Edexcel IGCSE 4MA1

Paper 2H (Higher Tier) – November 2024

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A (4MA1), Paper 2H (Higher Tier), November 2024 – 100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Show that $1\frac{5}{7} \times 2\frac{3}{16} = 3\frac{3}{4}$

Show every stage of your working. *[3 marks]*

[Total 3 marks]

2. A carpenter measures the width of a bookshelf as 1.4 metres, correct to one decimal place.

(a) Write down the upper bound of the width of the bookshelf. *[1 mark]*

(b) Write down the lower bound of the width of the bookshelf. *[1 mark]*

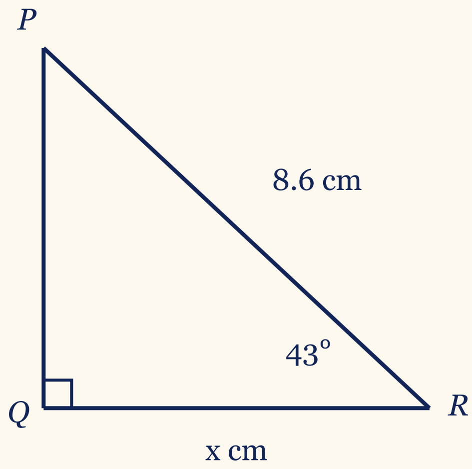
(a) metres ....

(b) metres ....

[Total 2 marks]

3. Triangle PQR is shown in the diagram below.

Diagram NOT
accurately drawn



Calculate the value of x.
Give your answer correct to one decimal place. *[3 marks]*

x = ...

[Total 3 marks]

4. N is a number.
17% of N is 357

(a) Work out the value of N *[2 marks]*

In 2019, the number of members of a cycling club was 650
In 2020, the number of members of the club was 806

(b) Work out the percentage increase in the number of members. *[3 marks]*

(a) N = ...

(b) % ....

[Total 5 marks]

5. Dylan has made a biased five-sided spinner to use in a board game.
The five sections of the spinner are numbered $1, 2, 3, 4, 5$

Number	1	2	3	4	5
Probability		0.14	0.17		0.21

The table gives the probabilities that, when the spinner is spun, it will land on 2 or on 3 or on 5

The probability that the spinner will land on 1 is the same as the probability that the spinner will land on 4

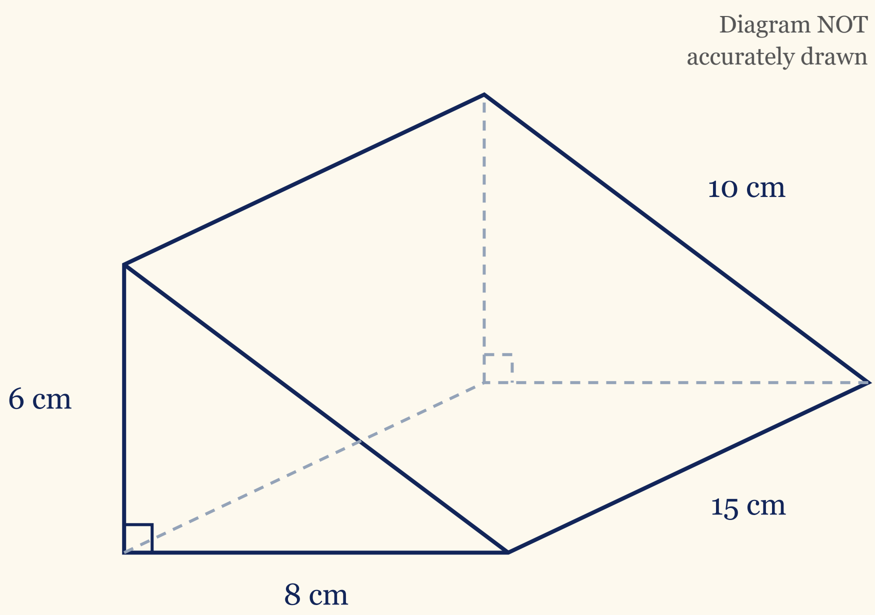
Dylan is going to spin the spinner 400 times.

Work out an estimate for the number of times the spinner will land on 4 *[4 marks]*

....

[Total 4 marks]

6. The diagram shows a solid prism.
The cross-section of the prism is a right-angled triangle.



Work out the total surface area of the prism. *[3 marks]*

.... cm^2

[Total 3 marks]

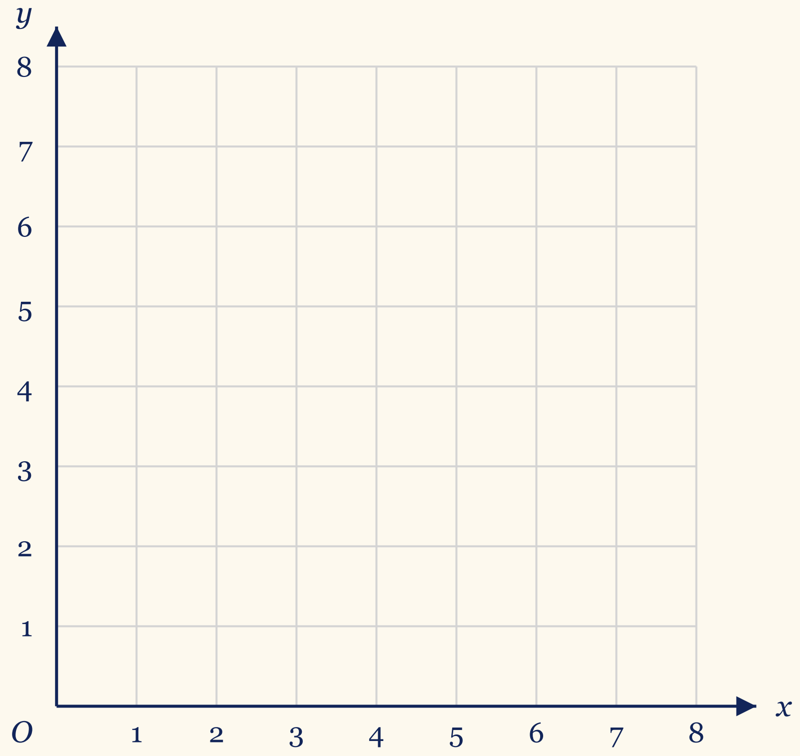
7. (a) On the grid below, draw each of these three straight lines.

(i) $x = 3$

(ii) $y = 1$

(iii) $x + y = 7$

Write the equation of each line beside the line you have drawn. *[3 marks]*



(b) On the same grid, shade the one region where all three of these inequalities hold.

$x \geq 3$, $y \geq 1$ and $x + y \leq 7$

Mark this region with the letter R. *[1 mark]*

[Total 4 marks]

8. Rosie puts 4 apples in a basket.
The mean weight of the 4 apples in the basket is 145 grams.

Martin puts one more apple into the basket.
The mean weight of the 5 apples in the basket is 142 grams.

Work out the weight of the apple that Martin puts into the basket. *[3 marks]*

.... grams

[Total 3 marks]

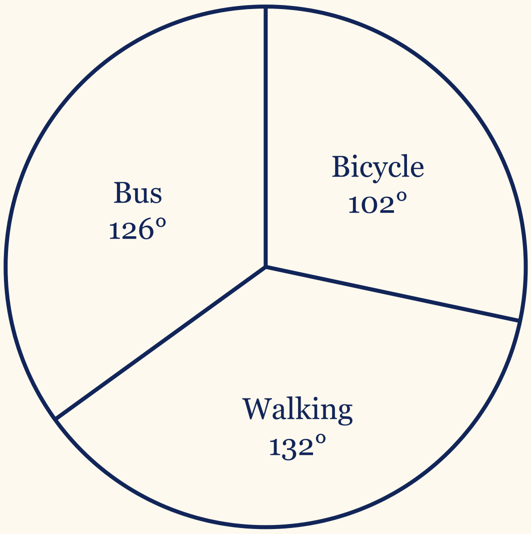
9. Meera puts $20\,000$ euros into a savings bond for 3 years.
The bond pays 3.5% per year compound interest.

Work out how much money Meera will have in the bond at the end of the 3 years.
Give your answer correct to the nearest euro. *[3 marks]*

.... euros

[Total 3 marks]

10. A company has two offices, Riverside and Eastgate.
Everyone who works at the Riverside office and everyone who works at the Eastgate office named their preferred way of travelling to work from bus, bicycle and walking.



travel to work	**number of people**
Bus	$3x + 6$
Bicycle	$5x + 8$
Walking	$7x - 9$

The pie chart shows information about the results for the Riverside office.
The table shows information about the results for the Eastgate office.

There are 300 people who work at the Riverside office.
There are 320 people who work at the Eastgate office.

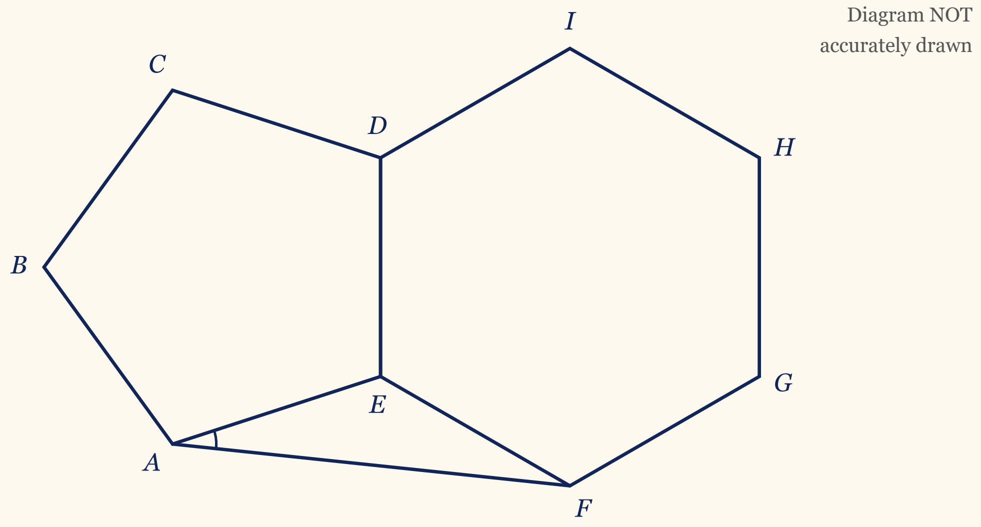
More people at the Riverside office than at the Eastgate office said bus was their preferred way of travelling to work.

How many more? *[5 marks]*

....

[Total 5 marks]

11. The diagram shows a regular pentagon $ABCDE$ joined to a regular hexagon $DEFGHI$ along the edge DE that the two shapes share.



AF is a straight line.

Work out the size of angle EAF. *[5 marks]*

.... °

[Total 5 marks]

12. (a) Solve $\dfrac{3x+2}{5} - \dfrac{2x+1}{3} = x$

You must show clear algebraic working. *[3 marks]*

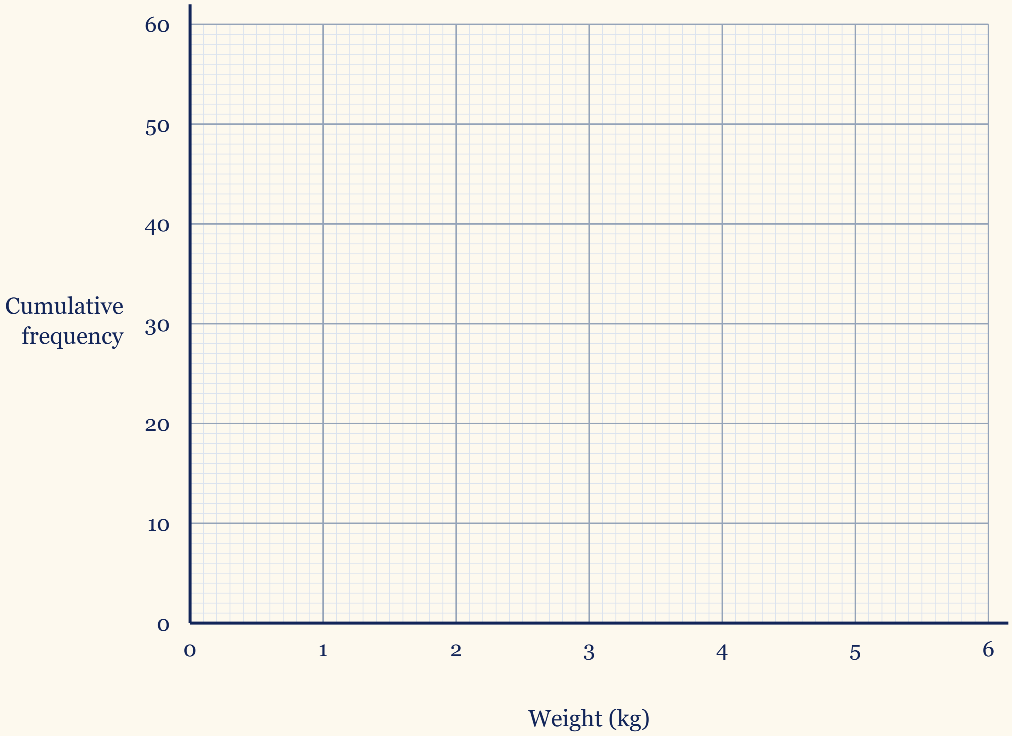
(b) Rearrange the formula $f = \sqrt{\dfrac{a+bc}{c-d}}$ to make c the subject. *[4 marks]*

(a) x = ...

(b) ....

[Total 7 marks]

13. The frequency table gives information about the weights, in kilograms, of 60 bags of flour on a market stall.



Weight (w kilograms)	**Frequency**
$0 < w \leq 1$	4
$1 < w \leq 2$	15
$2 < w \leq 3$	20
$3 < w \leq 4$	11
$4 < w \leq 5$	6
$5 < w \leq 6$	4

(a) Complete the cumulative frequency table.

Weight (w kilograms)	**Cumulative frequency**
$0 < w \leq 1$	
$0 < w \leq 2$	
$0 < w \leq 3$	
$0 < w \leq 4$	
$0 < w \leq 5$	
$0 < w \leq 6$	

[1 mark]

(b) On the grid below, draw a cumulative frequency graph for your table. *[2 marks]*

(c) Use your graph to find an estimate for the median weight of the 60 bags. *[1 mark]*

(d) Use your graph to find an estimate for the number of these bags that weigh more than 3.7 kilograms. *[2 marks]*

(c) kilograms ....

(d) ....

[Total 6 marks]

14. Given that

$$\frac{3^{2n+3}}{3^4} = 3^3 \times 3^{1-2n}$$

work out the value of n.
You must show clear working. *[3 marks]*

n = ...

[Total 3 marks]

15. Using algebra, show that the recurring decimal $0.7\dot{6}\dot{3}$ is equal to the fraction $\frac{42}{55}$

You must show your working clearly. *[2 marks]*

[Total 2 marks]

16. The points A, B, C and D all lie on a circle whose centre is O

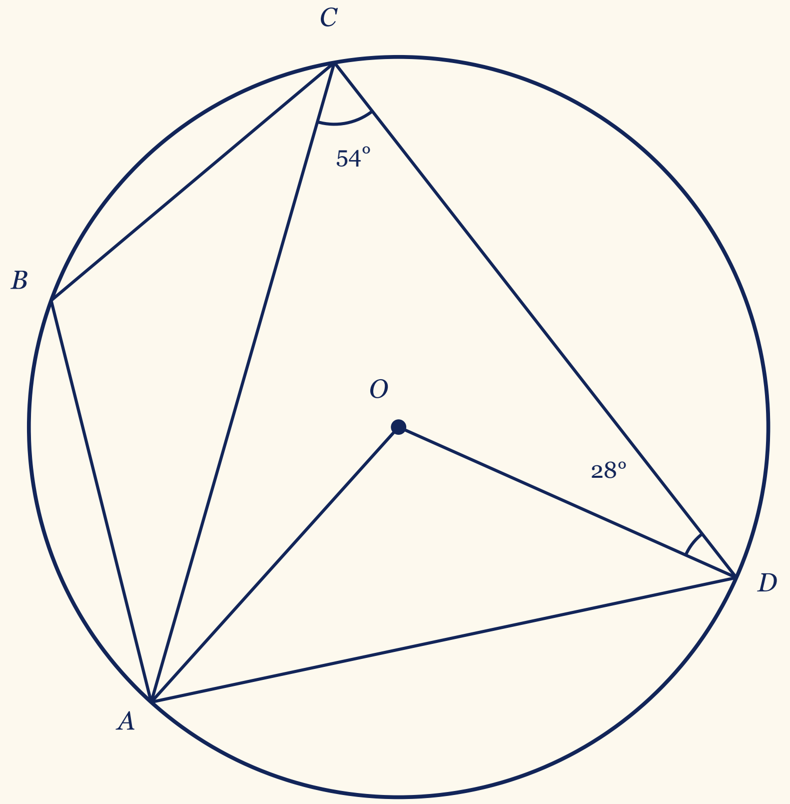


Diagram NOT accurately drawn

(a) (i) Work out the size of angle AOD
[1 mark]
(ii) Give a reason for your answer to part (a)(i). *[1 mark]*

(b) Work out the size of angle CAO *[1 mark]*

(c) Work out the size of angle ABC *[2 marks]*

(a)(i) ° ....

(a)(ii) ....

(b) ° ....

(c) ° ....

[Total 5 marks]

17. Two vectors are given below.

$$\overrightarrow{FG} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad \overrightarrow{HG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$$

Work out the magnitude of the vector $\overrightarrow{HF}$ *[3 marks]*

....

[Total 3 marks]

18. The straight line L is perpendicular to the line with equation $2x + y = 9$
The line L passes through the point with coordinates $(8, 11)$

Find an equation for L
Give your answer in the form $y = mx + c$ *[4 marks]*

....

[Total 4 marks]

19. The curve C has equation $y = f(x)$.
There is exactly one minimum point on C, and its coordinates are $(5,\ 4)$.

For each equation below, write down the coordinates of the minimum point on that curve.

(i) $y = f(x + 5)$
[1 mark]
(ii) $y = 3f(x)$
[1 mark]
(iii) $y = f(x) - 7$ *[1 mark]*

(i) ....

(ii) ....

(iii) ....

