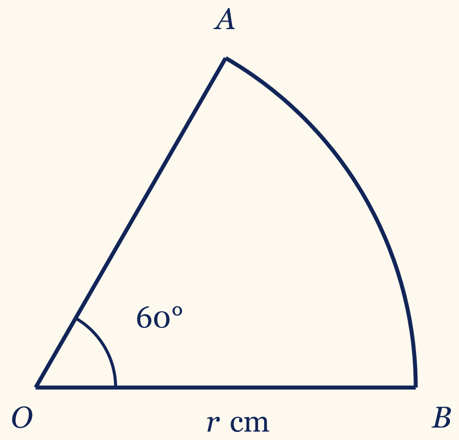


Diagram NOT accurately drawn

Angle $AOB = 60°$
The perimeter of the sector is P cm.

Work out a formula for P in terms of r.
Write your answer in the form $P = r(c\pi + k)$, where c and k are numbers to be found. *[3 marks]*

....

[Total 3 marks]

14. Camila is going to spin a biased spinner and drop a bent drawing pin.
The spinner has six sections, numbered 1 to 6.
The drawing pin will land either point up or point down.

The probability that the drawing pin will land point up is 0.8
The probability that the spinner will land on 6 and the drawing pin will land point up is 0.24

Work out the probability that the spinner will land on 6 and the drawing pin will land point down. *[3 marks]*

....

[Total 3 marks]

15. The diagram shows three sides of a regular pentagon together with a triangle.

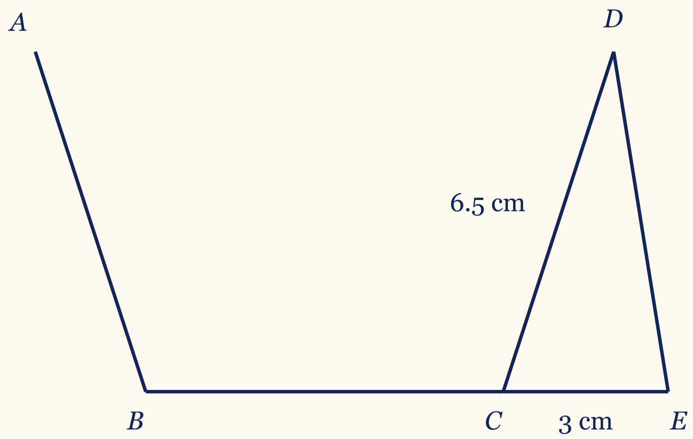


Diagram NOT accurately drawn

AB, BC and CD are three sides of a regular pentagon and CDE is a triangle.
BCE is a straight line.

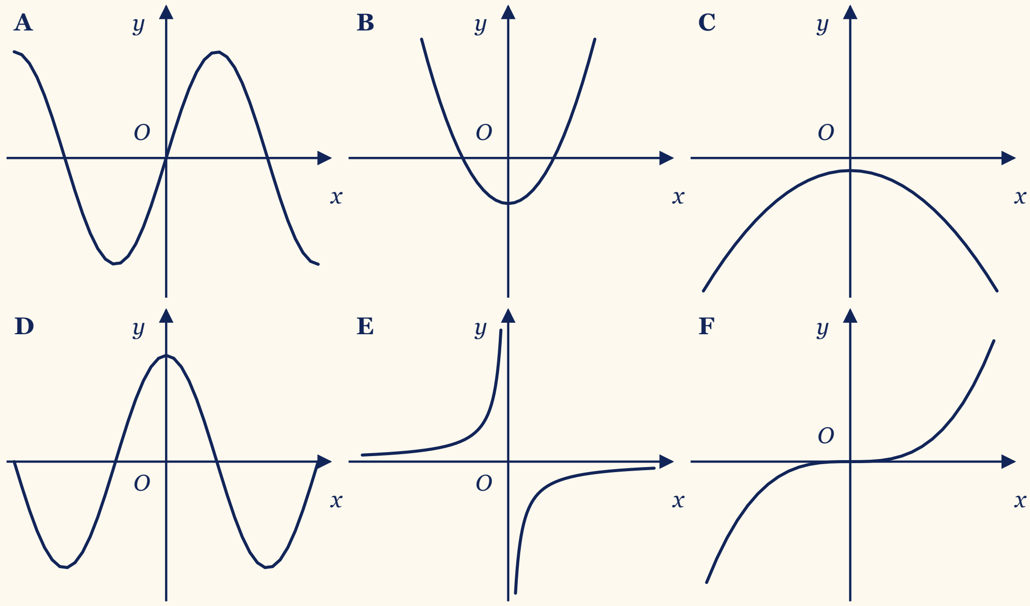
$CD = 6.5$ cm
$CE = 3$ cm

Calculate the area of triangle CDE.
Give your answer correct to 3 significant figures. *[3 marks]*

.... cm²

[Total 3 marks]

16. Six graphs are sketched below, labelled A to F.



For each equation, write down the letter of the graph that could have that equation.

(i) $y = -\dfrac{1}{x}$ *[1 mark]*

(ii) $y = \sin x^\circ$ *[1 mark]*

(i) ....

(ii) ....

[Total 2 marks]

17. The functions f and g are given by

$$\mathrm{f}(x) = \frac{x}{2x-4}$$

$$\mathrm{g}(x) = 3x + 1$$

Given that $\mathrm{fg}(k) = 2$

work out the value of k *[3 marks]*

k = ...

[Total 3 marks]

18. The recurring decimal $0.\dot{3}0\dot{6}$ can be written as the fraction $\frac{34}{111}$.

Use algebra to show that this is correct.
You must show clear algebraic working. *[2 marks]*

[Total 2 marks]

19. Noam goes on an electric scooter ride along a coastal path.

For the scooter ride

average speed $=19$ km/h correct to the nearest whole number
time $=1.5$ hours correct to one decimal place

Work out the upper bound for the distance Noam travels.
Give your answer correct to 3 significant figures. *[3 marks]*

.... km

[Total 3 marks]

20. Solve the inequality $6x^2-7x-20>0$
You must show clear algebraic working. *[4 marks]*

....

[Total 4 marks]

21. $ABCD$ is a square.

A is the point $(-5, 2)$
B is the point $(3, 5)$

Work out an equation of the line that passes through B and C
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. *[4 marks]*

....

[Total 4 marks]

22. Solve the simultaneous equations
$x^2 + y^2 = y + 11$
$y = 3x - 1$
You must show clear algebraic working. *[5 marks]*

....

[Total 5 marks]

23. The curve C has equation $y = \mathrm{f}(x)$
The minimum point of C has coordinates $(6, -3)$

Write down the coordinates of the minimum point of the curve with equation

(i) $y = \mathrm{f}(x) + 10$ *[1 mark]*

(ii) $y = \mathrm{f}(3x)$ *[1 mark]*

(i) ....

(ii) ....

[Total 2 marks]

24. The diagram shows a solid, S, formed by joining a cone to a hemisphere.

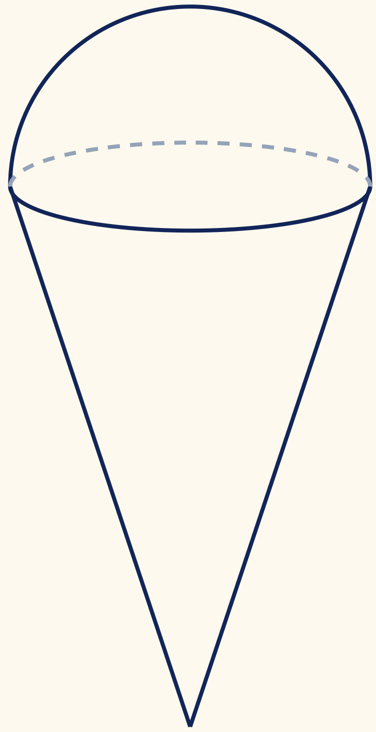


Diagram NOT accurately drawn

The circular face of the cone and the flat face of the hemisphere have the same centre.

The radius of the circular face of the cone is x cm, and it is equal to the radius of the hemisphere.

The total height of S is 4 times the radius of the hemisphere.

A different sphere has radius kx cm.
The volume of this sphere is 12.5 times the volume of S

(a) Work out the value of k *[4 marks]*

A solid, T, is mathematically similar to solid S

The volume of T is 512 times the volume of S

The total surface area of T is d times the total surface area of S

(b) Find the value of d *[1 mark]*

(a) k = ...

(b) d = ...

[Total 5 marks]

25. $OPQR$ is a parallelogram.

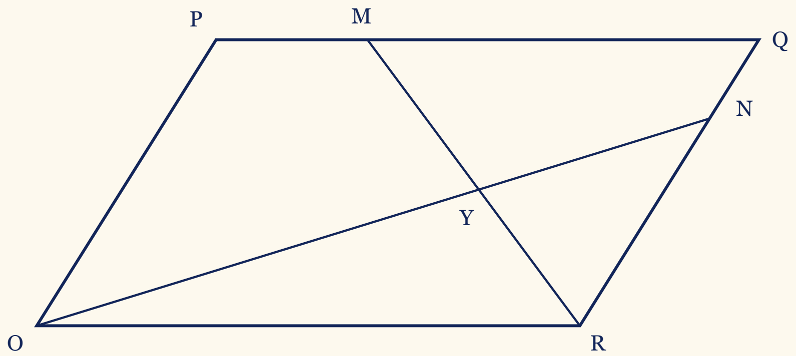


Diagram NOT accurately drawn

$\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$

The point M lies on PQ so that $PM = \frac{1}{4}PQ$

The point N lies on RQ so that $RN = \frac{4}{5}RQ$

(a) Work out, in terms of **a** and **b**, giving each answer in its simplest form

(i) $\overrightarrow{ON}$

[1 mark]

(ii) $\overrightarrow{MR}$ *[1 mark]*

MR and ON cross at the point Y.

Given that

$OY = k \times ON$

(b) use a vector method to work out the value of k. *[4 marks]*

(a)(i) ....

(a)(ii) ....

k = ...

[Total 6 marks]

26.

Express $4 - \left[\dfrac{3x-5}{\dfrac{3x^2+x-10}{4x-1}}\right]$ as a single fraction in its simplest form.

[4 marks]

....

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The number $1\,400$.
Working has to be shown, so the marks are for the factorising and not only for the last line.

Find

$1\,400$ written as a product of powers of its prime factors. The answer must be in index form, so equal primes are collected into one power each.

Plan the Solution

- Divide by the smallest prime that goes in exactly, then divide the quotient again, and keep going until the quotient reaches one.
- Start with 2, because $1\,400$ is even. Move up to the next prime only when the one being used stops dividing exactly.
- Write the primes out as a full list first, then collect the equal ones into powers.
- Multiply the powers back together, and factorise from a different starting split, to check.

Worked Solution [3 marks]

> Rule - Prime factorisation: divide by the smallest prime that goes in exactly, keep dividing each new quotient until it reaches 1, then collect equal primes into powers.

Step 1: take out the twos

$1\,400 = 2 \times 700$

$700 = 2 \times 350$

$350 = 2 \times 175$

(Reason: Each of these is even, so 2 divides it exactly. The quotient 175 is odd, so this is where the twos stop.)

Step 2: move up to the next prime that divides

$175 = 5 \times 35$

$35 = 5 \times 7$

(Reason: 175 is odd, and its digits add to 13, so neither 2 nor 3 divides it. It ends in 5, so 5 does.)

Step 3: stop, and write the full list

$1\,400 = 2 \times 2 \times 2 \times 5 \times 5 \times 7$

(Reason: The last quotient is 7, which is prime, so the dividing is finished. The list holds the six primes that were divided out, in the order they came out.)

Step 4: collect equal primes into powers

$2 \times 2 \times 2 = 2^3$

$5 \times 5 = 5^2$

$1\,400 = 2^3 \times 5^2 \times 7$

(Reason: There are three 2s and two 5s, so each becomes a power. A prime that appears once, here the 7, is written with no index.)

$$1\,400 = 2^3 \times 5^2 \times 7$$

Verification

✓ **Check 1:** Multiply the powers back together: $2^3 = 8$ and $5^2 = 25$. $8 \times 25 = 200$, and $200 \times 7 = 1\,400$, the number the question started from.

✓ **Check 2:** Start from a completely different split, $1\,400 = 14 \times 100$, and factorise each piece on its own. $14 = 2 \times 7$ and $100 = 2^2 \times 5^2$, which between them give three 2s, two 5s and one 7 again.

✓ **Check 3:** Count what the indices claim. They should add up to the number of primes in the list, and every base should itself be prime. $3 + 2 + 1 = 6$, which is the length of the list, and 2, 5 and 7 are all prime, with no factor of 1 left in.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two correct stages of the factorisation, for example $1\,400 = 2 \times 700$ and then $700 = 2 \times 350$.	M1	Award for 2 correct stages with no incorrect stage, or for at least 3 stages with no more than one incorrect stage. Each stage gives two factors and may be a list, a factor tree or a ladder, so 2, 2, 350 earns it. Other correct openings include $2 \times 7 \times 100$, $2 \times 5 \times 140$, $5 \times 7 \times 40$ and $5 \times 5 \times 56$.	✓

Step	Mark	Description	Got it?
The complete list of prime factors, $2 \times 2 \times 2 \times 5 \times 5 \times 7$.	**M1 dep**	Dependent on the first M1. Award for $2 \times 2 \times 2 \times 5 \times 5 \times 7$, or for the list 2^3, 5^2, 7 , or for $2^3 + 5^2 + 7$. Ignore any 1s. It may be seen inside a fully correct factor tree or ladder rather than written out on the answer line.	✓
The answer in index form, $2^3 \times 5^2 \times 7$.	**A1**	Dependent on both method marks. Any order is accepted, and $2^3 \cdot 5^2 \cdot 7$ is accepted, but the answer must be in index form because that is what the question asks for. Do not accept a 1 in the final answer.	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

Shape A has vertices $(2, 1)$, $(2, 2)$, $(3, 2)$ and $(4, 1)$
Shape B has vertices $(5, -4)$, $(5, -3)$, $(6, -3)$ and $(7, -4)$
Both shapes are drawn on the same grid, and part (b) gives the centre $(-1, 0)$ and the angle $180°$

Find

(a) the single transformation that maps A onto B, described fully (b) the image of shape A after that rotation, drawn on the grid and labelled C

Plan the Solution

- Part (a): line the vertices up in pairs. Same size, same way round and same way up rules out rotation, reflection and enlargement, so it is a translation; subtracting a pair of coordinates gives the vector.
- Describe fully means the name of the transformation AND everything that pins it down. For a translation that is the vector, and nothing else.
- Part (b): a rotation of $180°$ about (a, b) sends (x, y) to $(2a - x,\ 2b - y)$. Apply it to each vertex in turn, plot the four images and join them in the same order.

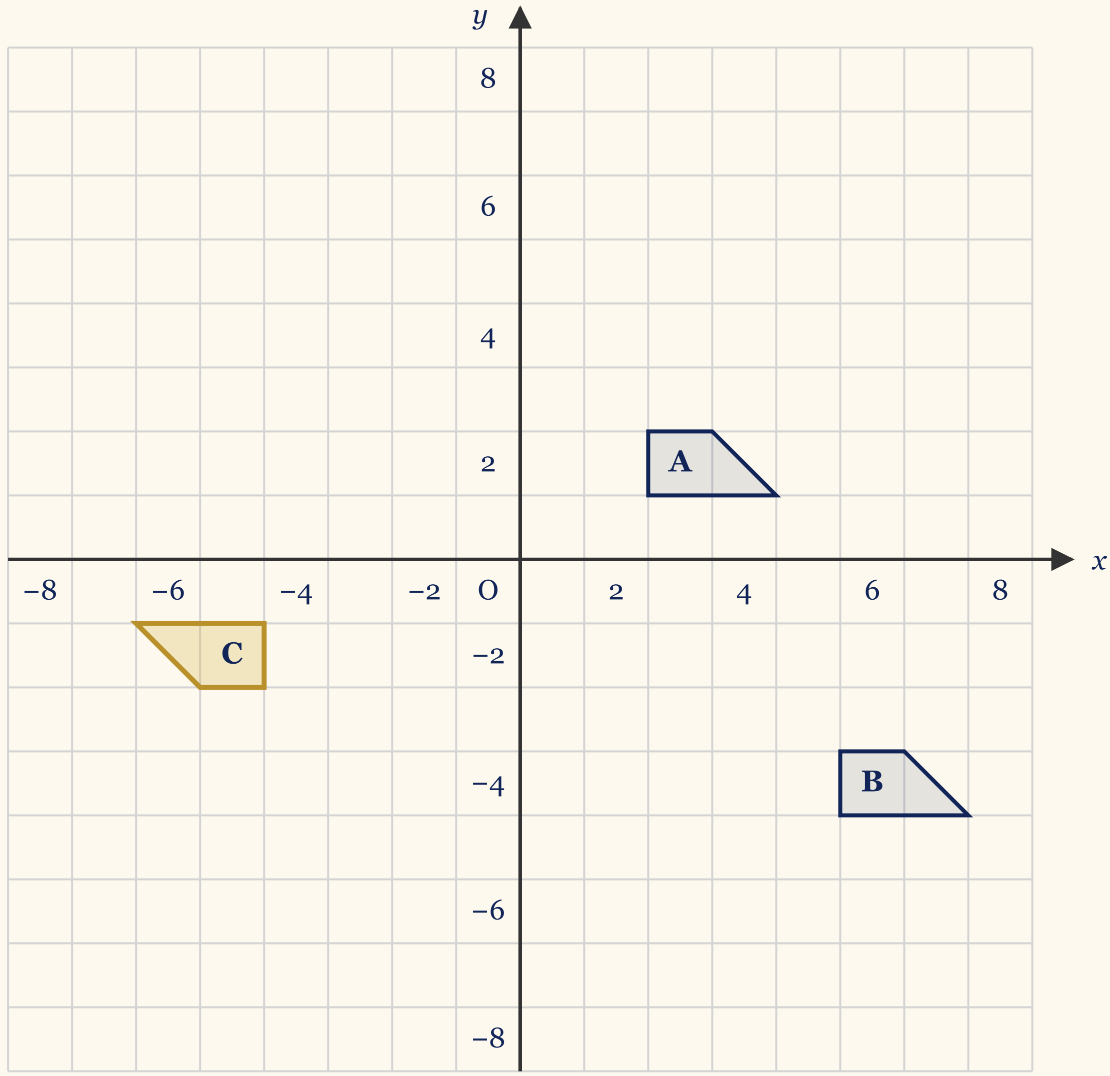
Worked Solution [4 marks]

Rule - Translation: a translation is described by one column vector, the top number the movement in x and the bottom number the movement in y. Rule - Half turn: a rotation of 180° about (a, b) maps (x, y) to $(2a - x,\ 2b - y)$, so the centre is the midpoint of every point and its image.

