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Edexcel IGCSE 4MA1

4MA1/1H (Higher Tier) – Thursday 15 May 2025

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/1H (Higher Tier), June 2025 series, sat Thursday 15 May 2025 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. The table gives information about the distances 100 members of a swimming club travel to reach the pool.

Distance (d **km**)	**Frequency**
$0 < d \leq 5$	26
$5 < d \leq 10$	40
$10 < d \leq 15$	16
$15 < d \leq 20$	10
$20 < d \leq 25$	8

(a) Write down the modal class.

[1 mark]

(b) Work out an estimate for the mean distance. *[4 marks]*

(a) ....

(b) km ....

[Total 5 marks]

2. Elise makes candles.
Each candle costs 6 euros to make.

Elise packs the candles into boxes to sell.
Each box contains 4 candles.

Elise sells 80 boxes of candles for a total of 2160 euros.

(a) Work out the percentage profit Elise makes.
Show your working clearly. *[4 marks]*

The height of each candle is 9 cm, correct to the nearest cm

(b) Write down the lower bound of the height. *[1 mark]*

The weight of each candle is 120 g, correct to the nearest 10 g

(c) Write down the upper bound of the weight. *[1 mark]*

(a) % ....

(b) cm ....

(c) g ....

[Total 6 marks]

3. AB and BC are two sides of a regular polygon that has n sides.

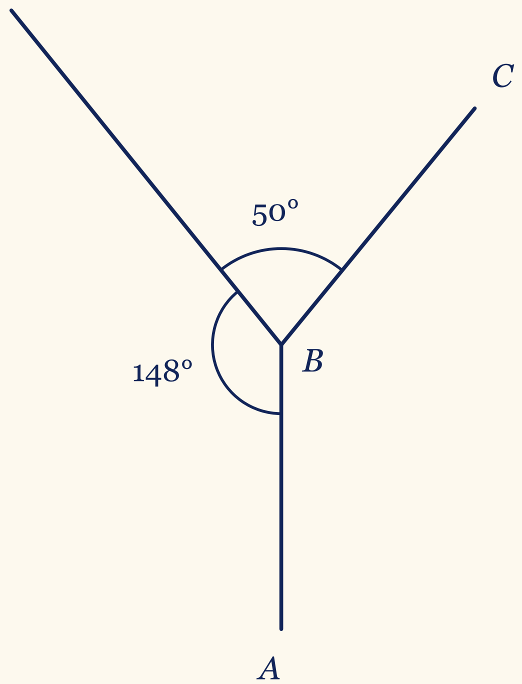


Diagram NOT accurately drawn

Find the value of n.

You must show your working clearly. *[4 marks]*

n = ...

[Total 4 marks]

4. $x^5 \times x^7 = x^m$

(a) Work out the value of m *[1 mark]*

$$\frac{y^8}{y^3} = y^n$$

(b) Work out the value of n *[1 mark]*

(c) Simplify fully $(5a^4r^2)^3$ *[2 marks]*

(a) m = ...

(b) n = ...

(c) ....

[Total 4 marks]

5. A music shop is having a sale.
In the sale, normal prices are reduced by 28%
The sale price of a guitar is 198 euros.

Work out the normal price of the guitar. *[3 marks]*

.... euros

[Total 3 marks]

6. (a) Solve the equation $x - 4 = \dfrac{3 + 2x}{6}$

You must show clear algebraic working. *[3 marks]*

(b) (i) Factorise $y^2 - 11y + 30$
[2 marks]
(ii) Hence solve $y^2 - 11y + 30 = 0$ *[1 mark]*

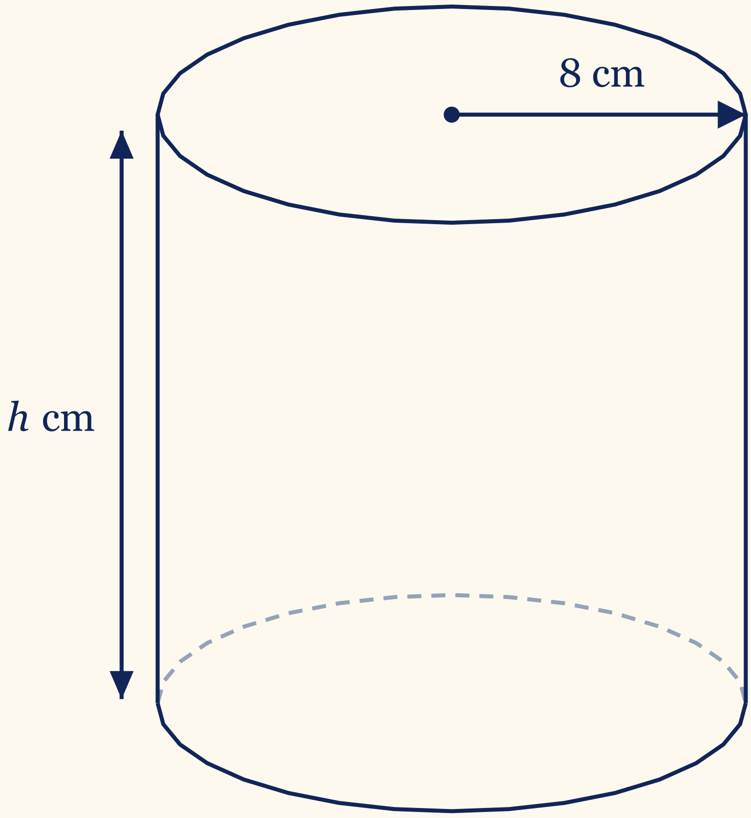
(a) x = ...

(b)(i) ....

(b)(ii) ....

[Total 6 marks]

7. The diagram shows a solid cylinder.



Not drawn accurately

The radius of the cylinder is 8 cm.
The height of the cylinder is h cm.
The volume of the cylinder is 3892 cm^3.

Find the value of h.
Give your answer correct to one decimal place. *[3 marks]*

h = ...

[Total 3 marks]

8. (a) Express 520 million in standard form. *[1 mark]*

(b) Convert 8.79×10^{-5} to an ordinary number. *[1 mark]*

(c) Find the value of $(5 \times 10^{42}) \times (7 \times 10^{-180})$

Give your answer in standard form. *[2 marks]*

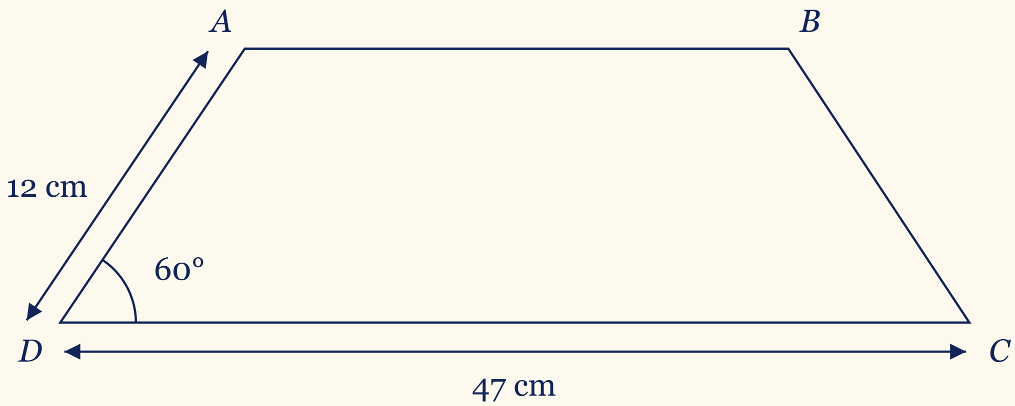
(a) ....

(b) ....

(c) ....

[Total 4 marks]

9. The diagram shows a trapezium $ABCD$ that has one line of symmetry.



Not drawn accurately

angle $ADC = 60°$ $AD = 12$ cm $DC = 47$ cm

Find the area of the trapezium.

Give your answer correct to 3 significant figures.

You must show your working clearly. *[5 marks]*

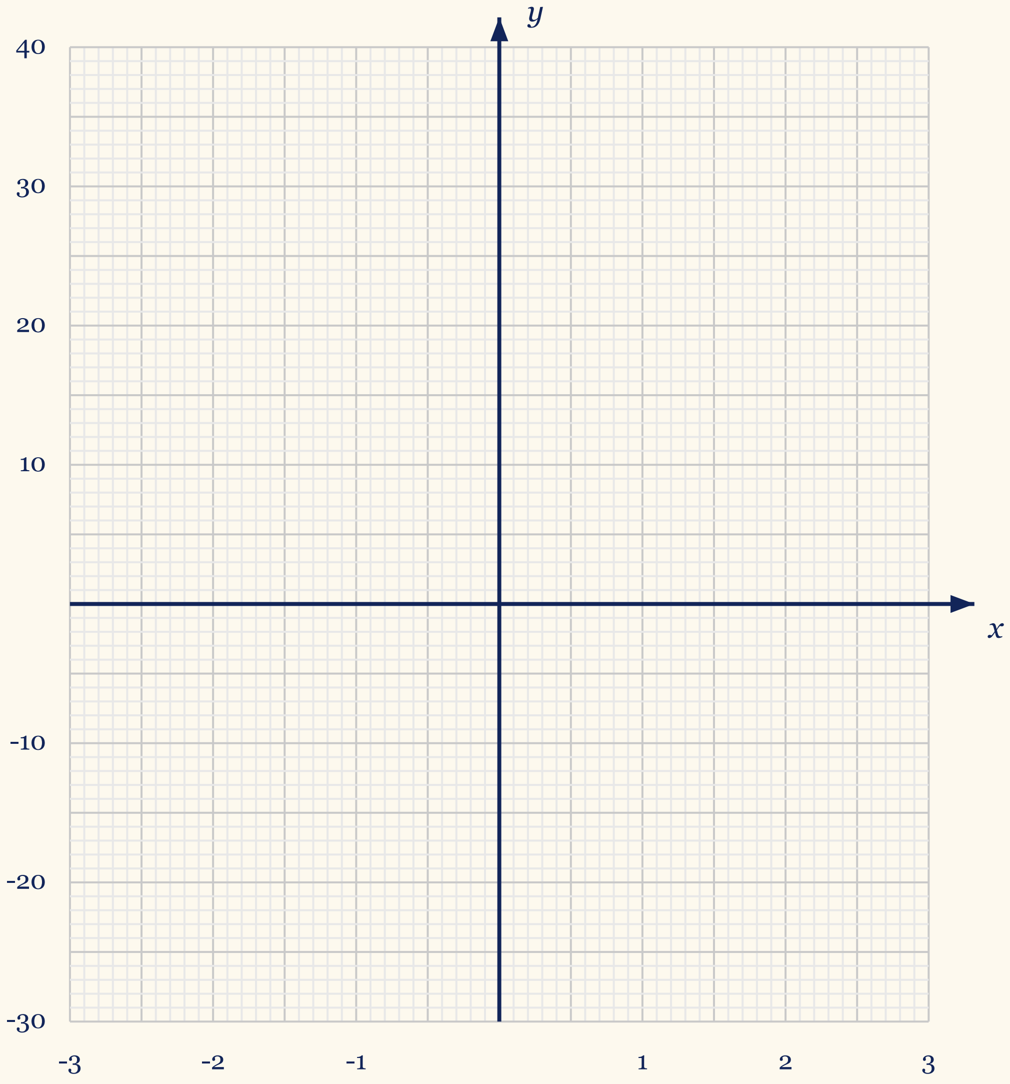
.... cm²

[Total 5 marks]

10. (a) The table below gives values of $y = x^3 + 2x + 3$ for values of x from -3 to 3

x	-3	-2	-1	0	1	2	3
y		-9	0	3	6		36

Work out the two missing values and complete the table. *[1 mark]*

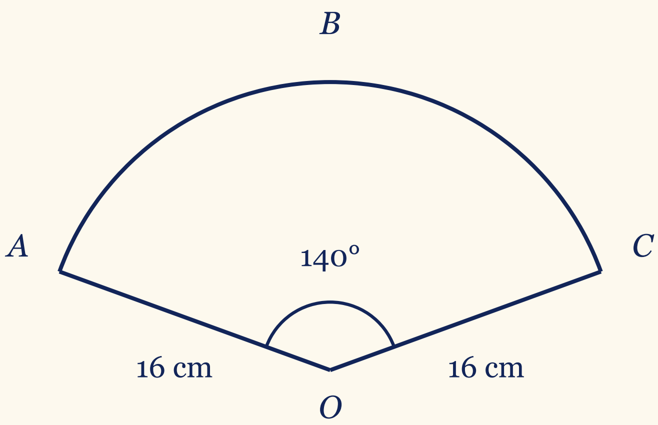


(b) Draw the graph of $y = x^3 + 2x + 3$ on the grid below, for $-3 \leq x \leq 3$ *[2 marks]*

[Total 3 marks]

11. $OABC$ is a sector of a circle with centre O.

Not drawn accurately



In the sector, angle $AOC = 140°$ and $OA = OC = 16$ cm.

Work out the area of the sector.
Give your answer correct to 3 significant figures. *[2 marks]*

.... cm²

[Total 2 marks]

12. Write $\frac{5}{4} + \frac{x-3}{6x}$ as a single fraction in its simplest form. *[3 marks]*

....

[Total 3 marks]

13. Beatrice bought a lakeside cabin for $\$750\,000$

In the first year, the value of the cabin fell by 4%
In the second year, the value of the cabin fell by 6.5%

In the third year, the value of the cabin rose by $x\%$

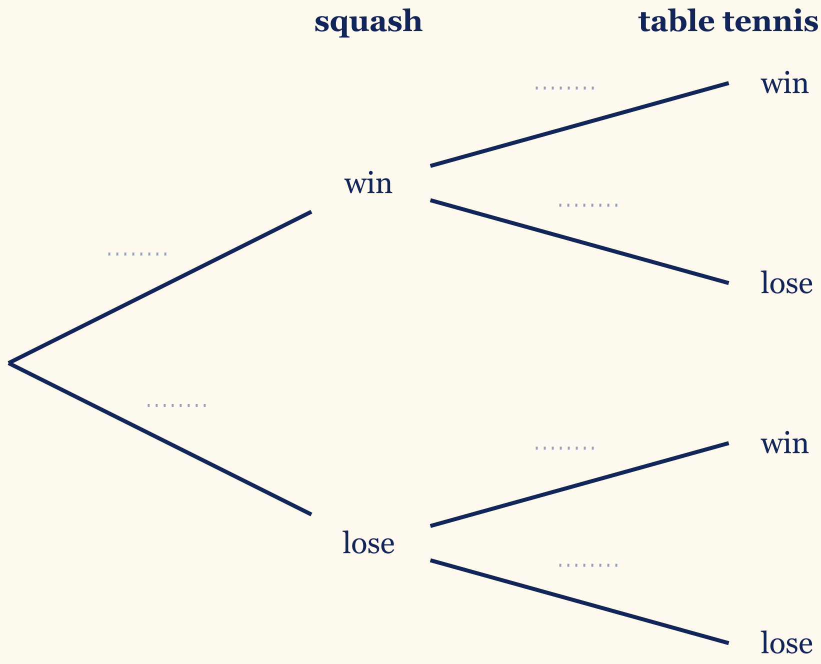
At the end of the third year, the value of Beatrice's cabin was $\$698\,445$

Work out the value of x *[3 marks]*

x = ...

[Total 3 marks]

14. Lorenzo is going to play one game of squash and one game of table tennis.



He can either win or lose each game.

The probability that he will win the game of squash is 0.7
The probability that he will win the game of table tennis is 0.4

(a) Complete the probability tree diagram. *[2 marks]*

(b) Work out the probability that Lorenzo loses both games. *[2 marks]*

(b) ....

[Total 4 marks]

15. $\left(\sqrt{3}\right)^5 = k\sqrt{3}$ where k is an integer.

(a) Work out the value of k *[1 mark]*

(b) Show that $\dfrac{21}{3-\sqrt{2}}$ can be written in the form $c+\sqrt{d}$

where c and d are integers.

Set out each stage of your working clearly. *[3 marks]*

k = ...

[Total 4 marks]

16. $\mathrm{f}(x) = 9 - \sqrt{x}$ where $x \geqslant 0$

$\mathrm{g}(x) = 4x^2$

(a) Work out $\mathrm{f}(9)$ *[1 mark]*

(b) Solve $\mathrm{fg}(x) < 0$

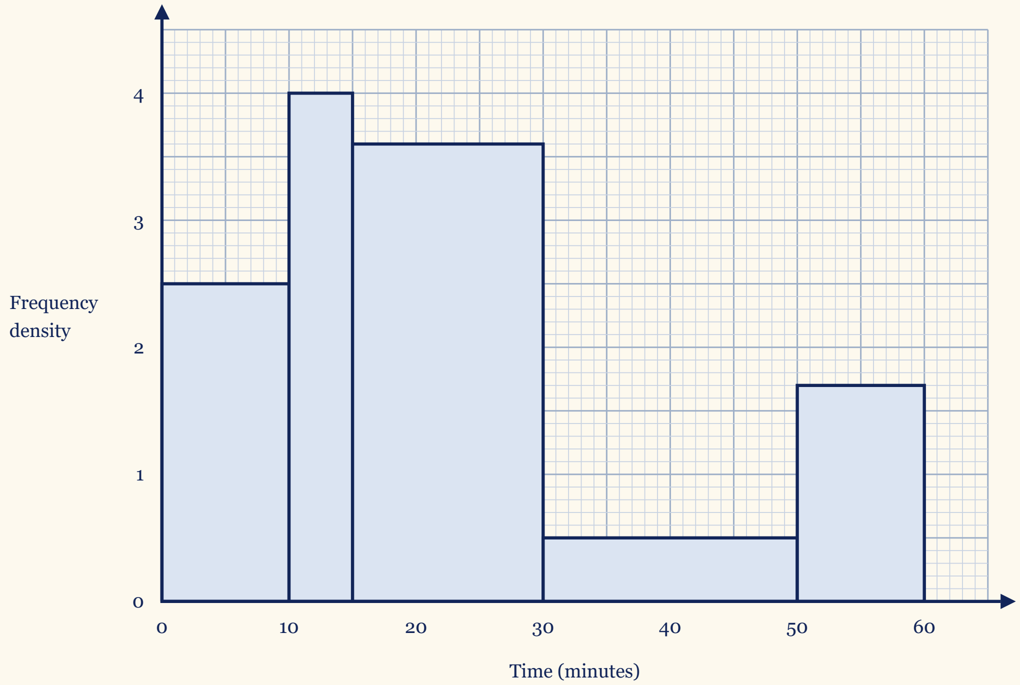
You must show clear algebraic working. *[3 marks]*

(a) ....