[Total 3 marks]

20. Find the set of values of x for which

$10x^2 + 11x - 21 < 0$

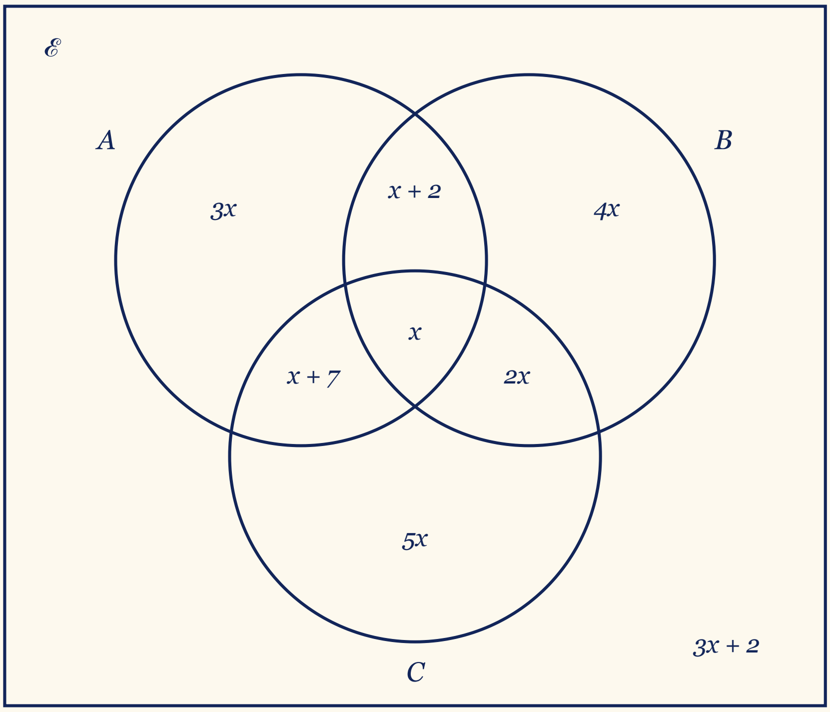
You must show clear algebraic working. *[3 marks]*

....

[Total 3 marks]

21. The Venn diagram below records how many items lie in each region of set A, set B and set C.

In the diagram x is an integer.



Given that $\mathrm{n}(A \cup B)' = 26$

work out $\mathrm{n}(A' \cap C)$ *[4 marks]*

$\mathrm{n}(A' \cap C) =$ ....

[Total 4 marks]

22. Find all the solutions of the simultaneous equations

$$x^2 + y^2 + y = 3$$
$$x + 2 = y$$

You must show clear algebraic working. *[5 marks]*

....

[Total 5 marks]

23. Show that

$$\frac{\dfrac{16x^2 - 36}{x - 7}}{\dfrac{2x^2 + 7x + 6}{x^2 - 5x - 14}} - (7 + 8x) = n$$

where n is an integer to be found.

You must show clear algebraic working. *[4 marks]*

[Total 4 marks]

24. A, B and C are three control posts on level ground at an orienteering course.

$AB = 8.4$ metres, $BC = 9.2$ metres

B is on a bearing of $067°$ from A
C is on a bearing of $129°$ from B

Calculate the bearing of A from C.
Give your answer correct to the nearest degree. *[6 marks]*

.... °

[Total 6 marks]

25. The diagram shows an equilateral triangle ABC and a circle, centre O

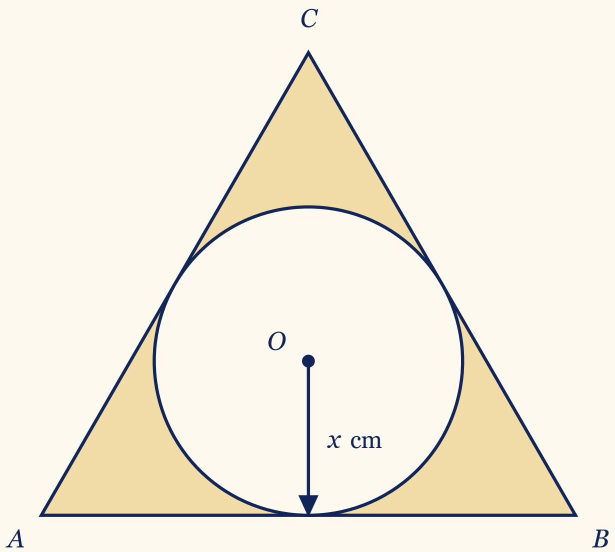


Diagram NOT
accurately drawn

Each of AB, BC and CA is a tangent to the circle.
The circle has radius x cm
The three regions shown shaded have a total area of nx^2 cm^2

Work out the value of n
Give your answer correct to 3 significant figures. *[5 marks]*

n = ...

[Total 5 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The product of two mixed numbers, $1\dfrac{5}{7}$ and $2\dfrac{3}{16}$.

The result that has to be reached: $3\dfrac{3}{4}$.

Find

Show that the left-hand side works out to exactly $3\dfrac{3}{4}$. The working is the answer, so every stage has to be written down.

Plan the Solution

- Write each mixed number as an improper fraction, because a mixed number cannot be multiplied a piece at a time.
- Multiply the numerators together and the denominators together to get one fraction, $\dfrac{420}{112}$.
- Cancel that fraction to lowest terms, then turn it back into a mixed number and compare it with $3\dfrac{3}{4}$.

Worked Solution [3 marks]

> Rule - Multiplying mixed numbers: write every mixed number as an improper fraction first, then $\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{a \times c}{b \times d}$. Common factors may be cancelled before multiplying or after.

Step 1: Write both mixed numbers as improper fractions

$$1\frac{5}{7} = \frac{1 \times 7 + 5}{7} = \frac{12}{7}$$

$$2\frac{3}{16} = \frac{2 \times 16 + 3}{16} = \frac{35}{16}$$

(Reason: Multiply the whole number by the denominator and add the numerator; the denominator itself never changes. So $1\frac{5}{7}$ has numerator $1 \times 7 + 5 = 12$ and $2\frac{3}{16}$ has numerator $2 \times 16 + 3 = 35$.)

Step 2: Multiply the numerators, and multiply the denominators

$$\frac{12}{7} \times \frac{35}{16} = \frac{12 \times 35}{7 \times 16} = \frac{420}{112}$$

(Reason: The product of two fractions is the product of the numerators over the product of the denominators: $12 \times 35 = 420$ on top and $7 \times 16 = 112$ underneath.)

Step 3: Cancel the fraction to its lowest terms

$$\frac{420}{112} = \frac{15}{4}$$

(Reason: Both parts divide by 28, since $\frac{420}{28} = 15$ and $\frac{112}{28} = 4$. Cancelling before multiplying gives the same fraction: $\frac{12}{7} \times \frac{35}{16} = \frac{3}{4} \times \frac{5}{1} = \frac{15}{4}$.)

Step 4: Write the improper fraction as a mixed number

$$\frac{15}{4} = \frac{3 \times 4 + 3}{4} = 3\frac{3}{4}$$

(Reason: 4 goes into 15 three times with 3 left over, so $\frac{15}{4} = 3\frac{3}{4}$, which is exactly the result the question asked for.)

$$1\frac{5}{7} \times 2\frac{3}{16} = \frac{420}{112} = \frac{15}{4} = 3\frac{3}{4} \text{ as required}$$

Verification

✓ **Check 1:** Compare the two sides over the same denominator. The target is $3\frac{3}{4} = \frac{15}{4}$, and scaling top and bottom by 28 gives $\frac{15 \times 28}{4 \times 28}$. $\frac{15 \times 28}{4 \times 28} = \frac{420}{112}$, the unsimplified product from Step 2, so both sides are the same number

✓ **Check 2:** Work backwards. Dividing the result by one factor must return the other, so $\frac{15}{4}$ divided by $\frac{35}{16}$ should give $1\frac{5}{7}$. $\frac{15}{4} \times \frac{16}{35} = \frac{240}{140} = \frac{12}{7} = 1\frac{5}{7}$, the first factor recovered

✓ **Check 3:** Use decimals, which avoids fractions altogether: $2\frac{3}{16} = 2.1875$ and $3\frac{3}{4} = 3.75$, so $\frac{3.75}{2.1875}$ must come to $1\frac{5}{7}$. $\frac{3.75}{2.1875} = 1.714285\ldots = \frac{12}{7} = 1\frac{5}{7}$, agreeing with the fraction working

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Both mixed numbers written as improper fractions: $\frac{12}{7} \times \frac{35}{16}$	**M1**	For both fractions written as improper fractions. The multiplication sign may be implied, and any equivalent pair of improper fractions scores this mark.	✓
Multiplying across, or cancelling: $\frac{12 \times 35}{7 \times 16} = \frac{420}{112}$	**M1**	For multiplying the numerators and multiplying the denominators, or for cancelling the fractions fully, or for cancelling partially and then multiplying across. Writing both fractions over a common denominator also scores it, for example $\frac{192}{112} \times \frac{245}{112} = \frac{47\,040}{12\,544}$.	✓
Completion: $\frac{420}{112} = \frac{15}{4} = 3\frac{3}{4}$	**A1**	Completion to the given result. Working is required, so an unsupported statement of the answer scores nothing here.	✓
Note	**note**	If the working shows clearly that $3\frac{3}{4} = \frac{15}{4}$, it is enough to show that the left-hand side comes to $\frac{15}{4}$; the mixed number need not be written again.	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

A measured width of 1.4 metres, correct to one decimal place.

Find

(a) The upper bound of the width, in metres. (b) The lower bound of the width, So the true width is any value that rounds to 1.4 at one decimal place.

in metres.

Plan the Solution

- Find the rounding unit. One decimal place means the width was rounded to the nearest 0.1 of a metre.
- Halve that unit. Half of 0.1 is 0.05, the greatest amount the true width can differ from the measurement.
- Add the half-unit to 1.4 for the upper bound, and subtract it for the lower bound.
- Keep the two the right way round: the upper bound is the larger of the pair.

Worked Solution [2 marks]

> Rule - Bounds of a rounded measurement: if x is given correct to the nearest u, the lower bound is $x - \dfrac{u}{2}$ and the upper bound is $x + \dfrac{u}{2}$.

Step 1: find the rounding unit, and half of it

$$\text{one decimal place} \implies u = 0.1$$

$$\frac{u}{2} = \frac{0.1}{2} = 0.05$$

(Reason: Rounding to one decimal place means the width was rounded to the nearest 0.1 m, so the true width lies within half of 0.1 m of the 1.4 m that was written down.)

Step 2: (a) add the half-unit to get the upper bound

$$1.4 + 0.05 = 1.45$$

(Reason: Every width below 1.45 m rounds down to 1.4 m, while 1.45 m itself rounds up to 1.5 m, so this is where the interval stops.)

Step 3: (b) subtract the half-unit to get the lower bound

$$1.4 - 0.05 = 1.35$$

(Reason: Going the same distance the other way from the measurement gives the smallest width that still rounds to 1.4 m. Anything smaller would be measured as 1.3 m instead.)

(a) 1.45 metres

(b) 1.35 metres

Verification

✓ **Check 1:** Round each bound back to one decimal place. 1.35 rounds up to 1.4, and any width just below 1.45 rounds down to 1.4. $1.35 \leq \text{width} < 1.45$, and every width in that interval is measured as 1.4 m

✓ **Check 2:** The gap between the two bounds should be exactly one rounding unit, that is one whole step at this degree of accuracy. $1.45 - 1.35 = 0.1$, which is one step of one decimal place

✓ **Check 3:** The measurement should sit exactly halfway between the two bounds. $\frac{1.35 + 1.45}{2} = 1.4$, the width the carpenter wrote down

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The upper bound of the width	B1	1.45. Allow $1.44\dot{9}$ or $1.44999(9\ldots)$, which are the same number as 1.45.	✓
(b) The lower bound of the width	B1	1.35, cao. No working is needed for either part, so nothing else earns this mark.	✓
Special case - the two bounds interchanged	SCB1	If (a) is given as 1.35 and (b) as 1.45, the pair of bounds is right but the labels are swapped. Score B0 then B1, so one mark in total.	✓

Full marks: 2/2

Question 3 - Exam Solution

Understanding the Question

Given

Triangle PQR, with the right angle at Q.
$PR = 8.6$ cm - the side facing the right angle, so it is the hypotenuse.
The angle at R is $43°$.
$QR = x$ cm - the side running along one arm of that angle.

Find

The value of x, correct to one decimal place.

Plan the Solution

- Stand at the 43° angle and name the two sides the question involves.
- QR lies along an arm of that angle, so it is the adjacent side, and PR faces the right angle, so it is the hypotenuse.
- Adjacent with hypotenuse is the cosine ratio, so no third side has to be found first.
- Rearrange for x, evaluate with the calculator in degree mode, and round only at the very end.

Worked Solution [3 marks]

> Rule - Cosine ratio in a right-angled triangle: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

Step 1: Name the two sides at the 43° angle

$\text{hypotenuse} = PR = 8.6$

$\text{adjacent} = QR = x$

(Reason: The hypotenuse is always the side opposite the right angle, and the right angle is at Q, so it is PR. The side QR runs from R along the angle rather than across from it, so it is the adjacent side.)

Step 2: Write down the cosine ratio

$$\cos 43^\circ = \frac{x}{8.6}$$

(Reason: Adjacent over hypotenuse is cosine. The side PQ never appears, which is exactly why cosine is the efficient choice here.)

Step 3: Rearrange to make x the subject

$x = 8.6 \times \cos 43^\circ$

(Reason: Multiplying both sides by 8.6 clears the fraction. This single line is worth both method marks on its own.)

Step 4: Evaluate, with the calculator in degree mode

$\cos 43^\circ = 0.7313537\ldots$

$x = 8.6 \times 0.7313537\ldots = 6.2896418\ldots$

(Reason: A calculator left in radian mode gives a completely different value, so check the DEG indicator before pressing cosine. Keep the full display value and do not round yet.)

Step 5: Round to one decimal place

$x = 6.2896418\ldots$

$x = 6.3 \text{ (to 1 d.p.)}$

(Reason: The second decimal digit is 8, so the first decimal digit rounds up from 2 to 3.)

$$x = 6.3$$

Verification

✓ **Check 1 - Pythagoras closes the triangle:** Work out the third side a different way: $PQ = 8.6 \sin 43° = 5.8651858\ldots$ cm. Squaring both legs and adding them must give the square of the hypotenuse. $6.2896418^2 + 5.8651858^2 \approx 73.96$, and $8.6^2 = 73.96$

✓ **Check 2 - run the trigonometry backwards:** Put the answer back into the ratio. If x is right, the angle at R must come back out as $43°$. $\cos^{-1}\left(\frac{6.2896418}{8.6}\right) = 43.0°$

✓ **Check 3 - is the size sensible:** No trigonometry needed. A leg is shorter than the hypotenuse, and because $43°$ is less than $45°$ the side along the angle is the longer of the two legs, so x must sit between $\frac{8.6}{\sqrt{2}}$ and 8.6. $6.0811\ldots < 6.3 < 8.6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\cos 43° = \frac{x}{8.6}$	**M1**	A correct trig statement for x or for QR, or a correct Pythagoras statement for x^2. Also accept $\tan 43° = \frac{8.6 \sin 43°}{x}$, $\sin(90 - 43) = \frac{x}{8.6}$, $\frac{x}{\sin(90-43)} = \frac{8.6}{\sin 90}$ or $x^2 = 8.6^2 - (8.6 \sin 43°)^2$.	✓
$x = 8.6 \cos 43°$	**M1**	A fully correct calculation to find x. Also accept $x = \frac{8.6 \sin 43°}{\tan 43°}$, $x = 8.6 \sin(90 - 43)$, $x = \frac{8.6 \sin 47°}{\sin 90°}$ or $x = \sqrt{8.6^2 - (8.6 \sin 43°)^2}$. A student who goes straight to this line scores both method marks.	✓
$x = 6.3$	**A1**	Award for anything which rounds to 6.3, seen even if it is then rounded incorrectly afterwards. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

17% of N is 357

The club had 650 members in 2019 and 806 members in 2020

Two separate percentage skills: one works backwards from a percentage, the other measures a change

Find

(a) The value of N (b) The percentage increase from 650 members to 806 members

Plan the Solution

- Part (a) is a reverse percentage. 17% of N means $0.17 \times N$, so the equation is $0.17N = 357$.
- Undo the multiplication by dividing 357 by 0.17. Do not take 17% of 357 - that would make the answer smaller, when it must be far larger.
- Part (b) is a percentage change. Find the increase first, then write it as a fraction of the original amount.
- The original amount is the 2019 figure, so 650 goes on the bottom of that fraction. Multiply by 100 to turn it into a percentage.

Worked Solution [5 marks]

Rule - Reverse percentage: if $p\%$ of N is A, then $N = \dfrac{100A}{p}$.

Rule - Percentage change: $\text{percentage change} = \dfrac{\text{change}}{\text{original amount}} \times 100$

Step 1 - Part (a): write 17% of N as an equation

$$17\% = \frac{17}{100} = 0.17$$

$$0.17 \times N = 357$$

(Reason: Per cent means out of 100, so 17% of N is $0.17 \times N$. Writing the equation first makes it clear that N has been multiplied, so a division will undo it.)

Step 2 - Part (a): divide to undo the multiplication

$$N = \frac{357}{0.17}$$

$$N = 2100$$

(Reason: Dividing both sides by 0.17 leaves N on its own. Without a calculator the same value comes from $\frac{357 \times 100}{17}$, or from 1% of N being $\frac{357}{17} = 21$.)

Step 3 - Part (b): find the increase

$806 - 650 = 156$

(Reason: The increase is what has been added on: the 2020 figure minus the 2019 figure. 156 extra members is the change the percentage has to describe.)

Step 4 - Part (b): write the increase as a percentage of the original

$$\frac{156}{650} \times 100 = 24$$

(Reason: A percentage increase is always measured against the original amount, so the 2019 figure goes on the bottom of the fraction. Multiplying by 100 turns the decimal into a percentage.)

(a) $N = 2100$

(b) 24%

Verification

✓ **Check 1:** Put $N = 2100$ back into part (a) and take 17% of it. $0.17 \times 2100 = 357$, the amount the question gives.

✓ **Check 2:** Reach part (a) a second way, with no decimal at all. 1% of N is $\frac{357}{17} = 21$, so N is 100 times that. $21 \times 100 = 2100$, the same value as the division gave.

✓ **Check 3:** Test part (b) forwards. An increase of 24% multiplies the original by 1.24. $1.24 \times 650 = 806$, the 2020 membership.

