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Edexcel IGCSE 4MA1

Paper 1H (Higher Tier) – November 2024

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A (4MA1), Paper 1H (Higher Tier), November 2024 – 100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. The table gives information about the hourly rates of pay of the 60 workers at a fruit-packing plant.

Hourly rate of pay (p dollars)	Frequency
$10 < p \leq 15$	18
$15 < p \leq 20$	16
$20 < p \leq 25$	14
$25 < p \leq 30$	8
$30 < p \leq 35$	4

(a) Write down the modal class. *[1 mark]*

(b) Work out an estimate for the mean hourly rate of pay, in dollars, of the 60 workers. *[4 marks]*

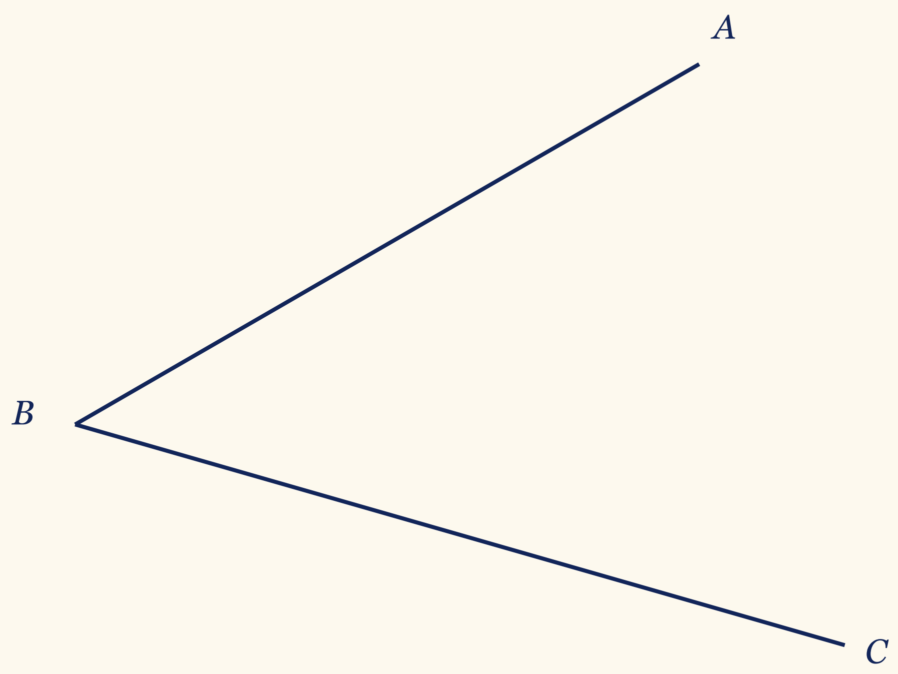
(a) ....

(b) dollars ....

[Total 5 marks]

2. Use a ruler and a pair of compasses only to construct the bisector of angle ABC.

You must leave in all your construction arcs. *[2 marks]*



[Total 2 marks]

3. (a) Write $(p^3)^5$ as a single power of p. *[1 mark]*

(b) Multiply out the brackets and simplify $2n(4n+3)+n(n-4)$ *[2 marks]*

(c) Solve $\dfrac{2x+5}{3}=4-x$

You must show clear algebraic working. *[3 marks]*

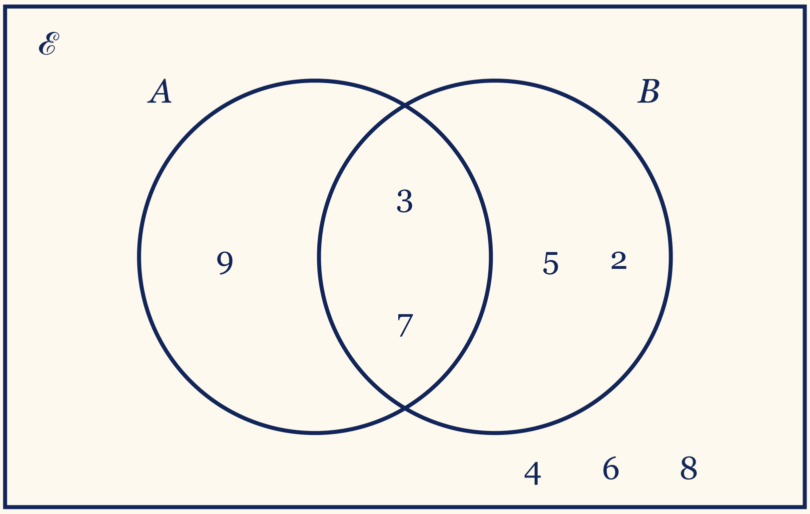
(a) ....

(b) ....

(c) x = ...

[Total 6 marks]

4. The Venn diagram shows the universal set $\mathscr{E}$ and the two sets A and B.



(a) Write down the members of the set B *[1 mark]*

(b) Write down the members of the set $A \cap B$ *[1 mark]*

(c) Write down the members of the set A' *[1 mark]*

(a) ....

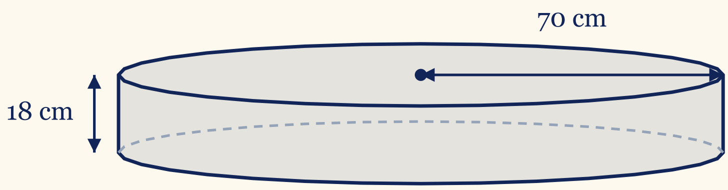
(b) ....

(c) ....

[Total 3 marks]

5. A cylinder is shown in the diagram below.

Diagram NOT accurately drawn



The cylinder has radius 70 cm and height 18 cm.

Find the volume of the cylinder.

Give your answer in litres, correct to the nearest litre. *[4 marks]*

.... litres

[Total 4 marks]

6. Three numbers are written here as products of their prime factors.

$A = 2^3 \times 5^4 \times 7 \times 11$

$B = 2^2 \times 5^2 \times 7^2$

$C = 2^2 \times 5^3 \times 7^4$

Work out the highest common factor (HCF) of A, B and C

Give your answer as a product of prime factors. *[2 marks]*

....

[Total 2 marks]

7. Shop A and Shop B are both selling the same model of bicycle, and each shop has an offer on it.

Shop A	**Shop B**
Our normal price	Get 15% off our normal price
£475	Only pay
Get 16% off our normal price	**£408**

The normal price of the bicycle in Shop A is not the same as the normal price of the bicycle in Shop B.

Which shop takes more money off its normal price?
Show your working clearly. *[4 marks]*

[Total 4 marks]

8. (a) (i) Write $x^2 + 5x - 24$ as a product of two brackets.
[2 marks]
(ii) Hence solve the equation $x^2 + 5x - 24 = 0$.
[1 mark]
(b) Find the range of values of y for which $3y + 5 > 7y - 10$.
You must show clear algebraic working. *[3 marks]*

(a)(i) ....

(a)(ii) ....

(b) ....

[Total 6 marks]

9. (a) The number 8.4×10^{-5} is written in standard form. Write it as an ordinary number. *[1 mark]*

(b) Work out the product $(6.5 \times 10^{-40}) \times (8 \times 10^{185})$. Give your answer in standard form. *[2 marks]*

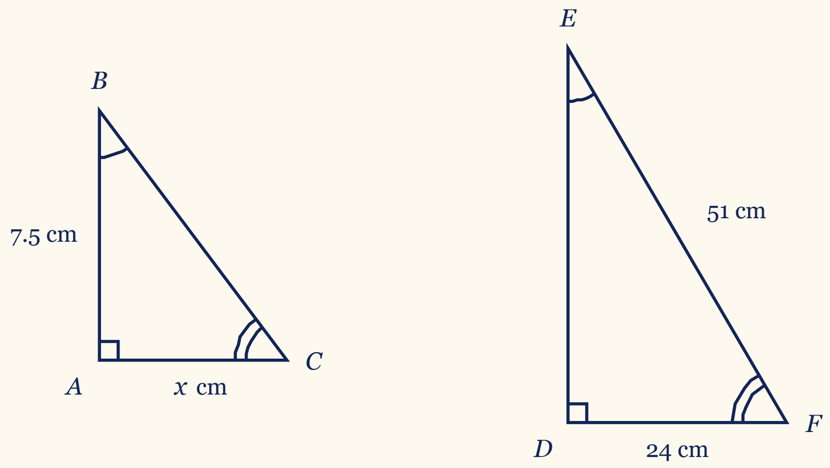
(a) ....

(b) ....

[Total 3 marks]

10. The two triangles below are similar.

Diagrams not accurately drawn



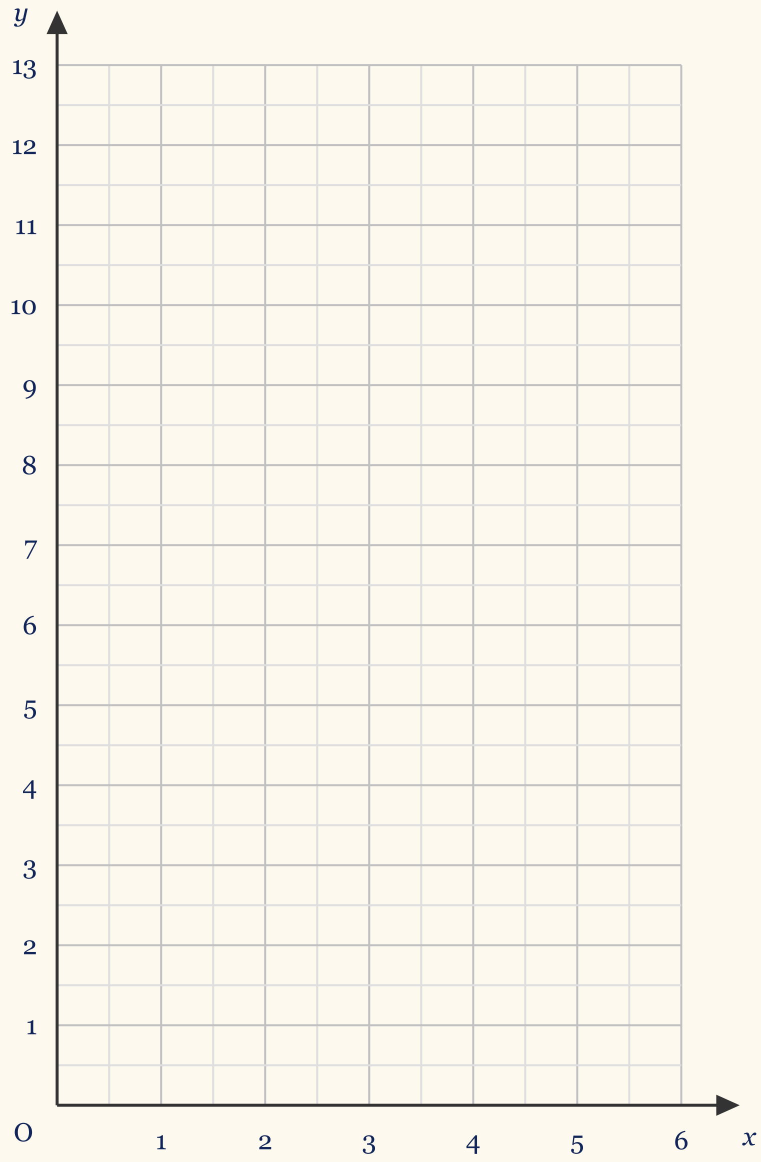
Work out the value of x

Show your working clearly. *[5 marks]*

x = ...

[Total 5 marks]

11. (a) The table below is for the function $y = 2(x + \frac{1}{x})$. Work out the four missing values of y. *[2 marks]*



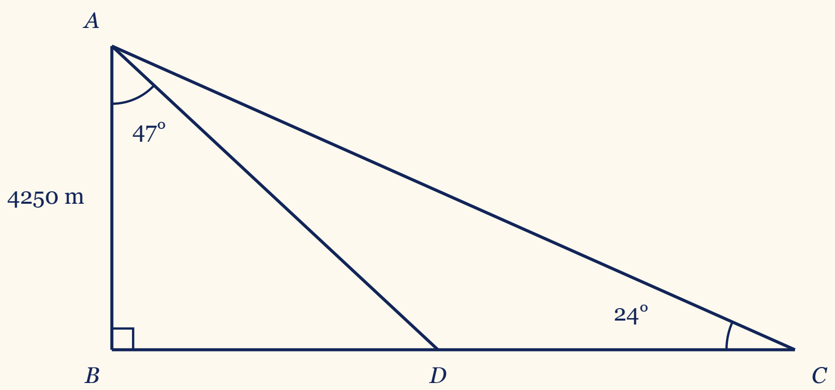
x	0.5	1	2	3	4	5	6
y			5	6.7			12.3

(b) Using your completed table, draw the graph of $y = 2(x + \frac{1}{x})$ on the grid for $0.5 \leqslant x \leqslant 6$ *[2 marks]*

[Total 4 marks]

12. ABC is a triangle with a right angle at B, and D is a point on the side BC.

Diagram NOT accurately drawn



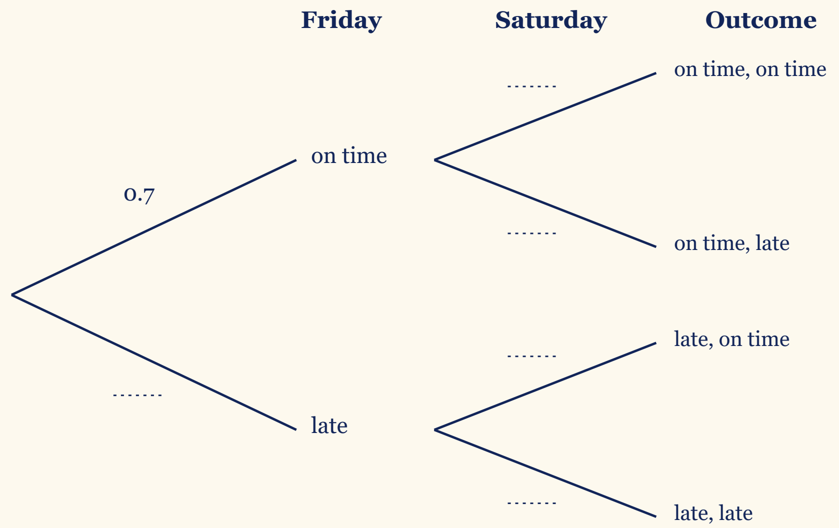
$AB = 4\,250$ m, angle $BAD = 47°$ and angle $BCA = 24°$.

Work out the length of DC. Give your answer correct to the nearest integer. *[4 marks]*

.... m

[Total 4 marks]

13. Martin drives a delivery van to a bakery. The probability that Martin arrives on time on Friday is 0.7, the probability that he arrives on time on Saturday after arriving on time on Friday is 0.9, and the probability that he arrives on time on Saturday after arriving late on Friday is 0.6



(a) Use this information to complete the probability tree diagram. *[2 marks]*

(b) Work out the probability that Martin arrives on time on both Friday and Saturday. *[2 marks]*

(b) ....

[Total 4 marks]

14. B is inversely proportional to the square of d

B takes the value 0.25 when $d = 12$

Find a formula for B in terms of d *[3 marks]*

....

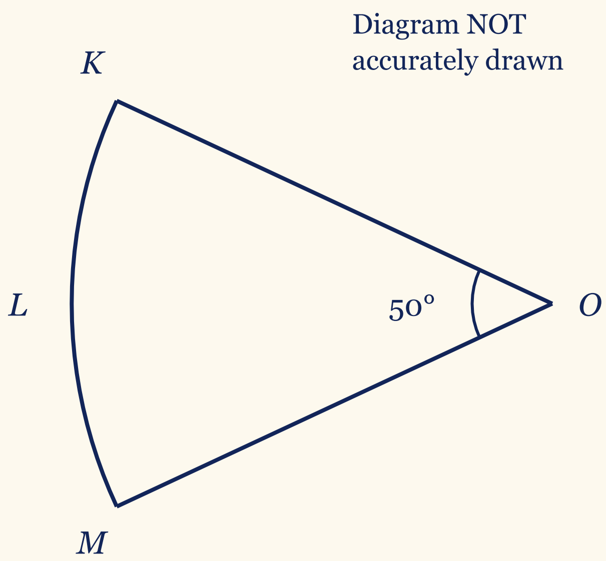
[Total 3 marks]

15. Expand and simplify $3x(2x-1)(5x+4)$, giving your answer in the form ax^3+bx^2+cx where a, b and c are integers. *[3 marks]*

....

[Total 3 marks]

16. $OKLM$ is a sector of a circle, centre O.



Angle KOM is 50°

The area of triangle OKM is $120\ \text{cm}^2$

Work out the area of the sector $OKLM$.

Give your answer correct to 3 significant figures. *[4 marks]*

.... cm²

[Total 4 marks]

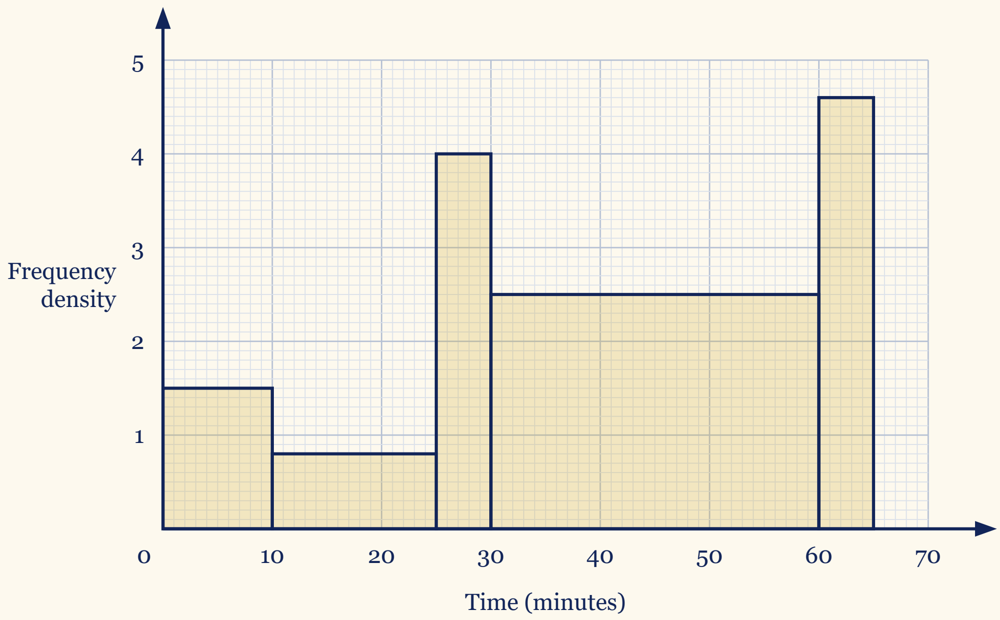
17. (a) Write $\sqrt{675}$ in the form $n\sqrt{27}$, where n is a positive integer. [1 mark]

(b) Show that $\dfrac{5-\sqrt{2}}{\sqrt{2}-1}$ can be expressed in the form $a + b\sqrt{2}$, where a and b are integers. *[3 marks]*

(a) ....

[Total 4 marks]

18. The histogram gives information about the times taken by some visitors to complete a maze at a country park.



Work out an estimate for the fraction of these visitors who took between 20 minutes and 60 minutes to complete the maze. *[4 marks]*

....

[Total 4 marks]

19. An arithmetic series has first term a and common difference d.

Adding the first 30 terms of this series gives $4\,395$

The 10th term and the 20th term have a sum of 284

Find the sum of the first 45 terms of the series. You must show clear algebraic working. *[5 marks]*

....

[Total 5 marks]

20. A curve C has equation $y = 2x^4 - 64x$
The curve C has a minimum point.
Work out an equation of the tangent to C at this minimum point.
You must show clear algebraic working. *[4 marks]*

....

[Total 4 marks]

21. $T = \dfrac{x^2 + y^2}{w}$

Each of the three values below has been rounded.
$x = 28.4$ correct to 1 decimal place
$y = 17$ correct to 2 significant figures
$w = 90$ correct to the nearest 5
Work out the upper bound for the value of T.
Give your answer correct to 3 significant figures.
You must show your working clearly. *[3 marks]*

....

[Total 3 marks]

22. The diagram shows a garden bird bath in the shape of a hemisphere. The bird bath is cast from bronze.

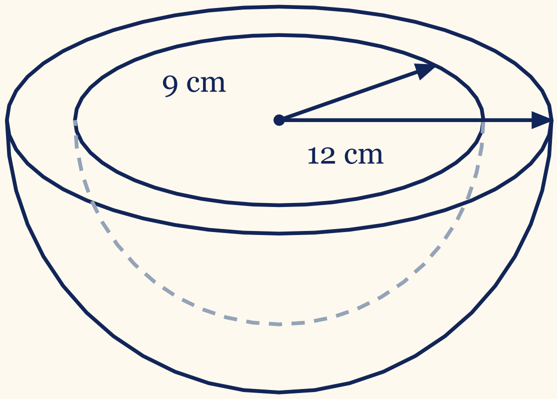


Diagram NOT accurately drawn

The outer radius of the bird bath is 12 cm

The inner radius of the bird bath is 9 cm

The thickness of the bird bath is uniform.