(b) ....

[Total 4 marks]

17. The histogram gives information about the times, in minutes, that some visitors spent in an art gallery.



Work out an estimate for the proportion of these visitors who spent more than 40 minutes in the gallery. *[3 marks]*

....

[Total 3 marks]

18. y is inversely proportional to x^3

When $x = 3, y = 4$

Find the value of x when $y = 864$ *[4 marks]*

x = ...

[Total 4 marks]

19. A curve has equation $y = \mathrm{f}(x)$
The curve has exactly one minimum point.
The coordinates of that minimum point are $(8, -12)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = \mathrm{f}(x) + 3$ *[1 mark]*

(ii) $y = \mathrm{f}(2x)$ *[1 mark]*

(i) ....

(ii) ....

[Total 2 marks]

20. The diagram shows a quadrilateral $ABCD$.
The diagonal BD is drawn.

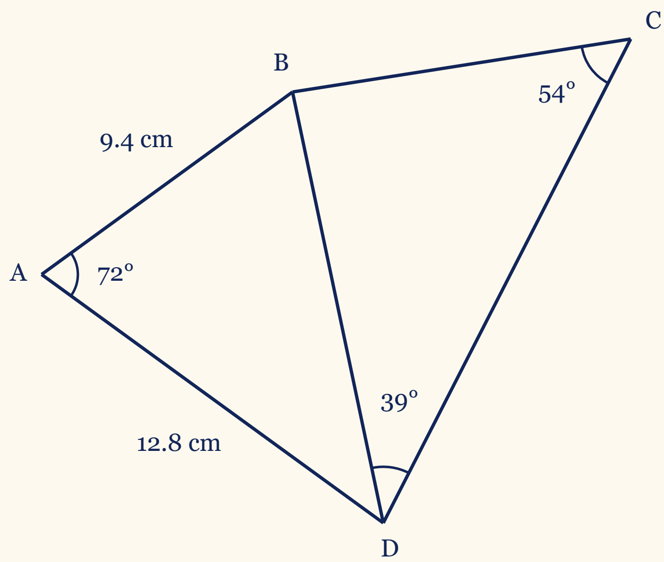


Diagram NOT accurately drawn

Find the length of BC.
Give your answer correct to 3 significant figures. *[5 marks]*

.... cm

[Total 5 marks]

21. A jar contains 20 beads.

9 of the beads are red
7 of the beads are yellow
4 of the beads are green

Callum takes at random three beads from the jar.

Work out the probability that exactly two of the three beads are the same colour. *[3 marks]*

....

[Total 3 marks]

22. Solve the simultaneous equations

$$x^2 + 3y + y^2 = 7$$
$$y = x + 2$$

You must show clear algebraic working. *[5 marks]*

....

[Total 5 marks]

23. In the diagram, OAB is a triangle.
P is the midpoint of OA
Q is a point on OB

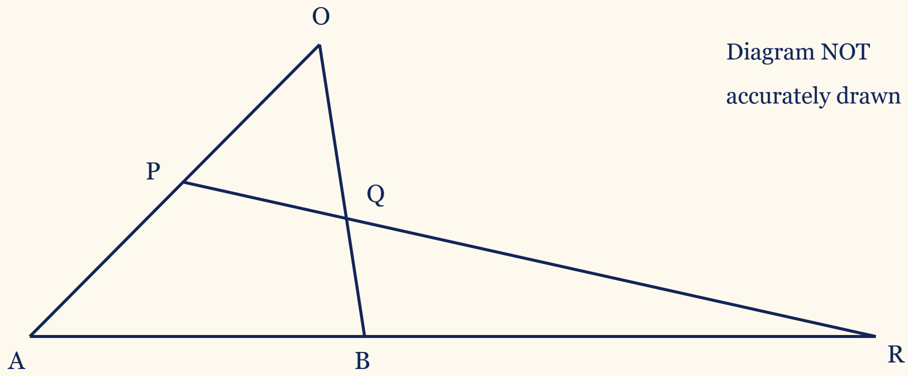


Diagram NOT accurately drawn

ABR and PQR are straight lines.

$\overrightarrow{OA} = 12\mathbf{a}$ $\overrightarrow{OB} = 8\mathbf{b}$

(a) Write $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ *[1 mark]*

$AB : BR = 1 : 2$ $\overrightarrow{OQ} = n\mathbf{b}$

(b) Using a vector method, work out the value of n *[4 marks]*

(a) ....

(b) n = ...

[Total 5 marks]

24. An arithmetic series has 30 terms.

The first term of the series is a
The common difference between consecutive terms is d

The 20th term of the series is 123
The sum of all 30 terms of the series is 2880

Find the value of a and the value of d
You must show clear algebraic working. *[5 marks]*

a = ...

d = ...

[Total 5 marks]

25. $PQRS$ is a square.
PR is a diagonal of the square.

The vertex P has coordinates $(4, 7)$
The vertex R has coordinates $(8, -5)$

Work out an equation of the straight line that passes through Q and S
Give your answer in the form $ay = bx + c$ where a, b and c are integers.
[5 marks]

....

[Total 5 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The distances d km travelled by 100 members, grouped into five classes that are each 5 km wide.
The frequencies are 26, 40, 16, 10 and 8, and they add to 100.
Only the class each distance falls in is known, never the distance itself.

Find

(a) The modal class. (b) An estimate for the mean distance, in km.

Plan the Solution

- (a) Pick out the class with the highest frequency, and give the class interval as the answer, not the frequency.
- (b) Let the midpoint of each class stand for every distance in that class. Multiply each midpoint by its frequency, add the five products, then divide by the total frequency 100.
- The word estimate is the warning: midpoints have replaced the real distances, so the answer cannot be exact.

Worked Solution [5 marks]

> Estimated mean of grouped data: the midpoint x of a class stands for every value in it, so the estimate is the total of the fx products divided by the total of the frequencies f.

Step 1: Compare the five frequencies

$26 \quad 40 \quad 16 \quad 10 \quad 8$

(Reason: The mode is what occurs most often, so with grouped data the modal class is the class holding the highest frequency. Here 40 is the largest of the five.)

Step 2: Name the class that frequency belongs to

$5 < d \leq 10$

(Reason: The answer is the class interval itself, not the frequency 40 and not a single distance. Every class here is 5 km wide, so no class can be top merely by being wider than the rest.)

Step 3: Find the midpoint of each class

$$\frac{0+5}{2} = 2.5$$

$$\frac{5+10}{2} = 7.5$$

$$\frac{10+15}{2} = 12.5$$

$$\frac{15+20}{2} = 17.5$$

$$\frac{20+25}{2} = 22.5$$

(Reason: The table never says where inside a class a distance falls, so the midpoint is used as the best single stand-in for all of them. This is exactly what makes the mean an estimate.)

Step 4: Multiply each midpoint by its frequency

$2.5 \times 26 = 65$

$7.5 \times 40 = 300$

$12.5 \times 16 = 200$

$17.5 \times 10 = 175$

$22.5 \times 8 = 180$

(Reason: Each product is the distance one whole class contributes: 26 members at roughly 2.5 km each contribute roughly 65 km between them.)

Step 5: Add the five products

$65 + 300 + 200 + 175 + 180 = 920$

(Reason: This total is an estimate of the distance travelled by all 100 members together, in km.)

Step 6: Divide by the total frequency

$$\frac{920}{100} = 9.2$$

(Reason: A mean is a total shared out equally, so the estimated total distance is divided by the number of members, 100. Divide by the total frequency, never by the 5 classes.)

(a) $5 < d \leq 10$

(b) 9.2 **km**

Verification

✓ **Check 1:** Trap the answer between two totals that need no midpoints. Put every distance at the bottom of its class and the estimate is $\frac{670}{100} = 6.7$; put every distance at the top of its class and it is $\frac{1170}{100} = 11.7$. A midpoint estimate has to land between the two.
$6.7 < 9.2 < 11.7$

✓ **Check 2:** Rebuild the mean without ever forming the total 920. Take 12.5 off every midpoint, leaving -10, -5, 0, 5 and 10. Weighted by the frequencies these total -330, and the 12.5 is then added back. $12.5 + \frac{-330}{100} = 12.5 - 3.3 = 9.2$

✓ **Check 3:** Test part (a) by the definition that survives unequal classes. The frequency density of $5 < d \leq 10$ is $\frac{40}{5} = 8$ members per km, and the nearest rival, $0 < d \leq 5$, has $\frac{26}{5} = 5.2$ members per km. The same class, $5 < d \leq 10$, comes top whether frequencies or densities are compared.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $5 < d \leq 10$	**B1**	Allow 5 to 10, or $5 < d < 10$, or $5 \leq d \leq 10$, or $5 \leq d < 10$.	✓
(b) $2.5 \times 26 + 7.5 \times 40 + 12.5 \times 16 + 17.5 \times 10 + 22.5 \times 8\ (= 920)$ or $65 + 300 + 200 + 175 + 180\ (= 920)$	**M2**	For at least 4 correct products added. They need not be evaluated, ie the work may be left in the form $2.5 \times 26 + 7.5 \times 40 + \ldots$ If not M2 then award M1 for consistent use of values within the interval, including end points, for at least 4 products added. Again they need not be evaluated, ie the work may be left in the form $5 \times 26 + 10 \times 40 + \ldots$ Or award M1 for correct midpoints used for at least 4 products and not added.	✓
$\frac{920}{100}$, where either value may be the candidate's own	**M1**	Dependent on at least M1. Allow division by their Σf provided the addition, or a total under the frequency column, is seen.	✓

Step	Mark	Description	Got it?
9.2	**A1**	Or equivalent, eg $9\frac{1}{5}$ or $\frac{46}{5}$. A correct answer scores full marks, unless it comes from obvious incorrect working.	✓
Answer of 6.7 or 9.7 or 11.7	**SC B2**	For an answer of 6.7 or 9.7 or 11.7	✓
Where those three special-case answers come from	**Note**	Each is one of the totals in the working column divided by 100. Lower class bounds give products $0, 200, 160, 150, 160$ and a total of 670, so 6.7. The values $3, 8, 13, 18, 23$ give a total of 970, so 9.7. Upper class bounds give a total of 1170, so 11.7.	✓

Full marks: 5/5

Question 2 - Exam Solution

Understanding the Question

Given

Each candle costs 6 euros to make.
Each box holds 4 candles.
80 boxes are sold for 2160 euros in total.
The height of a candle is 9 cm, correct to the nearest cm.
The weight of a candle is 120 g, correct to the nearest 10 g.

Find

(a) the percentage profit (b) the lower bound of the height (c) the upper bound of the weight

Plan the Solution

- Work out what the whole order cost to make, then compare it with the money taken in.
- Percentage profit is measured against the cost, never against the money taken in - so the profit is divided by the cost.
- A measurement given to the nearest 1 cm or the nearest 10 g can be up to half of that step away from the value written down. Halve the step, then move that far from the stated value.

Worked Solution [6 marks]

Rule - Percentage profit: $\text{profit} = \text{income} - \text{cost}$, then

$$\text{percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100.$$

Rule - Bounds: a measurement given to the nearest u lies within $\frac{u}{2}$ of the value written down.

Step 1: what the whole order cost to make

$4 \times 6 = 24$

$24 \times 80 = 1920$

(Reason: Each box holds 4 candles at 6 euros each, so filling one box costs 24 euros, and there are 80 boxes.)

Step 2: the profit

$2160 - 1920 = 240$

(Reason: Profit is what is left of the 2160 euros taken in once the 1920 euros the candles cost to make has been taken off.)

Step 3: the profit as a fraction of the cost

$$\frac{240}{1920} = 0.125$$

(Reason: Percentage profit compares the profit with what was spent, so the 240 euros of profit is divided by the 1920 euros of cost - not by the money taken in.)

Step 4: write that decimal as a percentage

$0.125 \times 100 = 12.5$

(Reason: A decimal is turned into a percentage by multiplying it by 100.)

Step 5: the lower bound of the height

$9 - 0.5 = 8.5$

(Reason: The height is given to the nearest cm, and half of 1 cm is 0.5 cm. Every height from 8.5 cm up to 9.5 cm is recorded as 9 cm, so 8.5 cm is the smallest it can be.)

Step 6: the upper bound of the weight

$120 + 5 = 125$

(Reason: The weight is given to the nearest 10 g, and half of 10 g is 5 g. Every weight from 115 g up to 125 g is recorded as 120 g, so 125 g is the upper bound.)

(a) 12.5 **%**

(b) 8.5 **cm**

(c) 125 **g**

Verification

✓ **Check 1:** Do part (a) one box at a time instead of all 80 at once. One box brings in $\frac{2160}{80} = 27$ euros and costs 24 euros to fill, so it makes 3 euros of profit.
$\frac{3}{24} \times 100 = 12.5$

✓ **Check 2:** Compare the money taken in with the cost as one multiplier: $\frac{2160}{1920} = 1.125$. That says the income is 112.5 per cent OF the cost, and a percentage profit measures how far ABOVE the cost that is. $112.5 - 100 = 12.5$

✓ **Check 3:** Round both bounds back. To the nearest cm, 8.5 cm goes up to 9 cm while 8.4 cm goes down to 8 cm. To the nearest 10 g, 124.9 g goes to 120 g while 125.1 g goes to 130 g. The height cannot be below 8.5 cm, and 125 g is where the weight stops being recorded as 120 g.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$4 \times 6 \times 80$ $(= 1920)$ or 4×6 $(= 24)$ or $\frac{2160}{80}$ $(= 27)$ or $\frac{2160}{80 \times 4}$ $(= 6.75)$	M1	(a) for method to work out total income or income from one box or expenditure for one box or income per candle	✓
$2160 - 1920$ $(= 240)$ or $\frac{2160}{1920}$ $(= 1.125)$ or $27 - 24$ $(= 3)$ or $\frac{27}{24}$ $(= 1.125)$ or $6.75 - 6$ $(= 0.75)$ or $\frac{6.75}{6}$ $(= 1.125)$	M1	for working out the profit or income divided by expenditure. The candidate's own value from the previous mark may be used in place of 1920, 27, 24 or 6.75	✓

Step	Mark	Description	Got it?
$\frac{240}{1920}$ $(\times 100)$ or 0.125 $(\times 100)$ or $\left(\frac{2160}{1920} - 1\right)$ $(\times 100)$ or $(1.125 - 1)$ $(\times 100)$ or 1.125×100 $(= 112.5)$	**M1**	for a method to reach one step from the answer ie getting to $\frac{1}{8}$ oe or 0.125 or 112.5. The candidate's own earlier values may be used in place of 240, 1920, 3, 24, 0.75 or 1.125	✓
12.5	**A1**	Working required	✓
8.5	**B1**	(b) cao	✓
125	**B1**	(c) allow 124.9 or $124.99\ldots$	✓

Full marks: 6/6

Question 3 - Exam Solution

Understanding the Question

Given

Three straight lines meet at the point B.
The angle between BA and the third line is $148°$.
The angle between that third line and BC is $50°$.
AB and BC are two sides of a regular polygon with n sides.

Find

The value of n, the number of sides of the polygon.

Plan the Solution

- The three lines leaving B fill a whole turn, so taking the two marked angles away from $360°$ leaves the polygon's own angle at B.
- That angle is an interior angle of the regular polygon. Its exterior angle is whatever is left over to $180°$.
- Every polygon's exterior angles add to $360°$, and in a regular polygon they are all the same size, so dividing counts how many there are.

- The figure is not accurately drawn, so nothing may be measured off it. Only the two marked values are usable.

Worked Solution [4 marks]

> Rule - Regular polygon: $\text{interior angle} + \text{exterior angle} = 180°$, and $n = \dfrac{360°}{\text{exterior angle}}$.

Step 1: Find the polygon's angle at B

$\angle ABC = 360° - (148° + 50°)$

$\angle ABC = 360° - 198° = 162°$

(Reason: The three lines drawn from B use up a full turn, and angles at a point add to $360°$. What is left after the two marked angles is the angle inside the polygon.)

Step 2: Turn that interior angle into the exterior angle

$180° - 162° = 18°$

(Reason: An interior angle and the exterior angle beside it lie along a straight line, so the two of them together make $180°$.)

Step 3: Divide a whole turn by one exterior angle

$$n = \frac{360°}{18°} = 20$$

(Reason: The exterior angles of any polygon add to $360°$. The polygon is regular, so all n of them are equal and the division counts them.)

$$n = 20$$

Verification

✓ **Check 1:** Work back from the answer. A regular polygon with 20 sides has exterior angle $\dfrac{360°}{20} = 18°$, so its interior angle is what is left to $180°$. $180° - 18° = 162°$

✓ **Check 2:** Use the interior-angle sum instead, which never mentions exterior angles. The angles of a 20-sided polygon add to $180° \times (20 - 2) = 3240°$, shared equally between 20 corners. $\dfrac{3240°}{20} = 162°$

✓ **Check 3:** Put the three angles at B back together and see whether they close the turn. $148° + 50° + 162° = 360°$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $360-(148+50)$ $(=162)$ or $180-50$ $(=130)$ or $180-148$ $(=32)$	**M1**	for method to interior angle of the polygon or start to the method of finding the exterior angle of the polygon	✓
eg $180-162$ $(=18)$ or $148-130$ $(=18)$ or $50-32$ $(=18)$ or $180(n-2)=162n$ or $\dfrac{180(n-2)}{n}=162$ The printed scheme puts the first row's value in quotation marks in each of these, so the candidate's own 162, 130 or 32 is accepted in its place.	**M1**	for method to find the exterior angle or for setting up an equation using sum of interior angles formula	✓
eg $\dfrac{360}{18}$ or $(n=)$ $\dfrac{360}{180-162}$ The printed scheme quotes the 18 and the 162 here as well, so the candidate's own values are accepted.	**M1**	for a complete method	✓
20 Working required	**A1**	dep on M1	✓

Full marks: 4/4

Question 4 - Exam Solution

Understanding the Question

Given

$x^5\times x^7=x^m$

$\dfrac{y^8}{y^3}=y^n$

Find

The value of m. The value of n. $(5a^4r^2)^3$ written as a single simplified term.

The expression $(5a^4r^2)^3$: a number and two powers inside one bracket, all raised to the power 3.

Plan the Solution

- Parts (a) and (b) use the two index laws for powers of the same letter: a product adds the indices, a quotient subtracts them.
- In part (c) three things sit inside the bracket, so cube each of them in turn: the number 5, the power a^4 and the power r^2.
- Raising a power to a power multiplies the two indices, so the 3 outside the bracket multiplies the 4 and the 2. It is not added to them.

