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Edexcel IGCSE 4MA1

4MA1/1FR (Foundation Tier) – Thursday 15 May 2025

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/1FR (Foundation Tier), June 2025 series, sat Thursday 15 May 2025 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. An online atlas gives the land area, in km^2, of six states in the USA.

State	Land area (km^2)
Arizona	294 207
Florida	138 887
Montana	376 962
New Mexico	314 161
Oregon	248 608
Washington	172 119

(a) Write down the name of the state with the greatest land area. *[1 mark]*

(b) Round the number 314 161 to the nearest hundred. *[1 mark]*

(c) Write down the value of the digit 4 in the number 294 207 *[1 mark]*

(d) Work out the total land area of Oregon and Washington. *[1 mark]*

The land area of Vermont is 23 871 km^2

(e) Write the number 23 871 in words. *[1 mark]*

(a) ....

(b) ....

(c) ....

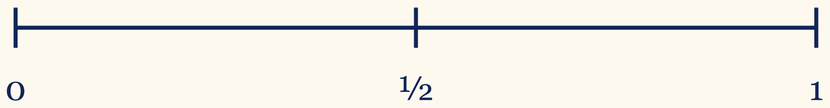
(d) km^2 ....

(e) ....

[Total 5 marks]

2. Astrid has four tiles.
There is a number on each tile.

1	2	4	5



3	3	5	6		

Astrid is going to pick at random one of these tiles.

(a) Circle the word in the box below that best describes the likelihood that Astrid will pick a tile with the number 5 on it.

impossible	unlikely	evens	likely	certain

[1 mark]

(b) On the probability scale below, mark with a cross (×) the probability that Astrid will pick a tile with a number less than 6 on it. *[1 mark]*

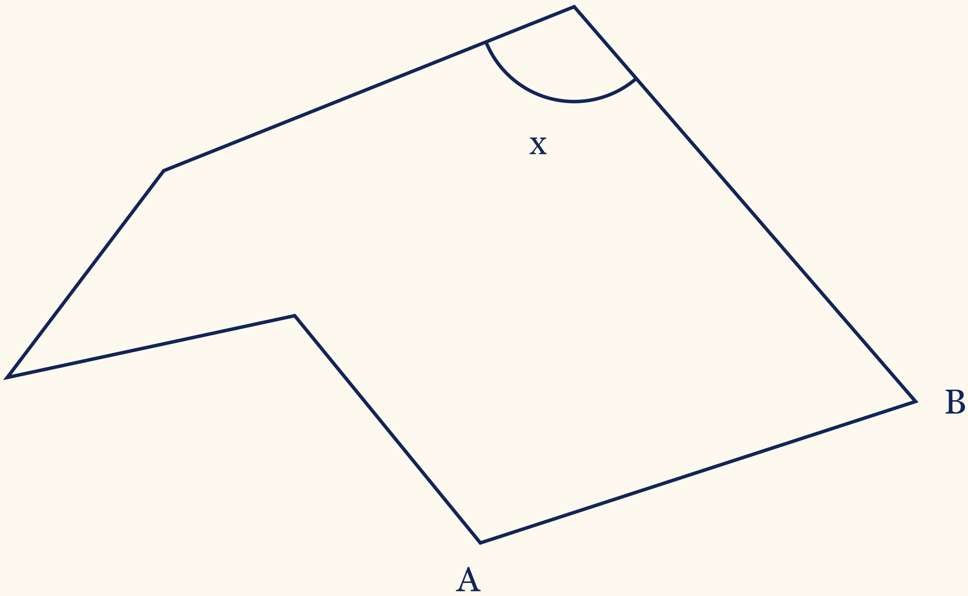
Meera has six tiles each with a number on it.
Four of these numbers are shown below.

When she picks at random one of the six tiles, the probability that she picks a tile with an even number on it is $\frac{1}{2}$

(c) Write a number on each of the blank tiles to show one possible set of six tiles that Meera could have. *[1 mark]*

[Total 3 marks]

3. The diagram shows a polygon with 6 sides.



(a) Measure the length of the side AB
Write down the units of your answer. *[2 marks]*

(b) Measure the size of the angle marked x *[1 mark]*

(c) Write down the mathematical name for a polygon with 6 sides. *[1 mark]*

(a) ....

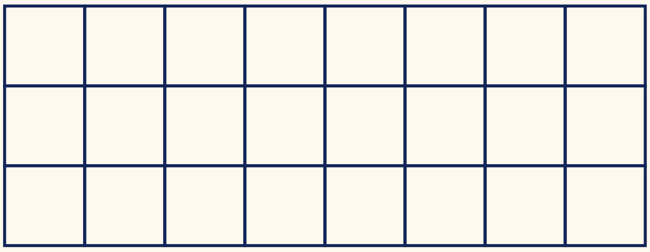
(a) ....

(b) degrees ....

(c) ....

[Total 4 marks]

4. (a) Write $\frac{9}{10}$ as a decimal. *[1 mark]*



(b) Write $\frac{3}{50}$ as a percentage. *[1 mark]*

Here is a shape made from identical squares.

(c) Shade $\frac{5}{8}$ of the shape. *[1 mark]*

(d) One of these fractions is not equivalent to $\frac{2}{5}$

Which one?

$\frac{20}{50}$ $\frac{4}{10}$ $\frac{2}{500}$ $\frac{10}{25}$ $\frac{60}{150}$ *[1 mark]*

A choir has 32 members.
8 of the members sing tenor.

(e) What fraction of the members do not sing tenor?
Give your fraction in its simplest form. *[2 marks]*

(a) ....

(b) % ....

(d) ....

(e) ....

[Total 6 marks]

5. (a) Write $4e \times 7f$ in its simplest form. *[1 mark]*

(b) Write $d \times d \times d \times d \times d$ in index form. *[1 mark]*

(c) Write $3a + 2k + a - 7k$ in its simplest form. *[2 marks]*

(d) Solve the equation $x + 6 = 15$ *[1 mark]*

(e) Solve the equation $2r - 9 = 14$ *[2 marks]*

(a) ....

(b) ....

(c) ....

(d) x = ...

(e) r = ...

[Total 7 marks]

6. Zubair has some sacks of flour and some tins of oil.
Each sack has the same weight.
Each tin has the same weight.

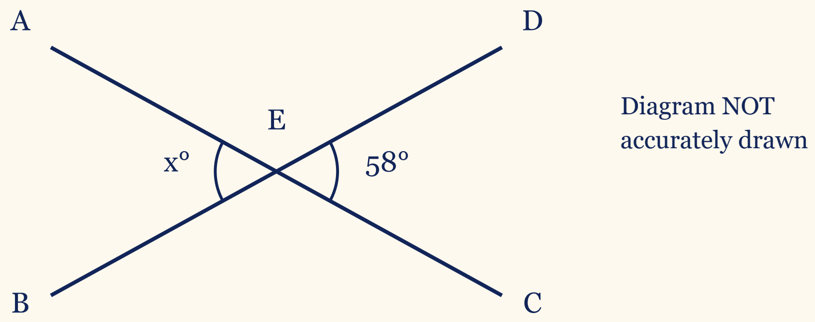
The total weight of 7 sacks and 2 tins is 27.3 kg
The total weight of 4 sacks and 2 tins is 16.8 kg

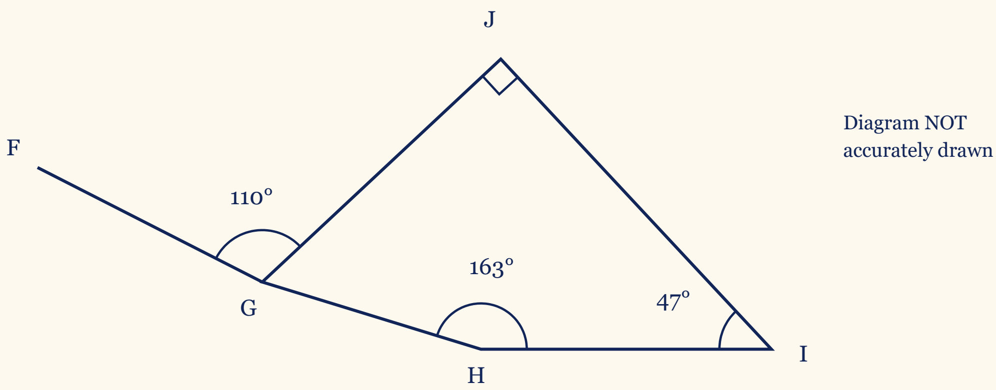
Work out the weight of one tin. *[4 marks]*

.... kg

[Total 4 marks]

7. AEC and BED are straight lines.





Mateo says that the value of x is 58

(a) Give a reason why Mateo is correct. *[1 mark]*

In the diagram, $GHIJ$ is a quadrilateral.

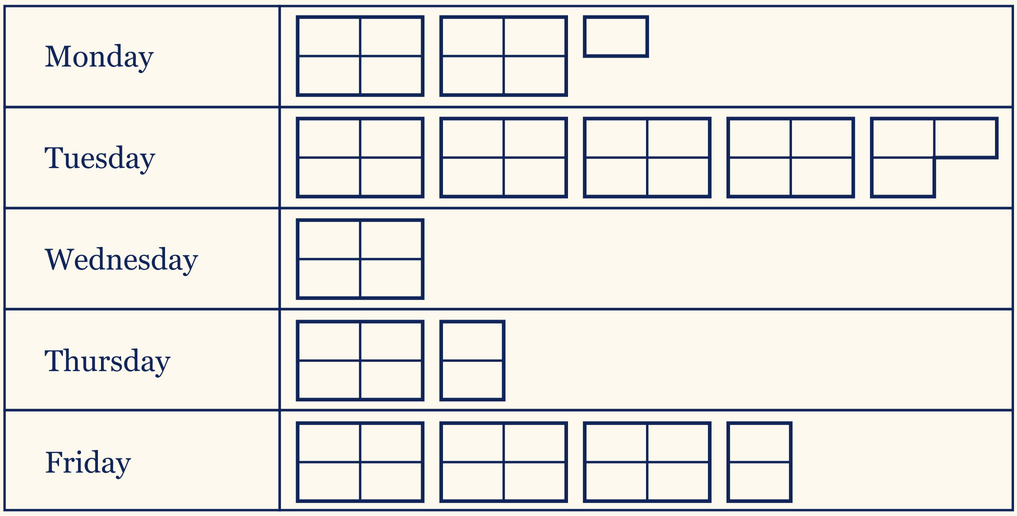
Elena says that FGH is a straight line.

(b) Show that Elena is wrong.
Give a reason for each stage of your working. *[4 marks]*

(a) ....

[Total 5 marks]

8. The pictogram gives information about the number of loaves of bread a bakery sold on each of five days.



The number of loaves sold on Friday was 16 more than the number of loaves sold on Thursday.

Work out the number of loaves sold on Monday. *[3 marks]*

....

[Total 3 marks]

9. The first four terms of a number sequence are shown below.

5 12 19 26

(i) Write down the fifth term of this sequence. *[1 mark]*

(ii) Explain how you found that term. *[1 mark]*

(i) ....

(ii) ....

[Total 2 marks]

10. A music shop keeps a record of how many guitars it sells each week.
The table gives information about the number of guitars sold in each of 30 weeks.

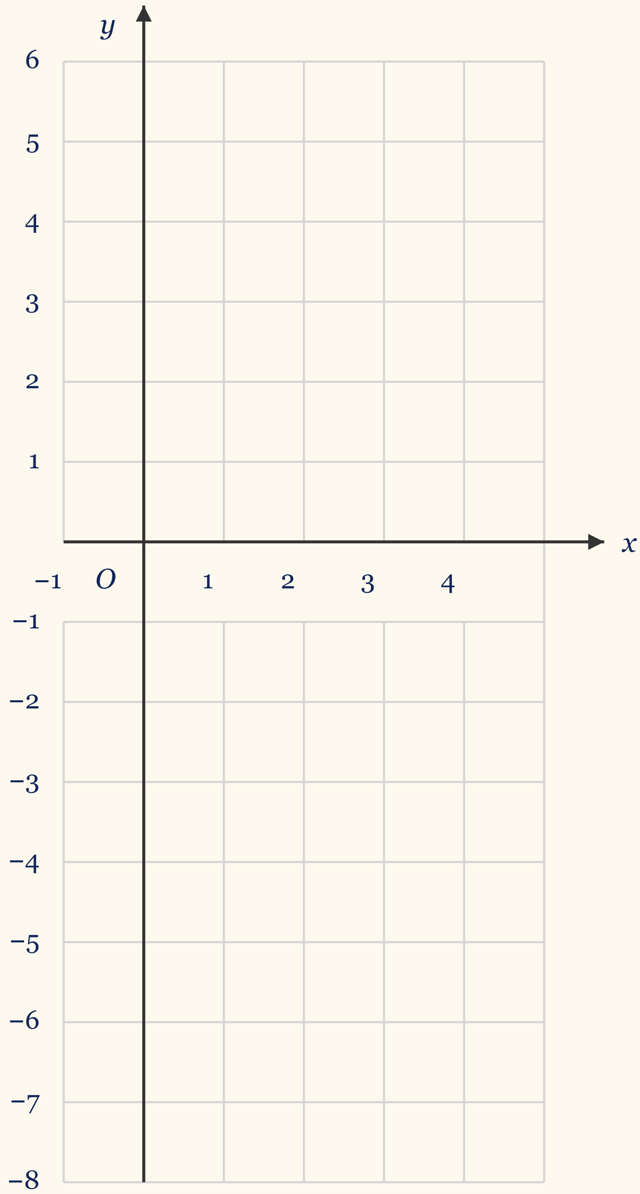
Number of guitars sold	Frequency
1	4
2	10
3	5
4	7
5	4

Work out the mean number of guitars sold per week. *[3 marks]*

....

[Total 3 marks]

11. Draw the graph of $y = 2x - 5$ on the grid below, taking values of x from -1 to 4. *[3 marks]*



[Total 3 marks]

12. A sports shop in a Swiss ski resort accepts payment in pounds (£) or in Swiss francs.

In the shop, a fleece costs £35 or 42 Swiss francs.
The cost of a rucksack is £54

Claire works out the cost of the rucksack in Swiss francs.
She uses the same exchange rate that was used for the cost of the fleece.

What is the cost of the rucksack in Swiss francs? *[3 marks]*

.... Swiss francs

[Total 3 marks]

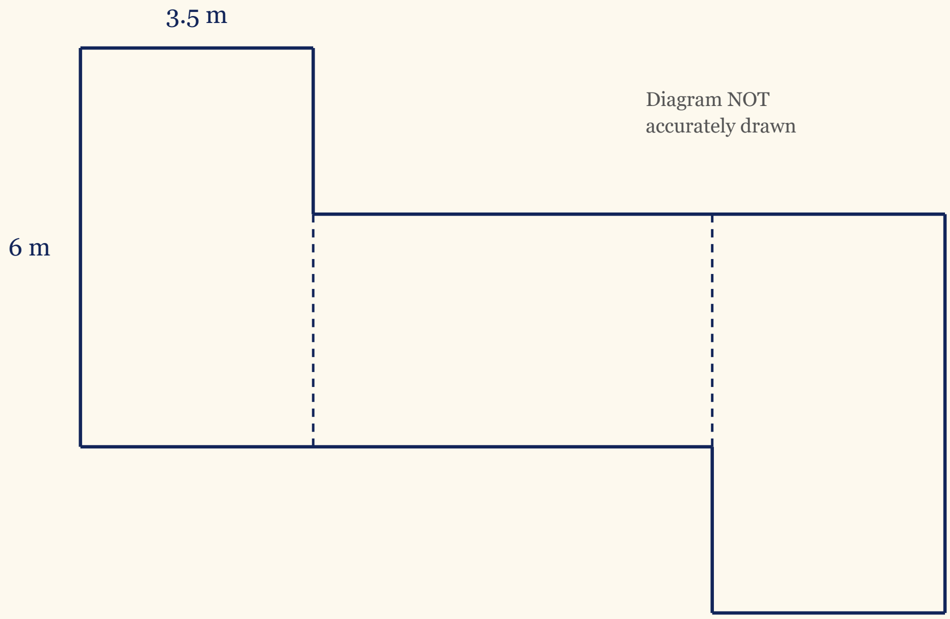
13. The radius of a circle is 6.4 cm.

Work out the area of this circle.
Give your answer correct to 3 significant figures.
[2 marks]

.... cm^2

[Total 2 marks]

14. The diagram shows a plan of a paddock made from three identical rectangles.



The length of each rectangle is 6 metres.
The width of each rectangle is 3.5 metres.

Martin puts a fence around the perimeter of the paddock.
He charges 7.60 euros for each 1 metre of fence.

Work out how much Martin charges in total for the fence. *[4 marks]*

.... euros

[Total 4 marks]

15. Fiona took the written test to qualify as a football referee.
Fiona scored 119 out of 140 marks on the test.

Work out Fiona's score as a percentage. *[2 marks]*

.... %

[Total 2 marks]

16. Work out the lowest common multiple (LCM) of 45 and 70. *[2 marks]*

....

[Total 2 marks]

17. The length of a footbridge is 142.8 m, correct to 1 decimal place.

(i) Write down the lower bound of the length of the footbridge. *[1 mark]*

(ii) Write down the upper bound of the length of the footbridge. *[1 mark]*

(i) ....

(ii) ....