Step 1: Pair the vertices of A with the vertices of B

$A : (2, 1),\ (2, 2),\ (3, 2),\ (4, 1)$

$B : (5, -4),\ (5, -3),\ (6, -3),\ (7, -4)$



(Reason: Both shapes are trapeziums of the same size, the same way round and the same way up, so nothing has been turned, flipped or resized. A translation is the only single transformation left.)

Step 2: Count the move on one pair of matching vertices

$5 - 2 = 3$

$-4 - 1 = -5$

(Reason: The vertex $(2, 1)$ of shape A goes to $(5, -4)$ of shape B. The x-coordinate goes up by 3, which is 3 squares right; the y-coordinate goes down by 5, which is 5 squares down.)

Step 3: Check the same move on a second vertex

$$4 + 3 = 7$$
$$1 - 5 = -4$$

(Reason: The vertex $(4, 1)$ lands on $(7, -4)$, which is the matching vertex of shape B. Every vertex moves by the same amount, and that is what makes it one single translation.)

Step 4: Write the translation as a column vector

$$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

(Reason: The top number is the movement in the x-direction, the bottom number the movement in the y-direction, and a negative bottom number means downwards. The word and the vector are worth a mark each, so an answer with only one of them scores 1 out of 2.)

Step 5: Write down what a rotation of 180° about $(-1, 0)$ does

$$(x, y) \to (2 \times (-1) - x,\ 2 \times 0 - y)$$
$$(x, y) \to (-2 - x,\ -y)$$

(Reason: A half turn sends every point to the point the same distance the other side of the centre, so the centre sits exactly halfway between a point and its image. Doubling the centre and subtracting the point is that sentence written as arithmetic, one coordinate at a time. Note that the y-coordinate is negated as well as the x-coordinate.)

Step 6: Turn each vertex of shape A

$$(2, 1) \to (-4, -1)$$
$$(2, 2) \to (-4, -2)$$
$$(3, 2) \to (-5, -2)$$
$$(4, 1) \to (-6, -1)$$

(Reason: Take $(2, 1)$ as the pattern: $-2 - 2 = -4$ for the x-coordinate and -1 for the y-coordinate. Work through the vertices in the order they are joined, so the images can be joined in that order too.)

Step 7: Plot the four images, join them and label the shape C

$$C : (-6, -1),\ (-4, -1),\ (-4, -2),\ (-5, -2)$$

(Reason: Joining the images in the same order as the vertices of shape A puts the sloping side of C on the far side from the sloping side of A, which is what a half turn does. The label is part of the instruction, although the mark scheme condones it being missing.)

(a) Translation by the vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$

(b) Shape C drawn at $(-6,-1),\ (-4,-1),\ (-4,-2),\ (-5,-2)$

Verification

✓ **Check 1:** Add the vector to every vertex of shape A: $(2,1) \to (5,-4)$, $(2,2) \to (5,-3)$, $(3,2) \to (6,-3)$, $(4,1) \to (7,-4)$. the four vertices of shape B, so the vector maps the whole shape and not just one corner

✓ **Check 2:** The centre of a rotation is the midpoint of a point and its image. Halfway between $(2,2)$ and $(-4,-2)$ is $\left(\dfrac{2+(-4)}{2},\ \dfrac{2+(-2)}{2}\right)$. $(-1,0)$, the centre the question gives

✓ **Check 3:** Turn shape C through 180° about $(-1,0)$ as well. A half turn undoes a half turn, so the images should come back to where they started, and the four sides should still measure 1, 1, $\sqrt{2}$ and 2. shape A again, side for side

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Translation	**B1**	for translation, with none of reflection, rotation, enlargement, mirrored, turned or flipped also stated. Allow translated, translating, translate, and a misspelling such as translat. Move with translation is acceptable.	✓
(a) (vector =) $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$	**B1**	for the vector. Both marks are needed for a full description: the word on its own, or the vector on its own, scores 1.	✓
(b) Shape drawn at $(-6,-1),\ (-4,-1),\ (-4,-2),\ (-5,-2)$	**B2**	condone a missing label C.	✓

Step	Mark	Description	Got it?
(b) Partial credit	**B1**	if not B2, then B1 for a correct trapezium drawn with the correct orientation in the wrong position, or for 3 points plotted correctly. The wrong position usually comes from turning about the origin instead of about $(-1, 0)$.	✓

Full marks: 4/4

Question 3 - Exam Solution

Understanding the Question

Given

The four numbers x, x, y, 15, already written in ascending order of size.
Both x and y are integers.
The median is 12.5 and the range is 4.

Find

The value of x and the value of y. Two numbers are wanted, and the paper prints a separate answer line for each of them.

Plan the Solution

- The list is already in order, so the smallest number is x and the largest is 15. The range is built from those two alone, so it gives x by itself. Start there.
- A list of four numbers has no single middle number, so the median is the mean of the middle two, x and y. Use that second, once x is known.
- Finish by writing the four numbers out in full and reading the median and the range back off them.

Worked Solution [2 marks]

Rule - Median and range of an ordered list: for four numbers in order the median is the mean of the middle two, and the range is the largest minus the smallest.

Step 1: read what the order already tells you

$x \leq x \leq y \leq 15$

(Reason: The four numbers are written in ascending order of size, so the smallest is x and the largest is 15. The middle two, the ones the median is built from, are x and y.)

Step 2: use the range to find x

$15 - x = 4$

$x = 15 - 4 = 11$

(Reason: The range is the largest minus the smallest, which here is $15 - x$. That equation holds only x, so it can be solved on its own, before anything is known about y.)

Step 3: use the median to find y

$\frac{x + y}{2} = 12.5$

$x + y = 2 \times 12.5 = 25$

$y = 25 - 11 = 14$

(Reason: With four numbers the median is the mean of the middle two, so multiplying the median by 2 gives the total of that middle pair. Taking the known x away from 25 leaves y.)

Step 4: write the list out and check it is a legal list

$11 \quad 11 \quad 14 \quad 15$

(Reason: The question sets two conditions that are easy to forget: the numbers are integers, and they rise from left to right. Both hold here, so this list is a legitimate answer to the question as it is set.)

$$x = 11$$

$$y = 14$$

Verification

✓ **Check 1:** Read the range straight off the finished list $11, 11, 14, 15$: largest minus smallest. $15 - 11 = 4$, which is the range the question gives.

✓ **Check 2:** Read the median off the same list. The two middle numbers are 11 and 14. $\frac{11 + 14}{2} = \frac{25}{2} = 12.5$, which is the median the question gives.

✓ **Check 3:** Test the pair the other way round, $x = 14$ and $y = 11$, to confirm that only one pair fits. The list would then read $14 \quad 14 \quad 11 \quad 15$, which is not in ascending order of size, so it is rejected. The ordering is the condition that separates the two pairs.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Both values correct: $x = 11$ and $y = 14$.	**B2**	Award both marks for the pair $x = 11$ and $y = 14$, however they are reached. The question does not ask for working, so a correct pair with nothing written down still scores 2. The values may appear on the answer lines in either order of writing, as long as each is against its own letter.	✓
Only one of the two values correct.	**B1**	If the answer does not score B2, award one mark for $x = 11$ or for $y = 14$, with the other value wrong or missing.	✓
Special case: the two values written the wrong way round, $x = 14$ and $y = 11$.	**SC B1**	One mark, and no more, for the pair reversed. This is worth a mark because the two numbers themselves are right: the middle pair still totals 25 and the smallest is still 4 below 15. What fails is the ordering, since 14, 14, 11, 15 does not rise from left to right, and the question states that the list is in ascending order of size.	✓

Full marks: 2/2

Question 4 - Exam Solution

Understanding the Question

Given

$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ - the universal set, so every member of every set here comes from this list

$A = \{\text{factors of } 6\}$ and $B = \{\text{prime numbers}\}$, both taken from $\mathcal{E}$

C has 4 members, with $n(A \cap C) = 2$ and $n(B \cap C) = 2$, and those two sets share no member

Find

(a)(i) the members of $A \cup B$ (a)(ii) the members of A' (b) why the claim $A \cap B = \varnothing$ is wrong (c) the 4 members of set C

Plan the Solution

- Write A and B out as lists first. Every part of the question is then read straight off those two lists.
- A factor of 6 divides 6 exactly, and a prime number has exactly two factors - so 1 is a factor of 6 but is not prime.
- For (c), ask what a member of $A \cap C$ is not allowed to be. If it were prime it would sit in $B \cap C$ too, and the question says those two sets share nothing.

Worked Solution [5 marks]

> Rule - $A \cup B$ is every member that is in A or in B, each written once; A' is every member of $\mathcal{E}$ that is not in A; and $A \cap B$ is what the two sets share. The empty set $\varnothing$ has no members at all.

Step 1: List the factors of 6

$6 = 1 \times 6$

$6 = 2 \times 3$

$A = \{1, 2, 3, 6\}$

(Reason: The factor pairs give every whole number that divides 6 exactly, and all four of them are in $\mathcal{E}$.)

Step 2: List the prime numbers in $\mathcal{E}$

$B = \{2, 3, 5, 7\}$

(Reason: A prime has exactly two factors. So 1 is not prime (it has only one factor), and $9 = 3 \times 3$ is not prime either.)

Step 3: Part (a)(i) - take the union

$A \cup B = \{1, 2, 3, 5, 6, 7\}$

(Reason: The union collects everything that is in A or in B. 2 and 3 belong to both sets, but each member is written only once.)

Step 4: Part (a)(ii) - take the complement

$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 6\}$

$A' = \{4, 5, 7, 8, 9, 10\}$

(Reason: The complement is everything left in $\mathcal{E}$ once the members of A are crossed out. Work along $\mathcal{E}$ in order, so that nothing at the end of the list is missed.)

Step 5: Part (b) - find what A and B share

$A \cap B = \{2, 3\}$

(Reason: 2 and 3 are factors of 6 and are also prime, so the intersection has two members. The empty set has none, so $A \cap B = \varnothing$ cannot be right.)

Step 6: Part (c) - which members can be in $A \cap C$

$A = \{1, 2, 3, 6\}$
$A \cap C = \{1, 6\}$

(Reason: A member of $A \cap C$ must not be prime: if it were, it would be in $B \cap C$ as well, and those two sets share nothing. That rules out 2 and 3, leaving exactly the 2 members needed.)

Step 7: Part (c) - which members can be in $B \cap C$

$B = \{2, 3, 5, 7\}$
$B \cap C = \{5, 7\}$

(Reason: The same argument the other way round. 2 and 3 are factors of 6, so they would fall in $A \cap C$ too, which leaves 5 and 7.)

Step 8: Part (c) - put set C together

$C = \{1, 6\} \cup \{5, 7\} = \{1, 5, 6, 7\}$

(Reason: The two pairs have no member in common, so together they give the 4 members C is allowed. Any other choice breaks one of the three conditions.)

(a)(i) $A \cup B = \{1, 2, 3, 5, 6, 7\}$

(a)(ii) $A' = \{4, 5, 7, 8, 9, 10\}$

(b) 2 and 3 are in both sets, so $A \cap B = \{2, 3\}$, which is not empty

(c) $C = \{1, 5, 6, 7\}$

Verification

✓ **Check 1:** Count the union a second way, with $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. $4 + 4 - 2 = 6$, and the listed union has 6 members

✓ **Check 2:** A set and its complement must fill $\mathcal{E}$ between them with no overlap, so their sizes add to 10. $4 + 6 = 10$, and no number appears in both A and A'

✓ **Check 3:** Test $C = \{1, 5, 6, 7\}$ against all three conditions in the question. $A \cap C = \{1, 6\}$ has 2 members, $B \cap C = \{5, 7\}$ has 2 members, and the two share none

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $A \cup B = \{1, 2, 3, 5, 6, 7\}$	**B1**	In any order, with no repeats.	✓
(a)(ii) $A' = \{4, 5, 7, 8, 9, 10\}$	**B1**	In any order, with no repeats.	✓
(b) 2 (or 3, or both) is a member of A and of B	**B1**	For naming the shared element with an explanation that shows the meaning of intersection and of the empty set. $A \cap B = \{2, 3\}$ on its own also scores. If a number is named as common, that number must be correct. The word sector is allowed in place of set. This is not an exhaustive list of acceptable answers.	✓
(c) $C = \{1, 5, 6, 7\}$	**B2**	For all four correct members, in any order.	✓
(c) partial credit, if B2 is not earned	**(B1)**	For three correct values with no more than one incorrect, or for four correct values with no more than one incorrect.	✓

Full marks: 5/5

Question 5 - Exam Solution

Understanding the Question

Given

A circle sits inside the square $ABCD$ and touches all four sides, at E, F, G and H

The area of the square is 81 cm²

The wire has to make both outlines - the square and the circle - so the two lengths are added

Find

The total length of wire, in cm, correct to 3 significant figures

Plan the Solution

- The area of a square is side times side, so the side is the square root of 81.
- Four equal sides give the perimeter, which is the wire that makes the square.
- The circle touches all four sides, so it fits exactly across the square: its diameter is the side of the square. That single sentence is what the whole question turns on.
- Add the perimeter and the circumference, and round only at the very end - rounding part way through moves the answer.

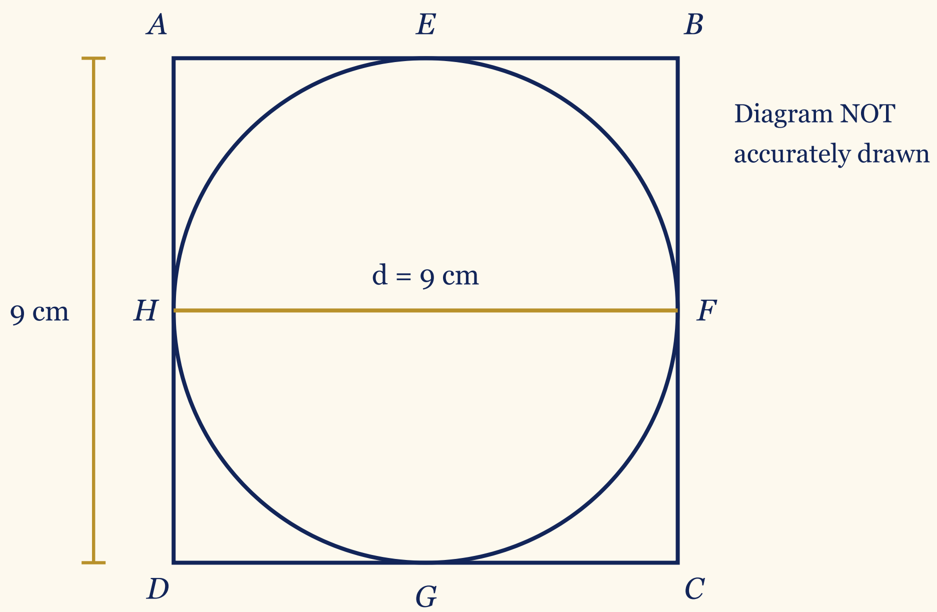
Worked Solution [4 marks]

> Rule - A square of side s has area s^2 and perimeter $4s$. A circle of diameter d has circumference πd, which is the same as $2\pi r$. A circle that touches all four sides of a square has d equal to the side of that square.

Step 1: Work out the side of the square

$$s^2 = 81$$

$$s = \sqrt{81} = 9$$



(Reason: The area of a square is side times side, so the side is the square root of the area. A length is positive, so the negative root is not used here. The side is 9 cm.)

Step 2: Work out the perimeter of the square

$$4 \times 9 = 36$$

(Reason: All four sides of a square are equal, so the perimeter is 4 lots of 9 cm. That is 36 cm of wire for the square on its own.)

Step 3: Find the diameter of the circle

$d = s = 9$

(Reason: The circle touches the square at E, F, G and H, so HF runs right across the circle through its centre and lies along the full width of the square. The diameter is therefore the side of the square, 9 cm, and the radius is half of that, 4.5 cm.)

Step 4: Work out the circumference of the circle

$C = \pi d$

$C = \pi \times 9 = 28.2743\ldots$

(Reason: The circumference formula multiplies π by the DIAMETER - the full distance across the circle, not the distance from the centre to the edge. Keep the decimal running on the calculator; rounding it here would move the final answer.)

Step 5: Add the two lengths of wire

$36 + 28.2743\ldots = 64.2743\ldots$

(Reason: The wire makes the square and the circle, so the total is the perimeter plus the circumference. The two shapes touch, but no wire is shared between them - each outline is drawn completely.)

Step 6: Round to 3 significant figures

$64.2743\ldots \to 64.3$

(Reason: The first three significant figures are 6, 4 and 2. The next digit is 7, which is 5 or more, so the 2 rounds up to 3. The unit is centimetres, because every length in the question is in centimetres.)