Worked Solution [4 marks]

> Rule - Index laws: $x^p \times x^q = x^{p+q}$, $\dfrac{x^p}{x^q} = x^{p-q}$, and $(kx^p)^q = k^q x^{pq}$.

Step 1: part (a), a product of two powers of x

$x^5 \times x^7 = x^{5+7} = x^{12}$

$m = 12$

(Reason: Multiplying powers of the same letter adds the indices: 5 factors of x followed by 7 more gives 12 factors of x altogether.)

Step 2: part (b), a quotient of two powers of y

$$\frac{y^8}{y^3} = y^{8-3} = y^5$$

$n = 5$

(Reason: Dividing powers of the same letter subtracts the indices: 3 of the 8 factors of y on top cancel with the 3 underneath, leaving 5.)

Step 3: part (c), cube every factor inside the bracket

$(5a^4r^2)^3 = 5^3 \times (a^4)^3 \times (r^2)^3$

(Reason: The bracket holds three factors multiplied together, and the power 3 applies to every one of them, the number 5 included.)

Step 4: part (c), work out each of the three factors

$5^3 = 125$

$(a^4)^3 = a^{4\times 3} = a^{12}$

$(r^2)^3 = r^{2\times 3} = r^6$

(Reason: Cubing the number means $5 \times 5 \times 5$, and raising a power to a power multiplies the two indices.)

Step 5: part (c), put the three factors back together

$(5a^4r^2)^3 = 125a^{12}r^6$

(Reason: The number and the two powers are multiplied, so they are written side by side as one term with the number in front.)

(a) $m = 12$

(b) $n = 5$

(c) $125a^{12}r^6$

Verification

✓ **Check 1:** Part (a) with a number instead of a letter. Put $x = 2$: the left-hand side is $2^5 \times 2^7 = 32 \times 128 = 4096$, and $2^{12} = 4096$. $m = 12$

✓ **Check 2:** Part (b) the same way. Put $y = 2$: the left-hand side is $\dfrac{256}{8} = 32$, and $2^5 = 32$. $n = 5$

✓ **Check 3:** Part (c) with numbers. Put $a = 2$ and $r = 3$. Inside the bracket, $5 \times 16 \times 9 = 720$, so the question gives $720^3 = 373\,248\,000$, while the answer gives $125 \times 4096 \times 729 = 373\,248\,000$. Both sides give $373\,248\,000$.

✓ **Check 4:** Part (c) counted rather than calculated. Writing the bracket out three times gives $5 \times 5 \times 5$ for the number, $4 + 4 + 4$ factors of a and $2 + 2 + 2$ factors of r. $125a^{12}r^6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	12. Accept x^{12}.	✓
(b)	**B1**	5. Accept y^5.	✓
(c)	**B2**	For $125a^{12}r^6$.	✓
(c)	**B1**	For a product in the form ka^pr^q where 2 from k, p or q are correct, eg $5a^{12}r^6$. Allow $125a^{12}$ or $125r^6$ or $a^{12}r^6$ so as long as not added to any other terms.	✓

Full marks: 4/4

Question 5 - Exam Solution

Understanding the Question

Given

In the sale, normal prices are reduced by 28%.
The sale price of the guitar is 198 euros.

Find

The normal price of the guitar - the price before the reduction.

Plan the Solution

- The reduction is measured against the normal price, not against the sale price, so start by asking what percentage of the normal price a shopper actually pays: $100\% - 28\%$.
- Turn that percentage into a multiplier, so the sale price is the normal price multiplied by it.
- Undo that multiplication by dividing 198 by the multiplier. This is a reverse percentage, so never take 28% off 198 and never add it back on.

Worked Solution [3 marks]

> Rule - Reverse percentage: after a reduction, sale price = normal price $\times$ multiplier, so normal price = $\dfrac{\text{sale price}}{\text{multiplier}}$.

Step 1: Find what percentage of the normal price is actually paid

$100\% - 28\% = 72\%$

$1 - 0.28 = 0.72$

(Reason: The 28% comes off the normal price, so the sale price is the 72% that is left, and 0.72 is that percentage written as a multiplier.)

Step 2: Write the sale price as a multiplication

$0.72n = 198$

(Reason: Let n be the normal price in euros. Taking 72% of it means multiplying it by 0.72, and the result is the sale price of 198 euros.)

Step 3: Divide by the multiplier to get the normal price

$$n = \frac{198}{0.72} = 275$$

(Reason: Dividing by the multiplier undoes the multiplication in Step 2, taking the sale price back up to the price the shop charged before the reduction.)

275 euros

Verification

✓ **Check 1:** Put the answer back into the question and take the reduction off it: $0.28 \times 275 = 77$ euros comes off. $275 - 77 = 198$, which is the sale price the question gives.

✓ **Check 2:** Reach it a different way, one per cent at a time. If 198 euros is 72%, then one per cent is $\frac{198}{72} = 2.75$ euros. $2.75 \times 100 = 275$ euros, the same normal price by a method that never uses the multiplier.

✓ **Check 3:** Is the size sensible? A reduction makes a price smaller, so the normal price must be the larger of the two, and the amount taken off should be a little over a quarter of it. 275 is larger than 198, and $\frac{77}{275} = 0.28$, which is the reduction the question states.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $1 - 0.28$ $(= 0.72)$ oe or $0.72x = 198$ or $100(\%) - 28(\%)$ $(= 72(\%))$ or $\frac{198}{72}$ $(= 2.75)$ oe	M1	for a correct first step	✓
eg $(x =)$ $\frac{198}{0.72}$ oe or $\frac{198}{72} \times 100$ oe or 2.75×100. The printed scheme writes the first step's value in quotation marks, so the candidate's own value from the first step may be used here.	M1	for a complete method	✓
275 - correct answer scores full marks (unless from obvious incorrect working)	A1	cao	✓

Full marks: 3/3

Question 6 - Exam Solution

Understanding the Question

Given

Find

(a) $x - 4 = \frac{3 + 2x}{6}$, a linear equation with a fraction on the right-hand side
(b) $y^2 - 11y + 30$, a quadratic expression whose y^2 coefficient is 1

(a) the value of x (b)(i) $y^2 - 11y + 30$ written as a product of two brackets (b)(ii) the values of y that make that product zero. A quadratic factorising into two different brackets has two of them.

Plan the Solution

- (a) Multiply every term on both sides by 6 so the fraction disappears, then collect the x terms on one side and the numbers on the other.
- (b)(i) Because the y^2 coefficient is 1, look for two numbers with product 30 and sum -11. Each becomes the constant in its own bracket.
- (b)(ii) The word "Hence" means reuse part (i). A product is zero only when one of the factors is zero, so set each bracket to zero in turn.

Worked Solution [6 marks]

> Rule - Clear a fraction by multiplying EVERY term by its denominator. Factorise $y^2 + by + c$ by finding two numbers with product c and sum b.

Step 1 - Multiply every term by 6

$6(x - 4) = 3 + 2x$

$6x - 24 = 3 + 2x$

(Reason: Multiplying both sides by 6 cancels the denominator on the right. The whole of the left-hand side is multiplied, so it is bracketed first and then expanded; the 4 is inside that bracket, so it is multiplied too.)

Step 2 - Collect the x terms on one side

$6x - 2x = 3 + 24$

$4x = 27$

(Reason: Subtracting $2x$ from both sides and adding 24 to both sides leaves one x term on the left and one number on the right. This is the rearrangement the second method mark is for.)

Step 3 - Divide by 4

$x = \frac{27}{4} = 6.75$

(Reason: 27 does not divide exactly by 4, so the answer is left as the fraction $\frac{27}{4}$. The decimal 6.75 is the same value and is equally acceptable.)

Step 4 - Find two numbers with product 30 and sum -11

$(-6) \times (-5) = 30$
$(-6) + (-5) = -11$

(Reason: The product is positive and the sum is negative, so both numbers are negative. The negative pairs multiplying to 30 are -1 and -30, -2 and -15, -3 and -10, -5 and -6. Only the last pair adds to -11.)

Step 5 - Write the two brackets

$y^2 - 11y + 30 = (y-6)(y-5)$

(Reason: Each number found becomes the constant in its own bracket, and each bracket starts with y because the y^2 coefficient is 1.)

Step 6 - Set each bracket to zero

$(y-6)(y-5) = 0$
$y - 6 = 0 \quad \text{or} \quad y - 5 = 0$
$y = 6 \quad \text{or} \quad y = 5$

(Reason: Two numbers multiply to zero only when at least one of them is zero, so each bracket is taken to be zero in turn. Solving the two one-step equations gives the two solutions.)

$$\textbf{(a) } x = \frac{27}{4} = 6.75$$

$$\textbf{(b)(i) } (y-6)(y-5)$$

$$\textbf{(b)(ii) } y = 6 \textbf{ or } y = 5$$

Verification

✓ **Check 1:** Put $x = 6.75$ back into the original equation and work out each side on its own. They must come out the same.

On the right, $2 \times 6.75 = 13.5$. $6.75 - 4 = 2.75$ and $\dfrac{3 + 13.5}{6} = 2.75$, so both sides agree.

✓ **Check 2:** Expand the brackets from (b)(i) and compare the result with the expression printed in the question. $y^2 - 5y - 6y + 30 = y^2 - 11y + 30$, which is the expression given.

✓ **Check 3:** Substitute each solution from (b)(ii) into $y^2 - 11y + 30$. A genuine solution makes it zero. $36 - 66 + 30 = 0$ and $25 - 55 + 30 = 0$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $6x - 24 = 3 + 2x$ or $x - 4 = \frac{3}{6} + \frac{2}{6}x$ oe	M1	for correct removal of fraction and expansion of bracket in a correct equation or separating fraction (RHS) in an equation	✓
(a) $6x - 2x = 3 + 24$ or $4x = 27$ or $-24 - 3 = 2x - 6x$ or $-27 = -4x$ oe or $x - \frac{2}{6}x = \frac{3}{6} + 4$ oe or $-4 - \frac{3}{6} = \frac{2}{6}x - x$ oe	M1ft	(dep on 4 terms) correctly rearranging their 4 term equation for terms in x on one side of equation and number terms on the other	✓
(a) $\frac{27}{4}$ Working required	A1	oe eg 6.75 or $6\frac{3}{4}$, dep on M1	✓
(b)(i) $(y \pm 6)(y \pm 5)$ or $(6 \pm y)(5 \pm y)$ or $y(y - 6) - 5(y - 6)$ or $y(y - 5) - 6(y - 5)$	M1	for $(y \pm 6)(y \pm 5)$ or $(6 \pm y)(5 \pm y)$ or for $(y + a)(y + b)$ where $ab = 30$ or $a + b = -11$ or $y(y + a) + b(y + a)$ or $y(y + b) + a(y + b)$ where $ab = 30$ or $a + b = -11$	✓
(b)(i) $(y - 6)(y - 5)$ Correct answer scores full marks (unless from obvious incorrect working)	A1	oe, allow any letter for y	✓
(b)(ii) $(y =)6$, $(y =)5$	B1	must ft from their answer in (b)(i) ft from their factors in the form $(y + a)(y + b)$	✓

Full marks: 6/6

Question 7 - Exam Solution

Understanding the Question

Given

A solid cylinder with radius 8 cm and height h cm

Find

The value of h, correct to one decimal place

Volume of the cylinder: 3892 cm³

Plan the Solution

- Start from the volume formula for a cylinder, $V = \pi r^2 h$.
- Substitute $r = 8$ and $V = 3892$, which leaves h as the only unknown.
- Work out the circular cross-section area $\pi \times 8^2$ first, then divide the volume by it.
- Keep the full calculator value all the way through, and round only at the end.

Worked Solution [3 marks]

> Rule - Volume of a cylinder: $V = \pi r^2 h$ - the area of the circular cross-section multiplied by the height.

Step 1: Put the given values into the volume formula

$V = \pi r^2 h$

$3892 = \pi \times 8^2 \times h$

(Reason: The radius is 8 cm and the volume is 3892 cm³, so h is the only letter left in the equation.)

Step 2: Work out the area of the circular cross-section

$8^2 = 64$

$\pi \times 64 = 201.0619\ldots$

(Reason: Square the radius first, then multiply by π. Keep the full calculator value - rounding at this point would move the final answer.)

Step 3: Rearrange to make h the subject

$$h = \frac{3892}{\pi \times 8^2}$$

$$h = \frac{3892}{201.0619\ldots}$$

(Reason: Dividing both sides by the cross-section area leaves h on its own.)

Step 4: Divide, then round to one decimal place

$h = 19.3572\ldots$

$h = 19.4$ (to 1 d.p.)

(Reason: The digit after the first decimal place is a 5, so $19.35\ldots$ rounds up to 19.4.)

$h = 19.4$ (to one decimal place)

Verification

✓ **Check 1:** Put the unrounded h back into $\pi r^2 h$ and rebuild the volume. $\pi \times 8^2 \times 19.3572\ldots = 3892$, the volume the question gives.

✓ **Check 2:** Divide by the 8^2 and by the π as two separate steps, in the other order - the alternative the mark scheme prints. $\frac{3892}{64} = 60.8125$ and $\frac{60.8125}{\pi} = 19.3572\ldots$, the same value as before.

✓ **Check 3:** Repeat the division with the two approximations for π that the mark scheme allows. 3.14 gives $19.36\ldots$ and $\frac{22}{7}$ gives $19.34\ldots$, so both land inside the 19.3 to 19.4 the scheme accepts.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$3892 = \pi \times 8^2 \times h$ or $\pi \times 8^2\ (= 64\pi = 201.0\ldots)$	M1	Allow the use of $3.14\ldots$ or $\frac{22}{7}$ for π.	✓
$(h =)\frac{3892}{\pi \times 8^2}$ oe, for example $\frac{3892}{64} = 60.8\ldots$ and $\frac{60.8\ldots}{\pi}$	M1	Allow the use of $3.14\ldots$ or $\frac{22}{7}$ for π.	✓
19.4	A1	Allow 19.3 to 19.4.	✓
A correct answer scores full marks.	Note	Unless it comes from obviously incorrect working.	✓

Full marks: 3/3

Question 8 - Exam Solution

Understanding the Question

Given

(a) the number 520 million
(b) 8.79×10^{-5}
(c) $(5 \times 10^{42}) \times (7 \times 10^{-180})$

Find

(a) the same number written as $a \times 10^n$
(b) the same number written out in full,

Three separate pieces of standard-form work: going into it, coming back out of it, and multiplying inside it.

with no power of ten (c) the product, given in standard form

Plan the Solution

- Standard form is one front number times a power of ten, and the front number must be at least 1 and less than 10. Every part below comes down to that one condition.
- For (a), write the number out in full first. Then the index is a matter of counting how far the decimal point travels, not of guessing.
- For (b), a negative index sends the point the other way, so expect an answer smaller than 1 .
- For (c), multiply the front numbers, add the indices, and only then ask whether the front number still obeys the condition. It will not, and fixing it changes the index.

Worked Solution [4 marks]

> Rule - Standard form: a number is written as $a \times 10^n$, where a is at least 1 and less than 10, and n is a whole number. To multiply two such numbers, multiply the front numbers and add the indices: $10^m \times 10^n = 10^{m+n}$.

Part (a): write 520 million out in full

$$520 \times 1\,000\,000 = 520\,000\,000$$

(Reason: One million is $1\,000\,000$, so 520 million is 520 lots of it. Writing the number out in full turns the next step into counting rather than guessing.)

Part (a): move the decimal point until one digit stands in front of it

$$520\,000\,000 = 5.2 \times 10^8$$

(Reason: The point starts after the last zero and moves 8 places to the left, to sit between the 5 and the 2. Each place is one power of ten, so the index is 8. The front number 5.2 is at least 1 and less than 10, which is what standard form asks for.)

Part (b): read what the negative index is telling you

$$8.79 \times 10^{-5} = \frac{8.79}{10^5}$$

(Reason: A negative index means divide, not subtract - the answer is not a negative number. Dividing by 10^5 sends every digit 5 places to the right, so the result must be smaller than 1.)

Part (b): move the point 5 places and fill the gaps with zeros

$$\frac{8.79}{100\,000} = 0.0000879$$

(Reason: The 8 starts in the units column and finishes five columns to its right, so it lands in the fifth decimal place and four zeros sit between the point and it. Count them in the answer: 0.0000879.)

Part (c): multiply the front numbers

$5 \times 7 = 35$

(Reason: Multiplication can be done in any order, so gather the ordinary numbers together and the powers of ten together, and deal with each pair once.)

Part (c): add the indices

$10^{42} \times 10^{-180} = 10^{42+(-180)} = 10^{-138}$

(Reason: Multiplying powers of the same base adds the indices. Adding a negative index is a subtraction, so $42 - 180 = -138$, and the answer so far is 35×10^{-138}.)

Part (c): put the answer back into standard form

$35 \times 10^{-138} = 3.5 \times 10 \times 10^{-138}$
$3.5 \times 10 \times 10^{-138} = 3.5 \times 10^{-137}$

(Reason: The front number has to be less than 10, and 35 is not, so the work is not finished at the previous line. Moving the point of 35 one place to the left divides the front number by 10, so the power of ten has to go up by one to leave the value unchanged.)

(a) 5.2×10^{8}

(b) 0.0000879

(c) 3.5×10^{-137}

Verification

- ✓ **Check 1:** Work part (a) backwards. Move the point of 5.2 eight places to the right and count the digits that come out. $5.2 \times 10^{8} = 520\,000\,000$, which reads as 520 million.
- ✓ **Check 2:** Work part (b) backwards. Moving the point five places the other way must give the front number the question started from. $0.0000879 \times 10^{5} = 8.79$
- ✓ **Check 3:** Compare the two ways of writing part (c). The unfinished form and the standard form must be the same number, and the index must differ by exactly the one place the front number moved. $35 \times 10^{-138} = 3.5 \times 10^{-137}$, and $-138 + 1 = -137$.
- ✓ **Check 4:** Is the size sensible? The 10^{-180} is far stronger than the 10^{42}, so the product has to be a tiny positive number rather than a large one, and the indices on their own say roughly where it lands. $42 - 180 = -138$, and two front numbers below 10 can only shift that by one place, so an index of -137 is the size to expect.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	5.2×10^8	✓
(b)	**B1**	0.0000879	✓
(c)	**M1**	35×10^{-138} or $3.5 \times 10 \times 10^{-138}$ or 3.5×10^n where $n \neq -137$	✓
(c)	**A1**	3.5×10^{-137}. A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 9 - Exam Solution

Understanding the Question

Given

A trapezium $ABCD$ with one line of symmetry, so AB is parallel to DC and $AD = BC$
angle $ADC = 60^\circ$
$AD = 12$ cm and $DC = 47$ cm

Find

The area of the trapezium, correct to 3 significant figures

Plan the Solution

- Drop a perpendicular from A down to DC. It cuts off a right-angled triangle whose hypotenuse is $AD = 12$ cm, with the 60° angle at D.
- Use $\sin 60^\circ$ on that triangle for the height of the trapezium, and $\cos 60^\circ$ for the piece of DC it stands on.
- The one line of symmetry makes the triangle at the C end congruent to it, so the same piece is used up at each end of DC.
- Take both pieces off 47 to get AB, then put the two parallel sides and the height into the trapezium formula.
- Keep the full calculator value of the height all the way through, and round only at the very end.