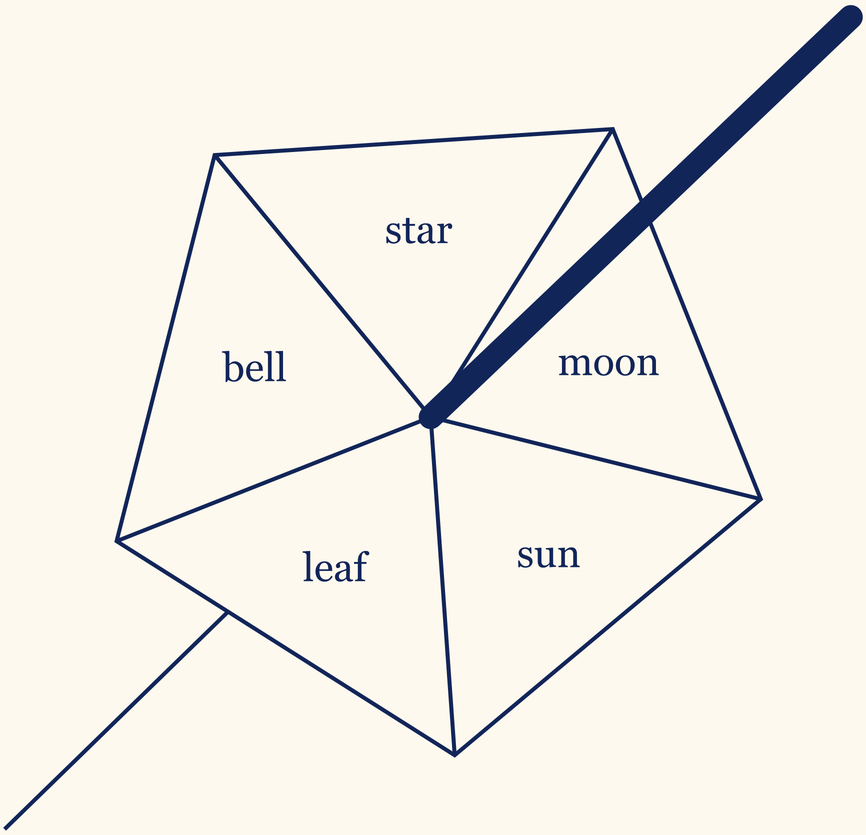
[Total 2 marks]

18. Show that $2\frac{1}{4} \times 1\frac{5}{7} = 3\frac{6}{7}$

You must show all your working. *[3 marks]*

[Total 3 marks]

19. Here is a biased 5-sided spinner.
When the spinner is spun, it can land on a star or on a moon or on a sun or on a leaf or on a bell.



The table gives information about the probability of the spinner landing on each symbol.

Symbol	star	moon	sun	leaf	bell
Probability	0.12	0.20	0.38	$4x$	x

Hannah spins the spinner once.

(a) Work out the probability that the spinner lands on a star or on a moon or on a sun. *[1 mark]*

Oliver spins the spinner 350 times.

(b) Work out an estimate for the number of times the spinner lands on a leaf. *[4 marks]*

(a) ....

(b) ....

[Total 5 marks]

20. $\mathcal{E} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\}$
$A = \{2,\ 4,\ 6,\ 8,\ 10,\ 12\}$
$B = \{3,\ 6,\ 9,\ 12\}$
$C = \{1,\ 3,\ 5,\ 7,\ 9,\ 11\}$

(a) Write down all the members of the set
(i) $A \cup B$
(ii) B' *[2 marks]*

$\mathcal{E}$	$\cap$	$\cup$	$\varnothing$	$\in$	$\notin$

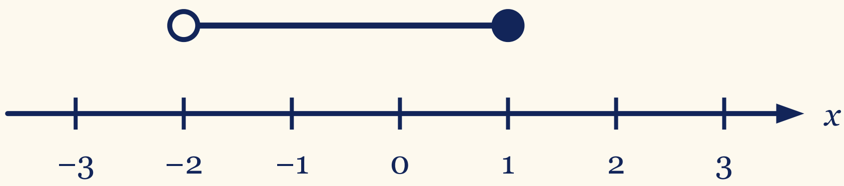
(b) Complete each statement below by writing one symbol from the box on the dotted line, so that the statement is true.
(i) $A \cap C$ = ...
(ii) 13 .... $\mathcal{E}$ *[2 marks]*

(a)(i) ....

(a)(ii) ....

[Total 4 marks]

21. (a) The number line above shows an inequality.
Write down this inequality. *[2 marks]*



(b) Solve the inequality $7a - 5 \leq 3a + 28$
You must show clear algebraic working. *[2 marks]*

(a) ....

(b) ....

[Total 4 marks]

22. A coach travels a distance of 1262 km from Lahore to Karachi.

The coach takes $17\frac{1}{2}$ hours.

(a) Work out the average speed of the coach.
Give your answer, in km/h, correct to the nearest whole number. *[2 marks]*

(b) Change a speed of $50x$ metres per second to a speed in kilometres per hour. *[3 marks]*

(a) km/h ....

(b) km/h ....

[Total 5 marks]

23. (a) Multiply out the brackets in $x(x-3)$ *[1 mark]*

(b) Rearrange the formula $m = \frac{t+4}{5}$ to make t the subject. *[2 marks]*

(c) Simplify $a^6 \times a^{10}$ *[1 mark]*

(d) Simplify $\frac{c^{30}}{c^{12}}$ *[1 mark]*

(e) (i) Factorise $y^2 - 10y + 21$ *[2 marks]*

(ii) Hence solve $y^2 - 10y + 21 = 0$ *[1 mark]*

(a) ....

(b) ....

(c) ....

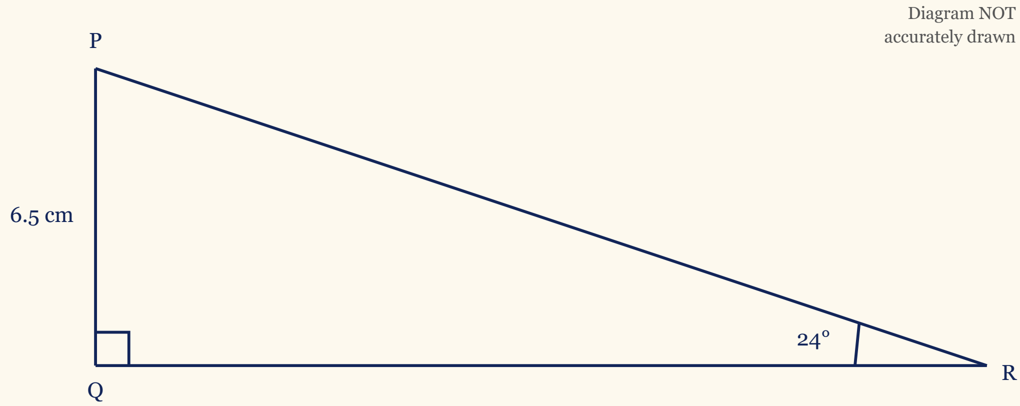
(d) ....

(e)(i) ....

(e)(ii) ....

[Total 8 marks]

24. Triangle PQR is shown in the diagram.
The right angle is at Q.

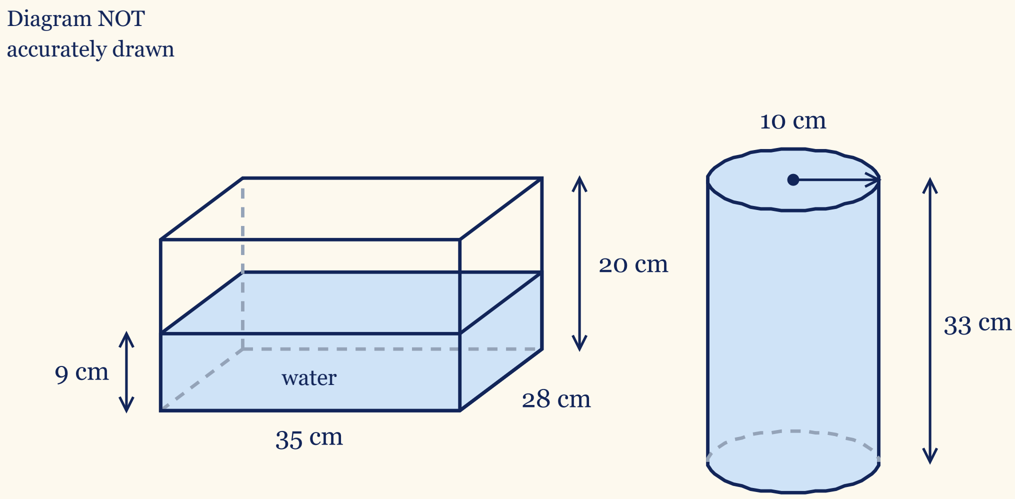


Work out the length of QR.
Give your answer correct to 3 significant figures. *[3 marks]*

.... cm

[Total 3 marks]

25. The diagram shows two rainwater tanks at a plant nursery.
One tank is a cuboid and the other tank is a cylinder.



The cuboid tank measures 35 cm by 28 cm by 20 cm
The surface of the water in the cuboid tank is 9 cm above the base of that tank.

The cylindrical tank has a radius of 10 cm and a height of 33 cm
The cylindrical tank is completely full of water.

Rowena is going to pour all the water from the cylindrical tank into the cuboid tank.

Show that the cuboid tank will not be completely full of water. *[3 marks]*

[Total 3 marks]

26. Wei puts money into two savings plans. Each plan runs for 2 years.
He puts \$2500 into the Harbour plan and \$3000 into the Meridian plan.

Harbour plan
Invests \$2500
amount invested : interest after 2 years = 20 : 3

Meridian plan
Invests \$3000
4% per year compound interest for 2 years

Wei receives more interest from the Harbour plan than from the Meridian plan.

How much more? *[5 marks]*

\$ ....

[Total 5 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Six land areas, in km^2: Arizona $294\,207$, Florida $138\,887$, Montana $376\,962$, New Mexico $314\,161$, Oregon $248\,608$, Washington $172\,119$.
Vermont has a land area of $23\,871\ \text{km}^2$.
Every land area in the table is a six-digit whole number, so every comparison is a place-value comparison.

Find

(a) The state with the greatest land area. (b) $314\,161$ to the nearest hundred. (c) What the digit 4 is worth in $294\,207$. (d) The two land areas added together. (e) $23\,871$ written out in words.

Plan the Solution

- Line the six land areas up by place value and read from the left, so the largest is found by comparing digits and nothing is added.
- For the nearest hundred, look only at the tens digit: 5 or more sends the hundreds up, and anything less leaves them alone.
- For place value, name the column the digit is sitting in, then multiply the digit by that column.
- For the total, add in columns from the right, carrying into the next column whenever a column reaches ten.
- For the words, split the number at the thousands and write out each block in turn.

Worked Solution [5 marks]

> Rule - Place value: in a whole number the columns, reading from the right, are units, tens, hundreds, thousands, ten thousands and hundred thousands. A digit is worth the digit multiplied by its own column, and comparing, rounding, adding and writing in words all read off those same columns.

Step 1: (a) Compare the six land areas

$$376\,962 > 314\,161 > 294\,207 > 248\,608 > 172\,119 > 138\,887$$

(Reason: every land area has six digits, so the hundred thousands column decides first. Montana and New Mexico both have 3 there, and Montana then wins on the ten thousands, 7 against 1)

Step 2: (b) Round $314\,161$ to the nearest hundred

$$314\,100 < 314\,161 < 314\,200$$
$$314\,161 - 314\,100 = 61$$
$$314\,200 - 314\,161 = 39$$
$$314\,161 \approx 314\,200$$

(Reason: the tens digit is 6, which is 5 or more, so the hundreds go up. The two gaps say the same thing a second way: 39 is smaller than 61, so the higher hundred is the nearer one)

Step 3: (c) Read the column the 4 is sitting in

$$294\,207 = 200\,000 + 90\,000 + 4000 + 200 + 7$$
$$4 \times 1000 = 4000$$

(Reason: counting from the right the columns are units, tens, hundreds, thousands, ten thousands and hundred thousands, so this 4 is in the thousands column and is worth 4 thousands. The digit is a 4 wherever it sits; the column is what gives it its value)

Step 4: (d) Add the two land areas in columns

$$248\,608 + 172\,119 = 420\,727$$

(Reason: add from the right. The units give $8 + 9 = 17$, so write 7 and carry 1 into the tens. The thousands then give $8 + 2 = 10$, so write 0 and carry again into the ten thousands)

Step 5: (e) Split $23\,871$ into thousands and the rest

$$23\,871 = 23 \times 1000 + 871$$

(Reason: say the two blocks in turn: 23 thousand, then 871 as eight hundred and seventy one. The comma in the written answer sits exactly where the number splits)

(a) Montana

(b) $314\,200$

(c) 4 **thousands, or** 4000

(d) $420\,727$ km^2

(e) Twenty three thousand, eight hundred and seventy one

Verification

✓ **Check 1:** Part (a): take the next largest land area, New Mexico's, away from Montana's. A positive difference means Montana really is ahead. $376\,962 - 314\,161 = 62\,801$, which is positive.

✓ **Check 2:** Part (b): measure the distance from $314\,161$ to each of the two hundreds either side of it. 39 up against 61 down, so $314\,200$ is the nearer hundred.

✓ **Check 3:** Part (c): replace the 4 with a zero and subtract. Whatever is left is what the 4 was worth. $294\,207 - 290\,207 = 4000$

✓ **Check 4:** Part (d): undo the addition by taking Washington's land area back off the total. $420\,727 - 172\,119 = 248\,608$, which is Oregon's land area.

✓ **Check 5:** Part (d) again, roughly: round both land areas to the nearest thousand and add those instead. $249\,000 + 172\,000 = 421\,000$, only 273 above the exact total.

✓ **Check 6:** Part (e): read the written answer back as a number. $23\,000 + 800 + 71 = 23\,871$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	Montana	✓
(b)	**B1**	$314\,200$	✓
(c)	**B1**	4 thousands. Accept 4000, thousands	✓
(d)	**B1**	$420\,727$	✓
(e)	**B1**	Twenty three thousand, eight hundred (and) seventy one	✓

Full marks: 5/5

Question 2 - Exam Solution

Understanding the Question

Given

Astrid's four tiles show 1, 2, 4 and 5, and one tile is picked at random.
Meera has six tiles: four of them show 3, 3, 5 and 6, and two are blank.

Find

(a) The word that best describes the likelihood of picking the tile showing 5.
(b) Where the cross goes on the probability scale for a number less than 6.

For Meera's tiles, $P(\text{even}) = \dfrac{1}{2}$.

(c) A number for each of the two blank tiles.

Plan the Solution

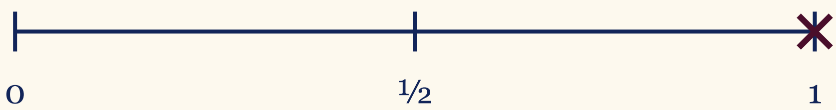
- Count the tiles that succeed, then write the probability as a fraction of the total number of tiles.
- Measure that fraction against 0, $\dfrac{1}{2}$ and 1 to choose the word in (a) and the place on the scale in (b).
- For (c), turn the probability into a number of tiles first, then count the even numbers already on show.

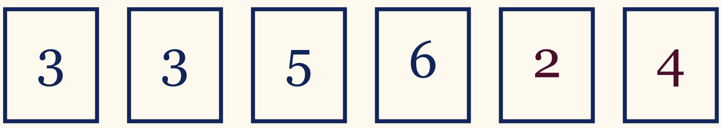
Worked Solution [3 marks]

> Rule - Probability of an event: $P(\text{event}) = \dfrac{\text{successful outcomes}}{\text{all outcomes}}$, which runs from 0 (impossible) to 1 (certain).

Step 1: (a) count the tiles that show 5

$$P(5) = \frac{1}{4} = 0.25$$





(Reason: Only one of the four tiles has 5 on it, so one outcome out of four is successful.)

Step 2: (a) measure 0.25 against a half

$$0 < 0.25 < 0.5$$

(Reason: The chance is bigger than 0, so it is not impossible, and smaller than $\frac{1}{2}$, so it is less than evens. That band is the word unlikely.)

Step 3: (b) count the tiles showing a number less than 6

$$P(\text{less than } 6) = \frac{4}{4} = 1$$

(Reason: Every one of $1, 2, 4$ and 5 is smaller than 6, so no tile can fail. An event that cannot fail is certain, and certain is the right-hand end of the scale.)

Step 4: (c) turn the probability into a number of tiles

$$\frac{1}{2} \times 6 = 3$$

(Reason: A probability of $\frac{1}{2}$ out of six tiles means 3 of the six tiles must have an even number on them.)

Step 5: (c) count the even numbers already there

$$3 - 1 = 2$$

(Reason: Of $3, 3, 5$ and 6 only the 6 is even, so two more even numbers are needed - and there are exactly two blank tiles. Writing 2 and 4 on them is one possible set.)

(a) unlikely

(b) a cross at 1, the right-hand end of the scale

(c) 2 and 4 (any two even numbers)

Verification

✓ **Check 1:** (a) from the other side. Three of the four tiles are not the 5, and the two probabilities have to add to 1. The smaller of the two shares is the unlikely one. $0.25 + 0.75 = 1$

✓ **Check 2:** (b) by listing. The numbers smaller than 6 are $1, 2, 4$ and 5 - all four of the tiles - so no tile is left out. $\frac{4}{4} = 1$

✓ **Check 3:** (c) by counting the finished set. Meera's six tiles would read $3, 3, 5, 6, 2, 4$, and three of those numbers are even. $\frac{3}{6} = \frac{1}{2}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	unlikely	✓
(b)	**B1**	× at 1	✓

Step	Mark	Description	Got it?
(c)	**B1**	2 numbers which are even	✓

Full marks: 3/3

Question 3 - Exam Solution

Understanding the Question

Given

A polygon with 6 sides, drawn accurately, with two of its vertices labelled A and B. One angle of the polygon is marked x, with a small arc drawn inside it.
No length and no angle is printed on the figure, so both readings have to be taken off the drawing itself.