Work out the volume of the bronze.

Give your answer correct to the nearest whole number. *[3 marks]*

.... cm^3

[Total 3 marks]

23. The curve C has equation $y = x^2 - 8x - 9$

The straight line L has equation $y = k$, where k is an integer.

C and L meet at the points A and B.

Point A has coordinates (p, k) and point B has coordinates (q, k).

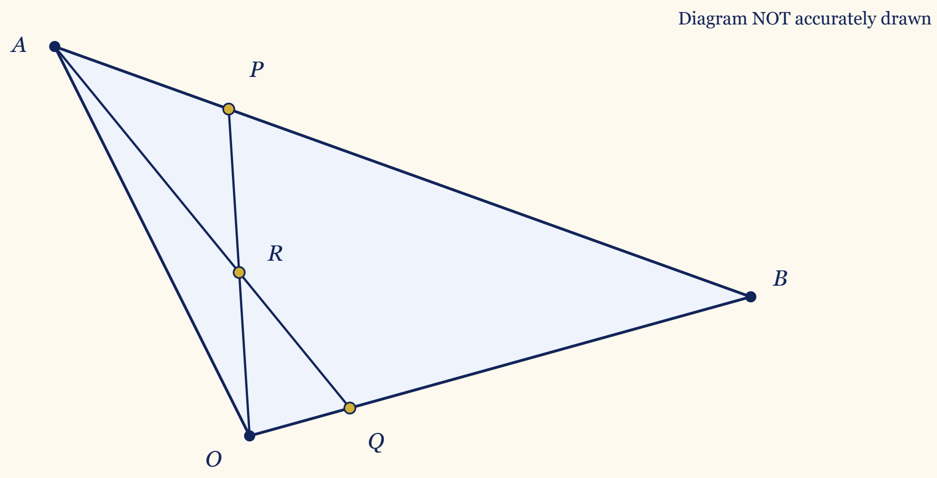
Given that $p - q = 14$, work out the value of k.

You must show clear algebraic working. *[5 marks]*

k = ...

[Total 5 marks]

24. The diagram shows the triangle OAB.



$\overrightarrow{OA} = 10\mathbf{a}$ and $\overrightarrow{OB} = 10\mathbf{b}$

P is the point on AB for which $\overrightarrow{AP} = \dfrac{1}{4}\overrightarrow{AB}$, and Q is the point on OB for which $\overrightarrow{OQ} = \dfrac{1}{5}\overrightarrow{OB}$.

R is the point where the straight line ARQ crosses the straight line ORP .

Write each of these vectors in terms of $\mathbf{a}$ and $\mathbf{b}$, giving each answer in its simplest form.

(i) $\overrightarrow{AQ}$ *[1 mark]*

(ii) $\overrightarrow{OP}$ *[1 mark]*

(iii) $\overrightarrow{OR}$ *[4 marks]*

(i) ....

(ii) ....

(iii) ....

[Total 6 marks]

25. A function f has domain $x \geq 6$ and is defined by

$\mathrm{f}(x) = 2x^2 - 24x + 7$

Work out the inverse function $\mathrm{f}^{-1}(x)$ *[4 marks]*

$\mathrm{f}^{-1}(x) =$ ....

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The hourly rates of pay, in dollars, of 60 workers, grouped into five classes of width 5

$18 + 16 + 14 + 8 + 4 = 60$

Find

(a) the modal class (b) an estimate for the mean hourly rate of pay of the 60 workers

Plan the Solution

- Part (a) needs no arithmetic. The modal class is the class with the greatest frequency, so it is read straight from the table.
- Part (b) can only be an estimate, because a grouped table does not record what any one worker earns. Assume every worker in a class is paid the midpoint of that class.
- Multiply each midpoint by its frequency, add the five products, then divide by the total frequency of 60.

Worked Solution [5 marks]

> Rule - Estimated mean of grouped data: the estimate is $\dfrac{\sum fx}{\sum f}$, taking x as the midpoint of each class.

Part (a): compare the five frequencies

$18,\ 16,\ 14,\ 8,\ 4$

$10 < p \leq 15$

(Reason: The modal class is the class that occurs most often, so it is the class with the greatest frequency. That frequency is 18, in the first class, and the answer is written as the class itself rather than as the frequency.)

Part (b): take the midpoint of each class

$$\frac{10+15}{2} = 12.5$$
$$\frac{15+20}{2} = 17.5$$
$$\frac{20+25}{2} = 22.5$$
$$\frac{25+30}{2} = 27.5$$
$$\frac{30+35}{2} = 32.5$$

(Reason: The table says how many workers fall in each class, not what any one of them is paid, so the midpoint stands in for every rate in that class. Using the midpoint is what makes the estimate a fair one, because rates below it and above it balance out.)

Multiply each midpoint by its frequency, then add

$12.5 \times 18 + 17.5 \times 16 + 22.5 \times 14 + 27.5 \times 8 + 32.5 \times 4 = 225 + 280 + 315 + 220 + 130$

$225 + 280 + 315 + 220 + 130 = 1170$

(Reason: Each product is an estimate of the total hourly pay of one class, so adding the five products estimates the total hourly pay of all 60 workers together.)

Divide by the total frequency

$18 + 16 + 14 + 8 + 4 = 60$

$$\frac{1170}{60} = 19.5$$

(Reason: A mean is a total shared between everybody it covers. Adding the frequency column first confirms the 60 workers the question states, which is the number the total must be divided by.)

(a) $10 < p \leq 15$; (b) 19.5 dollars

Verification

✓ **Check 1 - the frequency column:** Add the frequencies: $18 + 16 + 14 + 8 + 4\ 60$, which is the number of workers the question states, so nothing has been missed from the table

✓ **Check 2 - the assumed-mean method, counting each midpoint in steps of 5 from 22.5:** $18 \times (-2) + 16 \times (-1) + 14 \times 0 + 8 \times 1 + 4 \times 2 = -36$

$$22.5 + \frac{5 \times (-36)}{60} = 19.5$$

✓ **Check 3 - the estimate must sit between the two extreme estimates:** Paying every worker the lower bound of their class gives $\frac{1020}{60}$, and paying every worker the upper bound gives $\frac{1320}{60}$ 17 dollars and 22 dollars, and the estimate 19.5 dollars lies between them

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Write down the class with the greatest frequency	**B1**	$10 < p \leq 15$. Accept $10 \leq p \leq 15$, $10 < p < 15$, $10 \leq p < 15$ or $10 - 15$	✓
(b) At least four correct products, added. They need not be evaluated	**M2**	$12.5 \times 18 + 17.5 \times 16 + 22.5 \times 14 + 27.5 \times 8 + 32.5 \times 4 \ (= 1170)$	✓
(b) If M2 is not earned: at least four products, each using a consistent value from within its class (end points allowed), added - for example the lower bound of every class. Correct midpoints for at least four products, not added, also earns this mark	**M1**	$10 \times 18 + 15 \times 16 + 20 \times 14 + 25 \times 8 + 30 \times 4 \ (= 1020)$	✓
(b) Divide the total by the total frequency	**dM1**	$\frac{1170}{60} \ (= 19.5)$	✓
(b) The estimated mean hourly rate of pay	**A1**	19.5, or any equivalent form such as 19.50	✓
(b) Note on the two most common wrong methods	**note**	Paying every worker the lower bound of their class totals 1020, and the upper bound totals 1320. Neither can reach the correct estimate, and each scores M1 at most. A fully correct answer scores all four marks unless it clearly follows from incorrect working.	✓

Full marks: 5/5

Question 2 - Exam Solution

Understanding the Question

Given

Angle ABC, drawn as the two arms BA and BC meeting at the point B

A ruler and a pair of compasses, and nothing else - no protractor, and no measuring of the angle

Find

The bisector of angle ABC, drawn accurately, with every construction arc left on the page

Plan the Solution

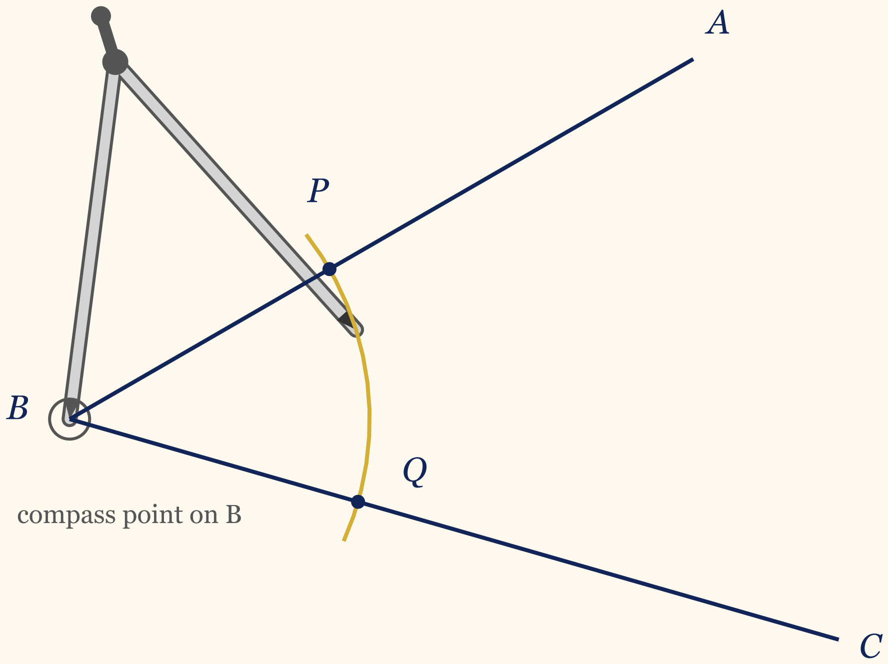
- The bisector is the line through B that splits the angle into two equal halves, so the way to find it without measuring is to build a shape that is symmetrical about it.
- Mark one point on each arm, both the same distance from B. One compass setting does this in a single sweep.
- Find a point that is the same distance from each of those two points. A second compass setting, used twice, does this.
- Join B to that point. The two halves of the figure are then mirror images, so the angle has been bisected.
- Leave every arc on the page. Half the marks here are for the arcs, not for the line.

Worked Solution [2 marks]

> Rule - Bisecting an angle: with one compass setting, cut both arms from B at P and at Q; with a second setting, draw an arc from P and an arc from Q so that they cross at R; the bisector is the straight line through B and R.

Cut both arms with one compass setting

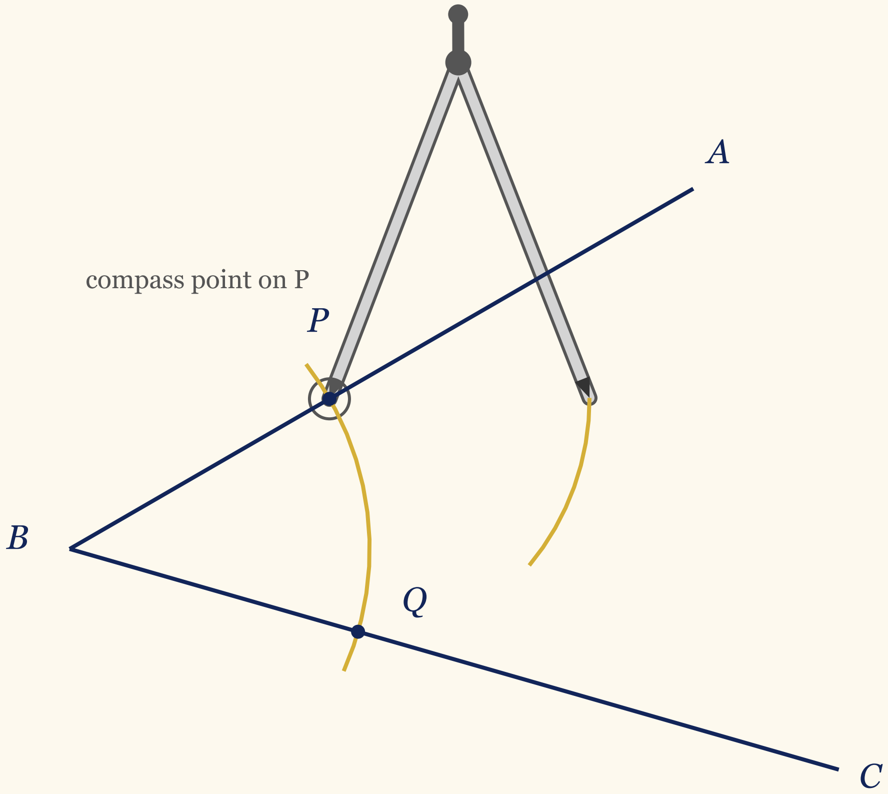
$BP = BQ$

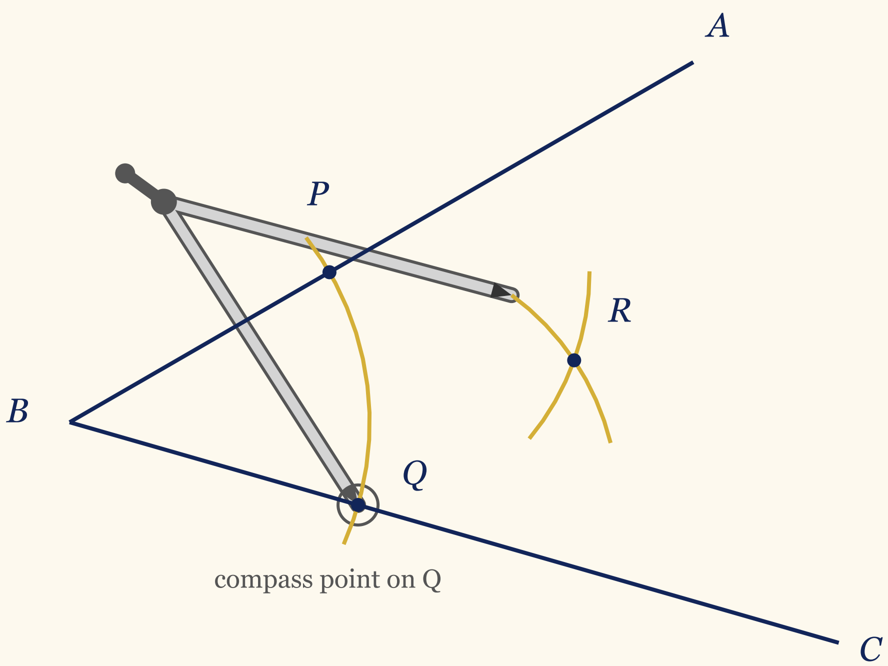


(Reason: Put the compass point on B, open the compasses to any convenient width, and sweep an arc that crosses both arms - at P on BA and at Q on BC. Because the setting is not touched between the two crossings, BP and BQ are the same length, and that single fact is what makes everything drawn after it symmetrical about the line being looked for.)

Cross a pair of arcs from P and from Q

$$PR = QR$$



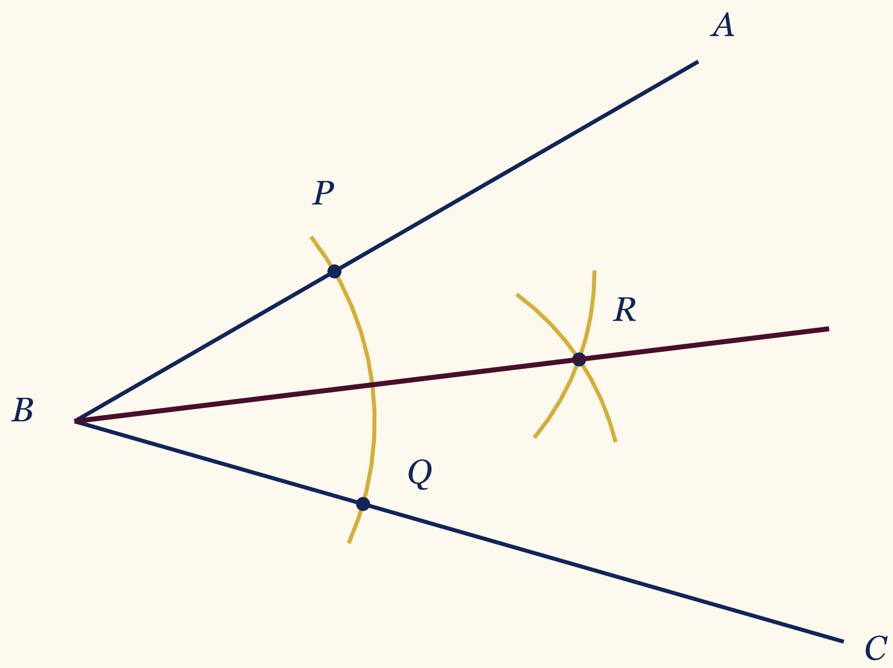


(Reason: Open the compasses to a second setting, comfortably more than half of PQ, and draw a short arc from P into the angle. Without changing the setting, draw a second arc from Q. They meet

at R. A setting smaller than half of PQ leaves the two arcs short of each other and there is no R to find, which is the one thing that can go wrong at this stage.)

Join B to R and extend the line

$\angle ABR = \angle RBC$



(Reason: Line the ruler up on B and on R, draw the line, and carry it past R so that it clearly cuts across the angle. Leave the arcs exactly where they are - rubbing them out throws away the mark they earn.)

Why the two halves have to be equal

$BP = BQ, \ PR = QR, \ BR = BR$

$\frac{46}{2} = 23$

(Reason: Triangles BPR and BQR have three pairs of equal sides, so they are congruent, and congruent triangles have equal angles at B. That argument uses neither compass setting, which is why the construction works on any angle at all. Halving the angle drawn at B on this page gives the size each half must come out at, and a protractor laid on the finished drawing agrees with it.)

The straight line through B and R, with the arc from B and the two arcs from P and Q all left on the diagram

Verification

✓ **Check 1 - measure the two halves:** The angle ABC printed on this page measures 46 degrees, so measure angle ABR and angle RBC with a protractor each half measures 23 degrees, and $23 + 23 = 46$, so the whole angle has been accounted for

✓ **Check 2 - the shape $BPRQ$is a kite:** $BP = BQ$ came from the first compass setting and $PR = QR$ came from the second, so $BPRQ$ is a kite whose long diagonal is BR a kite is symmetrical about that diagonal, so BR is a mirror line of the figure and the two parts of the angle at B have to match

✓ **Check 3 - the distance from $R$to each arm:** Measure the perpendicular distance from R to the arm BA, then from R to the arm BC the two distances are the same, which is the property that defines every point lying on the bisector

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A fully correct bisector of angle ABC, with all the relevant construction arcs left on the diagram	**B2**	The bisector and the arcs may cross anywhere on or inside the overlay guidelines	✓
All the construction arcs drawn, but no bisector drawn	**B1**	The arcs on their own show the method, even though the line that answers the question is missing	✓
A correct bisector, on or within the guidelines, but with no arcs, or with too few arcs	**B1**	A line in the right place with no evidence of how it was found. Measuring the angle with a protractor and halving it lands here at best	✓
Note on the overlay	**note**	An overlay is supplied with the official mark scheme. It carries the guidelines that both the B2 row and the B1 rows refer to, and it is what decides whether a drawn bisector is close enough to be correct	✓

Full marks: 2/2

Question 3 - Exam Solution

Understanding the Question

Given

$(p^3)^5$ - a power raised to another power

$2n(4n+3)+n(n-4)$ - two brackets, each with a term outside it

$\frac{2x+5}{3}=4-x$ - a linear equation with the fraction on the left

Find

(a) $(p^3)^5$ written as a single power of p. (b) the expansion in its simplest form. (c) the value of x, with the algebra shown.

Plan the Solution

- (a) A power raised to another power keeps its base, and the two indices multiply.
- (b) Multiply every term inside each bracket by the term outside it, then collect the squared terms and the terms in n separately.
- (c) Clear the fraction first by multiplying both sides by 3, then gather the terms in x on one side and the numbers on the other.
- In (c) every line keeps the two sides balanced, which is what the instruction to show clear algebraic working is asking for.

Worked Solution [6 marks]

> Rule - index law, expanding a bracket, and balance: $(p^m)^n = p^{mn}$, $a(b+c)=ab+ac$, and an equation stays true when both sides are treated in exactly the same way.

Part (a) - multiply the indices

$$(p^3)^5 = p^{3\times 5} = p^{15}$$

(Reason: A power raised to another power is that power used over and over, so p cubed used five times uses the index 3 five times. The base stays as p and the two indices multiply.)

Part (b) - multiply out each bracket

$$2n(4n+3) = 8n^2 + 6n$$

$$n(n-4) = n^2 - 4n$$

(Reason: Each term inside a bracket is multiplied by the term outside it, so $2n$ multiplies the $4n$ and the 3, and n multiplies the n and the minus 4. Four terms come out of the two brackets.)

Part (b) - collect the like terms

$$8n^2 + 6n + n^2 - 4n = 9n^2 + 2n$$

(Reason: The two squared terms add to give 9 lots of n squared, and the two terms in n add to give 2 lots of n. A term in n squared and a term in n are unlike, so nothing further combines.)