✓ **Check 4:** Compare the two figures directly instead of using the increase: $\frac{806}{650} = 1.24$, so 2020 is 124% of 2019. $124\% - 100\% = 24\%$, so the increase is 24%.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): $\frac{357}{0.17}$ or $0.17N = 357$ or	M1	A correct calculation for N, or a correct equation in N, or equivalent. Not $17\% \times N = 357$.	✓

Step	Mark	Description	Got it?
$\frac{17}{100} \times N = 357$ or $\frac{357 \times 100}{17}$			
$N = 2100$	**A1**	cao. A correct answer scores both marks, unless it comes from obviously incorrect working.	✓
Part (b): $806 - 650 = 156$ or $\frac{806}{650} = 1.24$	**M1**	Either the increase, or the multiplier from 2019 to 2020, or equivalent.	✓
$\frac{806 - 650}{650} \times 100$ or $1.24 \times 100 = 124$ or $1.24 - 1 = 0.24$	**M1**	A correct calculation for the percentage increase, or 124 or 0.24 seen as the answer or in part of the working.	✓
24 per cent	**A1**	cao. A correct answer scores all three marks, unless it comes from obviously incorrect working.	✓
An answer between 19.3 and 19.4, when no other mark in part (b) has been earned	**SCB1**	This special case is for one specific error: dividing the increase by the 2020 figure, 806, instead of by the original 650. The method is right and only the base is wrong, so it is worth one mark.	✓

Full marks: 5/5

Question 5 - Exam Solution

Understanding the Question

Given

A biased five-sided spinner, its sections numbered $1, 2, 3, 4, 5$

Probability of landing on 2 is 0.14

Probability of landing on 3 is 0.17

Find

An estimate for the number of times the spinner will land on 4

Probability of landing on 5 is 0.21
Landing on 1 and landing on 4 are equally likely, so those two probabilities are equal
The spinner is going to be spun 400 times

Plan the Solution

- The five sections are the only outcomes there are, so the five probabilities must total 1.
- Add the three probabilities the table gives, then subtract from 1 to see how much is left for 1 and 4 together.
- Halve that leftover, because landing on 1 and landing on 4 are equally likely.
- Multiply the probability by 400 to turn it into an expected number of spins.

Worked Solution [4 marks]

> Rule - Total probability and expected frequency: the probabilities of all the possible outcomes add up to 1, and an estimate for the number of times one outcome happens in n trials is $p \times n$.

Step 1: add the three probabilities the table gives

$0.14 + 0.17 + 0.21 = 0.52$

Number	1	2	3	4	5
Probability	**0.24**	0.14	0.17	**0.24**	0.21

(Reason: landing on 2, on 3 and on 5 cannot happen together, so their probabilities simply add)

Step 2: find the probability left over for 1 and 4

$1 - 0.52 = 0.48$

(Reason: the five sections are the only places the spinner can stop, so all five probabilities must total 1, and whatever is not already used belongs to 1 and 4)

Step 3: split that leftover into two equal parts

$$\frac{0.48}{2} = 0.24$$

(Reason: the spinner is just as likely to land on 1 as on 4, so the probability left over is shared equally between the two of them)

Step 4: turn that probability into a number of spins

$0.24 \times 400 = 96$

(Reason: an estimate for how often an outcome happens is its probability multiplied by the number of trials, and here there are 400 trials)

96 times

Verification

✓ **Check 1:** Put the answer back into the table and add all five probabilities. A complete set of outcomes must total 1. $0.14 + 0.17 + 0.24 + 0.21 + 0.24 = 1$

✓ **Check 2:** Work in spins rather than probabilities. Multiply each given probability by 400, then share the spins that are left between 1 and 4. $56 + 68 + 84 + 96 + 96 = 400$

✓ **Check 3:** Run the question backwards: turn the answer back into a probability by dividing by the number of spins, and see whether it matches step 3. $\frac{96}{400} = 0.24$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1 - (0.14 + 0.17 + 0.21)$ $(= 0.48)$	**M1**	Correct use of the fact that the probabilities total 1. Accept instead $0.14 + 0.17 + 0.21 + x + x = 1$ oe, or a correct calculation for an estimate for the number of times the spinner lands on 2 or on 3 or on 5, eg 0.14×400 $(= 56)$ or 0.52×400 $(= 208)$.	✓
$\frac{0.48}{2}$ $(= 0.24)$	**M1**	A completely correct method to find the probability that the spinner lands on 4 - the value may be written straight into the table. Accept instead a completely correct method for the number of times it lands on 1 or on 4, eg $400 - 56 - 68 - 84$ $(= 192)$ oe or 0.48×400 $(= 192)$.	✓
0.24×400	**M1**	A correct calculation to find the estimate required, or an answer leading from 96 seen, eg $\frac{96}{400}$. Accept instead $\frac{192}{2}$.	✓
Correct answer scores full marks, unless it comes from	**A1**	96 cao	✓

Step	Mark	Description	Got it?
obviously incorrect working			
Special case - halving the total that was just added instead of the total that is left	SC	$B1$ for an answer of 104 if no other marks have been awarded. It comes from $\frac{0.52}{2} = 0.26$ and then $0.26 \times 400 = 104$, which halves the probability already accounted for rather than the probability still to be shared.	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

A solid prism whose cross-section is a right-angled triangle with shorter sides 6 cm and 8 cm, and hypotenuse 10 cm.
The prism is 15 cm long.
The figure is not accurately drawn, so every length must come from the labels and not from measuring.

Find

The total surface area of the prism, in square centimetres. Total means every face: the prism is solid, so there are 5 faces on the outside - two triangular ends and three rectangles.

Plan the Solution

- Confirm the cross-section really is right-angled, because half base times height only applies then: test whether $6^2 + 8^2 = 10^2$.
- Work out the area of one triangular end, then double it for the two ends.
- Sweep each side of the triangle along the prism. Each gives a rectangle 15 cm long and as wide as that side, so the three widths are 8 cm, 6 cm and 10 cm.
- Add the five face areas.
- Finish in square centimetres, because an area is being asked for.

Worked Solution [3 marks]

Rule - Surface area of a prism: add the areas of every face. A triangular prism has five of them - two identical triangular ends, and one rectangle for each side of the triangle, each rectangle as long as the prism.

Step 1: check that the cross-section is right-angled

$6^2 + 8^2 = 36 + 64 = 100$

$10^2 = 100$

(Reason: The three sides satisfy Pythagoras, so the 6 cm and 8 cm sides meet at the right angle. That is what makes half base times height the correct area rule here.)

Step 2: the two triangular ends

$\frac{1}{2} \times 8 \times 6 = 24$

$2 \times 24 = 48$

(Reason: The base and the height are the two sides that meet at the right angle, 8 cm and 6 cm, never the 10 cm hypotenuse. The two ends are identical, so one area is worked out and then doubled.)

Step 3: the three rectangular faces

$15 \times 8 = 120$

$15 \times 6 = 90$

$15 \times 10 = 150$

(Reason: Every rectangle runs the whole length of the prism, so each is 15 cm by the width of the side it came from. The 10 cm side gives the sloping face, which is easy to forget because it faces away in the drawing.)

Step 4: add the areas of all five faces

$48 + 120 + 90 + 150 = 408$

(Reason: Two triangular ends and three rectangles, each counted exactly once. Adding gives the total surface area in cm^2, because every term is an area.)

408 cm^2

Verification

✓ **Check 1 - factor the length out:** All three rectangles are 15 cm long, so together they cover the perimeter of the triangle multiplied by the length: $(6 + 8 + 10) \times 15$. $24 \times 15 = 360$ for the rectangles, then $360 + 48 = 408$ with the two ends - the same total, reached by grouping the faces a different way.

✓ **Check 2 - half of a cuboid:** Two of these prisms fit together along their sloping faces to make a cuboid 6 cm by 8 cm by 15 cm, whose surface area is $2 \times (6 \times 8) + 2 \times (6 \times 15) + 2 \times (8 \times 15) = 516$. Cutting it in half gives each prism half of that surface, plus the new sloping face the cut creates, 10×15. $\frac{516}{2} = 258$, then $258 + 150 = 408$ - the same total, with no face area added one at a time.

✓ **Check 3 - is the size sensible?** The whole cuboid that contains the prism has surface area 516 cm^2, and the prism is that cuboid with a corner sliced off, so its surface must be smaller than 516 but comfortably larger than the three rectangles alone, 360. $360 < 408 < 516$, so the answer sits exactly where a surface area for this solid should.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method for the areas of two different faces, for example 15×8 together with $\frac{1}{2} \times 8 \times 6$	**M1**	For a correct method to find the areas of 2 different faces (that is, not 2 triangles). Allow 8×6 as one area, and allow this mark even when the areas themselves are incorrect.	✓
$0.5 \times 8 \times 6 = 24$ (and $\times 2 = 48$) oe, with $15 \times 8 = 120$, $15 \times 6 = 90$, $15 \times 10 = 150$, written with the intention of adding them	**M1**	For adding together 4 or 5 values for area (condone 48 as 1 or 2 areas), at least 3 of which come from a correct method.	✓
408	**A1**	cao. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
An answer of 456	**SC**	Special case: B2 for an answer of 456 if no other marks are awarded. This is the total with the triangular ends left unhalved - each end counted as the whole rectangle, $2 \times 8 \times 6 = 96$ in place of 48.	✓
Note - three faces only	**Note**	$(6 + 8 + 10) \times 15$ (the sum must be seen) accounts for 3 faces. Award the second method mark for it only when it is clearly not intended as the volume - for example when the area of a triangular end is then added.	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

A grid on which x and y both run from 0 to 8.

Three equations to draw: $x = 3$, $y = 1$ and $x + y = 7$.

Three inequalities to satisfy at the same time: $x \geq 3$, $y \geq 1$ and $x + y \leq 7$.

Find

(a) The three straight lines, drawn on the grid, each labelled with its equation. (b) The one region where all three inequalities hold, shaded and marked R.

Plan the Solution

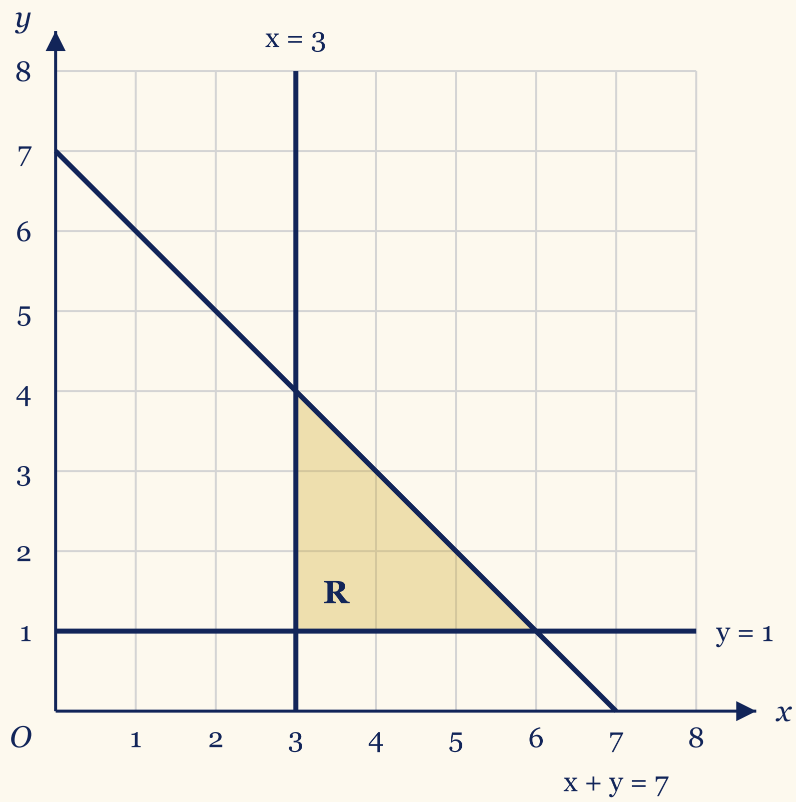
- An equation with only x in it gives a vertical line and an equation with only y in it gives a horizontal line, so draw those two straight away.
- For $x + y = 7$, work out where it crosses each axis and join those two points with a ruler.
- Decide which side of each line to keep by testing one point that is clearly on one side of it.
- The region wanted is where the three kept sides overlap. Find the corners where the boundaries cross, shade the overlap, and write R inside it.

Worked Solution [4 marks]

> Rule - draw the boundary from the equation, then choose the side with a test point. An inequality written with $\geq$ or $\leq$ includes its own boundary, so the lines themselves belong to the region.

Step 1: Draw $x = 3$

$x = 3$ passes through $(3, 0)$ and $(3, 8)$



(Reason: Every point whose x-coordinate is 3 lies on this line, whatever y is, so the line is vertical and runs the full height of the grid.)

Step 2: Draw $y = 1$

$y = 1$ passes through $(0, 1)$ and $(8, 1)$

(Reason: This time the y-coordinate is fixed and x is free, so the line is horizontal, one square above the x-axis.)

Step 3: Draw $x + y = 7$

$x = 0$ gives $y = 7$

$y = 0$ gives $x = 7$

join $(0, 7)$ to $(7, 0)$

(Reason: Two points fix a straight line, and the two axis crossings are the easiest pair to read off. The gradient is -1, so the line drops one square for every square it moves across, which is a quick way to check it.)

Step 4: Choose the side of each line to keep

$x \geq 3$: keep the side to the right of $x = 3$

$y \geq 1$: keep the side above $y = 1$

$x + y \leq 7$: keep the side below $x + y = 7$

(Reason: Test the point $(4, 2)$: $4 \geq 3$ is true, $2 \geq 1$ is true, and $4 + 2 = 6$, which is not more than 7. One point on the correct side of all three lines is enough to fix which side each inequality keeps.)

Step 5: Find the corners of the region

$x = 3$ and $y = 1$ meet at $(3, 1)$

$y = 1$ and $x + y = 7$ meet where $x = 6,$ so $(6, 1)$

$x = 3$ and $x + y = 7$ meet where $y = 4,$ so $(3, 4)$

(Reason: A corner sits where two of the boundaries cross, so take the three lines in pairs. All three inequalities allow equality, so the edges and the corners are part of the region rather than excluded from it.)

Step 6: Shade the region and label it R

the triangle $(3, 1), (6, 1), (3, 4)$

(Reason: The three kept sides overlap in a single triangle. Shade it and write R inside, so there is no doubt which region is meant.)

(a) $x = 3$ vertical, $y = 1$ horizontal, $x + y = 7$ from $(0, 7)$ to $(7, 0)$

(b) R is the triangle with corners $(3, 1)$, $(6, 1)$ and $(3, 4)$

Verification

✓ **Check 1:** Take a point inside the shading, $(4, 2)$, and test all three inequalities: $4 \geq 3$, $2 \geq 1$ and $4 + 2 = 6$, which is at most 7. All three hold, so $(4, 2)$ does belong to R.

✓ **Check 2:** Take a point just outside the sloping edge, $(5, 3)$. Now $5 \geq 3$ and $3 \geq 1$ still hold, but $5 + 3 = 8$. 8 is more than 7, so $(5, 3)$ is correctly left outside the shading.

✓ **Check 3:** Count the shape instead. The base runs from $(3, 1)$ to $(6, 1)$, so it is 3 squares long, and the height from $(3, 1)$ up to $(3, 4)$ is 3 squares. Area $\frac{1}{2} \times 3 \times 3 = 4.5$ squares, which is what counting the squares inside the shading gives.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) The line $x = 3$ drawn	B1	A vertical line through 3 on the x-axis. Dashed or solid is accepted, of minimum length 2 squares.	✓
(a)(ii) The line $y = 1$ drawn	B1	A horizontal line through 1 on the y-axis. Dashed or solid is accepted, of minimum length 2 squares.	✓

Step	Mark	Description	Got it?
(a)(iii) The line $x + y = 7$ drawn	B1	The line joining $(0, 7)$ to $(7, 0)$. Dashed or solid is accepted, of minimum length 2 squares.	✓
Note on labelling in part (a)	-	A missing label is condoned when the line is unambiguous. Where the lines are unlabelled the scheme awards on what has been drawn: $x = 3$ and $y = 3$ score B1 B0; $y = 1$ and $x = 1$ score B0 B1; $x = 3$, $x = 1$ and $y = 1$ score B0 B1; $x = 3$, $y = 1$ and $y = 3$ score B1 B0; and $x = 3$, $x = 1$, $y = 1$ and $y = 3$ score B0 B0.	✓
(b) The correct region shaded and labelled R	B1	Shaded in or shaded out, labelled R or with a clear intention that it is the region wanted. Follow through is allowed only for one vertical line (not $x = 0$), one horizontal line (not $y = 0$) and one line of negative gradient, for example $x = 1, y = 3$ and $x + y = 7$.	✓
Note on the shading in part (b)	-	Lines must be at least 2 cm long, and the shaded area must be fully enclosed before the mark in part (b) can be given.	✓

Full marks: 4/4

Question 8 - Exam Solution

Understanding the Question

Given

A basket holds 4 apples whose mean weight is 145 grams.
One more apple is put in, and the mean weight of the 5 apples is then 142 grams.
Nothing is said about any individual apple. The two means are the only facts given.

Find

The weight, in grams, of the apple that Martin adds. A mean on its own says nothing about one single item, so each mean has to be turned back into a TOTAL weight before the extra apple can be picked out.

Plan the Solution

- Rearrange the mean formula into $\text{total} = \text{mean} \times \text{number of apples}$.
- Use it once with 4 apples and a mean of 145 grams, then again with 5 apples and a mean of 142 grams. The number of apples changes as well as the mean.
- Subtract the first total from the second. The only difference between the two baskets is the apple Martin added, so that difference is its weight.
- Sanity-check the size at the end: the mean fell when the apple went in, so the new apple must be lighter than 142 grams.

Worked Solution [3 marks]

Rule - Mean and total: $\text{mean} = \dfrac{\text{total weight}}{\text{number of apples}}$, so $\text{total weight} = \text{mean} \times \text{number of apples}$. A mean is a total in disguise, and going back to totals is what makes two groups of different sizes comparable.

Step 1: the total weight of the first 4 apples

$4 \times 145 = 580$

(Reason: Four apples averaging 145 grams weigh the same altogether as four apples of exactly 145 grams each, so the total is 580 grams however the four are shared out.)

Step 2: the total weight of all 5 apples

$5 \times 142 = 710$

(Reason: The second mean is a mean of 5 apples, not 4, so the count in the multiplication changes as well as the mean itself.)