64.3 cm

Verification

✓ **Check 1:** Square the side to get back to the area the question gives. $9 \times 9 = 81$, the area printed in the question, so the side is right

✓ **Check 2:** Work backwards. Take the perimeter off the total, then divide what is left by the diameter - it should give π. $64.2743 - 36 = 28.2743$ and $\dfrac{28.2743}{9} = 3.1416$, which is π to 4 decimal places

✓ **Check 3:** A rough estimate, done without a calculator: π is a little over 3, so the circumference is a little over 3 diameters. $36 + 3 \times 9 = 63$, so the answer should be a little above 63 cm - and 64.3 cm is

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\sqrt{81} = 9$ or 9 or $9 \times 9 = 81$	M1	For a method to find the length of the side of the square. A bare 9 with no working also earns it. It may be seen on the diagram rather than in the working.	✓
$4 \times 9 = 36$	M1	For the perimeter of the square, or any equivalent method. The scheme writes the 9 in quotation marks, so the candidate's own value for the side follows through. The first M mark can be implied by 36.	✓
$\pi \times 9 = 28.2743\ldots$ or 9π	M1	For a correct expression for the circumference, from $2\pi r$ or πD. The scheme writes the 9 in quotation marks here too, so the candidate's own side value follows through. The first M mark can be implied by $28.2(743\ldots)$ rounded or truncated to 1 decimal place, so by 28.2 or 28.3, or by 9π.	✓
64.3	A1	Accept any value from 64.26 to 64.3. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working.	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

Part (a): the equation $\dfrac{2f}{3} = 4f - 17$, with the algebraic working itself asked for
Part (b): the expression $(e + 12)^0$, together with the condition $e > 0$
Part (c): the fraction $\dfrac{12a^4h^6}{4ah^2}$, to be simplified fully
Part (d): the expression $20x^5y + 12x^3y^4$, to be factorised fully

Find

(a) The value of f that makes the two sides equal (b) The value of $(e + 12)^0$ (c) The fraction written as one term, with each letter appearing once (d) The expression written as a product, with the highest common factor outside the bracket

Plan the Solution

- Part (a): a fraction in an equation is cleared by multiplying every term on BOTH sides by the denominator. Then gather the letter terms on one side and the numbers on the other, and divide by whatever is multiplying f.
- Part (b): the condition $e > 0$ is not decoration. It guarantees the bracket is not zero, and only a number that is not 0 can be raised to the power 0.
- Part (c): deal with the numbers and with each letter separately. The numbers divide; powers of the same letter subtract their indices.
- Part (d): the highest common factor is the highest common factor of the numbers, together with the LOWEST power of each letter that appears in both terms. Divide each term by it to find what goes inside the bracket.

Worked Solution [8 marks]

> Rule - Balance: multiplying both sides of an equation by the same number that is not 0 keeps it true, and every term must be multiplied. Indices with the same base: $\frac{x^m}{x^n} = x^{m-n}$ and $x^0 = 1$ for every x that is not 0. Factorising fully: outside the bracket goes the highest common factor of the numbers together with the lowest power of each letter that appears in every term.

Step 1: Part (a) - clear the fraction

$$\frac{2f}{3} = 4f - 17$$

$$3 \times \frac{2f}{3} = 3 \times (4f - 17)$$

$$2f = 12f - 51$$

(Reason: The only denominator is 3, so multiplying both sides by 3 clears it. Multiply every term on the right, not just the first one: $3 \times 4f = 12f$ and $3 \times 17 = 51$. Leaving the 17 untouched is exactly the slip the mark scheme carries a follow-through for.)

Step 2: Part (a) - collect the letter terms

$$2f - 12f = -51$$

$$-10f = -51$$

(Reason: Subtract $12f$ from both sides, so the f terms sit together on the left and the number sits on the right. Since $2 - 12 = -10$, the left-hand side becomes $-10f$. This is the two-term equation the second method mark is for.)

Step 3: Part (a) - divide by the coefficient

$$f = \frac{-51}{-10} = \frac{51}{10}$$

$$\frac{51}{10} = 5.1$$

(Reason: Divide both sides by -10. A negative divided by a negative is positive, so the two minus signs cancel and f comes out positive. The mark scheme gives $\frac{51}{10}$ and allows any equivalent form, so the decimal 5.1 scores the same mark.)

Step 4: Part (b) - use the zero index

$$e > 0 \implies e + 12 > 12$$
$$(e+12)^0 = 1$$

(Reason: Any number except 0 raised to the power 0 is 1. The condition $e > 0$ makes the bracket larger than 12, so it certainly is not 0 and the rule applies. The size of the bracket makes no difference whatsoever: the answer is 1 for every allowed value of e.)

Step 5: Part (c) - divide the numbers, subtract the indices

$$\frac{12a^4h^6}{4ah^2} = \frac{12}{4} \times \frac{a^4}{a^1} \times \frac{h^6}{h^2}$$
$$\frac{12}{4} = 3$$
$$\frac{a^4}{a^1} = a^{4-1} = a^3$$
$$\frac{h^6}{h^2} = h^{6-2} = h^4$$
$$\frac{12a^4h^6}{4ah^2} = 3a^3h^4$$

(Reason: Split the fraction into a number part and one part for each letter. The numbers are divided; the letters follow the index law, so their indices are subtracted. Remember that a written on its own means a^1, so its index is 1 and the a does not vanish from the answer.)

Step 6: Part (d) - find the highest common factor

$$20x^5y = 4x^3y \times 5x^2$$
$$12x^3y^4 = 4x^3y \times 3y^3$$

(Reason: The highest common factor of 20 and 12 is 4. The lowest power of x in the two terms is x^3, and the lowest power of y is y itself, so the highest common factor of the whole expression is $4x^3y$. Taking out something smaller, $2x^3y$ say, does factorise the expression but not fully, and fully is what is asked for.)

Step 7: Part (d) - write it as a product

$$20x^5y + 12x^3y^4 = 4x^3y(5x^2 + 3y^3)$$

(Reason: The highest common factor goes outside the bracket and what is left of each term goes inside. Then check the bracket: $5x^2$ and $3y^3$ share no number factor and no letter, so nothing further can

come out and the factorisation is complete.)

$$\textbf{(a) } f = \frac{51}{10} \textbf{ (that is, } f = 5.1\textbf{)}$$

$$\textbf{(b) } 1$$

$$\textbf{(c) } 3a^3h^4$$

$$\textbf{(d) } 4x^3y(5x^2 + 3y^3)$$

Verification

✓ **Check 1:** Part (a): put $f = 5.1$ back into the equation the question printed, and work the two sides out separately. $\dfrac{2 \times 5.1}{3} = \dfrac{10.2}{3} = 3.4$ and $4 \times 5.1 - 17 = 20.4 - 17 = 3.4$, so the two sides agree

✓ **Check 2:** Part (b): try an allowed value. Take $e = 4$, which satisfies $e > 0$, and work the bracket out before applying the index. $(4 + 12)^0 = 16^0 = 1$, and every other positive value of e gives 1 in exactly the same way

✓ **Check 3:** Part (c): multiply the answer back by the denominator. It must return the numerator the question printed. $3a^3h^4 \times 4ah^2 = 12a^4h^6$, which is the numerator in the question

✓ **Check 4:** Part (d): expand the brackets again and compare, term by term, with the expression the question printed. $4x^3y \times 5x^2 = 20x^5y$ and $4x^3y \times 3y^3 = 12x^3y^4$, which add to give the original expression

✓ **Check 5:** Parts (c) and (d) with numbers in place of the letters: take $a = 2$ and $h = 1$ in part (c), and $x = 2$ and $y = 1$ in part (d). $\dfrac{12 \times 16}{4 \times 2} = 24$ and $3 \times 8 = 24$; then $20 \times 32 + 12 \times 8 = 736$ and $32 \times 23 = 736$, so both answers survive a numerical test

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$2f = 12f - 51$ or $2f = -51 + 12f$	M1	For a correct first step: multiplying both sides by 3 correctly and expanding. Writing the right-hand side as two terms each over 2 also earns it, for example $\frac{f}{3} = 2f - \frac{17}{2}$ or $0.3f = 2f - 8.5$. Decimals are allowed to 1 decimal place or better, rounded or truncated.	✓
$-10f = -51$ or $10f = 51$	M1	For a correct two-term equation in the form $af = b$, for example $\frac{5f}{3} = \frac{17}{2}$ or $17 = \frac{10f}{3}$ or $3.3f = 17$. Follow through is allowed from these three equations only: $2f = 12f - 17, 2f = 4f - 51$ and $6f = 12f - 51$, or any equivalent of them. Decimals are allowed to 1 decimal place or better, rounded or truncated.	✓
$f = \frac{51}{10}$	A1	Or any equivalent, for example 5.1. Dependent on at least one of the method marks. Working is required on this part, so the answer alone scores nothing.	✓
1	B1	The single mark is for the answer, and no working is expected for it.	✓
$3a^3h^4$	B2	Or any equivalent. B1 for a product in the form ka^ph^q in which two of k, p and q are correct, with multiplication signs allowed, for example $5a^3h^4$ or $\frac{12a^3h^4}{4}$. Also allow $3a^3$ or a^3h^4 or $3h^4$ for B1, as long as it is not added to any other term.	✓
$4x^3y(5x^2 + 3y^3)$	B2	B1 for any correct factorisation whose factor outside the bracket has at least two parts to it, for example $2x^3y(10x^2 + 6y^3)$ or $x^3y(20x^2 + 12y^3)$ or $2x(10x^4y + 6x^2y^4)$ or $4y(5x^5 + 3x^3y^3)$ or $4x^3(5x^2y + 3y^4)$. B1 is also given for the correct highest common factor with a two-term bracket in which at most one term is wrong, for example $4x^3y(5x^2 + \ldots)$ or $4x^3y(\ldots + 3y^3)$.	✓

Full marks: 8/8

Question 7 - Exam Solution

Understanding the Question

Given

The equation $\dfrac{3^{-2} \times 3^5}{3^{10}} = 3^n$

Every power on both sides has the same base, 3, which is what makes the index laws usable here

Find

The value of n: the single index the whole left-hand side collapses to

Plan the Solution

- Take the top line first. Multiplying two powers of the same base ADDS their indices, so $3^{-2} \times 3^5$ becomes one power of 3.
- Then divide. Dividing powers of the same base SUBTRACTS the indices, and the one that comes off is the index downstairs.
- Both sides are then a single power of 3. Two powers of the same base are equal only when their indices are equal, so n can be read straight off.

Worked Solution [2 marks]

> Rule - Index laws with a common base: $a^m \times a^n = a^{m+n}$ and $\dfrac{a^m}{a^n} = a^{m-n}$. A negative index is not a mistake to be tidied away: $a^{-k} = \dfrac{1}{a^k}$, so an index below 0 is a perfectly good answer.

Step 1: Add the indices on the top line

$3^{-2} \times 3^5 = 3^{-2+5} = 3^3$

(Reason: Multiplying two powers of the same base adds their indices, so the -2 and the 5 are added rather than the numbers $\frac{1}{9}$ and 243 being multiplied out. A negative index behaves as a subtraction inside that sum, which is why $-2+5$ comes to 3 and not to 7. This single line is what the mark scheme's method mark is for.)

Step 2: Take away the index on the bottom

$$\frac{3^3}{3^{10}} = 3^{3-10} = 3^{-7}$$

(Reason: Dividing powers of the same base subtracts the indices, and the index that comes off is the one downstairs. Both powers here have base 3, so the whole fraction collapses into a single power of 3. The index that results must be SMALLER than the one it started from, because the bottom of the fraction is the bigger power.)

Step 3: Match the indices on the two sides

$3^n = 3^{-7}$

$n = -7$

(Reason: The left-hand side is now one power of 3 and the right-hand side always was one. Two powers of the same base are equal only when their indices are equal, so the index on the left IS n. Nothing is calculated on this line; it is a comparison.)

$$n = -7$$

Verification

✓ **Check 1:** Count all three indices in one sum instead of in two steps. Multiplying puts an index in and dividing takes one out, so the -2, the 5 and the 10 combine on a single line. $-2 + 5 - 10 = -7$, the same index the two-step working reached

✓ **Check 2:** Put the index laws aside and work each side out as an ordinary fraction. If the answer is right, the two sides must be the same number. $\dfrac{3^{-2} \times 3^5}{3^{10}} = \dfrac{27}{59049} = \dfrac{1}{2187}$ and $3^{-7} = \dfrac{1}{2187}$, so the two sides agree exactly

✓ **Check 3:** Test the SIGN on its own, without finding the index at all. The bottom of the fraction, 3^{10}, is far bigger than the top, so the left-hand side is less than 1 - and only a negative power of 3 is less than 1. $\dfrac{1}{2187}$ is less than 1, so n is negative, which rules out 7 and leaves -7

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$3^{-2} \times 3^5 = 3^3$ or $-2 + 5 - 10$	M1	For a correct application of an index rule as a first step, or a correct calculation for n. Any equivalent earns it, whichever pair of powers is combined first: 3^3 or $(3^{-2}) \times 3^{-5}$ or $\dfrac{3^3}{3^{10}}$ or $\dfrac{3^5}{3^{12}}$ or $\dfrac{3^{-2}}{3^5}$ or 3^{-12} with the 3^5 still to be multiplied in.	✓

Step	Mark	Description	Got it?
		The index sums score it just as well: $-2+5-10$ or $-12+5$ or $3-10$.	
$n=-7$	A1	For -7. The scheme also allows the answer written as the power itself, 3^{-7}.	✓
Working is not required on this question	Note	A correct answer on its own scores both marks, unless it has clearly come from incorrect working.	✓

Full marks: 2/2

Question 8 - Exam Solution

Understanding the Question

Given

In the sale, every normal price is reduced by 17%
Sale price of the table fan: $6\,225$ rupees

Find

The normal price of the table fan, in rupees.

Plan the Solution

- A reduction of 17% leaves 83% of the normal price, so the sale price is 0.83 of it.
- The normal price has been multiplied by 0.83, so divide by 0.83 to undo that. This is a reverse percentage: never add 17% back on to the sale price.
- Finish by taking 17% off the answer and looking for the sale price again.

Worked Solution [3 marks]

> Rule - Reverse percentage: $\text{normal price} = \dfrac{\text{sale price}}{\text{multiplier}}$, and the multiplier for a reduction of 17% is 0.83.

Step 1: Turn the reduction into a multiplier

$1-0.17=0.83$

(Reason: Start from the whole normal price, which is 1*, and account for the* 17% *the sale takes off it. What is left is the fraction of the normal price a shopper actually pays.)*

Step 2: Divide the sale price by the multiplier

$$\frac{6\,225}{0.83}$$

(Reason: The normal price was multiplied by 0.83 *to give* $6\,225$ *rupees, so dividing* $6\,225$ *by* 0.83 *undoes the multiplication and gives the normal price back.)*

Step 3: Work out the division

$$\frac{6\,225}{0.83} = 7\,500$$

(Reason: The division is exact, so the normal price is $7\,500$ *rupees, which is larger than the sale price of* $6\,225$ *rupees, as a reduction requires.)*

$7\,500$ rupees

Verification

✓ **Check 1:** Take 17% of the answer and subtract it. The sale price should come back. $0.17 \times 7\,500 = 1\,275$ and $7\,500 - 1\,275 = 6\,225$

✓ **Check 2:** Reach the answer a different way. The sale price is 83 per cent of the normal price, so divide by 83 to get one per cent, then multiply by 100. $\dfrac{6\,225}{83} \times 100 = 7\,500$

✓ **Check 3:** Test the trap. Adding 17% on to the sale price is a different calculation, so it must give a different number. $1.17 \times 6\,225 = 7\,283.25$, which is not $7\,500$, so a reduction is undone by dividing, never by adding the same percentage on.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Find the multiplier for the reduction, or one per cent of the normal price	M1	for $1 - 0.17$ or 0.83 or $\dfrac{83}{100}$, or $100 - 17 = 83$ per cent, or $\dfrac{6\,225}{83} = 75$ or equivalent	✓
Divide the sale price by the multiplier	M1	for $\dfrac{6\,225}{0.83}$ or $\dfrac{6\,225}{83} \times 100$ or $6\,225 \times \dfrac{100}{83}$ or 75×100 or equivalent. The multiplier here	✓

Step	Mark	Description	Got it?
		may be the candidate's own value from the first mark.	
State the normal price	**A1**	for $7\,500$. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given

Part (a): 6.04×10^5, already written in standard form.
Part (b): $0.000\,07$, written as an ordinary number.
Part (c): $\dfrac{7.6 \times 10^{10}}{4 \times 10^5 - 2 \times 10^4}$, with a subtraction in the denominator.

Find

Part (a): the same number written out in full. Part (b): the same number written in standard form. Part (c): the value of the fraction, in standard form.

Plan the Solution

- Standard form is $a \times 10^n$, where a is at least 1 and less than 10, and n is a whole number. All three parts are that one definition, read in three different directions.
- In part (a) the index is positive, so the ordinary number is the bigger of the two: the decimal point moves 5 places to the right.
- In part (b) the number is smaller than 1, so the index is negative. Count the places from the decimal point to the first significant figure, not the zeros.
- In part (c) the denominator has to be worked out before anything is divided. 4×10^5 and 2×10^4 carry different powers of ten, so their front numbers cannot be subtracted while they stand as they are.
- Then divide: front number by front number, and the indices subtracted. Finish by checking the front number still lies between 1 and 10.

Worked Solution [4 marks]

Rule - Standard form: $a \times 10^n$ with $1 \leq a < 10$. To divide, $\dfrac{a \times 10^m}{b \times 10^n} = \dfrac{a}{b} \times 10^{m-n}$.

Step 1: Part (a) - move the decimal point five places to the right

$10^5 = 100\,000$

$6.04 \times 10^5 = 6.04 \times 100\,000 = 604\,000$

(Reason: Multiplying by 10^5 is multiplying by $100\,000$, so every digit moves five columns to the left and the decimal point ends up five places further right. The 6 lands in the hundred-thousands column, and the columns left empty behind it are filled with zeros.)

Step 2: Part (b) - count the places to the first significant figure

$0.000\,07 = \dfrac{7}{100\,000}$

$\dfrac{7}{100\,000} = \dfrac{7}{10^5} = 7 \times 10^{-5}$

(Reason: The only non-zero digit is the 7, so the front number is 7. It sits five places after the decimal point, which makes the index -5, and the index is negative because the number is smaller than 1. There are only four zeros after the point, so counting zeros instead of places gives 10^{-4} and an answer ten times too big.)

Step 3: Part (c) - write both terms of the denominator out in full

$4 \times 10^5 = 400\,000$

$2 \times 10^4 = 20\,000$

(Reason: The two terms carry different powers of ten, so their front numbers cannot simply be subtracted. Writing each one out in full lines the digits up in the right columns, and the subtraction then becomes ordinary arithmetic.)

Step 4: Part (c) - subtract, then put the denominator back into standard form

$400\,000 - 20\,000 = 380\,000$

$380\,000 = 3.8 \times 10^5$

(Reason: Taking twenty thousand away from four hundred thousand leaves three hundred and eighty thousand. This is the value the mark scheme wants for its method mark, in whichever of its forms it is written.)

Step 5: Part (c) - divide the front numbers, and subtract the indices

$\dfrac{7.6}{3.8} = 2$

$\dfrac{10^{10}}{10^5} = 10^{10-5} = 10^5$

(Reason: A division in standard form splits into two easy divisions: the front numbers divide to give 2, and the powers of ten are handled by subtracting the indices. Dividing the indices instead of

subtracting them is the slip to avoid here.)

