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Edexcel IGCSE 4MA1

4MA1/1HR (Higher Tier) – Thursday 15 May 2025

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/1HR (Higher Tier), June 2025 series, sat Thursday 15 May 2025 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Work out the lowest common multiple (LCM) of 45 and 70 *[2 marks]*

....

[Total 2 marks]

2. The length of a footbridge is 142.8 m, correct to 1 decimal place.

(i) Write down the lower bound of the length of the footbridge. *[1 mark]*

(ii) Write down the upper bound of the length of the footbridge. *[1 mark]*

(i) ....

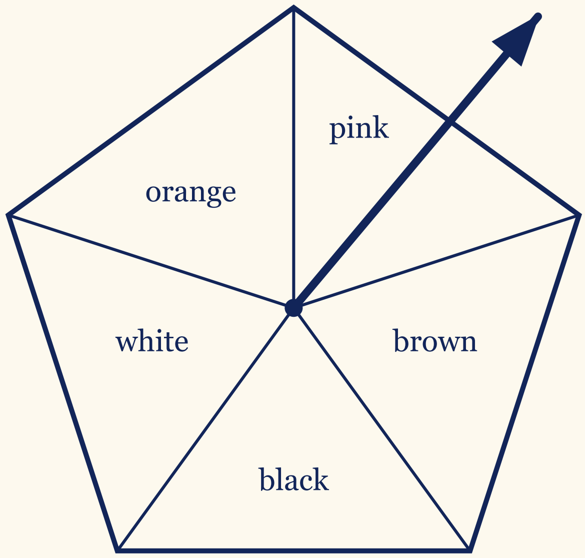
(ii) ....

[Total 2 marks]

3. Show that the product of $2\frac{1}{4}$ and $1\frac{5}{7}$ is $3\frac{6}{7}$. *[3 marks]*

[Total 3 marks]

4. A stall at a school summer fair uses the biased 5-sided spinner shown below.
When the spinner is spun, it can land on orange or on pink or on brown or on black or on white.



The table gives information about the probability of the spinner landing on each colour.

Colour	orange	pink	brown	black	white
Probability	0.12	0.20	0.38	$4x$	x

Chloe spins the spinner once.

(a) Work out the probability that the spinner lands on orange or on pink or on brown. *[1 mark]*

Oliver spins the spinner 350 times.

(b) Work out an estimate for the number of times the spinner lands on black. *[4 marks]*

(a) ....

(b) ....

[Total 5 marks]

5. $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{2, 4, 6, 8, 10, 12\}$
$B = \{3, 6, 9, 12\}$
$C = \{1, 3, 5, 7, 9, 11\}$

(a) Write down the members of the set
(i) $A \cup B$
(ii) B' *[2 marks]*

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{2, 4, 6, 8, 10, 12\}$
$B = \{3, 6, 9, 12\}$
$C = \{1, 3, 5, 7, 9, 11\}$

$\mathscr{E}$	$\cap$	$\cup$	$\varnothing$	$\in$	$\notin$

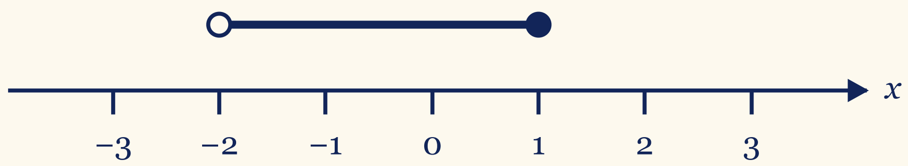
(b) Choose a symbol from the box and write it on each dotted line so that each statement below is true.
(i) $A \cap C =$ ....
(ii) 13 .... $\mathscr{E}$ *[2 marks]*

(a)(i) ....

(a)(ii) ....

[Total 4 marks]

6. (a) Write down the inequality that the number line above shows. *[2 marks]*



(b) Solve the inequality $7a - 5 \leq 3a + 28$
You must show clear algebraic working. *[2 marks]*

(a) ....

(b) ....