Worked Solution [5 marks]

Rule - Area of a trapezium: $\text{Area} = \frac{1}{2}(a+b)h$ - half the sum of the two parallel sides, multiplied by the perpendicular height between them.

Step 1: Work out the perpendicular height of the trapezium

$h = 12\sin 60^\circ$

$h = 6\sqrt{3} = 10.3923\ldots$

(Reason: In the right-angled triangle the height is the side opposite the 60° angle and 12 cm is the hypotenuse, so the height is $12\sin 60^\circ$. This value is not exact as a decimal, so carry it in the calculator rather than rounding it now.)

Step 2: Work out how much of DC that triangle stands on

$12\cos 60^\circ = 6$

(Reason: The base of the same triangle lies along DC and is the side next to the 60° angle, so it is $12\cos 60^\circ$. Because $\cos 60^\circ = \frac{1}{2}$, this length is exactly 6 cm.)

Step 3: Work out the length of AB

$AB = 47 - 6 - 6 = 35$

(Reason: The line of symmetry makes the triangle at the C end congruent to the one at the D end, so 6 cm of DC is used up at each end. What is left between the two feet is AB.)

Step 4: Put both parallel sides and the height into the formula

$\text{Area} = \frac{1}{2} \times (47+35) \times 10.3923\ldots$

$\frac{1}{2} \times (47+35) = 41$

$\text{Area} = 41 \times 10.3923\ldots$

(Reason: The parallel sides are $DC = 47$ cm and $AB = 35$ cm. Half of their sum is 41, so the area is 41 lots of the height.)

Step 5: Multiply out, then round to three significant figures

$\text{Area} = 426.0844\ldots$

$\text{Area} = 426 \text{ cm}^2$ (to 3 s.f.)

(Reason: The first three significant figures are 4, 2 and 6, and the digit after them is a 0, so the 426 is left as it stands.)

426 cm^2 (to 3 significant figures)

Verification

✓ **Check 1:** Rebuild the shape as a rectangle with one triangle at each end - the second complete method the mark scheme prints - and add the three areas. $35 \times 10.3923\ldots = 363.73\ldots$ and each end triangle is $\frac{1}{2} \times 6 \times 10.3923\ldots = 31.17\ldots$, so the total is $363.73\ldots + 2 \times 31.17\ldots = 426.08\ldots$

✓ **Check 2:** Cut the trapezium along the diagonal AC instead and use $\frac{1}{2}ab\sin C$ on each triangle. This route never works out a height at all. Triangle ADC is $\frac{1}{2} \times 12 \times 47 \times \sin 60^\circ = 244.21\ldots$ and triangle ABC is $\frac{1}{2} \times 12 \times 35 \times \sin 120^\circ = 181.86\ldots$, giving $426.08\ldots$ again.

✓ **Check 3:** Find the height a second way, by Pythagoras on the same right-angled triangle rather than by the sine. $\sqrt{12^2 - 6^2} = \sqrt{108} = 10.3923\ldots$, the same height as in Step 1.

✓ **Check 4:** Check the size is sensible. The trapezium must be larger than a rectangle on the short parallel side and smaller than one on the long parallel side, both at the same height. $35 \times 10.3923\ldots = 363.7\ldots$ and $47 \times 10.3923\ldots = 488.4\ldots$, and 426 lies between them.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$12\sin 60\ (= 6\sqrt{3} = 10.3(9\ldots))$ or $\sqrt{12^2 - 6^2}\ (= 6\sqrt{3} = 10.3(9\ldots))$ or (Area ADC =) $\frac{1}{2} \times 12 \times 47 \times \sin 60\ (= 244.2\ldots)$	M1	for a method to find the height of the trapezium or the area of triangle ADC. The printed scheme puts the 6 in quotation marks, so a candidate's own value from the base row may be used. The first two M1 marks can be awarded in either order.	✓
$12\cos 60\ (= 6)$ or $\sqrt{12^2 - (6\sqrt{3})^2}\ (= 6)$	M1	(indep) for a method to find the base of the triangle, condoning missing brackets around the $6\sqrt{3}$, which the printed scheme puts in	✓

Step	Mark	Description	Got it?
		quotation marks so a candidate's own height may be used. The first two M1 marks can be awarded in either order.	
$(AB =)\ 47 - 6 - 6\ (= 35)$	**M1**	(dep on previous M1) for a method to find the length of AB. The printed scheme puts each 6 in quotation marks, so a candidate's own value may be used.	✓
(Trapezium =) $\frac{1}{2} \times (47 + 35) \times 10.3(9\ldots)$ or (Rectangle + two triangles =) $35 \times 10.3(9\ldots) + 2 \times \frac{1}{2} \times 6 \times 10.3(9\ldots)$ or $35 \times 10.3(9\ldots) + 2 \times \frac{1}{2} \times 6 \times 12 \times \sin 60$ or (Triangle ADC + Triangle ABC =) $244.2\ldots + \frac{1}{2} \times 12 \times 35 \times \sin 120$ oe, for example $(47 - 6) \times 10.3(9\ldots)$	**M1**	for a complete method. Each value the printed scheme puts in quotation marks may be a candidate's own. There are other methods, and marks should be awarded for a complete method that should give the correct area.	✓
426	**A1**	(dep on M1) allow 420 to 427 from correct working. The printed scheme also states that working is required.	✓

Full marks: 5/5

Question 10 - Exam Solution

Understanding the Question

Given

Find

The curve $y = x^3 + 2x + 3$
A table of values running from $x = -3$ to $x = 3$, with the cells for $x = -3$ and $x = 2$ left blank
A grid running from -3 to 3 across and from -30 to 40 up

The two missing y values The curve, drawn on the grid for $-3 \leq x \leq 3$

Plan the Solution

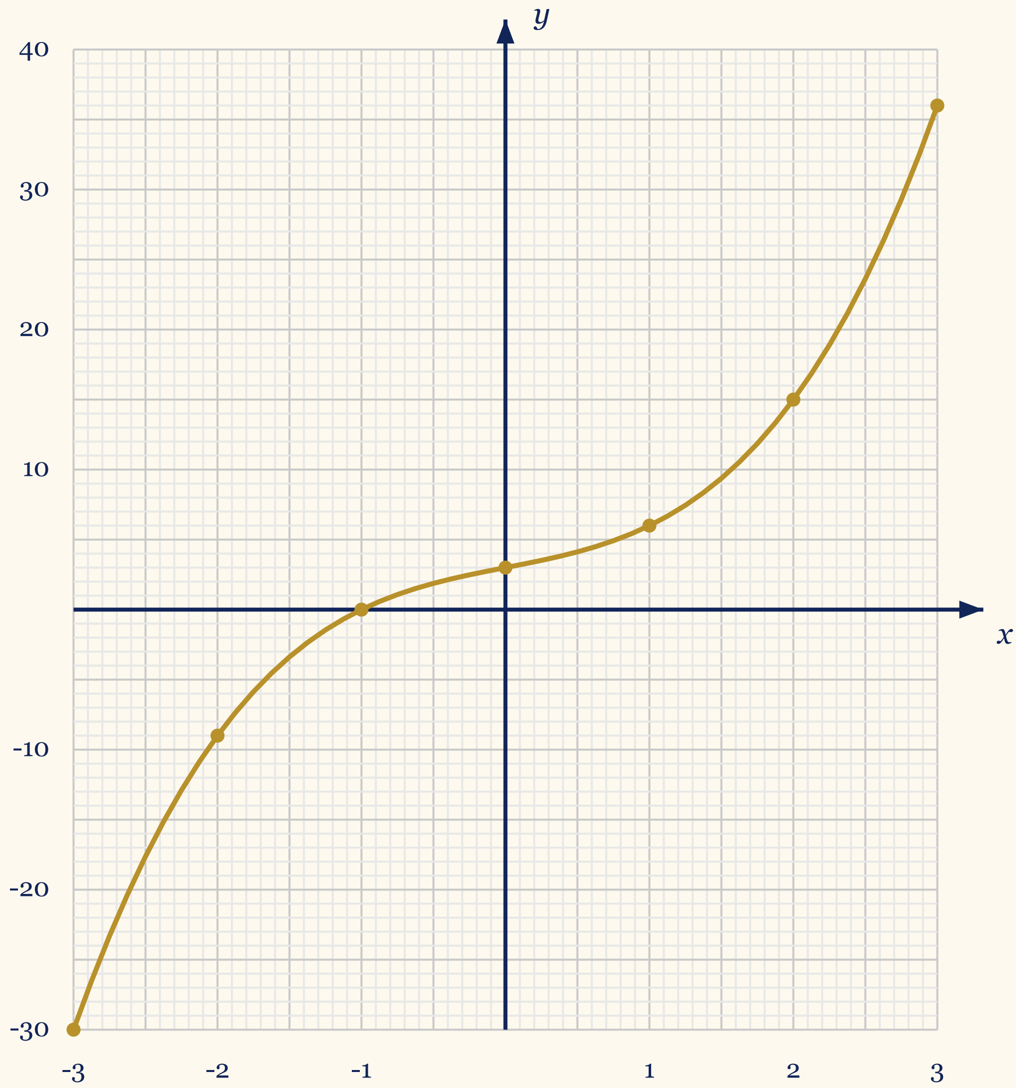
- Put each missing x value into $y = x^3 + 2x + 3$ and work the answer out.
- Cube first, then double, then add 3. An odd power of a negative number stays negative, so the sign matters at $x = -3$.
- Plot all seven points from the finished table.
- Join them with one smooth curve - a cubic has no corners and no straight stretches.

Worked Solution [3 marks]

> Rule - Table of values: work out y for every x the table asks for, plot each pair (x, y), then join the points with a single smooth curve.

Step 1: substitute $x = -3$

$$(-3)^3 + 2 \times (-3) + 3 = -27 - 6 + 3 = -30$$



(Reason: Reason: take the cube first, then the $2x$ term, then the 3. Cubing is an odd power, so a negative number stays negative when it is cubed.)

Step 2: substitute $x = 2$

$$2^3 + 2 \times 2 + 3 = 8 + 4 + 3 = 15$$

(Reason: Reason: 2^3 means $2 \times 2 \times 2$, which is 8 - it is not 2×3.)

Step 3: write the completed table out as seven points

$(-3, -30)$ $(-2, -9)$ $(-1, 0)$ $(0, 3)$
$(1, 6)$ $(2, 15)$ $(3, 36)$

(Reason: Reason: the five values the table already carries are used exactly as printed. Only the two blanks were ours to fill.)

Step 4: plot the points and join them

$$-3 \leq x \leq 3$$

(Reason: Reason: one mark is for the plotting and one is for the curve. Draw it only between $x = -3$ and $x = 3$, and keep it curved throughout - joining the points with straight lines loses the accuracy

mark.)

(a) -30 and 15

(b) a smooth curve through all seven points, from $(-3, -30)$ to $(3, 36)$

Verification

✓ **Check 1:** Test the reading of the formula on a value the table already gives. Put $x = -1$ in and see whether it returns the printed value.
$(-1)^3 + 2 \times (-1) + 3 = -1 - 2 + 3 = 0$, and the table prints 0 there

✓ **Check 2:** Take the 3 off every y. What is left is $x^3 + 2x$, which is an odd expression, so the values at x and $-x$ must be equal and opposite. $-30 - 3 = -33$ against $36 - 3 = 33$, and $15 - 3 = 12$ against $-9 - 3 = -12$ - both pairs match

✓ **Check 3:** The gradient is $3x^2 + 2$, which is never smaller than 2, so y has to rise all the way across the table and the curve can have no turning point. the y row reads -30, -9, 0, 3, 6, 15, 36 - rising throughout, and every value sits between -30 and 40, so the whole curve fits on the grid

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) -30 and 15 written in the two empty cells	**B1**	for -30 and 15 in the correct place. This may be awarded if they are plotted correctly on the graph.	✓
(b) at least 6 points plotted correctly	**M1ft**	for at least 6 points plotted correctly (within the circles on the overlay). Follow through their incorrect table.	✓
(b) correct curve between $x = -3$ and $x = 3$	**A1**	for a correct curve between $x = -3$ and $x = 3$ (clear intention to go through all the points, and it must be curved). Ignore to the left of $x = -3$ and to the right of $x = 3$.	✓
Guidance on a blank table	**Note**	If a fully correct graph is shown but a blank table is shown in (a), then award the mark for (a).	✓
Guidance on the answer alone	**Note**	Correct answer scores full marks (unless from obvious incorrect working).	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

A sector $OABC$ of a circle with centre O
angle $AOC = 140°$
$OA = OC = 16$ cm, so the radius is $r = 16$ cm

Find

The area of the sector, correct to 3 significant figures.

Plan the Solution

- Write the angle at the centre as a fraction of a full turn of $360°$.
- Work out the area of the whole circle from the radius.
- Multiply the two, and round only on the last line.

Worked Solution [2 marks]

Rule - Area of a sector: $\text{Area} = \dfrac{\theta}{360} \times \pi r^2$, where θ is the angle at the centre, in degrees.

Step 1: Write the sector as a fraction of the whole circle

$$\frac{140}{360} = \frac{7}{18}$$

(Reason: The angle at the centre is $140°$ out of a full turn of $360°$, and 140 and 360 both divide by 20.)

Step 2: Work out the area of the whole circle

$\pi \times 16^2 = 256\pi$

$256\pi \approx 804.2477$

(Reason: OA and OC are radii, so $r = 16$ and $r^2 = 256$. Keeping the π here means nothing is rounded early.)

Step 3: Take that fraction of the whole circle

$$\frac{7}{18} \times 256\pi \approx 312.763$$

(Reason: The sector is that share of the whole circle, so the two multiply. On a calculator the whole line goes in at once.)

Step 4: Round to 3 significant figures

$312.763\ldots \approx 313$

$\text{Area} \approx 313 \text{ cm}^2$

(Reason: The first three significant figures are 3, 1 and 2. The next digit is 7, so the 2 rounds up to 3. Area is measured in cm^2.)

313 cm^2

Verification

✓ **Check 1:** Turn $140°$ into radians, $\dfrac{7\pi}{9}$, and use the other sector formula, $\dfrac{1}{2}r^2\theta$. A different formula must give the same area. $\dfrac{1}{2} \times 16^2 \times \dfrac{7\pi}{9} \approx 312.763$

✓ **Check 2:** Work backwards. Divide the sector area by the area of the whole circle and the fraction of the turn should come back. $\dfrac{312.763}{804.2477} \approx 0.3889$, and $\dfrac{140}{360} \approx 0.3889$

✓ **Check 3:** The mark scheme allows 3.14 or $\dfrac{22}{7}$ in place of π, so neither may change the answer. 3.14 gives $312.60\ldots$ and $\dfrac{22}{7}$ gives $312.88\ldots$, both 313 to 3 significant figures.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\pi \times 16^2 \times \dfrac{140}{360}$ oe eg $256\pi \times \dfrac{7}{18}$	M1	allow use of $3.14\ldots$ or $\dfrac{22}{7}$ for π. $\dfrac{140}{360}$ may be seen as an equivalent fraction or decimal eg $\dfrac{7}{18}$ or $0.3\dot{8}$, $0.388(8\ldots)$ or 0.389	✓
313	A1	accept 311.8 to 313	✓
Correct answer scores full marks (unless from obvious incorrect working)	Note	Printed on the official scheme for this question, and it applies to the whole question.	✓

Full marks: 2/2

Question 12 - Exam Solution

Understanding the Question

Given

The sum of two fractions, $\frac{5}{4}$ and $\frac{x-3}{6x}$

The denominators 4 and $6x$ are different, and one of them contains x

Find

One single fraction, written in its simplest form. So the final numerator and denominator must share no factor, not even x

Plan the Solution

- Find the lowest common denominator of 4 and $6x$.
- Rewrite each fraction over that denominator, multiplying the top by whatever the bottom was multiplied by.
- Add the two numerators, expanding the bracket in full.
- Check whether anything cancels before writing the answer down.

Worked Solution [3 marks]

> Rule - Adding algebraic fractions: rewrite both fractions over the lowest common denominator, add the numerators, then cancel any factor common to the top and the bottom.