Part (c) - clear the fraction

$$\frac{2x+5}{3} = 4 - x$$

$$2x + 5 = 3(4 - x)$$

(Reason: Multiplying both sides by 3 undoes the division on the left. The whole of the right hand side is multiplied by 3, so it is written inside a bracket first.)

Part (c) - multiply out the right hand side

$$2x + 5 = 12 - 3x$$

(Reason: The 3 outside the bracket multiplies both terms inside it, not only the first one. This is the step the mark scheme singles out, and it is where marks are most often lost.)

Part (c) - gather the terms in x

$$2x + 3x = 12 - 5$$

$$5x = 7$$

(Reason: Adding $3x$ to both sides and taking 5 from both sides puts every term in x on the left and every number on the right. That rearrangement is what the second method mark is for.)

Part (c) - divide to finish

$$x = \frac{7}{5} = 1.4$$

(Reason: Dividing both sides by 5 leaves x on its own. The fraction is exact and 1.4 is the same value written as a decimal, and the mark scheme accepts either.)

(a) p^{15}

(b) $9n^2 + 2n$

(c) $x = 1.4$

Verification

✓ **Check 1 - part (a):** Put $p = 2$ into each form and work the two out as plain numbers: $(2^3)^5 = 8^5 = 32\,768$ and $2^{15} = 32\,768$, so the two forms are the same expression.

✓ **Check 2 - part (b):** Put $n = 3$ into the expression given and into the answer: $2 \times 3 \times 15 + 3 \times (-1) = 87$ and $9 \times 9 + 6 = 87$, so the two agree.

✓ **Check 3 - part (c):** Put $x = 1.4$ back into the equation given and work out each side separately: the left side is $\dfrac{2 \times 1.4 + 5}{3} = \dfrac{7.8}{3} = 2.6$ and the right side is $4 - 1.4 = 2.6$, so the two sides are equal.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Multiply the indices	**B1**	cao for p^{15}. No other form earns this mark.	✓
(b) Expand both brackets	**M1**	for expanding with at least 3 correct terms, for example $8n^2 + 6n + n^2 - 4n$. Seeing $8n^2$ is what counts, not $2n \times 4n$ left unworked. Where the terms are written in a list or a table, a term with no sign in front of it may be taken as a plus.	✓
(b) Collect the like terms	**A1**	oe for $9n^2 + 2n$. The forms $2n + 9n^2$, $n(9n + 2)$ and $n(2 + 9n)$ are all accepted.	✓
(c) Clear the fraction	**M1**	for removal of the fraction and multiplying out the right hand side correctly by 3, giving $2x + 5 = 12 - 3x$, or for separating the fraction on the left in an equation, giving $\frac{2}{3}x + \frac{5}{3} = 4 - x$ oe.	✓
(c) Rearrange for the terms in x	**M1ft**	dep on 4 terms, for correctly rearranging their four term equation so that the terms in x are on one side and the number terms on the other, for example $2x + 3x = 12 - 5$ or $5x = 7$.	✓
(c) Divide to finish	**A1**	dep on M2 for $\frac{7}{5}$ oe, for example 1.4 or $1\frac{2}{5}$. Working is required, so an answer standing on its own does not score.	✓

Full marks: 6/6

Question 4 - Exam Solution

Understanding the Question

Given

The region inside A only holds 9
The overlap of A and B holds 3 and 7
The region inside B only holds 5 and 2

Find

The members of B, of $A \cap B$ and of A', read straight off the diagram

The rectangle holds 4, 6 and 8 outside both circles
So the universal set holds the eight numbers $\{2, 3, 4, 5, 6, 7, 8, 9\}$

Plan the Solution

- Read the diagram one region at a time: A only, the overlap, B only, and the numbers outside both circles.
- A set is everything drawn inside its own circle, so B is the overlap together with the B-only region.
- An intersection is the overlap region on its own, where the two circles cross.
- The complement A' is everything in the universal set that is not inside A, so it is the B-only region together with the numbers outside both circles.
- Each part is worth one mark for the complete listing, so every member must appear, with no repeats and nothing extra.

Worked Solution [3 marks]

> Rule - reading a Venn diagram: a set is the union of every region inside its own circle, an intersection is the overlapping region, and A' is everything in the universal set that lies outside A

Part (a) - take both of the regions inside circle B

$B = \{3, 7\} \cup \{5, 2\}$
$B = \{2, 3, 5, 7\}$

(Reason: Circle B covers two regions - its overlap with A, and the part of B on its own - so a number in either region is a member of B. The 3 and the 7 are in B even though they are in A as well.)

Part (b) - take the overlap only

$A \cap B = \{3, 7\}$

(Reason: $A \cap B$ is the region where the two circles cross, so only a number inside both circles qualifies. The 9 is inside A alone and the 5 and the 2 are inside B alone, so none of them belongs to the intersection.)

Part (c) - take everything outside circle A

$A = \{3, 7, 9\}$
$A' = \{2, 4, 5, 6, 8\}$

(Reason: A' is everything in the universal set that is not inside circle A. The 5 and the 2 sit inside B, but they are still outside A, so they join the 4, 6 and 8 from outside both circles.)

(a) $2, 3, 5, 7$; (b) $3, 7$; (c) $2, 4, 5, 6, 8$

Verification

✓ **Check 1 - the four regions account for every number:** Count the numbers region by region - one in A only, two in the overlap, two in B only and three outside both circles - and compare the total with the eight numbers printed inside the rectangle $1 + 2 + 2 + 3 = 8$

✓ **Check 2 - a set and its complement:** Every number in the universal set is either inside A or in A' and never in both, so the three members of A and the five members of the answer to part (c) must account for the whole universal set $3 + 5 = 8$

✓ **Check 3 - the overlap is counted once in each set:** Adding the size of A to the size of B counts the two members of the overlap twice, so taking the overlap off again must leave the five numbers that are drawn inside the two circles $3 + 4 - 2 = 5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The members of the set B	B1	cao $2\ 3\ 5\ 7$. All four numbers must be present, with no repeats and no other numbers. Any order is accepted, and so is any separator between them (commas, colons and so on)	✓
(b) The members of the set $A \cap B$	B1	cao $3\ 7$. Both numbers must be present, with no repeats and no other numbers. Any order, any separator	✓
(c) The members of the set A'	B1	cao $2\ 4\ 5\ 6\ 8$. All five numbers must be present, with no repeats and no other numbers. Any order, any separator. Because nothing may be missing, a listing of $4\ 6\ 8$ alone - the numbers outside both circles, with the members of B that lie outside A forgotten - scores zero	✓

Full marks: 3/3

Question 5 - Exam Solution

Understanding the Question

Given

A cylinder with radius $r = 70$ cm and height $h = 18$ cm

Both lengths are in centimetres, so the volume comes out in cubic centimetres

The conversion the answer needs at the end: $1 \text{ litre} = 1000 \text{ cm}^3$

Find

The volume of the cylinder, in litres, correct to the nearest litre

Plan the Solution

- Work out the volume in cubic centimetres first, from the radius and the height.
- Leave the volume as a multiple of π while the working goes on, so that no rounding creeps in early.
- There are 1000 cubic centimetres in one litre, so divide the volume in cubic centimetres by 1000 to turn it into litres.
- Round to the nearest whole litre only at the very end, after the calculator has done the last multiplication.
- The four marks split two and two: two for the volume in cubic centimetres, and two for the conversion and the rounded answer.

Worked Solution [4 marks]

> Rule - Volume of a cylinder: $V = \pi r^2 h$, where r is the radius of the circular face and h is the height. A litre is 1000 cm^3, so $\dfrac{\text{volume in cm}^3}{1000}$ is the volume in litres

Write down the rule, then put in the radius and the height

$V = \pi r^2 h$

$V = \pi \times 70^2 \times 18$

(Reason: The volume of a cylinder is the area of its circular face multiplied by its height. Only the radius is squared, never the height, so the 70 is squared and the 18 is not.)

Work out the volume in cubic centimetres

$70^2 = 4900$

$4900 \times 18 = 88\,200$

$V = 88\,200\pi \text{ cm}^3$

(Reason: Squaring the radius and then multiplying by the height leaves the volume as a multiple of π. Keeping it as $88\,200\pi$ means nothing has been rounded yet, and the mark scheme awards this second mark for $88\,200\pi$ or for the decimal $277\,088.472$ that it comes to.)

Change the cubic centimetres into litres

1 litre $= 1000 \text{ cm}^3$

$$V = \frac{88\,200\pi}{1000} = 88.2\pi \text{ litres}$$

(Reason: A litre is 1000 cubic centimetres, so the number of litres is the volume in cubic centimetres divided by 1000. Dividing the exact form $88\,200\pi$ keeps the working exact, and the mark scheme gives this third mark on the volume the candidate actually found, so an earlier slip does not stop it.)

Work out the decimal, then round to the nearest litre

$V = 88.2\pi = 277.088\ldots$ litres

$V \approx 277$ litres

(Reason: The calculator turns 88.2π into 277.088 and so on. The first decimal place is a 0, which is below 5, so the value rounds down to 277 whole litres.)

277 litres

Verification

- ✓ **Check 1 - work backwards from the answer:** Turn the answer in litres back into cubic centimetres, then divide by the height and by π, and see whether the square of the radius comes back: $277.088\ldots \times 1000 = 277\,088.472\ldots \text{ cm}^3$, and $\dfrac{277\,088.472\ldots}{18\pi} = 4900 = 70^2$, so the radius is 70 cm again
- ✓ **Check 2 - units that need no conversion at all:** Work in decimetres instead. One cubic decimetre is exactly one litre, so this route never divides by 1000 anywhere. The radius is 7 dm and the height is 1.8 dm: $\pi \times 7^2 \times 1.8 = 88.2\pi = 277.088\ldots$, which is 277 litres to the nearest litre
- ✓ **Check 3 - the two usual approximations for π:** Replace π by 3.14, then by $\dfrac{22}{7}$, which is where the band in the mark scheme comes from: $3.14 \times 88\,200 = 276\,948 \text{ cm}^3$ and $\dfrac{22}{7} \times 88\,200 = 277\,200 \text{ cm}^3$, that is 276.948 litres and 277.2 litres, and both round to 277

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Use of the volume of a	**M1**	for use of $\pi r^2 h$, for example $\pi \times 70^2 \times 18$ oe, or $\pi \times 0.7^2 \times 0.18$ oe where the lengths are changed into	✓

Step	Mark	Description	Got it?
cylinder		metres first	
The volume, before any conversion	A1	for $88\,200\pi$ or $277\,088.472$ in cubic centimetres, or for 0.0882π or $\frac{441}{5000}\pi$ or 0.277088472 in cubic metres. Allow $276\,948$ to $277\,200$ in cubic centimetres, or 0.276948 to 0.277200 in cubic metres	✓
Change the volume into litres	M1	for dividing their volume in cubic centimetres by 1000, for example $\frac{277\,088.472}{1000}$, or $\frac{88\,200\pi}{1000}$ giving 88.2π or $\frac{441}{5}\pi$, or for multiplying their volume in cubic metres by 1000. Allow any value for their volume that contains π, 70 and 18 to be divided by 1000, and any value that contains π, 0.7 and 0.18 to be multiplied by 1000	✓
The volume in litres, to the nearest litre	A1	awrt 277. Any value that rounds to 277 scores, so $277.088\ldots$ left on the answer line is not penalised here, although the question asks for the nearest litre and 277 is the answer wanted	✓
Note - a correct answer standing on its own	note	A correct answer scores full marks, unless it comes from obviously incorrect working	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

$A = 2^3 \times 5^4 \times 7 \times 11$, which is built from the primes 2, 5, 7 and 11
$B = 2^2 \times 5^2 \times 7^2$, which has no 11 in it
$C = 2^2 \times 5^3 \times 7^4$, which has no 11 in it either

Find

The highest common factor of A, B and C The answer left as a product of prime factors, not turned into a single number

All three numbers are already written as products of prime factors, so there is no factorising to do first

Plan the Solution

- A factor shared by all three numbers can only be built from primes that appear in all three, so the first job is to see which primes those are.
- The prime 11 appears in A alone, so it cannot be part of any factor of B or of C.
- For each prime that does appear in all three, take the lowest power of it that appears - going any higher would stop it dividing the number that has the fewest of them.
- Multiply those lowest powers together and leave the answer in that form, because a product of prime factors is what the question asks for.
- Both marks are for the answer itself. There is no method mark in this question, so the product has to be right.

Worked Solution [2 marks]

> Rule - HCF from prime factors: with each number written as a product of prime powers, the HCF is the product of the LOWEST power of every prime that appears in all of them. A prime missing from one of the numbers counts there as that prime to the power 0, and since $p^0 = 1$ it multiplies nothing into the answer

Line the three numbers up prime by prime

$A = 2^3 \times 5^4 \times 7^1 \times 11^1$
$B = 2^2 \times 5^2 \times 7^2 \times 11^0$
$C = 2^2 \times 5^3 \times 7^4 \times 11^0$

(Reason: Writing all three numbers with the same four primes lets the powers be read straight down a column. The single 7 in A is written as 7 to the power 1, and the 11 that is missing from B and from C is written as 11 to the power 0, so that no prime is left out of the comparison by accident.)

Take the lowest power of each prime

$2^3, 2^2, 2^2 \implies 2^2$
$5^4, 5^2, 5^3 \implies 5^2$
$7^1, 7^2, 7^4 \implies 7^1$
$11^1, 11^0, 11^0 \implies 11^0$
$11^0 = 1$

(Reason: A common factor has to divide all three numbers, so it can use each prime only as many times as the number with the fewest of them has it. B has just two 2s and just two 5s, and A has just

one 7. The lowest power of 11 is the power 0, and 11 to the power 0 is 1, so 11 contributes nothing at all.)

Multiply the lowest powers together

$\text{HCF} = 2^2 \times 5^2 \times 7$

(Reason: Multiplying the lowest powers gives the largest number that divides all three. This product is the answer the question asks for, so it is left exactly as it stands, and no 1 is written into it.)

See what that product comes to as an ordinary number

$2^2 \times 5^2 \times 7 = 4 \times 25 \times 7 = 700$

(Reason: Multiplying the product out is a useful size check, and the mark scheme is content to see the number in the working space beside the product. It is not the form the answer line wants, though. The prime 11 belongs to A alone, so it takes no part in a factor shared by all three numbers, and a bare number left on its own with no product anywhere beside it scores one mark instead of two.)

$$2^2 \times 5^2 \times 7$$

Verification

- ✓ **Check 1 - divide each number by the answer:** The product comes to 700. Divide each of the three numbers by 700 and see whether every answer is a whole number: $A = 385\,000 = 700 \times 550$, $B = 4900 = 700 \times 7$ and $C = 1\,200\,500 = 700 \times 1715$, so 700 divides all three exactly
- ✓ **Check 2 - nothing larger can work:** Any common factor must be a factor of the smallest of the three numbers, $B = 4900$. Its factors above 700 are 980, 1225, 2450 and 4900, so test each of those against $A = 385\,000$: the four divisions leave remainders of 840, 350, 350 and 2800, so not one of them divides A, and no common factor above 700 exists
- ✓ **Check 3 - what is left after dividing by 700:** Divide all three numbers by 700 and factorise what is left: $550 = 2 \times 5^2 \times 11$, then 7, then $1715 = 5 \times 7^3$: those three share no prime at all, so their own highest common factor is 1, which means 700 cannot be scaled up any further and is the highest common factor of A, B and C

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The HCF as a product of prime factors	**B2**	for $2^2 \times 5^2 \times 7$. Allow $2 \times 2 \times 5 \times 5 \times 7$, and the factors may be written in any order, so $2^2.5^2.7$ is accepted. The answer must be a product of prime factors, and 1 is not allowed in the final answer	✓

Step	Mark	Description	Got it?
A product that is nearly right, or the value 700	**B1**	for $2^p \times 5^q \times 7^r$ where two of p, q, r are correct, or for one mistake in their product, for example $2^2 \times 5^2$ oe, $2^2 \times 7$ oe, $5^2 \times 7$ oe, $2^2 \times 5 \times 7$ oe, $2 \times 5^2 \times 7$ oe or $2^2 \times 5^2 \times 7 \times 11$ oe, or for 700	✓
Note - 700 on the answer line	**note**	$2 \times 2 \times 5 \times 5 \times 7$ in the working space with 700 on the answer line is awarded B2, and $2^2 \times 5^2 \times 7$ in the working space with 700 on the answer line is awarded B2. It is 700 with no product anywhere that drops to B1	✓

Full marks: 2/2

Question 7 - Exam Solution

Understanding the Question

Given

Shop A's notice prints its normal price, £ 475, and takes 16% off that price
Shop B's notice prints no normal price at all - only 15% off, and the £408 you pay
The normal price in Shop A is not the same as the normal price in Shop B, so the two percentages are percentages of different amounts

Find

Which shop takes more money off its own normal price The money off in each shop, in pounds, because two percentages of two different prices cannot be compared as percentages Both amounts written down: the mark scheme awards the final mark only when 72 and 76 are both seen, so the letter on its own scores nothing

Plan the Solution

- Shop A is one step. Its normal price is printed on the notice, so the money off is 16% of £ 475.
- Shop B is a reverse percentage. The £408 is not its normal price - it is what is left after 15% has gone, so £408 is 85% of the normal price.
- Divide the £408 by 0.85 to get back to the full 100%, then take 15% of that normal price to get the money off. Subtracting £408 from it does the same job.
- Compare the two amounts of money, not the two percentages. A bigger percentage taken off a smaller price is not always more money.
- Say which shop, and leave both amounts in the working where the examiner can see them.

Worked Solution [4 marks]

> Rule - Reverse percentage: taking $p\%$ off a price multiplies it by $\frac{100-p}{100}$, so dividing the price paid by that multiplier gives the normal price back. The money off is always that percentage of the NORMAL price, never of the price paid.

Shop A: take 16% of the price on its notice

$$0.16 \times 475 = 76$$

(Reason: Shop A is the easy one, because it prints its normal price. Taking 16% off means multiplying that normal price by 0.16 to find the money off, and 0.16 of 475 is 76. So Shop A takes £76 off, and a customer there pays £399.)

Shop B: work out what the £408 is a percentage of

$$x - 0.15x = 408$$
$$0.85x = 408$$

(Reason: Shop B never prints its normal price, so call it x. Taking 15% off leaves 85% behind, which means the £408 on the notice is 0.85 of the normal price rather than the normal price itself. Everything else in this question depends on reading the £408 that way.)

Divide by the multiplier to get Shop B's normal price

$$\frac{408}{0.85} = 480$$

(Reason: Multiplying by 0.85 took the price from 100% down to 85%, so dividing by 0.85 undoes it and brings the price back up to the full 100%. Shop B's normal price is £480, which is £5 more than Shop A's - exactly the sort of difference the question warns about in its second line.)

Shop B: take 15% of that normal price

$$0.15 \times 480 = 72$$

(Reason: The 15% comes off Shop B's own normal price of £480, not off the £408 a customer actually hands over, so the money off is 15% of 480. Subtracting gives the same amount: £480 less the £408 paid leaves £72.)

Compare the two amounts of money

$$76 > 72$$
$$76 - 72 = 4$$

(Reason: The question asks which shop takes more MONEY off, so the comparison is between £76 and £72 and not between 16% and 15%. Shop A takes £4 more off. The percentages alone could not have settled it: 16% is the bigger percentage but it is taken off the smaller normal price, so the two effects pull against each other.)