[Total 4 marks]

7. Convert a speed of $50x$ metres per second into kilometres per hour. *[3 marks]*

.... km/h

[Total 3 marks]

8. (a) Simplify $a^6 \times a^{10}$ *[1 mark]*

(b) Simplify $\frac{c^{30}}{c^{12}}$ *[1 mark]*

(c) (i) Factorise $y^2 - 10y + 21$
[2 marks]
(ii) Hence solve the equation $y^2 - 10y + 21 = 0$ *[1 mark]*

(a) ....

(b) ....

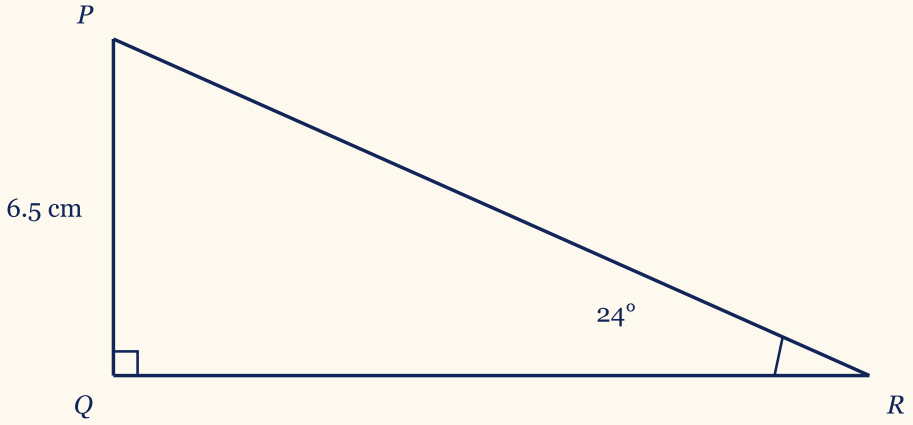
(c)(i) ....

(c)(ii) ....

[Total 5 marks]

9. The diagram shows triangle PQR, in which angle PQR is a right angle.

Diagram NOT accurately drawn

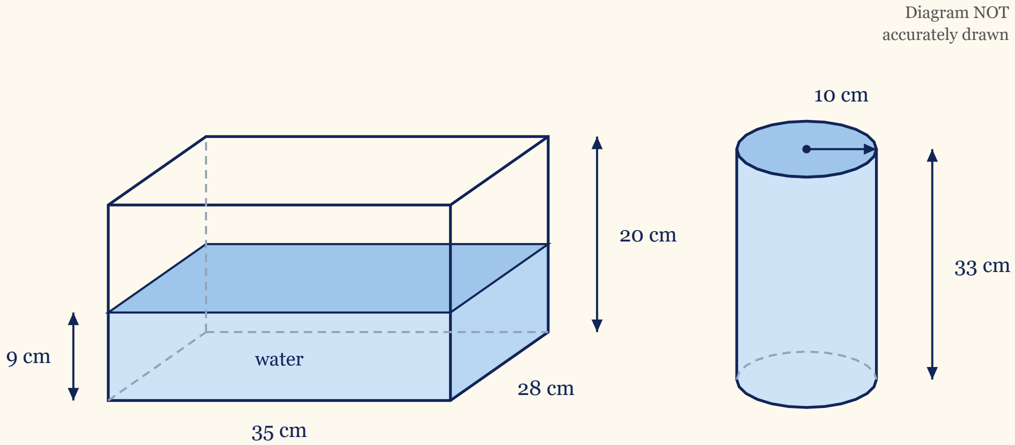


Calculate the length of QR.
Give your answer correct to 3 significant figures. *[3 marks]*

.... cm

[Total 3 marks]

10. The diagram shows two water containers at a garden centre.
One is a cuboid trough and the other is a cylindrical drum.



The trough measures 35 cm by 28 cm by 20 cm
The surface of the water in the trough is 9 cm above the base of the trough.

The drum has a radius of 10 cm and a height of 33 cm
The drum is completely full of water.

Freya is going to pour all the water from the drum into the trough.

Show that the trough will not be completely full of water. *[3 marks]*

[Total 3 marks]

11. Jian invests money for 2 years.
The amount invested with Meridian Bank is \$2500 and the amount invested with Halewood Bank is \$3000.