Step 6: Part (c) - put the two halves back together

$$\frac{7.6 \times 10^{10}}{3.8 \times 10^{5}} = 2 \times 10^{5}$$

(Reason: The front number 2 lies between 1 and 10, so the answer is already in standard form and nothing more needs doing to it. Writing $200\,000$ instead is the same number in the wrong form, and the question asked for standard form.)

(a) $604\,000$

(b) 7×10^{-5}

(c) 2×10^{5}

Verification

✓ **Check 1:** Reverse part (a). Divide the answer by $100\,000$ and the number the question printed should come back. $\frac{604\,000}{100\,000} = 6.04$

✓ **Check 2:** Reverse part (b). Turn the answer back into an ordinary number and compare it with the number the question printed. $7 \times 10^{-5} = \frac{7}{100\,000}$, and $\frac{7}{100\,000}$ written out is $0.000\,07$

✓ **Check 3:** Reverse part (c). Multiplying the answer by the denominator must give the numerator back. $2 \times 10^{5} \times 3.8 \times 10^{5} = 7.6 \times 10^{10}$

✓ **Check 4:** Reach part (c) a second way, in ordinary numbers only, so that nothing depends on the index laws. $\frac{76\,000\,000\,000}{380\,000} = 200\,000$, which is 2×10^{5}

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): the number written out in full	B1	for $604\,000$	✓
Part (b): the number written in standard form	B1	for 7×10^{-5}	✓

Step	Mark	Description	Got it?
Part (c): simplify the denominator	**M1**	for 380 000 or 3.8×10^5 or 38×10^4 or equivalent	✓
Part (c): the answer, in standard form	**A1**	for 2×10^5. Accept 2.0×10^5 or 2.00×10^5 and so on, and accept a dot or a comma in place of the multiplication sign. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Part (c): special case - the right digits, in the wrong form	**SC B1**	for 200 000 or 20×10^4 or 0.2×10^6 or equivalent, or for 2×10^n with n other than 5, when it is given as the final answer. The first three are the right number written in a form that is not standard form. Not for incorrect simplification of the denominator.	✓

Full marks: 4/4

Question 10 - Exam Solution

Understanding the Question

Given

A hexagon $ABCDEF$ with $AB = 11$ cm, $BC = 5$ cm, $ED = 23$ cm and $EF = 4.7$ cm
Angle $BCF = 30°$, and AB, FC and ED are parallel
Right angles at E and at D, marked on the diagram

Find

The area of the hexagon $ABCDEF$, in cm^2

Plan the Solution

- The dashed line FC cuts the hexagon into two shapes you already have formulas for: the rectangle $FCDE$ underneath and the trapezium $ABCF$ on top.
- The right angles at E and D are what make $FCDE$ a rectangle, so FC is the same length as ED.
- The trapezium's height is not BC itself. Drop a perpendicular from B to FC to make a right-angled triangle with BC as its hypotenuse, and use the $30°$ angle.

- Work out the two areas and add them.

Worked Solution [5 marks]

> Rule - Rectangle: $\text{Area} = \text{length} \times \text{width}$. Trapezium: $\text{Area} = \dfrac{1}{2}(a+b)h$, where a and b are the two parallel sides and h is the perpendicular distance between them.

Step 1: Cut the hexagon along FC

$FC = ED = 23$ cm

(Reason: FC is parallel to ED, and the right angles at E and D make $FCDE$ a rectangle, so its opposite sides are equal)

Step 2: Area of the rectangle $FCDE$

$23 \times 4.7 = 108.1 \text{ cm}^2$

(Reason: The rectangle is 23 cm long and 4.7 cm high, because EF is one of its sides)

Step 3: Height of the trapezium $ABCF$

$\sin 30^\circ = \dfrac{h}{5}$

$h = 5 \sin 30^\circ = 2.5$ cm

(Reason: Dropping a perpendicular from B to FC gives a right-angled triangle in which $BC = 5$ cm is the hypotenuse and h is the side opposite the 30° angle)

Step 4: Area of the trapezium $ABCF$

$\dfrac{1}{2} \times (11 + 23) \times 2.5 = 42.5 \text{ cm}^2$

(Reason: The parallel sides are $AB = 11$ cm and $FC = 23$ cm, and the height is the perpendicular distance between them from Step 3)

Step 5: Add the two areas

$108.1 + 42.5 = 150.6 \text{ cm}^2$

(Reason: The rectangle and the trapezium meet along FC and do not overlap, so together they make up the whole hexagon)

$$150.6 \text{ cm}^2$$

Verification

✓ **Check 1:** Build the area a different way. The hexagon sits inside a rectangle 23 cm by 7.2 cm, with one triangle cut off above FA and another above BC. The two triangle bases add up to $23 - 11 = 12$ cm and both triangles have height 2.5 cm.

$$23 \times 7.2 - \frac{1}{2} \times 12 \times 2.5 = 150.6$$

✓ **Check 2:** A trapezium's area is also its mean width times its height. The mean of the parallel sides is $\frac{11+23}{2} = 17$ cm, which should give the same trapezium area as Step 4.

$$17 \times 2.5 = 42.5$$

✓ **Check 3:** Check the size is sensible. The hexagon contains the rectangle $FCDE$ and fits inside the 23 cm by 7.2 cm rectangle, so its area must lie between those two.

$$108.1 < 150.6 < 165.6$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$23 \times 4.7 = 108.1$	**B1**	The area of the rectangle $FCDE$, or equivalent. Awarded whatever else is done, and it may be embedded in $23 \times (4.7 + 2.5) = 165.6$.	✓
$\sin 30^\circ = \frac{x}{5}$	**M1**	A correct trigonometric statement for the height x of the trapezium, or equivalent, for example $\frac{x}{\sin 30^\circ} = \frac{5}{\sin 90^\circ}$. The mark is also earned without trigonometry for the height itself: $5 \cos 30^\circ$ $(= 4.33\ldots)$ for the horizontal run, then $x^2 = 5^2 - (5 \cos 30^\circ)^2$ $(= 6.25)$.	✓
$x = 5 \sin 30^\circ = 2.5$	**M1**	The height of the trapezium worked out correctly. Accept $x = \sqrt{5^2 - (5 \cos 30^\circ)^2}$ reaching 2.5.	✓
$\frac{1}{2} \times (11 + 23) \times 2.5 = 42.5$	**M1**	A correct method for the area of the trapezium $ABCF$, or for the whole shape, using their own height.	✓
$108.1 + 42.5 = 150.6$	**A1**	The area of the hexagon. Accept anything that rounds to 150.6, allow 151, and accept $\frac{753}{5}$.	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

A cumulative frequency table over 60 members, in classes five hours wide.

The running totals are $6, 17, 27, 42, 53, 60$ at $5, 10, 15, 20, 25$ and 30 hours.

25 members trained for more than W hours.

Find

(a) the cumulative frequency graph (b) an estimate for the interquartile range of the times (c) the value of W (d) the probability that $5 < H \leq 10$

Plan the Solution

- Plot each running total at the upper end of its class, then join the six points in order.
- Read the quartiles off that line at $\dfrac{60}{4} = 15$ and $\dfrac{3 \times 60}{4} = 45$, then subtract the smaller time from the larger.
- For (c), turn 25 members above W into $60 - 25 = 35$ at or below it, because the graph only ever counts upwards.
- For (d), the frequency of a single class is the difference of two neighbouring running totals.

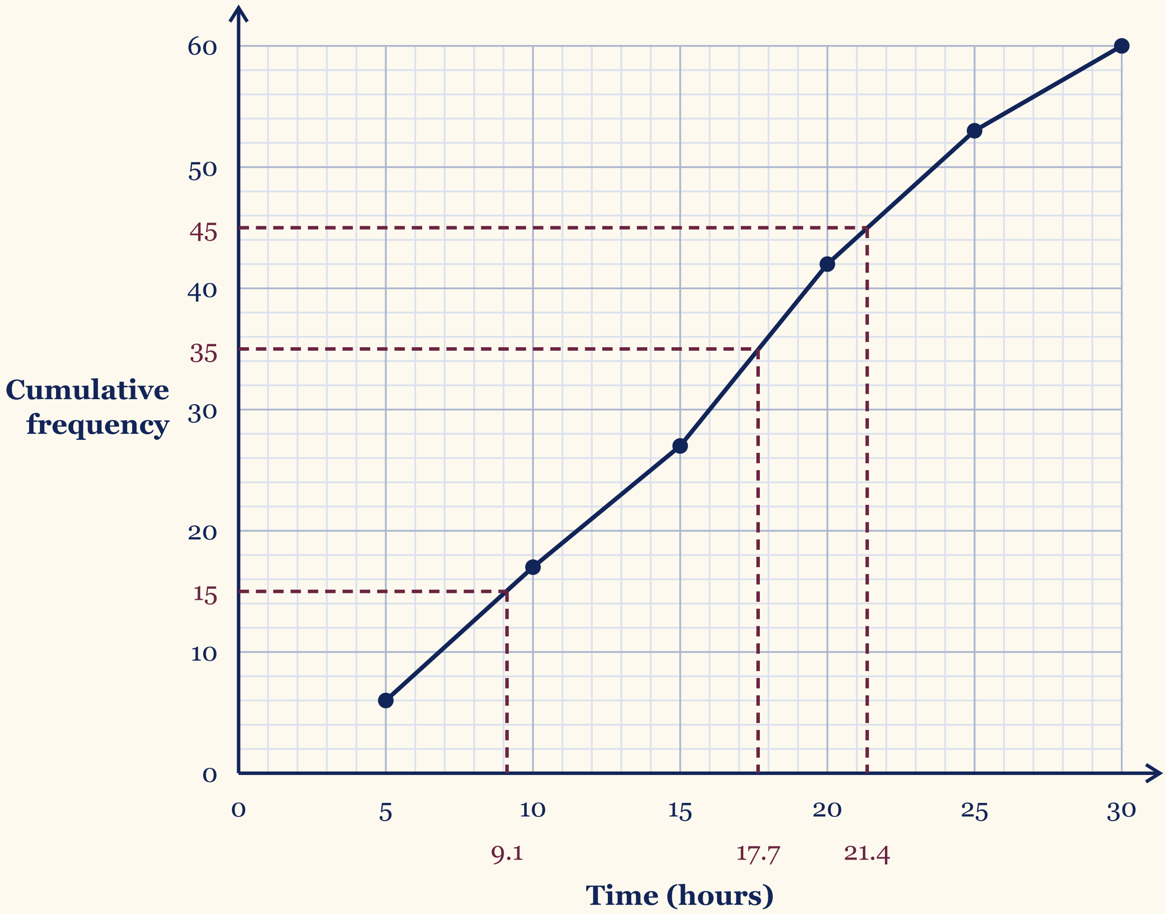
Worked Solution [7 marks]

Rule - Cumulative frequency: plot each running total against the upper boundary of its class. The lower quartile is read at $\dfrac{n}{4}$ on the cumulative frequency axis, the upper quartile at $\dfrac{3n}{4}$, and the interquartile range is the upper quartile minus the lower quartile.

Step 1: plot each running total at the top of its class

$(5, 6),\ (10, 17),\ (15, 27)$

$(20, 42),\ (25, 53),\ (30, 60)$



(Reason: 27 members trained for 15 hours or less, so the height 27 belongs at the end of that class and not in the middle of it)

Step 2: join the six points in order

$6 < 17 < 27 < 42 < 53 < 60$

(Reason: a running total never falls, so the six points climb from left to right; join them with a smooth curve or with straight line segments, and every later answer is read off that line)

Step 3: find the two quartile positions on the cumulative frequency axis

$$\frac{60}{4} = 15$$

$$\frac{3 \times 60}{4} = 45$$

(Reason: with 60 members, a quarter of the way up the cumulative frequency axis is 15 and three quarters of the way up is 45)

Step 4: read across to the line, down to the time axis, and subtract

$Q_1 \approx 9.1$

$Q_3 \approx 21.4$

$21.4 - 9.1 = 12.3$

(Reason: the mark scheme allows anything from 8 to 9.5 for the lower quartile and 21 to 23 for the upper, so a careful reading of your own graph is enough)

Step 5: turn 25 members above W into a cumulative frequency

$60 - 25 = 35$

$W \approx 17.7$

(Reason: the graph counts members at or below a time, never above it, so 25 above W means 35 at or below W, and 35 is the height to read across from)

Step 6: pick one class out of the running totals

$17 - 6 = 11$

$\frac{11}{60}$

(Reason: 17 members trained for 10 hours or less and 6 of those for 5 hours or less, so the difference is the number of members in the class $5 < H \le 10$)

(a) the line through $(5, 6)$ up to $(30, 60)$

(b) 12.3 hours

(c) $W = 17.7$

(d) $\frac{11}{60}$

Verification

- ✓ **Check 1 - the column adds back up:** Difference the cumulative column into class frequencies and add them: $6 + 11 + 10 + 15 + 11 + 7$
  $6 + 11 + 10 + 15 + 11 + 7 = 60$, the stated number of members
- ✓ **Check 2 - the median falls between the quartiles:** Read the median at $\frac{60}{2} = 30$, which lands in the class $15 < t \le 20$ the median is 16 hours, which sits between 9.1 and 21.4 as it must
- ✓ **Check 3 - count above W instead of below it:** Take the reading used for (c) back off the total: $60 - 35 = 25$ 25 members above W, which is what the question states
- ✓ **Check 4 - the three probabilities cover everybody:** Below 5 hours, between 5 and 10 hours, and above 10 hours: $\frac{6}{60} + \frac{11}{60} + \frac{43}{60}$ $\frac{6}{60} + \frac{11}{60} + \frac{43}{60} = 1$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) the cumulative frequency graph	**B2**	a fully correct graph: the six points at the ends of the intervals, joined with a curve or with line segments	✓
(a) partly correct	**(B1)**	5 correct points plotted and joined, or 6 correct points plotted but not joined, or 5 or 6 points plotted consistently within each interval rather than at its upper end, at their correct heights and joined, eg plotted at $2.5, 7.5, 12.5, 17.5, 22.5$ and 27.5	✓
(a) guidance	**Note**	a bar chart type graph scores zero marks. Ignore any part of the graph drawn before $(5, 6)$	✓
(b) a correct method for the two readings	**M1ft**	readings taken on the time axis from cumulative frequency 45 (or 45.75) and from 15 (or 15.25), or equivalent, shown by lines or by marks on the time axis or just by the correct readings. Follow through from their own graph	✓
(b) the interquartile range	**A1ft**	a single value in the range 11.5 to 13.5, follow through from their own graph. The two readings themselves are 8 to 9.5 and 21 to 23	✓
(c) a correct method for W	**M1ft**	for using or stating 35, or for lines or marks showing cumulative frequency 35 used on the graph, or for an indication on the time axis at the correct point, or just for the correct reading. Follow through from an incorrect graph if the method is shown	✓
(c) the value of W	**A1ft**	a value in the range 16.5 to 18.5, follow through from their own graph	✓
(d) the probability	**B1**	$\frac{11}{60}$. Accept $0.18(333\ldots)$ or $18.(333\ldots)\%$, the bracketed digits being optional, so 0.18 or better, or 18% or better	✓

Full marks: 7/7

Question 12 - Exam Solution

Understanding the Question

Given

ABC and AED are straight lines, and BE is parallel to CD

$AE = 10$ cm

$CD = 1.5 \times BE$

$AB = (2x + 5)$ cm and

$BC = (3x - 5)$ cm

Find

(a) the length of ED (b) the value of x

Plan the Solution

- BE is parallel to CD, so triangle ABE and triangle ACD are similar.
- The scale factor from the small triangle to the large one is $\dfrac{CD}{BE} = 1.5$, so every length measured from A is 1.5 times as long in the large triangle.
- Part (a): scale AE up to get AD, then take AE off it, because E sits between A and D.
- Part (b): AC can be written two ways, as $AB + BC$ and as $1.5 \times AB$. Setting them equal gives one equation in x.

Worked Solution [4 marks]

Rule - Similar triangles: a line parallel to one side of a triangle cuts the other two sides in the same ratio, so $AD = k \times AE$ and $AC = k \times AB$, where $k = \dfrac{CD}{BE}$.

Step 1: Find the scale factor, and use it on AD

$$k = \frac{CD}{BE} = 1.5$$

$$AD = 1.5 \times AE = 1.5 \times 10 = 15$$

(Reason: BE is parallel to CD, so the angle at A is shared and the angles at B and C are equal corresponding angles. That makes the two triangles similar, and AD is the image of AE under the same 1.5.)

Step 2: Take AE off AD

$$ED = AD - AE$$

$$ED = 15 - 10 = 5$$

(Reason: E lies on AD between A and D, so AD splits into AE and ED. The answer is in centimetres because AE was.)

Step 3: Write AC in two different ways

$AC = AB + BC = (2x + 5) + (3x - 5) = 5x$

$AC = 1.5 \times AB = 1.5(2x + 5) = 3x + 7.5$

(Reason: B lies on AC, so $AC = AB + BC$ and the $+5$ and the -5 cancel. The same scale factor 1.5 also takes AB to AC, which is where the second expression comes from.)

Step 4: Set the two expressions equal, and solve

$5x = 3x + 7.5$

$2x = 7.5$

$x = \frac{7.5}{2} = 3.75$

(Reason: Take $3x$ from both sides, then halve. x carries no unit here: it is the number that makes the two lengths fit together.)

(a) $ED = 5$ cm

(b) $x = 3.75$

Verification

✓ **Check 1:** Put $x = 3.75$ back into the two expressions: $AB = 2 \times 3.75 + 5 = 12.5$ and $BC = 3 \times 3.75 - 5 = 6.25$, so $AC = 12.5 + 6.25 = 18.75$. $\frac{AC}{AB} = \frac{18.75}{12.5} = 1.5$, the scale factor the question gives

✓ **Check 2:** A parallel line cuts both sides in the same ratio, so $ED : AE$ must match $BC : AB$. The first pair comes from part (a), the second from part (b), so this ties the two answers together. $\frac{5}{10} = \frac{6.25}{12.5} = 0.5$

✓ **Check 3:** Solve part (b) a different way. BC is half of AB, so $\frac{3x - 5}{2x + 5} = \frac{1}{2}$. Cross-multiplying gives $2(3x - 5) = 2x + 5$. $6x - 10 = 2x + 5$, so $4x = 15$ and $x = \frac{15}{4} = 3.75$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $(AD =)10 \times 1.5$ $(= 15)$ oe	**M1**	for a complete method to find AD, the scale factor 1.5 applied to AE. Any equivalent working scores it.	✓
(a) 5	**A1**	cao. Working is not required, so a correct answer scores both marks, unless it comes from obviously incorrect working.	✓
(b) $(2x+5)+(3x-5) = 1.5(2x+5)$ oe	**M1**	for a correct equation in x. Accept any equivalent form, eg $5x = 1.5(2x+5)$, or $5x = 3x + 7.5$, or $\dfrac{3x-5}{2x+5} = \dfrac{1}{2}$.	✓
(b) 3.75	**A1**	oe, eg $\dfrac{15}{4}$ or $3\dfrac{3}{4}$. Working is not required, so a correct answer scores both marks, unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 13 - Exam Solution

Understanding the Question

Given

A sector OAB of a circle with centre O and radius r cm

The angle at the centre: $AOB = 60^\circ$

The perimeter of the sector is P cm

Find

A formula for P in terms of r It must be written as $P = r(c\pi + k)$, so the two numbers c and k are what the answer really asks for.