Shop A - it takes £76 off, while Shop B takes £72 off

Verification

- ✓ **Check 1 - put Shop B's normal price back through its own offer:** Take 15% off £ 480 and see whether the £408 printed on the notice comes back: $0.85 \times 480 = 408$ the notice's £408 is reproduced exactly, so £480 is the right normal price, and $480 - 408 = 72$ gives the money off a second way, by subtraction rather than by taking a percentage
- ✓ **Check 2 - Shop B by the 1%method:** The £408 is 85% of the normal price, so 1% of that price is $\frac{408}{85} = 4.8$, which makes 15% equal to $4.8 \times 15 = 72$ £72 again, this time without dividing by 0.85 anywhere, and 100% comes to $4.8 \times 100 = 480$, which agrees with Shop B's normal price
- ✓ **Check 3 - Shop A from the other end:** Taking 16% off leaves 84%, so the price paid in Shop A is $0.84 \times 475 = 399$, and the money off is $475 - 399 = 76$ £76 again, reached without multiplying by 0.16, so £76 against £72 holds and Shop A is £4 the better offer

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Working for Shop A	M1	for 475×0.16 $(= 76)$ oe, or $475 \times (1 - 0.16)$ $(= 399)$ oe	✓
Working for Shop B - reading what the £408 is a percentage of	M1	for $1 - 0.15$ $(= 0.85)$, or $x - 0.15x = 408$, or $100(\%) - 15(\%)$ $(= 85(\%))$, or $\frac{408}{85}$ $(= 4.8)$ oe	✓
Working for Shop B - the normal price, or the money off	M1	for $\frac{408}{0.85}$ $(= 480)$, or $\frac{408}{85} \times 100$ $(= 480)$, or $408 \times \frac{100}{85}$ $(= 480)$ oe, or their own 1% value 4.8×100 $(= 480)$, or $\frac{408}{85} \times 15$ $(= 72)$	✓
The decision, with both amounts shown	A1	dep on M2, for A, with correct working and with 72 and 76 both seen	✓

Step	Mark	Description	Got it?
Note - working required	note	The answer column reads Working required, so A written on its own, with no working, earns nothing at all. Both amounts must appear: 76 for Shop A and 72 for Shop B	✓

Full marks: 4/4

Question 8 - Exam Solution

Understanding the Question

Given

The quadratic expression $x^2 + 5x - 24$, with 1 as the coefficient of x^2
The equation $x^2 + 5x - 24 = 0$, the same expression set equal to zero
The inequality $3y + 5 > 7y - 10$, which is linear in y

Find

Part (a)(i): the expression written as a product of two brackets, each of the form $(x + a)$ with a an integer Part (a)(ii): the values of x that satisfy the equation - a quadratic, so expect two solutions Part (b): every value of y that makes the inequality true, given as a single inequality in y

Plan the Solution

- Factorise by hunting for two integers whose product is the constant term -24 and whose sum is 5, the coefficient of x
- The word hence means part (a)(ii) is free once part (a)(i) is done: set each bracket equal to zero and read off the two solutions
- Treat the inequality exactly like an equation, but reverse the inequality sign at the moment both sides are divided by a negative number
- As a safeguard, collect the y terms on the side that keeps their coefficient positive, where no reversal is needed, and confirm the same answer appears

Worked Solution [6 marks]

> Factorising $x^2 + bx + c$: find integers p and q with $pq = c$ and $p + q = b$, so that $x^2 + bx + c = (x + p)(x + q)$. A product of two brackets equals zero only when one of

the brackets equals zero. For an inequality, dividing or multiplying both sides by a negative number reverses the direction of the inequality sign.

Set up the pair of numbers the brackets need

$x^2 + 5x - 24 = (x + p)(x + q)$

$p \times q = -24$

$p + q = 5$

(Reason: Expanding $(x + p)(x + q)$ gives $x^2 + (p + q)x + pq$, so the constant term fixes the product of p and q and the coefficient of x fixes their sum.)

Test the integer pairs whose product is -24

$12 \times (-2) = -24$ but $12 + (-2) = 10$

$6 \times (-4) = -24$ but $6 + (-4) = 2$

$8 \times (-3) = -24$ and $8 + (-3) = 5$

(Reason: Every pair here has the right product, so the sum is what decides. Only 8 and -3 give the 5 that the coefficient of x demands.)

Write down the factorised form

$x^2 + 5x - 24 = (x + 8)(x - 3)$

(Reason: With $p = 8$ and $q = -3$, the bracket $(x + (-3))$ is written as $(x - 3)$. Both numbers inside the brackets are integers, which is the form the mark scheme requires.)

Use the factors to solve the equation

$(x + 8)(x - 3) = 0$

$x + 8 = 0$ or $x - 3 = 0$

$x = -8$ or $x = 3$

(Reason: A product is zero only when at least one factor is zero, so each bracket is set to zero in turn and the resulting one-step equation is solved.)

Collect the y terms on one side of the inequality

$3y + 5 > 7y - 10$

$3y - 7y > -10 - 5$

$-4y > -15$

(Reason: Subtracting $7y$ from both sides and subtracting 5 from both sides keeps the inequality balanced, exactly as it would keep an equation balanced.)

Divide both sides by -4

$-4y > -15$

$y < \dfrac{15}{4}$

(Reason: Dividing by a negative number reverses the direction of the inequality, so the greater-than sign becomes a less-than sign. The two negatives in the division also cancel, leaving a positive 15 *over* 4*.)*

Check it another way, keeping the coefficient of y positive

$3y + 5 > 7y - 10$

$5 + 10 > 7y - 3y$

$15 > 4y$

$\frac{15}{4} > y$

(Reason: Moving the y terms to the right instead leaves $4y$*, whose coefficient is positive, so dividing by* 4 *needs no reversal. Reading* 15 *over* 4 *is greater than y from the other end gives the same solution as before.)*

(a)(i) $(x+8)(x-3)$ (ii) $x = -8$ or $x = 3$ (b) $y < 3.75$

Verification

- ✓ **Check 1:** Expand the two brackets and compare with the expression in the question $(x+8)(x-3) = x^2 - 3x + 8x - 24 = x^2 + 5x - 24$, the original expression
- ✓ **Check 2:** Substitute each solution into $x^2 + 5x - 24$ $(-8)^2 + 5(-8) - 24 = 64 - 40 - 24 = 0$ and $3^2 + 5(3) - 24 = 9 + 15 - 24 = 0$
- ✓ **Check 3:** Compare the sum and the product of the two solutions with the coefficients of the quadratic $-8 + 3 = -5$, the negative of the coefficient of x, and $(-8) \times 3 = -24$, the constant term
- ✓ **Check 4:** Test one value inside the solution set and one outside it in $3y + 5 > 7y - 10$ $y = 0$ gives $5 > -10$, which is true, and $y = 4$ gives $17 > 18$, which is false
- ✓ **Check 5:** Test the boundary value $y = 3.75$ in both sides of the inequality $3(3.75) + 5 = 16.25$ and $7(3.75) - 10 = 16.25$, so the two sides are equal rather than one being greater, and 3.75 itself is not part of the solution

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Start to factorise the quadratic	M1	$(x \pm 8)(x \pm 3)$, or a partial factorisation $x(x-3) + 8(x-3)$ or $x(x+8) - 3(x+8)$, or any $(x+a)(x+b)$ with $ab = -24$ or $a + b = 5$, where a and b are integers	✓

Step	Mark	Description	Got it?
Give the fully correct factorisation	A1	$(x+8)(x-3)$. Any letter may be used in place of x, and the answer must be in the form $(x+a)(x+b)$ with a and b integers	✓
Guidance on a bare correct answer in part (a)(i)	note	A correct answer scores full marks unless it follows obvious incorrect working	✓
Solve the equation from the factorised form	B1ft	-8 and 3, followed through from the candidate's own answer to part (a)(i) provided their factors are of the form $(x+a)(x+b)$. Award B0 for -8 and 3 if no marks were scored in part (a)(i)	✓
Collect the y terms and the number terms	M1	$3y - 7y > -10 - 5$, or $5 + 10 > 7y - 3y$. The use of $=$ is allowed and an incorrect inequality sign is condoned at this stage	✓
Simplify to a single inequality in y	M1	$-4y > -15$, or $15 > 4y$, or $y = \frac{15}{4}$ or equivalent. Again $=$ is allowed and an incorrect inequality sign is condoned	✓
State the solution with the correct inequality sign	A1	$y < \frac{15}{4}$ or equivalent, for example $y < 3.75$ or $\frac{15}{4} > y$ or $3.75 > y$. Algebraic working is required, and the answer line must carry the correct inequality sign	✓
Guidance on an answer line that drops the inequality	note	Sight of the correct answer in the working space, with just $(y =)\frac{15}{4}$ or equivalent on the answer line, gains M2 only	✓

Full marks: 6/6

Question 9 - Exam Solution

Understanding the Question

Given	Find

Part (a) gives the number 8.4×10^{-5}, already written in standard form
Part (b) gives the product $(6.5 \times 10^{-40}) \times (8 \times 10^{185})$, with both numbers written in standard form
A number is in standard form when its number part is at least 1 and less than 10, multiplied by a whole-number power of 10

Part (a): the same value written out in full as an ordinary number, with no power of 10 left in it Part (b): the value of the product, written in standard form, so its number part must again be at least 1 and less than 10

Plan the Solution

- Part (a): a negative index means a division, so read the power of 10 as dividing by $100\,000$, then move the decimal point five places to the left.
- Part (b): multiply the two number parts, and separately add the two indices, because powers of the same base add when they are multiplied.
- Then test the result against the definition of standard form. The number part comes out as 52, which is too big, so it has to be adjusted.
- Adjust it by making the number part ten times smaller and the power of 10 one higher, which leaves the value unchanged.
- Check part (b) by dividing the final answer by one of the two numbers in the question and confirming the other one comes back.

Worked Solution [3 marks]

> Multiplying in standard form: multiply the two number parts together, and add the two indices, because powers of the same base add when they are multiplied. Then look at the number part. If it is 10 or more the answer is not yet in standard form, so move its decimal point one place to the left and add 1 to the index. That makes the number part ten times smaller and the power of 10 ten times bigger, so the value itself does not change.

Read what the negative index in part (a) means

$$8.4 \times 10^{-5} = \frac{8.4}{10^5}$$

$$10^5 = 100\,000$$

(Reason: A negative index means the reciprocal of the matching positive power, so multiplying by 10 to the power negative 5 is the same as dividing by 10 to the power 5. That power of 10 is $100\,000$.)

Divide 8.4 by $100\,000$

$$\frac{8.4}{100\,000} = 0.000084$$

(Reason: Dividing by $100\,000$ moves the decimal point five places to the left: 8.4 becomes 0.84, then 0.084, then 0.0084, then 0.00084, and finally 0.000084. Counting the five places one at a time is what stops a zero being lost, and the 8 finishes in the fifth decimal place.)

Split part (b) into number parts and powers of 10

$$(6.5 \times 10^{-40}) \times (8 \times 10^{185}) = (6.5 \times 8) \times (10^{-40} \times 10^{185})$$

$$6.5 \times 8 = 52$$

(Reason: Multiplication can be carried out in any order, so the two number parts are collected together and the two powers of 10 are collected together. The number parts give 52.)

Add the two indices

$$10^{-40} \times 10^{185} = 10^{-40+185}$$

$$-40 + 185 = 145$$

(Reason: Powers of the same base are multiplied by adding their indices, not by multiplying them. Adding a negative index is a subtraction, so this is 185 take away 40, which is 145.)

Put the two halves back together

$$(6.5 \times 10^{-40}) \times (8 \times 10^{185}) = 52 \times 10^{145}$$

(Reason: This is the line the mark scheme awards the method mark for. The value is already right, but it is not yet an answer, because standard form needs a number part between 1 and 10 and 52 is not one.)

Rewrite 52 in standard form

$$52 = 5.2 \times 10$$

$$52 \times 10^{145} = 5.2 \times 10^{146}$$

(Reason: The number part has to be at least 1 and less than 10, so the decimal point in 52 moves one place to the left to give 5.2. That makes the number part ten times smaller, so the power of 10 must be ten times bigger for the value to stay the same: the index goes up by one, from 145 to 146.)

(a) 0.000084 **(b)** 5.2×10^{146}

Verification

✓ **Check 1:** Multiply the ordinary number from part (a) back by $100\,000$ and see whether 8.4 returns $0.000084 \times 100\,000 = 8.4$

✓ **Check 2:** Reach the same ordinary number a second way, as a fraction over a power of ten, with no decimal point moved at all $\frac{84}{1\,000\,000} = 0.000084$

✓ **Check 3:** Undo part (b) by dividing the answer by the second number in the question, number parts first $\dfrac{5.2}{8} = 0.65$

✓ **Check 4:** Now the indices of that same division. With the 0.65 above it, this rebuilds the first number of the question, since 0.65 times ten to the power negative 39 is 6.5 times ten to the power negative 40 $146 - 185 = -39$

✓ **Check 5:** Add the two indices the other way round, reading the negative one as a subtraction, and confirm the same total appears $185 - 40 = 145$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write the standard-form number in part (a) as an ordinary number	**B1**	0.000084, which the paper prints with a digit space as $0.000\,084$	✓
Reach a correct product in part (b), in any form	**M1**	Sight of 52×10^{145}, or of 5.2 multiplied by any power of 10, or of any number from 1 up to but not including 10 multiplied by 10^{146}	✓
Give the answer in standard form	**A1**	5.2×10^{146}, with the number part between 1 and 10 and the index correct	✓
Guidance on a bare correct answer in part (b)	**note**	A correct answer scores full marks unless it comes from obviously incorrect working	✓

Full marks: 3/3

Question 10 - Exam Solution

Understanding the Question

Given

Triangle ABC and triangle DEF are similar, and each one has a right angle: at A in the small triangle and at D in the large one

The small triangle has $AB = 7.5$ cm and $AC = x$ cm

Find

The value of x, which is the length of AC in centimetres. Only one length of the large triangle is given alongside its hypotenuse, so the side that matches AB has to be worked out before the two triangles can be compared.

The large triangle has $DF = 24$ cm and hypotenuse $EF = 51$ cm
The marked angles pair the vertices up: the single arc puts B with E, and the double arc puts C with F

Plan the Solution

- Pair the vertices up first, using the right angles and the arcs. That is what decides which side matches which, and every later step depends on it.
- The side matching AB is DE, and DE is not given, so find it from Pythagoras theorem in triangle DEF, where EF is the hypotenuse and DF is the other short side.
- Divide DE by AB to get the scale factor from the small triangle to the large one.
- The side x matches DF, and x is on the small triangle, so divide DF by that scale factor to come back down.
- Check the answer on the pair of sides that has not been used, and again on the angles, which must match if the triangles really are similar.

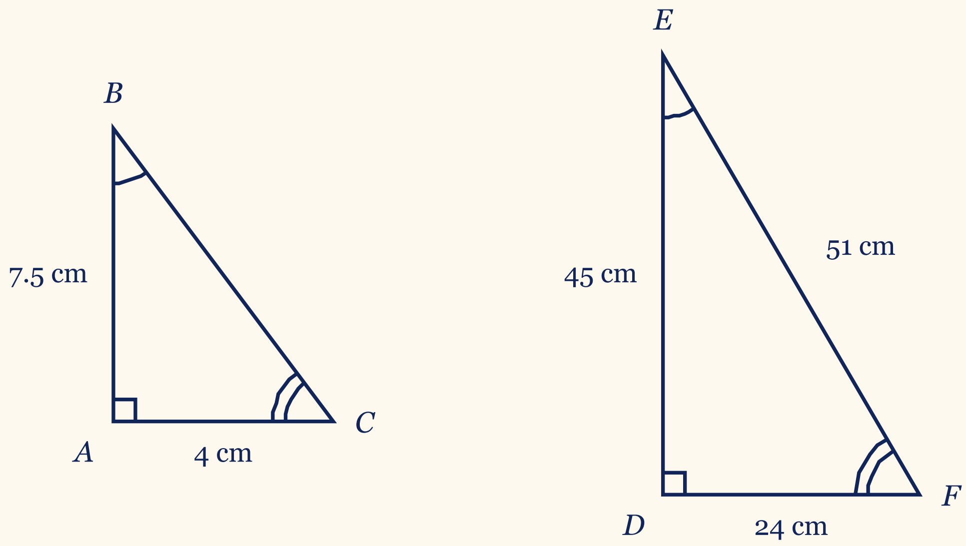
Worked Solution [5 marks]

> Similar triangles: matching sides are all in the same ratio, so the scale factor from the small triangle to the large one is $k = \dfrac{DE}{AB}$, and dividing a length of the large triangle by k gives the length that matches it on the small one.

Match the two triangles up, vertex by vertex

AB matches DE
AC matches DF
BC matches EF

Diagrams not accurately drawn



(Reason: The right angles are at A and at D, so those two vertices go together. The single arc pairs B with E, and the double arc pairs C with F. Reading the letters in that order pairs the sides as well. The known side of the small triangle is AB, so the length it must be compared with is DE, and DE is not given.)

Find DE using Pythagoras theorem

$$DE^2 = 51^2 - 24^2$$

$$51^2 - 24^2 = 2601 - 576 = 2025$$

$$DE = \sqrt{2025} = 45$$

(Reason: In triangle DEF the right angle is at D, so EF is the hypotenuse and DE is one of the two shorter sides. Squaring the hypotenuse and taking away the square of the other short side leaves the square of DE. 2025 is a perfect square, so DE is exactly 45 cm and nothing has to be rounded.)

Work out the scale factor from the small triangle to the large one

$$k = \frac{DE}{AB} = \frac{45}{7.5}$$

$$\frac{45}{7.5} = 6$$

(Reason: A scale factor is a length on the large triangle divided by the length that matches it on the small triangle. DE and AB are such a pair, so 45 divided by 7.5 gives it. Every side of the large triangle is 6 times the matching side of the small one.)

Divide DF by the scale factor to reach x

$$x = \frac{DF}{k}$$

$$\frac{24}{6} = 4$$

(Reason: AC matches DF, and AC belongs to the small triangle, so the large triangle length is divided by the scale factor rather than multiplied by it. Multiplying instead is the commonest slip on this question: it would make the small triangle bigger than the large one, which the figure rules out at a glance.)

State the value of x

$x = 4$

(Reason: The question asks for the value of x, so the answer is the number 4, and the length AC is 4 cm. It is sensible: x is the shorter of the two sides meeting at the right angle in triangle ABC, exactly as 24 is shorter than 45 in triangle DEF.)

$$x = 4$$

Verification

✓ **Check 1:** Scale the answer back up. A length on the small triangle multiplied by 6 must give the length that matches it on the large one, so 4 must return the 24 the question gives $4 \times 6 = 24$

✓ **Check 2:** Test the pair of sides that has not been used. The hypotenuse of the small triangle is 8.5 cm, because 8.5 squared is 72.25 and that is 7.5 squared added to 4 squared. Scaled up by 6 it must give the hypotenuse of the large triangle $8.5 \times 6 = 51$

✓ **Check 3:** Compare the two triangles as a ratio instead of a scale factor, which never forms 6 at all. Each small side divided by the large side matching it must give the same fraction $\frac{7.5}{45} = \frac{4}{24}$

✓ **Check 4:** Check the angles rather than the sides. The tangent of the angle at F is DE over DF, and the tangent of the angle at C, which is marked as equal to it, is AB over AC, so the two must come to the same value $\frac{45}{24} = \frac{7.5}{4}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Apply Pythagoras theorem to triangle DEF	**M1**	For example $51^2 = DE^2 + 24^2$, or $DE^2 = 51^2 - 24^2$, or $2601 = DE^2 + 576$. A trigonometric start earns the same mark: $\cos DFE = \frac{24}{51}$ or $\sin DEF = \frac{24}{51}$	✓

Step	Mark	Description	Got it?
Square root to reach the missing side of the large triangle	**M1**	$DE = \sqrt{51^2 - 24^2} = \sqrt{2025} = 45$, or, from the trigonometric start, angle $DFE = 61.9$ or angle $DEF = 28.0$, both as the mark scheme prints them, cut off after the first decimal place	✓
A correct method for the scale factor	**M1**	For example $\frac{45}{7.5} = 6$ from their own DE, or $\frac{7.5}{45} = \frac{1}{6}$, for which a decimal of 0.17 or better is accepted, or the ratio written as $\frac{x}{24} = \frac{7.5}{45}$. Correct use of the sine rule, the cosine rule or Pythagoras theorem is allowed here. Special case: a candidate who adds the squares instead of subtracting them reaches $\sqrt{3177} = 3\sqrt{353}$, about 56.36, and may still earn this mark by using that value in place of 45	✓
Use the scale factor to find x	**dM1**	Dependent on the previous method mark. For example $\frac{24}{6}$ with their own scale factor, or $24 \times \frac{1}{6}$, or $\frac{7.5}{45} \times 24$. Finding the small hypotenuse first earns it too: $\frac{51}{6} = 8.5$ followed by $\sqrt{8.5^2 - 7.5^2}$	✓
The value of x	**A1**	4, and it must come from correct figures	✓
Working must be shown	**note**	The mark scheme prints Working required beside the answer, and the question itself asks for the working to be shown clearly, so an answer of 4 standing on its own scores nothing	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

Find

$y = 2(x + \frac{1}{x})$
A table of values with $x = 0.5, 1, 2, 3, 4,$ 5 and 6
Three values of y are already printed: 5 at $x = 2$, 6.7 at $x = 3$ and 12.3 at $x = 6$
A grid running from 0 to 6 across and 0 to 13 up

the four missing values of y in the table, then the graph of $y = 2(x + \frac{1}{x})$ for $0.5 \leqslant x \leqslant 6$

Plan the Solution

- Read the function as an instruction: take the reciprocal of x, add x to it, then double the total.
- Work out the four missing values one at a time. All four come out exactly, so nothing has to be rounded.
- Use the three values the table already prints as a running check on the method.
- Plot all seven points, then join them with one smooth curve that starts at $x = 0.5$ and stops at $x = 6$.

Worked Solution [4 marks]

> Rule - Substitution into $y = 2(x + \frac{1}{x})$: work out $\frac{1}{x}$ first, add x to it, then multiply the total by 2.

Substitute $x = 0.5$

$2 \times (0.5 + 2) = 2 \times 2.5 = 5$

(Reason: One divided by 0.5 is 2, so the bracket comes to 2.5. Doubling that gives $y = 5$. Notice how large y is at this end: the reciprocal term is what pushes the curve back up for small x.)