Meridian Bank
Invests \$2500
amount of money invested : total interest after 2 years = 20 : 3

Halewood Bank
Invests \$3000
4% per year compound interest for 2 years

Jian receives more interest from Meridian Bank than from Halewood Bank.
How much more? *[5 marks]*

\$ ....

[Total 5 marks]

12. The table gives information about the ages of the 80 people who visited an art gallery one Saturday.

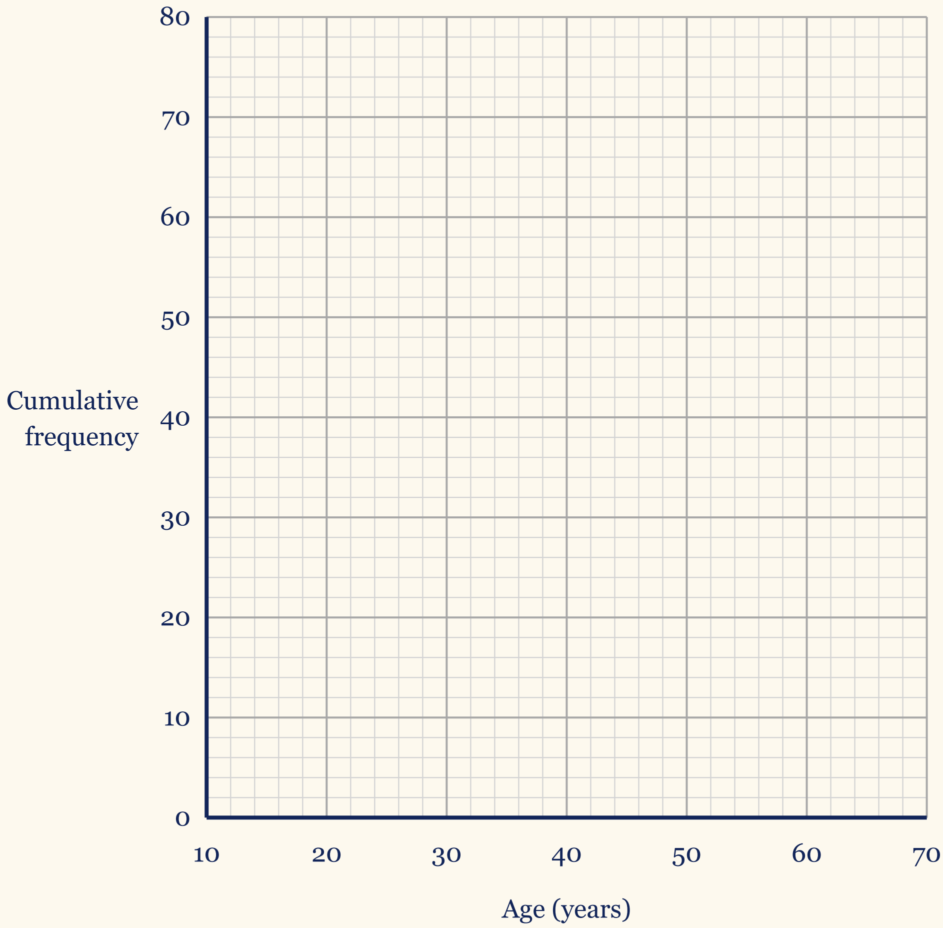
Age (n years)	**Frequency**
$10 < n \leq 20$	12
$20 < n \leq 30$	15
$30 < n \leq 40$	20
$40 < n \leq 50$	18
$50 < n \leq 60$	9
$60 < n \leq 70$	6

(a) Complete the cumulative frequency table.

Age (n years)	**Cumulative frequency**
$10 < n \leq 20$	
$10 < n \leq 30$	
$10 < n \leq 40$	
$10 < n \leq 50$	
$10 < n \leq 60$	
$10 < n \leq 70$	

[1 mark]

(b) On the grid below, draw a cumulative frequency graph for your completed table. *[2 marks]*



(c) Use your graph to work out an estimate for the percentage of these 80 people who are more than 46 years of age.
Give your answer correct to the nearest whole number. *[3 marks]*

.... %

[Total 6 marks]

13. Here are the numbers of bicycles hired from a seaside kiosk on each of 11 days in July.

9 10 12 15 17 18 19 19 20 25

Work out the interquartile range of the numbers of bicycles hired. *[2 marks]*

....