Find

(a) the length of AB, and the units it is measured in (b) the size of the angle marked x (c) the mathematical name for a polygon with 6 sides

Plan the Solution

- Nothing here is worked out. Parts (a) and (b) are read off the page with a ruler and a protractor, and part (c) is recall.
- For (a), lay the ruler along AB with its zero mark exactly on A, and read the mark that B falls on. Read to the nearest millimetre.
- Then write the units beside the number. The number on its own is only half of that answer, which is why part (a) carries two marks and not one.
- For (b), put the centre of the protractor on the vertex where x is marked, lay the zero line along one arm of the angle, and read the scale where the other arm crosses it.
- A protractor carries two scales, running in opposite directions. The angle marked x is obtuse - it opens wider than a right angle - so its reading must be more than 90, and that is which of the two scales to take.
- For (c), count the sides round the outline and name the polygon from that count.

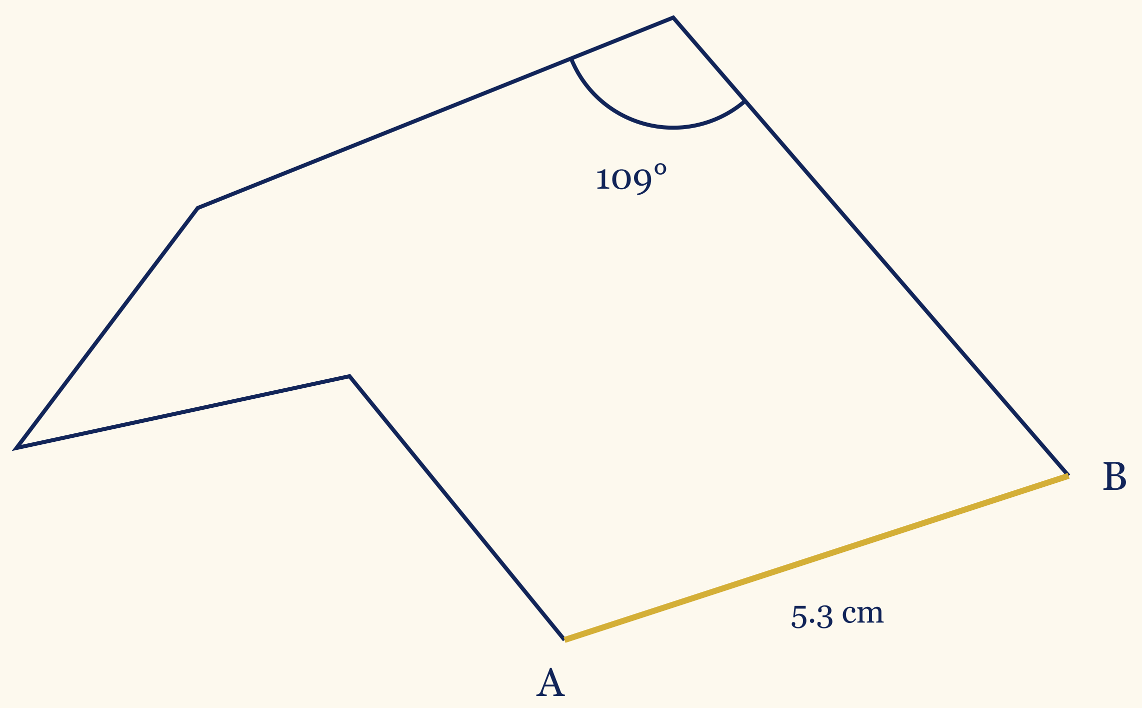
Worked Solution [4 marks]

Ruler: 10 mm make 1 cm, so a reading of 5 cm and 3 mm is 5.3 cm, or 53 mm.
Protractor: an acute angle reads under 90 and an obtuse angle reads over it, and the two

scales at any one position always add to 180. Polygons are named by their number of sides: 3 triangle, 4 quadrilateral, 5 pentagon, 6 hexagon, 7 heptagon.

(a) Measure AB with a ruler

$AB = 5 \text{ cm } 3 \text{ mm}$



(Reason: With the zero mark on A, the vertex B falls between the 5 cm and 6 cm marks, three of the small millimetre marks past the 5. Read the small marks rather than guessing at the gap.)

(a) Write the reading with its units

$5 \text{ cm} = 50 \text{ mm}$

$50 + 3 = 53$

(Reason: The two marks here are for two different things: one for the value and one for the units. Centimetres and millimetres are both accepted, so 5.3 cm and 53 mm are worth the same. A bare 5.3 with nothing after it is not.)

(b) Measure the angle marked x with a protractor

$x = 109°$

(Reason: The centre of the protractor goes on the vertex the arc is drawn at, and the zero line goes along one arm. Both scales are printed on the protractor and only one of them is the answer: the arms open wider than a right angle, so the reading to take is the one above 90, not the one below it.)

(c) Name the polygon

6 sides $\rightarrow$ hexagon

(Reason: Count the straight edges round the outline: there are 6 of them. A polygon is named by that count, and the prefix hexa- means six. The corner that turns in towards the middle of the shape does not change the name - a polygon may have a corner like that and still be a hexagon.)

(a) 5.3 cm (or 53 mm)

(b) $x = 109°$

(c) hexagon

Verification

✓ **Check 1:** Measure the same side in millimetres instead of centimetres, then convert. The ruler reads 53 mm, and dividing by 10 turns millimetres into centimetres. $\frac{53}{10} = 5.3$ cm, which is the reading part (a) gave

✓ **Check 2:** Read the protractor's other scale at the same position. The two scales run in opposite directions, so the two readings must account for a straight line between them. $109 + 71 = 180$, so the obtuse reading and the acute one fit together, and the obtuse one is the angle marked

✓ **Check 3:** Name the shape from its angles rather than from its sides. The angles inside the drawn figure add up to 720, and the angles of a polygon with n sides add up to $(n - 2) \times 180$. $(6 - 2) \times 180 = 720$, so the figure has 6 sides, and the name for 6 sides is hexagon

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B2**	5.3 cm or 53 mm or 5 cm 3 mm	✓
(a)	**B1**	for 5.3 (allow 5.1 - 5.5) or 53 (allow 51 - 55) or cm with a value from 4.8 - 5.8 or mm with a value from 48 - 58	✓
(b)	**B1**	109	✓
(c)	**B1**	hexagon	✓

Full marks: 4/4

Question 4 - Exam Solution

Understanding the Question

Given

Two fractions to rewrite: $\frac{9}{10}$ as a decimal, and $\frac{3}{50}$ as a percentage.

A shape made from identical squares, 8 across and 3 down.

Five fractions: $\frac{20}{50}, \frac{4}{10}, \frac{2}{500}, \frac{10}{25}$ and $\frac{60}{150}$. Four of them are equivalent to $\frac{2}{5}$ and one is not.

A choir of 32 members, of whom 8 sing tenor.

Find

(a) $\frac{9}{10}$ written as a decimal (b) $\frac{3}{50}$ written as a percentage (c) how much of the shape to shade so that $\frac{5}{8}$ of it is shaded (d) the one fraction that is not equivalent to $\frac{2}{5}$ (e) the fraction of the members who do not sing tenor, in its simplest form

Plan the Solution

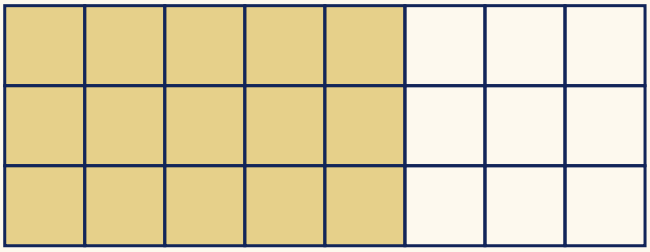
- For (a), read the fraction as a number of tenths. Tenths are the first place after the decimal point, so a fraction with 10 underneath needs no working at all once that is seen.
- For (b), a percentage is a number of hundredths, so turn the fraction into hundredths. 50 doubles to 100, and whatever is done to the bottom is done to the top.
- For (c), count the squares first. The denominator 8 says how many equal parts the shape is cut into, and the numerator 5 says how many of those parts to shade.
- For (d), cancel each fraction down and see which one does not become $\frac{2}{5}$. Cross-multiplying is a second way to test the same thing.
- For (e), subtract first and simplify second. Find how many members do not sing tenor, write that over 32, then cancel by the largest number that divides both.
- Every part here is about the same idea: multiplying or dividing the top and the bottom of a fraction by the same number leaves its value unchanged.

Worked Solution [6 marks]

Equivalent fractions: multiplying or dividing top and bottom by the same number does not change a fraction's value, so $\frac{2}{5} = \frac{4}{10} = \frac{20}{50}$. Decimals: the first place after the point is tenths and the second is hundredths. Percentages: per cent means out of 100, so a fraction becomes a percentage once its denominator is 100, or by multiplying it by 100. A fraction of an amount: divide by the denominator to get one part, then multiply by the numerator. Simplest form: cancel until the only number dividing top and bottom is 1.

(a) Read $\frac{9}{10}$ as tenths

$$\frac{9}{10} = 0.9$$



(Reason: The first column after the decimal point is the tenths column, so 9 tenths is a 9 written in that column. Said as a division instead: the fraction bar means divide, and dividing by 10 moves the digit one place to the right, out of the units and into the tenths.)

(b) Scale $\frac{3}{50}$ to a denominator of 100

$$\frac{3}{50} = \frac{3 \times 2}{50 \times 2} = \frac{6}{100}$$

(Reason: Per cent means out of 100, so the useful denominator is 100. 50 doubles to 100, so the 3 on top doubles as well. Doubling both parts leaves the value of the fraction alone.)

(b) Write those hundredths as a percentage

$$\frac{3}{50} \times 100 = 6$$

(Reason: Six hundredths is 6 per cent, so the answer is read straight off the scaled fraction. Multiplying the original fraction by 100 is the same conversion done in one line, and it gives the same 6.)

(c) Count the squares in the shape

$$8 \times 3 = 24$$

(Reason: The shape is a rectangle of identical squares, 8 across and 3 down, so there are 24 of them altogether. Nothing can be shaded until this total is known, because the fraction is a fraction of these squares.)

(c) Work out how many squares make $\frac{5}{8}$

$$\frac{24}{8} = 3$$

$5 \times 3 = 15$

(Reason: The denominator 8 cuts the shape into 8 equal parts, and 24 squares shared into 8 parts is 3 squares per part. The numerator 5 says to take five of those parts. A column of the grid is exactly one of them, so shading five whole columns is the tidiest way to show it, though any 15 squares would earn the mark.)

(d) Cancel each fraction and compare it with $\frac{2}{5}$

$$\frac{20}{50} = \frac{2 \times 10}{5 \times 10} = \frac{2}{5}$$
$$\frac{4}{10} = \frac{2 \times 2}{5 \times 2} = \frac{2}{5}$$
$$\frac{10}{25} = \frac{2 \times 5}{5 \times 5} = \frac{2}{5}$$
$$\frac{60}{150} = \frac{2 \times 30}{5 \times 30} = \frac{2}{5}$$

(Reason: A fraction is equivalent to $\frac{2}{5}$ when it is $\frac{2}{5}$ with both parts multiplied by the same number. Four of the five are, with the multiplier $10, 2, 5$ and 30 in turn.)

(d) Name the one that is left

$$\frac{2}{500} = \frac{1}{250}$$

(Reason: In $\frac{2}{500}$ the denominator has been multiplied by 100 but the numerator has not, so the two parts have not been scaled by the same number and the value has changed. It cancels to $\frac{1}{250}$, which is nowhere near $\frac{2}{5}$.)

(e) Count the members who do not sing tenor

$32 - 8 = 24$

(Reason: The question asks about the members who do not sing tenor, so those are counted first: the whole choir less the 8 who do. Answering with the 8 themselves is the one mistake this part is testing for.)

(e) Write that as a fraction of the choir and simplify it

$$\frac{24}{32} = \frac{3 \times 8}{4 \times 8} = \frac{3}{4}$$

(Reason: A fraction is the part over the whole, so it is 24 over 32. Simplest form means cancelling by the largest number that divides both, and that is 8: it goes into 24 three times and into 32 four times.

Cancelling by 4 instead gives $\frac{6}{8}$, which is the right value but not yet the simplest form the question asks for.)

(a) 0.9

(b) 6%

(c) 15 **squares shaded, which is** 5 **of the** 8 **columns**

(d) $\dfrac{2}{500}$

(e) $\dfrac{3}{4}$

Verification

✓ **Check 1:** Turn part (a)'s decimal back into a fraction. One decimal place is tenths, so multiplying by 10 should give the numerator the question started with. $0.9 \times 10 = 9$, so the decimal is $\dfrac{9}{10}$ again

✓ **Check 2:** Take part (b)'s percentage back the other way. 6 per cent is 6 hundredths, and both parts of $\dfrac{6}{100}$ halve. $\dfrac{6}{100} = \dfrac{3 \times 2}{50 \times 2} = \dfrac{3}{50}$, the fraction the question started from

✓ **Check 3:** Count what is left unshaded in part (c) instead of what is shaded. Three eighths of the shape should be left, and three eighths of 24 is 9. $24 - 15 = 9$ squares left, and $\dfrac{9}{24} = \dfrac{3}{8}$, so the part shaded really is five eighths

✓ **Check 4:** Test part (d) by cross-multiplying rather than by cancelling. For two equivalent fractions the two cross-products match. $20 \times 5 = 100$ and $2 \times 50 = 100$, which match, and the same happens for $\dfrac{4}{10}$, $\dfrac{10}{25}$ and $\dfrac{60}{150}$; but $2 \times 5 = 10$ against $2 \times 500 = 1000$, which do not

✓ **Check 5:** Work part (e) backwards. Take three quarters of the choir and see whether it is the number who do not sing tenor. $\dfrac{3}{4} \times 32 = 24$, and $32 - 24 = 8$, which is the number who do sing tenor

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	0.9	✓
(b)	**B1**	6	✓
(c)	**B1**	15 squares shaded	✓
(d)	**B1**	$\frac{2}{500}$	✓
(e)	**M1**	eg $\frac{32-8}{32}$ oe or $1-\frac{8}{32}$ oe or $\frac{32}{32}-\frac{8}{32}$ oe or $\frac{24}{32}$ or 0.75 or $\frac{1}{4}$	✓
(e)	**A1**	$\frac{3}{4}$	✓

Full marks: 6/6

Question 5 - Exam Solution

Understanding the Question

Given

Two products to simplify: $4e \times 7f$ and $d \times d \times d \times d \times d$.
One sum with two different letters in it: $3a + 2k + a - 7k$.
Two equations to solve: $x + 6 = 15$ and $2r - 9 = 14$.

Find

(a) $4e \times 7f$ written in its simplest form (b) $d \times d \times d \times d \times d$ written in index form (c) $3a + 2k + a - 7k$ written in its simplest form (d) the value of x (e) the value of r

Plan the Solution

- Parts (a) and (b) are products, so nothing is added anywhere in them. In (a) the numbers multiply and the letters are written side by side. In (b) the same letter is multiplied by itself, and that is counted with an index rather than a coefficient.
- Part (c) is a sum, so here things do add, but only terms carrying the SAME letter may be put together. Deal with the a terms and the k terms separately, and let each term keep the sign printed in front of it.
- Parts (d) and (e) are solved by doing the same thing to both sides until the letter stands alone. In (d) one operation has been applied to x; in (e) two have been applied to r, so

they are undone in reverse order.

- A calculator is allowed, but every part here is quicker by hand than by keying it in. What the calculator is worth is the check at the end.

Worked Solution [7 marks]

> Products: multiply the numbers, then write the letters after them, so $3p \times 5q = 15pq$. Index form: an index COUNTS how many times a letter is multiplied by itself, so $p \times p \times p = p^3$. Like terms: only terms with exactly the same letter can be combined, and each term carries the sign written in front of it. Solving: do the same thing to both sides, undoing the operations in the reverse of the order they were applied.

(a) Multiply the numbers, then write the letters together

$4e \times 7f = 4 \times 7 \times e \times f$

$4 \times 7 = 28$

$4e \times 7f = 28ef$

(Reason: Multiplication may be done in any order, so the two numbers can be brought together and the two letters can be brought together. 4 times 7 is 28, and $e \times f$ is written ef with no sign between the letters. Because the letters are different, there is nothing further to collect and $28ef$ is as simple as it gets.)

(b) Count how many times d is multiplied by itself

$d \times d \times d \times d \times d = d^5$

(Reason: An index is a count, not a total: it records how many copies of the letter are being multiplied. There are five copies of d here, so the index is 5. Read as the index law, each d is d^1 and multiplying powers of the same letter adds the indices, which gives $1 + 1 + 1 + 1 + 1 = 5$ again. Answering $5d$ is the slip this part is testing for: $5d$ means $d + d + d + d + d$, which is addition, not multiplication.)

(c) Collect the a terms and the k terms separately

$3a + a = 4a$

$2k - 7k = -5k$

$3a + 2k + a - 7k = 4a - 5k$

(Reason: Only like terms combine. $3a$ and a are both counts of a, and a letter written on its own means one of it, so $3 + 1 = 4$ of them. Each term keeps the sign printed in front of it, so the second pair is $2k$ take away $7k$, and $2 - 7 = -5$ leaves $-5k$. The two totals cannot then be added to each other, because a and k stand for different things, so the answer is two terms and not one.)