Substitute $x = 1$

$2 \times (1 + 1) = 2 \times 2 = 4$

(Reason: At $x = 1$ the reciprocal is 1 as well, so the bracket is just 2. This is the smallest value anywhere in the table, and it is where the curve turns.)

Substitute $x = 4$

$2 \times (4 + 0.25) = 2 \times 4.25 = 8.5$

(Reason: A quarter is 0.25, so the bracket is 4.25 and doubling gives 8.5 exactly. From here on the reciprocal is small and y is only a little more than $2x$.)

Substitute $x = 5$

$2 \times (5 + 0.2) = 2 \times 5.2 = 10.4$

(Reason: Work out the reciprocal of 5 first, add it on, then double the bracket. This value is exact too, so no rounding is needed.)

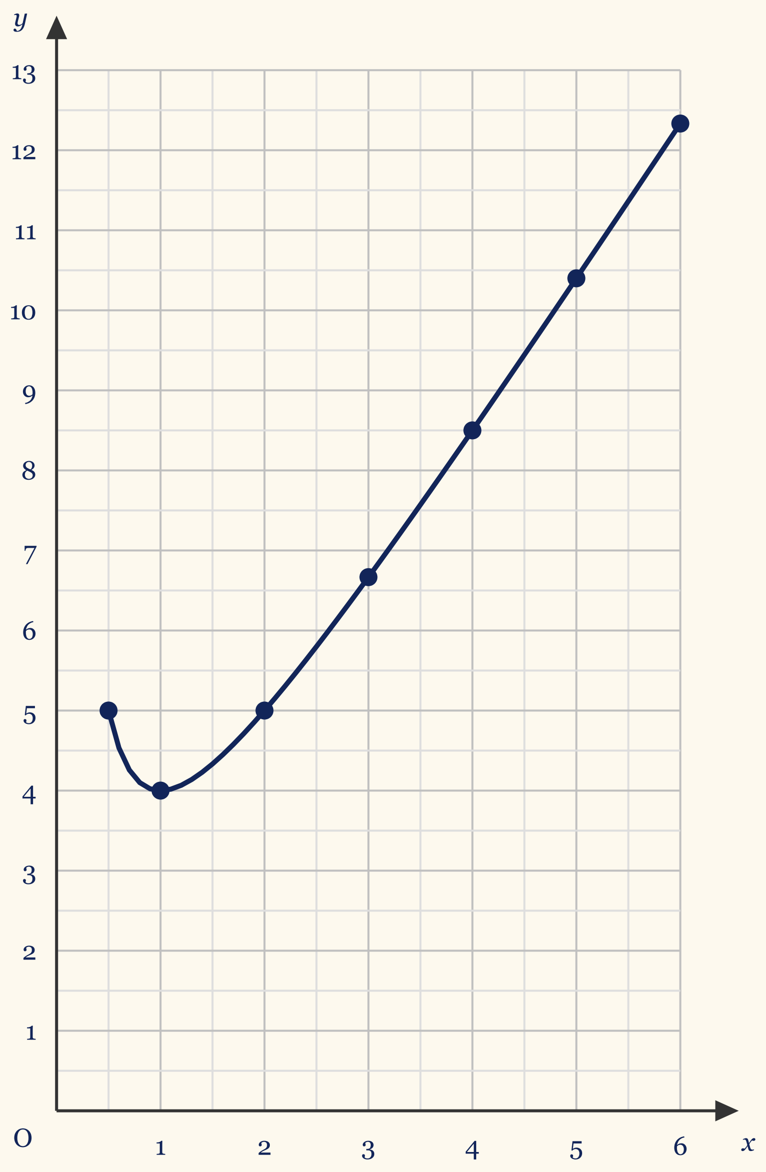
The completed table

x	0.5	1	2	3	4	5	6
y	5	4	5	6.7	8.5	10.4	12.3

(Reason: The four values worked out above are 5, 4, 8.5 and 10.4. The other three were already printed, and 6.7 and 12.3 are the only rounded entries in the row.)

Plot the seven points and join them

$(0.5, 5),\ (1, 4),\ (2, 5),\ (3, 6.7)$
$(4, 8.5),\ (5, 10.4),\ (6, 12.3)$



(Reason: Plot every point from the completed row, then join them with a single smooth curve. The curve dips to its lowest point at $(1, 4)$, turns, and climbs steadily to $(6, 12.3)$. Two things lose the accuracy mark: joining the points with straight line segments, and letting the curve run on past $x = 0.5$ or $x = 6$.)

Missing values of y: 5, 4, 8.5 and 10.4 - then a smooth curve through all seven points

Verification

✓ **Check 1:** Rewrite the function as the single fraction $\dfrac{2(x^2+1)}{x}$, which needs no bracket at all, and test it on $x = 5$ $\dfrac{2 \times (5^2+1)}{5} = \dfrac{52}{5} = 10.4$

✓ **Check 2:** The values the table already prints have to come out of the same rule, correct to 1 decimal place $2 \times (3 + \frac{1}{3}) = 6.7$

✓ **Check 3:** The smallest number in the completed row should sit exactly where the curve turns, at $x = 1$, and every other value in the row is larger than it $2 \times (1 + \frac{1}{1}) = 4$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The four missing values of y	**B2**	B2 oe for all four values: $5, 4, 8.5$ and 10.4. B1 for 2 or 3 correct values of y. The marks may also be awarded if the values are plotted correctly on the graph.	✓
(b) The seven points plotted	**M1ft**	M1 ft their table, dep on B1, for at least 6 points plotted correctly, within or on the circles on the overlay.	✓
(b) The curve itself	**A1**	A1 for a correct smooth curve between $x = 0.5$ and $x = 6$.	✓
Notes	**Note**	A correct answer scores full marks unless it comes from obviously incorrect working. If a fully correct graph is drawn but the table in part (a) is left incomplete, the marks for part (a) are still awarded. Any curve drawn for x below 0.5 or above 6 is ignored.	✓

Full marks: 4/4

Question 12 - Exam Solution

Understanding the Question

Given

ABC is a triangle with a right angle at B
D is a point on BC, so BD and DC together make up BC
$AB = 4\,250$ m
angle $BAD = 47^\circ$
angle $BCA = 24^\circ$

Find

The length of DC, correct to the nearest integer

Plan the Solution

- The right angle at B belongs to two triangles at once: ABD and the whole triangle ABC. AB is the side next to the known angle in each of them, so the tangent ratio reaches everything.
- Use the 47° angle at A to find BD, the near part of the base.
- Use the 24° angle at C to find BC, the whole base.
- D lies between B and C, so DC is what is left when BD is taken away from BC.
- Keep the unrounded values on the calculator and round only at the very end.

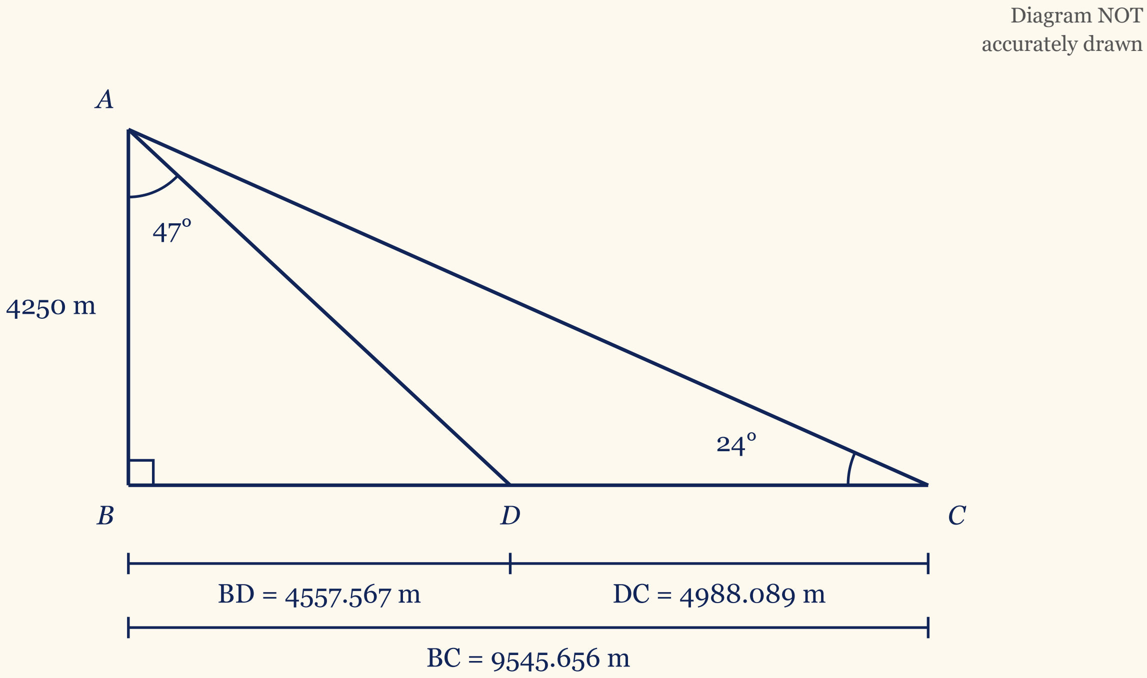
Worked Solution [4 marks]

Tangent ratio in a right-angled triangle: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Find BD in triangle ABD

$\tan 47^\circ = \dfrac{BD}{4\,250}$

$BD = 4\,250 \times \tan 47^\circ = 4\,557.567\ldots$



(Reason: Angle ABD is a right angle, BD is opposite the 47° angle at A, and AB is adjacent to it. So BD and AB are linked by the tangent of 47°.)

Find the whole of BC in triangle ABC

$$\tan 24^\circ = \frac{4\,250}{BC}$$

$$BC = \frac{4\,250}{\tan 24^\circ} = 9\,545.656\ldots$$

(Reason: Angle ABC is the same right angle. This time the known angle is the 24° at C, and AB is the side opposite it while BC is the side adjacent to it, so BC is reached by dividing by the tangent instead of multiplying by it.)

Subtract to leave DC

$$DC = BC - BD$$

$$9\,545.656 - 4\,557.567 = 4\,988.089$$

(Reason: D is between B and C, so BD and DC make up the whole of BC. Taking BD away from BC leaves the part that is asked for.)

Round to the nearest integer

$$DC = 4\,988 \text{ m}$$

(Reason: $4988.089\ldots$ lies between 4988 and 4989, and the first digit after the decimal point is 0, so it rounds down to 4988.)

$$DC = 4988 \textbf{ m}$$

Verification

✓ **Check 1:** Put the two parts of the base back together and compare the total with BC.
$4\,557.567 + 4\,988.089 = 9\,545.656$ m

✓ **Check 2:** Work DC out a second way, from triangle ADC alone. Its angles are $19°$ at A, with $137°$ at D and $24°$ at C, and $AD = \dfrac{4\,250}{\cos 47°} = 6\,231.686\ldots$ The sine rule gives $DC = AD \times \dfrac{\sin 19°}{\sin 24°} = 4\,988.089\ldots$, which is the same length.

✓ **Check 3:** The two angles at A must fill angle BAC, and angle BAC is the complement of the $24°$ angle at C. $47 + 19 = 66 = 90 - 24$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct trigonometric statement in one of the right-angled triangles	M1	eg $\tan 47° = \dfrac{BD}{4\,250}$ or $\tan 24° = \dfrac{4\,250}{BC}$ or $\tan 66° = \dfrac{BC}{4\,250}$ or $AD = \dfrac{4\,250}{\cos 47°}$ or $AC = \dfrac{4\,250}{\sin 24°}$	✓
A correct value for one of the lengths the method needs	M1	eg $BD = 4\,250 \times \tan 47° = 4\,557.567\ldots$ or $BC = \dfrac{4\,250}{\tan 24°} = 9\,545.656\ldots$ or $AD = 6\,231.686\ldots$ or $AC = 10\,449.021\ldots$	✓
A complete method for DC	M1	eg the subtraction $9\,545.656 - 4\,557.567$, or $DC = \dfrac{6\,231.686 \times \sin 19°}{\sin 24°}$, or $DC = \dfrac{10\,449.021 \times \sin 19°}{\sin 137°}$, or the cosine rule $6\,231.686^2 + 10\,449.021^2 - 2 \times 6\,231.686 \times 10\,449.021 \times \cos 19°$ followed by a square root	✓
The answer	A1	The answer $4\,988$ m, and anything in the range $4\,932$ to $4\,990$ is allowed, which absorbs answers built from figures rounded at an earlier stage. A correct answer scores full marks unless it comes from obvious incorrect working.	✓

Full marks: 4/4

Question 13 - Exam Solution

Understanding the Question

Given

The probability that Martin arrives on time on Friday is 0.7
If he arrives on time on Friday, the probability that he arrives on time on Saturday is 0.9
If he arrives late on Friday, the probability that he arrives on time on Saturday is 0.6
A two stage probability tree: Friday first, then Saturday

Find

Part (a): the five probabilities missing from the tree. Part (b): the probability that Martin arrives on time on both days.

Plan the Solution

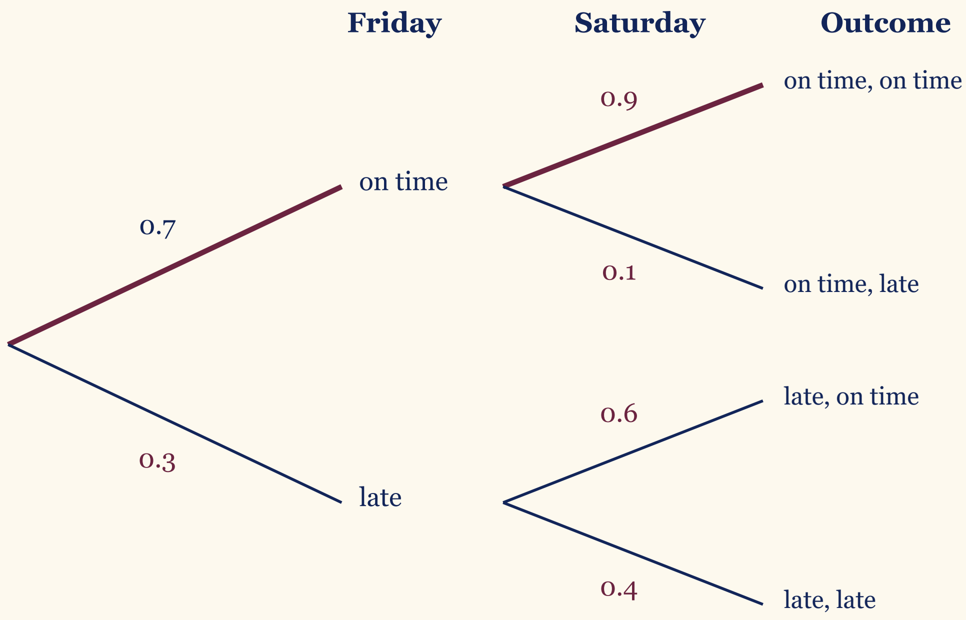
- Write the given Saturday probabilities on the pairs they belong to: 0.9 follows an on time Friday, 0.6 follows a late Friday.
- Every pair of branches leaving the same point covers all the outcomes, so subtract each given probability from 1 to get its partner.
- For part (b), pick out the single path that is on time on Friday and then on time on Saturday.
- Multiply along that path, then check the four complete paths add up to 1.

Worked Solution [4 marks]

> Rule - Tree diagrams: the branches leaving any one point add up to 1, and the probability of a whole path is the product of the probabilities along it.

Complete the tree

$1 - 0.7 = 0.3$
$1 - 0.9 = 0.1$
$1 - 0.6 = 0.4$



(Reason: On time and late are the only two ways each day can go, so each pair of branches adds up to 1*. The* 0.9 *belongs to the upper Saturday pair because that pair follows an on time Friday, and the* 0.6 *belongs to the lower pair, which follows a late Friday.)*

Read the completed tree back

$0.7 + 0.3 = 1$

$0.9 + 0.1 = 1$

$0.6 + 0.4 = 1$

(Reason: A pair that does not add up to 1 *is the quickest sign that a probability has been written on the wrong branch, so this is worth doing before any multiplying starts.)*

Multiply along the on time, on time path

$0.7 \times 0.9 = 0.63$

(Reason: Following a single path through the tree means both things happen, so the probabilities along that path are multiplied together.)

Give the answer in the forms the mark scheme accepts

$$0.63 = \frac{63}{100}$$

(Reason: The accuracy mark is awarded for any equivalent form, so the fraction and 63% *earn it too.)*

0.63

Verification

✓ **Check 1:** Work out the other three paths and add all four path probabilities: $0.63 + 0.07 + 0.18 + 0.12 = 1$

✓ **Check 2:** Count 1000 weekends instead of using decimals: 700 arrive on time on Friday, and 9 out of every 10 of those arrive on time on Saturday as well, which is 630 weekends out of 1000: $\frac{630}{1000} = 0.63$

✓ **Check 3:** Sanity check the size of the answer: arriving on time on both days must be harder than arriving on time on Friday alone. 0.63 is smaller than 0.7, as it has to be

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) All three pairs of probabilities correct	**B2**	All 3 correct pairs of probabilities on the correct branches: 0.7 and 0.3 on Friday, 0.9 and 0.1 on the upper Saturday pair, 0.6 and 0.4 on the lower Saturday pair. Equivalent fractions are allowed.	✓
(a) One correct pair of probabilities	**B1**	If B2 is not earned, one correct pair of probabilities on a correct branch. The given 0.7 together with 0.3 counts as a pair.	✓
(b) A complete method involving one product	**M1ft**	A complete method involving one product, 0.7×0.9 or equivalent.	✓
(b) The probability	**A1ft**	0.63, or any equivalent form, for example $\frac{63}{100}$ or 63%.	✓
(b) Note	**note**	A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 14 - Exam Solution

Understanding the Question

Given

Find

B is inversely proportional to the square of d
B takes the value 0.25 when $d = 12$, and that is the only pair of values the question supplies

a formula for B in terms of d - an equation in d with no unknown constant left in it

Plan the Solution

- Write inverse square proportion as an equation, using a letter for the constant of proportionality. Without that letter there is nothing for the given values to fix.
- Substitute the one pair of values the question supplies, squaring d first so the equation carries a single number rather than a power.
- Solve for the constant, then put its value back into the formula and leave d as a letter.

Worked Solution [3 marks]

Rule - Inverse proportion to a square: when one quantity is inversely proportional to the square of another, it equals a constant divided by that square, so the formula has the shape $\frac{k}{d^2}$ for some constant. Substitute the pair of values given to find that constant, then write the formula out with the constant in place of the letter.

Write the proportion as an equation with a constant

$$B = \frac{k}{d^2}$$

(Reason: Inversely proportional to the square of d means B is a constant divided by d squared. The constant has to appear as a letter at this stage; a version with no constant in it cannot be made to fit the pair of values, and the mark scheme awards nothing for it.)

Substitute the pair of values the question gives

$$0.25 = \frac{k}{12^2}$$
$$0.25 = \frac{k}{144}$$

(Reason: The value of B goes on the left and the value of d goes into the denominator. Squaring the 12 before going any further leaves a single number under the constant, so what is left to solve is a one-step equation.)

Clear the fraction and read off the constant

$$k = 0.25 \times 144$$
$$k = 36$$

(Reason: Multiplying both sides by 144 clears the fraction and leaves the constant on its own. A quarter of 144 is 36.)

Put the constant back into the formula

$$B = \frac{36}{d^2}$$

(Reason: The question asks for a formula for B in terms of d, so d stays a letter and only the constant is replaced by its value. Stopping at the value of the constant answers a different question from the one asked.)

$$\boxed{B = \frac{36}{d^2}}$$

Verification

✓ **Check 1:** Put $d = 12$ back into the finished formula. Squaring the 12 makes the denominator 144, and the fraction has to return the value of B the question gives.
$\frac{36}{144} = 0.25$

✓ **Check 2:** Halve d from 12 to 6. Inverse square proportion must then multiply B by 4, so the formula has to return 4 lots of 0.25, which is 1. $\frac{36}{6^2} = \frac{36}{36} = 1$

✓ **Check 3:** Work out B multiplied by the square of d at $d = 12$, at $d = 6$ and at $d = 3$. For an inverse square relationship that product has to come to the same number every time, and that number is the constant in the formula.
$0.25 \times 144 = 1 \times 36 = 4 \times 9 = 36$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write the relationship as a formula carrying a constant	**M1**	for $B = \frac{k}{d^2}$ oe, or the alternative form $Bk = \frac{1}{d^2}$ oe. The constant of proportionality must be written as a symbol - k is the usual choice, and D or b are allowed instead - so a version with no constant in it, $B = \frac{1}{d^2}$, earns nothing here.	✓
Substitute the given pair into a correct formula	**M1**	for substituting B and d into a correct formula: $0.25 = \frac{k}{12^2}$ oe, or $0.25 = \frac{k}{144}$ oe, or $k = 36$. Both	✓

Step	Mark	Description	Got it?
		method marks are available together for $0.25 = \frac{k}{12^2}$ oe. In the alternative form the right-hand side is $\frac{1}{12^2}$ oe or $\frac{1}{144}$ oe, and the constant comes to $\frac{1}{36}$.	
The finished formula	A1	for the finished formula $B = \frac{36}{d^2}$, or any equivalent - $B = 36 \times \frac{1}{d^2}$ and the negative-index form both count. Full marks are also awarded when $B = \frac{k}{d^2}$ is written on the answer line and $k = 36$ is clearly given in the body of the working.	✓
General guidance	note	A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Rearranged, but not a formula for B	SC	M2A0 where the constant has been found but the answer is left as a rearrangement rather than as a formula for B - for example an answer that gives the square of d as $\frac{36}{B}$, or d as $\frac{6}{\sqrt{B}}$. Both method marks are earned and the accuracy mark is lost.	✓

Full marks: 3/3

Question 15 - Exam Solution

Understanding the Question

Given

The expression $3x(2x - 1)(5x + 4)$, a single term multiplied by two linear brackets

a, b and c are integers

Find

The expansion, simplified, in the form $ax^3 + bx^2 + cx$ the values of a, b and c

Plan the Solution

- Multiply the two brackets together first, so that only one bracket is left to deal with.
- Collect the two x terms straight away, so three terms are carried forward instead of four.
- Multiply each of those three terms by $3x$, adding the indices.
- Never share the $3x$ out between the two brackets - that answers a different question and the mark scheme refuses the first method mark for it.
- Check by substituting a value of x into the original product and into the expansion.