[Total 2 marks]

14. Find the value of $6.7 \times 10^{135} + 3 \times 10^{134}$
Give your answer in standard form. *[2 marks]*

....

[Total 2 marks]

15. (a) Solve the equation

$$\frac{5a+8}{3} - \frac{2a+5}{4} = 23$$

Show clear algebraic working. *[4 marks]*

(b) Write $\left(\frac{\sqrt{y}}{3}\right)^{-1}$ in the form cy^n, where c and n are numbers to be found. *[2 marks]*

(a) a = ...

(b) ....

[Total 6 marks]

16. Show, using algebra, that the recurring decimal $0.6\dot{1}\dot{2}$ is equal to $\frac{101}{165}$. *[2 marks]*

[Total 2 marks]

17. Three numbers are consecutive multiples of 4.
Prove that the difference between the square of the largest number and the square of the smallest number is always a multiple of 64.

You must show clear algebraic working. *[3 marks]*

[Total 3 marks]

18. F varies inversely with the cube of r
When $r = 2$, $F = 6$

(a) Work out a formula for F in terms of r *[3 marks]*

(b) Work out the value of r when $F = 3072$ *[2 marks]*

(a) ....

(b) r = ...

[Total 5 marks]

19. Malee has 9 tiles.
There is a number on each tile.

1 1 2 3 3 3 5 6 6

Malee puts the 9 tiles in a box.
She takes a tile from the box at random and does not put the tile back.
She then takes a second tile from the box at random.

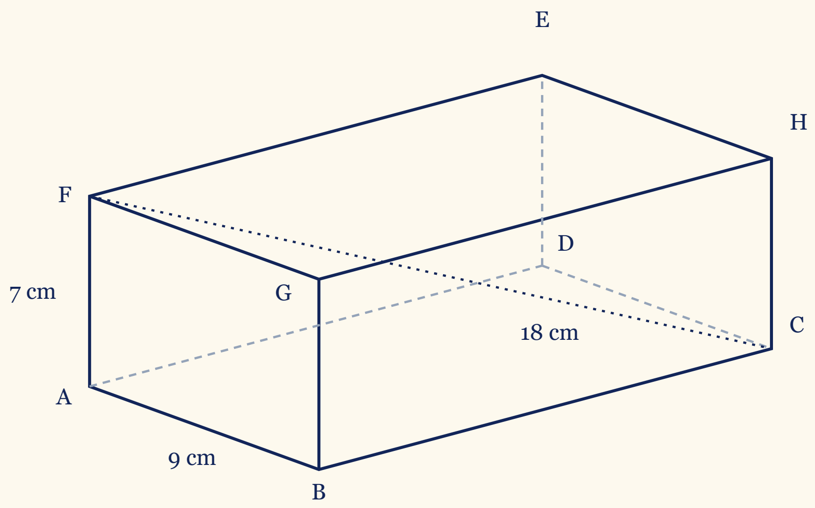
Work out the probability that the sum of the numbers on the two tiles is less than 5 *[3 marks]*

....

[Total 3 marks]

20. The diagram shows a cuboid $ABCDEFGH$.

Diagram NOT accurately drawn



$AB = 9$ cm
$AF = 7$ cm
$FC = 18$ cm

Work out the length of BC.
Give your answer correct to 3 significant figures. *[3 marks]*

.... cm

[Total 3 marks]

21. A curve has equation $y = f(x)$ and it has exactly one turning point.
The coordinates of that turning point are $(-6, 9)$

(a) Write down the coordinates of the turning point of the curve with equation $y = f(3x)$ *[1 mark]*

The curve C, with equation $y = g(x)$, is transformed to give the curve S, with equation $y = g(x + a) + b$
Under this transformation the point $(4, -5)$ on C is mapped to the point $(1, -16)$ on S

(b) Write down the value of a and the value of b *[2 marks]*

(a) ....

a = ...

b = ...