Step 3: subtract the two totals

$710 - 580 = 130$

(Reason: The two baskets hold the same first four apples, so everything they share cancels in the subtraction. What is left over is exactly the apple Martin added, in grams.)

130 grams

Verification

✓ **Check 1 - put the answer back in:** Add the answer to the first total and take the mean of the 5 apples: $580 + 130 = 710$, then divide by 5. $\dfrac{710}{5} = 142$, which is the mean the question states for the 5 apples.

✓ **Check 2 - balance the weights about the new mean:** Distances from the new mean must cancel out. Each of the first four apples averages $145 - 142 = 3$ grams ABOVE the new mean, which is $4 \times 3 = 12$ grams of surplus between them, so the new apple must sit 12 grams BELOW it. $142 - 12 = 130$ grams - the same answer, with no total worked out anywhere.

✓ **Check 3 - is the size sensible?** Adding the apple pulled the mean DOWN, from 145 grams to 142 grams, so the new apple has to weigh less than the new mean - and it must of course be a positive weight. $0 < 130 < 142$, so the answer sits exactly where an apple that drags the mean down should sit. An answer above 142 would have been impossible.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$4 \times 145\ (= 580)$ or $5 \times 142\ (= 710)$, or $\dfrac{145 + 145 + 145 + 145 + x}{5} = 142$ oe	**M1**	For one correct product, or for a correct equation for the weight of the last apple.	✓
$5 \times 142 - 4 \times 145$ or $710 - 580$ using their own two totals, or $145 + 145 + 145 + 145 + x = 5 \times 142$	**M1**	For a fully correct method to find the weight of the fifth apple, or a fully correct equation to find the missing weight with no denominator.	✓
130	**A1**	cao	✓
Note - a correct answer on its own	**Note**	A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given

Amount put into the bond: $20\,000$ euros
Compound interest: 3.5% per year
Time in the bond: 3 years

Find

The value of the bond after 3 years, correct to the nearest euro

Plan the Solution

- Compound interest adds the same percentage each year to whatever is in the bond at the time, so turn 3.5% growth into a single multiplier for one year.
- One year is one multiplication, so 3 years is that multiplier to the power 3.
- Multiply the $20\,000$ euros by the three-year multiplier, then round the result to the nearest whole euro.

Worked Solution [3 marks]

> Compound interest: $\text{final amount} = \text{starting amount} \times \text{multiplier}^n$, where n is the number of years and the multiplier is 1 plus the yearly rate as a decimal.

Step 1: turn the percentage into a one-year multiplier

$$100\% + 3.5\% = 103.5\%$$

$$\text{multiplier} = \frac{103.5}{100} = 1.035$$

(Reason: (Reason: the bond keeps what it already had and adds 3.5% on top, so each year's value is 103.5% of the year before. Writing that as 1.035 replaces a two-stage percentage calculation with one multiplication.))

Step 2: raise the multiplier to the power 3

$$1.035^3 = 1.108717875$$

(Reason: (Reason: the money stays in the bond for 3 years, so it is multiplied by 1.035 three times. Keep every decimal place here; rounding this multiplier is what produces the wrong answers $22\,160$ and $22\,180$.))

Step 3: multiply the amount invested by the three-year multiplier

$$20\,000 \times 1.035^3 = 20\,000 \times 1.108717875 = 22\,174.3575$$

(Reason: (Reason: the multiplier scales the whole balance, so the $20\,000$ euros is multiplied by it once for the whole three-year period.))

Step 4: round to the nearest euro

$$22\,174.3575 \approx 22\,174$$

(Reason: (Reason: the question asks for the nearest euro, and the first decimal place is 3, which is less than 5, so the euros digit stays as it is.))

$22\,174$ euros

Verification

✓ **Check 1:** Grow the bond one year at a time instead of using a power: $20\,000 \times 1.035 = 20\,700$, then $20\,700 \times 1.035 = 21\,424.5$, then $21\,424.5 \times 1.035 = 22\,174.3575$. the same total, $22\,174.3575$, so the power of 3 was applied the right number of times

✓ **Check 2:** Look at the interest on its own. The bond has earned $22\,174.3575 - 20\,000 = 2\,174.3575$ euros. Simple interest at the same rate would earn only $3 \times 700 = 2\,100$ euros, so compound interest must come out a little higher. $2\,174.3575 > 2\,100$, and the extra 74.3575 euros is the interest earned on earlier interest

✓ **Check 3:** Work backwards. Dividing the final amount by the three-year multiplier must return the amount that was put in: $\dfrac{22\,174.3575}{1.108717875} = 20\,000$. the original $20\,000$ euros comes back exactly, so nothing was multiplied one time too many or too few

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$20\,000 \times 1.035 = 20\,700$ oe, or $20\,000 \times 0.035 = 700$ oe	**M1**	For finding 103.5% of $20\,000$, or 3.5% of $20\,000$. Accept $(1 + \frac{3.5}{100})$ for 1.035, but not $(1 + 3.5\%)$.	✓
$20\,700 \times 1.035 = 21\,424.5$, then $21\,424.5 \times 1.035$ oe	**M1**	Dependent on the first method mark, for a complete method that carries the multiplier through all three years.	✓
The value of the bond after 3 years, to the nearest euro	**A1**	$22\,174$. Allow $22\,174$ to $22\,175$. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Both method marks earned in one line	**Note**	M2 for $20\,000 \times 1.035^3$, where $1.035^3 = 1.108717875$. M2 is also earned by $20\,000 \times 1.035^4 = 22\,950.4600125$, which is a complete method run for four years instead of three, so the A mark is lost.	✓

Step	Mark	Description	Got it?
The interest given instead of the total	**Note**	If the correct answer is seen and $20\,000$ is then subtracted to leave $2\,174$ to $2\,175$, award full marks. The same range with no working gains 2 marks.	✓
Misread: 2000 invested in place of $20\,000$	**SC**	B2 for $2000 \times 1.035^3 = 2217.43575$. The compound-interest method is completely right and only the amount invested was misread.	✓
The three-year multiplier cut short to 3 decimal places	**SC**	B2 for $22\,160$, from $20\,000 \times 1.108$, or B2 for $22\,180$, from $20\,000 \times 1.109$. Both come from rounding 1.108717875 before multiplying.	✓
One recognisable piece of the method only	**SC**	B1 if no other marks are awarded and any of these is seen, whether or not it is given as the answer: $20\,000 \times 1.035^n$, $20\,000 \times 0.965^3 = 17\,972.6425$ (a decrease instead of an increase), $20\,000 \times 0.105 = 2\,100$, $20\,000 \times 1.105 = 22\,100$, or $20\,000 \times 1.105^2$.	✓

Full marks: 3/3

Question 10 - Exam Solution

Understanding the Question

Given

Riverside office: 300 people, shown on a pie chart with sector angles bus 126, bicycle 102 and walking 132 degrees.
Eastgate office: 320 people, given in a table as bus $3x + 6$, bicycle $5x + 8$ and walking $7x - 9$.
The three ways of travelling are the only choices, so in each office they account for everybody.

Find

How many more people chose the bus at the Riverside office than at the Eastgate office. Two figures are needed before that subtraction can happen: one read from the table, one read from the pie chart.

Plan the Solution

- The three Eastgate expressions must add up to 320, so form an equation and solve it for x.

- Substitute that value back into the bus row, $3x + 6$, to get the Eastgate bus figure.
- For Riverside, the bus sector is 126 degrees out of 360, so take that fraction of 300 people.
- Subtract the smaller figure from the larger one.

Worked Solution [5 marks]

> Rule - Pie chart: a category's frequency is its angle out of 360 degrees, multiplied by the total. And when the parts of a total are given as expressions in x, add the parts and set the sum equal to that total.

Step 1: add the three Eastgate expressions and set the total to 320

$(3x + 6) + (5x + 8) + (7x - 9) = 320$

$15x + 5 = 320$

(Reason: Everyone at the Eastgate office chose exactly one way of travelling, so the three expressions must account for all 320 people. Collecting like terms gives $3x + 5x + 7x = 15x$ and $6 + 8 - 9 = 5$.)

Step 2: solve the equation for x

$15x = 320 - 5 = 315$

$$x = \frac{315}{15} = 21$$

(Reason: Take 5 from both sides, then divide both sides by 15. A whole number is expected here, because it counts people.)

Step 3: work out how many at the Eastgate office chose the bus

$3 \times 21 + 6 = 69$

(Reason: The bus row of the table reads $3x + 6$, so it is that expression, and no other row, that $x = 21$ goes into.)

Step 4: work out how many at the Riverside office chose the bus

$$\frac{126}{360} \times 300 = 105$$

(Reason: The whole pie chart is 360 degrees and stands for all 300 people, so the bus sector of 126 degrees stands for the same fraction of those 300 people.)

Step 5: subtract to find how many more

$105 - 69 = 36$

(Reason: The question asks how many more chose the bus at Riverside, so the Eastgate figure comes off the Riverside figure.)

36 more people

Verification

✓ **Check 1:** With $x = 21$ the three Eastgate rows come to 69, 113 and 138 people. $69 + 113 + 138 = 320$, the number of people at the Eastgate office.

✓ **Check 2:** Turn the other two Riverside sectors into people the same way: $\frac{102}{360} \times 300 = 85$ and $\frac{132}{360} \times 300 = 110$. $105 + 85 + 110 = 300$, the number of people at the Riverside office.

✓ **Check 3:** Do the Riverside step a completely different way. The whole chart is 360 degrees for 300 people, which is 1.2 degrees per person, so divide the bus angle by that. $\frac{126}{1.2} = 105$, the same Riverside bus figure as before.

✓ **Check 4:** Put the difference back on to the smaller of the two groups. $69 + 36 = 105$, which is the Riverside bus group, so the subtraction was the right way round.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$3x + 6 + 5x + 8 + 7x - 9 = 320$ oe, for example $15x + 5 = 320$	**M1**	A correct method to find the correct value of x for the Eastgate office, for example an equation. It can be implied by $\frac{320 - 5}{15}$ oe.	✓
$(x =)21$ or $(3x =)63$	**A1**	For the correct value of x or of $3x$. A correct answer of 21 or 63 on its own scores these first two marks, unless it comes from obviously incorrect working.	✓
$3 \times 21 + 6 = 69$ or $63 + 6 = 69$	**M1ft**	Dependent on the first M1. A correct method for the number at the Eastgate office who chose the bus, following through their value of x as long as only one value is offered and it is clearly intended as x. Look for 69 written beside the table.	✓
$\frac{126}{360} \times 300 = 105$ oe, for example $\frac{300}{360} = \frac{5}{6}$	**M1**	Independent of the marks above. A correct method for the number at the Riverside office whose preferred way of travelling is the bus.	✓

Step	Mark	Description	Got it?
then $\frac{5}{6} \times 126 = 105$, or $\frac{360}{300} = 1.2$ then $\frac{126}{1.2} = 105$		Because $\frac{300}{360}$ is 0.83 recurring, a rounded 0.83 is allowed here.	
$105 - 69 = 36$	A1	cao. Dependent on the A1 already scored, so the 36 must come from a correct x.	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

$ABCDE$ is a regular pentagon
$DEFGHI$ is a regular hexagon
The two shapes are joined along the edge DE, so every edge in the figure is the same length
AF is a straight line

Find

The size of angle EAF, in degrees

Plan the Solution

- Work out the interior angle of a regular pentagon, then the interior angle of a regular hexagon.
- Three angles meet at E: the pentagon's angle AED, the hexagon's angle DEF, and angle AEF underneath them. They fill one full turn, so subtracting gives angle AEF.
- Angle EAF is a base angle of triangle AEF, which is isosceles. Take angle AEF out of the triangle's angle sum, then halve what is left.

Worked Solution [5 marks]

Rule - interior angle of a regular polygon with n sides: $\frac{(n-2) \times 180}{n}$ degrees. Angles at a point add to 360 degrees, the angles of a triangle add to 180 degrees, and the two base angles of an isosceles triangle are equal.

Step 1: the interior angle of the regular pentagon $ABCDE$

$$\frac{(5-2)\times 180}{5} = \frac{540}{5} = 108$$

(Reason: A pentagon can be cut into three triangles from one vertex, so its five interior angles add to $(5-2)\times 180 = 540$ degrees. The pentagon is regular, so those five angles are equal, and angle AED is one of them.)

Step 2: the interior angle of the regular hexagon $DEFGHI$

$$\frac{(6-2)\times 180}{6} = \frac{720}{6} = 120$$

(Reason: A hexagon can be cut into four triangles, so its interior angles add to $(6-2)\times 180 = 720$ degrees. The hexagon is regular, so divide by the six equal angles it has to get the size of angle DEF.)

Step 3: the angle AEF that is left at E

$$360 - 108 - 120 = 132$$

(Reason: Angle AED in the pentagon, angle DEF in the hexagon and angle AEF below them fit round the point E with no gap and no overlap, so all three together make one full turn of 360 degrees. What is left is obtuse, which is what the figure suggests.)

Step 4: what is left for the other two angles of triangle AEF

$$180 - 132 = 48$$

(Reason: The three angles of triangle AEF add to 180 degrees, so angle EAF and angle EFA share the 48 degrees that are left over.)

Step 5: halve it, because the triangle is isosceles

$$\frac{48}{2} = 24$$

(Reason: AE is an edge of the pentagon and EF is an edge of the hexagon, and the two shapes are joined along the edge DE, so every edge in the figure is the same length. That makes AE and EF equal, so triangle AEF is isosceles and its base angles EAF and EFA are equal. Each one is half of what was left.)

Angle EAF is $24°$

Verification

✓ **Check 1:** Reach angle AEF a second way, from exterior angles. A regular pentagon has exterior angle $\dfrac{360}{5} = 72$ degrees and a regular hexagon has exterior angle $\dfrac{360}{6} = 60$

degrees. Taking each interior angle off the full turn leaves exactly those two exterior angles. $72 + 60 = 132$

✓ **Check 2:** Put the three angles at E back together. If they do not close the full turn, one of them is wrong. $108 + 120 + 132 = 360$

✓ **Check 3:** Put the answer back into triangle AEF. Both base angles are 24 degrees, so with the obtuse angle they must come to the angle sum of a triangle. $24 + 24 + 132 = 180$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Interior angle of the pentagon $\frac{3 \times 180}{5} = 108$, or exterior angle of the pentagon $\frac{360}{5} = 72$	M1	Allow this in the working, but not if it is labelled in the wrong place on the diagram, unless the candidate clearly starts again. Any equivalent method scores.	✓
Interior angle of the hexagon $\frac{4 \times 180}{6} = 120$, or exterior angle of the hexagon $\frac{360}{6} = 60$	M1	Allow this in the working, but not if it is labelled in the wrong place on the diagram, unless the candidate clearly starts again. Any equivalent method scores.	✓
A complete method for the obtuse angle AEF: $360 - (108 + 120) = 132$, or $72 + 60 = 132$, or $(180 - 108) + (180 - 120) = 132$	M1	A fully correct method for the size of the obtuse angle AEF, but not if it is labelled in the wrong place on the diagram. A value carried forward must come from correct working.	✓
A complete method for angle EAF: $\frac{180 - 132}{2} = 24$, or $\frac{180 - (72 + 60)}{2} = 24$	M1	A fully correct method for the size of angle EAF. A value carried forward must come from correct working.	✓
Answer 24	A1	Correct answer only. A correct answer scores full marks unless it clearly comes from incorrect working.	✓

Full marks: 5/5

Question 12 - Exam Solution

Understanding the Question

Given

(a) The equation $\dfrac{3x+2}{5} - \dfrac{2x+1}{3} = x$, whose denominators are 5 and 3.

(b) The formula $f = \sqrt{\dfrac{a+bc}{c-d}}$, in which c appears twice, once on the top of the fraction and once on the bottom.

Find

(a) The value of x, with the algebra shown. (b) A formula for c in terms of a, b, d and f.

Plan the Solution

- (a) Multiply every term by 15, the lowest common multiple of 5 and 3. The x on the right must be multiplied too.
- (a) Expand the brackets, collect the x terms on one side and the numbers on the other, then divide.
- (b) Square both sides first. The square root is the outermost operation, so undoing it is the opening move.
- (b) Multiply by $c - d$ to clear the fraction, expand, then gather every term containing c on one side and everything else on the other.
- (b) Factorise the c out of that side and divide by the bracket. Because c appears twice, it can only be made the subject after factorising.

Worked Solution [7 marks]

> Rule - Clearing denominators: multiply every term by the lowest common denominator (or, for a square root, square first and then multiply by the denominator); then collect every term in the unknown on one side and factorise it out.

Step 1 (a): multiply every term by 15

$$15 \times \frac{3x+2}{5} - 15 \times \frac{2x+1}{3} = 15 \times x$$

$$3(3x+2) - 5(2x+1) = 15x$$

(Reason: (Reason: 15 is the lowest common multiple of 5 and 3, so each denominator cancels. $\frac{15}{5} = 3$ is what the first bracket is multiplied by, and $\frac{15}{3} = 5$ is what the second bracket is multiplied by.))

Step 2 (a): expand both brackets

$$9x + 6 - 10x - 5 = 15x$$

(Reason: (Reason: the second bracket is being subtracted, so -5 multiplies both terms inside it: $-5 \times 2x = -10x$ and $-5 \times 1 = -5$. Losing that second sign is the commonest slip here.))

Step 3 (a): collect like terms on the left

$$-x + 1 = 15x$$

(Reason: (Reason: $9x - 10x = -x$ and $6 - 5 = 1$. The equation now has no brackets and no fractions, which is what the second method mark is for.))

Step 4 (a): gather the x terms

$$1 = 15x + x$$
$$1 = 16x$$

(Reason: (Reason: adding x to both sides keeps every x term positive, so $15x + x = 16x$.))

Step 5 (a): divide by 16

$$x = \frac{1}{16}$$

(Reason: (Reason: dividing both sides by 16 leaves x on its own. As a decimal this is 0.0625, which the mark scheme also accepts.))

Step 6 (b): square both sides

$$f^2 = \frac{a + bc}{c - d}$$

(Reason: (Reason: squaring is the inverse of taking a square root, so the root disappears and the fraction underneath is left untouched.))

Step 7 (b): multiply by $c - d$ and expand

$$f^2(c - d) = a + bc$$
$$cf^2 - df^2 = a + bc$$

(Reason: (Reason: multiplying both sides by the denominator clears the fraction. Expanding puts the c term into the open, which is what the next step needs.))