Worked Solution [3 marks]

> Expanding a triple product: expand one pair of brackets first, $(px+q)(rx+s) = prx^2 + (ps+qr)x + qs$, then multiply every term of that quadratic by the single term outside, using $x^m \times x^n = x^{m+n}$

Expand the two brackets

$(2x-1)(5x+4) = 10x^2 + 8x - 5x - 4$

$10x^2 + 8x - 5x - 4 = 10x^2 + 3x - 4$

(Reason: Each term in the first bracket multiplies each term in the second, so there are four products; the two x terms are then collected into a single term. Taking this pair of brackets first is the shorter road, because it leaves only one bracket for the $3x$ to multiply.)

Multiply every term by $3x$

$3x \times 10x^2 = 30x^3$

$3x \times 3x = 9x^2$

$3x \times (-4) = -12x$

$3x(10x^2 + 3x - 4) = 30x^3 + 9x^2 - 12x$

(Reason: The $3x$ multiplies all three terms of the quadratic, and the indices add, so $3x$ times $10x^2$ gives $30x^3$. The $3x$ is not shared out between the two original brackets: doing that gives $6x^2 - 3x + 15x^2 + 12x = 21x^2 + 9x$, which is not even a cubic, and the mark scheme names it as an expansion that earns nothing.)

Read off a, b and c

$30x^3 + 9x^2 - 12x$

$a = 30, b = 9, c = -12$

(Reason: The three terms are already in descending powers of x and there is no constant term, which is exactly the form the question asks for, so the three integers can be read straight off.)

$$30x^3 + 9x^2 - 12x$$

Verification

✓ **Check 1:** Substitute $x = 1$ into the original product, $3 \times 1 \times (2 - 1) \times (5 + 4)$, and into the expansion the product gives 27, and $30 + 9 - 12 = 27$

✓ **Check 2:** Substitute $x = 2$, so the product is $6 \times 3 \times 14$ and the expansion is $240 + 36 - 24$ both give 252

✓ **Check 3:** Expand in the other order: $3x(2x - 1) = 6x^2 - 3x$ first, then multiply that by $(5x + 4)$ $30x^3 + 24x^2 - 15x^2 - 12x = 30x^3 + 9x^2 - 12x$, the same expansion

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Expand one pair of brackets	**M1**	For an expansion with only one error, eg $(2x - 1)(5x + 4) = 10x^2 + 8x - 5x - 4$, or $3x(2x - 1) = 6x^2 - 3x$, or $3x(5x + 4) = 15x^2 + 12x$. This mark is not awarded for $6x^2 - 3x + 15x^2 + 12x$, which shares the $3x$ out between the two brackets instead of multiplying all three factors together.	✓
Complete the expansion	**M1ft**	Follow through on the candidate's own quadratic, dependent on the first M1, allowing one further error, eg $3x(10x^2 + 3x - 4) = 30x^3 + 9x^2 - 12x$, or $(6x^2 - 3x)(5x + 4) = 30x^3 + 24x^2 - 15x^2 - 12x$, or $(15x^2 + 12x)(2x - 1) = 30x^3 - 15x^2 + 24x^2 - 12x$.	✓
The simplified cubic	**A1**	cao $30x^3 + 9x^2 - 12x$. Terms may be in any order but must be simplified, dependent on the first M1. Accept $a = 30$, $b = 9$, $c = -12$. Ignore subsequent working for a correct factorisation, eg $3(10x^3 + 3x^2 - 4x)$; do not ignore an incorrect simplification.	✓
Partial credit when the two brackets containing the $3x$ are multiplied out	**note**	Expanding $(6x^2 - 3x)(5x + 4)$ has four products to get right: M2 for three of the four correct, giving $30x^3 + 24x^2 - 15x^2 - 12x$, and M1 for two of the four correct.	✓

Full marks: 3/3

Question 16 - Exam Solution

Understanding the Question

Given

$OKLM$ is a sector of a circle, centre O
Angle KOM is 50°
The area of triangle OKM is 120 cm^2

Find

The area of the sector $OKLM$, correct to 3 significant figures

Plan the Solution

- The two straight edges of a sector are radii of the same circle, so the two sides of triangle OKM that meet at O are the same length. Call that length r.
- Use the given area of the triangle to work out r^2, then square root it to get the radius.
- The sector is 50 parts out of 360 of the whole circle, so take that fraction of the circle area.
- Keep every decimal on the calculator to the very last line, then round once, to 3 significant figures.

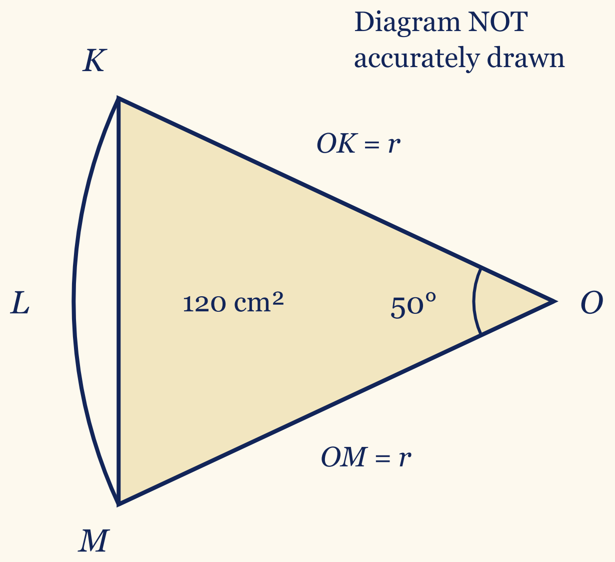
Worked Solution [4 marks]

> Rule - Area of a triangle from two sides and the angle between them: $\frac{1}{2} \times \text{side} \times \text{side} \times \sin(\text{angle})$. Area of a sector: $\frac{\text{angle}}{360} \times \pi \times \text{radius}^2$

Turn the triangle into an equation

$$120 = \frac{1}{2} \times OK \times OM \times \sin 50^\circ$$

$$120 = \frac{1}{2} \times r^2 \times \sin 50^\circ$$



(Reason: OK and OM are radii of the same circle, so they are equal. Writing both of them as r leaves r^2 as the only unknown, which is exactly what the sector formula needs.)

Make r^2 the subject

$$r^2 = \frac{2 \times 120}{\sin 50^\circ} = 313.2977\ldots$$

(Reason: Rearranging isolates r^2. Nothing is rounded here - the full figure stays on the calculator.)

Square root for the radius

$$r = \sqrt{313.2977\ldots} = 17.7002\ldots \text{ cm}$$

(Reason: This is the length of each radius, and the mark scheme awards a method mark for reaching it. Carry the full accuracy forward rather than working on from 17.7.)

Put the radius into the sector formula

$$\text{Area of sector } OKLM = \frac{50}{360} \times \pi \times r^2$$

$$\frac{50}{360} \times \pi \times 313.2977\ldots = 136.7019\ldots$$

(Reason: A whole circle has area π times r^2, and this sector is 50 parts out of 360 of it. Squaring the radius simply gives r^2 back, so the value from step 2 can go straight in.)

Round to 3 significant figures

$$136.7019\ldots \approx 137 \text{ cm}^2$$

(Reason: The first three significant figures are 1, 3 and 6. The next digit is 7, so the 6 rounds up to 7.)

137 cm²

Verification

✓ **Check 1:** Square the radius again. If step 3 undid step 2 correctly, multiplying 17.7002 by itself has to land back on r^2, give or take the four decimal places it was written to.
$17.7002 \times 17.7002 = 313.2971$

✓ **Check 2:** Work all the way back to the question. If r^2 is right, putting it through the triangle formula must return the area the question gives.
$\frac{1}{2} \times 313.2977\ldots \times \sin 50^\circ = 120 \text{ cm}^2$

✓ **Check 3:** Reach the answer without the radius at all. Dividing the sector formula by the triangle formula cancels r^2, so the sector is the same multiple of the triangle whatever the radius happens to be. To six decimal places that multiple is 1.139183.
$120 \times 1.139183 = 136.702$

✓ **Check 4:** Check the size against the whole circle. Since 360 divided by 50 is 7.2, a 50 degree sector is one part in 7.2 of the circle, and this circle has area 984.2539 cm² to four decimal places. $\frac{984.2539}{7.2} = 136.7019$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct equation or expression for the square of the radius	**M1**	for a correct equation or expression for the square of the radius, such as $120 = \frac{1}{2} \times \text{OK} \times \text{OM} \times \sin 50^\circ$ or $\frac{2 \times 120}{\sin 50^\circ}$, or for $313(.2977494)$. Any pair of letters may be used for the two sides.	✓
Rearrange to reach the radius	**M1**	for a correct rearrangement to find the radius, or for square rooting $313(.2977494)$, or for $17.7(0021891)$	✓
Substitute into the area of a sector	**M1**	for the area of sector $OKLM$ written as $\frac{50}{360} \times \pi \times 17.7^2$ or an equivalent	✓
The answer, to 3 significant figures	**A1**	awrt 137. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 17 - Exam Solution

Understanding the Question

Given

$\sqrt{675}$, to be written in the form $n\sqrt{27}$ with n a positive integer

$\dfrac{5-\sqrt{2}}{\sqrt{2}-1}$, to be written in the form $a+b\sqrt{2}$ with a and b integers

Find

the positive integer n in part (a) the integers a and b in part (b), with the working shown

Plan the Solution

- Part (a): the target form keeps 27 under the root, so divide 675 by 27 and check that what is left is a perfect square.
- Part (b): the denominator $\sqrt{2}-1$ is irrational, so rationalise it by multiplying the top and the bottom by the conjugate $\sqrt{2}+1$.
- Expand the numerator to four terms and turn the denominator into a whole number, then collect the rational part and the $\sqrt{2}$ part to read off a and b.

Worked Solution [4 marks]

> Rule - Surds: $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$, and a denominator is rationalised by multiplying the top and the bottom by its conjugate, because $(\sqrt{k}-1)(\sqrt{k}+1)=k-1$

Part (a) - find the square factor that leaves 27 behind

$$\frac{675}{27}=25$$

$$675=25\times27$$

(Reason: The answer must keep 27 under the root, so 27 has to be one of the factors. Dividing shows the other factor is 25, and 25 is a perfect square, which is exactly what is needed for n to be a whole number.)

Part (a) - split the root and take the square root of 25

$$\sqrt{675}=\sqrt{25\times27}=\sqrt{25}\times\sqrt{27}$$

$$\sqrt{675}=5\sqrt{27}$$

(Reason: The root of a product is the product of the roots, so the square factor comes outside as the whole number 5 while 27 stays inside. That gives $n = 5$.)

Part (b) - multiply the top and the bottom by the conjugate

$$\frac{5-\sqrt{2}}{\sqrt{2}-1} = \frac{5-\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

(Reason: The conjugate of $\sqrt{2}-1$ is $\sqrt{2}+1$. Multiplying by $\frac{\sqrt{2}+1}{\sqrt{2}+1}$ is multiplying by 1, so the value of the fraction is unchanged and only its appearance changes.)

Part (b) - expand the denominator

$$(\sqrt{2}-1)(\sqrt{2}+1) = 2 - 1 = 1$$

(Reason: The two middle terms, $+\sqrt{2}$ and $-\sqrt{2}$, cancel each other, so only the difference of the two squares survives. Clearing the surd out of the denominator is the whole purpose of the conjugate.)

Part (b) - expand the numerator to four terms

$$(5-\sqrt{2})(\sqrt{2}+1) = 5\sqrt{2} + 5 - 2 - \sqrt{2}$$

(Reason: Every term in the first bracket multiplies every term in the second, which is why four terms appear. Note that $-\sqrt{2} \times \sqrt{2} = -2$, because squaring a square root removes the root and leaves the number underneath it.)

Part (b) - collect the like terms

$$5\sqrt{2} - \sqrt{2} = 4\sqrt{2}$$

$$5 - 2 = 3$$

$$\frac{3+4\sqrt{2}}{1} = 3 + 4\sqrt{2}$$

(Reason: Whole numbers collect with whole numbers and $\sqrt{2}$ terms with $\sqrt{2}$ terms. The denominator has come out as 1, so the fraction is simply its numerator, and comparing with $a + b\sqrt{2}$ gives $a = 3$ and $b = 4$.)

(a) $\sqrt{675} = 5\sqrt{27}$, so $n = 5$.

(b) $3 + 4\sqrt{2}$, so $a = 3$ and $b = 4$.

Verification

✓ **Check 1:** Reverse part (a) by squaring the whole number and putting it back under the root: work out $5^2 \times 27$ $5^2 \times 27 = 25 \times 27 = 675$, which is the number the question starts with

- ✓ **Check 2:** Reduce both forms in part (a) to the simplest surd, using $675 = 225 \times 3$ and $27 = 9 \times 3$ $\sqrt{675} = 15\sqrt{3}$ and $5\sqrt{27} = 5 \times 3\sqrt{3} = 15\sqrt{3}$, so the two forms agree
- ✓ **Check 3:** Reverse part (b) by multiplying the answer back by the original denominator: expand $(3 + 4\sqrt{2})(\sqrt{2} - 1)$ $3\sqrt{2} - 3 + 8 - 4\sqrt{2} = 5 - \sqrt{2}$, which is the original numerator, so the denominator really was 1
- ✓ **Check 4:** Evaluate the original fraction and the answer to part (b) as decimals and compare $\dfrac{5 - \sqrt{2}}{\sqrt{2} - 1} \approx 8.65685$ and $3 + 4\sqrt{2} \approx 8.65685$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $5\sqrt{27}$	B1	for $5\sqrt{27}$. Allow $n = 5$. Do not accept 5 by itself.	✓
(b) Multiply the numerator and the denominator by the conjugate	M1	for rationalising the denominator by multiplying numerator and denominator by $\sqrt{2} + 1$ or $-\sqrt{2} - 1$	✓
(b) Expand, eg $\dfrac{5\sqrt{2} + 5 - 2 - \sqrt{2}}{2 - 1}$	M1	The numerator must be expanded to 4 terms. The denominator may be shown as 4 terms, which then need to be all correct, eg $\dfrac{5\sqrt{2} + 5 - 2 - \sqrt{2}}{\sqrt{4} + \sqrt{2} - \sqrt{2} - 1}$ oe, or the numerator $5\sqrt{2} + 5 - 2 - \sqrt{2}$ alone. Accept 1 in the denominator without working.	✓
(b) $3 + 4\sqrt{2}$	A1	for $3 + 4\sqrt{2}$, or for stating $a = 3$ and $b = 4$. Working required.	✓

Full marks: 4/4

Question 18 - Exam Solution

Understanding the Question

Given

A histogram of the times, in minutes, that some visitors took to complete a maze
The five bars have frequency densities 1.5, 0.8, 4, 2.5 and 4.6
Their class boundaries are at 0, 10, 25, 30, 60 and 65 minutes
No total is printed anywhere, so the number of visitors has to be built from the histogram

Find

An estimate for the fraction of these visitors whose time was between 20 minutes and 60 minutes

Plan the Solution

- On a histogram the frequency is the AREA of a bar, not its height, so multiply each frequency density by its own class width.
- Add the five frequencies. That total is the denominator of the fraction, and it is nowhere in the question.
- 20 minutes is not a class boundary. It cuts the bar running from 10 to 25, so take only the 5 minute piece from 20 to 25, at the same frequency density.
- Add the three pieces that make up 20 to 60 minutes, write that over the total, and leave the answer as a fraction.

Worked Solution [4 marks]

> Rule - Frequency from a histogram: $\text{frequency} = \text{frequency density} \times \text{class width}$, which is the area of the bar.

Turn every bar into a frequency

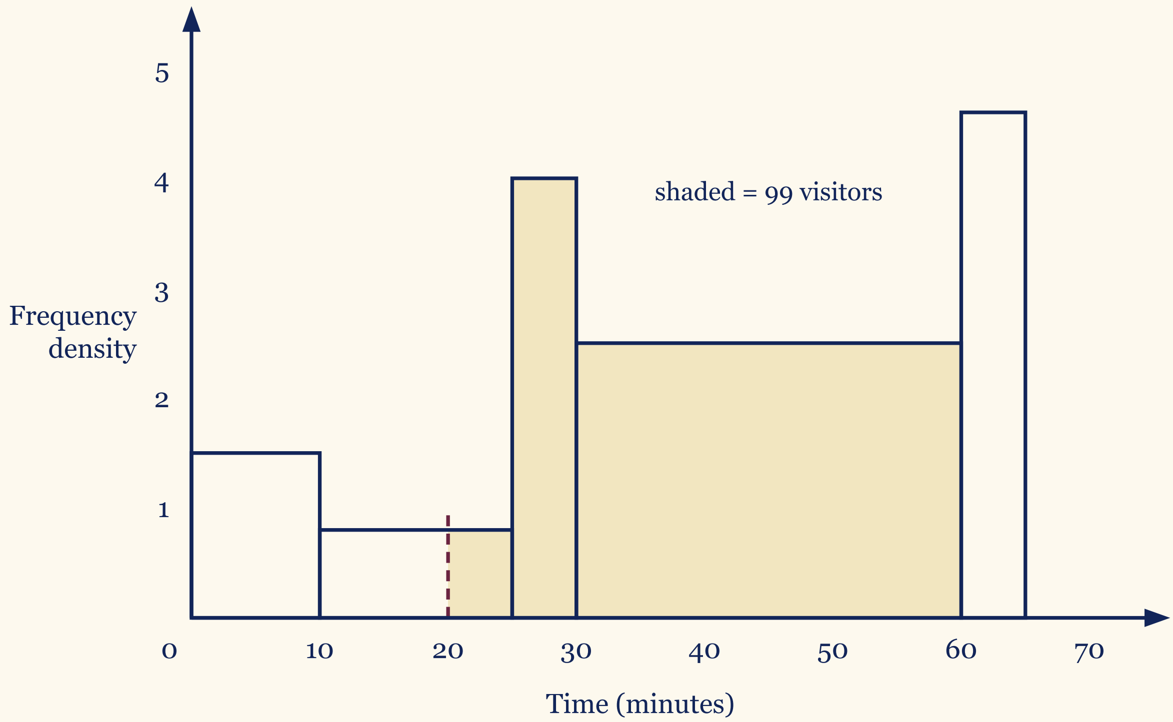
$1.5 \times 10 = 15$

$0.8 \times 15 = 12$

$4 \times 5 = 20$

$2.5 \times 30 = 75$

$4.6 \times 5 = 23$



(Reason: The height of a histogram bar is frequency density, not frequency, so every bar has to be multiplied by its own class width - which is the same as reading off the area of the bar. The five widths come from the horizontal axis: $10, 15, 5, 30$ *and* 5 *minutes.)*

Add the five frequencies for the total

$15 + 12 + 20 + 75 + 23 = 145$

(Reason: Every visitor is counted in exactly one bar, so the five frequencies add up to the whole group. This total is the denominator of the fraction, and the question never states it.)