[Total 3 marks]

22. Solve these simultaneous equations.

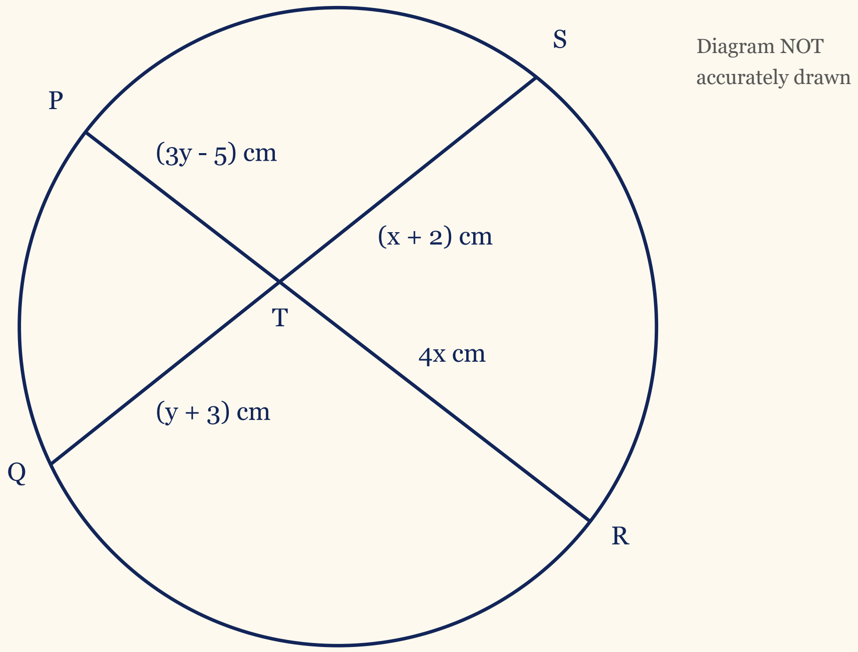
$x^2 + y^2 = 41$
$2x + y = 3$

You must show clear algebraic working. *[5 marks]*

....

[Total 5 marks]

23. The chords PTR and QTS of a circle cross at the point T.



$PT = (3y - 5)$ cm, $QT = (y + 3)$ cm, $RT = 4x$ cm and $ST = (x + 2)$ cm.

Work out an expression for y in terms of x. *[5 marks]*

y = ...

[Total 5 marks]

24. The opening 3 terms of an arithmetic series are

$$(2x + 5) \qquad (3y - 4) \qquad (4x - 2)$$

in which x and y are constants.

The first 9 terms of the series have a sum of 216

Work out the value of x and the value of y.
You must show clear algebraic working. *[6 marks]*

x = ...

y = ...

[Total 6 marks]

25. The diagram shows two sectors of circles that meet at C.

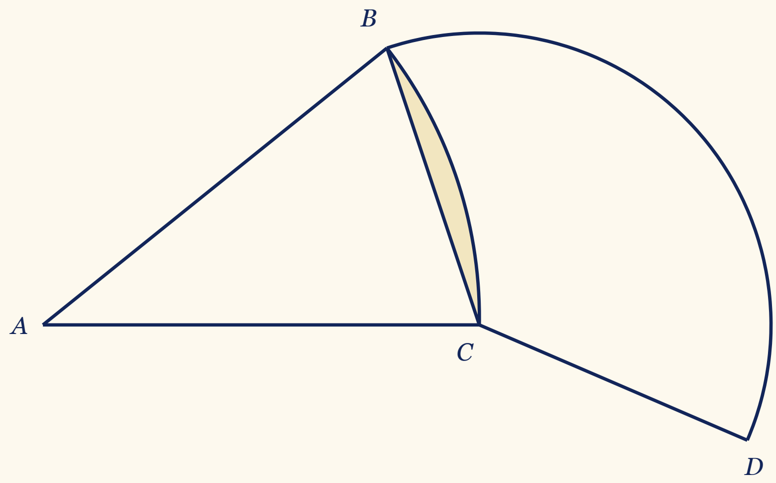


Diagram NOT accurately drawn

BAC is a sector of a circle, centre A

BCD is a sector of a circle, centre C

Angle $BAC = 40°$
Angle $BCD = 130°$
Area of the shaded segment $= 28 \text{ cm}^2$