Plan the Solution

- The perimeter of a sector is the curved arc plus the two straight edges, and both straight edges are radii, so start by writing $P = \text{arc} + 2r$.
- 60° out of 360° is one sixth of a full turn, so the arc is one sixth of the whole circumference $2\pi r$.
- Add the two radii, then take a factor of r out of both terms to reach the form the question asks for.

Worked Solution [3 marks]

Rule - Arc length and sector perimeter: an arc that subtends θ at the centre has length $\dfrac{\theta}{360} \times 2\pi r$, and the perimeter of the sector is that arc plus the two radii, $2r$.

Step 1: Write down what the perimeter is made of

$$P = \text{arc } AB + OA + OB$$

$$OA = OB = r$$

$$P = \text{arc } AB + 2r$$

(Reason: Both straight edges of a sector run from the centre to the circle, so each one is a radius. Together they contribute $2r$, and only the arc is left to find.)

Step 2: Find the length of the arc

$$\text{arc } AB = \frac{60}{360} \times 2\pi r$$

$$\frac{60}{360} = \frac{1}{6}$$

$$\frac{1}{6} \times 2\pi r = \frac{\pi r}{3}$$

(Reason: The arc is the same fraction of the circumference as 60° is of a full turn. Since 60° is one sixth of 360°, the arc is one sixth of $2\pi r$.)

Step 3: Add the two radii

$$P = \frac{\pi r}{3} + 2r$$

(Reason: The arc and the two radii together make the whole way round the sector, so the two pieces from Step 1 and Step 2 are simply added.)

Step 4: Take out the factor r

$$P = r\left(\frac{1}{3}\pi + 2\right)$$

$$c = \frac{1}{3} \text{ and } k = 2$$

(Reason: Both terms contain r, and $\frac{\pi r}{3} = r \times \frac{1}{3}\pi$, so taking r outside a bracket gives exactly the form $P = r(c\pi + k)$ that was asked for.)

$$P = r\left(\frac{1}{3}\pi + 2\right)$$

with $c = \frac{1}{3}$ and $k = 2$

Verification

✓ **Check 1:** Put a number in. With $r = 6$, work the perimeter out directly: the arc is one sixth of $2\pi \times 6$ and the two radii add 12. Then put $r = 6$ into the formula and compare. $\frac{1}{6} \times 2\pi \times 6 + 12 = 2\pi + 12$ and $6\left(\frac{1}{3}\pi + 2\right) = 2\pi + 12 \approx 18.28$

✓ **Check 2:** Six of these sectors fit round a point, so six of the arcs must make up the whole circumference. Multiply the arc by 6 and see whether the circumference comes back. $6 \times \frac{\pi r}{3} = 2\pi r$, which is the full circumference, so the fraction of the circle used is right.

✓ **Check 3:** Measure the angle in radians instead, which never uses 360 at all. $60^\circ = \frac{\pi}{3}$ radians, and an arc of angle θ radians has length $r\theta$. $r \times \frac{\pi}{3} + 2r = r\left(\frac{1}{3}\pi + 2\right)$, the same formula by a different route.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{60}{360} \times 2 \times \pi \times r$ oe, or $\frac{1}{6} \times 2 \times \pi \times r$ oe	M1	for finding the length of the arc	✓
their $\frac{60}{360} \times 2 \times \pi \times r + 2r$ oe	M1	dep on M1 for a complete expression from correct working for a method for the perimeter	✓
$P = r\left(\frac{1}{3}\pi + 2\right)$	A1	oe, eg $P = r(0.33\ldots\pi + 2)$ or $P = \left(\frac{1}{3}\pi + 2\right)r$ or	✓

Step	Mark	Description	Got it?
		$P = \left(2 + \dfrac{120}{360}\pi\right) r$ or $P = \left(\dfrac{120}{360}\pi + 2\right) r$	
Working not required, so a correct answer scores full marks, unless it comes from obviously incorrect working.	**Note**	guidance printed beside this question in the official mark scheme; it awards nothing on its own	✓

Full marks: 3/3

Question 14 - Exam Solution

Understanding the Question

Given

The drawing pin lands point up with probability 0.8, so point up and point down are its only two outcomes.
The spinner lands on 6 and the pin lands point up with probability 0.24.
The spin and the drop do not affect each other, so the two events are independent.

Find

The probability that the spinner lands on 6 and the drawing pin lands point down.

Plan the Solution

- The 0.24 is already a product of two probabilities, so work backwards from it to the probability of a 6.
- The pin has only two outcomes, so point down is 1 minus 0.8.
- Multiply the two probabilities, because the spinner and the pin are independent.

Worked Solution [3 marks]

> Rule - Independent events: $P(A \text{ and } B) = P(A) \times P(B)$, and reading that backwards gives $P(A) = \dfrac{P(A \text{ and } B)}{P(B)}$.

Step 1: Work back to the probability of a 6

$$P(6) \times 0.8 = 0.24$$

$$P(6) = \frac{0.24}{0.8} = 0.3$$

(Reason: The spin and the drop are independent, so the given 0.24 is the probability of a 6 multiplied by 0.8. Dividing 0.24 by 0.8 undoes that multiplication and leaves the probability of a 6 on its own.)

Step 2: Write down the probability that the pin lands point down

$$P(\text{point down}) = 1 - 0.8 = 0.2$$

(Reason: The pin lands point up or point down and there is no third outcome, so those two probabilities add to 1.)

Step 3: Multiply the two probabilities together

$$P(6 \text{ and point down}) = 0.3 \times 0.2 = 0.06$$

(Reason: Independent events multiply, so the probability of a 6 and point down is the probability of a 6 times the probability of point down.)

$$0.06$$

Verification

- ✓ **Check 1:** One spin and one drop have four outcomes in total, so their probabilities must add to 1. The other two are $P(\text{not } 6) = 1 - 0.3 = 0.7$, then $0.7 \times 0.8 = 0.56$ and $0.7 \times 0.2 = 0.14$. $0.24 + 0.56 + 0.14 + 0.06 = 1$
- ✓ **Check 2:** Reach the answer without finding the probability of a 6 at all. Point down is $\frac{0.2}{0.8} = 0.25$ as likely as point up, so scale the given 0.24 by that same ratio. $0.24 \times 0.25 = 0.06$
- ✓ **Check 3:** Redo the whole question in fractions, where nothing can be lost to a decimal point. The probability of a 6 is $\frac{3}{10}$ and point down is $\frac{1}{5}$. $\frac{3}{10} \times \frac{1}{5} = \frac{3}{50} = 0.06$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{0.24}{0.8} = 0.3$	M1	for a correct method to find the probability that the spinner lands on 6, or equivalent	✓

Step	Mark	Description	Got it?
$0.3 \times (1 - 0.8)$ or 0.3×0.2	M1	for a complete method, or equivalent, eg $1 - (0.3 \times 0.8 + 0.7 \times 0.8 + 0.7 \times 0.2)$ or $1 - 0.94$. The candidate's own value may be used in place of 0.3	✓
0.06	A1	or equivalent, eg $\frac{3}{50}$ or $\frac{6}{100}$ or 6%. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working	✓

Full marks: 3/3

Question 15 - Exam Solution

Understanding the Question

Given

AB, BC and CD are three sides of a regular pentagon, so they are equal and the angles at B and C inside the pentagon are equal.
BCE is a straight line, so the angle inside the pentagon at C and angle DCE sit side by side on it.
$CD = 6.5$ cm and $CE = 3$ cm - two sides of triangle CDE with the unknown angle between them.

Find

The area of triangle CDE, correct to 3 significant figures.

Plan the Solution

- Work out one interior angle of a regular pentagon. Its 5 angles are equal and they add to $(5 - 2) \times 180°$.
- Angle BCD is one of those interior angles, and BCE is straight, so angle DCE is what is left of $180°$.
- Triangle CDE then has two known sides with a known angle between them, which is exactly what $\frac{1}{2}ab\sin C$ asks for. No height has to be found.

Worked Solution [3 marks]

> Rule - Area from two sides and the included angle: $\text{Area} = \frac{1}{2}ab\sin C$, where a and b are two sides of the triangle and C is the angle BETWEEN them. The angle must be the one the two sides enclose, not either of the other two.

Step 1: Work out one interior angle of the regular pentagon

$(5-2)\times 180^\circ = 540^\circ$

$$\text{interior angle} = \frac{540^\circ}{5} = 108^\circ$$

(Reason: A polygon with n sides splits into $n-2$ triangles, so its angles add to $(n-2)\times 180^\circ$. A pentagon gives 540°, and in a REGULAR pentagon all 5 angles are equal, so each one takes a fifth of it.)

Step 2: Use the straight line BCE to find angle DCE

$\angle DCE = 180^\circ - 108^\circ = 72^\circ$

(Reason: Angle BCD is the interior angle just found. Angles on a straight line add to 180°, so angle DCE is the rest of that half turn. This is the exterior angle of the pentagon, which is why $\frac{360^\circ}{5}$ reaches the same 72° in one step.)

Step 3: Put the two sides and the angle between them into the area formula

$$\text{Area} = \frac{1}{2}\times 6.5 \times 3 \times \sin 72^\circ = 9.2728\ldots$$

(Reason: The two sides are $CD = 6.5$ cm and $CE = 3$ cm, and the angle between them at C is the 72° found in Step 2. Make sure the calculator is in degree mode.)

Step 4: Round to 3 significant figures

$9.2728\ldots \to 9.27$

(Reason: The first three significant figures are 9, 2 and 7. The next digit is 2, which is below 5, so the 7 stays as it is. The unit is square centimetres, because both lengths are in centimetres.)

$$9.27\ \text{cm}^2$$

Verification

✓ **Check 1:** Do it without the area formula. Drop a perpendicular from D to the straight line BCE. Its height is $6.5\times\sin 72^\circ$, and the base CE is 3 cm, so use half base times height. $h = 6.1818\ldots$ and $\frac{1}{2}\times 3\times 6.1818\ldots = 9.2728\ldots$

✓ **Check 2:** A route with no sine in it at all. Find the third side with the cosine rule, $DE^2 = 6.5^2 + 3^2 - 2 \times 6.5 \times 3 \times \cos 72°$, then put all three sides into Heron's formula $\sqrt{s(s-a)(s-b)(s-c)}$. $DE = 6.2608\ldots$, $s = 7.8804\ldots$ and the formula gives $9.2728\ldots$

✓ **Check 3:** A size check. Two sides of 6.5 cm and 3 cm enclose the largest area they can when the angle between them is $90°$, and that area is $\frac{1}{2} \times 6.5 \times 3 = 9.75$ cm². The angle here is $72°$, not far below $90°$, so the answer must be a little under 9.75. It is: 9.27 cm² is just under the 9.75 cm² ceiling, and far above zero, which is where a very small angle would put it.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A method for an exterior or an interior angle of a regular pentagon: $\frac{360}{5} = 72$ oe, or $\frac{(5-2) \times 180}{5} = 108$ oe, or $\frac{540}{5} = 108$ oe	**M1**	Do not award this mark if 108 is assigned as an exterior angle, or if 72 is assigned as an interior angle. Angles written on the diagram other than the exterior or interior angles of the pentagon are ignored, even where they are labelled incorrectly.	✓
Substitute into $\frac{1}{2} \times 6.5 \times 3 \times \sin[\text{angle } DCE]$ oe, or find the height first, $h = 6.5 \times \sin[\text{angle } DCE]$ (= 6.18...), and then $\frac{1}{2} \times 3 \times h$ oe	**M1ft**	Follow through on the candidate's own angle DCE when it is substituted in, provided that angle is less than $90°$.	✓
Working is not required, so a correct answer scores full marks (unless it comes from obviously incorrect working).	**A1**	For 9.27. Accept anything from 9.26 to 9.28, which covers rounding the angle or the area part way through.	✓
Special case: $\frac{1}{2} \times 6.5 \times 3 \times \sin 108° = 9.27$ ...	**SC B2**	The named error is using the pentagon's interior angle as the angle inside triangle CDE, instead of the angle beside it on the straight line. It still produces 9.27, because $\sin 108° = \sin 72°$, so the	✓

Step	Mark	Description	Got it?
		arithmetic hides the mistake and the work scores 2 of the 3 marks rather than all of them. This row carries no mark of its own.	

Full marks: 3/3

Question 16 - Exam Solution

Understanding the Question

Given

Six sketch graphs, labelled A to F. The axes carry no numbers, so only the SHAPE of each curve can be used.

Two equations to place: $y = -\dfrac{1}{x}$ and $y = \sin x^\circ$.

Find

One letter for each equation - the sketch that could be its graph.

Plan the Solution

- Name the family each equation belongs to. $y = -\dfrac{1}{x}$ is a reciprocal curve; $y = \sin x^\circ$ is a wave that repeats.
- Write down the feature that family must show, then look for it. A reciprocal has no value at $x = 0$, so it is drawn in two separate branches; a sine wave passes through the origin and turns over both above and below the axis.
- Two sketches can belong to the same family, so finish each part with one tested value: the sign of y on each side of the y-axis for the reciprocal, and the height at $x = 0$ for the wave.

Worked Solution [2 marks]

> Rule - Match a graph by its family first, then by one tested value. $y = \dfrac{k}{x}$ has two separate branches and no value at $x = 0$, and the sign of k decides which pair of quadrants they lie in. $y = \sin x^\circ$ passes through $(0, 0)$, reaches 1 at $x = 90$, returns to 0 at $x = 180$, falls to -1 at $x = 270$ and then repeats every 360°.

Step 1: Read off what $y = -\dfrac{1}{x}$ must look like

$y = -\dfrac{1}{x},\ x \neq 0$

(Reason: Dividing by 0 has no meaning, so the curve has a break at $x = 0$ and is drawn in two separate branches, one on each side of the y-axis. Only one of the six sketches is drawn in two pieces; the other five are single unbroken curves.)

Step 2: Test the sign of y on each side of the y-axis

$x = -2 \implies y = -\dfrac{1}{-2} = \dfrac{1}{2}$

$x = 2 \implies y = -\dfrac{1}{2}$

(Reason: A negative divided by a negative is positive, so to the left of the y-axis the curve lies ABOVE the x-axis, and to the right of it the curve lies below. The two branches therefore sit in the second and fourth quadrants: top left and bottom right.)

Step 3: Pick the sketch with those two branches

left branch: $y > 0$ right branch: $y < 0$

(Reason: Sketch E is the only one drawn in two pieces, and its pieces sit where the signs say: above the axis on the left, below it on the right. Both pieces flatten towards the x-axis as x grows, which is what $-\dfrac{1}{x}$ does once the denominator is large - at $x = 10$ the height is only -0.1.)

Step 4: Read off what $y = \sin x°$ must look like

$\sin 0° = 0$

$\sin 90° = 1$

$\sin 180° = 0$

$\sin 270° = -1$

$\sin 360° = 0$

(Reason: The wave starts at the origin, climbs to 1, comes back down through the axis, drops to -1 and returns - then does the same again. So the graph must pass THROUGH the origin, and be rising as it does so. Two of the six sketches are waves.)

Step 5: Separate the two wave sketches

$\sin 0° = 0$

$\cos 0° = 1$

(Reason: Sketches A and D are both waves. D is at the top of a crest where it meets the y-axis, so its graph is at its greatest height when $x = 0$ - that is the shape of $y = \cos x°$. Sketch A cuts the axis at the origin and rises, which is what $\sin 0° = 0$ requires. So A is the sine curve.)

(i) E

(ii) A

Verification

✓ **Check 1:** Test more points on $y = -\frac{1}{x}$, two close to the y-axis and one far from it: $x = -0.5$, $x = 0.5$ and $x = 10$. $y = 2$, $y = -2$ and $y = -0.1$. The left branch climbs steeply near the y-axis and the right branch flattens towards the x-axis far from it, which is sketch E.

✓ **Check 2:** Rule the other five sketches out of part (i). B and C are single curves with a value at $x = 0$, A and D are waves that cross the x-axis, and F climbs through the origin without a break. None of the five has the break at $x = 0$ that $-\frac{1}{x}$ must have, so E is the only sketch left.

✓ **Check 3:** Use symmetry for part (ii). Sine is an odd function: $\sin(-30)^\circ = -0.5$ while $\sin 30^\circ = 0.5$, so the curve has half-turn symmetry about O - whatever it does to the right of the y-axis, it does upside down to the left. Sketch A has a crest to the right of O and a matching trough the same distance to the left, so it has that symmetry. Sketch D is a mirror image in the y-axis instead, which is the symmetry of a cosine curve, so A is the sine.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) E	**B1**	For E. Working is not required, so the letter alone earns the mark. This is a B mark, given for the answer itself, so there is no method mark on this part and a correct description carrying the wrong letter earns nothing.	✓
(ii) A	**B1**	For A. Again the letter alone earns the mark. D is the near miss this part is built around: it is the same wave shifted by 90°, so it is the graph of $y = \cos x^\circ$ and it scores nothing.	✓

Full marks: 2/2

Question 17 - Exam Solution

Understanding the Question

Given

$f(x) = \dfrac{x}{2x-4}$ and $g(x) = 3x+1$,

two functions of x

The composite function satisfies

$fg(k) = 2$

Find

The value of k, a single number

Plan the Solution

- fg means g first and f second, so work out $g(k) = 3k+1$ and put the whole of it in place of x in f
- Multiply out the bracket in the denominator so the composite becomes one single algebraic fraction.
- Set that fraction equal to 2, multiply both sides by the denominator to clear the fraction, then solve the linear equation that is left

Worked Solution [3 marks]

> Rule - Composite functions: $fg(x)$ means $f(g(x))$. The inner function g is applied first, and the whole of its output replaces every x in f.