Cut the second bar at 20 minutes

$0.8 \times 5 = 4$

(Reason: 20 *minutes is not a class boundary. It falls inside the bar that runs from* 10 *to* 25*, and a histogram assumes the times are spread evenly across a bar, so this piece keeps the frequency density* 0.8 *and only the width changes, from* 15 *minutes down to* 5*.)*

Add the three pieces that make up 20 to 60 minutes

$4 + 20 + 75 = 99$

(Reason: The interval from 20 *to* 60 *minutes is made of three pieces: the part of the* 10 *to* 25 *bar beyond* 20 *minutes, the whole* 25 *to* 30 *bar and the whole* 30 *to* 60 *bar. The last bar is left out because it starts exactly where the interval ends.)*

Write the estimate as a fraction

$$\frac{4 + 20 + 75}{15 + 12 + 20 + 75 + 23} = \frac{99}{145}$$

(Reason: The question asks for a fraction, so it is left as one rather than turned into a decimal. 99 is 9×11 and 145 is 5×29, so the two share no factor and the fraction is already in its simplest form.)

$$\frac{99}{145}$$

Verification

✓ **Check 1 - the visitors left out:** Count everybody OUTSIDE 20 to 60 minutes instead and subtract. That is the whole first bar, 15, then the 10 to 20 piece of the second bar at 0.8×10, then the whole last bar, 23. $145 - (15 + 8 + 23) = 99$

✓ **Check 2 - counting blocks instead of using the formula:** Take one block of the grid to be 5 minutes wide and 0.5 of a unit of frequency density tall, so one block stands for 2.5 visitors. The five bars are then 6, 4.8, 8, 30 and 9.2 blocks, and the 20 to 60 region is $1.6 + 8 + 30$ blocks. $\frac{39.6}{58} = \frac{99}{145}$

✓ **Check 3 - working in tenths:** Multiply every frequency by 10 so that nothing is a decimal. The bars become 150, 120, 200, 750 and 230, a total of 1450, and the 20 to 60 region becomes $40 + 200 + 750$. Cancelling the 10 has to return the same fraction. $\frac{990}{1450} = \frac{99}{145}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
At least two correct frequencies	M1	for at least two correct frequencies, for example $1.5 \times 10 (= 15)$ or $0.8 \times 15 (= 12)$ or $4 \times 5 (= 20)$ or $2.5 \times 30 (= 75)$ or $4.6 \times 5 (= 23)$, or for any two of 15, 12, 20, 75 and 23. Counting squares or blocks earns it just as well, so any two of 150, 120, 200, 750 and 230 , or any two of 6, 4.8, 8, 30 and 9.2, also score.	✓
A method to find the number of visitors in each interval, with the intention to add	M1	for $1.5 \times 10 + 0.8 \times 15 + 4 \times 5 + 2.5 \times 30 + 4.6 \times 5 (= 145)$, or for $15 + 12 + 20 + 75 + 23 (= 145)$, with one error allowed. The block totals $150 + 120 + 200 + 750 + 230 (= 1450)$ and $6 + 4.8 + 8 + 30 + 9.2 (= 58)$ score the same mark, and so does a correct method for the 20 to 60 minute frequency such as $4 + 20 + 75 (= 99)$.	✓

Step	Mark	Description	Got it?
A complete method for the fraction	**M1**	for a complete method, for example $\dfrac{0.8 \times 5 + 4 \times 5 + 2.5 \times 30}{15 + 12 + 20 + 75 + 23}$, or the block form $\dfrac{1.6 + 8 + 30}{58}$. The one error already allowed in the previous method mark may follow through into this one.	✓
The fraction	**A1**	for $\dfrac{99}{145}$, or for any equivalent fraction such as $\dfrac{990}{1450}$. The answer must be left as a fraction. A correct answer scores full marks unless it has come from obviously incorrect working.	✓
Note - reaching 99 without listing the frequencies	**note**	A value of 99 for the 20 to 60 minute interval scores both of the first two method marks (M2), even when the individual frequencies are never written down.	✓

Full marks: 4/4

Question 19 - Exam Solution

Understanding the Question

Given

An arithmetic series with first term a and common difference d

The sum of the first 30 terms is $4\,395$

The 10th term added to the 20th term is 284

Find

the sum of the first 45 terms of the same series

Plan the Solution

- Put $n = 30$ into the sum formula, so the first fact becomes an equation in a and d.
- Put $n = 10$ and $n = 20$ into the nth term formula, add the two expressions, and the second fact becomes a second equation in a and d.
- Both equations contain $2a$, so subtracting one from the other removes a and leaves d on its own.

- Substitute d back to get a, then put a, d and $n = 45$ into the sum formula.

Worked Solution [5 marks]

> Rule - Arithmetic series: the nth term is $a + (n-1)d$, and the sum of the first n terms is $\frac{n}{2}[2a + (n-1)d]$

Turn the sum of the first 30 terms into an equation

$S_{30} = \frac{30}{2}[2a + (30-1)d] = 4\,395$

$15(2a + 29d) = 4\,395$

$\frac{4\,395}{15} = 293$

$2a + 29d = 293$

(Reason: A total made from 30 terms can only come from the sum formula, so putting $n = 30$ into it turns the first fact into an equation in the two unknowns. Dividing both sides by 15 straight away keeps the numbers small, which makes the elimination later much easier.)

Turn the 10th and 20th terms into a second equation

$u_{10} = a + (10-1)d = a + 9d$

$u_{20} = a + (20-1)d = a + 19d$

$(a + 9d) + (a + 19d) = 284$

$2a + 28d = 284$

(Reason: Every term is the first term plus a number of common differences, and that number is always one less than the position, so the 10th term carries 9 lots of d and the 20th carries 19. Adding the two expressions collects them into a second equation in the same two unknowns.)

Subtract the two equations to find the common difference

$(2a + 29d) - (2a + 28d) = 293 - 284$

$293 - 284 = 9$

$d = 9$

(Reason: Both equations contain exactly $2a$, so subtracting one from the other cancels the first term completely. On the left, 29 lots of d take away 28 lots of d leaves a single d, so the right hand side is the value of d itself.)

Substitute the common difference to find the first term

$2a + 28 \times 9 = 284$

$28 \times 9 = 252$

$2a = 284 - 252 = 32$

$a = 16$

(Reason: The second equation is the simpler of the two, so put $d = 9$ into that one. Taking 252 from both sides leaves $2a$ on its own, and halving 32 gives the first term.)

Put the first term, the common difference and 45 terms into the sum formula

$$S_{45} = \frac{45}{2}\,[2 \times 16 + (45 - 1) \times 9]$$

$$\frac{45}{2} \times (32 + 396) = \frac{45}{2} \times 428$$

$$\frac{45}{2} \times 428 = 45 \times 214 = 9\,630$$

(Reason: The sum formula needs the number of terms, the first term and the common difference, and all three are now known. Watch the multiplier of the common difference: with 45 terms it is 45 minus 1, which is 44, not 45.)

9630

Verification

✓ **Check 1:** Rebuild the first given fact from the values found, by working out the sum of the first 30 terms with $a = 16$ and $d = 9$ $\frac{30}{2} \times (2 \times 16 + 29 \times 9) = 15 \times 293 = 4\,395$

✓ **Check 2:** Rebuild the second given fact by adding the 10th term to the 20th term, again using $a = 16$ and $d = 9$ $(16 + 9 \times 9) + (16 + 19 \times 9) = 97 + 187 = 284$

✓ **Check 3:** Sum the 45 terms a completely different way, as half the number of terms times the first term plus the last term, where the 45th term is 412

$$\frac{45}{2} \times (16 + 412) = 22.5 \times 428 = 9\,630$$

✓ **Check 4:** Split the sum instead: add the given total for the first 30 terms to the 15 terms that run from the 31st term, 286, to the 45th term, 412

$$4\,395 + \frac{15}{2} \times (286 + 412) = 4\,395 + 5\,235 = 9\,630$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Use the sum formula with $n = 30$	M1	for using the sum formula $\frac{n}{2}\,[2a + (n-1)d]$ with $n = 30$. Accept $\frac{30}{2}\,[2a + (30-1)d] = 4\,395$, or the expanded	✓

Step	Mark	Description	Got it?
		$30a + 435d = 4\,395$, or the simplified $2a + 29d = 293$	
Use the nth term formula to form a second equation	**M1**	for using the nth term formula $a + (n-1)d$ correctly to form an equation. Accept $a + (10-1)d + a + (20-1)d = 284$, or $a + 9d + a + 19d = 284$, or $2a + 28d = 284$, or $a + 14d = 142$	✓
A correct method to eliminate a or d	**dM1**	dependent on both earlier method marks, for a correct method to eliminate a or d: the coefficients of a or of d made equal and the correct operator used on the chosen variable, condoning any one arithmetic error. Accept $2a + 29d = 293$ minus $2a + 28d = 284$ giving $d = 9$, or $28a + 406d = 4\,102$ minus $29a + 406d = 4\,118$ giving $a = 16$. Writing a or d in terms of the other variable and substituting correctly also earns it	✓
Use the sum formula again, now with $n = 45$	**dM1**	dependent on the previous method mark, for using the sum formula correctly with $a = 16$ and $d = 9$, eg $\frac{45}{2}[2(16) + (45-1)9]$	✓
The sum of the first 45 terms, with working shown	**A1**	for $9\,630$. Working is required, so a bare answer scores nothing	✓

Full marks: 5/5

Question 20 - Exam Solution

Understanding the Question

Given

The curve C has equation

$y = 2x^4 - 64x$

C has a minimum point

Find

An equation of the tangent to C at that minimum point

Plan the Solution

- Differentiate the curve to get an expression for the gradient, $\frac{dy}{dx}$
- At a minimum point the gradient is zero, so solve $\frac{dy}{dx} = 0$ to find the x-coordinate
- Substitute that x back into the equation of the curve to get the y-coordinate
- A tangent at a minimum point is horizontal, so its equation has the form $y = k$ with no x in it

Worked Solution [4 marks]

> Rule - Stationary points: solve $\frac{dy}{dx} = 0$ for x, substitute back into the curve for y, and note that a tangent at a minimum has gradient 0, so it is a horizontal line $y = k$.

Differentiate the equation of the curve

$y = 2x^4 - 64x$

$\frac{dy}{dx} = 8x^3 - 64$

(Reason: Multiply by the power and then reduce the power by one. The term $2x^4$ gives $4 \times 2x^3 = 8x^3$, and the term $-64x$ gives -64, because x differentiates to 1. One correct term is already worth the first method mark.)

Set the gradient equal to zero

$8x^3 - 64 = 0$

(Reason: At a minimum point the curve is momentarily flat, so the gradient there is 0. Solving $\frac{dy}{dx} = 0$ is what locates the stationary point.)

Solve the equation for x

$8x^3 = 64$

$x^3 = \frac{64}{8} = 8$

$x = \sqrt[3]{8} = 2$

(Reason: Divide both sides by 8, then take the cube root. A cube root has just one real value, so $x = 2$ is the only stationary point on this curve, and it must therefore be the minimum point the question describes.)

Work out the y-coordinate of the minimum point

$y = 2 \times 2^4 - 64 \times 2 = 32 - 128 = -96$

(Reason: Substitute $x = 2$ back into the equation of the curve, $y = 2x^4 - 64x$, to get the height of the minimum point. The derivative has done its job already and is not the expression to substitute into here.)

Write down the equation of the tangent

$\text{gradient of the tangent} = 0$

$y = -96$

(Reason: A tangent at a minimum point is horizontal, so its gradient is 0. Every point on a horizontal line through $(2, -96)$ has y-coordinate -96, so the tangent is $y = -96$. The question asks for an equation, so -96 on its own, or the coordinates $(2, -96)$, would score nothing.)

$$y = -96$$

Verification

- ✓ **Check 1:** Differentiate a second time and test the sign at $x = 2$, using $\dfrac{d^2y}{dx^2} = 24x^2$ $24 \times 2^2 = 96$, which is positive, so the stationary point really is a minimum and not a maximum
- ✓ **Check 2:** Work out the height of the curve just either side of $x = 2$ and compare it with -96 $y = -95.5358$ at $x = 1.9$ and $y = -95.5038$ at $x = 2.1$, both above -96, so -96 is the lowest value the curve reaches
- ✓ **Check 3:** Subtract the tangent from the curve and factorise the result, $2x^4 - 64x + 96$ $2(x-2)^2(x^2+4x+12)$, and $x^2 + 4x + 12 = (x+2)^2 + 8$ is always positive, so the difference is never negative and is zero only at $x = 2$ - the line $y = -96$ touches the curve there and never crosses it, which is exactly what a tangent does

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Differentiate one term of the curve correctly	M1	$4 \times 2x^3$ or $8x^3$ or 64 with either sign	✓
Form the equation that locates the stationary point	dM1	$8x^3 - 64 = 0$ or equivalent. The equation must be of the form $ax^3 - 64 = 0$ with a not equal to 0, or $8x^3 + b = 0$ with b not equal to 0, where a and b are constants	✓

Step	Mark	Description	Got it?
Solve the equation for x	dM1	$x = \sqrt[3]{\frac{64}{8}}$, leading to $x = 2$. Again the equation being solved must be of the form $ax^3 - 64 = 0$ with a not equal to 0, or $8x^3 + b = 0$ with b not equal to 0	✓
State an equation for the tangent	A1	$y = -96$ or equivalent, eg $y + 96 = 0$ or $y = 0x - 96$ or $-y = 96$. Working is required, and the answer must be an equation in terms of y: -96 on its own, or the coordinates $(2, -96)$, scores nothing	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

$$T = \frac{x^2 + y^2}{w}$$

$x = 28.4$ correct to 1 decimal place
$y = 17$ correct to 2 significant figures
$w = 90$ correct to the nearest 5

Find

The upper bound for the value of T, correct to 3 significant figures

Plan the Solution

- Turn each rounded value into an interval: a value rounded to a unit lies half a unit either side of the figure given
- Decide which end of each interval makes T biggest. x and y are on the top, so both take their UPPER bounds; w is on the bottom, so it takes its LOWER bound
- Substitute those three bounds into the formula and work out the numerator first
- Divide, then round the result to 3 significant figures

Worked Solution [3 marks]

Rule - Bounds of a calculation: a value rounded to a unit u lies within $\frac{u}{2}$ of the figure given, and a fraction is biggest when its numerator is biggest and its denominator is smallest, so the upper bound of $\frac{x^2+y^2}{w}$ comes from the upper bounds of x and y over the LOWER bound of w.

Write down the interval each letter lies in

$28.35 \leq x < 28.45$

$16.5 \leq y < 17.5$

$87.5 \leq w < 92.5$

(Reason: Each rounding has its own unit. One decimal place is a unit of 0.1, so x is within 0.05 of 28.4. On 17 the second significant figure is the units digit, so 2 significant figures here means the nearest 1, and y is within 0.5 of 17. The nearest 5 is a unit of 5, so w is within 2.5 of 90. Half the unit each way is what produces all six numbers, and the first mark is for writing down any one of them.)

Choose the bound of each letter that makes T biggest

upper bound of $x = 28.45$

upper bound of $y = 17.5$

lower bound of $w = 87.5$

(Reason: x^2 and y^2 sit on the top of the fraction, so the bigger they are the bigger T is. But w sits on the bottom, and dividing by a smaller number gives a bigger answer, so w must take its LOWEST value. Reaching for 92.5 because the question says 'upper bound' is the single most common way this mark is lost. The true x never actually reaches 28.45, which is why the mark scheme also accepts the upper bound written as the recurring decimal 28.449 recurring.)

Substitute the three bounds into the formula

$$T = \frac{28.45^2 + 17.5^2}{87.5}$$

(Reason: This substitution is what the method mark is for, before any arithmetic is done. Every figure in it came out of step 1, and the mark scheme wants to see the upper bounds of x and y over the lower bound of w.)

Work out the numerator

$28.45^2 = 809.4025$

$17.5^2 = 306.25$

$809.4025 + 306.25 = 1115.6525$

(Reason: Square each upper bound, then add. Squaring keeps the order for positive numbers, so the biggest x and the biggest y really do give the biggest numerator - there is no need to test any other combination.)

Divide by the lower bound of w

$$\frac{1115.6525}{87.5} \approx 12.75031429$$

(Reason: The lower bound of w is 87.5, so 87.5 is the number to divide by. The division is written as a fraction rather than with a division sign, and the mark scheme shows this value as 12.75031429.)

Round to 3 significant figures

$12.75031429 \approx 12.8$

(Reason: The first three significant figures are 1, 2 and 7, and the digit after them is 5, so the 7 rounds up to 8. The mark scheme asks for awrt 12.8 and insists the working is shown, so an answer written down with no substitution scores nothing here.)

Upper bound of $T = 12.8$

Verification

- ✓ **Check 1:** Take values strictly inside all three intervals, $x = 28.4499$, $y = 17.4999$ and $w = 87.5001$, and work out T $T = 12.75019469$, which is below 12.75031429, so a value the three letters are actually allowed to take falls short of the bound, exactly as an upper bound requires
- ✓ **Check 2:** Reverse the division: multiply the unrounded answer back up by the lower bound of w and compare it with the numerator $\frac{446261}{35000} \times 87.5 = 1115.6525$, which is $809.4025 + 306.25$, so the division was carried out on the right numbers
- ✓ **Check 3:** Test the other bound of w, and work out where the LOWER bound of T sits, to see that 12.8 lands at the top of the range using 92.5 instead gives 12.1 to 3 significant figures, not 12.8, and the lower bound $\frac{28.35^2 + 16.5^2}{92.5}$ is 11.6 to 3 significant figures, so T lies between 11.6 and 12.8 and 12.8 is the top of that range

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Write down a correct bound for x, y or w	B1	28.35 or 28.45 or 16.5 or 17.5 or 87.5 or 92.5. Accept 28.449 recurring for 28.45, and 17.49 recurring for 17.5	✓

Step	Mark	Description	Got it?
Substitute the correct bounds into the formula for T	M1	$T = \dfrac{(UB_x)^2 + (UB_y)^2}{LB_w}$, for example $\dfrac{28.45^2 + 17.5^2}{87.5}$, where $28.4 < UB_x \leq 28.45$ and $17 < UB_y \leq 17.5$ and $87.5 \leq LB_w < 90$	✓
Complete the calculation and round it	A1	awrt 12.8. The answer must come from the correct figures 28.45, 17.5 and 87.5, and working is required, so an unsupported 12.8 scores nothing	✓

Full marks: 3/3

Question 22 - Exam Solution

Understanding the Question

Given

A bird bath in the shape of a hemisphere, cast in bronze to a uniform thickness

The outer radius is 12 cm and the inner radius is 9 cm

The volume of a sphere is $\dfrac{4\pi r^3}{3}$

Find

The volume of the bronze, correct to the nearest whole number

Plan the Solution

- The bronze is what is left when the hollow is taken out of the solid outer hemisphere.
- Work out the outer hemisphere, radius 12 cm, then the hollow inner hemisphere, radius 9 cm.
- Subtract one from the other, keep the result as a multiple of π, and round only on the last line.

Worked Solution [3 marks]

Volume of a hemisphere (half a sphere): $\dfrac{2\pi r^3}{3}$

Volume of the outer hemisphere

$12^3 = 1728$

$$V_{\text{outer}} = \frac{2}{3} \times \pi \times 1728 = 1152\pi \approx 3619.115$$

(Reason: A hemisphere is half a sphere, so its volume is two thirds of π times the radius cubed. The outside of the bird bath has radius 12 cm.)

Volume of the hollow inside

$9^3 = 729$

$$V_{\text{inner}} = \frac{2}{3} \times \pi \times 729 = 486\pi \approx 1526.814$$

(Reason: The hollow is a hemisphere as well, of radius 9 cm, and it shares its centre with the outside because the thickness is uniform.)

Subtract the hollow from the outer hemisphere

$V = 1152\pi - 486\pi = 666\pi$

$666\pi \approx 2092.3007$

(Reason: The bronze is the shell left between the two hemispheres. Working in multiples of π keeps the value exact until the very last line.)

Round to the nearest whole number

$2092.3007 \approx 2092$

(Reason: The first figure after the decimal point is 3, which is below 5, so the whole-number part stays as it is.)

$$2092 \text{ cm}^3$$

Verification

✓ **Check 1:** Take the cubes apart first: $1728 - 729 = 999$, then $\frac{2}{3} \times 999 = 666$ $666\pi \approx 2092.3$, the same volume as before

✓ **Check 2:** Use whole spheres instead: $\frac{4}{3} \times \pi \times 1728 = 2304\pi$ and $\frac{4}{3} \times \pi \times 729 = 972\pi$, so the shell between the two spheres is 1332π The bird bath is half of that shell, and half of 1332π is 666π, which is 2092 to the nearest whole number

✓ **Check 3:** Is the size sensible? The bronze must come to less than the whole outer hemisphere, which is about 3619 cm^3 2092 cm^3 is about 58 per cent of it, which suits a shell whose hollow is three quarters as wide as the outside

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The volume of one sphere or one hemisphere	M1	For the volume of a sphere or of a hemisphere, for example $\frac{2}{3} \times \pi \times 12^3 (= 1152\pi = 3619.114)$ or $\frac{2}{3} \times \pi \times 9^3 (= 486\pi = 1526.814)$, or a full sphere $\frac{4}{3} \times \pi \times 12^3 (= 2304\pi = 7238.229)$ or $\frac{4}{3} \times \pi \times 9^3 (= 972\pi = 3053.628)$	✓
A complete method for the metal	M1	For a complete method, for example $1152\pi - 486\pi (= 666\pi)$, or $\frac{2304\pi - 972\pi}{2} (= 666\pi)$	✓
The rounded answer	A1	For 2092; the scheme accepts anything from 2091 to awrt 2093	✓
Note	note	A correct answer scores full marks unless it comes from obviously incorrect working	✓

Full marks: 3/3

Question 23 - Exam Solution

Understanding the Question

Given

The curve C has equation $y = x^2 - 8x - 9$

The straight line L has equation $y = k$, where k is an integer

C and L meet at $A(p, k)$ and $B(q, k)$, so the two points share one y value

$p - q = 14$

Find

The value of k

Plan the Solution

- Where L meets C the two y values are equal, so p and q are the two roots of $x^2 - 8x - 9 = k$.
- A horizontal line cuts a parabola at two points that sit the same distance either side of its line of symmetry, so find that line of symmetry first.
- The line of symmetry is $x = 4$, so the gap of 14 splits into 7 on each side, which fixes p and q.
- Put either root back into the equation of C, because the y value it returns is k itself.