Step 8 (b): put the c terms on one side

$$cf^2 - bc = a + df^2$$

(Reason: (Reason: subtract bc from both sides and add df^2 to both sides. Everything containing c is now on the left and everything else is on the right.))

Step 9 (b): factorise and divide

$c(f^2 - b) = a + df^2$

$$c = \frac{a + df^2}{f^2 - b}$$

(Reason: (Reason: c is a common factor of both terms on the left, so it can be taken outside a bracket. Dividing by that bracket makes c the subject.))

$$\textbf{(a) } x = \frac{1}{16}$$

$$\textbf{(b) } c = \frac{a + df^2}{f^2 - b}$$

Verification

✓ **Check 1:** (a) Put $x = \dfrac{1}{16}$ back into the left-hand side. The numerators become $3x + 2 = \dfrac{35}{16}$ and $2x + 1 = \dfrac{9}{8}$, so the two fractions are $\dfrac{7}{16}$ and $\dfrac{6}{16}$. $\dfrac{7}{16} - \dfrac{6}{16} = \dfrac{1}{16}$, which is exactly the value of x

✓ **Check 2:** (a) Work the same equation in decimals with $x = 0.0625$: the numerators are 2.1875 and 1.125, giving $\dfrac{2.1875}{5} = 0.4375$ and $\dfrac{1.125}{3} = 0.375$. $0.4375 - 0.375 = 0.0625$, the same answer reached without fractions

✓ **Check 3:** (b) Test the rearrangement on numbers. Take $a = 2$, $b = 3$, $d = 1$ and $f = 4$. The new formula gives $c = \dfrac{2 + 1 \times 16}{16 - 3} = \dfrac{18}{13}$. Feeding that back into the original gives $a + bc = \dfrac{80}{13}$ and $c - d = \dfrac{5}{13}$. $\sqrt{\dfrac{80}{5}} = \sqrt{16} = 4$, the value of f that was chosen at the start

✓ **Check 4:** (b) Multiply the numerator and the denominator of the answer by -1. A fraction is unchanged when both parts change sign, so the two forms must be the same formula. $c = \dfrac{-a - df^2}{b - f^2}$, which is the alternative form the mark scheme allows

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Writing both fractions over the common denominator 15, or multiplying every term by 15 to remove the denominators, or splitting the left-hand side into $\frac{3}{5}x + \frac{2}{5} - \frac{2}{3}x - \frac{1}{3}$	M1	A correct method for dealing with the two denominators. If the student expands at this stage, allow one of the four terms on the left to be wrong. If the denominators are removed here, a correct method must be shown or implied.	✓
(a) An equation with no brackets and no fractions, e.g. $9x + 6 - 10x - 5 = 15x$ or $1 - x = 15x$	M1	Or an equation with a common denominator for all terms and the numerators simplified, e.g. $\frac{-x+1}{15} = \frac{15x}{15}$. Allow one error in the four terms on the left across the two method marks, and follow through for the simplifying.	✓
(a) $x = \frac{1}{16}$	A1	Or equivalent, e.g. 0.0625 (allow 0.062 or 0.063). Working is required.	✓
(b) Squaring both sides: $f^2 = \frac{a+bc}{c-d}$	M1	For squaring both sides in a correct equation.	✓
(b) Multiplying by the denominator and expanding: $cf^2 - df^2 = a + bc$	M1	For multiplying by the denominator and expanding in a correct equation.	✓
(b) Isolating the terms in c: $cf^2 - bc = a + df^2$	M1	For isolating the terms in c on one side and the other terms on the other side, in a correct equation.	✓
(b) $c = \frac{a+df^2}{f^2-b}$	A1	Or equivalent, e.g. $c = \frac{-a-df^2}{b-f^2}$. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 7/7

Question 13 - Exam Solution

Understanding the Question

Given

The weights, in kilograms, of 60 bags of flour, grouped into six classes each 1 kilogram wide.

The frequencies $4, 15, 20, 11, 6, 4$, which total 60.

Find

(a) The cumulative frequency for each class. (b) The cumulative frequency graph. (c) An estimate of the median weight, read from the graph. (d) An estimate of how many bags weigh more than 3.7 kilograms.

Plan the Solution

- Add the frequencies down the column. Each running total counts every bag at or below that weight, so the last total must be the whole sample.
- Plot each total at the upper end of its class, then join the points. A class is only complete at its top boundary, so that is where its total belongs.
- The median is the 30th value, because $\frac{60}{2} = 30$. Read across from 30 and down to the weight axis.
- For part (d), read up from 3.7 to the graph and across. That gives how many bags are 3.7 kilograms or less, so subtract it from 60.

Worked Solution [6 marks]

Rule - Cumulative frequency: each entry is a running total of the frequencies, plotted at the UPPER end of its class. For n values the median is read at $\frac{n}{2}$ on the cumulative frequency axis.

Step 1: add the frequencies down the column

$0 + 4 = 4$

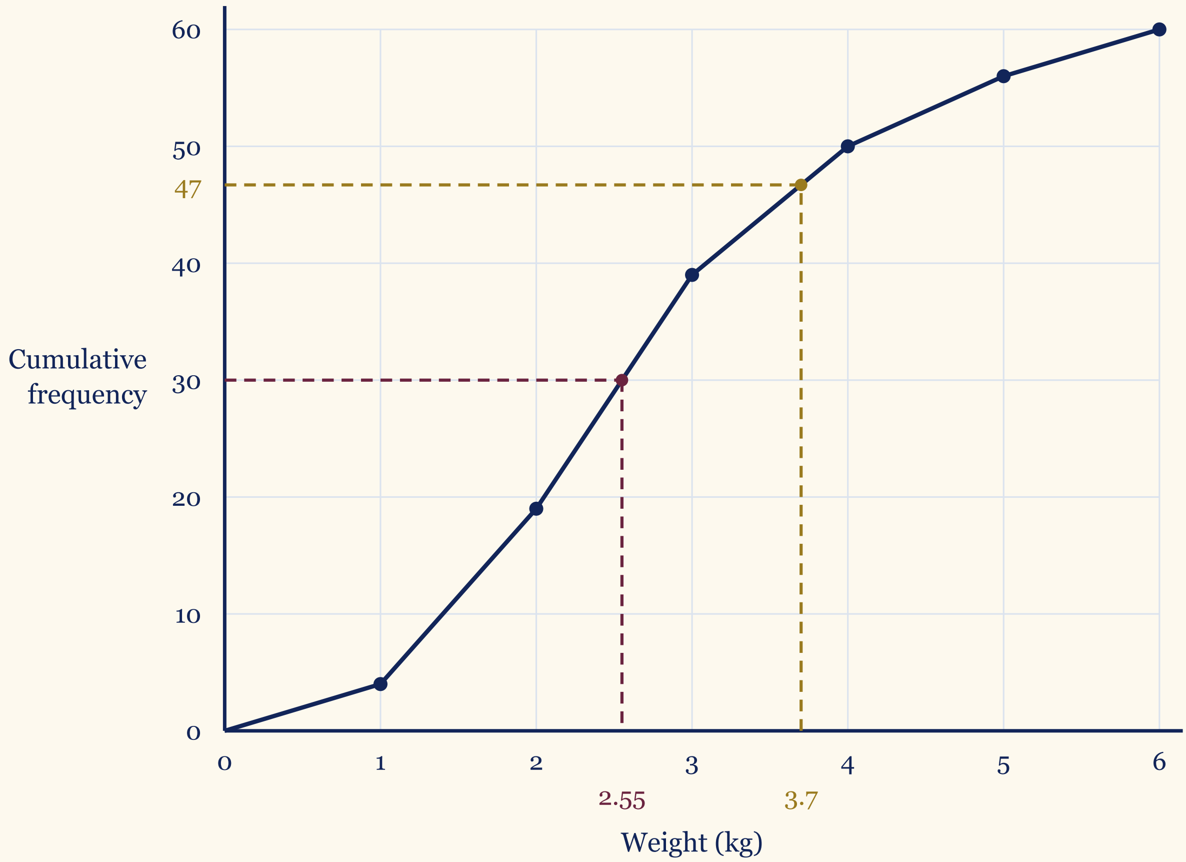
$4 + 15 = 19$

$19 + 20 = 39$

$39 + 11 = 50$

$50 + 6 = 56$

$56 + 4 = 60$



(Reason: Each row of the cumulative table asks how many bags weigh at most that much, so every frequency so far is included. The column reads 4, 19, 39, 50, 56, 60.)

Step 2: plot each total at the upper end of its class

$(1, 4)\quad (2, 19)\quad (3, 39)$

$(4, 50)\quad (5, 56)\quad (6, 60)$

(Reason: The class $0 < w \leq 1$ is only complete at $w = 1$, so its total of 4 belongs at 1 kilogram and not in the middle of the class. The graph is taken back to $(0, 0)$ because no bag weighs 0 kilograms or less, then the points are joined with a smooth curve.)

Step 3: read the median at a cumulative frequency of 30

$$\frac{60}{2} = 30$$

$$2 + \frac{11}{20} = 2.55$$

(Reason: 19 bags weigh 2 kilograms or less and 39 weigh 3 kilograms or less, so the 30th value lies in the class $2 < w \leq 3$. Reading across at 30 and down gives about 2.55 kilograms; the mark scheme accepts anything from 2.3 to 2.7.)

Step 4: read the graph at 3.7 kilograms

$$39 + 0.7 \times 11 = 46.7$$

(Reason: Reading up from 3.7 to the graph and across to the cumulative frequency axis gives about 47 bags weighing 3.7 kilograms or less. The mark scheme allows any reading from 45 to 48, which is one square either way.)

Step 5: subtract from the total

$60 - 47 = 13$

(Reason: The graph gives the number of bags at or below 3.7 kilograms, so take it from 60 to get the number above. The answer counts bags, so it must be a whole number - the mark scheme accepts 12, 13, 14 or 15.)

(a) $4, 19, 39, 50, 56, 60$

(b) the six points plotted at the upper end of each class, joined with a curve

(c) 2.55 **kilograms**

(d) 13 **bags**

Verification

✓ **Check 1:** Add the whole frequency column. The last cumulative total has to be the entire sample, and nothing else can be. $4 + 15 + 20 + 11 + 6 + 4 = 60$, which is the last entry in the cumulative column

✓ **Check 2:** Test the median against the class it should sit in. 19 bags weigh 2 kilograms or less and 39 weigh 3 kilograms or less, so the 30th value must lie between 2 and 3. 2.55 lies between 2 and 3, and it is just over halfway, as 30 is just over halfway from 19 to 39

✓ **Check 3:** Work part (d) from the frequency column instead of the graph: 0.3 of the 11 bags in $3 < w \leq 4$, plus all 6 and all 4 in the two classes above it. $3.3 + 6 + 4 = 13.3$, which is 13 bags to the nearest whole bag

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The cumulative frequencies $4, 19, 39, 50, 56, 60$	**B1**	All six entries correct.	✓
(b) The cumulative frequency graph	**B2**	Fully correct graph: the points plotted at the ends of the intervals and joined with a curve or line segments. If not B2 then B1 for 5 or 6 of their points at the ends of the intervals and joined, or	✓

Step	Mark	Description	Got it?
		for 5 or 6 points plotted correctly at the ends of the intervals but not joined, or for 5 or 6 points from a table with only one arithmetic error plotted consistently within each interval at their correct heights and joined. Ignore the part from 0 to the first plotted point.	
(c) Read across at 30 and down to the weight axis: 2.55	**B1ft**	Any value from 2.3 to 2.7. Follow through an increasing graph, reading across at 30 or 30.5.	✓
(d) A reading taken at 3.7 kilograms, about 47	**M1**	A correct method to take a reading at 3.7 kg, for example 45 to 48, or a correct value for their own graph, within one square.	✓
$60 - 47 = 13$	**A1ft**	12, 13, 14 or 15. Follow through their increasing graph if the value falls outside that range. It must be a whole number: a non-integer answer earns the M1 only.	✓

Full marks: 6/6

Question 14 - Exam Solution

Understanding the Question

Given

$$\frac{3^{2n+3}}{3^4} = 3^3 \times 3^{1-2n}$$

Every power on the page has the same base, 3, and every index is linear in n.

Find

The value of n.

Plan the Solution

- Use $\frac{a^p}{a^q} = a^{p-q}$ to turn the left-hand side into one power of 3.
- Use $a^p \times a^q = a^{p+q}$ to turn the right-hand side into one power of 3.
- With a single power of 3 on each side, the two indices must be equal. That gives a linear equation in n to solve.

- The answer is not a whole number here, so leave it as an exact fraction rather than rounding.

Worked Solution [3 marks]

> Rule - Index laws: $\frac{a^p}{a^q} = a^{p-q}$ and $a^p \times a^q = a^{p+q}$. If $a^p = a^q$ and a is a positive number other than 1, then $p = q$.

Step 1: make the left-hand side a single power of 3

$$\frac{3^{2n+3}}{3^4} = 3^{2n+3-4} = 3^{2n-1}$$

(Reason: dividing two powers of the same base subtracts the indices, so the 4 comes off the index $2n+3$)

Step 2: make the right-hand side a single power of 3

$$3^3 \times 3^{1-2n} = 3^{3+1-2n} = 3^{4-2n}$$

(Reason: multiplying two powers of the same base adds the indices, and $3+1=4$)

Step 3: equate the two indices

$$3^{2n-1} = 3^{4-2n}$$

$$2n - 1 = 4 - 2n$$

(Reason: the base 3 is now the same on both sides, so the equation can only hold when the indices match)

Step 4: solve the equation for n

$$2n + 2n = 4 + 1$$

$$4n = 5$$

$$n = \frac{5}{4}$$

(Reason: collect the n terms on one side and the numbers on the other, then divide by 4)

$$n = \frac{5}{4} = 1.25$$

Verification

✓ **Check 1:** Put $n = \frac{5}{4}$ into each index. The left-hand index is $2n - 1$ and the right-hand index is $4 - 2n$. $2 \times \frac{5}{4} - 1 = \frac{3}{2}$ and $4 - 2 \times \frac{5}{4} = \frac{3}{2}$, so the two indices agree.

✓ **Check 2:** Work each side of the original equation out on a calculator with $n = 1.25$, so the numerator index is $2 \times 1.25 + 3 = 5.5$ and the second factor index is $1 - 2 \times 1.25 = -1.5$. $\frac{3^{5.5}}{3^4} = 3^{1.5}$ and $3^3 \times 3^{-1.5} = 3^{1.5}$, which is 5.196 to 3 decimal places on both sides.

✓ **Check 3:** Reach the answer a second way. Multiply both sides by 3^4 first, which clears the fraction and gives one power of 3 on each side straight away. $3^{2n+3} = 3^{8-2n}$, so $2n + 3 = 8 - 2n$, giving $4n = 5$ and the same $n = \frac{5}{4}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $3^{2n+3-4} \left(= 3^3 \times 3^{1-2n}\right)$ or $3^{2n-1} \left(= 3^3 \times 3^{1-2n}\right)$ or $\left(3^{2n+3} =\right) 3^7 \times 3^{1-2n}$ or $\left(\frac{3^{2n+3}}{3^4} =\right) 3^{4-2n}$ or $\left(3^{2n+3} =\right) 3^3 \times 3^{5-2n}$ or $\frac{3^{2n}}{3^4} = 3^{1-2n}$ (division by 3^3). This is not an exhaustive list. Or one of the indices $2n + 3 - 4, 3 + 1 - 2n$, $2n - 1, 4 - 2n$.	**M1**	For one rule of indices used to correctly combine two or more of the given expressions. The part in brackets is not needed, and the working must include algebra. Or for one of the four indices listed, provided it is clear that it applies to the left-hand side or to the right-hand side.	✓
eg $3^{2n+3-4} = 3^{3+1-2n}$ or $3^{2n+3} = 3^{8-2n}$ or $3^{2n-1} = 3^{4-2n}$ or $2n + 3 - 4 = 3 + 1 - 2n$ oe eg $2n - 1 = 4 - 2n$	**M1**	A correct single power of 3 on both sides, or a correct equation in n without indices. Some students go straight to this line and gain M2.	✓
$n = \frac{5}{4}$, with working shown	**A1**	oe, dep on M1. Working is required, so an answer of $\frac{5}{4}$ on its own does not earn this mark.	✓

Full marks: 3/3

Question 15 - Exam Solution

Understanding the Question

Given

The recurring decimal $0.7\dot{6}\dot{3}$, which is written out in full as $0.7636363\ldots$ The dots sit on the 6 and the 3, so the block 63 repeats for ever, while the 7 does not repeat.

Find

An algebraic proof that this decimal is exactly $\frac{42}{55}$ This is a show that question, so the working is the answer: every line has to be written down, and a calculator display on its own earns nothing.

Plan the Solution

- Give the decimal a letter, x, so that it can be worked with algebraically.
- Multiply by 10 so that the repeating block starts immediately after the decimal point.
- Multiply by 1000 as well, so a second line has its repeating block in exactly the same position.
- Subtract one line from the other. The two recurring tails are identical, so they cancel and leave a whole number.
- Solve the equation that is left, then cancel the fraction to its lowest terms.

Worked Solution [2 marks]

> Rule - Recurring decimals: let x be the decimal, then multiply it by two powers of ten that place the repeating block in the same position and subtract. Multipliers 10^n and 10^m line the tails up when $n - m$ is a whole number of blocks and $10^m x$ has already passed the digits that do not repeat.

Step 1: give the decimal a letter

$x = 0.7\dot{6}\dot{3}$

$x = 0.7636363\ldots$

(Reason: Naming the decimal x is what turns this into algebra, and the mark scheme insists on algebra for the final mark.)

Step 2: move the repeating block up to the decimal point

$10x = 7.636363\ldots$

(Reason: Multiplying by 10 shifts every digit one place left, so the 7 that does not repeat is now in front of the point and the tail begins with the block.)

Step 3: make a second line with the block in the same position

$1000x = 763.636363\ldots$

(Reason: The block 63 is two digits long, so multiplying by a further 100 shifts it by exactly one whole block and the two tails now match digit for digit.)

Step 4: subtract, so the recurring tails cancel

$1000x - 10x = 763.636363\ldots - 7.636363\ldots$

$990x = 756$

(Reason: Both decimals end in the same never-ending tail, so the tails subtract away and a whole number is left. This is the whole point of choosing those two multipliers.)

Step 5: solve for x

$$x = \frac{756}{990}$$

(Reason: Dividing both sides by 990 leaves x on its own as a single fraction. The decimal has already become a fraction here, but it is not yet in its lowest terms.)