(d) Undo the $+6$

$x + 6 = 15$

$x = 15 - 6$

$x = 9$

(Reason: The equation says 6 has been added to x. Taking 6 off both sides undoes that and leaves x by itself on the left, with $15 - 6$ on the right. Both sides were treated the same way, so the equation still balances.)

(e) Undo the -9 first, then undo the $\times 2$

$2r - 9 = 14$

$2r = 14 + 9 = 23$

$r = \dfrac{23}{2} = 11.5$

(Reason: Two things have been done to r: it was multiplied by 2 and then 9 was taken off. They are undone in the reverse order, so the 9 goes back on first by adding 9 to both sides, and only after that are both sides halved. Halving 23 does not give a whole number, and it does not have to: 11.5 is a perfectly good solution, and the mark scheme accepts it as 11.5, $\frac{23}{2}$ or $11\frac{1}{2}$.)

(a) $28ef$

(b) d^5

(c) $4a - 5k$

(d) $x = 9$

(e) $r = 11.5$

Verification

✓ **Check 1:** Test parts (a) and (b) with numbers. Two expressions that are really the same give the same value whatever is put in, so take $e = 2$, $f = 5$ and $d = 2$. $4 \times 2 \times 7 \times 5 = 280$ and $28 \times 2 \times 5 = 280$; $2 \times 2 \times 2 \times 2 \times 2 = 32$ and $2^5 = 32$, while the wrong answer $5d$ would give only $5 \times 2 = 10$

✓ **Check 2:** Test part (c) the same way, with $a = 2$ and $k = 3$. A sign error in the k terms is the likeliest mistake, and it would show here at once. $3 \times 2 + 2 \times 3 + 2 - 7 \times 3 = -7$ and $4 \times 2 - 5 \times 3 = -7$

✓ **Check 3:** Put each solution back into the equation it came from. A solution is correct exactly when it makes the left-hand side equal the right-hand side. $9 + 6 = 15$ and $2 \times 11.5 - 9 = 14$

✓ **Check 4:** Solve part (e) a second way, halving every term first instead of moving the 9. A different order of working should reach the same value. $r - 4.5 = 7$, so $r = 7 + 4.5 = 11.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	$28ef$	✓
(b)	**B1**	d^5	✓
(c)	**B2**	$4a - 5k$	✓
(c)	**(B1)**	for $4a$ or $-5k$ or $4a + -5k$	✓
(d)	**B1**	9	✓
(e)	**M1**	$2r = 14 + 9$ or $2r = 23$ oe or $\dfrac{14+9}{2}$ or $2 \times 11.5 - 9 = 14$, for a correct first step or a correct calculation for r	✓
(e)	**A1**	for 11.5 or $\dfrac{23}{2}$ or $11\dfrac{1}{2}$	✓

Full marks: 7/7

Question 6 - Exam Solution

Understanding the Question

Given

Two kinds of item, and every item of one kind has the same weight: sacks of flour, and tins of oil.
7 sacks and 2 tins weigh 27.3 kg altogether.
4 sacks and 2 tins weigh 16.8 kg altogether.

Find

The weight of ONE tin, in kilograms.

Plan the Solution

- Give each unknown weight a letter, so that each printed total becomes an equation. Let s be the weight of one sack in kilograms and t the weight of one tin in kilograms.
- Look at what the two totals have in COMMON. Both carry 2 tins, so the tins contribute the same amount to each. Taking the smaller total away from the larger therefore removes the

tins completely and leaves only sacks.

- What is left is the weight of the extra sacks: $7 - 4 = 3$ of them. From that comes the weight of one sack.
- Put the weight of one sack back into either printed total. The sacks in it can then be accounted for, and whatever is left over is the weight of the 2 tins. Halve it for one tin.
- A calculator is allowed, so keep the decimals as they are printed. Nothing here needs converting into grams.

Worked Solution [4 marks]

> Rule - Elimination: when two totals contain the SAME number of one item, subtracting one total from the other removes that item entirely, and what is left is a statement about the other item alone. Here both totals contain 2 tins, so the subtraction leaves sacks and nothing else.

Step 1: write each printed total as an equation

$$7s + 2t = 27.3$$
$$4s + 2t = 16.8$$

(Reason: Every sack weighs the same, so 7 of them weigh $7s$; every tin weighs the same, so 2 of them weigh $2t$. Adding those two amounts gives the printed total of 27.3 kg, and the second total is written the same way. Both letters stand for a weight in kilograms, so both equations are in the same units.)

Step 2: subtract, because both totals carry the same 2 tins

$$(7s + 2t) - (4s + 2t) = 27.3 - 16.8$$
$$27.3 - 16.8 = 10.5$$
$$3s = 10.5$$

(Reason: The 2 tins weigh the same in both totals, so subtracting cancels them: $2t - 2t = 0$. The sacks do not cancel, because there are 7 in one total and only 4 in the other, leaving $7 - 4 = 3$ of them. So the whole gap between the two printed totals, 10.5 kg, is the weight of 3 sacks.)

Step 3: divide by 3 to reach one sack

$$3s = 10.5$$
$$s = \frac{10.5}{3} = 3.5$$

(Reason: The 10.5 kg is shared equally between the 3 extra sacks, so it is divided by 3 and not by either of the printed counts. Dividing by the 7 of the first total is the slip this step is easiest to make: the 10.5 kg is not the weight of 7 sacks, it is the weight of the 3 that the first load has and the second does not.)

Step 4: take the 4 sacks off the second total

$4 \times 3.5 = 14$

$16.8 - 14 = 2.8$

$2t = 2.8$

(Reason: The second load is 4 sacks and 2 tins. The sacks in it weigh $4 \times 3.5 = 14$ kg, so removing them from the printed 16.8 kg leaves only the tins. The first total would do just as well and is used as a check below.)

Step 5: halve 2.8 to reach one tin

$$t = \frac{2.8}{2} = 1.4$$

(Reason: The 2.8 kg is the weight of 2 tins, and the question asks for one. Every tin has the same weight, so halving is all that is left to do. The answer is a weight, so it is given in kilograms.)

1.4 kg

Verification

✓ **Check 1:** A pair of weights is only right if it fits BOTH printed totals, not just the one it was worked out from. Put 3.5 kg for a sack and 1.4 kg for a tin into each of them. $7 \times 3.5 + 2 \times 1.4 = 27.3$ and $4 \times 3.5 + 2 \times 1.4 = 16.8$

✓ **Check 2:** Work the question a second way, removing the SACKS instead of the tins. Multiplying the first total by 4 and the second by 7 puts 28 sacks in both, so subtracting now cancels the sacks. This route never divides by 3 and never finds the weight of a sack, so a slip made above cannot repeat itself here. $28s + 14t = 117.6$ take away $28s + 8t = 109.2$ gives $6t = 8.4$, so $t = 1.4$

✓ **Check 3:** Use the other printed total, the one the working did not use. Taking the 7 sacks out of the 27.3 kg load should leave the same 2 tins. $7 \times 3.5 = 24.5$, so $27.3 - 24.5 = 2.8$ for 2 tins, giving 1.4 kg each

✓ **Check 4:** Test the size of the gap between the two loads. The only difference between them is 3 extra sacks, so 3 sacks must account for the whole of it. $3 \times 3.5 = 10.5$ and $27.3 - 16.8 = 10.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Subtract the two totals	M1	$27.3 - 16.8$ $(= 10.5)$ or $7s + 2t = 27.3$ and $4s + 2t = 16.8$ subtracted to give $3s = 10.5$, for a correct first step to find the weight of 3 sacks	✓

Step	Mark	Description	Got it?
Weight of one sack	M1	$\frac{10.5}{7-4}$ $(= 3.5)$ or $s = 3.5$, for a method to find the weight of one sack. The printed scheme puts the 10.5 in quotation marks, so the candidate's own value from the first step may be used	✓
Weight of 2 tins	M1	eg $27.3 - 7 \times 3.5$ $(= 2.8)$ or $16.8 - 4 \times 3.5$ $(= 2.8)$ or $2t = 16.8 - 4 \times 3.5$ $(= 2.8)$, for a method to find the weight of 2 tins. The printed scheme puts the 3.5 in quotation marks, so the candidate's own value may be used	✓
The weight of one tin	A1	1.4, or an equivalent such as $\frac{7}{5}$	✓

Full marks: 4/4

Question 7 - Exam Solution

Understanding the Question

Given

AEC and BED are straight lines that cross at E, with one angle there marked $58°$ and the angle facing it across the crossing marked $x°$.
$GHIJ$ is a quadrilateral with $163°$ at H, a right angle at J and $47°$ at I. The angle at G inside the quadrilateral is not given.
F lies outside the quadrilateral, beyond G, and the angle between GF and GJ is $110°$.

Find

(a) A reason why $x = 58$ is correct. (b) Working, with a reason at every stage, showing that FGH is not a straight line.

Plan the Solution

- (a) Nothing needs calculating. The two marked angles are made by the same pair of straight lines crossing, so name the angle fact that connects them.
- (b) The angle $\angle JGH$ is missing, so find it first from the quadrilateral, whose four angles add up to $360°$.

- (b) Then add the two angles that meet at G and compare the total with the 180° that a straight line needs. If it misses, the line is not straight.

Worked Solution [5 marks]

> Rule - Angle facts: vertically opposite angles are equal; angles on a straight line add up to 180°; the four angles of a quadrilateral add up to 360°.

Step 1: Part (a), name the fact that links the two marked angles

$x = 58$

(Reason: The straight lines AEC and BED cross at E, and the two marked angles face each other across that crossing point, so they are vertically opposite. Vertically opposite angles are equal, which is why x must be 58 and Mateo is right. No calculation is needed, and the mark here is for the reason, not for the number.)

Step 2: Part (b), find the angle at G inside the quadrilateral

$163 + 90 + 47 = 300$

$360 - 300 = 60$

(Reason: The four angles of quadrilateral $GHIJ$ add up to 360°. Taking the three given angles away from 360° leaves the fourth one, so $\angle JGH = 60^\circ$. Note that this is the angle inside the quadrilateral, not the whole angle at G.)

Step 3: Part (b), add the two angles that meet at G

$\angle FGH = \angle FGJ + \angle JGH$

$110 + 60 = 170$

(Reason: The angle $\angle FGH$ is made of two pieces that sit side by side at G: the 110° marked outside the quadrilateral and the angle found in Step 2 inside it.)

Step 4: Part (b), compare with a straight line and conclude

$170 \neq 180$

(Reason: Angles on a straight line add up to 180°. The two angles at G come to 170°, which is 10° short of a half turn, so F, G and H do not lie on one straight line and Elena is wrong.)

(a) Vertically opposite angles are equal, so $x = 58$

(b) $\angle FGH = 110^\circ + 60^\circ = 170^\circ$, not 180°, so FGH is not a straight line

Verification

✓ **Check 1:** (a) Reach x through the straight lines instead of the opposite-angle rule. On the straight line BED, $\angle BEC = 180° - 58° = 122°$. On the straight line AEC, the angle marked $x°$ is what is left after that same $122°$. $180 - 122 = 58$, so $x = 58$, reached without using the rule it is being checked against

✓ **Check 2:** (b) Take Elena at her word. If FGH really were straight then $\angle JGH = 180° - 110° = 70°$, and the four angles of the quadrilateral would have to add up to $360°$. $70 + 163 + 90 + 47 = 370$, which is 10 too many, so the assumption cannot hold

✓ **Check 3:** (b) Put the angle found in Step 2 back into the quadrilateral and confirm the four angles close. $60 + 163 + 90 + 47 = 360$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Reason given	**B1**	vertically opposite angles are equal, or opposite to 58 (with or without the degree sign). The printed row underlines vertically, opposite, opposite angles and 58 as the words the answer must carry.	✓
(b) A method for angle JGH	**M1**	for a method to find angle JGH either using the quadrilateral or assuming line FGH is straight: (JGH =) $360 - 163 - 90 - 47$ (= 60) or (JGH =) $180 - 110$ (= 70)	✓
(b) The figure that shows Elena is wrong	**A1**	($110 + 60$ =) 170 or $110 + 60 \neq 180$ or ($180 - 60$ =) 120 or ($70 + 163 + 90 + 47$ =) 370 or $70 + 163 + 90 + 47 \neq 360$ or for ($180 - 110$ =) 70 and ($360 - 163 - 90 - 47$ =) 60	✓
(b) Reason for the straight-line stage	**B1**	angles on a straight line add to $180°$	✓
(b) Reason for the quadrilateral stage	**B1**	angles in a quad(rilateral) add up to 360 (Accept a 4-sided shape)	✓
(b) Guidance printed with the scheme	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 5/5

Question 8 - Exam Solution

Understanding the Question

Given

The pictogram gives the number of loaves sold on each of five days, and it prints no key

A whole symbol is a rectangle ruled into 4 small squares, and a part symbol draws only some of those four

Friday's row draws 3 whole symbols and one half symbol; Thursday's row draws 1 whole symbol and one half symbol

Monday's row draws 2 whole symbols and one small square

Friday's number of loaves was 16 more than Thursday's

Find

The number of loaves the bakery sold on Monday

Plan the Solution

- The pictogram has no key, so the first job is to build one. Count each row in SMALL SQUARES rather than in whole symbols, because every part symbol here is a whole number of small squares.
- Friday and Thursday are the only two rows the question ties together, so the gap between their counts must be worth the 16 loaves it names.
- Divide to find what one small square is worth, then count Monday's small squares and multiply.

Worked Solution [3 marks]

> Rule - the key of a pictogram is how many items one symbol stands for. When no key is printed, a stated difference between two rows supplies it: divide the difference in items by the difference in squares, then read every row with the value that gives.

Step 1: count the small squares in the Friday row and in the Thursday row

Friday: $3 \times 4 + 2 = 14$

Thursday: $1 \times 4 + 2 = 6$

(Reason: A whole symbol is 4 small squares and a half symbol is 2. Friday draws 3 whole symbols and one half symbol, and Thursday draws one whole symbol and one half symbol, so count four for each whole one and two for each half.)

Step 2: the difference between the two rows

$$14 - 6 = 8$$

(Reason: Friday's row carries 8 small squares more than Thursday's, and the question says that gap is worth 16 loaves.)

Step 3: the value of one small square

$$\frac{16}{8} = 2$$

(Reason: Sharing the 16 loaves equally between the 8 extra small squares says what one small square stands for. That is the key the pictogram does not print.)

Step 4: count Monday's small squares

$$\text{Monday: } 2 \times 4 + 1 = 9$$

(Reason: Monday draws 2 whole symbols, worth 4 small squares each, and then one small square on its own.)

Step 5: turn Monday's small squares into loaves

$$9 \times 2 = 18$$

(Reason: Every small square stands for 2 loaves, so multiply Monday's count of small squares by the key.)

18 loaves

Verification

- ✓ **Check 1:** Work in whole symbols instead of small squares. A whole symbol is 4 small squares, so it stands for 8 loaves. Friday draws 3.5 symbols, Thursday draws 1.5, and Monday draws 2.25. $3.5 \times 8 - 1.5 \times 8 = 16$ and $2.25 \times 8 = 18$
- ✓ **Check 2:** Work in half symbols, a third unit again. Friday is 7 halves and Thursday is 3, so the 16 loaves are spread over four halves. Monday is 4.5 halves. $\frac{16}{7-3} = 4$ and $4.5 \times 4 = 18$
- ✓ **Check 3:** Read the whole week with the same key and add it up. The rows hold 9, 19, 4, 6 and 14 small squares, each worth 2 loaves, so every day comes out a whole number of loaves and the two totals must agree. $18 + 38 + 8 + 12 + 28 = 104$ and $(9 + 19 + 4 + 6 + 14) \times 2 = 104$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg 8 small squares $= 16$ or 2 large squares $= 16$ or 2 small squares $= 4$ or [small square] $= \frac{16}{8}$ $(= 2)$ or [large square] $= \frac{16}{2}$ $(= 8)$ or [small square] $= \frac{4}{2}$ $(= 2)$ or [2 small squares] $= \frac{16}{4}$ $(= 4)$ or Friday $= 28$ (loaves) and Thursday $= 12$ (loaves)	**M1**	for starting to work with proportion. May be seen in a square on the pictogram or in working or implied by correct working, or for finding the correct number of loaves sold on Thursday and Friday.	✓
eg 9×2 oe or 2.25×8 oe or 4.5×4 oe. The printed scheme puts the 2, the 8 and the 4 in quotation marks, so the candidate's own value from the first mark may be used here.	**M1**	for a complete method to find the loaves sold on Monday.	✓
18	**A1**	cao. A correct answer scores full marks, unless it comes from obvious incorrect working.	✓
No key is printed on this pictogram, so the first mark is the one for producing a key at all.	**Note**	A common wrong answer is 20, from reading Monday's third symbol as a half rather than as a single small square: $2.5 \times 8 = 20$. That earns the first mark and the second, but not the accuracy mark.	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given	Find

A number sequence whose first four terms are 5, 12, 19 and 26.
The terms are listed in order, so each one follows on from the term before it.

(i) the fifth term of the sequence. (ii) an explanation, in words, of how that term is reached.

Plan the Solution

- Look at what happens from one term to the next, and check that the same thing happens every time.
- If the sequence climbs by the same amount each time it is a linear sequence, so the fifth term is the fourth term plus one more of those steps.
- Part (ii) asks for the rule in words, so say what is done to a term to reach the one after it.

Worked Solution [2 marks]

> Rule - Linear sequences: when the difference between one term and the next is the same all the way along, that difference is the common difference, and the next term is the last term given plus one common difference.

Step 1: find what is added from one term to the next

$12 - 5 = 7$

$19 - 12 = 7$

$26 - 19 = 7$

(Reason: All three gaps are the same, so the sequence climbs in equal steps of 7. That common difference is what generates every later term.)