Step 1: Work out $g(k)$ and feed it into f

$$g(k) = 3k+1$$

$$fg(k) = f(3k+1) = \frac{3k+1}{2(3k+1)-4}$$

(Reason: $fg(k)$ means $f(g(k))$, so the whole bracket $3k+1$ replaces x in the numerator and in the denominator of f at the same time)

Step 2: Simplify the denominator

$$2(3k+1) - 4 = 6k + 2 - 4 = 6k - 2$$

$$fg(k) = \frac{3k+1}{6k-2}$$

(Reason: The 2 outside the bracket multiplies both terms inside it, and only then are the two numbers collected, so the composite is a single fraction ready to be solved)

Step 3: Set $fg(k) = 2$ and clear the fraction

$$\frac{3k+1}{6k-2} = 2$$

$3k + 1 = 2(6k - 2)$

$3k + 1 = 12k - 4$

(Reason: Multiplying both sides by $6k - 2$ removes the fraction and leaves an ordinary linear equation, which is the step the mark scheme calls correctly removing the denominator)

Step 4: Solve the linear equation for k

$1 + 4 = 12k - 3k$

$5 = 9k$

$k = \frac{5}{9}$

(Reason: Collecting the k terms on the right, where the coefficient is larger, keeps that coefficient positive and avoids a sign slip)

$$k = \frac{5}{9}$$

Verification

✓ **Check 1 - put $k = \frac{5}{9}$ back through both functions:** Inner function first: $\mathrm{g}\left(\frac{5}{9}\right) = 3 \times \frac{5}{9} + 1 = \frac{5}{3} + 1 = \frac{8}{3}$. Feeding that into f gives the denominator $2 \times \frac{8}{3} - 4 = \frac{4}{3}$. $\mathrm{fg}\left(\frac{5}{9}\right) = \frac{8}{3} \times \frac{3}{4} = 2$

✓ **Check 2 - work backwards through the two functions instead:** Ask first which input f sends to 2: $\frac{x}{2x - 4} = 2$ gives $x = 2(2x - 4) = 4x - 8$, so $3x = 8$ and $x = \frac{8}{3}$. That input has to be $\mathrm{g}(k)$. $3k + 1 = \frac{8}{3} \implies 3k = \frac{5}{3} \implies k = \frac{5}{9}$

✓ **Check 3 - the value is one the composite is allowed to take:** The composite $\frac{3k + 1}{6k - 2}$ is undefined only where its denominator is zero, that is at $k = \frac{1}{3}$. At $k = \frac{5}{9}$ the denominator is $6 \times \frac{5}{9} - 2 = \frac{4}{3}$, which is not zero, so the answer is a genuine solution

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Form $\mathrm{fg}(k)$ by substituting $\mathrm{g}(k)$ into f	**M1**	For a correct expression for $\mathrm{fg}(k)$ or $\mathrm{fg}(x)$, e.g. $\dfrac{3k+1}{2(3k+1)-4}$ or $\dfrac{3k+1}{6k-2}$, with or without $=2$. Or for starting from $\mathrm{f}(x)=2$ and reaching $x=2(2x-4)$ or $x=4x-8$ or $x=\dfrac{8}{3}$. Allow x instead of k for all marks.	✓
Clear the denominator to form a correct equation	**M1 dep**	Dependent on the first M1, for correctly removing the denominator to form a correct equation, e.g. $3k+1=2(6k-2)$ or $3k+1=2\left(2(3k+1)-4\right)$ or $3k+1=12k-4$. Or, by the backwards route, for $\mathrm{g}(k)=\dfrac{8}{3}$, i.e. $3k+1=\dfrac{8}{3}$.	✓
Solve the linear equation for k	**A1**	For $k=\dfrac{5}{9}$ or any equivalent, e.g. $0.55(555\ldots)$ rounded or truncated, the bracketed digits being optional, so 0.55 or better, or $0.\dot{5}$ with the recurring dot shown.	✓
A correct answer written down with no working	**Note**	Working is not required in this question, so a fully correct answer scores all 3 marks, unless it has come from obviously incorrect working.	✓

Full marks: 3/3

Question 18 - Exam Solution

Understanding the Question

Given

The recurring decimal $0.\dot{3}0\dot{6}$. The dots mark the first and last digits of the block that repeats, so the digits after the point run 306 over and over without stopping.

The fraction it is claimed to equal, $\dfrac{34}{111}$.

Find

An algebraic argument that $0.\dot{3}0\dot{6}$ is exactly $\dfrac{34}{111}$. The working is the answer here, so a decimal read off a calculator proves nothing on its own.

Plan the Solution

- Give the decimal a name: let $x = 0.\dot{3}0\dot{6}$.
- Count the digits in the repeating block. There are three of them, so multiply by 1000 to move the point past exactly one whole block.
- Subtract the original equation from the shifted one. The two never-ending tails are identical, so they cancel and a whole number is left.
- Solve that equation, then cancel the fraction by the HCF of its numerator and its denominator.

Worked Solution [2 marks]

> Rule - Recurring decimal to fraction: if the repeating block is n digits long, multiply by 10^n, subtract the original, and divide by $10^n - 1$. The subtraction is what removes the tail that never ends.

Step 1: give the decimal a name

$x = 0.\dot{3}0\dot{6} = 0.306306306\ldots$

(Reason: Naming the decimal is what makes algebra possible. The dots say the block 306 repeats for ever, so the digits after the point are 306 again and again.)

Step 2: shift the point past one whole block

$1000x = 306.306306\ldots$

(Reason: The block is three digits long, so multiplying by 1000 moves the point three places. Everything after the point is left exactly as it was, and that is the whole trick.)

Step 3: subtract the original from the shifted copy

$1000x - x = 306.306306\ldots - 0.306306\ldots$

$999x = 306$

(Reason: The two tails are identical, so they cancel exactly and a whole number is left behind. Nothing has been rounded, which is what makes this a proof rather than a check.)

Step 4: divide to leave x on its own

$$x = \frac{306}{999}$$

(Reason: Both sides are divided by the number standing in front of x. The fraction that comes out is not yet in its lowest terms.)

Step 5: cancel to lowest terms

$$\frac{306}{999} = \frac{34}{111}$$

(Reason: The HCF of 306 and 999 is 9. Dividing top and bottom by that same 9 gives 34 over 111, which is the fraction the question asked for.)

$$x = 0.\dot{3}0\dot{6} = \frac{306}{999} = \frac{34}{111}$$

Verification

✓ **Check 1 - divide the fraction back out:** Work out 34 divided by 111 and read the digits after the point. $\frac{34}{111} = 0.306306306\ldots$, the block 306 repeating, which is the decimal the question started from.

✓ **Check 2 - multiply the fraction back up:** Step 3 left $999x = 306$. Putting the fraction back in for x must give exactly 306, with nothing left over. $999 \times \frac{34}{111} = \frac{33\,966}{111} = 306$

✓ **Check 3 - the fraction is in its lowest terms:** Factorise both parts: $34 = 2 \times 17$ and $111 = 3 \times 37$. They share no factor at all, so $\frac{34}{111}$ cannot be cancelled any further and is the finished fraction.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two correct equations, written ready to subtract, e.g. $1000x = 306.306306\ldots$ and $x = 0.306306\ldots$	**M1**	M1 for two correct algebraic equations involving the recurring decimal that, when subtracted, give a whole number or a terminating decimal (306 or $306\,000$), with the intention to subtract. The larger pair $1\,000\,000x = 306\,306.306306\ldots$ and $1000x = 306.306306\ldots$ earns it just as well.	✓
Subtract, solve and cancel: $999x = 306$ and $\frac{306}{999} = \frac{34}{111}$	**A1**	A1 for completion to $\frac{34}{111}$, dep on M1. The larger pair finishes the same way: $999\,000x = 306\,000$ and $\frac{306\,000}{999\,000} = \frac{34}{111}$.	✓
Working required	**Note**	If the recurring dots are not written on both numbers, at least one of them must be shown to at least six significant figures before the M1 can be given. A bare $\frac{34}{111}$ with no algebra behind it	✓

Step	Mark	Description	Got it?
		scores nothing: the question asks for the algebra, and the algebra is what is being marked.	

Full marks: 2/2

Question 19 - Exam Solution

Understanding the Question

Given

An average speed of 19 km/h, correct to the nearest whole number.
A time of 1.5 hours, correct to one decimal place.
Both figures have been rounded, so each one stands for a whole interval of possible values rather than a single exact number.

Find

The upper bound of the distance travelled, correct to 3 significant figures.
An upper bound is the value the distance can be pushed as close to as you like without ever quite reaching it, so it is the top end of an interval rather than a distance anyone actually rides.

Plan the Solution

- Write s for the speed, t for the time and d for the distance.
- Turn each rounded figure back into the interval it came from. A whole number of km/h carries 0.5 either side; one decimal place carries only 0.05 either side.
- Decide which end of each interval makes the distance as large as it can be.
- Multiply those two values, then round the product to 3 significant figures.

Worked Solution [3 marks]

> Rule - Bounds of a product: a figure rounded to a unit u lies within $\frac{u}{2}$ of the figure written down, so its bounds are $\text{figure} - \frac{u}{2}$ and $\text{figure} + \frac{u}{2}$. A product of two positive quantities grows whenever either one of them grows, so $UB_d = UB_s \times UB_t$.

Step 1: the interval the speed came from

$19 - 0.5 = 18.5$

$19 + 0.5 = 19.5$

$18.5 \leq s < 19.5$

(Reason: The speed was rounded to the nearest whole number, so the rounding unit is 1 km/h and half of it is 0.5. Every speed from 18.5 up to but not including 19.5 rounds to 19, which makes 19.5 the upper bound.)

Step 2: the interval the time came from

$1.5 - 0.05 = 1.45$

$1.5 + 0.05 = 1.55$

$1.45 \leq t < 1.55$

(Reason: One decimal place makes the rounding unit 0.1 hours, not 1 hour, so half of it is 0.05 and not 0.5. A time of 1.55 would itself round up to 1.6, so the interval stops just short of 1.55 and that value is the upper bound.)

Step 3: multiply the two bounds that make the distance largest

$d = s \times t$

$UB_d = UB_s \times UB_t = 19.5 \times 1.55 = 30.225$

(Reason: Distance is speed multiplied by time, and a product of two positive numbers gets bigger whenever either number gets bigger. The largest distance therefore comes from the largest speed together with the largest time.)

Step 4: round to 3 significant figures

$30.225 \approx 30.2$

(Reason: The first three significant figures of 30.225 are 3, 0 and 2. The digit after them is 2, which is below 5, so the tenths digit is left as it stands.)

30.2 km

Verification

✓ **Check 1 - a pair from inside both intervals:** Take a speed and a time from just inside each interval, 19.49 km/h and 1.549 hours, and round each one the way the question rounds it. $19.49 \approx 19$ to the nearest whole number and $1.549 \approx 1.5$ to one decimal place, so this ride fits the question. Its distance is $19.49 \times 1.549 = 30.19001$ km, which sits just below the bound and never above it.

✓ **Check 2 - divide the product back out:** Undo the multiplication: divide the unrounded distance by each upper bound in turn and see whether the other one comes back. $\dfrac{30.225}{1.55} = 19.5$ and $\dfrac{30.225}{19.5} = 1.55$, the two upper bounds the product was built from.

✓ **Check 3 - every other pairing of the bounds is smaller:** The two intervals allow four corner products. Work out the other three: 18.5×1.45, 19.5×1.45 and 18.5×1.55.

They come to 26.825, 28.275 and 28.675, all smaller than 30.225. So the distance lies between 26.825 km and 30.225 km, and pairing an upper bound with a lower one always falls short of the top of that interval.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One correct bound: 18.5 or 19.5 or 1.45 or 1.55	**B1**	B1 for one correct bound. Allow $19.4\dot{9}$ for 19.5, and allow $1.54\dot{9}$ for 1.55. Those recurring forms are equal to the bounds exactly, not merely close to them.	✓
(distance $=$) 19.5×1.55	**M1**	M1 for $UB_s \times UB_t$, where $19 < UB_s \leq 19.5$ and $1.5 < UB_t \leq 1.55$. The upper limits are what let a candidate who writes $19.4\dot{9}$ or 1.549 keep the method mark.	✓
30.2	**A1**	A1 for 30.2. Accept 30.225 or 30.23. The answer must come from the correct figures, 19.5 and 1.55.	✓
Working not required	**Note**	A correct answer written on the answer line scores all 3 marks on its own, unless it has obviously come from incorrect working.	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

The quadratic inequality $6x^2 - 7x - 20 > 0$.
The coefficient of x^2 is 6, which is positive, so the curve $y = 6x^2 - 7x - 20$ is U-shaped.

Find

Every value of x that makes $6x^2 - 7x - 20$ greater than 0. A quadratic gives two critical values, so expect an answer in two separate pieces.

Plan the Solution

- Factorise $6x^2 - 7x - 20$ by splitting the middle term, using the product $6 \times (-20) = -120$ and the sum -7.

- Set each bracket equal to 0 to get the two critical values.
- Test one value from each of the three regions to see where the expression is positive.
- Write the answer as two strict inequalities, because a U-shaped curve sits above the x-axis outside its roots.

Worked Solution [4 marks]

> Rule - Quadratic inequality: factorise, find the critical values, then read the regions off the shape of the curve. When the coefficient of x^2 is positive, the expression is positive outside the critical values and negative between them.

Step 1: Factorise $6x^2 - 7x - 20$

$6 \times (-20) = -120$

$-15 \times 8 = -120$

$-15 + 8 = -7$

$6x^2 - 15x + 8x - 20$

$3x(2x - 5) + 4(2x - 5)$

$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$

(Reason: Multiply the 6 by the -20 and look for two numbers with that product whose sum is -7. They are -15 and 8, so the middle term splits and the four terms pair into two brackets.)

Step 2: Find the two critical values

$(3x + 4)(2x - 5) = 0$

$3x + 4 = 0 \implies x = -\dfrac{4}{3}$

$2x - 5 = 0 \implies x = \dfrac{5}{2}$

(Reason: A product is zero only when one of its factors is zero, so each bracket gives one critical value. These are the two points where the curve meets the x-axis.)

Step 3: Test one value in each region

$(3 \times (-2) + 4)(2 \times (-2) - 5) = 18$

$(3 \times 0 + 4)(2 \times 0 - 5) = -20$

$(3 \times 3 + 4)(2 \times 3 - 5) = 13$

(Reason: The two critical values cut the number line into three regions. Testing $x = -2$, $x = 0$ and $x = 3$ gives positive, then negative, then positive - which is exactly what a U-shaped curve does.)

Step 4: Write down the solution

$$x < -\frac{4}{3} \quad \text{or} \quad x > \frac{5}{2}$$

(Reason: The two outside regions are the positive ones. The inequality is strict, so the critical values themselves are left out: at $x = -\frac{4}{3}$ and $x = \frac{5}{2}$ the expression equals 0, which is not greater than 0.)

$$x < -\frac{4}{3} \quad \text{or} \quad x > \frac{5}{2}$$

Verification

✓ **Check 1:** Expand $(3x+4)(2x-5)$ again and compare it with the expression the question gives. $6x^2 - 15x + 8x - 20 = 6x^2 - 7x - 20$

✓ **Check 2:** Reach the critical values a second way, with the quadratic formula. The discriminant is $(-7)^2 - 4 \times 6 \times (-20) = 529$, and $\sqrt{529} = 23$. $x = \frac{7+23}{12} = \frac{5}{2}$ and $x = \frac{7-23}{12} = -\frac{4}{3}$

✓ **Check 3:** Substitute a value from each piece of the answer, and one from between the critical values, into $6x^2 - 7x - 20$. $6(-3)^2 - 7(-3) - 20 = 55$, $6(4)^2 - 7(4) - 20 = 48$, $6(0)^2 - 7(0) - 20 = -20$

✓ **Check 4:** Confirm the critical values really are excluded, by putting the upper one back into the expression. $6\left(\frac{5}{2}\right)^2 - 7\left(\frac{5}{2}\right) - 20 = 0$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A first step towards the critical values: factorising to $(3x+4)(2x-5)$, or substituting into $\frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 6 \times (-20)}}{2 \times 6}$ oe, or completing the square to $6\left[\left(x - \frac{7}{12}\right)^2 - \left(\frac{7}{12}\right)^2\right] - 20$ oe.	M1	If factorising in the form $(ax+b)$ with a and b integers, allow brackets which expand to give 2 out of 3 terms correct. If using the formula or completing the square, allow one sign error and some simplification, as far as $\frac{7 \pm \sqrt{49+480}}{12}$ oe, or $6\left(x - \frac{7}{12}\right)^2 - \frac{529}{24}$ oe, or $\left(x - \frac{7}{12}\right)^2 - \frac{529}{144}$ oe.	✓

Step	Mark	Description	Got it?
Both critical values: $x = -\frac{4}{3}$ and $x = \frac{5}{2}$ oe.	A1	Dependent on the M1, and only for two correct critical values. Accept $-1.3\ldots$. The candidate may write $<, \leq, >$ or $\geq$ instead of $=$.	✓
Both regions written in the correct form: $x < a$ and $x > b$, where a is their lower critical value and b is their upper critical value.	M1ft	Dependent on the M1 and on two critical values having been found. Follow through on the candidate's own values. Also award for $x > \frac{5}{2}$ oe alone, or $x < -\frac{4}{3}$ oe alone, or $-\frac{4}{3} > x > \frac{5}{2}$ oe.	✓
$x < -\frac{4}{3}$ and $x > \frac{5}{2}$ oe.	A1	Dependent on the previous M1. Working is required. Accept $-1.3\ldots$, or $\left(-\infty, -\frac{4}{3}\right), \left(\frac{5}{2}, (+)\infty\right)$, or $\left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{5}{2}, (+)\infty\right)$. Do not ignore subsequent working.	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

$ABCD$ is a square, so all four sides are the same length and each pair of adjacent sides meets at a right angle.

Find

An equation of the line through B and C, written in the form $ax + by + c = 0$ with a, b and c integers. BC is the side next to AB, so the line BC is

Two adjacent vertices are given: $A(-5, 2)$ and $B(3, 5)$, so AB is one side of the square.

perpendicular to AB and it passes through B. A gradient and a point are all a line needs, so C itself never has to be found.