Step 1: Find the lowest common denominator

$4 = 2 \times 2$

$6x = 2 \times 3 \times x$

$2 \times 2 \times 3 \times x = 12x$

(Reason: $12x$ is the smallest expression that both 4 and $6x$ divide into exactly, so it is the lowest common denominator)

Step 2: Rewrite each fraction with denominator $12x$

$$\frac{5}{4} = \frac{5 \times 3x}{4 \times 3x} = \frac{15x}{12x}$$

$$\frac{x-3}{6x} = \frac{2(x-3)}{12x} = \frac{2x-6}{12x}$$

(Reason: multiplying the top and the bottom by the same amount leaves a fraction's value unchanged, and the bracket must be multiplied out in full)

Step 3: Add the numerators

$$\frac{15x}{12x} + \frac{2x-6}{12x} = \frac{15x+2x-6}{12x}$$
$$\frac{15x+2x-6}{12x} = \frac{17x-6}{12x}$$

(Reason: the denominators now match, so only the numerators are added, and $15x + 2x = 17x$)

Step 4: Check that nothing cancels

$$\frac{17x-6}{12x}$$

(Reason: no whole number divides both 17 and 6 except 1, so nothing can be taken out of the numerator, and x is not a factor of $17x - 6$ because of the -6)

$$\frac{17x-6}{12x}$$

Verification

✓ **Check 1 - substitute $x = 3$:** At $x = 3$ the second fraction is zero, so the original sum is just $\frac{5}{4}$. The answer must give the same value. $\frac{5}{4} + \frac{3-3}{6 \times 3} = \frac{5}{4}$ and $\frac{17 \times 3 - 6}{12 \times 3} = \frac{45}{36} = \frac{5}{4}$

✓ **Check 2 - substitute $x = 2$:** A second value, chosen so the second fraction is negative and the two sides cannot agree by accident. $\frac{5}{4} + \frac{2-3}{6 \times 2} = \frac{5}{4} - \frac{1}{12} = \frac{7}{6}$ and $\frac{17 \times 2 - 6}{12 \times 2} = \frac{28}{24} = \frac{7}{6}$

✓ **Check 3 - split the answer back apart:** Divide each term of the numerator by $12x$ and rebuild the original sum the other way round. $\frac{17x-6}{12x} = \frac{17}{12} - \frac{1}{2x}$ and $\frac{5}{4} + \frac{x-3}{6x} = \frac{5}{4} + \frac{1}{6} - \frac{1}{2x} = \frac{17}{12} - \frac{1}{2x}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\dfrac{5(6x)}{24x}+\dfrac{4(x-3)}{24x}$ oe, or $\dfrac{5(6x)}{4(6x)}+\dfrac{4(x-3)}{4(6x)}$ oe, or $\dfrac{30x}{24x}+\dfrac{4(x-3)}{24x}$ oe, or $\dfrac{30x+4(x-3)}{24x}$ oe, or $\dfrac{15x}{12x}+\dfrac{2(x-3)}{12x}$ oe, or $\dfrac{15x+2(x-3)}{12x}$ oe	**M1**	for two correct fractions with common denominator or a single correct fraction	✓
eg $\dfrac{30x+4x-12}{24x}$ oe, or $\dfrac{30x}{24x}+\dfrac{4x-12}{24x}$ oe, or $\dfrac{30x}{24x}+\dfrac{4x}{24x}-\dfrac{12}{24x}$ oe, or $\dfrac{34x}{24x}-\dfrac{12}{24x}$ oe, or $\dfrac{34x-12}{24x}$ oe, or $\dfrac{15x+2x-6}{12x}$ oe	**M1**	for correct fraction(s) with bracket(s) expanded correctly	✓
$\dfrac{17x-6}{12x}$	**A1**	oe but must be simplified, eg $\dfrac{-6+17x}{12x}$. Do not ISW incorrect simplification, eg $\dfrac{17x-6}{12x}=\dfrac{11}{12}$ is M2A0	✓
Correct answer scores full marks (unless from obvious incorrect working)	**Note**	The printed scheme awards all three marks for a correct simplified single fraction on the answer line even where no working is shown, unless the working that is shown is obviously incorrect. This row carries no mark of its own.	✓

Full marks: 3/3

Question 13 - Exam Solution

Understanding the Question

Given

The cabin was bought for $\$750\,000$
First year: a fall of 4%
Second year: a fall of 6.5%
Third year: a rise of $x\%$
After the three years the cabin was worth $\$698\,445$

Find

The value of x, the percentage rise in the third year. The three changes act one after another, so expect a chain of multipliers rather than one percentage added on at the end.

Plan the Solution

- Write each fall as a multiplier: a fall of 4% leaves 96% and a fall of 6.5% leaves 93.5%.
- Apply the two falls in order to get the value the third year starts from.
- Divide the final value by that value to get the third year's multiplier.
- Turn that multiplier back into a percentage to get x.

Worked Solution [3 marks]

Rule - percentage change as a multiplier: a fall of $p\%$ multiplies by $1 - \dfrac{p}{100}$ and a rise of $p\%$ multiplies by $1 + \dfrac{p}{100}$. Changes in successive years multiply together, so the whole chain is $750\,000 \times 0.96 \times 0.935 \times \left(1 + \dfrac{x}{100}\right) = 698\,445$.

Step 1: Write each fall as a multiplier

$$1 - \frac{4}{100} = 0.96$$

$$1 - \frac{6.5}{100} = 0.935$$

(Reason: (Reason: a fall of 4% leaves 96% of the value and a fall of 6.5% leaves 93.5% of it, so each year becomes a single multiplication instead of a subtraction.))

Step 2: Work down to the value at the end of the second year

$$750\,000 \times 0.96 = 720\,000$$

$$720\,000 \times 0.935 = 673\,200$$

(Reason: (Reason: the second fall is taken off the $\$720\,000$ *that is left after the first year, never off the original* $\$750\,000$*. Adding the two percentages to get* 10.5% *is the standard slip here.))*

Step 3: Find the multiplier for the third year

$$\frac{698\,445}{673\,200} = 1.0375$$

(Reason: (Reason: dividing the value at the END of a year by the value at the START of that same year leaves exactly the multiplier for that year, because the start value cancels.))

Step 4: Turn the multiplier back into a percentage

$1.0375 - 1 = 0.0375$

$0.0375 \times 100 = 3.75$

(Reason: (Reason: the rise multiplier is $1 + \frac{x}{100}$*, so taking away* 1 *leaves* $\frac{x}{100}$ *and multiplying by* 100 *gives* x *itself.))*

$$x = 3.75$$

Verification

✓ **Check 1:** Put the answer back on to the value the third year started from: raise $673\,200$ by 3.75%. $673\,200 \times 1.0375 = 698\,445$, which is the value the question gives.

✓ **Check 2:** Do all three years in one go, as a single multiplier: $0.96 \times 0.935 \times 1.0375 = 0.93126$. $750\,000 \times 0.93126 = 698\,445$, so the chain agrees with the year-by-year working.

✓ **Check 3:** Check the cash rise instead of the multiplier: work out 3.75% of $673\,200$ and add it on. $0.0375 \times 673\,200 = 25\,245$ and $673\,200 + 25\,245 = 698\,445$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$750\,000 \times (1 - 0.04)$ $(= 720\,000)$ oe and $720\,000 \times (1 - 0.065)$ $(= 673\,200)$ or $750\,000 \times (1 - 0.04) \times (1 - 0.065)$ $(= 673\,200)$ or $750\,000 \times 0.96 \times 0.935$ $(= 673\,200)$	M1	NB: accept $\left(1 - \frac{4}{100}\right)$ for 0.96 but not $(1 - 4\%)$, and accept $\left(1 - \frac{6.5}{100}\right)$ for 0.935 but not $(1 - 6.5\%)$. Calculations may be seen as part of an equation, eg	✓

Step	Mark	Description	Got it?
		$750\,000 \times 0.96 \times 0.935 \times \left(1 + \frac{x}{100}\right) = 698\,445$ 1.0375 or $25\,245$ imply M1.	
eg $\frac{698\,445 - 673\,200}{673\,200}$ $(\times 100)$ $(= 0.0375)$ or $\frac{698\,445}{673\,200} - 1$ $(= 0.0375)$ or $1.0375 - 1$ $(= 0.0375)$ oe or $\frac{698\,445}{673\,200} \times 100$ $(= 103.75)$	M1	For a method to reach a value one step away from x, ie a method leading to 0.0375 or 103.75. The printed scheme writes $673\,200$ in quotation marks in this row, so the candidate's own value from the first mark may be used in its place.	✓
3.75	A1	oe eg $3\frac{3}{4}$ or $\frac{15}{4}$. Correct answer scores full marks (unless from obvious incorrect working).	✓

Full marks: 3/3

Question 14 - Exam Solution

Understanding the Question

Given

Lorenzo plays one game of squash and then one game of table tennis.
He either wins or loses each game - neither game can be drawn.
The probability that he wins the squash is 0.7.
The probability that he wins the table tennis is 0.4.

Find

(a) The three pairs of probabilities that belong on the branches of the tree. (b) The probability that he loses both games. Losing both games is one single path through the tree, so expect one multiplication rather than a sum.

Plan the Solution

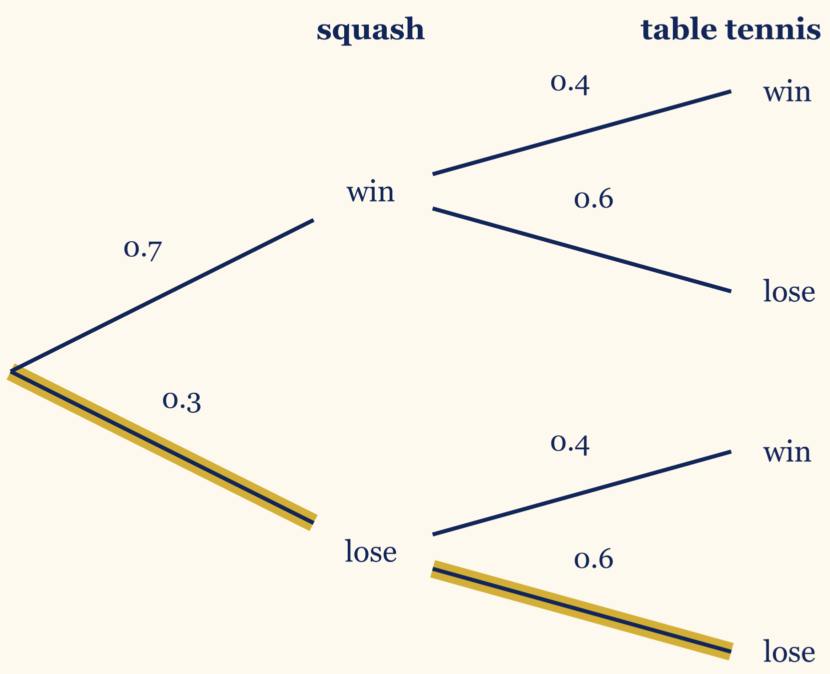
- The two branches leaving any one point cover everything that can happen there, so their probabilities add to 1. Subtract to fill in the missing one of each pair.
- The squash result does not change how likely he is to win the table tennis, so the same pair of probabilities goes on both table tennis pairs.
- For (b), find the one path that is lose and then lose, and multiply the probabilities along it.

Worked Solution [4 marks]

> Rule - a probability tree: the branches leaving any one point cover everything that can happen there, so their probabilities add to 1; and the probability of a whole path is the probabilities along that path multiplied together.

Step 1: Complete the squash pair

$1 - 0.7 = 0.3$



(Reason: (Reason: he either wins or loses, so those two branches take up the whole of the probability at that point. Taking 0.7 away from 1 leaves the probability that he loses the squash.))

Step 2: Complete both table tennis pairs

$1 - 0.4 = 0.6$

(Reason: (Reason: the same subtraction fills a table tennis pair - and because the squash result does not change how likely he is to win the table tennis, 0.4 and 0.6 go on the lower pair as well as the upper one. That is the third pair the mark scheme is looking for.))

Step 3: Multiply along the lose, lose path

$0.3 \times 0.6 = 0.18$

(Reason: (Reason: losing both games is one single route through the tree - down at the first pair of branches, then down again at the second - and the probability of a whole route is the probabilities along it multiplied together.))

(a) 0.3 on the lose squash branch, and 0.4 and 0.6 on each table tennis pair

(b) 0.18

Verification

✓ **Check 1:** Multiply along all four paths of the completed tree and add the four results. Every way the two games can turn out is one of those four paths, so they must total 1. $0.28 + 0.42 + 0.12 + 0.18 = 1$, and the last of those four, 0.18, is the lose then lose path.

✓ **Check 2:** Do the same multiplication in fractions instead of decimals: 0.3 is $\frac{3}{10}$ and 0.6 is $\frac{6}{10}$. $\frac{3}{10} \times \frac{6}{10} = \frac{18}{100} = \frac{9}{50} = 0.18$, which is the same answer in two of the forms the mark scheme accepts.

✓ **Check 3:** Count instead of multiplying. Imagine $1\,000$ repeats of this pair of games: he loses the squash in about 300 of them, and he goes on to lose the table tennis in 60% of those. $0.6 \times 300 = 180$ repeats out of $1\,000$, and $\frac{180}{1\,000} = 0.18$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) All three pairs of probabilities on the correct branches: 0.7 and 0.3 on the squash pair, 0.4 and 0.6 on each table tennis pair	**B2**	For all 3 correct pairs of probabilities on the correct branches.	✓
For 1 or 2 correct pairs of probabilities on the correct branches	**(B1)**	The printed scheme gives this row in brackets: it is the partial credit inside part (a)'s two marks, not a mark on top of them.	✓
Allow equivalent fractions or percentages	**Note**	Printed on the official scheme under part (a), so it covers both of the rows above it: a branch labelled with a fraction or a	✓

Step	Mark	Description	Got it?
		percentage is marked exactly as a decimal one is.	
0.3×0.6	**M1ft**	(b) The printed scheme writes both values in quotation marks, so a candidate's own two probabilities, read off their own tree, may be used in place of 0.3 and 0.6. Both probabilities must be less than 1.	✓
0.18	**A1ft**	oe eg $\frac{18}{100}$ or $\frac{9}{50}$ or $\frac{0.18}{1}$ or 18%.	✓
Correct answer scores full marks (unless from obvious incorrect working)	**Note**	Printed in italics on the official scheme beside part (b)'s answer.	✓

Full marks: 4/4

Question 15 - Exam Solution

Understanding the Question

Given

$\left(\sqrt{3}\right)^5 = k\sqrt{3}$, with k an integer

Part (b) starts from $\frac{21}{3-\sqrt{2}}$, a fraction carrying a surd in its denominator

Find

(a) the integer k (b) integers c and d for which $\frac{21}{3-\sqrt{2}} = c + \sqrt{d}$

Plan the Solution

- (a) Split the power. $\left(\sqrt{3}\right)^5$ is $\left(\sqrt{3}\right)^4$ multiplied by one spare $\sqrt{3}$, and an even power of a square root is a whole number.
- (b) Clear the surd out of the denominator by multiplying the top and the bottom by $3+\sqrt{2}$. That pairing is a difference of two squares, so the roots cancel and the denominator becomes a whole number.
- (b) Cancel the fraction, expand the bracket, then write the surd term as one square root, because the form $c+\sqrt{d}$ carries no number in front of the root.

Worked Solution [4 marks]

Rule - Rationalising a denominator: multiply by $\frac{a+\sqrt{b}}{a+\sqrt{b}}$, because $(a-\sqrt{b})(a+\sqrt{b}) = a^2 - b$ has no surd left in it. A coefficient enters a root squared: $m\sqrt{n} = \sqrt{m^2 n}$.

Step 1: split the fifth power of the root

$$\left(\sqrt{3}\right)^5 = \left(\sqrt{3}\right)^4 \times \sqrt{3}$$

$$\left(\sqrt{3}\right)^4 = \left(\left(\sqrt{3}\right)^2\right)^2 = 3^2 = 9$$

(Reason: Squaring a square root undoes it, so $\left(\sqrt{3}\right)^2 = 3$. Taking the even part of the power first leaves a whole number multiplied by one single $\sqrt{3}$.)

Step 2: read off k

$$\left(\sqrt{3}\right)^5 = 9\sqrt{3}$$

$$9\sqrt{3} = k\sqrt{3} \implies k = 9$$

(Reason: Both sides are now a whole number multiplied by $\sqrt{3}$, so those whole numbers must be the same.)

Step 3: multiply the top and the bottom by the conjugate

$$\frac{21}{3-\sqrt{2}} = \frac{21}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

(Reason: $\frac{3+\sqrt{2}}{3+\sqrt{2}}$ is equal to 1, so the value of the fraction is untouched. The conjugate is the multiplier to reach for because $3+\sqrt{2}$ and $3-\sqrt{2}$ multiply to give a whole number.)

Step 4: expand the denominator

$$(3-\sqrt{2})(3+\sqrt{2}) = 9 + 3\sqrt{2} - 3\sqrt{2} - 2 = 7$$

$$\frac{21}{3-\sqrt{2}} = \frac{21(3+\sqrt{2})}{7}$$

(Reason: The two middle terms are opposites and cancel, which is the difference of two squares: $3^2 - \left(\sqrt{2}\right)^2 = 9 - 2 = 7$. The denominator now holds no surd.)

Step 5: cancel, then expand the numerator

$$\frac{21(3+\sqrt{2})}{7} = 3(3+\sqrt{2})$$

$$3(3+\sqrt{2}) = 9 + 3\sqrt{2}$$

(Reason: $\frac{21}{7} = 3$, so cancelling before the bracket is expanded keeps the numbers small. Expanding first would give $\frac{63+21\sqrt{2}}{7}$, which reaches the same place.)

Step 6: write the surd term as a single square root

$$3\sqrt{2} = \sqrt{3^2 \times 2} = \sqrt{18}$$

(Reason: The form $c + \sqrt{d}$ carries no number in front of the root, so the 3 has to be taken inside. A coefficient enters a square root squared.)

Step 7: state the result in the form the question asks for

$$\frac{21}{3-\sqrt{2}} = 9 + \sqrt{18}$$

(Reason: Comparing with $c + \sqrt{d}$ gives $c = 9$ and $d = 18$, and both of those are integers, which is what the question demands.)

(a) $k = 9$

(b) $\frac{21}{3-\sqrt{2}} = 9 + \sqrt{18}$ **so** $c = 9$ **and** $d = 18$

Verification

✓ **Check 1:** Square both sides of part (a). Squaring clears the root, so the check becomes whole-number arithmetic. $\left(\sqrt{3}\right)^{10} = 3^5 = 243$ and $\left(9\sqrt{3}\right)^2 = 81 \times 3 = 243$

✓ **Check 2:** Work both sides of part (a) out as decimals on the calculator.
$\left(\sqrt{3}\right)^5 \approx 15.5885$ and $9\sqrt{3} \approx 15.5885$

✓ **Check 3:** Multiply the answer to part (b) back by the original denominator. If the rationalising is sound, the product is the original numerator.
$\left(9+\sqrt{18}\right)\left(3-\sqrt{2}\right) = 27 - 9\sqrt{2} + 9\sqrt{2} - 6 = 21$

✓ **Check 4:** Work the original fraction and the final answer out as decimals.
$\frac{21}{3-\sqrt{2}} \approx 13.2426$ and $9 + \sqrt{18} \approx 13.2426$

✓ **Check 5:** Read the answer back against the form the question demands, $c + \sqrt{d}$ with both letters integers. $c = 9$ and $d = 18$, and both are integers

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) 9	**B1**	Allow $9\sqrt{3}$.	✓
(b) $\dfrac{21}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$ or $\dfrac{21}{3-\sqrt{2}} \times \dfrac{-3-\sqrt{2}}{-3-\sqrt{2}}$	**M1**	For explicitly multiplying the numerator and the denominator by $3+\sqrt{2}$ or $-3-\sqrt{2}$.	✓
eg $\dfrac{21\left(3+\sqrt{2}\right)}{9-3\sqrt{2}+3\sqrt{2}-2}$ or $\dfrac{21\left(3+\sqrt{2}\right)}{3^2-2}$ or $\dfrac{21\left(3+\sqrt{2}\right)}{9-2}$ or $\dfrac{21\left(3+\sqrt{2}\right)}{7}$ or $\dfrac{63+21\sqrt{2}}{9-2}$ or $\dfrac{63+21\sqrt{2}}{7}$	**M1**	Dependent on the first M1. The denominator may be 4 terms, which all need to be correct. $\dfrac{21}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}} = 9+3\sqrt{2}$ scores M1M0.	✓
Working required $9+\sqrt{18}$	**A1**	Dependent on M2.	✓
$9+\sqrt{18}$ gained with no method marks awarded	**SC B1**	Special case.	✓
$9+\sqrt{18}$ gained if you would award the first M1 but not the second M1	**SC B2**	Special case, total 2 marks.	✓

Full marks: 4/4

Question 16 - Exam Solution

Understanding the Question

Given

Find

$f(x) = 9 - \sqrt{x}$ for $x \geqslant 0$

$g(x) = 4x^2$

Two functions: one built on a square root, one a quadratic.