Step 6: cancel to lowest terms

$756 = 18 \times 42$

$990 = 18 \times 55$

$$\frac{756}{990} = \frac{18 \times 42}{18 \times 55} = \frac{42}{55}$$

(Reason: The highest common factor of 756 and 990 is 18, so cancelling that factor from top and bottom gives the fraction the question asked for.)

$$0.7\dot{6}\dot{3} = \frac{756}{990} = \frac{42}{55} \text{ as required}$$

Verification

✓ **Check 1:** Go the other way. Turn $\frac{42}{55}$ back into a decimal by long division and read the digits. $\frac{42}{55} = 0.7\dot{6}\dot{3}$, the decimal the question started from

✓ **Check 2:** Use a different pair of multipliers, $100x$ and x, which the mark scheme also allows. The subtraction now leaves a terminating decimal instead of a whole number.

$99x = 75.6$, so $x = \frac{75.6}{99} = \frac{756}{990} = \frac{42}{55}$

✓ **Check 3:** Build the decimal from place value instead of by subtracting: 0.7 plus the recurring part $0.0\dot{6}\dot{3}$, which is $\frac{63}{990}$ because a two digit block over 99 is shifted one place further along. $\frac{7}{10} + \frac{63}{990} = \frac{693 + 63}{990} = \frac{756}{990} = \frac{42}{55}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two multiples of x whose recurring tails match, written down with the intention of subtracting, eg $1000x = 763.63\ldots$ and $10x = 7.63\ldots$	**M1**	Any pair that subtracts to a whole number or a terminating decimal, eg $10\,000x = 7636.36\ldots$ with $100x = 76.36\ldots$, or $100x = 76.363\ldots$ with $x = 0.763\ldots$. If the recurring is not shown, at least one of the numbers must be given to at least 5 significant figures. The difference itself may be 75.6, 756, 7560 and so on.	✓
Complete the working to $\frac{42}{55}$, eg $990x = 756$ leading to $x = \frac{756}{990} = \frac{42}{55}$	**A1**	Dependent on the M1, and algebra must be used for this final mark to be awarded. Allow, for instance, $99x = 75.6$ and then $\frac{756}{990} = \frac{42}{55}$.	✓
Note - no algebra used	**-**	A response that reaches $\frac{42}{55}$ without using algebra scores a maximum of 1 mark. Working is required throughout.	✓

Full marks: 2/2

Question 16 - Exam Solution

Understanding the Question

Given

Find

A, B, C and D lie on a circle with centre O, in that order round the circle.
The angle marked at C, between the chords CA and CD, is $\angle ACD = 54°$
The angle marked at D, between the chord DC and the radius DO, is $\angle ODC = 28°$
OA, OC and OD are all radii of the same circle, so the triangles OAD, OCD and OAC are every one of them isosceles.

The size of $\angle AOD$ The circle theorem that justifies that answer, written in words The size of $\angle CAO$ The size of $\angle ABC$

Plan the Solution

- Start at the marked $54°$. It stands on the arc AD, and so does the angle at the centre, so doubling gives $\angle AOD$ straight away.
- Use the isosceles triangle OCD to turn the marked $28°$ into the second angle at the centre, $\angle COD$.
- Two of the three angles at O are then known, so the third, $\angle AOC$, is what is left of the full turn.
- Triangle OAC is isosceles as well, so halving what is left of $180°$ gives part (b).
- For part (c), split $\angle ADC$ at the radius DO, then use the fact that $ABCD$ is a cyclic quadrilateral.

Worked Solution [5 marks]

> Rule - Angles in a circle: the angle at the centre is twice the angle at the circumference when both stand on the same arc, so $\angle AOD = 2 \times \angle ACD$. Angles round the point O add to $360°$, the angles of a triangle add to $180°$, and opposite angles of a cyclic quadrilateral add to $180°$.

Step 1: double the marked angle, for part (a)(i)

$\angle AOD = 2 \times \angle ACD$

$\angle AOD = 2 \times 54° = 108°$

(Reason: $\angle ACD$ has its vertex on the circle and $\angle AOD$ has its vertex at the centre, and both of them stand on the same arc AD, so the one at the centre is double the one at the circumference.)

Step 2: say which theorem that was, for part (a)(ii)

$$\angle ACD = \frac{1}{2} \times \angle AOD$$

(Reason: The mark here is for the words, not for the arithmetic a second time. Either direction of the same theorem earns it: the angle at the centre is twice the angle at the circumference, or the angle at the circumference is half the angle at the centre. Whichever wording is used, it must be clear that $\angle ACD$ and $\angle AOD$ stand on the same arc.)

Step 3: turn the marked 28 degrees into an angle at the centre

$$\angle OCD = \angle ODC = 28^\circ$$

$$\angle COD = 180^\circ - 28^\circ - 28^\circ = 124^\circ$$

(Reason: OC and OD are both radii, so triangle OCD is isosceles and its two base angles are equal. The three angles of that triangle then add to 180°.)

Step 4: the third angle at the centre

$$\angle AOC = 360^\circ - 108^\circ - 124^\circ = 128^\circ$$

(Reason: $\angle AOD$, $\angle COD$ and $\angle AOC$ fill the whole turn at O with nothing left over, so the three of them add to 360°.)

Step 5: halve what is left in triangle OAC, for part (b)

$$\angle CAO = \angle ACO = \frac{180^\circ - 128^\circ}{2} = 26^\circ$$

(Reason: OA and OC are radii, so triangle OAC is isosceles with its apex at O. The apex angle is 128°, and the two equal base angles share what is left of 180°.)

Step 6: the base angles of triangle OAD

$$\angle ODA = \angle OAD = \frac{180^\circ - 108^\circ}{2} = 36^\circ$$

(Reason: OA and OD are radii as well, so triangle OAD is isosceles too, with its apex angle the 108° found in step 1. Taking that apex angle off 180° leaves an amount that the two equal base angles share between them.)

Step 7: put the two pieces of the angle at D together

$$\angle ADC = \angle ODA + \angle ODC$$

$$\angle ADC = 36^\circ + 28^\circ = 64^\circ$$

(Reason: O lies inside the angle at D, so the radius DO cuts $\angle ADC$ into exactly the two pieces already found, and they add.)

Step 8: use the cyclic quadrilateral, for part (c)

$$\angle ABC + \angle ADC = 180^\circ$$

$$\angle ABC = 180^\circ - 64^\circ = 116^\circ$$

(Reason: A, B, C and D lie on the circle in that order, so $ABCD$ is a cyclic quadrilateral. $\angle ABC$ and $\angle ADC$ are a pair of opposite angles in it, and opposite angles of a cyclic quadrilateral add to 180°.)

(a)(i) $\angle AOD = 108^\circ$

(a)(ii) The angle at the centre is twice the angle at the circumference, both standing on the arc AD

(b) $\angle CAO = 26^\circ$

(c) $\angle ABC = 116^\circ$

Verification

✓ **Check 1:** Add the three angles at the centre. If $\angle AOD$, $\angle COD$ and $\angle AOC$ are all right, they must make exactly one full turn. $108^\circ + 124^\circ + 128^\circ = 360^\circ$

✓ **Check 2:** Reach part (c) the second way the mark scheme allows, with no cyclic quadrilateral at all. From B the arc through D is covered by the two angles at the centre 108° and 124°, so the angle at B is half of their total. $\dfrac{108^\circ + 124^\circ}{2} = \dfrac{232^\circ}{2} = 116^\circ$

✓ **Check 3:** Add up triangle ACD. The radius AO cuts $\angle CAD$ into the 26° of part (b) and the 36° of step 6, giving $\angle CAD = 62^\circ$. The other two angles of that triangle are the marked 54° and the 64° of step 7. $62^\circ + 54^\circ + 64^\circ = 180^\circ$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) 108	B1	Accept 252 as well, which is the reflex angle AOD at O. No working is required for this mark.	✓
(a)(ii) A correct reason	B1	Dependent on part (a)(i) being correct. Accept: the angle at the centre (midpoint, origin, middle) is twice the angle at the circumference (side, edge, arc), or any equivalent, for instance that the inscribed angle is half of the central angle. The angle symbol is accepted in place of the word angle, and $2\times$ or the word double is accepted in place of twice.	✓
(b) 26	B1	No working is required for this mark.	✓
(c) $\angle ABC = 180^\circ - 64^\circ$ or $\dfrac{108^\circ + 124^\circ}{2}$	M1ft	Follow through their $\dfrac{180^\circ - 108^\circ}{2}$ used in $28^\circ + 36^\circ = 64^\circ$. Only their 108 is followed through.	✓

Step	Mark	Description	Got it?
(c) 116	**A1**	Correct answer only.	✓
Note - a correct answer with no working	-	A correct answer scores full marks, unless it plainly follows from obviously incorrect working.	✓

Full marks: 5/5

Question 17 - Exam Solution

Understanding the Question

Given

$\overrightarrow{FG} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

$\overrightarrow{HG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$

Two column vectors joining the three points F, G and H. Neither of them runs from H to F, so that vector has to be built first.

Find

The magnitude $\left|\overrightarrow{HF}\right|$ - a length, so expect one positive number, and no unit is given here.

Plan the Solution

- Travel from H to F by way of G, which gives $\overrightarrow{HF} = \overrightarrow{HG} + \overrightarrow{GF}$.
- Only $\overrightarrow{FG}$ is given, so reverse it: $\overrightarrow{GF} = -\overrightarrow{FG}$.
- Add the two column vectors component by component to get $\overrightarrow{HF}$.
- Treat those two components as the legs of a right-angled triangle and apply Pythagoras. The magnitude is a length, so take the positive root.

Worked Solution [3 marks]

Rule - Magnitude of a column vector: $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2 + b^2}$. Reversing a vector reverses the sign of both of its components, so $\overrightarrow{GF} = -\overrightarrow{FG}$.

Step 1: build a route from H to F

$$\overrightarrow{HF} = \overrightarrow{HG} + \overrightarrow{GF}$$

$$\overrightarrow{GF} = -\overrightarrow{FG} = -\begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

(Reason: Going from H to G and then from G to F finishes at F, so the two journeys add to give $\overrightarrow{HF}$. Walking a vector backwards changes the sign of both components.)

Step 2: work out the components of $\overrightarrow{HF}$

$$\overrightarrow{HF} = \begin{pmatrix} 4 \\ 14 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

(Reason: Add the top components together and the bottom components together. This is the same as $\overrightarrow{HG} - \overrightarrow{FG}$, which is the form the mark scheme states.)

Step 3: apply Pythagoras to the two components

$$\left|\overrightarrow{HF}\right| = \sqrt{9^2 + 12^2}$$

$$\left|\overrightarrow{HF}\right| = \sqrt{81 + 144} = \sqrt{225}$$

(Reason: The components 9 and 12 are the two legs of a right-angled triangle, and the magnitude is its hypotenuse.)

Step 4: take the positive square root

$$\left|\overrightarrow{HF}\right| = \sqrt{225} = 15$$

(Reason: 225 is a square number, so the magnitude is exact and no rounding is needed. A magnitude is a length, so the negative root is discarded.)

$$\left|\overrightarrow{HF}\right| = 15$$

Verification

✓ **Check 1 - travel the other way:** Build the reverse journey instead: $\overrightarrow{FH} = \overrightarrow{FG} + \overrightarrow{GH} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ -14 \end{pmatrix} = \begin{pmatrix} -9 \\ -12 \end{pmatrix}$. A vector and its reverse must have the same length. $\sqrt{(-9)^2 + (-12)^2} = \sqrt{225} = 15$, the same magnitude.

✓ **Check 2 - an enlarged 3, 4, 5 triangle:** The components 9 and 12 are three times 3 and three times 4, so this is a $3, 4, 5$ triangle enlarged by scale factor 3. $3 \times 5 = 15$, which agrees with the square root, and no calculator is needed to see it.

✓ **Check 3 - put the points on a grid:** Pin H at the origin. Then G is at $(4, 14)$, and since $\overrightarrow{FG}$ runs from F to G, the point F is at $(4-(-5),\ 14-2)=(9,12)$. Now use the distance formula. $\sqrt{(9-0)^2+(12-0)^2}=\sqrt{225}=15$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct calculation for $\overrightarrow{HF}$ or $\overrightarrow{FH}$, for example $\begin{pmatrix}4\\14\end{pmatrix}-\begin{pmatrix}-5\\2\end{pmatrix}=\begin{pmatrix}9\\12\end{pmatrix}$ or equivalent.	M1	For this mark, allow the vector written as coordinates. Also allow, for example, $9\mathbf{i}+12\mathbf{j}$. The reverse vector $\begin{pmatrix}-9\\-12\end{pmatrix}$ scores this mark too.	✓
$\sqrt{9^2+12^2}$ or $\sqrt{(-9)^2+(-12)^2}$	M1 indep	Awarded independently of the first mark. Allow a complete method using their own $\overrightarrow{HF}$ or $\overrightarrow{FH}$, provided it came from $\begin{pmatrix}\pm4\\\pm14\end{pmatrix}-\begin{pmatrix}\pm5\\\pm2\end{pmatrix}$, allowing any sign error. If $(-9)^2$ and $(-12)^2$ are used, condone missing brackets if they are recovered.	✓
15	A1	From fully correct figures. Use of $\begin{pmatrix}9\\-12\end{pmatrix}$ would give the correct answer of 15, because the squares are the same, but it would not gain this accuracy mark.	✓
Note - no mark of its own	-	A correct answer scores full marks unless it comes from obviously incorrect working. Watch out for a correct answer from wrong working, for example $-5+2+4+14=15$: on these particular vectors the four components happen to total the right number, and that earns nothing.	✓

Full marks: 3/3

Question 18 - Exam Solution

Understanding the Question

Given

The line L is perpendicular to the line $2x + y = 9$.
L passes through the point $(8, 11)$.
Two facts, and each one fixes a different part of the answer.

Find

An equation for L, written in the form $y = mx + c$. So one gradient m and one intercept c are needed.

Plan the Solution

- Rearrange $2x + y = 9$ into $y = mx + c$ form, so its gradient can be read off.
- Turn that gradient into the perpendicular gradient using the negative reciprocal.
- Substitute $x = 8$ and $y = 11$ to find c.
- Write the finished equation in the form the question asks for.

Worked Solution [4 marks]

> Rule - Perpendicular gradients: if two lines are perpendicular then $m_1 \times m_2 = -1$, so $m_2 = \dfrac{-1}{m_1}$ - the negative reciprocal. The gradient of a line is only visible once its equation is in $y = mx + c$ form.

Step 1: Rearrange the given line into $y = mx + c$ form

$2x + y = 9$

$y = -2x + 9$

(Reason: taking $2x$ from both sides leaves y on its own, and the number in front of x is then the gradient, so the given line has gradient -2)

Step 2: Turn that into the gradient of L

$m_L \times (-2) = -1$

$m_L = \dfrac{-1}{-2} = \dfrac{1}{2}$

(Reason: perpendicular gradients multiply to give -1, so divide -1 by the gradient of the given line - the negative reciprocal)

Step 3: Use the point $(8, 11)$ to find c

$11 = \frac{1}{2} \times 8 + c$

$11 = 4 + c$

$c = 7$

(Reason: the point lies on L, so putting $x = 8$ and $y = 11$ into $y = mx + c$ leaves c as the only unknown)

Step 4: Write the equation in the form asked for

$y = \frac{1}{2}x + 7$

(Reason: the question asks for $y = mx + c$, so the gradient $\frac{1}{2}$ and the intercept 7 go straight into that form)

$$y = \frac{1}{2}x + 7$$

Verification

✓ **Check 1:** Put $x = 8$ into the answer and see whether y comes back as 11.
$\frac{1}{2} \times 8 + 7 = 4 + 7 = 11$, so L does pass through $(8, 11)$

✓ **Check 2:** Multiply the two gradients together. Perpendicular lines must give -1.
$-2 \times \frac{1}{2} = -1$, so the two lines really are at right angles

✓ **Check 3:** Compare directions instead of gradients. At $x = 0$ the answer gives $y = 7$, and the given line steps 1 across for every 2 down. From $(0, 7)$ to $(8, 11)$ is 8 across and 4 up, and $8 \times 1 + 4 \times (-2) = 0$, which is what perpendicular directions do

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$y = -2x + 9$ or gradient of the given line $= -2$	M1	a rearrangement with the correct x term, or a statement that the gradient of the given line is -2	✓
gradient of the perpendicular $= \frac{1}{2}$	M1ft	for a statement that the gradient of the perpendicular line is $\frac{1}{2}$, or for it being implied by an equation of a line with gradient $\frac{1}{2}$. A student who	✓

Step	Mark	Description	Got it?
oe, eg $\frac{-1}{-2}$		goes straight to this stage is awarded M2. The mark is also available for the perpendicular gradient of whatever they indicate the gradient of the original line to be	
eg $11 = \frac{1}{2} \times 8 + c$ or $y - 11 = \frac{1}{2}(x - 8)$ oe, or $c = 7$	**M1dep**	dep on the previous M1 being awarded; a correct method to find the equation of the perpendicular line, using their gradient of the perpendicular line and $(8, 11)$	✓
$y = \frac{1}{2}x + 7$	**A1**	a correct equation for the line in the form $y = mx + c$, as requested. oe	✓
If no other marks are awarded: $y = -\frac{1}{2}x + 15$	**SC B1**	the named error behind this special case is taking the reciprocal of -2 without changing its sign, which gives gradient $-\frac{1}{2}$ and then $c = 15$. This row carries no mark of its own; it records what that one wrong answer is worth	✓
Note	**note**	A correct answer scores full marks, unless it comes from obviously incorrect working. This row earns nothing on its own.	✓

Full marks: 4/4

Question 19 - Exam Solution

Understanding the Question

Given

A curve C with equation $y = f(x)$. The function f itself is never stated.

C has exactly one minimum point, at $(5,\ 4)$.

Find

(i) the coordinates of the minimum point on $y = f(x + 5)$ (ii) the coordinates of the minimum point on $y = 3f(x)$ (iii) the coordinates of the minimum point on $y = f(x) - 7$

Plan the Solution

- For each equation, ask one question first: is the change inside the bracket of $f(...)$, or outside it?
- Inside the bracket acts on x and does the opposite of what it looks like, so $f(x+5)$ moves the curve 5 to the left.
- Outside the bracket acts on y and does exactly what it looks like, so $3f(x)$ multiplies every y-coordinate by 3, and $f(x)-7$ lowers every y-coordinate by 7.
- Then move the single point $(5,\ 4)$ three times, once per equation. Nothing else about f is needed, and none of the three parts depends on the others.