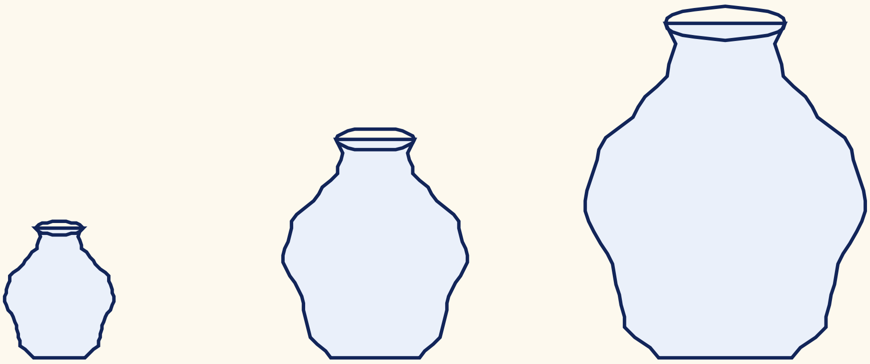
Work out the length of the arc BD.
Give your answer correct to 3 significant figures. *[6 marks]*

.... cm

[Total 6 marks]

26. R, S and T are three mathematically similar storage jars.

Not drawn accurately



R S T

The volume of jar S is 72.8% more than the volume of jar R

The height of jar R is h cm
The height of jar T is $6h$ cm

The surface area of jar S is A cm^2
The surface area of jar T is kA cm^2

Work out the value of k *[4 marks]*

k = ...

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

45 and 70
Two whole numbers, and neither one divides into the other.

Find

The lowest common multiple of 45 and 70 That is the smallest positive whole number that both 45 and 70 divide into exactly.

Plan the Solution

- Split 45 and 70 into products of prime factors.
- Collect every prime that appears in either list.
- For each of those primes, keep the higher of its two powers.
- Multiply the kept factors together.
- Check by dividing the answer by each of the two numbers.

Worked Solution [2 marks]

> Rule - LCM from prime factors: write each number as a product of primes, then multiply together the highest power of every prime that appears in either number.

Step 1: write 45 as a product of primes

$45 = 3 \times 15$
$15 = 3 \times 5$
$45 = 3^2 \times 5$

(Reason: Keep dividing by the smallest prime that goes in. The factor 3 is used twice, so it is written as 3^2.)

Step 2: write 70 as a product of primes

$70 = 2 \times 35$
$35 = 5 \times 7$

$70 = 2 \times 5 \times 7$

(Reason: 70 is even, so it starts with 2. Here every prime is used only once, so there are no powers to record.)

Step 3: take the highest power of every prime

primes used: $2,\ 3,\ 5,\ 7$

$\text{LCM} = 2 \times 3^2 \times 5 \times 7$

(Reason: The prime 3 appears twice in 45 and not at all in 70, so 3^2 is kept, not 3. Each of 2, 5 and 7 appears once at most, so one of each is kept.)

Step 4: multiply the kept factors together

$2 \times 3^2 \times 5 \times 7 = 630$

(Reason: Multiply the kept factors in any order. The product is the smallest number that both 45 and 70 divide into exactly.)

630

Verification

✓ **Check 1:** A common multiple must divide exactly by both numbers, leaving no remainder.
$\dfrac{630}{45} = 14$ and $\dfrac{630}{70} = 9$

✓ **Check 2:** Count up in 70s and stop at the first number that 45 divides into. Stopping at the first one is what makes it the LOWEST common multiple. The ninth multiple of 70 is 630, and none of the eight before it is a multiple of 45.