Step 2: add the common difference to the last term given

$26 + 7 = 33$

(Reason: The fifth term comes straight after the fourth term that is printed, so one more step of 7 is added on.)

Step 3: put the rule into words for part (ii)

$7n - 2$

$7 \times 5 - 2 = 33$

(Reason: The step is the same every time, so the explanation is simply that 7 is added to the term before. The position-to-term rule $7n - 2$ says the same thing in symbols, and it gives the same fifth term.)

(i) 33

(ii) Add 7 to the term before

Verification

✓ **Check 1:** Work backwards. Taking one step of 7 off the fifth term must land on the fourth term that is printed. $33 - 7 = 26$, which is the last term printed.

✓ **Check 2:** Count on from the first term instead. Getting from the first term to the fifth takes four steps of 7. $5 + 4 \times 7 = 33$, the same term reached a different way.

✓ **Check 3:** Test the position-to-term rule $7n - 2$ on the terms that are printed. $7 \times 1 - 2 = 5, 7 \times 2 - 2 = 12, 7 \times 3 - 2 = 19, 7 \times 4 - 2 = 26$, so the rule fits, and $7 \times 5 - 2 = 33$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) 33	**B1**	For the answer 33.	✓
(ii) Added 7	**B1**	Accept eg add 7, $(n) + 7$, $7n - 2$.	✓

Full marks: 2/2

Question 10 - Exam Solution

Understanding the Question

Given

A frequency table covering 30 weeks at a music shop, with the number of guitars sold in a week running from 1 to 5. The frequency column counts weeks: 4 weeks sold one guitar, 10 weeks sold two guitars, and so on down the table.

Find

The mean number of guitars sold per week.

Plan the Solution

- Add the frequency column first and check that it comes to 30, so the table really does account for every week in the record.
- Multiply each number of guitars sold by the frequency beside it, because that frequency says how many weeks sold that many.
- Add those products together to get the total number of guitars sold across the whole record.
- Divide that total by the total frequency, which shares the guitars equally between the weeks.

Worked Solution [3 marks]

Rule - Mean from a frequency table: $\text{mean} = \dfrac{\text{sum of the products}}{\text{total frequency}}$. Multiply every value by its frequency, add the products, then divide by the total frequency. The frequencies are what make the busy weeks count more than the quiet ones.

Step 1: check the frequencies account for every week

$4 + 10 + 5 + 7 + 4 = 30$

(Reason: The frequency column counts weeks, so adding it gives the length of the record. It comes to 30, which matches the 30 weeks in the question, so no week is missing and this total is the divisor to use later.)

Step 2: multiply each number of guitars by its frequency

$1 \times 4 = 4$

$2 \times 10 = 20$

$3 \times 5 = 15$

$4 \times 7 = 28$

$5 \times 4 = 20$

(Reason: The second row stands for 10 separate weeks that each sold 2 guitars, so between them those weeks account for $2 \times 10 = 20$ guitars. Every row is treated the same way.)

Step 3: add the products to get the total sold

$4 + 20 + 15 + 28 + 20 = 87$

(Reason: Adding the five products gives the number of guitars sold over all 30 weeks together, which is 87. Adding the left-hand column instead would only count the different sales figures, not the weeks.)

Step 4: divide the total sold by the total frequency

$$\frac{87}{30} = 2.9$$

(Reason: The mean shares the guitars equally between the weeks, so the total sold is divided by the total frequency found in step 1. A mean does not have to be a whole number, and a decimal is what a division like this normally gives.)

2.9 guitars per week

Verification

✓ **Check 1:** Reverse the last step. Multiplying the mean by the 30 weeks has to give the total number of guitars back. $2.9 \times 30 = 87$, the total found in step 3.

✓ **Check 2:** Work it out a different way, from an assumed mean of 3. Measure every value against 3, weight those differences by the frequencies, then correct the assumption. $4 \times (-2) + 10 \times (-1) + 5 \times 0 + 7 \times 1 + 4 \times 2 = -3$, so the mean is $3 + \dfrac{-3}{30} = 2.9$, reached without ever forming the total.

✓ **Check 3:** Sanity check the size of the answer. A mean must lie between the smallest and the largest value in the table, and it should sit near the values that come up most often. $1 < 2.9 < 5$, and the most common number of sales is 2, with a second cluster at 4, so a mean a little below 3 is exactly what to expect.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1 \times 4 + 2 \times 10 + 3 \times 5 + 4 \times 7 + 5 \times 4$ $(= 87)$ or $4 + 20 + 15 + 28 + 20$ $(= 87)$	**M1**	For at least 4 correct products and intention to add. Products may be seen by the side of the table.	✓
"87" divided by 30 oe	**M1**	Dep on M1. Allow use of their "30" from adding the frequencies from the table.	✓
2.9	**A1**	Correct answer scores full marks (unless from obvious incorrect working). Accept an answer of 3 if correct working seen, eg 87 divided by 30 oe.	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

The equation $y = 2x - 5$
The values of x to use: from -1 to 4
A printed grid, which is where the answer goes

Find

The graph of $y = 2x - 5$ drawn on the grid, from $x = -1$ to $x = 4$

Plan the Solution

- Work out y for every whole-number value of x in the range, so the graph is built from points rather than guessed.
- Plot each point on the grid, counting the squares across first and then up or down.
- Rule one straight line through the points, running it right out to $x = -1$ at one end and $x = 4$ at the other.

Worked Solution [3 marks]

> Straight-line graph: $y = 2x - 5$ is in the form $y = mx + c$, so its graph is a straight line with gradient $m = 2$ and y-intercept $c = -5$. Two points fix that line and the others check it.

Step 1: Work out y for each value of x

$2 \times (-1) - 5 = -7$

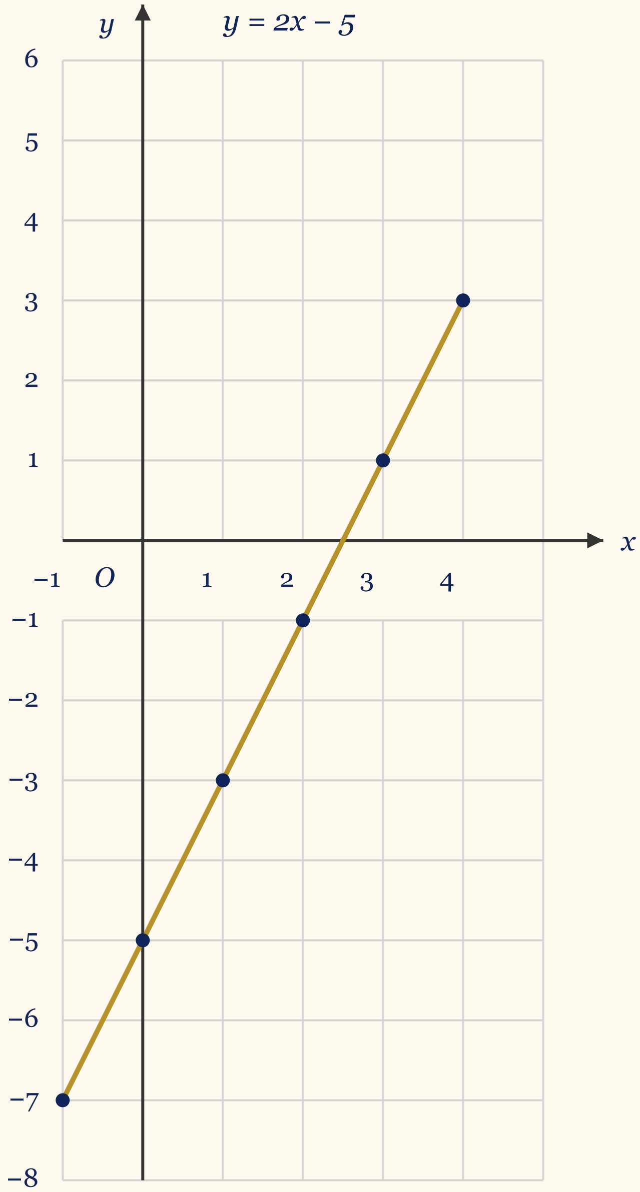
$2 \times 0 - 5 = -5$

$2 \times 1 - 5 = -3$

$2 \times 2 - 5 = -1$

$2 \times 3 - 5 = 1$

$2 \times 4 - 5 = 3$



(Reason: Each whole-number value of x from -1 to 4 goes into $y = 2x - 5$ in turn: double it, then subtract 5. Keep the bracket round (-1) so the minus sign survives the doubling.)

Step 2: Collect the points into a table

x	-1	0	1	2	3	4
y	-7	-5	-3	-1	1	3

(Reason: Each column of the table is one point to plot: $(-1, -7)$, $(0, -5)$, $(1, -3)$, $(2, -1)$, $(3, 1)$ and $(4, 3)$. Every one of them fits on the printed grid, which is a sign the table is right.)

Step 3: Plot the points and rule one straight line through them

$$\frac{3-(-7)}{4-(-1)} = \frac{10}{5} = 2$$

(Reason: The points climb 2 squares for every 1 square across, so they lie on one straight line and a ruler through them is the graph. Draw it right across the range, from $x = -1$ to $x = 4$, with no gap

at either end: the marks are for the whole segment.)

The straight line through $(-1, -7)$ and $(4, 3)$, ruled from $x = -1$ to $x = 4$

Verification

✓ **Check 1:** Read the gradient off the drawn line. From $(-1, -7)$ to $(4, 3)$ it rises 10 squares for a run of 5. $\frac{10}{5} = 2$, which is the gradient $y = 2x - 5$ asks for

✓ **Check 2:** Read where the drawn line crosses the y-axis, and compare it with the value the equation gives at $x = 0$. $2 \times 0 - 5 = -5$, and the line crosses at $(0, -5)$

✓ **Check 3:** Test a point that was not used to fix the two ends. Start at $(-1, -7)$ and move 4 squares across, which lifts y by 2×4. $-7 + 2 \times 4 = 1$, so the ruled line passes through $(3, 1)$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct line, drawn between $x = -1$ and $x = 4$	**B3**	for a correct line between $x = -1$ and $x = 4$	✓
A correct line that is short of the full range, or the points plotted and left unjoined	**B2**	for a correct straight line segment through at least 3 of $(-1, -7)$ $(0, -5)$ $(1, -3)$ $(2, -1)$ $(3, 1)$ $(4, 3)$ or for all of $(-1, -7)$ $(0, -5)$ $(1, -3)$ $(2, -1)$ $(3, 1)$ $(4, 3)$ plotted but not joined	✓
Some correct points, or a line with one of the two right features	**B1**	for at least 2 correct points stated (may be in a table) or for a line drawn with a positive gradient through $(0, -5)$ or for a line with a gradient of 2	✓

Full marks: 3/3

Question 12 - Exam Solution

Understanding the Question

Given

Find

A fleece, priced in both currencies: $£35$ or 42 Swiss francs A rucksack, priced in pounds only: $£54$ One exchange rate, used for both items	The cost of the rucksack in Swiss francs

Plan the Solution

- The fleece is the only item with a price in each currency, so it is the fleece that fixes the exchange rate. Work the rate out from its two prices first.
- Then apply that rate to the rucksack's price of $£54$.
- Decide which way round the rate goes before reaching for the calculator: francs for each pound, or pounds for each franc. The two are reciprocals, and they give very different answers.

Worked Solution [3 marks]

> Exchange rate: an item priced in both currencies fixes the rate for everything else in the shop. Dividing its price in francs by its price in pounds gives the number of francs one pound is worth, and every other price in pounds is then multiplied by that same number.

Step 1: Use the fleece to find the exchange rate

$$\frac{42}{35} = 1.2$$

(Reason: The fleece's two prices are the same amount of money, so $£35$ and 42 Swiss francs are worth exactly the same. Sharing those francs out over the pounds says how many francs a single pound is worth: 1.2 of them.)

Step 2: Apply the rate to the rucksack

$$54 \times 1.2 = 64.8$$

(Reason: Each of the 54 pounds the rucksack costs is worth 1.2 Swiss francs, so the price in francs is 54 lots of 1.2. The rate multiplies a price in pounds. Dividing by it would send the conversion back the other way and turn francs into pounds, which is the commonest slip on this question.)

64.8 Swiss francs

Verification

✓ **Check 1:** Turn the answer back into pounds at the same rate. If 64.8 Swiss francs really is the rucksack's price, dividing it by the rate must give back $£54$. $\frac{64.8}{1.2} = 54$, the price the question started from

✓ **Check 2:** Compare the two items instead of the two currencies. The rucksack costs $\frac{54}{35}$ times as much as the fleece, and that must hold whichever currency the prices are written in. $42 \times \frac{54}{35} = 64.8$

✓ **Check 3:** Cancel the rate to whole numbers and avoid decimals altogether. $£35$ to 42 francs cancels by 7 to $£5$ to 6 francs, and $£54$ is $\frac{54}{5}$ lots of $£5$. $6 \times \frac{54}{5} = 64.8$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\frac{42}{35}$ $(= 1.2)$ or $\frac{35}{42}$ $(= 0.833...)$ or $\frac{54}{35}$ $(= 1.54...)$ or $\frac{35}{54}$ $(= 0.648...)$	M1	a method to find a correct ratio	✓
eg 54×1.2 or $\frac{54}{0.833...}$ or $42 \times 1.54...$ or $\frac{42}{0.648...}$	M1	for a complete method, with the candidate's own ratio from the first mark used in place of each quoted rate	✓
64.8 - correct answer scores full marks (unless from obvious incorrect working)	A1	accept 64.80	✓

Full marks: 3/3

Question 13 - Exam Solution

Understanding the Question

Given

A circle whose radius is $r = 6.4$ cm. The length given is the radius, not the diameter, so nothing needs halving first.

Find

The area of the circle. The answer is wanted correct to 3 significant figures.

Plan the Solution

- Use the area formula for a circle, $A = \pi r^2$.
- Square the radius first, then multiply that by π.
- Keep the calculator's full value all the way through, and round only at the very end to 3 significant figures.

Worked Solution [2 marks]

> Rule - Area of a circle: $A = \pi r^2$, where r is the radius.

Step 1: Square the radius

$r^2 = 6.4^2 = 40.96$

(Reason: The formula multiplies π by r^2, and squaring means multiplying the radius by itself.)

Step 2: Multiply the squared radius by π

$A = \pi \times 40.96$

$A = 128.6796351\ldots$

(Reason: The calculator's own value for π is used, so every digit is still there when the rounding happens.)

Step 3: Round to 3 significant figures

$A = 129 \text{ cm}^2$

(Reason: The third significant figure of $128.6796351\ldots$ is the 8, and the digit after it is 6, so the 8 rounds up.)

$$129 \text{ cm}^2$$

Verification

✓ **Check 1:** Work the calculation backwards: $\sqrt{\dfrac{A}{\pi}}$ must give back the radius the question started with. The radius comes back as 6.4 cm, which is the length given.

✓ **Check 2:** Redo the calculation with $\dfrac{22}{7}$ in place of π, the approximation the mark scheme allows. $\dfrac{22}{7} \times 40.96 \approx 128.7$, which still rounds to 129.

✓ **Check 3:** Squeeze it. The circle fits inside a square of side 12.8 cm, and it contains the square whose diagonal is 12.8 cm, so the area must lie between those two square areas.

$81.92 < 129 < 163.84$, so the answer is the size a circle of this radius has to be.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\pi \times 6.4^2 \left(= \frac{1024}{25}\pi\right)$	M1	allow $3.14\ldots$ or $\frac{22}{7}$ for π	✓
129 Correct answer scores full marks (unless from obvious incorrect working)	A1	accept 128 to 129	✓

Full marks: 2/2

Question 14 - Exam Solution

Understanding the Question

Given

Three identical rectangles, each 6 m long and 3.5 m wide, joined to make the paddock in the diagram
A fence right round the outside of the paddock, charged at 7.60 euros for each 1 metre

Find

The total charge for the fence, in euros

Plan the Solution

- Work out the length of every side that goes round the outside of the shape.
- The two short steps are not printed on the diagram, so get them from $6 - 3.5$.
- Add the eight outside lengths to get the perimeter of the paddock.
- Multiply that perimeter by 7.60 euros for each metre.

Worked Solution [4 marks]

> Rule - Perimeter: the perimeter is the total distance all the way round the outside of a shape. Every outside edge is counted once, and a line drawn inside the shape is not counted at all.

Step 1: Find the short step at each end of the outline

$6 - 3.5 = 2.5$

(Reason: (Reason: where one rectangle drops below its neighbour, the exposed piece is what is left of a 6 m length once 3.5 m of it is covered by the rectangle beside it.))

Step 2: Find the two long sides of the outline

$6 + 3.5 = 9.5$

(Reason: (Reason: the top of the shape runs along the 6 m length of the middle rectangle and then the 3.5 m width of the right-hand one. The bottom does the same on the other two rectangles.))

Step 3: Add the eight sides that go round the outside

$3.5 + 2.5 + 9.5 + 6 + 3.5 + 2.5 + 9.5 + 6 = 43$

(Reason: (Reason: the fence follows the outline only, so each outside edge is counted once and the two dashed lines inside the paddock are not fence at all.))

Step 4: Charge for every metre of that perimeter

$43 \times 7.60 = 326.80$

(Reason: (Reason: the charge is 7.60 euros for each 1 metre, so the total is the number of metres multiplied by 7.60.))