Worked Solution [3 marks]

> Rule - transforming $y = f(x)$: a change inside the bracket is horizontal and reversed, and leaves y alone; a change outside the bracket is vertical and direct, and leaves x alone.

Step 1: (i) $y = f(x+5)$ - the change is inside the bracket

$x + 5 = 5 \implies x = 0$

$y = 4$

(Reason: The minimum of f happens when the number fed into f is 5. Here the number fed in is $x+5$, so $x+5=5$ and therefore $x=0$ - the curve has moved 5 to the left. Nothing has been done to the output of f, so the height stays at 4.)

Step 2: (ii) $y = 3f(x)$ - the change is outside the bracket

$x = 5$

$y = 3 \times 4 = 12$

(Reason: The number fed into f is untouched, so the lowest point is still reached at $x = 5$. What has changed is the output: it is multiplied by 3, so the y-coordinate is stretched away from the x-axis by scale factor 3.)

Step 3: (iii) $y = f(x) - 7$ - the change is outside the bracket

$x = 5$

$y = 4 - 7 = -3$

(Reason: Again the number fed into f is untouched, so $x = 5$. Subtracting 7 after f has acted lowers every point by 7, so the height 4 becomes -3. The minimum is now below the x-axis, which is allowed.)

(i) $(0,\ 4)$
(ii) $(5,\ 12)$
(iii) $(5,\ -3)$

Verification

✓ **Check 1:** Feed each answer back into its own equation. At $x = 0$, the first curve gives $f(0+5) = f(5) = 4$. At $x = 5$, the second gives $3f(5) = 3 \times 4 = 12$ and the third gives $f(5) - 7 = 4 - 7 = -3$. The three heights come out as 4, 12 and -3, which are the three answers.

✓ **Check 2:** Test a curve that really does have its only minimum at $(5,\ 4)$, for instance $f(x) = (x-5)^2 + 4$. Transforming it gives $f(x+5) = x^2 + 4$, $3f(x) = 3(x-5)^2 + 12$ and $f(x) - 7 = (x-5)^2 - 3$, and the minimum of each can be read straight off the completed square. The vertices are $(0,\ 4)$, $(5,\ 12)$ and $(5,\ -3)$, matching the answers. Repeating the test with $f(x) = |x-5| + 4$ gives the same three points, so the answers do not depend on which curve is chosen.

✓ **Check 3:** Count which coordinate each transformation is even allowed to change. A change inside the bracket can move x only; a change outside it can move y only. Part (i) keeps $y = 4$, and parts (ii) and (iii) keep $x = 5$. Each answer changes exactly one coordinate, as it must.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) $(0,\ 4)$	**B1**	Both coordinates correct, in the right order. Accept $x = 0$ and $y = 4$ written separately on the answer line. One coordinate alone scores nothing.	✓
(ii) $(5,\ 12)$	**B1**	Both coordinates correct, in the right order. $(5,\ 7)$ scores nothing: it comes from reading the multiplier as a translation up 3.	✓
(iii) $(5,\ -3)$	**B1**	Both coordinates correct, in the right order. $(5,\ 11)$ scores nothing: it comes from adding 7 instead of subtracting it. Each part is marked independently of the other two, so a wrong answer earlier costs nothing here.	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

The quadratic inequality $10x^2 + 11x - 21 < 0$, to be solved with algebraic working shown.
The coefficient of x^2 is 10, which is positive, so $y = 10x^2 + 11x - 21$ is a parabola that opens upwards.

Find

Every value of x that makes the expression negative. Expect an interval between two critical values, not a single number.

Plan the Solution

- Solve the matching equation $10x^2 + 11x - 21 = 0$ to get the two critical values. Factorising is quickest here, and the formula or completing the square give the same pair.
- Decide which region satisfies the inequality. An upwards parabola sits below the x-axis only between its two roots, and a test value settles it.
- Write the answer as one double inequality. The inequality is strict, so both critical values are excluded.

Worked Solution [3 marks]

> Rule, quadratic inequality: solve $ax^2 + bx + c = 0$ for the critical values, then when $a > 0$ and the inequality reads < 0, the solution is the region between them.

Step 1: factorise the quadratic

$$10x^2 + 11x - 21 = (10x + 21)(x - 1)$$
$$(10x + 21)(x - 1) = 10x^2 - 10x + 21x - 21 = 10x^2 + 11x - 21$$

(Reason: The brackets have to multiply to -21 and leave a middle term of $+11x$. Expanding straight back is the quickest way to be certain the pair chosen is the right one.)

Step 2: set each factor to zero to get the critical values

$$10x + 21 = 0 \implies x = -\frac{21}{10} = -2.1$$

$$x - 1 = 0 \implies x = 1$$

(Reason: A product is zero only when one of its factors is zero, so each bracket gives one critical value. These are the two places where the curve meets the x-axis.)

Step 3: choose the region where the expression is negative

$10(0)^2 + 11(0) - 21 = -21$

$-2.1 < x < 1$

(Reason: The parabola opens upwards, so it dips below the axis only between the two critical values. Testing $x = 0$, which lies between them, gives -21, and that is negative, so the middle region is the one wanted. Because the inequality is strict, neither endpoint belongs to the answer.)

$$-2.1 < x < 1$$

Verification

✓ **Check 1:** Reach the critical values a second way, with the quadratic formula on $10x^2 + 11x - 21 = 0$. The discriminant is $11^2 - 4 \times 10 \times (-21) = 121 + 840 = 961$, and $\sqrt{961} = 31$. $x = \dfrac{-11+31}{20} = 1$ and $x = \dfrac{-11-31}{20} = -2.1$, the same two critical values that factorising gave.

✓ **Check 2:** Test one value taken from each of the three regions: $x = -2.5$, $x = 0$ and $x = 1.5$. The three values of the expression are 14, -21 and 18. Only the middle one is negative, so only $-2.1 < x < 1$ satisfies the inequality.

✓ **Check 3:** Complete the square instead of factorising: $10(x+0.55)^2 - 3.025 - 21 < 0$. $(x+0.55)^2 < 2.4025$, so $-1.55 < x + 0.55 < 1.55$, which gives $-2.1 < x < 1$ once more.

✓ **Check 4:** Substitute the two critical values themselves, to confirm the endpoints are excluded rather than included. Each one gives exactly zero: $10(-2.1)^2 + 11(-2.1) - 21 = 44.1 - 23.1 - 21 = 0$ and $10 + 11 - 21 = 0$. Zero is not less than zero, so $x = -2.1$ and $x = 1$ are correctly left out.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method to solve the quadratic: the correct factors $(10x+21)(x-1)$, or correct substitution into the formula, or correctly completing the square	M1	Substitution into the formula may be simplified only as far as $\dfrac{-11 \pm \sqrt{121+840}}{20}$ and still earn this mark. Writing $(10)(x+2.1)(x-1)$ is not a correct factorisation: it is working backwards from calculator answers.	✓

Step	Mark	Description	Got it?
The correct critical values $x = 1$ and $x = -2.1$	**A1**	Dependent on the method mark. Critical values that did not come from a correct method earn nothing here.	✓
$-2.1 < x < 1$, with working shown	**A1**	Dependent on the method mark, and working is required. Accept any equivalent form: $x > -2.1$ and $x < 1$ (do not penalise 'or'), or $-\frac{21}{10} < x < 1$, or the open interval written $(-2.1, 1)$.	✓

Full marks: 3/3

Question 21 - Exam Solution

Understanding the Question

Given

One set only: $3x$ items in A alone, $4x$ in B alone, $5x$ in C alone
Overlaps: $x+2$ in A and B only, $x+7$ in A and C only, $2x$ in B and C only, x in all three
$3x+2$ items lie inside the rectangle but outside all three circles, and x is an integer
$\text{n}(A \cup B)' = 26$

Find

$\text{n}(A' \cap C)$, the number of items in C that are not in A

Plan the Solution

- Read $(A \cup B)'$ off the diagram: the two regions that touch neither A nor B
- Set their total equal to 26 and solve for x
- Read $A' \cap C$ off the diagram the same way, as a multiple of x
- Substitute the value of x into that multiple

Worked Solution [4 marks]

Rule - Complements and intersections are read as REGIONS: $\mathrm{n}(S')$ counts everything in the universal set that is not in S, and $A' \cap C$ is the part of C that lies outside A. Name the regions first, then add only those regions.

Step 1: pick out the regions that lie outside $A \cup B$

$\mathrm{n}(A \cup B)' = 5x + (3x + 2)$

$5x + 3x + 2 = 26$

(Reason: $A \cup B$ is everything inside circle A or circle B, so its complement is what is left of the universal set: the part of C outside both circles, $5x$, and the $3x + 2$ outside all three)

Step 2: solve the equation for x

$8x + 2 = 26$

$8x = 24$

$$x = \frac{24}{8} = 3$$

(Reason: Collect the x terms, take 2 from each side, then divide by 8. The value comes out a whole number, as the question promised)

Step 3: pick out the regions that make up $A' \cap C$

$\mathrm{n}(A' \cap C) = 5x + 2x = 7x$

(Reason: $A' \cap C$ is the part of C outside A: the $5x$ region and the $2x$ region that C shares with B. The $x + 7$ and x regions sit inside A, so they are left out)

Step 4: substitute $x = 3$

$7 \times 3 = 21$

(Reason: Multiply the coefficient found in step 3 by the value of x found in step 2)

$$\mathrm{n}(A' \cap C) = 21$$

Verification

✓ **Check 1:** Put $x = 3$ back into the two regions outside $A \cup B$: the C only region holds $5 \times 3 = 15$ and the region outside every circle holds $3 \times 3 + 2 = 11$ $15 + 11 = 26$, the given complement

✓ **Check 2:** Count $A' \cap C$ a different way: take the whole of C, which is $10 + 3 + 6 + 15 = 34$, then remove the part of C inside A, which is $10 + 3 = 13$ $34 - 13 = 21$

✓ **Check 3:** Add every region: $9 + 5 + 12 + 3 + 10 + 6 + 15 + 11 = 71$ items in the universal set, while the six regions inside $A \cup B$ hold $9 + 5 + 12 + 3 + 10 + 6 = 45$ $71 - 45 = 26$, so the diagram has been read consistently

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$5x + 3x + 2 = 26$ oe	**M1**	a correct equation for x	✓
$x = 3$	**A1**	the correct value of x. A correct value of x on its own scores M1A1, unless it comes from obviously incorrect working	✓
7×3, or $15 + 6$ oe	**M1ft**	use of their positive value of x in $7x$, i.e. use of the correct regions of the Venn diagram for the set required (15 being 5×3 and 6 being 2×3)	✓
21	**A1**	cao. A correct answer scores full marks, unless it comes from obviously incorrect working	✓

Full marks: 4/4

Question 22 - Exam Solution

Understanding the Question

Given

$x^2 + y^2 + y = 3$, an equation with a squared term in each letter

$x + 2 = y$, a linear equation

One curve and one straight line, so the pairs being looked for are where they meet

Find

Every pair of values of x and y that satisfies both equations at the same time. A quadratic appears, so expect two solution pairs Clear algebraic working is asked for, so the answer alone earns nothing

Plan the Solution

- Rearrange the linear equation so that y is written in terms of x
- Replace every y in the quadratic equation by $x + 2$, which leaves one letter only
- Expand, collect like terms and write the result as $ax^2 + bx + c = 0$

- Factorise to get the two values of x
- Put each value of x back into $y = x + 2$ so that every x leaves with its own y
- Test both pairs in both of the original equations

Worked Solution [5 marks]

> Rule - Substitution (line into curve): make one letter the subject of the LINEAR equation, substitute it into the quadratic equation, and solve the three-term quadratic that comes out. Each root is one solution PAIR, so every value found is sent back through the linear equation to collect its partner.

Step 1: make y the subject of the linear equation

$x + 2 = y$

$y = x + 2$

(Reason: The linear equation is the easy one to rearrange, and turning it round costs nothing: it now reads as an instruction, telling us what to write in place of every y in the other equation)

Step 2: substitute into the quadratic equation

$x^2 + (x+2)^2 + (x+2) = 3$

(Reason: y appears twice in the quadratic equation, once squared and once on its own, so BOTH are replaced by $x + 2$. Each replacement goes inside brackets, or the squaring and the signs come out wrong)

Step 3: expand the squared bracket

$(x+2)^2 = x^2 + 4x + 4$

$x^2 + (x^2 + 4x + 4) + (x + 2) = 3$

(Reason: $(x+2)^2$ means $(x+2)(x+2)$, which gives x^2, then two lots of $2x$, then 4. Writing $x^2 + 4$ and losing the middle term is the commonest slip in this question)

Step 4: collect the like terms and make one side zero

$2x^2 + 5x + 6 = 3$

$2x^2 + 5x + 3 = 0$

(Reason: The two squared terms add to $2x^2$. The x term from the expanded bracket and the single x that arrived with the curve's own y add to $5x$, and the two constants add to 6. Taking 3 from each side leaves the standard three-term form with zero on the right)

Step 5: factorise, then solve each bracket for x

$(2x+3)(x+1) = 0$

$$2x + 3 = 0 \implies x = -\frac{3}{2}$$

$x + 1 = 0 \implies x = -1$

(Reason: For $2x^2 + 5x + 3$ the two numbers needed multiply to give $2 \times 3 = 6$ and add to give 5, which are 2 and 3; splitting the middle term with them gives these brackets. A product is zero only when one factor is zero, so each bracket is set to zero in turn. The quadratic formula or completing the square would reach the same two values)

Step 6: pair each x with its own y

$x = -\frac{3}{2} \implies y = -\frac{3}{2} + 2 = \frac{1}{2}$

$x = -1 \implies y = -1 + 2 = 1$

(Reason: Each value of x goes back into $y = x + 2$, the simpler equation, so no new quadratic is needed. The values must stay in their pairs: $-\frac{3}{2}$ belongs with $\frac{1}{2}$, and swapping partners would give a point lying on neither graph)

$$x = -\frac{3}{2}, y = \frac{1}{2}$$
$$x = -1, y = 1$$

Verification

✓ **Check 1:** Test the first pair in the quadratic equation. Squaring $-\frac{3}{2}$ gives $\frac{9}{4}$, squaring $\frac{1}{2}$ gives $\frac{1}{4}$, and the separate y adds $\frac{1}{2}$ $\frac{9}{4} + \frac{1}{4} + \frac{1}{2} = 3$, and the line gives $-\frac{3}{2} + 2 = \frac{1}{2}$, so both equations hold

✓ **Check 2:** Test the second pair the same way, in both original equations rather than in any line of the working $1 + 1 + 1 = 3$, and $-1 + 2 = 1$, so both equations hold

✓ **Check 3:** Eliminate the other letter instead. Substituting $x = y - 2$ into the quadratic equation gives $(y-2)^2 + y^2 + y = 3$, which collects to $2y^2 - 3y + 1 = 0$ and factorises as $(2y-1)(y-1) = 0$ The y values come out as $\frac{1}{2}$ and 1 without using the quadratic in x at all

✓ **Check 4:** Use the sum and product of the roots of $2x^2 + 5x + 3 = 0$: they must be $-\frac{5}{2}$ and $\frac{3}{2}$ $-\frac{3}{2} + (-1) = -\frac{5}{2}$ and $-\frac{3}{2} \times (-1) = \frac{3}{2}$, so the two roots are the right pair for that quadratic

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$x^2 + (x+2)^2 + x + 2 = 3$, or $(y-2)^2 + y^2 + y = 3$	**M1**	substitution of the linear equation into the quadratic equation. Allow one sign error in the substituted expression	✓
$2x^2 + 5x + 3[= 0]$ oe (any form with three terms), or $2y^2 - 3y + 1[= 0]$ oe	**M1**	dep on the first M1. Simplified to a three-term quadratic with 2 or 3 of the 3 terms correct	✓
$(2x+3)(x+1)[= 0]$, or $x = \dfrac{-5 + \sqrt{5^2 - 4 \times 2 \times 3}}{2 \times 2}$ and $x = \dfrac{-5 - \sqrt{5^2 - 4 \times 2 \times 3}}{2 \times 2}$, or $2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 3 = 0$, leading to $x = -\frac{3}{2}$ and $x = -1$	**M1ft**	dep on the first M1, for solving their three-term quadratic by any correct method. If factorising, brackets which expand to give 2 of the 3 terms correct are enough; if using the formula, allow one sign error and only partial simplification; if completing the square, as far as the form shown. Leads to the correct values of x OR the correct values of y (the other letter may be used throughout)	✓
$(y =) -\frac{3}{2} + 2$ and $-1 + 2$ oe, or a correct pair of values	**M1**	dep on the previous M1, for a correct method to find both of the other two values, or for a correct pair of values	✓
$x = -\frac{3}{2}, y = \frac{1}{2}$ and $x = -1, y = 1$	**A1**	oe dep on M2, for all 4 values. Working is required, so a correct answer with no algebraic working scores nothing	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

Find

The value of n.

$$\frac{\frac{16x^2-36}{x-7}}{\frac{2x^2+7x+6}{x^2-5x-14}}-(7+8x)=n$$

Three quadratics, and every one of them factorises.

n is an integer, so every x must disappear by the end.

Plan the Solution

- Factorise all three quadratics. $16x^2-36$ has a common factor of 4 and then a difference of two squares.
- Dividing by a fraction is the same as multiplying by its reciprocal, so turn the second fraction upside down.
- Cancel every bracket that appears top and bottom. What is left is linear.
- Subtract $(7+8x)$ and collect like terms. The x terms cancel and a constant is all that survives.

Worked Solution [4 marks]

> Rule - Dividing algebraic fractions: factorise everything first, then $\dfrac{\frac{A}{B}}{\frac{C}{D}}=\dfrac{A}{B}\times\dfrac{D}{C}$, and cancel any bracket that appears in both a numerator and a denominator.

Step 1: Factorise $16x^2-36$

$16x^2-36=4(4x^2-9)$

$4(4x^2-9)=4(2x-3)(2x+3)$

(Reason: Take the common factor of 4 out first. What is left, $4x^2-9$, is a difference of two squares, because $4x^2=(2x)^2$ and $9=3^2$.)

Step 2: Factorise the other two quadratics

$2x^2+7x+6=(2x+3)(x+2)$

$x^2-5x-14=(x-7)(x+2)$

(Reason: For $2x^2+7x+6$ split the middle term: 4 and 3 multiply to 12 and add to 7. For $x^2-5x-14$ look for two numbers multiplying to -14 and adding to -5, which are -7 and 2. Notice that $(x+2)$ and $(2x+3)$ have both appeared twice - that is the hint that everything will cancel.)