✓ **Check 3:** The only prime shared by 45 and 70 is 5, so their HCF is 5. For any two numbers, the LCM multiplied by the HCF equals the product of the numbers.
$\dfrac{45 \times 70}{5} = 630$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$45, 90, 135, 180 \ldots$ and $70, 140, 210, 280 \ldots$ or $2, 5, 7$ and $3, 3, 5$ or a Venn diagram with 2 and 7 in one circle, 3 and 3 in the other, and 5 in the overlap	M1	For any correct valid method, eg for starting to list at least four multiples of each number, or $2, 5, 7$ and $3, 3, 5$ seen (may be in a factor tree, ignore 1), or a fully correct Venn diagram, or $5, 9, 14$ oe (could be in a table).	✓

Step	Mark	Description	Got it?
or $\dfrac{45 \times 70}{5}$ or $2, 3, 3, 5, 7$ oe or a table with 5 against 45 and 70, giving 9 and 14 or $5, 9, 14$ oe			
Correct answer scores full marks (unless from obvious incorrect working)	A1	630. Allow $2 \times 3^2 \times 5 \times 7$ oe, eg $5 \times 9 \times 14$.	✓

Full marks: 2/2

Question 2 - Exam Solution

Understanding the Question

Given

The length of a footbridge is 142.8 m, correct to 1 decimal place.
That is a rounded measurement, so the true length is not exactly 142.8 m.

Find

The lower bound of the length of the footbridge The upper bound of the length of the footbridge Each part asks for one value in metres, and neither asks for any working.

Plan the Solution

- Write down the values either side of 142.8 on a scale marked in tenths of a metre: 142.7 and 142.9.
- The gap between them is the unit the length was rounded to, 0.1 m.
- Halve that unit. The true length can be at most 0.05 m away from 142.8 m.
- Subtract 0.05 for the lower bound, then add 0.05 for the upper bound.
- Check by rounding each bound back, and by measuring the width of the interval between them.

Worked Solution [2 marks]

Rule - Bounds of a rounded measurement: halve the unit the measurement was rounded to, then subtract that half for the lower bound and add it for the upper bound.

Step 1: find the unit the length was rounded to

$142.7 \qquad 142.8 \qquad 142.9$

$142.9 - 142.8 = 0.1$

(Reason: Correct to 1 decimal place means the length was rounded to the nearest tenth of a metre, so the values it could have been rounded to sit 0.1 m apart.)

Step 2: halve the rounding unit

$$\frac{0.1}{2} = 0.05$$

(Reason: Every length between the two halfway points rounds to 142.8, and those halfway points sit 0.05 m either side of it.)

Step 3: subtract the half-unit for the lower bound

$142.8 - 0.05 = 142.75$

(Reason: The shortest length that still rounds up to 142.8 is the one exactly 0.05 m below it, so take the half-unit off and not the whole 0.1 m.)

Step 4: add the half-unit for the upper bound

$142.8 + 0.05 = 142.85$

(Reason: Going the other way, a length just under 142.85 m still rounds down to 142.8 m, so 142.85 is where that stops.)

Step 5: state the interval the true length lies in

$142.75 \leq \text{length} < 142.85$

(Reason: The lower bound is included, because 142.75 itself rounds up to 142.8. The upper bound is not, because 142.85 rounds up to 142.9. That is why the mark scheme also accepts $142.8499\ldots$ for part (ii).)

(i) 142.75 m

(ii) 142.85 m

Verification

✓ **Check 1:** Round the values at each edge back to 1 decimal place and see where the answer changes. 142.75 rounds to 142.8, while 142.749 rounds to 142.7. At the top, 142.8499 still rounds to 142.8.

✓ **Check 2:** Each bound must be the same distance from the measurement that was written down. $142.8 - 142.75 = 0.05$ and $142.85 - 142.8 = 0.05$

✓ **Check 3:** The interval must be exactly one rounding unit wide, with the given length at its midpoint. $142.85 - 142.75 = 0.1$ and $\dfrac{142.75 + 142.85}{2} = 142.8$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(i) lower bound	**B1**	142.75	✓
(ii) upper bound	**B1**	142.85. Accept $142.8499\ldots$ or $142.84\dot{9}$.	✓

Full marks: 2/2

Question 3 - Exam Solution

Understanding the Question

Given

The two mixed numbers $2\frac{1}{4}$ and $1\frac{5}{7}$.