(a) The value of $f(9)$ (b) Every value of x for which $fg(x) < 0$. The answer is a range, not a single number.

Plan the Solution

- (a) Replace x with 9 in $f(x)$ and work out the square root before subtracting.
- (b) $fg(x)$ means do g first, so put $4x^2$ in place of x in $f(x)$.
- Simplify $\sqrt{4x^2}$, which turns the expression into a linear one, then solve the inequality.
- Watch the inequality sign: adding to both sides never turns it, and dividing by a positive number never turns it either.

Worked Solution [4 marks]

> Rule - Composite functions: $fg(x) = f(g(x))$. Work g out first, then feed that result into f. The letter standing nearest the x acts first.

Step 1: put 9 into $f(x)$

$f(9) = 9 - \sqrt{9}$

$f(9) = 9 - 3 = 6$

(Reason: The square root sign means the positive square root, so $\sqrt{9} = 3$ and not -3. The root is worked out first, then subtracted from 9.)

Step 2: build $fg(x)$ by putting $g(x)$ inside f

$fg(x) = f(4x^2)$

$fg(x) = 9 - \sqrt{4x^2}$

(Reason: $fg(x)$ says do g first, then f, so every x in $9 - \sqrt{x}$ is replaced by $4x^2$. Nothing is squared or rooted yet - this is only the substitution.)

Step 3: take the square root out

$\sqrt{4x^2} = 2x$

$fg(x) = 9 - 2x$

(Reason: A root over a product splits into the root of each factor, so $\sqrt{4x^2} = \sqrt{4} \times \sqrt{x^2}$. Here $\sqrt{4} = 2$, and the root sign means the positive square root, so $\sqrt{x^2}$ is x for $x \geqslant 0$. The square root has gone, and what is left is linear.)

Step 4: solve the inequality

$9 - 2x < 0$

$9 < 2x$

$\dfrac{9}{2} < x$

$x > \dfrac{9}{2}$

(Reason: Add $2x$ to both sides so the x term is positive, then halve both sides. Neither move turns the sign round. Reading $\frac{9}{2} < x$ from right to left gives $x > \frac{9}{2}$, the same statement written the usual way round, and $\frac{9}{2} = 4.5$.)

(a) $\mathrm{f}(9) = 6$

(b) $x > \dfrac{9}{2}$

Verification

✓ **Check 1:** Run part (a) backwards. If $9 - \sqrt{x} = 6$ then $\sqrt{x} = 3$, so $x = 3^2 = 9$. The input comes back as 9, which is the number the question put in.

✓ **Check 2:** Test a value inside the answer. Take $x = 5$, which is bigger than 4.5. Then $4 \times 5^2 = 100$ and $9 - \sqrt{100} = 9 - 10 = -1$. $\mathrm{fg}(5) = -1$, which is less than 0, so $x = 5$ does belong to the solution set.

✓ **Check 3:** Test a value outside it and the boundary itself. For $x = 4$: $4 \times 4^2 = 64$ and $9 - \sqrt{64} = 9 - 8 = 1$. For $x = 4.5$: $4 \times 4.5^2 = 81$ and $9 - \sqrt{81} = 9 - 9 = 0$. $\mathrm{fg}(4) = 1$ and $\mathrm{fg}(4.5) = 0$, and neither is less than 0, so the boundary is excluded and the inequality is strict.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) 6	**B1**	cao	✓
(b) $9 - \sqrt{4x^2}$ (< 0)	**M1**	for substituting $\mathrm{g}(x)$ in $\mathrm{f}(x)$, allow incorrect inequality sign or $=$ sign	✓

Step	Mark	Description	Got it?
(b) eg $9 - 2x$ (< 0) or $9 < 2x$ or $81 < 4x^2$ or $9^2 < 4x^2$	M1	for removing the square root, allow incorrect inequality sign or $=$ sign	✓
(b) Working required. $x > \frac{9}{2}$	A1	(dep on M1) oe eg $x > 4.5$, $\frac{9}{2} < x$, $4.5 < x$	✓

Full marks: 4/4

Question 17 - Exam Solution

Understanding the Question

Given

A histogram of frequency density against time t, in minutes.
Five bars: $0 \leq t < 10$ at frequency density 2.5; $10 \leq t < 15$ at 4; $15 \leq t < 30$ at 3.6; $30 \leq t < 50$ at 0.5; $50 \leq t < 60$ at 1.7.

Find

An estimate for the proportion of these visitors with $t > 40$.

Plan the Solution

- On a histogram the frequency is the area of the bar, so multiply each frequency density by its class width.
- Add the five frequencies to get the total number of visitors.
- The bar from 30 to 50 has to be split at 40, because only the part above 40 counts.
- Write the number above 40 over the total, then give the fraction in its simplest form.

Worked Solution [3 marks]

> Rule - Histogram: $\text{frequency} = \text{frequency density} \times \text{class width}$. The area of a bar is the frequency for that class.

Step 1: Turn each bar into a frequency

$0 \text{ to } 10 \text{ minutes: } 10 \times 2.5 = 25$

$10 \text{ to } 15 \text{ minutes: } 5 \times 4 = 20$

15 to 30 minutes: $15 \times 3.6 = 54$

30 to 50 minutes: $20 \times 0.5 = 10$

50 to 60 minutes: $10 \times 1.7 = 17$

(Reason: The width of a bar is its class width in minutes and its height is the frequency density, so $\text{width} \times \text{height}$ *is the number of visitors in that class.)*

Step 2: Add the frequencies to find the total

$25 + 20 + 54 + 10 + 17 = 126$

(Reason: There were 126 *visitors altogether, and that total is the denominator of the proportion.)*

Step 3: Split the 30 to 50 bar at 40

40 to 50 minutes: $10 \times 0.5 = 5$

50 to 60 minutes: $10 \times 1.7 = 17$

$5 + 17 = 22$

(Reason: The bar from 30 *to* 50 *has a constant height, so the part from* 40 *to* 50 *is* 10 *minutes wide; the last bar lies entirely above* 40*.)*

Step 4: Write the proportion and simplify

$$\frac{22}{126} = \frac{11}{63}$$

(Reason: Both 22 *and* 126 *divide by* 2*.)*

$$\frac{11}{63}$$

Verification

✓ **Check 1:** Measure in centimetre squares on the printed grid instead. Across, 1 cm is 5 minutes; up, 1 cm is 0.5 of a unit of frequency density. The five bars are 10, 8, 21.6, 4 and 6.8 square centimetres, and above 40 minutes there are 2 and 6.8. $\frac{8.8}{50.4} = \frac{11}{63}$

✓ **Check 2:** Work out the other part instead. The visitors with $t \leq 40$ number $25 + 20 + 54 + 5 = 104$, and the two parts must make the whole. $104 + 22 = 126$ and $\frac{52}{63} + \frac{11}{63} = 1$

✓ **Check 3:** Count in small squares instead. One small square is 1 minute across and 0.1 of a unit of frequency density up, so the bars hold 250, 200, 540, 100 and 170 small squares, and above 40 minutes there are $50 + 170 = 220$. $\frac{220}{1260} = \frac{11}{63}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$10 \times 2.5 = 25$, $5 \times 4 = 20$, $15 \times 3.6 = 54$, $20 \times 0.5 = 10$, $10 \times 1.7 = 17$, $10 \times 0.5 = 5$ (area of the 40 to 50 bar)	**M1**	for finding the area of at least 2 bars, either using $\text{freq density} \times \text{mins}$ or use of counting small squares or cm^2. Values may be seen on the diagram. 22 or 220 or 8.8 implies M1	✓
$(10 \times 2.5) + (5 \times 4) + (15 \times 3.6) + (20 \times 0.5) + (10 \times 1.7) = 126$ or $25 + 20 + 54 + 10 + 17 = 126$ or $250 + 200 + 540 + 100 + 170 = 1260$ or $10 + 8 + 21.6 + 4 + 6.8 = 50.4$	**M1**	for method to find total number of visitors (allow one error or omission) or total number of small squares or cm^2 for the method used (allow one error or omission). The printed scheme puts those five values in quotation marks, so a candidate's own earlier frequencies may be used.	✓
$\frac{11}{63}$	**A1**	oe eg $\frac{22}{126}$ or $\frac{220}{1260}$ or $0.174(60\ldots)$ or 0.175 or $17.4(60\ldots)\%$ or 17.5% or 22 out of 126. If $\frac{22}{126}$ is seen in the workings and 22 is on the answer line, award M2A0	✓
On the answer line	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 3/3

Question 18 - Exam Solution

Understanding the Question

Given	Find

y is inversely proportional to x^3. One pair of values: $y = 4$ when $x = 3$.	The value of x when $y = 864$.

Plan the Solution

- Turn the words into an equation. Inversely proportional to x^3 means $y = \dfrac{k}{x^3}$ for some constant k.
- Use the pair of values the question gives to work out k.
- Put $y = 864$ into the finished formula and rearrange to leave x^3 on its own.
- Take the cube root to get x.

Worked Solution [4 marks]

> Rule - Inverse proportion: $y \propto \dfrac{1}{x^3}$ means $y = \dfrac{k}{x^3}$, where k is a constant. Find k from one pair of values, and the same formula then holds for every other pair.

Step 1: Write the statement as an equation

$$y \propto \frac{1}{x^3}$$

$$y = \frac{k}{x^3}$$

(Reason: Inversely proportional means y is a constant divided by x^3, not multiplied by it. The constant has to be written as a symbol such as k until its value is known.)

Step 2: Substitute $x = 3$ and $y = 4$ to find k

$$4 = \frac{k}{3^3}$$

$$k = 4 \times 3^3 = 108$$

(Reason: Multiplying both sides by 3^3 leaves k on its own, so the constant comes from the one pair of values the question gives.)

Step 3: Put $y = 864$ into the formula

$$864 = \frac{108}{x^3}$$

$$x^3 = \frac{108}{864}$$

$$\frac{108}{864} = \frac{1}{8}$$

(Reason: Multiply both sides by x^3 and then divide by 864. Both 108 and 864 divide by 108, which is what cancels the fraction down to $\frac{1}{8}$.)

Step 4: Take the cube root

$$x = \sqrt[3]{\frac{1}{8}}$$

$$x = \frac{1}{2} = 0.5$$

(Reason: Cube rooting undoes the cube. Since $2^3 = 8$, the cube root of $\frac{1}{8}$ is $\frac{1}{2}$, and a bigger y gives a smaller x, which is what inverse proportion should do.)

$$x = 0.5$$

Verification

✓ **Check 1:** Put $x = 0.5$ back into $y = \frac{108}{x^3}$. Cubing 0.5 gives 0.125, and the y that comes out has to be 864. $\frac{108}{0.5^3} = \frac{108}{0.125} = 864$

✓ **Check 2:** Work in multipliers and never find the constant at all. Here y is multiplied by $\frac{864}{4} = 216$, so x^3 is divided by 216 and x is divided by the cube root of 216. $6^3 = 216$ and $\frac{3}{6} = 0.5$

✓ **Check 3:** For inverse proportion to x^3 the product $y \times x^3$ is the same for every pair, so the given pair and the answer pair must produce the same number. $4 \times 27 = 108$ and $864 \times 0.5^3 = 108$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$y = \frac{k}{x^3}$ or $hy = \frac{1}{x^3}$	M1	do not award for $y = \frac{1}{x^3}$. Constant of proportionality must be a symbol such as k. Condone use of $\propto$ for method marks	✓

Step	Mark	Description	Got it?
$4 = \dfrac{k}{3^3}$ or $k = 4 \times 3^3\ (= 108)$ or $h \times 4 = \dfrac{1}{3^3}$ or $h = \dfrac{1}{4 \times 3^3}\ \left(= \dfrac{1}{108}\right)$	**M1**	for substitution of x and y into a correct formula. Condone use of $\propto$ for method marks	✓
eg $(x^3 =)\dfrac{4 \times 3^3}{864}\ \left(= \dfrac{1}{8}\right)$ or $(x^3 =)\dfrac{108}{864}\ \left(= \dfrac{1}{8}\right)$ oe	**M1**	for method to find x^3. Condone use of $\propto$ for method marks. The printed scheme writes the second form with 108 in quotation marks, so a candidate's own earlier constant may be used	✓
0.5	**A1**	oe	✓
On the answer line	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 4/4

Question 19 - Exam Solution

Understanding the Question

Given

A curve $y = \mathrm{f}(x)$ with exactly one minimum point.
That minimum point is $(8, -12)$.
No formula for $\mathrm{f}(x)$ is given, so the answer has to come from the transformation itself.

Find

(i) The minimum point of $y = \mathrm{f}(x) + 3$.
(ii) The minimum point of $y = \mathrm{f}(2x)$.

Plan the Solution

- For each new equation, ask whether the change happens after f has acted, or to the input before it acts.
- A change after f can only move the output, so only the y-coordinate shifts.

- A change to the input can only move the x-coordinate, so the height stays the same.
- Then apply that movement to the one point the question gives, $(8, -12)$, because the lowest point of the new curve is the image of the lowest point of the old one.

Worked Solution [2 marks]

> Rule - transforming $y = \mathrm{f}(x)$: the curve $y = \mathrm{f}(x) + a$ is the curve translated a units up, and the curve $y = \mathrm{f}(kx)$ is the curve stretched parallel to the x-axis with scale factor $\frac{1}{k}$.

Step 1: See where the $+3$ acts

$y = \mathrm{f}(x) + 3$

(Reason: The 3 is added after f has produced its output, so every height on the curve goes up by 3 and no x-coordinate changes.)

Step 2: Move the minimum point up 3

$y_{\text{new}} = -12 + 3 = -9 \implies (8, -9)$

(Reason: The whole curve slides up, so the lowest point of the new curve is the image of the lowest point of the old one. Its x-coordinate stays at 8 and only its height moves.)

Step 3: See where the 2 in $\mathrm{f}(2x)$ acts

$y = \mathrm{f}(2x)$

(Reason: Here the input is doubled before f acts, so the new curve reaches at x whatever the old curve reached at $2x$. Nothing is done to the output, so no height changes.)

Step 4: Halve the x-coordinate of the minimum

$$2x = 8 \implies x = \frac{8}{2} = 4 \implies (4, -12)$$

(Reason: The lowest value of f is still -12, and it happens when the input is 8. The input is now $2x$, so that is $x = 4$, with the height unchanged.)

(i) $(8, -9)$

(ii) $(4, -12)$

Verification

✓ **Check 1:** Test it on a curve that fits the question. $\mathrm{f}(x) = (x - 8)^2 - 12$ has one minimum, at $(8, -12)$, and $\mathrm{f}(x) + 3 = (x - 8)^2 - 9$, which is smallest when $(x - 8)^2 = 0$. Lowest at $x = 8$, height -9, giving $(8, -9)$.

✓ **Check 2:** The same curve for the second part: $f(2x) = (2x-8)^2 - 12$, which is smallest when $2x - 8 = 0$. Lowest at $x = 4$, height -12, giving $(4, -12)$.

✓ **Check 3:** Ask which coordinate each change is even able to move. Adding 3 happens to the output of f, while doubling happens to its input, so one answer must keep its x-coordinate and the other must keep its height. (i) keeps $x = 8$; (ii) keeps $y = -12$. Both answers do.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) Write down the coordinates	**B1**	for $(8, -9)$	✓
(ii) Write down the coordinates	**B1**	for $(4, -12)$	✓

Full marks: 2/2

Question 20 - Exam Solution

Understanding the Question

Given

In triangle ABD: $AB = 9.4$ cm, $AD = 12.8$ cm, and the angle BAD between them is $72°$

In triangle BCD: the angle BDC is $39°$ and the angle BCD is $54°$

The diagonal BD is a side of both triangles, so it is the only link between them

Find

The length of BC, correct to 3 significant figures

Plan the Solution

- Triangle ABD hands you two sides and the angle BETWEEN them, which is the cosine rule's shape. It gives BD.
- BD is the bridge. Once it is known, triangle BCD has two angles and one side, which is the sine rule's shape.
- Pair each side with the angle FACING it: BC faces $39°$ and BD faces $54°$.
- Keep the unrounded BD in the calculator and round once, right at the end.

Worked Solution [5 marks]

> Rule - Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$, where A is the angle between the sides b and c. Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$, where every side sits over the sine of the angle facing it.

Step 1: put triangle ABD into the cosine rule

$$BD^2 = 9.4^2 + 12.8^2 - 2 \times 9.4 \times 12.8 \times \cos 72^\circ$$

(Reason: Two sides and the angle between them is exactly what the cosine rule takes. The 72° sits between AB and AD, so the side it finds is the third one, BD.)

Step 2: work out BD

$$9.4^2 + 12.8^2 = 88.36 + 163.84 = 252.2$$

$$2 \times 9.4 \times 12.8 \times \cos 72^\circ = 240.64 \times \cos 72^\circ = 74.3618$$

$$252.2 - 74.3618 = 177.8382$$

$$BD = \sqrt{177.8382} = 13.3356$$

(Reason: The cosine rule gives BD^2, never BD, so the square root is a step in its own right and is easy to forget. Writing BD down as 13.3 here would drag the final answer with it, so carry the full display value forward.)

Step 3: match each side of triangle BCD to the angle facing it

$$\frac{BC}{\sin 39^\circ} = \frac{BD}{\sin 54^\circ}$$

(Reason: The angle at D is 39° and it looks straight across the triangle at BC. The angle at C is 54° and it looks across at BD. Pairing a side with the angle it touches, instead of the one facing it, is the commonest way to lose this question.)

Step 4: rearrange and work out BC

$$BC = \frac{BD}{\sin 54^\circ} \times \sin 39^\circ$$

$$BC = \frac{13.3356}{\sin 54^\circ} \times \sin 39^\circ = 10.3735$$

(Reason: Multiplying both sides of the sine rule by $\sin 39^\circ$ leaves BC on its own. Type it as one calculation so that nothing in the middle gets rounded.)

Step 5: round to 3 significant figures

$$BC = 10.3735\ldots$$

$$BC = 10.4 \text{ cm}$$

(Reason: The first three significant figures are 1, 0 and 3. The next digit is 7, so the 3 rounds up to 4.)