Worked Solution [5 marks]

> Rule - Symmetry of a parabola: $y = ax^2 + bx + c$ is symmetrical about the vertical line $x = \dfrac{-b}{2a}$, so the two points where a horizontal line cuts it lie the same distance either side of that line.

Set the equation of the curve equal to the equation of the line

$$x^2 - 8x - 9 = k$$
$$x^2 - 8x - 9 - k = 0$$

(Reason: At A and at B a point sits on the curve and on the line at the same moment, so its y value is $x^2 - 8x - 9$ and it is also k. That makes p and q the two roots of this one quadratic, and it is why both points carry the same second coordinate.)

Find the line of symmetry of the curve

$$x^2 - 8x - 9 = (x-4)^2 - 16 - 9 = (x-4)^2 - 25$$
$$x = 4$$

(Reason: Completing the square writes the curve as $(x-4)^2 - 25$, so its lowest point is $(4, -25)$ and the curve is symmetrical about the vertical line $x = 4$. Differentiating gives the gradient function $2x - 8$, which is zero at that same $x = 4$, and the roots of $x^2 - 8x - 9 = 0$, namely -1 and 9, have that same midpoint. Any one of the three earns the first mark.)

Place p and q either side of the line of symmetry

$$\frac{p+q}{2} = 4$$
$$\frac{p-q}{2} = \frac{14}{2} = 7$$
$$p = 4 + 7 = 11$$
$$q = 4 - 7 = -3$$

(Reason: A horizontal line meets the curve at two points the same distance from $x = 4$, so the midpoint of p and q is 4 and the gap of 14 between them is 7 on each side. p is the larger of the two, because the question gives $p - q$ as positive.)

Substitute a root back into the equation of the curve

$k = 11^2 - 8 \times 11 - 9$

$121 - 88 - 9 = 24$

(Reason: A lies on C, so putting $x = 11$ into $y = x^2 - 8x - 9$ returns the y value that A and B share, and that value is k. The question says k is an integer, and 24 is one, which is a small sign that nothing has gone wrong.)

$$k = 24$$

Verification

- ✓ **Check 1:** Use the other root. Substituting $q = -3$ into the curve gives $(-3)^2 - 8(-3) - 9$, and it must land on the same k if A and B really do lie on one horizontal line $9 + 24 - 9 = 24$
- ✓ **Check 2:** Work backwards. Put $k = 24$ into $x^2 - 8x - 9 = k$, so $x^2 - 8x - 33 = 0$, which factorises as $(x - 11)(x + 3) = 0$ $x = 11$ and $x = -3$, so $p - q = 11 - (-3) = 14$, which is exactly the gap the question gives
- ✓ **Check 3:** Avoid the roots altogether. From $k = (x - 4)^2 - 25$ the two solutions are $x = 4 + \sqrt{k + 25}$ and $x = 4 - \sqrt{k + 25}$, so the gap between them is $2\sqrt{k + 25}$ whatever k is $2\sqrt{k + 25} = 14$ gives $\sqrt{k + 25} = 7$, so $k + 25 = 49$ and $k = 24$ again, reached without ever naming p or q

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct start on the line of symmetry	M1	For using differentiation, completing the square or symmetry: $\dfrac{dy}{dx} = 2x - 8$ or $(x - 4)^2$ ..... or the roots of $x^2 - 8x - 9 = 0$ combined as $\dfrac{-1 + 9}{2}$	✓
The line of symmetry stated	M1	$x = 4$ or $(4, -25)$ or $(4, ...)$ or $\dfrac{p + q}{2} = 4$ oe	✓
One of the two roots	A1	$q = -3$ or $p = 11$. Either one alone is enough for this mark	✓

Step	Mark	Description	Got it?
A root substituted into the equation of C	**M1**	$(k =)(-3)^2 - 8(-3) - 9$ or $(k =)(11)^2 - 8(11) - 9$	✓
The value of k	**A1 dep on M2**	24. The paper asks for clear algebraic working, and the scheme prints Working required against this row, so 24 on its own scores nothing here	✓
Alternative routes	**note**	The official scheme prints three alternatives, each carrying the same five marks in the same places: substitute $p = q + 14$ and form one equation in one variable; or set $(q + 14)^2 - 8(q + 14) - 9$ equal to k directly; or solve $x^2 - 8x - 9 - k = 0$ by formula to reach $x = 4 + \sqrt{k + 25}$ and $x = 4 - \sqrt{k + 25}$, then $25 + k = \left(\frac{14}{2}\right)^2$	✓

Full marks: 5/5

Question 24 - Exam Solution

Understanding the Question

Given

$\overrightarrow{OA} = 10\mathbf{a}$ and $\overrightarrow{OB} = 10\mathbf{b}$, so $\mathbf{a}$ and $\mathbf{b}$ are the two directions everything else is built from.

P lies on AB with $\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$

Q lies on OB with $\overrightarrow{OQ} = \frac{1}{5}\overrightarrow{OB}$

ARQ and ORP are straight lines, so R lies on both of them.

Find

$\overrightarrow{AQ}$, $\overrightarrow{OP}$ and $\overrightarrow{OR}$, each in terms of $\mathbf{a}$ and $\mathbf{b}$. Parts (i) and (ii) are single journeys; part (iii) needs the two straight lines to be used together.

Plan the Solution

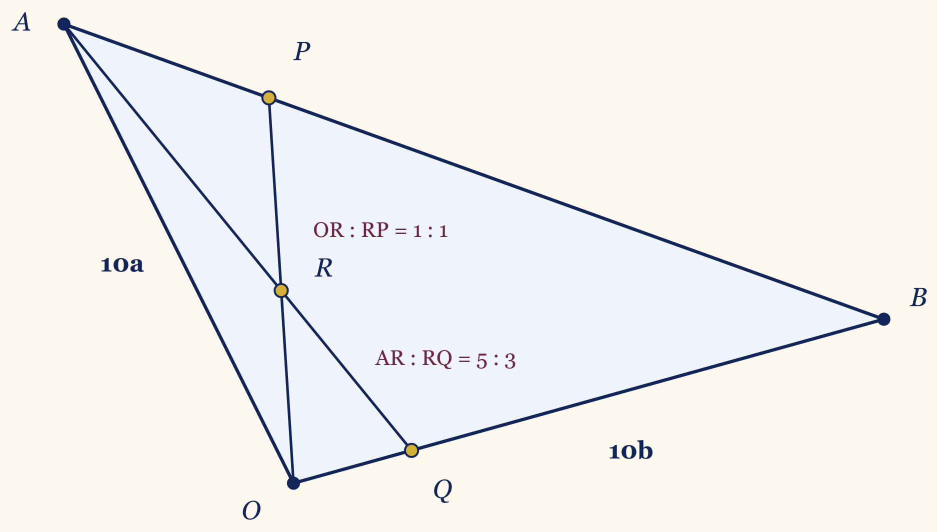
- Travel from one point to another along legs that are already known: to reach Q from A, go back along AO first and then forward along OQ.
- For OP, start at O, go to A, then take a quarter of AB.
- R is the only point lying on both straight lines, so write OR twice - once as a multiple of OP, once as OA plus a multiple of AQ.
- Two expressions for the same vector must have equal $\mathbf{a}$-parts and equal $\mathbf{b}$-parts. That gives two equations in the two unknown multipliers.
- Solve the pair, then substitute back into either expression.

Worked Solution [6 marks]

Rule - Two routes to one point: if R lies on both straight lines then $\overrightarrow{OR}$ can be written along either line, and comparing the $\mathbf{a}$ parts and the $\mathbf{b}$ parts of the two expressions gives a pair of simultaneous equations.

Part (i) - travel from A to Q through O

$$\overrightarrow{OQ} = \frac{1}{5} \times 10\mathbf{b} = 2\mathbf{b}$$
$$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ}$$
$$\overrightarrow{AQ} = -10\mathbf{a} + 2\mathbf{b}$$



(Reason: There is no single arrow from A to Q in the given information, so the journey is broken into legs that are given. Going from A to O is going along OA backwards, which reverses its sign.)

Part (ii) - travel from O to P through A

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -10\mathbf{a} + 10\mathbf{b}$$

$$\overrightarrow{AP} = \frac{1}{4}(-10\mathbf{a} + 10\mathbf{b}) = -\frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 10\mathbf{a} - \frac{5}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

$$\overrightarrow{OP} = \frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$$

(Reason: AB has to be found first, because AP is given as a fraction of it. Collecting the $\mathbf{a}$ terms at the end is what the word simplify is asking for.)

Part (iii), first expression - R is on the straight line ORP

$$\overrightarrow{OR} = k\overrightarrow{OP}$$

$$\overrightarrow{OR} = \frac{15k}{2}\mathbf{a} + \frac{5k}{2}\mathbf{b}$$

(Reason: O, R and P lie on one straight line, so OR points the same way as OP and is some multiple of it. That multiple is called k, and finding it is what the rest of the question is for.)

Part (iii), second expression - R is also on the straight line ARQ

$$\overrightarrow{OR} = \overrightarrow{OA} + \lambda\overrightarrow{AQ}$$

$$\overrightarrow{OR} = 10\mathbf{a} + \lambda(-10\mathbf{a} + 2\mathbf{b})$$

$$\overrightarrow{OR} = (10 - 10\lambda)\mathbf{a} + 2\lambda\mathbf{b}$$

(Reason: A, R and Q lie on one straight line, so AR is some fraction of AQ. Reaching R this way needs a second unknown, and part (i) has already supplied AQ.)

Part (iii) - compare the a parts and the b parts, then solve

$$\frac{15k}{2} = 10 - 10\lambda$$

$$\frac{5k}{2} = 2\lambda$$

$$\lambda = \frac{5k}{4}$$

$$\frac{15k}{2} = 10 - \frac{25k}{2}$$

$$20k = 10$$

$$k = \frac{1}{2}$$

(Reason: Both expressions describe the same vector OR, and $\mathbf{a}$ and $\mathbf{b}$ point in different directions, so the amount of $\mathbf{a}$ in each must match and the amount of $\mathbf{b}$ in each must match. The $\mathbf{b}$ comparison gives λ in terms of k, and substituting it into the $\mathbf{a}$ comparison leaves one equation in one unknown.)

Part (iii) - substitute k back into the first expression

$$\overrightarrow{OR} = \frac{1}{2}\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$$

$$\overrightarrow{OR} = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$$

(Reason: k is a half, so R sits exactly halfway along OP. Substituting into the OP route is quicker than the AQ route, and either one gives the same vector.)

(i) $AQ = -10\mathbf{a} + 2\mathbf{b}$; (ii) $OP = 7.5\mathbf{a} + 2.5\mathbf{b}$; (iii) $OR = 3.75\mathbf{a} + 1.25\mathbf{b}$

Verification

✓ **Check 1:** The two expressions for $\overrightarrow{OR}$ must agree. Put $k = \frac{1}{2}$ into the ORP route, and $\lambda = \frac{5}{8}$ into the ARQ route. The first gives $\frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$ and the second gives $(10 - 6.25)\mathbf{a} + 1.25\mathbf{b}$, the same vector.

✓ **Check 2:** Reach R along a third path, through Q instead of through A, using $\overrightarrow{OR} = \overrightarrow{OQ} + \mu\overrightarrow{QA}$ with $\mu = \frac{3}{8}$. $2\mathbf{b} + \frac{3}{8}(10\mathbf{a} - 2\mathbf{b}) = \frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$

✓ **Check 3:** Leave the vectors behind and use coordinates. With $\mathbf{a}$ and $\mathbf{b}$ as the two unit vectors the points are $O(0,0)$, $A(10,0)$, $B(0,10)$, $P(7.5,2.5)$ and $Q(0,2)$. Work out where the line through O and P meets the line through A and Q. They meet at $(3.75, 1.25)$, which reads back as $\frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) AQ	B1	for $-10\mathbf{a} + 2\mathbf{b}$ or $2\mathbf{b} - 10\mathbf{a}$. The answer must be simplified.	✓
(ii) OP	B1	for $\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}$ oe, eg $7.5\mathbf{a} + 2.5\mathbf{b}$. The answer must be simplified.	✓
(iii) first path to R, along ORP	M1ft	for $\overrightarrow{OR} = k\overrightarrow{OP} = k\left(\frac{15}{2}\mathbf{a} + \frac{5}{2}\mathbf{b}\right)$ oe. Here $\overrightarrow{OP}$ must be in terms of $\mathbf{a}$ and $\mathbf{b}$ (lower case).	✓

Step	Mark	Description	Got it?
(iii) second path to R, along ARQ or through Q	**M1ft**	for $\overrightarrow{OR} = \overrightarrow{OA} + \lambda\overrightarrow{AQ} = 10\mathbf{a} + \lambda(-10\mathbf{a} + 2\mathbf{b}) = (10 - 10\lambda)\mathbf{a} + 2\lambda\mathbf{b}$ oe, or $\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR} = 2\mathbf{b} - \mu(-10\mathbf{a} + 2\mathbf{b}) = 10\mu\mathbf{a} + (2 - 2\mu)\mathbf{b}$ oe. Here $\overrightarrow{AQ}$ must be in terms of $\mathbf{a}$ and $\mathbf{b}$ (lower case).	✓
(iii) both equations correct	**M1**	for $\frac{15}{2}k = 10 - 10\lambda$ and $\frac{5}{2}k = 2\lambda$ oe, or $\lambda = \frac{5}{8}$ oe; or $\frac{5}{2}k = 2 - 2\mu$ and $\frac{15}{2}k = 10\mu$ oe, or $k = 0.5$ oe.	✓
(iii) OR	**A1**	for $\frac{15}{4}\mathbf{a} + \frac{5}{4}\mathbf{b}$ oe, eg $3.75\mathbf{a} + 1.25\mathbf{b}$.	✓
Note	**note**	A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 6/6

Question 25 - Exam Solution

Understanding the Question

Given

$f(x) = 2x^2 - 24x + 7$

The domain is $x \geq 6$, and 6 is exactly where this parabola turns.

On that domain f only increases, so no two inputs share an output and the inverse really is a function.

Find

$f^{-1}(x)$, the inverse function. The mark scheme requires the answer in terms of x, so the working must end by swapping the letters back.

Plan the Solution

- An inverse undoes the function, so start from $y = f(x)$ and rearrange until x is the subject.

- x appears twice in $2x^2 - 24x + 7$, once squared and once not, so it cannot be peeled off one operation at a time. Completing the square rewrites the rule with x in only one place.
- Once it reads $y = 2(x-6)^2 - 65$, undo the operations in reverse order: add 65, halve, square root, add 6.
- Square rooting offers a plus and a minus. The domain $x \geq 6$ is what decides which one is kept.
- Finish by writing the answer in terms of x, because that is the letter an inverse function is written in.

Worked Solution [4 marks]

> Rule - Inverse by rearranging: write $y = \mathrm{f}(x)$, make x the subject, then rewrite the result in terms of x. Where x appears twice, complete the square first so that it appears only once.

Write $y = \mathrm{f}(x)$, then take the 2 out of the first two terms

$$y = 2x^2 - 24x + 7$$
$$y = 2(x^2 - 12x) + 7$$

(Reason: Making x the subject is the whole job, but x sits in the squared term and again in the $-24x$ term, so it cannot be undone one operation at a time. Taking the 2 out of the first two terms is the opening move of completing the square, and it is where the mark scheme awards its first method mark. The $+7$ is deliberately left outside the bracket, because only the x terms are being reshaped.)

Complete the square inside the bracket

$$x^2 - 12x = (x-6)^2 - 6^2$$
$$y = 2((x-6)^2 - 36) + 7$$
$$y = 2(x-6)^2 - 72 + 7 = 2(x-6)^2 - 65$$

(Reason: Half of 12 is 6, so $x^2 - 12x$ is $(x-6)^2$ short of 36. Multiplying the bracket out and collecting the constants leaves x in exactly one place, which is what makes the rearrangement possible at all. Watch the constant as it leaves the bracket: it is inside a bracket that is being multiplied by 2, so it must be doubled on the way out. This line is the mark scheme's second method mark, and it depends on the first.)

Make the squared bracket the subject

$$y + 65 = 2(x-6)^2$$
$$(x-6)^2 = \frac{y+65}{2}$$

(Reason: Undo the operations in the reverse of the order they were applied. The last thing done to the bracket was subtracting 65, so add 65 first, then undo the multiplication by 2 by dividing. This is the

third method mark, and the scheme accepts the equivalent form with $(y-7)$ over 2, plus 6^2, on the right instead.)

Take the square root, choose the sign, then swap the letters

$$x-6=\pm\sqrt{\frac{y+65}{2}}$$

$$x=6+\sqrt{\frac{y+65}{2}}$$

$$\mathrm{f}^{-1}(x)=6+\sqrt{\frac{x+65}{2}}$$

(Reason: Square rooting always offers two possibilities. The domain is $x \geq 6$, so $x-6$ can never be negative and only the positive root can be the inverse. That single decision is worth the accuracy mark on its own: the scheme awards M3A0, all three method marks and no accuracy mark, to an answer left as 6 plus-or-minus the root. The last line renames y as x, because an inverse function is written in terms of x.)

$$\mathrm{f}^{-1}(x)=6+\sqrt{\frac{x+65}{2}}$$

Verification

✓ **Check 1:** Take $x=10$, which is inside the domain. Work out the output, then feed that output into the inverse and see whether 10 comes back. $2\times100-240+7=-33$, and $6+\sqrt{\frac{-33+65}{2}}=6+\sqrt{16}=10$

✓ **Check 2:** Repeat with a different input, $x=9$, so that the first check is not being confirmed by the value that produced it. $2\times81-216+7=-47$, and $6+\sqrt{\frac{-47+65}{2}}=6+\sqrt{9}=9$

✓ **Check 3:** Test the edge of the domain. The smallest allowed input is 6, so the smallest possible output is the value at 6, and the inverse must send it straight back to 6.

$2\times36-144+7=-65$, and $6+\sqrt{\frac{-65+65}{2}}=6+0=6$

✓ **Check 4:** Confirm that the rejected root really is rejected. The equation has a second solution when the output is -33; work out what the minus sign gives. $6-\sqrt{16}=2$, and $2\times4-48+7=-33$, so 2 does solve the equation but lies outside the domain $x\geq6$, which is why only the plus sign survives

✓ **Check 5:** Show that the other form the mark scheme accepts is the same expression, by putting $\dfrac{x-7}{2}+36$ over a common denominator.

$\dfrac{x-7}{2}+36=\dfrac{x-7+72}{2}=\dfrac{x+65}{2}$, so $6+\sqrt{\dfrac{x-7}{2}+36}$ is the same function written a different way

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct first step towards completing the square	**M1**	for eg $(y=)2(x^2-12x)+7$ or $(y=)2(x^2-12x+\frac{7}{2})$ or $\dfrac{y-7}{2}=x^2-12x$. For each method mark the function must be correct.	✓
The square completed	**M1 dep**	for eg $(y=)2((x-6)^2-6^2)+7$ or $(y=)2((x-6)^2-6^2+\frac{7}{2})$ or $(y=)2(x-6)^2-65$ oe or $\dfrac{y-7}{2}=(x-6)^2-6^2$ oe	✓
The squared bracket made the subject	**M1**	for $(x-6)^2=\dfrac{y+65}{2}$ oe, or $(x-6)^2=\dfrac{y-7}{2}+6^2$ oe	✓
The inverse function	**A1**	for $6+\sqrt{\dfrac{x+65}{2}}$ oe, eg $6+\sqrt{\dfrac{x-7}{2}+36}$. The answer must be in terms of x. Award M3A0 for $6\pm\sqrt{\dfrac{x+65}{2}}$, for $6\pm\sqrt{\dfrac{y+65}{2}}$ and for $6+\sqrt{\dfrac{y+65}{2}}$.	✓
Note	**note**	A correct answer scores full marks, unless it comes from obviously incorrect working. Candidates are allowed to swap x and y while finding the inverse, provided the final answer is in terms of x.	✓
Alternative method	**note**	The scheme prints a full alternative. Rearranging to $2x^2-24x+7-y=0$ earns the first M1;	✓

Step	Mark	Description	Got it?
		$(x =)\dfrac{24 \pm \sqrt{576 - 8(7 - y)}}{4}$ earns the dependent M1; $(x =)6 \pm \sqrt{\dfrac{y + 65}{2}}$ earns the third M1; and the same A1 is awarded for the final answer.	

Full marks: 4/4

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