326.80 euros

Verification

✓ **Check 1 - count the fence in pieces of each size:** Going round the outline, four of the sides are 6 m, four are 3.5 m, and the two short steps are 2.5 m each. $4 \times 6 + 4 \times 3.5 + 2 \times 2.5 = 43$ m, the same perimeter

✓ **Check 2 - build the perimeter from three separate rectangles:** One rectangle on its own has perimeter $2 \times (6 + 3.5) = 19$ m, so three separate rectangles give 57 m. Each of the two joins hides a 3.5 m width from both of the rectangles that meet there. $57 - 4 \times 3.5 = 43$ m, the same perimeter

✓ **Check 3 - split the charge into two easier pieces:** Charge for the first 40 m, then for the remaining 3 m, and add the two. $40 \times 7.60 = 304$ and $3 \times 7.60 = 22.80$, so $304 + 22.80 = 326.80$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$6 - 3.5 = 2.5$ or $4 \times 6 + 4 \times 3.5 = 38$ or	**M1**	for a method to find the missing length (may be shown on the diagram) or for a method to	✓

Step	Mark	Description	Got it?
$6 \times 6 + 6 \times 3.5 = 57$ or $2 \times 9.5 \times 3 = 57$ or $19 \times 3 = 57$		find the length of the solid lines excluding the 2.5, may include extra sides added, or for a method to find the perimeter of the 3 rectangles	
$4 \times 6 + 4 \times 3.5 + 2 \times 2.5 = 43$ or $6 \times 6 + 6 \times 3.5 - 4 \times 3.5 = 43$ or $2 \times 9.5 \times 3 - 4 \times 3.5 = 43$ or $19 \times 3 - 4 \times 3.5 = 43$	**M1**	for a complete method to find the perimeter of the shape. The printed scheme writes the 2.5 in quotation marks, so the candidate's own earlier value may be used here.	✓
eg 43×7.6	**M1**	for a method to find the cost, allow use of their 43 as long as it is from adding at least 4 correct lengths including a length of 3.5 and a length of 6, eg $57 \times 7.6(0)$ or $38 \times 7.6(0)$	✓
$326.8(0)$	**A1**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 4/4

Question 15 - Exam Solution

Understanding the Question

Given

A written test marked out of 140 marks
Fiona's score on that test: 119 marks

Find

Fiona's score written as a percentage

Plan the Solution

- Write the score as a fraction of the total number of marks on the test.
- Cancel that fraction down: 119 and 140 share a factor of 7.
- Multiply the cancelled fraction by 100, because a percentage counts parts per 100.

Worked Solution [2 marks]

> Rule - A score as a percentage: put the score over the total, then multiply by 100. The fraction says what part of the whole test was earned, and multiplying by 100 re-counts that part in hundredths.

Step 1: Write the score as a fraction of the whole test

$$\frac{119}{140}$$

(Reason: (Reason: the whole test is 140 marks, so the total goes underneath and the 119 marks that were scored go on top.))

Step 2: Cancel the fraction to its simplest form

$$119 = 7 \times 17$$

$$140 = 7 \times 20$$

$$\frac{119}{140} = \frac{17}{20}$$

(Reason: (Reason: 7 is the highest common factor of the two numbers, and cancelling it leaves 20 underneath, which scales up to 100 in one step.))

Step 3: Scale the fraction up to a percentage

$$\frac{17}{20} \times 100 = 17 \times 5 = 85$$

(Reason: (Reason: 20 goes into 100 exactly 5 times, so each twentieth of the test is worth 5 per cent.))

$$85\%$$

Verification

✓ **Check 1 - turn the percentage back into marks:** Take 85 per cent of the 140 marks the test is out of. $0.85 \times 140 = 119$, the marks Fiona scored

✓ **Check 2 - count the marks that were dropped:** Fiona missed $140 - 119 = 21$ marks, and $\frac{21}{140} = \frac{3}{20}$, which is 15 per cent of the test. $100 - 15 = 85$, the same percentage

✓ **Check 3 - build the score from blocks of ten per cent:** A tenth of 140 is 14 marks, so 80 per cent is 112 marks and 5 per cent is 7 marks. $112 + 7 = 119$ marks, which is $80 + 5 = 85$ per cent

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\frac{119}{140} \times 100$ or 0.85×100 or $\frac{17}{20} \times 100$ oe	M1	for a correct method to write the score as a percentage	✓
85	A1	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 2/2

Question 16 - Exam Solution

Understanding the Question

Given

The two numbers 45 and 70
Both are whole numbers, so each one can be written as a product of prime factors

Find

The lowest common multiple (LCM) of 45 and 70 - the smallest number that both of them divide into exactly

Plan the Solution

- Write each number as a product of prime factors, using a factor tree or repeated division by $2, 3, 5, 7$.
- Build the LCM by multiplying together the highest power of every prime that appears in either list.
- Check the result divides exactly by both numbers, and cross-check it against the product of the two numbers divided by their highest common factor.

Worked Solution [2 marks]

> LCM from prime factors: write each number as a product of primes, then multiply together the highest power of every prime that appears in either list.

Step 1: Write 45 as a product of prime factors

$$45 = 3 \times 3 \times 5 = 3^2 \times 5$$

(Reason: 45 divides by 3 to give 15, and $15 = 3 \times 5$. Both 3 and 5 are prime, so the tree stops there.)

Step 2: Write 70 as a product of prime factors

$70 = 2 \times 35 = 2 \times 5 \times 7$

(Reason: 70 is even, so take out 2 first, leaving 35. Then $35 = 5 \times 7$, and both of those are prime as well.)

Step 3: Multiply the highest power of every prime

$2^1, \quad 3^2, \quad 5^1, \quad 7^1$

$2 \times 3^2 \times 5 \times 7 = 630$

(Reason: The LCM has to contain $45 = 3^2 \times 5$, so it needs two 3s, and it has to contain $70 = 2 \times 5 \times 7$, so it needs a 2 and a 7. One 5 is enough, because neither number uses more than one.)

630

Verification

✓ **Check 1:** Divide the answer by each of the two numbers. A common multiple must give a whole number both times. $\frac{630}{45} = 14$ and $\frac{630}{70} = 9$, both whole numbers

✓ **Check 2:** Use the other standard route: the product of the two numbers divided by their highest common factor. The only prime 45 and 70 share is 5, so the highest common factor is 5. $\frac{45 \times 70}{5} = \frac{3150}{5} = 630$

✓ **Check 3:** Confirm nothing smaller works: list every multiple of 70 below 630 and test each one for divisibility by 45. $70, 140, 210, 280, 350, 420, 490, 560$ - not one of them is a multiple of 45, so 630 really is the lowest

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Any correct valid method for the LCM of 45 and 70	**M1**	for any correct valid method, eg for starting to list at least four multiples of each number: $45, 90, 135, 180$ ... and $70, 140, 210, 280$ ... or $2, 5, 7$ and $3, 3, 5$ seen (may be in a factor tree, ignore 1) or a fully correct Venn diagram	✓

Step	Mark	Description	Got it?
		or $\frac{45 \times 70}{5}$ or $2, 3, 3, 5, 7$ oe or $5, 9, 14$ oe (could be in a table)	
630	A1	Allow $2 \times 3^2 \times 5 \times 7$ oe, eg $5 \times 9 \times 14$ Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 2/2

Question 17 - Exam Solution

Understanding the Question

Given

A footbridge whose length is 142.8 m, correct to 1 decimal place.
That length has been rounded, so the true length is not exactly 142.8 m.

Find

The lower bound of the length. The upper bound of the length.

Plan the Solution

- Find the rounding unit. Correct to 1 decimal place means rounded to the nearest 0.1 m.
- Halve that unit, because a rounded value can be at most half a unit away from the true length.
- Take half a unit off for the lower bound, and put half a unit on for the upper bound.

Worked Solution [2 marks]

> Rule - Bounds: a measurement written as x correct to the nearest u has lower bound $x - \frac{u}{2}$ and upper bound $x + \frac{u}{2}$.

Step 1: Find half of one rounding unit

$$\text{rounding unit} = 0.1$$

$$\frac{0.1}{2} = 0.05$$

(Reason: Correct to 1 decimal place means rounded to the nearest 0.1 m, so the true length is less than 0.05 m away from the value written down.)

Step 2: Take half a unit off for the lower bound

$142.8 - 0.05 = 142.75$

(Reason: The lower bound is the smallest length that still rounds to the stated length, so half a rounding unit comes off.)

Step 3: Put half a unit on for the upper bound

$142.8 + 0.05 = 142.85$

(Reason: Any length below 142.85 m rounds down to 142.8 m, while 142.85 m itself rounds up to 142.9 m, so it is the value the length stops short of.)

Step 4: Write the error interval

$142.75 \leq L < 142.85$

(Reason: Writing L for the true length in metres, the lower bound is included because it does round to 142.8 m, and the upper bound is not.)

(i) 142.75 m

(ii) 142.85 m

Verification

✓ **Check 1:** Round the lower bound back to 1 decimal place. A halfway value rounds up, so it should come back to the stated length. 142.75 rounds to 142.8, so the lower bound is a length the footbridge could actually have.

✓ **Check 2:** The two bounds must be one whole rounding unit apart, with the stated length exactly halfway between them. $142.85 - 142.75 = 0.1$ and $\frac{142.75 + 142.85}{2} = 142.8$.

✓ **Check 3:** Test a length just below the upper bound, and then the upper bound itself. 142.8499 rounds to 142.8, while 142.85 rounds to 142.9, which is why the upper bound is a value the length never reaches.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i)	**B1**	142.75	✓

Step	Mark	Description	Got it?
(ii)	**B1**	142.85. Accept $142.8499\ldots$ or $142.84\dot{9}$	✓

Full marks: 2/2

Question 18 - Exam Solution

Understanding the Question

Given

The product of two mixed numbers, $2\frac{1}{4} \times 1\frac{5}{7}$.

The value it is claimed to come to, $3\frac{6}{7}$.

Find

Working that shows the product really is $3\frac{6}{7}$. The answer is printed in the question, so every mark here is for the working, not for the number.

Plan the Solution

- Write each mixed number as an improper fraction. Mixed numbers cannot be multiplied whole part by whole part and fraction part by fraction part, so this has to come first.
- Multiply the numerators together and the denominators together. Cancelling a common factor before multiplying is allowed and keeps the numbers small - the mark scheme accepts either order.
- Simplify the result, then turn it back into a mixed number so that it matches the form the question states.

Worked Solution [3 marks]

> Rule - Multiplying mixed numbers: write each one as an improper fraction, then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, and finally write the result back as a mixed number.

Step 1: Write each mixed number as an improper fraction

$$2 \times 4 + 1 = 9$$

$$2\frac{1}{4} = \frac{9}{4}$$

$$1 \times 7 + 5 = 12$$

$$1\frac{5}{7} = \frac{12}{7}$$

(Reason: A mixed number is a whole number added to a fraction, so $2\frac{1}{4}$ *means* $2 + \frac{1}{4}$*. To collect that into a single fraction, multiply the whole number by the denominator and add the numerator, keeping the denominator unchanged. The same rule turns* $1\frac{5}{7}$ *into sevenths. This first stage carries a mark of its own because the whole method depends on it.)*

Step 2: Multiply the numerators, and multiply the denominators

$$\frac{9}{4} \times \frac{12}{7} = \frac{9 \times 12}{4 \times 7}$$

$$\frac{9 \times 12}{4 \times 7} = \frac{108}{28}$$

(Reason: Two fractions are multiplied straight across: numerator times numerator over denominator times denominator. A common denominator is not needed here - that is only for adding and subtracting - so $9 \times 12 = 108$ *goes on top and* $4 \times 7 = 28$ *underneath.)*

Step 3: Simplify the fraction

$$\frac{108}{28} = \frac{4 \times 27}{4 \times 7} = \frac{27}{7}$$

(Reason: The highest common factor of 108 *and* 28 *is* 4*, so each is written as* 4 *times something and the shared factor is removed. Cancelling that* 4 *into the* 12 *before multiplying gives the same* $\frac{27}{7}$ *with much smaller numbers, and the mark scheme accepts either order.)*

Step 4: Write the improper fraction as a mixed number

$$\frac{27}{7} = \frac{21 + 6}{7} = \frac{21}{7} + \frac{6}{7}$$

$$\frac{21}{7} + \frac{6}{7} = 3 + \frac{6}{7} = 3\frac{6}{7}$$

(Reason: Seven goes into 27 *three whole times, using up* 21 *and leaving* 6 *sevenths over. That is the form the question prints, so the working has now shown exactly what it was asked to show.)*

$$2\frac{1}{4} \times 1\frac{5}{7} = \frac{9}{4} \times \frac{12}{7} = \frac{108}{28} = \frac{27}{7}$$

$$\frac{27}{7} = 3\frac{6}{7}, \textbf{ as required}$$

Verification

✓ **Check 1:** Clear both denominators. Multiplying by $4 \times 7 = 28$ must leave a whole number on each side, and the two whole numbers must agree.

$\frac{9}{4} \times \frac{12}{7} \times 28 = 9 \times 12 = 108$ and $3\frac{6}{7} \times 28 = \frac{27}{7} \times 28 = 108$

✓ **Check 2:** Run it backwards. Dividing the product by $1\frac{5}{7}$ means multiplying by its reciprocal $\frac{7}{12}$, which must return the first factor. $\frac{27}{7} \times \frac{7}{12} = \frac{27}{12} = \frac{9}{4} = 2\frac{1}{4}$

✓ **Check 3:** Reach the same total without improper fractions at all, by expanding $\left(2 + \frac{1}{4}\right)\left(1 + \frac{5}{7}\right)$ as four separate products and putting them over 28.

$2 + \frac{10}{7} + \frac{1}{4} + \frac{5}{28} = \frac{56 + 40 + 7 + 5}{28} = \frac{108}{28}$

✓ **Check 4:** A size check. $1\frac{5}{7}$ lies between 1 and 2, so the product must lie between $2\frac{1}{4} \times 1$ and $2\frac{1}{4} \times 2$. $2\frac{1}{4} < 3\frac{6}{7} < 4\frac{1}{2}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Both mixed numbers written as improper fractions	**M1**	for $2\frac{1}{4}$ and $1\frac{5}{7}$ expressed as improper fractions, eg $\frac{9}{4}$ and $\frac{12}{7}$	✓
Correct cancelling, or the multiplication carried out without it	**M1**	correct cancelling or multiplication of numerators and denominators without cancelling, eg $\frac{9}{4} \times \frac{12}{7}$ with the 4 cancelled to 1 and the 12 cancelled to 3, or $\frac{9}{4} \times \frac{12}{7} = \frac{108}{28}$ oe eg $\frac{63}{28} \times \frac{48}{28} = \frac{3024}{784}$	✓
Conclusion reached from correct working	**A1**	dep on M2, for conclusion to $3\frac{6}{7}$ from correct working - either sight of the result of the multiplication, eg $\frac{108}{28}$ oe must be seen, or correct cancelling prior to the multiplication to $\frac{27}{7}$	✓

Step	Mark	Description	Got it?
Guidance printed with the scheme	**Note**	NB: use of decimals scores no marks unless as a check. Working required, and the answer column reads shown.	✓

Full marks: 3/3

Question 19 - Exam Solution

Understanding the Question

Given

The spinner is biased and has 5 sections: a star, a moon, a sun, a leaf and a bell.
$P(\text{star}) = 0.12$, $P(\text{moon}) = 0.20$ and $P(\text{sun}) = 0.38$
$P(\text{leaf}) = 4x$ and $P(\text{bell}) = x$, so the last two sections are in the ratio $4:1$
In part (b) the spinner is spun 350 times.

Find

(a) The probability that one spin lands on a star or on a moon or on a sun. (b) An estimate of how many of the 350 spins land on a leaf.

Plan the Solution

- One spin cannot land on two sections at once, so for part (a) the three probabilities simply add.
- Every spin lands on one of the five sections, so all five probabilities add to 1. Taking the part (a) total off 1 leaves the probability of a leaf or a bell.
- That leftover is $4x + x = 5x$, so divide it by 5 to get x, then take four of those to get $4x$.
- An estimate of how often something happens is its probability multiplied by the number of spins.

Worked Solution [5 marks]

> Rule - Mutually exclusive outcomes: their probabilities add, and the probabilities of all the possible outcomes add to 1. An expected frequency is the probability multiplied by the number of trials.

Step 1: Add the three probabilities the table gives

$0.12 + 0.20 + 0.38 = 0.7$

(Reason: One spin lands on a single section, so a star, a moon and a sun cannot happen together. For outcomes like that the probabilities add, and this is the answer to part (a).)

Step 2: Find what is left for the leaf and the bell

$4x + x = 1 - 0.7$

$5x = 0.3$

(Reason: The five sections are all that can happen, so their probabilities add to 1. Whatever is not a star, a moon or a sun must be a leaf or a bell.)

Step 3: Solve for x

$$x = \frac{0.3}{5} = 0.06$$

(Reason: The leftover 0.3 is shared as $4x + x$, which is 5 equal parts, so divide by the number of parts.)

Step 4: Write down the probability of a leaf

$P(\text{leaf}) = 4x = 4 \times 0.06 = 0.24$

(Reason: The table gives the leaf $4x$, which is four of those equal parts, and the bell keeps the remaining one at 0.06.)

Step 5: Turn the probability into an estimate for 350 spins

$0.24 \times 350 = 84$

(Reason: The spinner is biased, but the probability still says what share of the spins to expect, so multiplying by 350 estimates how many of them land on a leaf.)