$BC = 10.4$ cm

Verification

✓ **Check 1:** The sine rule says every side divided by the sine of the angle facing it gives the SAME number. Work the two out separately and compare them. $\dfrac{\sin 39°}{10.3735} = 0.06067$ and $\dfrac{\sin 54°}{13.3356} = 0.06067$

✓ **Check 2:** Work backwards through a different route. The third angle of triangle BCD is $180° - 39° - 54° = 87°$, and the sine rule then gives $DC = 16.4611$. Putting BC and DC into the cosine rule must hand BD^2 straight back.
$10.3735^2 + 16.4611^2 - 2 \times 10.3735 \times 16.4611 \times \cos 54° = 177.84$ and $13.3356^2 = 177.84$

✓ **Check 3:** A sanity check with no calculator. In any triangle the larger angle faces the longer side, and in triangle BCD the angle facing BC is $39°$ while the angle facing BD is $54°$. So BC has to come out SHORTER than BD, and 10.4 is shorter than 13.3.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(BD^2 =)9.4^2 + 12.8^2 - 2 \times 9.4 \times 12.8 \times \cos 72 \quad (= 177.8\ldots)$	M1	for applying the cosine rule. Also allow $(BD^2 =)88.36 + 163.84 - 2 \times 9.4 \times 12.8 \times \cos 72 \quad (= 177.8\ldots)$ oe, or $(BD =)\sqrt{9.4^2 + 12.8^2 - 2 \times 9.4 \times 12.8 \times \cos 72}$.	✓
$(BD =)13.3$	A1	allow 13.3 to 13.342, or $\sqrt{177.8\ldots}$, or $\sqrt{178}$.	✓
eg $\dfrac{BC}{\sin 39} = \dfrac{BD}{\sin 54}$ or $\dfrac{\sin 39}{BC} = \dfrac{\sin 54}{BD}$, where BD stands for the candidate's own earlier value	M1ft	for applying the sine rule. Allow use of their BD.	✓

Step	Mark	Description	Got it?
$(BC =)\dfrac{BD}{\sin 54} \times \sin 39$, where BD stands for the candidate's own earlier value	**M1ft**	for a method to find BC using the sine rule. Allow use of their BD.	✓
10.4	**A1**	allow 10.3 to 10.4.	✓
A correct answer scores full marks, unless it comes from obviously incorrect working.	**Note**	The printed scheme carries this in italics beside the final A1, and it awards nothing of its own.	✓

Full marks: 5/5

Question 21 - Exam Solution

Understanding the Question

Given

A jar of 20 beads: 9 red, 7 yellow and 4 green
Three beads are taken at random, and none is put back, so each draw changes what is left in the jar

Find

The probability that exactly two of the three beads are the same colour

Plan the Solution

- Exactly two the same means one matching pair and one bead of a different colour. All three alike does not count, and neither does all three different.
- There are only three colours, so there are only three cases: two reds, two yellows, two greens. Work each one out and add.
- Nothing is put back, so the denominators count down 20, 19, 18. Keeping every fraction over 6840 makes the adding easy.

- In each case the odd bead can be drawn first, second or third. Those 3 orders have the same probability, so work out one and multiply by 3.

Worked Solution [3 marks]

> Rule - Taking without replacement: each bead taken is gone, so the totals count down 20, 19, 18. A matching pair and one odd bead can come out in 3 orders of equal probability, so one product multiplied by 3 covers the whole case.

Step 1: check the colours, and pin down what the question allows

$$9 + 7 + 4 = 20$$

(Reason: The three colours account for every bead, so no fourth colour is hiding. Exactly two the same means one matching pair plus one bead of a different colour, which leaves three cases to work out: two reds, two yellows, two greens.)

Step 2: two reds and one bead that is not red

$$\frac{9}{20} \times \frac{8}{19} \times \frac{11}{18} = \frac{792}{6840}$$
$$3 \times \frac{792}{6840} = \frac{2376}{6840}$$

(Reason: There are 9 reds out of 20. The first red is gone, so the second is 8 out of 19, and the third bead must be one of the $20 - 9 = 11$ that are not red, out of the 18 still in the jar. That product is only the order red, red, other. The odd bead could just as easily come first or second, and each of those orders has the same probability, so multiply by 3.)

Step 3: two yellows and one bead that is not yellow

$$\frac{7}{20} \times \frac{6}{19} \times \frac{13}{18} = \frac{546}{6840}$$
$$3 \times \frac{546}{6840} = \frac{1638}{6840}$$

(Reason: The same shape with new numbers: 7 yellows, then 6 out of 19, then one of the $20 - 7 = 13$ beads that are not yellow out of the 18 left. Leaving it over 6840 rather than cancelling now means the three cases can simply be added at the end.)

Step 4: two greens and one bead that is not green

$$\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18} = \frac{192}{6840}$$
$$3 \times \frac{192}{6840} = \frac{576}{6840}$$

(Reason: Green is the scarce colour: only 4 to begin with, so the second green is 3 out of 19. The third bead is one of the $20 - 4 = 16$ beads that are not green. Take the complement from the colour you

have just paired, never from one of the other colours - that swap is the commonest way this question is lost.)

Step 5: add the three cases, then cancel

$$\frac{2376}{6840} + \frac{1638}{6840} + \frac{576}{6840} = \frac{4590}{6840}$$

$$\frac{4590}{6840} = \frac{51}{76}$$

(Reason: Two reds, two yellows and two greens cannot happen at the same time, so the three probabilities add. The highest common factor of 4590 and 6840 is 90, and dividing both by it leaves $\frac{51}{76}$, which is 0.671 to 3 significant figures.)

$\frac{51}{76}$ (0.671 to 3 significant figures)

Verification

✓ **Check 1:** Count the other way round. Every one of the $20 \times 19 \times 18 = 6840$ ordered picks falls into exactly one of three camps: all three the same colour, all three different, or exactly two the same. Work out the first two camps and take them away.
$9 \times 8 \times 7 + 7 \times 6 \times 5 + 4 \times 3 \times 2 = 738$, $6 \times 9 \times 7 \times 4 = 1512$, and $6840 - 738 - 1512 = 4590$

✓ **Check 2:** Throw order away completely and count selections instead. There are 1140 ways of choosing three beads from 20. A pair of reds with one other bead can be chosen in 36×11 ways, a pair of yellows in 21×13 ways and a pair of greens in 6×16 ways.
$396 + 273 + 96 = 765$ and $\frac{765}{1140} = \frac{51}{76}$

✓ **Check 3:** The three camps have to account for everything, so their probabilities must add to 1. Put the answer back beside the other two and see whether they close.
$$\frac{4590}{6840} + \frac{738}{6840} + \frac{1512}{6840} = 1$$

Mark Scheme Breakdown

Step

eg $(\mathrm{P}(RRY)=)\frac{9}{20}\times\frac{8}{19}\times\frac{7}{18}\left(=\frac{504}{6840}=\frac{7}{95}\right)$ oe or

$(\mathrm{P}(RRG)=)\frac{9}{20}\times\frac{8}{19}\times\frac{4}{18}\left(=\frac{288}{6840}=\frac{4}{95}\right)$ oe or

$(\mathrm{P}(RRR)=)\frac{9}{20}\times\frac{8}{19}\times\frac{7}{18}\left(=\frac{504}{6840}=\frac{7}{95}\right)$ oe or

$(\mathrm{P}(RRR')=)\frac{9}{20}\times\frac{8}{19}\times\frac{11}{18}\left(=\frac{792}{6840}=\frac{11}{95}\right)$ oe or

$(\mathrm{P}(YYR)=)\frac{7}{20}\times\frac{6}{19}\times\frac{9}{18}\left(=\frac{378}{6840}=\frac{21}{380}\right)$ oe or

$(\mathrm{P}(YYG)=)\frac{7}{20}\times\frac{6}{19}\times\frac{4}{18}\left(=\frac{168}{6840}=\frac{7}{285}\right)$ oe or

$(\mathrm{P}(YYY)=)\frac{7}{20}\times\frac{6}{19}\times\frac{5}{18}\left(=\frac{210}{6840}=\frac{7}{228}\right)$ oe or

$(\mathrm{P}(YYY')=)\frac{7}{20}\times\frac{6}{19}\times\frac{13}{18}\left(=\frac{546}{6840}=\frac{91}{1140}\right)$ oe or

$(\mathrm{P}(GGR)=)\frac{4}{20}\times\frac{3}{19}\times\frac{9}{18}\left(=\frac{108}{6840}=\frac{3}{190}\right)$ oe or

$(\mathrm{P}(GGY)=)\frac{4}{20}\times\frac{3}{19}\times\frac{7}{18}\left(=\frac{84}{6840}=\frac{7}{570}\right)$ oe or

$(\mathrm{P}(GGG)=)\frac{4}{20}\times\frac{3}{19}\times\frac{2}{18}\left(=\frac{24}{6840}=\frac{1}{285}\right)$ oe or

$(\mathrm{P}(GGG')=)\frac{4}{20}\times\frac{3}{19}\times\frac{16}{18}\left(=\frac{192}{6840}=\frac{8}{285}\right)$ oe or

$(\mathrm{P}(RGY)=)\frac{9}{20}\times\frac{7}{19}\times\frac{4}{18}\left(=\frac{252}{6840}=\frac{7}{190}\right)$ oe

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$(\mathrm{P}(RRR' \text{ or } YYY' \text{ or } GGG')=)\left(3\times\frac{9}{20}\times\frac{8}{19}\times\frac{11}{18}\right)+\left(3\times\frac{7}{20}\times\frac{6}{19}\times\frac{13}{18}\right)+\left(3\times\frac{4}{20}\times\frac{3}{19}\times\frac{16}{18}\right)$ oe

or $(\mathrm{P}(RRY \text{ or } RRG \text{ or } YYR \text{ or } YYG \text{ or } GGR \text{ or } GGY)=)\left(3\times\frac{9}{20}\times\frac{8}{19}\times\frac{7}{18}\right)+\left(3\times\frac{9}{20}\times\frac{8}{19}\times\frac{4}{18}\right)+\left(3\times\frac{7}{20}\times\frac{6}{19}\times\frac{9}{18}\right)+\left(3\times\frac{7}{20}\times\frac{6}{19}\times\frac{4}{18}\right)+\left(3\times\frac{4}{20}\times\frac{3}{19}\times\frac{9}{18}\right)+\left(3\times\frac{4}{20}\times\frac{3}{19}\times\frac{7}{18}\right)$ oe

Step
or $(1 - \mathrm{P}(RRR \text{ or } YYY \text{ or } GGG \text{ or } RGY) =)$ $1 - \left(\left(\frac{9}{20} \times \frac{8}{19} \times \frac{7}{18}\right) + \left(\frac{7}{20} \times \frac{6}{19} \times \frac{5}{18}\right) + \left(\frac{4}{20} \times \frac{3}{19} \times \frac{2}{18}\right) + \left(6 \times \frac{9}{20} \times \frac{7}{19} \times \frac{4}{18}\right.\right.$ oe
$\frac{51}{76}$
Correct answer scores full marks (unless from obvious incorrect working).
for an answer of $\frac{669}{1000}$ oe eg $0.66(9)$ or $66(.9)\%$
Where the three named wrong answers come from.

Full marks: 3/3

Question 22 - Exam Solution

Understanding the Question

Given

Find

$x^2 + 3y + y^2 = 7$, an equation with a squared term in each letter, so it is a curve
$y = x + 2$, a straight line, with y already on its own
Algebraic working is asked for, so the values on their own would earn nothing.

Every pair of values of x and y that fits both equations at the same time. A quadratic appears on the way, so expect two solution pairs.

Plan the Solution

- The line gives y in terms of x, so replace every y in the curve equation by $x + 2$. That leaves one letter instead of two.
- Expand, then collect everything on one side, to reach a three term quadratic in x alone.
- Factorise that quadratic and read off the two values of x.
- Put each value of x back into $y = x + 2$, the simpler of the two equations, and keep each y beside the x it came from.

Worked Solution [5 marks]

> Rule - Substitution (line into curve): where a linear equation gives one letter on its own, write that letter's expression everywhere the letter appears in the other equation. One letter is eliminated, and the quadratic left behind has two roots - each root carrying its own partner.

Step 1: substitute the line into the curve

$$x^2 + 3(x + 2) + (x + 2)^2 = 7$$

(Reason: The second equation says y and $x + 2$ are the same thing, so every y in the first equation may be written as $x + 2$. Both of them have to go - the y inside $3y$ as well as the y^2. Replacing only one of them leaves both letters in the equation, and nothing can be solved from there.)

Step 2: expand, then collect into a three term quadratic

$$x^2 + 3x + 6 + x^2 + 4x + 4 = 7$$
$$2x^2 + 7x + 10 = 7$$
$$2x^2 + 7x + 3 = 0$$

(Reason: $(x + 2)^2$ means $(x + 2)(x + 2) = x^2 + 4x + 4$, not $x^2 + 4$. Collecting then gives $x^2 + x^2 = 2x^2$, $3x + 4x = 7x$ and $6 + 4 = 10$. Taking 7 from both sides leaves zero on the right, which is the form a quadratic must be in before it can be factorised.)

Step 3: factorise, and read off the two values of x

$$2x^2 + 7x + 3 = (2x + 1)(x + 3)$$
$$2x + 1 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = -3$$

(Reason: The $2x^2$ can only come from $2x \times x$, and the $+3$ only from 1×3. What decides which bracket takes the 1 and which takes the 3 is the middle term: multiply the brackets out again and the two x terms must add to $7x$. A product is zero only when one of its brackets is zero, and that is what turns one factorised line into two small equations.)

Step 4: put each value of x back into the line

$$x = -\frac{1}{2} \implies y = -\frac{1}{2} + 2 = \frac{3}{2}$$
$$x = -3 \implies y = -3 + 2 = -1$$

(Reason: Use $y = x + 2$ rather than the curve: it is one addition each, with no squaring to go wrong, and the mark scheme allows either equation. Keep each y with the x that produced it - the marks are for pairs, not for four loose numbers.)

Step 5: write the two solution pairs

$$x = -\frac{1}{2},\ y = \frac{3}{2}$$
$$x = -3,\ y = -1$$

(Reason: Each root of the quadratic carries one value of y, so the answer is two pairs and not four separate values. Written as coordinates, $\left(-\frac{1}{2}, \frac{3}{2}\right)$ and $(-3, -1)$ are the two points where the line crosses the curve, and the mark scheme allows that form as well.)

$$x = -\frac{1}{2},\ y = \frac{3}{2}$$
$$x = -3,\ y = -1$$

Verification

✓ **Check 1, the pair $\left(-\frac{1}{2}, \frac{3}{2}\right)$:** Put both values into the curve equation $x^2 + 3y + y^2$, working the three terms out one at a time, then test the line as well. A pair has to fit both equations, not just the one it was found from.

$$\left(-\frac{1}{2}\right)^2 + 3 \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 = \frac{1}{4} + \frac{9}{2} + \frac{9}{4} = 7 \text{ and } -\frac{1}{2} + 2 = \frac{3}{2}$$

✓ **Check 2, the pair $(-3, -1)$:** The same test on the second pair. Squaring a negative gives a positive, so $(-3)^2$ is 9 and $(-1)^2$ is 1, while the middle term $3y$ stays negative.

$$(-3)^2 + 3 \times (-1) + (-1)^2 = 9 - 3 + 1 = 7 \text{ and } -3 + 2 = -1$$

✓ **Check 3:** Start from the other end. Rearranging the line gives $x = y - 2$, and substituting that into the curve leaves a quadratic in y instead. This is the mark scheme's own second method, and it must arrive at the same two pairs. $(y-2)^2 + 3y + y^2 = 7$ gives $2y^2 - y - 3 = 0$, so $(y+1)(2y-3) = 0$ and $y = -1$ or $y = \frac{3}{2}$, and $x = y - 2$ turns those into -3 and $-\frac{1}{2}$

✓ **Check 4:** Test the two roots against the quadratic without factorising anything. Whatever method solves $ax^2 + bx + c = 0$, its roots must add to $-\frac{b}{a}$ and multiply to $\frac{c}{a}$. Here $a = 2$, $b = 7$ and $c = 3$, so the sum should be $-\frac{7}{2}$ and the product $\frac{3}{2}$.
$-\frac{1}{2} + (-3) = -\frac{7}{2}$ and $-\frac{1}{2} \times (-3) = \frac{3}{2}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $x^2 + 3(x+2) + (x+2)^2 = 7$ or eg $(y-2)^2 + 3y + y^2 = 7$	M1	for substitution of $y = x + 2$ (or $x = \pm y \pm 2$) into $x^2 + 3y + y^2 = 7$ to obtain an equation in x only (or y only)	✓
eg $2x^2 + 7x + 3 (= 0)$ or $2x^2 + 7x = -3$ or eg $2y^2 - y - 3 (= 0)$ or $2y^2 - y = 3$	M1ft	dep on previous M1 for multiplying out and collecting terms, forming a three term quadratic in any form of $ax^2 + bx + c\,(= 0)$ where at least 2 coefficients (a or b or c) are correct	✓
eg $(2x+1)(x+3)(=0)$ or $x = \dfrac{-7 \pm \sqrt{(7)^2 - 4 \times 2 \times 3}}{2 \times 2}$ or $\left(x + \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = -\frac{3}{2}$ $\left(x = -\frac{1}{2} \text{ and } x = -3\right)$ or eg $(y+1)(2y-3)(=0)$ or	M1ft	dep on first M1 method to solve their 3 term quadratic using any correct method (allow one sign error and some simplification - allow as far as eg $\dfrac{-7 \pm \sqrt{49 - 24}}{4}$ or $\dfrac{1 \pm \sqrt{1 + 24}}{4}$ or if factorising allow brackets which expanded give 2 out of 3 terms correct) or	✓

Step	Mark	Description	Got it?
$y = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 2 \times -3}}{2 \times 2}$ or $\left(y - \dfrac{1}{4}\right)^2 - \left(\dfrac{1}{4}\right)^2 = \dfrac{3}{2}$ $\left(y = -1 \text{ and } y = \dfrac{3}{2}\right)$		correct values for x or correct values for y	
$y = " - \dfrac{1}{2} " + 2\left(= \dfrac{3}{2}\right)$ and $y = " - 3" + 2\,(= -1)$ or $x = " - 1" - 2\,(= -3)$ and $x = "\dfrac{3}{2}" - 2\left(= -\dfrac{1}{2}\right)$	**M1ft**	dep on previous M1 for substituting their 2 found values of x or y into one of the two given equations or fully correct values for the other variable (correct labels for x / y) or for one correct pair of values. The printed row puts the substituted values in quotation marks, so a candidate's own earlier values may be used here.	✓
Working required $x = -\dfrac{1}{2}$ and $y = \dfrac{3}{2}$ $x = -3$ and $y = -1$	**A1**	oe dep on M2 (allow coordinates)	✓
If they find the values of y but think they are the values of x then the maximum mark is 3	**Note**	The printed scheme carries this line beneath the table, and it awards nothing of its own.	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

OAB is a triangle with $\overrightarrow{OA} = 12\mathbf{a}$ and $\overrightarrow{OB} = 8\mathbf{b}$

Find

(a) $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ (b) the value of n

P is the midpoint of OA, and Q lies on OB
ABR and PQR are straight lines, with $AB : BR = 1 : 2$ and $\overrightarrow{OQ} = n\mathbf{b}$

Plan the Solution

- For (a), travel from A to B the long way round, through O.
- For (b), build $\overrightarrow{PR}$ out of $\overrightarrow{PA}$ and $\overrightarrow{AR}$. Read the ratio carefully: $AB : BR = 1 : 2$ makes AR three parts long, not two.
- Write $\overrightarrow{PQ}$ using n, then use the fact that P, Q and R lie on one straight line.
- Compare the $\mathbf{a}$ parts to find the multiplier, then the $\mathbf{b}$ parts give n.