Plan the Solution

- Work out the gradient of AB from the two coordinates the question gives.
- Turn that into the gradient of BC with the perpendicular rule.
- Substitute $B(3, 5)$ into $y = mx + c$ to find c.
- Multiply through by 3 to clear the fraction, then collect every term on one side so that a, b and c come out as integers.

Worked Solution [4 marks]

> Rule - Perpendicular gradients: adjacent sides of a square meet at $90°$, so their gradients multiply to -1. One gradient is therefore the negative reciprocal of the other: turn the fraction upside down and change its sign. Gradient from two points: divide the change in y by the change in x, taking both differences in the same order.

Step 1: Find the gradient of AB

$$m_{AB} = \frac{5-2}{3-(-5)} = \frac{3}{8}$$

(Reason: Subtract the y-coordinates and the x-coordinates in the same order, B take away A both times. The denominator is $3 - (-5) = 8$, not 2 - subtracting a negative adds.)

Step 2: Turn it into the gradient of BC

$$\frac{3}{8} \times m_{BC} = -1$$

$$m_{BC} = -\frac{8}{3}$$

(Reason: AB and BC are adjacent sides of a square, so they meet at B at a right angle and their gradients multiply to -1. Invert the fraction and change the sign.)

Step 3: Use $B(3, 5)$ to find the intercept

$$5 = -\frac{8}{3} \times 3 + c$$

$$-\frac{8}{3} \times 3 = -8$$

$$c = 5 - (-8) = 13$$

(Reason: The line passes through B, so $x = 3$ and $y = 5$ must fit $y = mx + c$. The 3 cancels the denominator, which is why this question was set with an x-coordinate of 3.)

Step 4: Rearrange into the form the question asks for

$$y = -\frac{8}{3}x + 13$$
$$3y = -8x + 39$$
$$8x + 3y - 39 = 0$$

(Reason: Multiplying every term by 3 clears the fraction, and moving the x-term and the number across leaves 0 on the right. That gives $a = 8$, $b = 3$ and $c = -39$, which are integers as required.)

$$8x + 3y - 39 = 0$$

Verification

✓ **Check 1:** Put $B(3, 5)$ back into the left-hand side. If B is on the line it must come out as 0. $8 \times 3 + 3 \times 5 - 39 = 24 + 15 - 39 = 0$

✓ **Check 2:** Find C for real, as a test of the answer rather than a way to get it. Going from A to B is 8 across and 3 up; turning that through a right angle at B puts C at either $(6, -3)$ or $(0, 13)$. Both are corners of a genuine square, and both must satisfy the equation - which is why the answer is the same either way.
$8 \times 6 + 3 \times (-3) - 39 = 48 - 9 - 39 = 0$ and
$8 \times 0 + 3 \times 13 - 39 = 39 - 39 = 0$

✓ **Check 3:** Read the gradient back off the answer: $3y = -8x + 39$ gives $y = -\frac{8}{3}x + 13$. Multiply that gradient by the gradient of AB - a right angle must give -1.
$-\frac{8}{3} \times \frac{3}{8} = -1$

✓ **Check 4:** Confirm the shape really is a square and not just a rectangle, by squaring both side lengths with Pythagoras. Going A to B is 8 across and 3 up; going B to $(6, -3)$ is 3 across and 8 down. $8^2 + 3^2 = 73$ and $3^2 + 8^2 = 73$, so both sides have length $\sqrt{73}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A method for the gradient of AB, eg $\frac{5-2}{3-(-5)} = \frac{3}{8} = 0.375$ oe, or	**M1**	For a method to find the gradient of AB, or for finding the possible coordinates of C.	✓

Step	Mark	Description	Got it?
from $2 = -5m + c$ and $5 = 3m + c$ leading to $(m =)\frac{3}{8}$ oe. Or the possible coordinates of C: $(C =)(6, -3)$ or $(0, 13)$.			
eg $\left[\frac{3}{8}\right] \times m = -1$ oe, or their $(m =) -\frac{8}{3}$ oe, or $\frac{5-(-3)}{3-6} = -\frac{8}{3}$, or $\frac{5-13}{3-0} = -\frac{8}{3}$, each of which is $-2.6(666\ldots)$ as a decimal.	**M1ft**	(indep) For finding the gradient of BC. Allow the perpendicular gradient to be truncated or rounded to 1 dp. $\left[\frac{3}{8}\right]$ means their gradient of AB.	✓
eg $5 = -\frac{8}{3} \times 3 + c$, or $c = 13$, or $y = -\frac{8}{3}x + 13$, or $y - 5 = -\frac{8}{3}(x - 3)$ oe, or $y - (-3) = -\frac{8}{3}(x - 6)$ oe, or $y - 13 = -\frac{8}{3}(x - 0)$ oe.	**M1ft**	(ft dep on the previous M1, on their own perpendicular gradient) For substitution to find c, or for finding an equation for BC. If students find the coordinates of D, which are $(-2, -6)$ or $(-8, 10)$, then allow this mark for $y - (-6) = -\frac{8}{3}(x - (-2))$ oe or $y - 10 = -\frac{8}{3}(x - (-8))$ oe.	✓
$8x + 3y - 39 = 0$	**A1**	oe, with a, b and c integers, eg $16x + 6y - 78 = 0$ or $-8x - 3y + 39 = 0$ or $3y = -8x + 39$. Working is not required, so a correct answer scores full marks unless it comes from obvious incorrect working.	✓

Full marks: 4/4

Question 22 - Exam Solution

Understanding the Question

Given

$x^2 + y^2 = y + 11$, a curve (it is a circle)
$y = 3x - 1$, a straight line
Clear algebraic working is asked for, so the values on their own would earn nothing.

Find

Every pair of values x and y that fits both equations at the same time. A quadratic appears on the way, so expect two solution pairs.

Plan the Solution

- The line already gives y in terms of x, so substitute it into the curve equation.
- Expand, then collect every term on one side to leave a three term quadratic in x alone.
- Solve that quadratic by factorising; the quadratic formula would give the same two values.
- Put each value of x back into $y = 3x - 1$ to find its partner value of y.
- Keep each pair together, then test both pairs in the original equations.

Worked Solution [5 marks]

Rule, substitution (line into curve): where a line $y = mx + c$ meets a curve, replace y in the curve equation by $mx + c$, solve the quadratic in x that follows, then use the line again to find each y.

Step 1: Substitute the line into the curve

$$x^2 + (3x - 1)^2 = (3x - 1) + 11$$

(Reason: The line gives y in terms of x, so every y in the curve equation can be replaced by $3x - 1$. That leaves one equation in x only, which is what earns the first method mark.)

Step 2: Expand the squared bracket

$$(3x - 1)^2 = 9x^2 - 6x + 1$$
$$x^2 + 9x^2 - 6x + 1 = 3x + 10$$

(Reason: $(3x - 1)^2$ means $(3x - 1)(3x - 1)$, which gives $9x^2$, two lots of $-3x$ and $+1$. Squaring each term separately is the classic slip here. On the right, $-1 + 11 = 10$.)

Step 3: Collect the terms on one side

$$10x^2 - 6x + 1 - 3x - 10 = 0$$
$$10x^2 - 9x - 9 = 0$$

(Reason: Take everything across so the equation reads $ax^2 + bx + c = 0$. Here $x^2 + 9x^2 = 10x^2$, $-6x - 3x = -9x$ and $1 - 10 = -9$.)

Step 4: Factorise and solve for x

$$(5x+3)(2x-3)=0$$

$$5x+3=0 \implies x=-\frac{3}{5}=-0.6$$

$$2x-3=0 \implies x=\frac{3}{2}=1.5$$

(Reason: Two brackets multiply to zero only when one of them is zero, so each bracket gives a value. Checking the factorisation: $5x \times 2x = 10x^2$, $3 \times (-3) = -9$, and the middle terms are $-15x + 6x = -9x$. The quadratic formula would give $\frac{9 \pm \sqrt{81+360}}{20} = \frac{9 \pm 21}{20}$, the same two values.)

Step 5: Find the matching y for each x

$$y = 3 \times (-0.6) - 1 = -2.8$$

$$y = 3 \times 1.5 - 1 = 3.5$$

(Reason: Each value of x has its own y. Substitute into the line rather than the curve: it is linear, so it produces one value and cannot introduce an extra one.)

Step 6: Pair the values correctly

$$x=-0.6,\ y=-2.8$$

$$x=1.5,\ y=3.5$$

(Reason: The pairs must stay together: $x=-0.6$ belongs with $y=-2.8$, not with $y=3.5$. The final mark is for four correct values, correctly labelled, or written as the two coordinate pairs.)

$$x=-0.6,\ y=-2.8$$

$$x=1.5,\ y=3.5$$

Verification

✓ **Check 1, the pair $(-0.6, -2.8)$:** Put both values into the curve equation $x^2+y^2=y+11$, working each side out separately, then test the line as well. $0.36+7.84=8.2$ and $-2.8+11=8.2$, so the two sides agree, and the line gives $3 \times (-0.6) - 1 = -2.8$.

✓ **Check 2, the pair $(1.5, 3.5)$:** The same test on the second pair: x^2+y^2 on the left, $y+11$ on the right, then the line. $2.25+12.25=14.5$ and $3.5+11=14.5$, so this pair fits too, and the line gives $3 \times 1.5 - 1 = 3.5$.

✓ **Check 3, the two roots against the quadratic:** A quadratic $ax^2+bx+c=0$ has roots that add to $-\frac{b}{a}$ and multiply to $\frac{c}{a}$. For $10x^2-9x-9=0$ those are $\frac{9}{10}$ and $-\frac{9}{10}$.

$-0.6 + 1.5 = 0.9$ and $-0.6 \times 1.5 = -0.9$, so the two values are exactly the roots of that quadratic, with nothing missing and nothing spare.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$x^2 + (3x-1)^2 = 3x - 1 + 11$	**M1**	For substituting $y = 3x - 1$ into $x^2 + y^2 = y + 11$ to obtain an equation in x only. Substituting $x = \dfrac{\pm y \pm 1}{3}$ to obtain an equation in y only scores the same mark, with either arrangement of the two signs.	✓
$10x^2 - 9x - 9 = 0$	**M1 dep**	Dependent on the previous M1. For multiplying out and collecting terms to form a three term quadratic in any form of $ax^2 + bx + c = 0$, with at least two of a, b, c correct. Or equivalent, such as $10x^2 - 9x = 9$. Working in y instead, the quadratic is $10y^2 - 7y - 98 = 0$.	✓
$(5x+3)(2x-3) = 0$	**M1ft**	Follow through, dependent on the first M1. For solving their three term quadratic by any correct method: factorising, completing the square, or the formula, allowing one sign error and some simplification, as far as $\dfrac{9 \pm \sqrt{81+360}}{20}$ or $\dfrac{7 \pm \sqrt{49+3920}}{20}$. If factorising, brackets which expand to give two of the three terms correct are enough. Correct values for x or for y with no method shown also score it, and the labels may be the wrong way round for this mark only.	✓
$y = 3 \times (-0.6) - 1 = -2.8$ and $y = 3 \times 1.5 - 1 = 3.5$	**M1ft**	Follow through, dependent on the previous M1. For substituting their two found values of x (or of y) into either of the two given equations, or for fully correct values of the other variable.	✓

Step	Mark	Description	Got it?
$x = -0.6,\ y = -2.8$ and $x = 1.5,\ y = 3.5$	**A1 dep on M2**	All four values correct and correctly labelled, or shown correctly as the coordinate pairs $(-0.6, -2.8)$ and $(1.5, 3.5)$. Or equivalent, so $-\frac{3}{5}, -\frac{14}{5}, \frac{3}{2}$ and $\frac{7}{2}$ are accepted.	✓
Working required	**Note**	The four values on their own score nothing. The question asks for clear algebraic working, so the substitution and the quadratic must be seen.	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

The curve C has equation $y = \mathrm{f}(x)$.
The minimum point of C is at $(6, -3)$.
No formula for f is given, so each answer has to come from the transformation alone.

Find

The minimum point of $y = \mathrm{f}(x) + 10$.
The minimum point of $y = \mathrm{f}(3x)$.

Plan the Solution

- Read each new equation as a change to $y = \mathrm{f}(x)$ and decide whether it acts on the output of f or on the input.
- A change outside f moves the curve vertically, so only the y-coordinate of the minimum point changes.
- A change inside f moves the curve horizontally, so only the x-coordinate of the minimum point changes.
- Both parts are write-down marks, so each is one mark for the pair of coordinates and no working is required.

Worked Solution [2 marks]

Rule - Transforming $y = \mathrm{f}(x)$: a constant added outside the function, $\mathrm{f}(x) + a$, translates the curve a units up, so a point (p, q) becomes $(p, q + a)$. A multiplier applied to x inside the function, $\mathrm{f}(ax)$, is a horizontal stretch of scale factor $\frac{1}{a}$ about the y-axis, so (p, q) becomes $\left(\frac{p}{a}, q\right)$.

Step 1: (i) the x-coordinate does not move

$$y = \mathrm{f}(x) + 10$$

(Reason: The 10 is added after f has produced its output, so the input is untouched. The whole curve is translated straight up, and the minimum stays above $x = 6$.)

Step 2: (i) add 10 to the y-coordinate

$$y = -3 + 10 = 7$$

$$(6, 7)$$

(Reason: Every output of f gains 10, so the least output -3 becomes 7. The lowest point of the curve is still the lowest point after it is lifted.)

Step 3: (ii) find the x that feeds 6 into f

$$y = \mathrm{f}(3x)$$

$$3x = 6$$

$$x = \frac{6}{3} = 2$$

(Reason: The new curve does at x whatever the old curve does at $3x$. The old curve was at its minimum when its input was 6, so the new curve is at its minimum where $3x = 6$.)

Step 4: (ii) the y-coordinate does not move

$$y = -3$$

$$(2, -3)$$

(Reason: Nothing has been added to the output of f and nothing multiplies it, so every y-value survives unchanged. The curve is squashed sideways towards the y-axis, which slides the minimum along but never up or down.)

(i) $(6, 7)$

(ii) $(2, -3)$

Verification

✓ **Check 1:** Use a curve that really does have its minimum at $(6, -3)$, say $y = (x-6)^2 - 3$. Adding 10 gives $y = (x-6)^2 + 7$, and a square is never negative, so the smallest value is 7 at $x = 6$. Minimum $(6, 7)$, which is the answer to part (i).

✓ **Check 2:** Replace x with $3x$ in the same model curve to get $y = (3x-6)^2 - 3$. The bracket is zero when $x = 2$ and positive everywhere else, so that is where the curve bottoms out, at $y = -3$. Minimum $(2, -3)$, which is the answer to part (ii).

✓ **Check 3:** Sanity-check the two directions. Adding 10 lifts the curve, so the minimum y must rise. Replacing x with $3x$ squashes the curve towards the y-axis by a factor of 3, so the minimum x must end up closer to 0, not further from it. $7 > -3$ and $2 < 6$, so both coordinates moved the way the transformations force them to.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) $(6, 7)$	B1	The coordinates of the minimum point of $y = \mathrm{f}(x) + 10$, written as a pair. No working is required for this mark.	✓
(ii) $(2, -3)$	B1	The coordinates of the minimum point of $y = \mathrm{f}(3x)$, written as a pair. No working is required for this mark.	✓

Full marks: 2/2

Question 24 - Exam Solution

Understanding the Question

Given

Solid S is a cone with a hemisphere on top, joined along one circular face of radius x cm.
The total height of S is $4x$ cm.
A sphere of radius kx cm has volume 12.5 times the volume of S.
Solid T is mathematically similar to S, with volume 512 times the volume of S.

Find

(a) The value of k. (b) The value of d, the number the total surface area of S is multiplied by to give the total surface area of T.

Plan the Solution

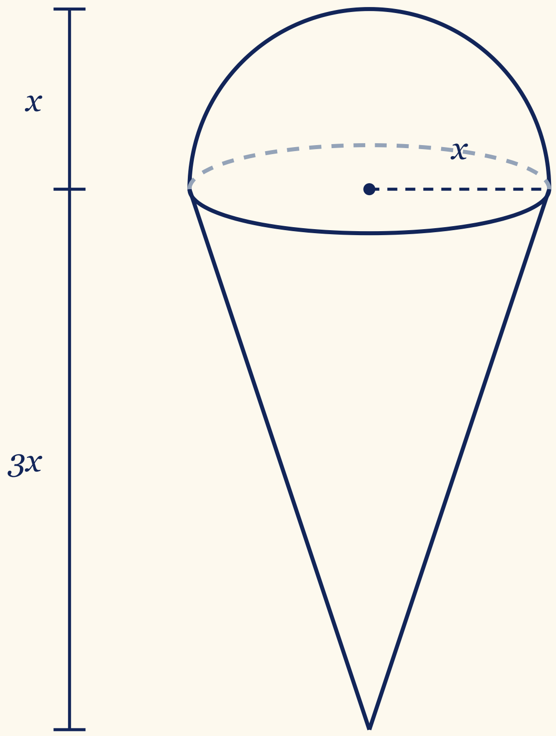
- A hemisphere stands its own radius above its flat face, so it takes x of the total height and leaves the rest to the cone.
- Write the volume of the cone and the volume of the hemisphere as multiples of πx^3, then add them.
- Set the sphere's volume equal to 12.5 times that total. Both π and x^3 cancel, leaving a number for k^3.
- For part (b) no volumes are needed: cube root the volume scale factor to get the length scale factor, then square it for the areas.

Worked Solution [5 marks]

> Rule - Volumes, and scale factors between similar solids: a cone is $\frac{1}{3}\pi r^2 h$, a sphere is $\frac{4}{3}\pi r^3$, and a hemisphere is half of that. If the lengths of two similar solids are in the ratio $n : 1$, their areas are in the ratio $n^2 : 1$ and their volumes in the ratio $n^3 : 1$.

Step 1: Split the total height of S

$h = 4x - x = 3x$



(Reason: The hemisphere rises exactly its own radius, x cm, above the circular face it shares with the cone. Taking that off the total height $4x$ leaves $3x$ for the height h of the cone.)

Step 2: The volume of the cone

$$\frac{1}{3}\pi x^2 \times 3x = \pi x^3$$

(Reason: The cone has base radius x and height $3x$, and the 3 cancels the $\frac{1}{3}$, so the cone is worth exactly πx^3.)