(a) 0.7

(b) 84 **times**

Verification

✓ **Check 1:** Put the two found probabilities back into the table and add all five. They must come to 1. $0.12 + 0.20 + 0.38 + 0.24 + 0.06 = 1$

✓ **Check 2:** Estimate the number of spins for every section and add them. They must come to 350. $42 + 70 + 133 + 84 + 21 = 350$

✓ **Check 3:** Reach part (b) without x at all. A leaf or a bell has probability 0.3, so about $0.3 \times 350 = 105$ spins land on one of the two, and the leaf takes 4 of every 5 of them. $\frac{105}{5} \times 4 = 84$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The probability of a star or a moon or a sun.	**B1**	0.7 oe eg $\frac{7}{10}$ oe or 70% or $\frac{0.7}{1}$. If probabilities are given as percentages then the $\%$ sign must be seen.	✓
(b) A correct first step.	**M1ft**	eg $1 - (0.12 + 0.2 + 0.38) = 0.3$ oe or $1 - 0.7 = 0.3$ oe or $0.12 + 0.20 + 0.38 + 4x + x = 1$ oe or $0.7 \times 350 = 245$ oe or $0.12 \times 350 = 42$ or $0.38 \times 350 = 133$. Follow through their 0.7 from part (a). If probabilities are given as percentages then the $\%$ sign must be seen.	✓
(b) A correct second step.	**M1**	eg $\frac{0.3}{5} = 0.06$ or $\frac{0.3}{5} \times 4 = 0.24$ or 0.24 or $x = 0.06$ or $4x = 0.24$ or $0.3 \times 350 = 105$ oe or $350 - 245 = 105$ oe or $350 - 42 - 0.20 \times 350 - 133 = 105$ oe, in each case using their 0.3, their 245, their 42 and their 133.	✓
(b) A correct third step.	**M1**	eg $0.06 \times 350 = 21$ oe or $\frac{105}{5} = 21$ oe or $0.06 \times 4 \times 350$ oe or 0.24×350, in each case using their 0.06, their 105 or their 0.24. Or for $\frac{21}{350}$ or $\frac{84}{350}$.	✓
(b) The estimate.	**A1**	84 cao. A correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 5/5

Question 20 - Exam Solution

Understanding the Question

Given

$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ - the universal set, the whole numbers from 1 to 12

Find

(a)(i) every member of $A \cup B$ (a)(ii) every member of B' (b)(i) the symbol from the box that makes the statement about $A \cap C$ true (b)(ii) the symbol

$A = \{2, 4, 6, 8, 10, 12\}$ - the even members of $\mathcal{E}$
$B = \{3, 6, 9, 12\}$ - the multiples of 3 in $\mathcal{E}$
$C = \{1, 3, 5, 7, 9, 11\}$ - the odd members of $\mathcal{E}$

from the box that correctly links 13 and $\mathcal{E}$

Plan the Solution

- Read $\cup$ as "in A, or in B, or in both", and write each member once however many sets it belongs to.
- Read the dash in B' as "not in B", then work through $\mathcal{E}$ keeping everything B leaves out.
- For (b)(i), compare A and C member by member and see what, if anything, they share.
- For (b)(ii), look for 13 among the members listed in $\mathcal{E}$.

Worked Solution [4 marks]

> Rule - Union, intersection and complement: $A \cup B$ holds every member of A together with every member of B; $A \cap C$ holds only the members A and C share; and B' holds every member of $\mathcal{E}$ that is not in B.

Step 1: list $A \cup B$

$A = \{2, 4, 6, 8, 10, 12\}$
$B = \{3, 6, 9, 12\}$
$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$

(Reason: The union collects everything that is in A or in B. 6 and 12 sit in both sets, but a member is written once however many sets it belongs to, so the list holds 8 members rather than 10. Writing them in order makes a missed member easy to spot.)

Step 2: list B'

$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$B = \{3, 6, 9, 12\}$
$B' = \{1, 2, 4, 5, 7, 8, 10, 11\}$

(Reason: The dash means "not in B", so start from $\mathcal{E}$ and cross out each of the four members B contains. Everything still standing belongs to B'.)

Step 3: work out $A \cap C$

$A = \{2, 4, 6, 8, 10, 12\}$
$C = \{1, 3, 5, 7, 9, 11\}$
$A \cap C = \varnothing$

(Reason: Every member of A is even and every member of C is odd, so no number can be in both. A set with no members at all is the empty set, written $\varnothing$, which is the symbol wanted from the box.)

Step 4: decide where 13 sits

$\mathcal{E} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\}$

$13 \notin \mathcal{E}$

(Reason: The universal set stops at 12, so 13 is not one of its members. $\notin$ is read as "is not a member of", so it is the symbol that makes the statement true.)

(a)(i) $A \cup B = \{2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12\}$

(a)(ii) $B' = \{1,\ 2,\ 4,\ 5,\ 7,\ 8,\ 10,\ 11\}$

(b)(i) $A \cap C = \varnothing$

(b)(ii) $13 \notin \mathcal{E}$

Verification

✓ **Check 1:** Count the union a second way instead of reading the list again: A has 6 members and B has 4, and the two members they share, 6 and 12, have been counted twice. $6 + 4 - 2 = 8$, and the answer to (a)(i) holds 8 members.

✓ **Check 2:** A set and its complement must share nothing and, between them, account for every member of $\mathcal{E}$ exactly once. Count B against B'. $4 + 8 = 12$, the number of members of $\mathcal{E}$, and no number appears in both lists.

✓ **Check 3:** Test the two symbols against the members themselves: hunt for a single number that is in both A and C, then hunt for 13 in the list for $\mathcal{E}$. Neither hunt finds anything, so $A \cap C = \varnothing$ and $13 \notin \mathcal{E}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $A \cup B$	**B1**	$2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12$	✓
(a)(ii) B'	**B1**	$1,\ 2,\ 4,\ 5,\ 7,\ 8,\ 10,\ 11$	✓
(b)(i)	**B1**	$\varnothing$	✓
(b)(ii)	**B1**	$\notin$	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

(a) A number line marked from -3 to 3, carrying an unfilled circle at -2, a filled circle at 1, and a solid line joining them. (b) The inequality $7a - 5 \leq 3a + 28$, with the unknown on both sides.

Find

(a) The inequality that the number line shows, written in symbols. (b) Every value of a for which the inequality is true.

Plan the Solution

- (a) Read the two ends separately. An unfilled circle leaves its own value out; a filled circle keeps it in.
- (a) Write the two ends either side of x so that one statement carries both.
- (b) Gather the a terms on one side of the sign and the plain numbers on the other.
- (b) Divide by the number in front of a. It is positive, so the sign still points the same way.

Worked Solution [4 marks]

> Rule - Reading a number line: an unfilled circle means that value is not included, so the sign is $<$ or $>$; a filled circle means it is included, so the sign is $\leq$ or $\geq$. Rule - Solving a linear inequality: add or subtract the same amount on both sides, then divide by the coefficient of the unknown. Dividing by a positive number leaves the sign pointing the same way; dividing by a negative number turns it round.

(a) Step 1: read the left-hand end

$-2 < x$

(Reason: The circle above -2 is not filled in, so -2 itself is not one of the values. Everything the line covers is greater than -2.)

(a) Step 2: read the right-hand end

$x \leq 1$

(Reason: The circle above 1 is filled in, so 1 is one of the values. Everything the line covers is less than or equal to 1.)

(a) Step 3: put the two ends into one inequality

$-2 < x \leq 1$

(Reason: The line runs from one end to the other with nothing missing in between, so x lies between the two values. Writing them either side of x keeps both ends, and both signs, in one statement.)

(b) Step 4: collect the a terms on one side and the numbers on the other

$7a - 5 \leq 3a + 28$

$7a - 3a \leq 28 + 5$

$4a \leq 33$

(Reason: Subtract $3a$ from both sides and add 5 to both sides. Adding or subtracting the same amount on both sides never changes the direction of the sign, so the $\leq$ is untouched.)

(b) Step 5: divide both sides by 4

$a \leq \dfrac{33}{4}$

$\dfrac{33}{4} = 8.25$

(Reason: 4 is positive, so dividing by it leaves the sign pointing the same way. The answer line wants a value, and $\frac{33}{4}$ written as a decimal is 8.25.)

(a) $-2 < x \leq 1$

(b) $a \leq 8.25$

Verification

- ✓ **Check 1 - part (a), the two ends:** Test each end against its own circle. The unfilled circle at -2 must be left out by the inequality, and the filled circle at 1 must be kept in. $-2 < -2$ is not true, so -2 is left out, and $1 \leq 1$ is true, so 1 is kept in. Both agree with the circles.
- ✓ **Check 2 - part (a), a value on the line and a value off it:** Take 0, which the drawn line passes over, and 2, which it does not reach. $-2 < 0 \leq 1$ is true, and 2 is greater than 1 so it fails the right-hand end. The inequality covers exactly what the picture shades.
- ✓ **Check 3 - part (b), a value inside and a value outside:** Put 8 in place of a in the original inequality, then 9. One is below 8.25 and one is above it, so the first should work and the second should not. $7 \times 8 - 5 = 51$ and $3 \times 8 + 28 = 52$, and $51 \leq 52$, so 8 works. But $7 \times 9 - 5 = 58$ and $3 \times 9 + 28 = 55$, and 58 is greater than 55, so 9 does not.
- ✓ **Check 4 - part (b), the boundary itself:** At the boundary the two sides should come to the same value, since that is where $\leq$ changes from true to false. Put 8.25 in place of a.

$7 \times 8.25 - 5 = 52.75$ and $3 \times 8.25 + 28 = 52.75$, so the two sides meet exactly at 8.25, which is why it is included in the answer.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $-2 < x \leq 1$	B2	accept $1 \geq x > -2$ or $x > -2, x \leq 1$ if not B2 then B1 for $-2 < x$ or $x \leq 1$ or $-2 \leq x < 1$ or $-2 \leq x \leq 1$ or $-2 < x < 1$ Condone use of a variable other than x but not 0	✓
(b) $7a - 3a \leq 28 + 5$ or $4a \leq 33$ or $-5 - 28 \leq 3a - 7a$ or $-33 \leq -4a$	M1	for a terms on one side and numbers on the other. Condone an equals sign rather than $\leq$, or any other sign, for this mark.	✓
(b) Working required. $a \leq 8.25$	A1	(dep on M1) oe eg $a \leq \frac{33}{4}$ or $a \leq 8\frac{1}{4}$ or $8.25 \geq a$ must have correct sign on answer line (sight of correct answer in working space and just 8.25 on answer line gains M1 only)	✓

Full marks: 4/4

Question 22 - Exam Solution

Understanding the Question

Given

(a) A journey of 1262 km that takes $17\frac{1}{2}$ hours, so a distance in kilometres and a time in hours.
(b) A speed of $50x$ metres per second, with the letter x standing for a number that is never given.

Find

(a) The average speed in kilometres per hour, rounded to the nearest whole number. (b) The same speed measured in kilometres per hour, still written in terms of x.

Plan the Solution

- (a) Turn the mixed number of hours into a decimal, so the division can be typed straight into the calculator.
- (a) Divide the distance by the time. Both are already in the units the answer asks for, so no conversion is needed afterwards.
- (b) Change the two units one at a time: seconds into hours first, then metres into kilometres.
- (b) Finish by noting that the two steps together are one multiplication by 3.6, which is worth remembering for any change from metres per second to kilometres per hour.

Worked Solution [5 marks]

> Rule - Average speed: $\text{average speed} = \dfrac{\text{distance}}{\text{time}}$, with the distance and the time measured in the units the answer asks for. Rule - Changing the units of a speed: change the distance unit and the time unit separately. Multiplying by 3600 turns a rate per second into a rate per hour, and dividing by 1000 turns metres into kilometres, so metres per second become kilometres per hour on multiplying by $\dfrac{3600}{1000} = 3.6$.

(a) Step 1: write the time as a decimal number of hours

$$17\frac{1}{2} = 17 + 0.5 = 17.5$$

(Reason: Half an hour is 0.5 of an hour, so the mixed number becomes 17.5. The calculator needs a single decimal number of hours before the division can be done.)

(a) Step 2: divide the distance by the time

$$\text{average speed} = \frac{\text{distance in km}}{\text{time in hours}}$$

$$\frac{1262}{17.5} \approx 72.11$$

(Reason: Speed is distance divided by time. The distance is already in kilometres and the time is now in hours, so the division gives a speed in kilometres per hour with nothing left to convert.)

(a) Step 3: round to the nearest whole number

$$\text{average speed} \approx 72 \text{ km/h}$$

(Reason: The first digit after the decimal point is 1, which is below 5, so the whole-number part is left as it stands. The working was already in km/h, which is the unit the answer line asks for.)

(b) Step 4: change the seconds into hours

1 hour $= 60 \times 60 = 3600$ seconds

$50x \times 3600 = 180\,000x$

(Reason: A speed of $50x$ metres every second covers 3600 times as far in an hour, because an hour is 3600 seconds. So the coach covers $180\,000x$ metres in one hour.)

(b) Step 5: change the metres into kilometres

$1 \text{ km} = 1000 \text{ m}$

$$\frac{180\,000x}{1000} = 180x$$

(Reason: There are 1000 metres in a kilometre, so dividing the number of metres by 1000 gives the number of kilometres. The x is only a multiplier and rides through both steps untouched.)

(b) Step 6: do the same thing in one multiplication

$$\frac{3600}{1000} = 3.6$$

$50x \times 3.6 = 180x$

(Reason: Multiplying by 3600 and then dividing by 1000 is one multiplication by 3.6, and it gives the same $180x$. Either route earns the marks; this one is quicker once the factor is known.)

(a) 72 km/h

(b) $180x$ km/h

Verification

✓ **Check 1 - part (a), put the rounded speed back into the journey:** Multiply the answer by the time and see how close the distance comes back. Rounding 72.11 down to 72 should leave the journey slightly short, not long. $72 \times 17.5 = 1260$, which is 2 km short of 1262. A shortfall that small over $17\frac{1}{2}$ hours is exactly what rounding down by 0.11 km/h produces.

✓ **Check 2 - part (a), work the time back out instead:** Turn the division round: divide the distance by the answer and see whether the time given comes back. $\frac{1262}{72} \approx 17.53$ hours, against the 17.5 hours the question gives. The gap is a few tenths of a minute and comes only from the rounding.

✓ **Check 3 - part (b), test it on one value:** Put $x = 1$, which makes the speed a plain 50 metres per second, and work out the distance covered in one hour from scratch.

$50 \times 3600 = 180\,000$ metres in an hour, and $\dfrac{180\,000}{1000} = 180$ kilometres, so the speed is 180 km/h. That is $180x$ with $x = 1$.

✓ **Check 4 - part (b), reverse the conversion:** Take the answer back the other way. Turning $180x$ km/h into metres per second must return the speed the question started with. $180x \times 1000 = 180\,000x$ metres in an hour, and $\dfrac{180\,000x}{3600} = 50x$ metres per second, which is the speed the question gives.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\dfrac{1262}{17.5}$ oe	**M1**	$\dfrac{1262}{\text{their time}}$ their time may be an incorrect conversion to a decimal time eg 17.3 or from an attempt at converting to minutes eg 1050	✓
(a) 72	**A1**	accept 72.1 or $72.11\ldots$	✓
(b) $\dfrac{50x}{1000}$ $(= 0.05x)$ oe or $50x \times 60 \times 60$ $(= 180\,000x)$ oe or $\dfrac{\frac{50x}{1}}{3600}$ $(= 180\,000x)$ oe or $\dfrac{50x}{1000} \times 60$ $(= 3x)$ or $\dfrac{3600}{1000}$ or $\dfrac{18}{5}$ or 3.6 or $\dfrac{1000}{3600}$ or $\dfrac{5}{18}$ or $0.277(77\ldots)$	**M1**	Condone omission of x for this mark	✓
(b) $\dfrac{50x \times 60 \times 60}{1000}$ oe or $50x \times 3.6$ oe	**M1**	for a complete method including x or for an answer of 180	✓

Step	Mark	Description	Got it?
or $\dfrac{\frac{50x}{1000}}{3600}$ oe or 180			
(b) $180x$	A1	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

A bracket to expand: $x(x-3)$

A formula: $m = \dfrac{t+4}{5}$

Two powers to simplify: $a^6 \times a^{10}$ and $\dfrac{c^{30}}{c^{12}}$

A quadratic expression: $y^2 - 10y + 21$

Find

(a) the bracket multiplied out (b) t written in terms of m (c) and (d) each written as a single power of a and of c (e) (i) the two brackets, and (ii) the values of y that make the expression zero

Plan the Solution

- (a) Multiply each term inside the bracket by the x outside it.
- (b) Clear the fraction first by multiplying both sides by 5, then subtract 4 from both sides.
- (c) and (d) Use the index laws: powers of the same letter are multiplied by adding the indices and divided by subtracting them.
- (e) Look for two numbers with product 21 and sum -10. Part (ii) says "hence", so read the solutions straight off the brackets rather than starting again.

Worked Solution [8 marks]

Rule - the four tools this question needs: expand by multiplying every term inside the bracket, $a^m \times a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$, and $y^2 + by + c = (y+p)(y+q)$ when

$pq = c$ and $p + q = b$.

Step 1: (a) multiply the bracket out

$$x(x-3) = x \times x - x \times 3$$
$$= x^2 - 3x$$

(Reason: Both terms inside the bracket are multiplied by the x outside, and $x \times x$ is x^2, not $2x$.)

Step 2: (b) clear the fraction

$$m = \frac{t+4}{5}$$
$$5m = t + 4$$

(Reason: The whole of $t+4$ is being divided by 5, so multiplying both sides by 5 undoes it in one move and leaves t on top.)

Step 3: (b) make t the subject

$$5m - 4 = t$$
$$t = 5m - 4$$

(Reason: Taking 4 from both sides leaves t on its own. Writing it with t on the left is what "make t the subject" asks for.)