Worked Solution [5 marks]

> Rule - Vectors on a straight line: if P, Q and R lie on one line, then $\overrightarrow{PQ} = \lambda\overrightarrow{PR}$ for some number λ. Because $\mathbf{a}$ and $\mathbf{b}$ are not parallel, the $\mathbf{a}$ parts and the $\mathbf{b}$ parts must then match separately.

Step 1: (a) Travel from A to B through O

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$\overrightarrow{AB} = -12\mathbf{a} + 8\mathbf{b}$$

(Reason: Going from A to O is $\overrightarrow{OA} = 12\mathbf{a}$ travelled backwards, which is $-12\mathbf{a}$. Adding $\overrightarrow{OB} = 8\mathbf{b}$ finishes the journey at B.)

Step 2: (b) Find $\overrightarrow{PA}$, and count how many parts long AR is

$$\overrightarrow{PA} = \frac{1}{2} \times 12\mathbf{a} = 6\mathbf{a}$$
$$AB : BR = 1 : 2 \implies \overrightarrow{AR} = 3\overrightarrow{AB}$$

(Reason: P is the midpoint of OA, so $\overrightarrow{PA}$ is half of $12\mathbf{a}$. Along ABR, AB is 1 part and BR is 2 more parts, so the whole of AR is 3 parts.)

Step 3: Build $\overrightarrow{PR}$ from those two pieces

$$\overrightarrow{PR} = \overrightarrow{PA} + \overrightarrow{AR}$$
$$\overrightarrow{PR} = 6\mathbf{a} + 3(-12\mathbf{a} + 8\mathbf{b}) = -30\mathbf{a} + 24\mathbf{b}$$

(Reason: Walk from P back to A, then straight along ABR all the way to R.)

Step 4: Write $\overrightarrow{PQ}$ using n

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$

$$\overrightarrow{PQ} = -6\mathbf{a} + n\mathbf{b}$$

(Reason: $\overrightarrow{PO}$ reverses $\overrightarrow{OP} = 6\mathbf{a}$, and $\overrightarrow{OQ} = n\mathbf{b}$ is given.)

Step 5: PQR is a straight line, so $\overrightarrow{PQ} = \lambda\overrightarrow{PR}$

$$-6\mathbf{a} + n\mathbf{b} = \lambda(-30\mathbf{a} + 24\mathbf{b})$$

$$-6 = -30\lambda \implies \lambda = \frac{1}{5}$$

(Reason: $\mathbf{a}$ and $\mathbf{b}$ are not parallel, so the two sides can only be equal if the $\mathbf{a}$ parts match and the $\mathbf{b}$ parts match. The $\mathbf{a}$ parts pin down λ.)

Step 6: Compare the $\mathbf{b}$ parts to get n

$$n = 24\lambda = 24 \times \frac{1}{5}$$

$$n = \frac{24}{5} = 4.8$$

(Reason: The $\mathbf{b}$ part of $\overrightarrow{PR}$ is 24, and $\lambda = \frac{1}{5}$ scales it down to $\frac{24}{5}$.)

(a) $\overrightarrow{AB} = -12\mathbf{a} + 8\mathbf{b}$

(b) $n = 4.8$

Verification

✓ **Check 1:** Put real vectors in: $\mathbf{a} = (3, 1)$ and $\mathbf{b} = (-1, 4)$. Then A is $(36, 12)$, B is $(-8, 32)$, P is $(18, 6)$ and R is $(-96, 72)$. The line PR crosses OB at $(-4.8, 19.2)$, which is $4.8\mathbf{b}$. $n = 4.8$

✓ **Check 2:** Take a different journey to Q, namely O to P to Q:

$n\mathbf{b} = 6\mathbf{a} + k(-30\mathbf{a} + 24\mathbf{b})$. There is no $\mathbf{a}$ on the left, so $0 = 6 - 30k$ and $k = \frac{1}{5}$.

$$n = 24 \times \frac{1}{5} = 4.8$$

✓ **Check 3:** Ask where this puts Q. Since $\overrightarrow{OQ} = 4.8\mathbf{b}$ and $\overrightarrow{OB} = 8\mathbf{b}$, the rest of the side is $\overrightarrow{QB} = 3.2\mathbf{b}$, giving $OQ : QB = 3 : 2.8 - 4.8 = 3.2$, so Q sits on OB nearer B, which is where the diagram puts it

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\overrightarrow{AB} = -12\mathbf{a} + 8\mathbf{b}$	B1	oe	✓
$(\overrightarrow{PR} =)\ 6\mathbf{a} + 3(-12\mathbf{a} + 8\mathbf{b})\ (= -30\mathbf{a} + 24\mathbf{b})$ oe or $(\overrightarrow{RP} =)\ -3(-12\mathbf{a} + 8\mathbf{b}) - 6\mathbf{a}\ (= 30\mathbf{a} - 24\mathbf{b})$	M1	for method to find $\overrightarrow{PR}$ or $\overrightarrow{RP}$, ft their $\overrightarrow{AB}$. The printed scheme puts $-12\mathbf{a} + 8\mathbf{b}$ in quotation marks, so a candidate's own answer to part (a) may be used here.	✓
$(\overrightarrow{PQ} =)\ -6\mathbf{a} + n\mathbf{b}$ oe eg $(\overrightarrow{PQ} =)\ -6\mathbf{a} + m \times 8\mathbf{b}$ or $(\overrightarrow{OQ} =)\ 6\mathbf{a} + k(-30\mathbf{a} + 24\mathbf{b})$ or $(\overrightarrow{AQ} =)\ -12\mathbf{a} + n\mathbf{b}$	M1	for method to find $\overrightarrow{PQ}$ or $\overrightarrow{OQ}$ or $\overrightarrow{AQ}$. The printed scheme puts $-30\mathbf{a} + 24\mathbf{b}$ in quotation marks in the $\overrightarrow{OQ}$ form, so a candidate's own earlier value may be used.	✓
$\overrightarrow{PQ} = \lambda\overrightarrow{PR}$ eg $-6\mathbf{a} + n\mathbf{b} = \lambda(-30\mathbf{a} + 24\mathbf{b})$ or $-6 = -30\lambda$ oe or $\lambda = \frac{1}{5}$ oe OR $\mu\overrightarrow{PQ} = \overrightarrow{PR}$ eg $\mu(-6\mathbf{a} + n\mathbf{b}) = -30\mathbf{a} + 24\mathbf{b}$ or $-6\mu = -30$ oe or $\mu = 5$ OR $\overrightarrow{OQ} = \overrightarrow{OP} + k\overrightarrow{PR}$ eg $n\mathbf{b} = 6\mathbf{a} + k(-30\mathbf{a} + 24\mathbf{b})$ or $0 = 6 - 30k$ or $k = \frac{1}{5}$ oe OR $\overrightarrow{AQ} = \overrightarrow{AR} + x\overrightarrow{RP}$ eg $-12\mathbf{a} + n\mathbf{b} = 3(-12\mathbf{a} + 8\mathbf{b}) + x(30\mathbf{a} - 24\mathbf{b})$ or $-36 + 30x = -12$ or $x = \frac{4}{5}$ oe	M1	for setting up an equation to find the value of the unknown coefficient(s)	✓
Working required. $n = 4.8$	A1	oe eg $\frac{24}{5}$, dep on M1	✓

Full marks: 5/5

Question 24 - Exam Solution

Understanding the Question

Given

An arithmetic series with 30 terms, first term a and common difference d
The 20th term: $U_{20} = 123$
The sum of all 30 terms: $S_{30} = 2880$

Find

The value of the first term a The value of the common difference d Two unknowns, so two equations are needed and they are solved together

Plan the Solution

- Turn the 20th term into an equation using $U_n = a + (n-1)d$
- Turn the sum of the 30 terms into a second equation using $S_n = \frac{n}{2}(2a + (n-1)d)$
- Make the coefficients of a match, then subtract to leave an equation in d only
- Substitute that value of d back into the simpler equation to find a

Worked Solution [5 marks]

Rule - Arithmetic series: the nth term is $U_n = a + (n-1)d$, and the sum of the first n terms is $S_n = \frac{n}{2}(2a + (n-1)d)$

Step 1: turn the 20th term into an equation

$U_{20} = a + (20-1)d$
$a + 19d = 123$

(Reason: the nth term of an arithmetic series is $U_n = a + (n-1)d$, so the 20th term sits 19 common differences after the first term, not 20)

Step 2: turn the sum of the 30 terms into a second equation

$$S_{30} = \frac{30}{2}(2a + (30-1)d)$$

$2880 = 15(2a + 29d)$
$2a + 29d = 192$

(Reason: the sum of the first n terms is $S_n = \frac{n}{2}(2a + (n-1)d)$, and here $\frac{30}{2} = 15$, so dividing both sides by 15 clears the bracket and leaves a second equation in a and d)

Step 3: match the coefficients of a and subtract

$2 \times (a + 19d) = 2 \times 123$

$2a + 38d = 246$

$(2a + 38d) - (2a + 29d) = 246 - 192$

$9d = 54$

$d = \frac{54}{9} = 6$

(Reason: both equations now begin with $2a$, so subtracting one from the other removes a completely and leaves an equation in d alone)

Step 4: substitute $d = 6$ back to find a

$a + 19 \times 6 = 123$

$a + 114 = 123$

$a = 123 - 114 = 9$

(Reason: either equation will do, and $a + 19d = 123$ is the simpler one; $19 \times 6 = 114$, which leaves a on its own)

Step 5: state both values clearly

$a = 9$

$d = 6$

(Reason: the final mark is awarded only when a and d are clearly identified, so both are written out and labelled rather than left inside the working)

$$a = 9$$
$$d = 6$$

Verification

✓ **Check 1:** Put $a = 9$ and $d = 6$ into the 20th term, $a + 19d$
$9 + 19 \times 6 = 9 + 114 = 123$

✓ **Check 2:** Put both values into $S_{30} = \frac{30}{2}(2a + 29d)$, where $2 \times 9 = 18$ and $29 \times 6 = 174$ $15 \times (18 + 174) = 15 \times 192 = 2880$

✓ **Check 3:** Total the series a different way. The last term is $9 + 29 \times 6 = 183$, and an arithmetic series also sums as $\frac{n}{2}(U_1 + U_n)$ $15 \times (9 + 183) = 15 \times 192 = 2880$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$123 = a + (20 - 1)d$ or $123 = a + 19d$	M1	for using $U_n = a + (n - 1)d$	✓
$2880 = \frac{30}{2}(2a + (30 - 1)d)$ or $2880 = \frac{30}{2}(2a + 29d)$ or $192 = 2a + 29d$	M1	for using $S_n = \frac{n}{2}(2a + (n - 1)d)$	✓
eg $192 = 2a + 29d$, $123 = a + 19d\,(\times 2)$, $246 = 2a + 38d$, subtracting $54 = 9d$ or $192 = 2(123 - 19d) + 29d$ oe or $d = 6$ or $192 = 2a + 29d\,(\times 19)$, $123 = a + 19d\,(\times 29)$, $3648 = 38a + 551d$, $3567 = 29a + 551d$, subtracting $81 = 9a$ or $192 = 2a + 29\left(\frac{123 - a}{19}\right)$ oe or $a = 9$	M1	(dep on M2) for a correct method to find a or d: coefficients of a or d the same in correct equations and correct operator to eliminate the selected variable, resulting in an equation in a only or in d only, or writing a or d in terms of the other variable and correctly substituting (condone missing brackets)	✓
eg $192 = 2a + 29("6")$ oe or $123 = a + 19("6")$ oe or $192 = 2("9") + 29d$ oe or $123 = "9" + 19d$ oe	M1	(dep on M3) for substituting their found value of a or d into a correct equation	✓
Working required $a = 9$ $d = 6$	A1	dep on M2, and a and d must be clearly identified	✓

Step	Mark	Description	Got it?
The quotation marks in the substitution row	**Note**	a value in quotation marks is the candidate's own earlier value, so the substitution mark is available on a follow-through even when that value is wrong	✓

Full marks: 5/5

Question 25 - Exam Solution

Understanding the Question

Given

$PQRS$ is a square, and PR is a diagonal of it
The vertex P is at $(4,\ 7)$
The vertex R is at $(8,\ -5)$
Q and S are the ends of the other diagonal, and their coordinates are not given

Find

An equation of the straight line through Q and S It must be written as $ay = bx + c$ with a, b and c integers, so any fraction has to be cleared before the answer is stated

Plan the Solution

- The diagonals of a square bisect each other at right angles, so QS is the perpendicular bisector of PR
- Find the midpoint of PR: that centre point lies on QS as well
- Find the gradient of PR, then take its negative reciprocal for the gradient of QS
- Put the gradient and the centre into $y - y_1 = m(x - x_1)$, then multiply through to clear the fraction

Worked Solution [5 marks]

> Rule - Diagonals of a square: they bisect each other at right angles, so QS passes through the midpoint of PR and $m_{PR} \times m_{QS} = -1$

Step 1: find the midpoint of PR

$$\left(\frac{4+8}{2}, \frac{7+(-5)}{2}\right) = (6,\ 1)$$

(Reason: the diagonals of a square bisect each other, so the midpoint of PR is the centre of the square and therefore lies on QS too; it is the one point of QS the question hands you)

Step 2: find the gradient of PR

$$m_{PR} = \frac{7-(-5)}{4-8} = \frac{12}{-4} = -3$$

(Reason: gradient is the change in y divided by the change in x; taking the two points in the other order gives $\frac{-5-7}{8-4} = \frac{-12}{4}$, the same value)

Step 3: turn that into the gradient of QS

$$m_{PR} \times m_{QS} = -1$$

$$-3 \times m_{QS} = -1$$

$$m_{QS} = \frac{-1}{-3} = \frac{1}{3}$$

(Reason: the diagonals of a square meet at right angles, and perpendicular gradients multiply to -1, so the gradient of QS is the negative reciprocal of -3; dividing -1 by a negative number gives a positive gradient)

Step 4: build the equation through the centre

$$y - 1 = \frac{1}{3}(x-6)$$

$$y - 1 = \frac{1}{3}x - 2$$

$$y = \frac{1}{3}x - 1$$

(Reason: a line through $(x_1,\ y_1)$ with gradient m is $y - y_1 = m(x - x_1)$, and here the point is the centre $(6,\ 1)$ with gradient $\frac{1}{3}$)

Step 5: clear the fraction to get integer coefficients

$$3 \times y = 3 \times \left(\frac{1}{3}x - 1\right)$$

$$3y = x - 3$$

(Reason: the question asks for $ay = bx + c$ with a, b and c integers, and multiplying every term by 3 removes the fraction, leaving $a = 3$, $b = 1$ and $c = -3$)

$$3y = x - 3$$

Verification

✓ **Check 1:** Find the other two vertices by turning the half-diagonal a quarter turn each way about the centre $(6,\ 1)$. They are $(12,\ 3)$ and $(0,\ -1)$. Put each one into $3y = x - 3$ $3 \times 3 = 9$ and $12 - 3 = 9$; then $3 \times (-1) = -3$ and $0 - 3 = -3$

✓ **Check 2:** Multiply the two diagonal gradients together. Perpendicular lines must give -1 $-3 \times \dfrac{1}{3} = -1$

✓ **Check 3:** Every point of QS must be the same distance from P as from R. Test the centre $(6,\ 1)$: squared distance to P is $(6-4)^2 + (1-7)^2$, and to R it is $(6-8)^2 + (1+5)^2$ $4 + 36 = 40$ and $4 + 36 = 40$

✓ **Check 4:** The two diagonals of a square are equal in length. Compare their squared lengths, PR first and then QS $(8-4)^2 + (-5-7)^2 = 16 + 144 = 160$ and $(12-0)^2 + (3-(-1))^2 = 144 + 16 = 160$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\left(\dfrac{4+8}{2},\ \dfrac{7-5}{2}\right)$ oe or $(6,\ 1)$	M1	for finding the midpoint of PR	✓
$\dfrac{7--5}{4-8}\left(=-\dfrac{12}{4}=-3\right)$ oe	M1	for method to find the gradient of PR	✓
$"-3" \times m = -1$ oe or $(m =)\dfrac{-1}{"-3"}$ or $(m =)\dfrac{1}{3}$	M1ft	for finding the gradient of QS, may be seen embedded in an equation, ft their gradient of PR	✓
$"1" = "\dfrac{1}{3}"("6") + c$ oe or $c = -1$ or $y - 1 = \dfrac{1}{3}(x-6)$ or $y = \dfrac{1}{3}x - 1$	M1ft	(dep on previous M1) for finding the equation through QS, ft their gradient of PR and their midpoint of PR, do not allow $(4,\ 7)$ or $(8,\ -5)$ as midpoint PR	✓
$3y = x - 3$	A1	oe eg $6y = 2x - 6$ or $3y - x + 3 = 0$ etc but must be	✓

Step	Mark	Description	Got it?
		integer coefficients, accept $a = 3$, $b = 1$, $c = -3$	
Correct answer scores full marks (unless from obvious incorrect working)	**Note**	the printed scheme carries this line beside the method rows, so a correct equation earns all five marks even when the midpoint and gradient working is not shown	✓
The quotation marks in the two follow-through rows	**Note**	a value in quotation marks is the candidate's own earlier value, so the gradient row and the equation row are both available on a follow-through even when the gradient of PR was found wrongly	✓

Full marks: 5/5

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