Step 3: Multiply by the reciprocal, then cancel

$$\frac{\dfrac{4(2x-3)(2x+3)}{x-7}}{\dfrac{(2x+3)(x+2)}{(x-7)(x+2)}} = \frac{4(2x-3)(2x+3)}{x-7} \times \frac{(x-7)(x+2)}{(2x+3)(x+2)}$$

$$= 4(2x-3) = 8x - 12$$

(Reason: Turning the divisor upside down puts $(x-7)$ on top and $(2x+3)$ underneath, so $(x-7)$, $(x+2)$ and $(2x+3)$ each cancel. Only the 4 and the bracket $(2x-3)$ survive.)

Step 4: Subtract $(7+8x)$ and collect like terms

$$8x - 12 - (7+8x) = 8x - 12 - 7 - 8x$$

$$= (8x - 8x) + (-12 - 7) = -19$$

(Reason: Watch the bracket: subtracting $(7+8x)$ changes the sign of both terms inside it. The $8x$ terms then cancel and only the constant -19 is left, which is exactly why the question can promise that n is an integer.)

$$n = -19$$

Verification

✓ **Check 1:** Put $x = 0$ into the original expression. The first fraction is $\dfrac{-36}{-7} = \dfrac{36}{7}$ and the second is $\dfrac{6}{-14} = -\dfrac{3}{7}$, so the division gives $\dfrac{36}{7} \times \left(-\dfrac{7}{3}\right) = -12$.
$-12 - (7+0) = -19$

✓ **Check 2:** Put $x = 1$ in instead, so a second, unrelated value tests the same identity. The first fraction is $\dfrac{-20}{-6} = \dfrac{10}{3}$ and the second is $\dfrac{15}{-18} = -\dfrac{5}{6}$, so the division gives
$\dfrac{10}{3} \times \left(-\dfrac{6}{5}\right) = -4$. $-4 - (7+8) = -19$

✓ **Check 3:** Do it without cancelling at all. Keep $2x+3$ underneath and combine the whole expression over it: $16x^2 - 36 - (7+8x)(2x+3) = -38x - 57$. If the answer really is a constant, that numerator must be that constant times $(2x+3)$.

$$\frac{-38x-57}{2x+3} = \frac{-19(2x+3)}{2x+3} = -19$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$16x^2 - 36 = 4(2x-3)(2x+3)$ or $(4x-6)(4x+6)$ or $(8x-12)(2x+3)$	**M1**	Factorise the first numerator, in any correct factorised form.	✓
$2x^2 + 7x + 6 = (2x+3)(x+2)$ and $x^2 - 5x - 14 = (x-7)(x+2)$	**M1**	Factorise both of the other quadratics. Allow $2x^2 + 7x + 6 = (4x+6)(0.5x+1)$, since that also cancels with $(4x+6)$.	✓
The two fractions, divided and cancelled, give $4(2x-3)$ or $2(4x-6)$ or $8x - 12$.	**Note**	Any one of these on its own gains the first two M marks. Award $M2$ for any fraction with a completely simplified non-linear numerator and non-linear denominator that will cancel to -19.	✓
$4(2x-3) - (7 + 8x)$ $(= n)$	**M1**	A linear expression that should give the correct value for n. Allow invisible brackets, ie $8x - 12 - 7 + 8x$. Alternatively, an expression clearly showing the numerator is -19 times the denominator, eg $\frac{-38x - 57}{2x + 3}$ $(= n)$. This mark implies the previous M marks, as not all of the factorising is necessary.	✓
Working required: $n = -19$	**A1**	The integer -19, with the algebraic working shown. An answer with no working scores no marks.	✓

Full marks: 4/4

Question 24 - Exam Solution

Understanding the Question

Given

Find

$AB = 8.4$ m
$BC = 9.2$ m
Bearing of B from A is $067°$
Bearing of C from B is $129°$
Level ground, and the paper supplies no figure, so the sketch is yours to draw

The bearing of A from C, correct to the nearest degree

Plan the Solution

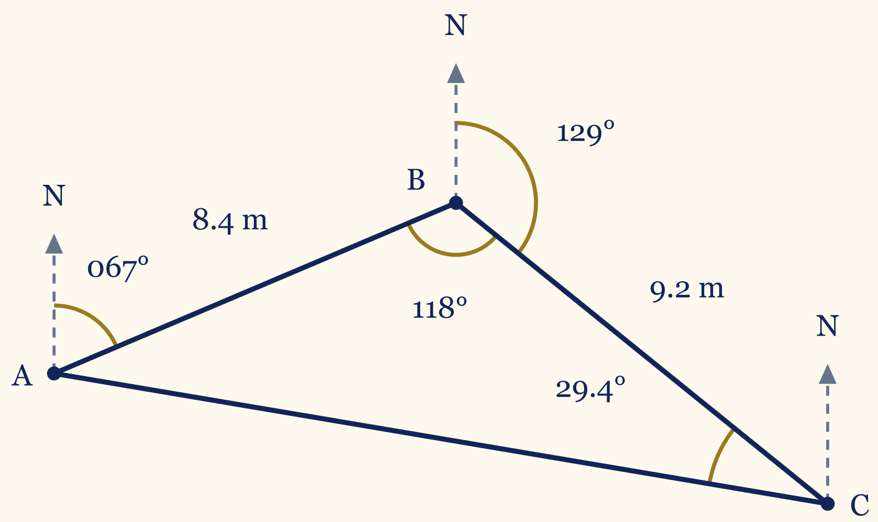
- Sketch it first: a north line at each post, then the two given bearings. Nothing can be worked out until the angle INSIDE the triangle at B is known.
- The north lines are parallel, so that angle comes straight from the two bearings: $67° + 51°$.
- Two sides with the angle between them is the cosine rule, which gives AC. Then the sine rule gives the angle at C.
- An angle is not a bearing. Reverse the $129°$ to get the bearing of B from C, then turn through the angle at C to reach A.

Worked Solution [6 marks]

Cosine rule: $b^2 = a^2 + c^2 - 2ac\cos B$. Sine rule: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$. A back bearing is the forward bearing $\pm 180°$.

Find the angle inside the triangle at B

$180 - 129 = 51$
$51 + 67 = 118$



(Reason: The north lines at A and B are parallel, so the 67° at A reappears at B between BA and the southward direction (alternate angles). North round to south at B is a straight 180°, so BC lies 51° from that same southward direction. The two pieces together make angle ABC.)

Use the cosine rule to find AC^2

$$AC^2 = 8.4^2 + 9.2^2 - 2 \times 8.4 \times 9.2 \times \cos 118^\circ$$
$$70.56 + 84.64 + 72.5615 = 227.7615$$

(Reason: Angle ABC sits between the two known sides, so the cosine rule applies with no extra work. $\cos 118^\circ$ is negative, which is why the third term ADDS 72.5615 instead of taking it away.)

Square root to get the length AC

$$AC = \sqrt{227.7615} = 15.0918 \text{ m}$$

(Reason: The mark scheme accepts anything from 15 to 15.1 here, but leave the unrounded value in the calculator: the marks that follow depend on figures that come from correct working.)

Set up the sine rule for angle ACB

$$\frac{\sin ACB}{8.4} = \frac{\sin 118^\circ}{15.0918}$$

(Reason: Angle ACB faces the side $AB = 8.4$, and the known angle $ABC = 118^\circ$ faces the side AC just found. A known angle opposite a known side is exactly what the sine rule needs.)

Solve for angle ACB

$$\sin ACB = \frac{8.4 \times \sin 118^\circ}{15.0918} = 0.4914$$
$$ACB = \sin^{-1} 0.4914 = 29.4^\circ$$

(Reason: Angle ABC is obtuse, so the other two angles are both acute and the inverse sine gives the one wanted with no ambiguity.)

Turn the angle at C into a bearing

$$129 + 180 = 309$$
$$309 - 29.4 = 279.6$$

(Reason: Reversing the 129° journey gives the bearing of B from C as 309°. On the sketch, turning from CB round to CA is an ANTICLOCKWISE turn of 29.4°, so the angle is taken away from 309°. Bearings are given to three figures, and to the nearest degree this is 280°.)

The bearing of A from C is 280°

Verification

✓ **Check 1:** Drop the triangle rules and use coordinates instead. Put A at the origin: B is $8.4 \sin 67^\circ = 7.73$ east and $8.4 \cos 67^\circ = 3.28$ north of it, and C is a further $9.2 \sin 129^\circ = 7.15$ east and $9.2 \cos 129^\circ = -5.79$ north of B. So A lies 14.88 west and 2.51 north of C, which puts the bearing in the final quarter turn.

$$360^\circ - \tan^{-1} \frac{14.88}{2.51} = 360^\circ - 80.4^\circ = 279.6^\circ$$

✓ **Check 2:** Go round the other way, through A instead of C. The cosine rule on the third angle gives $\cos BAC = \dfrac{8.4^2 + 227.7615 - 9.2^2}{2 \times 8.4 \times 15.0918} = 0.8428$, so $BAC = 32.6^\circ$. The bearing of C from A is therefore $67^\circ + 32.6^\circ$, and the answer is its back bearing.

$$(67^\circ + 32.6^\circ) + 180^\circ = 99.6^\circ + 180^\circ = 279.6^\circ$$

✓ **Check 3:** A cheap sanity check on both angles at once: the three angles of the triangle must close to 180°. If either of the two calculated angles were wrong, this sum would miss.

$$118^\circ + 29.4^\circ + 32.6^\circ = 180^\circ$$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$67 + 51 = 118$, or the angle at B split into 67 and 51	**M1**	A diagram showing 118, or 118 used in a further calculation.	✓
$AC^2 = 8.4^2 + 9.2^2 - 2 \times 8.4 \times 9.2 \times \cos 118^\circ$ $(= 227.7615\ldots)$	**M1**	A correct method to find AC^2. Accept 227 to 228.	✓
$AC = \sqrt{8.4^2 + 9.2^2 - 2 \times 8.4 \times 9.2 \times \cos 118^\circ}$ $(= 15.09\ldots)$	**M1**	A correct method to find the length AC. Accept 15 to 15.1.	✓
$\dfrac{\sin ACB}{8.4} = \dfrac{\sin 118^\circ}{15.09}$ or $\cos ACB = \dfrac{9.2^2 + 15.09^2 - 8.4^2}{2 \times 9.2 \times 15.09}$	**M1**	Dependent on the previous method marks. A correct statement of the sine rule, or of the cosine rule, to find angle ACB or angle BAC. The figures used must come from correct working.	✓

Step	Mark	Description	Got it?
$ACB = \sin^{-1}\left(\frac{8.4 \sin 118^\circ}{15.09}\right) \quad (= 29.4\ldots)$	M1	A completely correct statement for angle ACB, accept 29 to 30, or for angle BAC, accept 32 to 33.	✓
$309 - 29.4 = 279.6$, so the bearing is 280	A1	Allow 279 to 280. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Note - a scale drawing	note	A correct answer read off an accurate scale drawing gains full marks, but a slightly inaccurate one gains 0 marks. Look for angles and lengths written on the candidate's own diagram.	✓

Full marks: 6/6

Question 25 - Exam Solution

Understanding the Question

Given

An equilateral triangle ABC with a circle, centre O, inside it.
AB, BC and CA are all tangents, so the circle touches each side exactly once.
The radius of the circle is x cm.
The three shaded corner regions have total area nx^2 cm^2.

Find

The value of n, correct to 3 significant figures. Note that x never has to be given a value: it cancels.

Plan the Solution

- A radius drawn to a point of contact is perpendicular to the tangent, so the distance from O to each side is x. That makes O the centre of the inscribed circle.
- Join O to A, B and C. Each 60° vertex angle is bisected, so a right-angled triangle with a 30° angle sits at every corner.
- Use one of those right-angled triangles to get half a side, then the whole side, then the height.
- Shaded area = area of triangle minus area of circle. Divide by x^2 and what is left is n.

Worked Solution [5 marks]

> Rule - Tangent and inscribed circle: the radius to a point of contact is perpendicular to the tangent, and the line from a vertex to the centre bisects that vertex angle. Shaded area = area of ABC minus area of the circle.

Step 1: Work out half a side from the tangent

$$\angle OBM = 30^\circ$$

$$\tan 30^\circ = \frac{OM}{MB} = \frac{x}{MB}$$

$$MB = \frac{x}{\tan 30^\circ} = x\tan 60^\circ = \sqrt{3}\,x$$

(Reason: M is the point where the circle touches AB. The radius to a point of contact is perpendicular to the tangent, so $OM = x$ and $\angle OMB = 90^\circ$. OB bisects the 60° angle at B, leaving 30°.)

Step 2: Write down the side and the perpendicular height

$$AB = 2 \times \sqrt{3}\,x = 2\sqrt{3}\,x$$

$$CM = MB \times \tan 60^\circ = \sqrt{3}\,x \times \sqrt{3} = 3x$$

(Reason: By symmetry M is the midpoint of AB, so $AB = 2MB$. In triangle CMB the angle at B is 60°, which gives a height of $3x$ - three radii, which is what the picture suggests.)

Step 3: Area of triangle ABC

$$\text{Area of } ABC = \frac{1}{2} \times AB \times CM$$

$$= \frac{1}{2} \times 2\sqrt{3}\,x \times 3x = 3\sqrt{3}\,x^2$$

$$\frac{1}{2} \times 3.46410 \times 3 = 5.19615$$

(Reason: In decimals $2\sqrt{3} = 3.46410$ to 5 decimal places, so the area of ABC is $5.19615\,x^2$. Keeping the surd form $3\sqrt{3}\,x^2$ until the very end avoids rounding twice.)

Step 4: Area of the circle

$$\text{Area of circle} = \pi x^2$$
$$\pi x^2 = 3.14159\,x^2$$

(Reason: The radius is x, so the circle's area is πx^2. Both areas now carry a factor of x^2, which is exactly why the answer does not depend on the radius.)

Step 5: Subtract, then read off n

$$\text{Shaded area} = 3\sqrt{3}\,x^2 - \pi x^2 = (3\sqrt{3} - \pi)x^2$$
$$5.19615 - 3.14159 = 2.05456$$
$$n = 2.05\ (3\text{ s.f.})$$

(Reason: Dividing by x^2 leaves $n = 3\sqrt{3} - \pi$, a pure number: the shaded area is 2.05456 times x^2 whatever the radius happens to be. Rounding to 3 significant figures gives 2.05.)

$n = 2.05$ correct to 3 significant figures

Verification

✓ **Check 1:** Put a real radius in and work in numbers only. With $x = 100$, the side is $200\tan 60^\circ = 346.410$ and the area of ABC is $\frac{1}{2} \times 346.410^2 \times \sin 60^\circ = 51961.5$. The circle has area $\pi \times 100^2 = 31415.9$. $\dfrac{51961.5 - 31415.9}{100^2} = 2.05456$

✓ **Check 2:** Compare the two areas instead of subtracting them. The circle fills $\dfrac{\pi}{3\sqrt{3}} = 0.60460$ of the triangle, so the shaded part is 0.39540 of it.
$0.39540 \times 5.19615 = 2.05456$

✓ **Check 3:** Build the triangle from the six congruent right-angled triangles that OA, OB and OC create. Each has legs x and $\sqrt{3}\,x$, so each has area $\dfrac{\sqrt{3}}{2}x^2$. $6 \times \dfrac{\sqrt{3}}{2} = 3\sqrt{3}$, the same coefficient as Step 3.

✓ **Check 4:** Is the size sensible? The circle must fill most of the triangle but not all of it, so n must be a good deal smaller than 5.19615 and bigger than 0. $0 < 2.05 < 5.19615$, and the shaded fraction 0.39540 looks right for three corners.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct tangent ratio linking the radius to half a side, e.g. $\tan 30^\circ = \dfrac{x}{0.5AB}$ or $\tan 60^\circ = \dfrac{0.5AB}{x}$ oe	M1	Also allow $0.5AB = \dfrac{x}{\tan 30^\circ}$, $0.5AB = x\tan 60^\circ$ or $\dfrac{1}{2}AB = \sqrt{3}\,x$. The scheme also allows the off-spec route from a known height of $3x$: $\tan 60^\circ = \dfrac{3x}{0.5AB}$ or $\sin 60^\circ = \dfrac{3x}{BC}$.	✓
An expression for a side, e.g. $AB = 2x\tan 60^\circ$, $\dfrac{2x}{\tan 30^\circ}$, $2\sqrt{3}\,x$ or $3.46\ldots x$	M1	Expression for a side of the triangle (AB or BC or AC), OR the area of one or more of the six triangles, e.g. $\dfrac{1}{2} \times x\tan 60^\circ \times x$, $\dfrac{1}{2} \times \sqrt{3}\,x \times x$ or $\dfrac{\sqrt{3}}{2}x^2$.	✓
A correct expression for the area of ABC, e.g. $6 \times \dfrac{1}{2} \times x\tan 60^\circ \times x$, $\dfrac{1}{2}(2x\tan 60^\circ)^2 \sin 60^\circ$, $0.5 \times 2\sqrt{3}\,x \times 3x$, $3\sqrt{3}\,x^2$ or $5.19\ldots x^2$	M1	Any equivalent correct form scores, including $\dfrac{1}{2}\left(\dfrac{2x}{\tan 30^\circ}\right)^2 \sin 60^\circ$ and the Pythagoras route $\dfrac{1}{2} \times 2\sqrt{3}\,x \times \sqrt{(2\sqrt{3}x)^2 - (\sqrt{3}x)^2}$.	✓
A correct expression or equation for the shaded area, e.g. $3\sqrt{3}\,x^2 - \pi x^2$, $6 \times \dfrac{\sqrt{3}}{2}x^2 - \pi x^2$ or $5.19\ldots x^2 - \pi x^2$	M1	Triangle minus circle, correctly formed. An equation set equal to nx^2 scores here too, e.g. $3\sqrt{3}\,x^2 - \pi x^2 = nx^2$.	✓
The value of n	A1	Accept 2.05 to 2.06. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Guidance for the whole question	Note	Any value may be used for the radius provided it is used consistently: a solution that works throughout with $x = 100$ earns every mark, reaching $51961.52 - \pi \times 100^2 = 100^2 n$.	✓

Step	Mark	Description	Got it?
		Examiners are also told to look for values written on the diagram.	

Full marks: 5/5

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