Step 4: (c) multiply the powers of a

$$a^6 \times a^{10} = a^{6+10} = a^{16}$$

(Reason: The letter stays the same and the two indices are combined into a single power in one step.)

Step 5: (d) divide the powers of c

$$\frac{c^{30}}{c^{12}} = c^{30-12} = c^{18}$$

(Reason: Twelve of the c's on the top cancel with the twelve on the bottom, so the indices are subtracted.)

Step 6: (e)(i) find the pair of numbers, then factorise

$$(-3) \times (-7) = 21$$
$$(-3) + (-7) = -10$$
$$y^2 - 10y + 21 = (y-3)(y-7)$$

(Reason: The constant 21 is positive and the y term is negative, so both numbers must be negative. -3 and -7 are the only pair with product 21 and sum -10.)

Step 7: (e)(ii) solve using the brackets

$$(y-3)(y-7) = 0$$
$$y - 3 = 0 \quad \text{or} \quad y - 7 = 0$$

$y = 3 \quad$ or $\quad y = 7$

(Reason: Two numbers multiply to give zero only when one of them is zero, so each bracket is set to zero in turn. This is why part (i) was asked first.)

(a) $x^2 - 3x$

(b) $t = 5m - 4$

(c) a^{16}

(d) c^{18}

(e)(i) $(y-3)(y-7)$

(e)(ii) $y = 3$ **or** $y = 7$

Verification

✓ **Check 1 - part (a):** Put $x = 5$ into both forms. The bracket gives 5×2 and the expanded expression gives $25 - 15$. $10 = 10$

✓ **Check 2 - part (b):** Put the rearranged t back into the original formula. The -4 and the $+4$ cancel, leaving $5m$ over 5. $\frac{(5m-4)+4}{5} = m$

✓ **Check 3 - parts (c) and (d):** Count the letters instead of using the laws. a^6 is six a's and a^{10} is ten more, which is sixteen a's altogether. For (d), reverse it: multiplying the answer by c^{12} must return c^{30}. a^{16} and $c^{18} \times c^{12} = c^{30}$

✓ **Check 4 - part (e):** Multiply the brackets back out: $y^2 - 7y - 3y + 21$. Then substitute each solution into the original expression: $9 - 30 + 21$ and $49 - 70 + 21$. $y^2 - 10y + 21$, and both solutions give 0

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	B1	$x^2 - 3x$	✓
(b)	M1	for a correct first step, eg $5m = t + 4$ or $m = \frac{t}{5} + \frac{4}{5}$	✓
(b)	A1	$t = 5m - 4$, oe eg $t = 5\left(m - \frac{4}{5}\right)$ or $t = -4 + 5m$. $5m - 4$ only on the answer line scores M1 unless $t = 5m - 4$ is seen in the	✓

Step	Mark	Description	Got it?
		working, then score M1A1. Correct answer scores full marks (unless from obvious incorrect working).	
(c)	**B1**	a^{16}	✓
(d)	**B1**	c^{18}	✓
(e)(i)	**M1**	for $(y \pm 3)(y \pm 7)$ or for $(y \pm a)(y \pm b)$ with $ab = 21$ or $a + b = -10$	✓
(e)(i)	**A1**	for correct factors $(y - 3)(y - 7)$. Correct answer scores full marks (unless from obvious incorrect working).	✓
(e)(ii)	**B1**	$3, 7$ ft dep on factorising in the form $(y \pm p)(y \pm q)$	✓

Full marks: 8/8

Question 24 - Exam Solution

Understanding the Question

Given

Triangle PQR has its right angle at Q.
The diagram labels the vertical side: $PQ = 6.5$ cm.
The angle marked at R is 24°.
The figure is not drawn to scale, so nothing may be measured off it.

Find

The length of QR, correct to 3 significant figures.

Plan the Solution

- Stand at the 24° angle and name the two sides the question involves. PQ is across the triangle from it, so it is the opposite side; QR runs from it to the right angle, so it is the adjacent side.
- Opposite with adjacent is the tangent ratio, so the hypotenuse PR is never needed and Pythagoras is not the tool here.
- The unknown lands underneath the fraction, so rearrange first and press the calculator once.

- Keep the whole calculator display, and round to 3 significant figures only at the very end.

Worked Solution [3 marks]

Rule - Tangent: in a right-angled triangle $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$, where the two sides are named from the angle θ itself. Set the calculator to degrees before pressing anything.

Step 1: Choose the ratio that uses the two sides in the question

$$\tan R = \frac{PQ}{QR}$$

(Reason: Seen from R, the side PQ is opposite and the side QR is adjacent. Opposite over adjacent is the tangent, and the hypotenuse PR does not appear in it.)

Step 2: Put in the angle and the length the diagram gives

$$\tan 24^\circ = \frac{6.5}{QR}$$

(Reason: The angle at R is 24°, and the side opposite it is PQ, which is 6.5 cm. Everything in the equation is now a number except QR.)

Step 3: Rearrange to make QR the subject

$$QR \times \tan 24^\circ = 6.5$$

$$QR = \frac{6.5}{\tan 24^\circ}$$

(Reason: The unknown is underneath, so multiply both sides by QR to lift it up, then divide both sides by $\tan 24^\circ$ to leave it alone.)

Step 4: Work it out, with the calculator in degrees

$$QR = 14.5992\ldots$$

(Reason: Type the division in one go and keep the whole display. Rounding here and then carrying the rounded number on is how an answer drifts outside the range the mark scheme accepts.)

Step 5: Round to 3 significant figures

$$QR = 14.6 \text{ cm}$$

(Reason: The first three significant figures are 1, 4 and 5. The digit after them is 9, so the 5 rounds up to 6. The question asks for a length, so the unit goes on the answer line.)

$$QR = 14.6 \text{ cm}$$

Verification

✓ **Check 1:** Put the answer back where it came from. Multiplying $14.5992\ldots$ by $\tan 24°$ has to give back the 6.5 cm the diagram prints. $14.5992\ldots \times \tan 24° = 6.5$

✓ **Check 2:** Come at it from the other acute angle. The angles of a triangle add to $180°$, so angle QPR is $180 - 90 - 24 = 66$ degrees, and from P it is QR that is opposite and 6.5 cm that is adjacent. $QR = 6.5 \times \tan 66° = 14.5992\ldots$

✓ **Check 3:** Take a route with no tangent in it at all: the sine ratio gives the hypotenuse PR, and Pythagoras then gives QR. The answer must also come out longer than 6.5 cm, because $24°$ is less than $45°$. $PR = \dfrac{6.5}{\sin 24°} = 15.9808\ldots$

$QR = \sqrt{PR^2 - 6.5^2} = 14.5992\ldots$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\tan 24 = \dfrac{6.5}{QR}$ or $\dfrac{6.5}{\sin 24} = \dfrac{QR}{\sin(180-90-24)}$ oe or $\tan(180-90-24) = \dfrac{QR}{6.5}$ or $(PR =)\dfrac{6.5}{\sin 24}(= 15.9\ldots)$ and $6.5^2 + QR^2 = 15.9^2$	**M1**	for setting up a trig equation in QR or for a complete method to find PR and then setting up Pythagoras or a trig equation for QR. The printed scheme puts 15.9 in quotation marks, which means the candidate's own value for PR may be used there.	✓
eg $(QR =)\dfrac{6.5}{\tan 24}$ or $(QR =)\dfrac{6.5}{\sin 24} \times \sin 66$ or $(QR =)6.5 \tan 66$ [where $66 = 180 - 90 - 24$] or $(QR =)\sqrt{15.9^2 - 6.5^2}$	**M1**	for a complete method	✓
14.6 Correct answer scores full marks (unless from obvious incorrect working)	**A1**	accept 14.5 to 14.61	✓

Full marks: 3/3

Question 25 - Exam Solution

Understanding the Question

Given

Cuboid tank: 35 cm by 28 cm by 20 cm
Water already standing in the cuboid tank: 9 cm deep
Cylindrical tank: radius 10 cm, height 33 cm, completely full of water

Find

Show that pouring all the water from the cylindrical tank into the cuboid tank does not fill the cuboid tank. A show-that question, so the working is the answer: two volumes and a comparison between them.

Plan the Solution

- Find the depth of empty space above the water in the cuboid tank, then the volume of that space.
- Find the volume of water in the cylindrical tank.
- Compare the two. If the water poured in is less than the space waiting for it, the tank cannot end up full.

Worked Solution [3 marks]

> Volume of a cuboid: multiply the three dimensions together. Volume of a cylinder: $\pi r^2 h$.

Step 1: the depth of empty space above the water

$20 - 9 = 11$

(Reason: The tank is 20 cm deep and the water reaches 9 cm up it, so 11 cm of depth is still empty.)

Step 2: the volume of that empty space

$11 \times 35 \times 28 = 10\,780$

(Reason: The empty part of the tank is a cuboid in its own right: the same 35 cm by 28 cm base, with the empty depth from Step 1 as its height.)

Step 3: the volume of water in the cylindrical tank

$10^2 \times 33 = 3300$

$\pi \times 3300 = 10\,367.3$

(Reason: Volume of a cylinder is $\pi r^2 h$, with $r = 10$ and $h = 33$. Holding the π back until the last line keeps the rounding out of the working; to one decimal place the volume is $10\,367.3$ cm³.)

Step 4: compare the water with the space

$10\,367.3 < 10\,780$

$10\,780 - 10\,367.3 = 412.7$

(Reason: There is less water in the cylinder than there is space waiting for it, so all of it fits and 412.7 *cm³ of the cuboid tank is still empty afterwards.)*

$10\,367.3$ cm³ of water is poured into $10\,780$ cm³ of empty space, leaving about 412.7 cm³ unfilled, so the cuboid tank will not be completely full of water.

Verification

✓ **Check 1:** Work with totals rather than with the gap. The water already in the cuboid tank is $35 \times 28 \times 9 = 8\,820$ cm³, and the tank holds $35 \times 28 \times 20 = 19\,600$ cm³ altogether. After the pour there is $8\,820 + 10\,367.3 = 19\,187.3$ cm³ of water, and $19\,600 - 19\,187.3 = 412.7$ cm³ of the tank is still empty. The same shortfall, from the other end.

✓ **Check 2:** A bound, with no rounding in it at all. Since π is smaller than 3.15, the cylinder must hold less than $3300 \times 3.15 = 10\,395$ cm³. Under $10\,395$ cm³ of water going into $10\,780$ cm³ of space, so the tank cannot fill however the rounding is done.

✓ **Check 3:** The empty depth is 11 of the tank's 20 cm, so the empty space should be $\frac{11}{20}$ of the whole tank: $\frac{11}{20} \times 19\,600 = 10\,780$ cm³. The same $10\,780$ cm³ as Step 2, reached from the tank's capacity instead of from its base area.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(volume of water =) $9 \times 35 \times 28$ $(= 8\,820)$ or (total volume of cuboid =) $20 \times 35 \times 28$ $(= 19\,600)$ or (volume of space =) $(20 - 9) \times 35 \times 28$ $(= 10\,780)$	M1	for a method to find a relevant volume for the cuboid	✓

Step	Mark	Description	Got it?
$\pi \times 10^2 \times 33$ (= 3300π or $10\,367(.25\ldots)$) oe	**M1 indep**	(indep) for a method to find the volume of the cylinder, accept a volume in the range $10\,362$ to $10\,368.6$. Allow $3.14\ldots$ or $\frac{22}{7}$ for π	✓
(total volume of water =) "$8\,820$ " + "$10\,367(.25\ldots)$" (= $19\,187(.25\ldots)$) or (difference between the volumes of both solids =) " $19\,600$" − "$10\,367(.25\ldots)$" (= $9\,232(.74\ldots)$) or (volume not filled =) "$19\,600$ " − "$8\,820$" − "$10\,367(.25\ldots)$ " (= $412(.74\ldots)$) Shown	**A1**	correct workings with accurate figures, eg $10\,780$ with $10\,367(.25\ldots)$ (accept $10\,362$ to $10\,372$), or $19\,600$ with $19\,187(.25\ldots)$ (accept $19\,182$ to $19\,192$), or $8\,820$ with $9\,232(.74\ldots)$ (accept $9\,228$ to $9\,238$), or $412(.74\ldots)$ or 413 (accept 408 to 418) with no second value needed. The values in quotation marks in the step column may be the candidate's own earlier values.	✓
Working required	**Note**	A show-that question: the conclusion on its own earns nothing. Both volumes and the comparison between them have to be on the page. The printed scheme puts Shown in the answer column and Working required beneath the working.	✓

Full marks: 3/3

Question 26 - Exam Solution

Understanding the Question

Given

Harbour plan: $2500 invested, with amount invested : interest after 2 years in

Find

The interest the Harbour plan pays The interest the Meridian plan pays How

the ratio $20 : 3$
Meridian plan: $\$3000$ invested at 4% per year compound interest, for 2 years

much more the Harbour plan pays than the Meridian plan

Plan the Solution

- The Harbour ratio compares the amount invested with the interest, so the $\$2500$ invested is 20 equal shares. Divide by 20 for one share, then take 3 shares for the interest.
- Compound interest is not the same amount every year: each year's 4% is worked out on the new total, so multiply by 1.04 once for each year.
- That gives the Meridian TOTAL, not its interest, so subtract the $\$3000$ invested.
- Only then subtract: interest against interest, never a total against an interest.

Worked Solution [5 marks]

> Rule - Ratio share: one share is the amount divided by the number of shares in it. Rule - Compound growth: multiply by $1 + r$ once for each year, where r is the yearly rate written as a decimal, then subtract the amount invested to leave the interest.

Step 1: The interest the Harbour plan pays

$$\frac{2500}{20} = 125$$

$$3 \times 125 = 375$$

(Reason: the ratio $20 : 3$ compares the amount invested with the interest, so the $\$2500$ invested is 20 equal shares. One share is worth $\$125$, and the interest is 3 of those shares)

Step 2: The Meridian total after both years

$$1 + \frac{4}{100} = 1.04$$

$$3000 \times 1.04^2 = 3244.8$$

(Reason: adding 4% means keeping the 100% already there and adding 4 parts more, which is one multiplication by 1.04. The second year's 4% is charged on the new total, not on the original $\$3000$, so the multiplier is used twice)

Step 3: The interest the Meridian plan pays

$$3244.8 - 3000 = 244.8$$

(Reason: the $\$3244.80$ is everything in the plan, so it still contains the $\$3000$ Wei put in. Taking that out leaves the interest on its own)

Step 4: How much more the Harbour plan pays

$$375 - 244.8 = 130.2$$

(Reason: both figures are interest, so taking one from the other compares like with like; subtracting a total from an interest would compare two different things)

$130.20

Verification

✓ **Check 1 - rebuild the Harbour ratio:** Divide both parts of $2500 : 375$ by 125 and see whether the ratio really is $20 : 3$. $\frac{2500}{125} = 20$ and $\frac{375}{125} = 3$, so the ratio is $20 : 3$

✓ **Check 2 - the Meridian plan one year at a time:** Instead of squaring, add the interest twice over: $3000 \times 1.04 = 3120$ at the end of the first year. $3120 \times 1.04 = 3244.8$, the same total, so the interest is $3244.8 - 3000 = 244.8$

✓ **Check 3 - put the difference back:** Add the smaller interest on to the difference. It must give the larger interest. $244.8 + 130.2 = 375$, the Harbour interest

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Interest for the Harbour plan	**M1**	for a method to find the interest for the Harbour plan: $\frac{2500}{20} \times 3 = 375$ oe, or $125 \times 3 = 375$, or $\frac{7500}{20} = 375$ oe; or $\frac{3000}{20} \times 3 = 450$ oe, or $150 \times 3 = 450$, or $\frac{9000}{20} = 450$ oe. An answer of 2875 or 3450 implies this method mark.	✓
One year's growth on the Meridian plan	**M1**	for finding 4% or 104% of 3000 or of 2500: $0.04 \times 3000 = 120$ oe, or $0.04 \times 2500 = 100$ oe, or $1.04 \times 3000 = 3120$ oe, or $1.04 \times 2500 = 2600$ oe.	✓
The Meridian total in one step	**M2**	as an alternative to that method mark and the next one, for $3000 \times 1.04^2 = 3244.8$ or $2500 \times 1.04^2 = 2704$.	✓
The Meridian	**M1**	for completing the method to find the total amount for the Meridian plan: $1.04 \times 3120 = 3244.8$ oe, or $1.04 \times 2600 = 2704$ oe, where the candidate's own value	✓

Step	Mark	Description	Got it?
total after both years		from the previous mark may stand in place of 3120 or 2600; or $3000 \times 1.04^3 = 3374.59$ or $2500 \times 1.04^3 = 2812.16$.	
Interest for the Meridian plan	**M1**	for a complete method to find the interest for the Meridian plan, for example $3244.8 - 3000 = 244.8$ or $2704 - 2500 = 204$, where the candidate's own total may stand in place of 3244.8 or 2704.	✓
The answer	**A1**	for 130.2 or 130.20. A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Special case	**SC**	if none of the second or third method marks is gained, award M1 for $0.08 \times 3000 = 240$ oe, or $1.08 \times 3000 = 3240$, or $0.08 \times 2500 = 200$ oe, or $1.08 \times 2500 = 2700$, or $3000 \times (1 - 0.04)^2 = 2764.8$, or $2500 \times (1 - 0.04)^2 = 2304$. Any of 240, 3240, 200 or 2700 seen on its own is enough. This is the student who treats the yearly interest as the same amount every year, so the second year's interest is taken on the original amount again.	✓
Note	**Note**	accept $(1 + 0.04)$ or $\left(1 + \dfrac{4}{100}\right)$ as equivalent to 1.04 throughout.	✓

Full marks: 5/5

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