Step 3: The volume of the hemisphere, and of S

$$\frac{1}{2} \times \frac{4}{3}\pi x^3 = \frac{2}{3}\pi x^3$$

$$V_S = \pi x^3 + \frac{2}{3}\pi x^3 = \frac{5}{3}\pi x^3$$

(Reason: A hemisphere is half a sphere of the same radius, so halve $\frac{4}{3}\pi x^3$. Adding the two pieces writes the whole of S as one multiple of πx^3.)

Step 4: Set the sphere against S

$$\frac{4}{3}\pi(kx)^3 = 12.5 \times \frac{5}{3}\pi x^3$$

$$\frac{4}{3}k^3\pi x^3 = \frac{125}{6}\pi x^3$$

(Reason: The sphere has radius kx, so cubing the bracket gives k^3x^3. Keep the brackets until they are expanded - dropping them turns $(kx)^3$ into kx^3, which is a different thing.)

Step 5: Cancel, then cube root

$$k^3 = \frac{125}{6} \times \frac{3}{4} = \frac{125}{8}$$

$$k = \sqrt[3]{\frac{125}{8}} = \frac{5}{2} = 2.5$$

(Reason: Every term carries πx^3, so it divides out and the unknown x never has to be found. Both 125 and 8 are cubes, so the cube root is exact.)

Step 6: Part (b), from the volume scale factor

$$\sqrt[3]{512} = 8$$

$$8^2 = 64$$

(Reason: Volume scales by the cube of the length scale factor, so the lengths of T are 8 times those of S. Area scales by the square of that same factor, so $d = 64$. Nothing from part (a) is needed here.)

(a) $k = 2.5$

(b) $d = 64$

Verification

✓ **Check 1:** Put $k = 2.5$ back into the equation. Every volume is a multiple of πx^3, so only the coefficients need comparing: the sphere gives $\frac{4}{3} \times 2.5^3$ and the right-hand side gives $12.5 \times \frac{5}{3}$. $\frac{4}{3} \times 2.5^3 = \frac{125}{6}$ and $12.5 \times \frac{5}{3} = \frac{125}{6}$, so the two sides match.

✓ **Check 2:** Try a real size instead of a letter. With $x = 2$ cm the cone has radius 2 cm and height 6 cm, so S has volume $\frac{1}{3}\pi \times 4 \times 6 + \frac{2}{3}\pi \times 8 = \frac{40}{3}\pi$ cm³, and the sphere has radius $2.5 \times 2 = 5$ cm. The sphere holds $\frac{4}{3}\pi \times 125 = \frac{500}{3}\pi$ cm³, and $12.5 \times \frac{40}{3} = \frac{500}{3}$, which is the multiple the question asks for.

✓ **Check 3:** Test part (b) by building T. Multiplying every length of S by 8 multiplies its volume by 8^3, and the question says that multiplier is 512, so 8 is the right length scale factor. $8^3 = 512$ confirms the length scale factor, and $8^2 = 64$ is then the area multiplier.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) An expression for the volume of any one of the three solids: the cone $\frac{1}{3}\pi x^2 \times 3x$, the hemisphere $\frac{1}{2} \times \frac{4}{3}\pi x^3$, or the sphere $\frac{4}{3}\pi (kx)^3$	**M1**	Any one of the three earns it. Missing brackets around kx are ignored for this mark, and r may be written in place of x for all the method marks.	✓
(a) A correct equation for the volumes, $\frac{4}{3}\pi (kx)^3 = 12.5 \times \frac{5}{3}\pi x^3$ or equivalent	**M1**	The equation must be correct, so the volume of S must already be right. If $(kx)^3$ has not been expanded at this stage, the brackets must be seen.	✓
(a) A correct calculation for k or for k^3, such as $k^3 = \frac{125}{8}$ or $k = \sqrt[3]{\frac{125}{8}}$	**M1**	A correct equation for kx or for k^3x^3 earns this mark just as well, since the x cancels either way.	✓
(a) $k = 2.5$	**A1**	Or equivalent, for example $\frac{5}{2}$. Working is not required, so a correct answer scores full marks unless it follows obviously incorrect working.	✓
(b) $d = 64$	**B1**	Cube root the volume multiplier to get the length scale factor 8, then square it. Answering 512 or 8 scores nothing here.	✓

Full marks: 5/5

Question 25 - Exam Solution

Understanding the Question

Given

$OPQR$ is a parallelogram, with $\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$.

M lies on PQ with $PM = \frac{1}{4}PQ$, and

N lies on RQ with $RN = \frac{4}{5}RQ$.

MR and ON cross at Y, and $OY = k \times ON$.

Find

(a) $\overrightarrow{ON}$ and $\overrightarrow{MR}$, each in terms of $\mathbf{a}$ and $\mathbf{b}$ in simplest form. (b) the value of k, by a vector method.

Plan the Solution

- Start with the parallelogram itself: opposite sides are parallel and the same length, so each pair carries the same vector.
- Reach every point by travelling along edges. For $\overrightarrow{ON}$ go O to R to N; for $\overrightarrow{MR}$ go M back to O and on to R.
- For part (b) write $\overrightarrow{OY}$ twice - once as a fraction of $\overrightarrow{ON}$, once as a journey along $\overrightarrow{MR}$ - then match the $\mathbf{a}$ parts and the $\mathbf{b}$ parts to get two equations.
- Do not assume Y is the midpoint of MR. It turns out to be, but that is something the working produces, not something the figure gives you.

Worked Solution [6 marks]

> Rule - matching coefficients: $\mathbf{a}$ and $\mathbf{b}$ are not parallel, so if $p\mathbf{a} + q\mathbf{b} = r\mathbf{a} + s\mathbf{b}$ then $p = r$ and $q = s$. One vector equation therefore gives two ordinary equations.

Step 1: write the sides of the parallelogram as vectors

$$\overrightarrow{PQ} = \overrightarrow{OR} = 3\mathbf{b}$$

$$\overrightarrow{RQ} = \overrightarrow{OP} = 2\mathbf{a}$$

(Reason: In a parallelogram opposite sides are parallel and equal in length, so they carry exactly the same vector. Everything else in the question is measured along these two sides.)

Step 2: part (a)(i), travel from O to R and then on to N

$$\overrightarrow{RN} = \frac{4}{5}\overrightarrow{RQ} = \frac{4}{5} \times 2\mathbf{a} = \frac{8}{5}\mathbf{a}$$

$$\overrightarrow{ON} = \overrightarrow{OR} + \overrightarrow{RN} = 3\mathbf{b} + \frac{8}{5}\mathbf{a}$$

$$\overrightarrow{ON} = \frac{8}{5}\mathbf{a} + 3\mathbf{b}$$

(Reason: The fraction is taken along RQ, which is $2\mathbf{a}$ and not $\mathbf{a}$, so the $\frac{4}{5}$ multiplies the whole of it. Writing the $\mathbf{a}$ term first is what puts the answer in simplest form.)

Step 3: part (a)(ii), travel from M back to O and on to R

$$\overrightarrow{PM} = \frac{1}{4}\overrightarrow{PQ} = \frac{1}{4} \times 3\mathbf{b} = \frac{3}{4}\mathbf{b}$$

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM} = 2\mathbf{a} + \frac{3}{4}\mathbf{b}$$

$$\overrightarrow{MR} = \overrightarrow{MO} + \overrightarrow{OR} = -2\mathbf{a} - \frac{3}{4}\mathbf{b} + 3\mathbf{b}$$

$$\overrightarrow{MR} = \frac{9}{4}\mathbf{b} - 2\mathbf{a}$$

(Reason: Going backwards along a vector reverses its sign, so $\overrightarrow{MO} = -\overrightarrow{OM}$. The two $\mathbf{b}$ terms then combine, and $\overrightarrow{OM}$ is worth keeping because part (b) needs it again.)

Step 4: part (b), write OY in two different ways

$$\overrightarrow{OY} = k\overrightarrow{ON} = \frac{8}{5}k\mathbf{a} + 3k\mathbf{b}$$

$$\overrightarrow{OY} = \overrightarrow{OM} + \lambda\overrightarrow{MR} = 2\mathbf{a} + \frac{3}{4}\mathbf{b} + \lambda\left(\frac{9}{4}\mathbf{b} - 2\mathbf{a}\right)$$

$$\overrightarrow{OY} = (2 - 2\lambda)\mathbf{a} + \left(\frac{3}{4} + \frac{9}{4}\lambda\right)\mathbf{b}$$

(Reason: Y is on both lines, so it can be reached along either. The second journey uses a new letter λ for the fraction of $\overrightarrow{MR}$ travelled, because there is no reason yet to think it equals k.)

Step 5: match the a parts and the b parts

$$\frac{8}{5}k = 2 - 2\lambda$$

$$3k = \frac{3}{4} + \frac{9}{4}\lambda$$

(Reason: The same vector cannot be written two genuinely different ways in terms of $\mathbf{a}$ and $\mathbf{b}$, so the coefficients must agree one at a time. Two unknowns, two equations.)

Step 6: solve the pair for k

$$\lambda = 1 - \frac{4}{5}k$$

$$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$$

$$3k = \frac{3}{4} + \frac{9}{4} - \frac{9}{5}k$$

$$\frac{3}{4} + \frac{9}{4} = 3$$

$$3k + \frac{9}{5}k = 3$$

$$\frac{24}{5}k = 3$$

$$k = \frac{15}{24} = \frac{5}{8}$$

(Reason: Rearranging the $\mathbf{a}$ equation gives λ in terms of k, and substituting it into the $\mathbf{b}$ equation leaves one unknown. The fraction then cancels: $\frac{15}{24}$ has a common factor of 3.)

$$\textbf{(a)(i) } \overrightarrow{ON} = \frac{8}{5}\mathbf{a} + 3\mathbf{b}$$

$$\textbf{(a)(ii) } \overrightarrow{MR} = \frac{9}{4}\mathbf{b} - 2\mathbf{a}$$

$$\textbf{(b) } k = \frac{5}{8}$$

Verification

✓ **Check 1 - put the value back into both journeys:** With $k = \frac{5}{8}$ the first journey gives $\overrightarrow{OY} = \frac{5}{8}\left(\frac{8}{5}\mathbf{a} + 3\mathbf{b}\right)$, and the $\mathbf{a}$ term needs $\frac{5}{8} \times \frac{8}{5} = 1$. With $\lambda = \frac{1}{2}$ the second gives $\overrightarrow{OY} = 2\mathbf{a} + \frac{3}{4}\mathbf{b} + \frac{1}{2}\left(\frac{9}{4}\mathbf{b} - 2\mathbf{a}\right)$. Both journeys land on $\overrightarrow{OY} = \mathbf{a} + \frac{15}{8}\mathbf{b}$.

✓ **Check 2 - test it on numbers:** Let $\mathbf{a}$ be one step across and $\mathbf{b}$ one step up. Then O is the origin, P is $(2, 0)$, R is $(0, 3)$ and Q is $(2, 3)$, so M is $(2, 0.75)$ and N is $(1.6, 3)$. Crossing the two lines gives $(1, 1.875)$, and $\frac{5}{8} \times 1.6 = 1$ with $\frac{5}{8} \times 3 = 1.875$. The crossing point sits exactly $\frac{5}{8}$ of the way from O to N, which is what k means.

✓ **Check 3 - what the value of lambda tells you, and what it does not license:** λ came out as $\frac{1}{2}$, so Y is the midpoint of MR. On the numbers of check 2 the midpoint of $M(2, 0.75)$ and $R(0, 3)$ is $\frac{1}{2} \times (2+0) = 1$ across and $\frac{1}{2} \times (0.75+3) = 1.875$ up. The same point, $(1, 1.875)$. The midpoint is a result of the working and never a starting point - the mark scheme says as much.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $\overrightarrow{ON} = \overrightarrow{OR} + \frac{4}{5}\overrightarrow{RQ} = \frac{8}{5}\mathbf{a} + 3\mathbf{b}$	**B1**	or equivalent, but it must be in simplest form - for example $1.6\mathbf{a} + 3\mathbf{b}$ or $\frac{8\mathbf{a}+15\mathbf{b}}{5}$.	✓
(a)(ii) $\overrightarrow{MR} = \overrightarrow{MO} + \overrightarrow{OR} = \frac{9}{4}\mathbf{b} - 2\mathbf{a}$	**B1**	or equivalent, but it must be in simplest form - for example $2.25\mathbf{b} - 2\mathbf{a}$ or $\frac{9\mathbf{b}-8\mathbf{a}}{4}$.	✓
(b) A correct expression for one vector, for example $\overrightarrow{OM} = 2\mathbf{a} + \frac{3}{4}\mathbf{b}$, or $\overrightarrow{OY} = k\left(\frac{8}{5}\mathbf{a} + 3\mathbf{b}\right)$, or $\overrightarrow{YN} = (1-k)\left(\frac{8}{5}\mathbf{a} + 3\mathbf{b}\right)$	**M1ft**	Follow through the answers the candidate gave in part (a). The reversed forms $\overrightarrow{MO}$, $\overrightarrow{YO}$ and $\overrightarrow{NY}$ are equally acceptable, and on every method mark any letter may stand for k or for λ.	✓
Two independent expressions for the same vector, for example $\overrightarrow{OY} = k\left(\frac{8}{5}\mathbf{a} + 3\mathbf{b}\right)$ together with $\overrightarrow{OY} = 2\mathbf{a} + \frac{3}{4}\mathbf{b} + \lambda\left(\frac{9}{4}\mathbf{b} - 2\mathbf{a}\right)$	**M1ft**	The two expressions may be embedded in one correct equation rather than written out separately.	✓
A correct equation in k alone, for example	**M1**	The value of λ may not be assumed. A candidate who writes	✓

Step	Mark	Description	Got it?
$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$, or the correct value $\lambda = 0.5$		$\lambda = 0.5$ because Y looks like the midpoint of MR has assumed the very thing the vector method is there to establish.	
$k = \frac{5}{8}$	**A1 dep on M2**	or equivalent, for example 0.625 . Dependent on at least two of the method marks above.	✓
The question asks for a vector method, so no mark is available for a value reached by measuring the figure or by assuming a midpoint.	**Note**	The figure is not drawn accurately, so nothing may be read off it.	✓

Full marks: 6/6

Question 26 - Exam Solution

Understanding the Question

Given

The expression $4 - \left[\dfrac{3x-5}{\dfrac{3x^2+x-10}{4x-1}}\right]$

A division inside the square brackets, and a subtraction outside them. The quadratic $3x^2 + x - 10$ is the only part that can be factorised.

Find

One fraction, in its simplest form, with no fraction left inside a fraction.

Plan the Solution

- Factorise $3x^2 + x - 10$ first. Nothing can cancel while it is still expanded.
- Work inside the square brackets before the subtraction, so the division is dealt with first.
- Dividing by a fraction is multiplying by its reciprocal, so turn $\dfrac{3x^2+x-10}{4x-1}$ upside down.

- Cancel the bracket that appears top and bottom, then write 4 over the same denominator and subtract.

Worked Solution [4 marks]

> Rule - Dividing by a fraction: turn the fraction upside down and multiply.
> $$\frac{a}{\left(\frac{b}{c}\right)} = a \times \frac{c}{b}$$

Step 1: Factorise the quadratic

$$3x^2 + x - 10 = (3x-5)(x+2)$$

(Reason: Expanding $(3x-5)(x+2)$ gives $3x^2 + 6x - 5x - 10$, and the two middle terms combine to x. The bracket $3x-5$ is the one the numerator already has, which is why the factorising comes first.)

Step 2: Turn the second fraction upside down and cancel

$$(3x-5) \times \frac{4x-1}{(3x-5)(x+2)} = \frac{4x-1}{x+2}$$

(Reason: Dividing by $\frac{(3x-5)(x+2)}{4x-1}$ is multiplying by $\frac{4x-1}{(3x-5)(x+2)}$. The bracket $3x-5$ is now a factor of both the top and the bottom, so it cancels.)

Step 3: Write 4 over the same denominator

$$4 - \frac{4x-1}{x+2} = \frac{4(x+2) - (4x-1)}{x+2}$$

(Reason: Two parts can only be subtracted once they share a denominator, and $4 = \frac{4(x+2)}{x+2}$.)

Step 4: Simplify the numerator

$$\frac{4x+8-4x+1}{x+2} = \frac{9}{x+2}$$

(Reason: A whole bracket is being subtracted, so the sign of every term inside it changes. The x terms then cancel and a constant is left on top, which is why the answer has no x in its numerator.)

$$\frac{9}{x+2}$$

Verification

✓ **Check 1:** Put $x = 3$ into the original expression. The quadratic is $27 + 3 - 10 = 20$, the numerator is $3 \times 3 - 5 = 4$ and $4 \times 3 - 1 = 11$, so the bracket is $4 \times \frac{11}{20} = \frac{11}{5}$. $4 - \frac{11}{5} = \frac{9}{5}$, and the answer gives $\frac{9}{3+2} = \frac{9}{5}$

✓ **Check 2:** Put $x = 0$, where both signs are negative. The quadratic is -10 and the denominator under it is -1, so the fraction inside the brackets is $\frac{-10}{-1} = 10$, and $\frac{-5}{10} = -\frac{1}{2}$. $4 + \frac{1}{2} = \frac{9}{2}$, and the answer gives $\frac{9}{0+2} = \frac{9}{2}$

✓ **Check 3:** Add back the fraction that was subtracted. If the answer is right, the 4 must reappear: $\frac{9}{x+2} + \frac{4x-1}{x+2} = \frac{4x+8}{x+2}$. $\frac{4(x+2)}{x+2} = 4$, which is the 4 the question started from

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Factorise $3x^2 + x - 10$	**M1**	For $(3x-5)(x+2)$. It may be seen later on in the working. Alternatively, for combining the two parts into a correct single fraction.	✓
Invert and cancel	**M1**	For inverting and cancelling to give a correct fraction, $\frac{4x-1}{x+2}$, which implies the first M1. Alternatively, for a correct single fraction whose denominator is factorised.	✓
One fraction over a common denominator	**M1**	For a correct single fraction, or two correct fractions with a common denominator, $\frac{4(x+2)-(4x-1)}{x+2}$. Alternatively, for a correct fully factorised single fraction, $\frac{9(3x-5)}{(3x-5)(x+2)}$.	✓
Simplify to the final answer	**A1**	For $\frac{9}{x+2}$. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

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