The value their product is claimed to have, $3\frac{6}{7}$.

The answer is already printed, so the marks are for the working that reaches it, not for the value.

Find

Working that takes $2\frac{1}{4} \times 1\frac{5}{7}$ to $3\frac{6}{7}$, with every line shown.

Plan the Solution

- Write each mixed number as an improper fraction, because a whole number and a fraction cannot be multiplied separately and then joined back together.
- Multiply the numerators together and the denominators together, which gives $\dfrac{108}{28}$.
- Cancel that down to $\dfrac{27}{7}$, then turn it back into a mixed number.
- Finish on the value the question prints, so the last line of the working is the statement being shown. A calculator is allowed on this paper, but a decimal answer earns nothing

here.

Worked Solution [3 marks]

> Rule - Multiplying mixed numbers: write each one as an improper fraction, then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, and change the result back to a mixed number.

Step 1: write each mixed number as an improper fraction

$$2\frac{1}{4} = \frac{4 \times 2 + 1}{4} = \frac{9}{4}$$

$$1\frac{5}{7} = \frac{7 \times 1 + 5}{7} = \frac{12}{7}$$

(Reason: A mixed number is a whole number plus a fraction, so multiply the whole number by the denominator and add the numerator on. Four quarters make one whole, so $2\frac{1}{4}$ is nine quarters, and seven sevenths make one whole, so $1\frac{5}{7}$ is twelve sevenths. The denominator never changes.)

Step 2: multiply the numerators, and multiply the denominators

$$\frac{9}{4} \times \frac{12}{7} = \frac{9 \times 12}{4 \times 7} = \frac{108}{28}$$

(Reason: Two fractions are multiplied by multiplying the tops together and the bottoms together. This is the line the second mark is for, and the mark scheme asks to see either $\frac{108}{28}$ or the cancelling that replaces it.)

Step 3: cancel the fraction down

$$\frac{108}{28} = \frac{27 \times 4}{7 \times 4} = \frac{27}{7}$$

(Reason: The highest common factor of 108 and 28 is 4, and taking that factor out of the top and the bottom leaves $\frac{27}{7}$.)

Step 4: write the improper fraction back as a mixed number

$$\frac{27}{7} = \frac{21}{7} + \frac{6}{7} = 3 + \frac{6}{7}$$

$$3 + \frac{6}{7} = 3\frac{6}{7}$$

(Reason: Seven goes into 27 three times with 6 left over, so twenty-seven sevenths is three whole ones and $\frac{6}{7}$ of another. That is the value the question asks for, so the statement is shown.)

$$2\frac{1}{4} \times 1\frac{5}{7} = \frac{108}{28} = \frac{27}{7} = 3\frac{6}{7} \textbf{ as required}$$

Verification

✓ **Check 1:** Cancel the 4 into the 12 before multiplying instead of afterwards, which is the other order the mark scheme allows. $\frac{9}{4} \times \frac{12}{7} = \frac{9 \times 3}{1 \times 7} = \frac{27}{7}$, the same simplified fraction, reached without ever writing $\frac{108}{28}$.

✓ **Check 2:** Split each mixed number into its two parts and multiply out $\left(2 + \frac{1}{4}\right)\left(1 + \frac{5}{7}\right)$, a route that forms no improper fraction at all. $2 + \frac{10}{7} + \frac{1}{4} + \frac{5}{28} = \frac{108}{28}$, which is the same product as Step 2 reached.

✓ **Check 3:** Work backwards: turn the printed answer $3\frac{6}{7}$ into an improper fraction and compare it with the product. $\frac{7 \times 3 + 6}{7} = \frac{27}{7} = \frac{108}{28}$, so the two ends of the working meet.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Both mixed numbers written as improper fractions	M1	For $2\frac{1}{4}$ and $1\frac{5}{7}$ expressed as improper fractions, eg $\frac{9}{4}$ and $\frac{12}{7}$.	✓
Correct cancelling, or the multiplication carried out without cancelling	M1	Correct cancelling, or multiplication of numerators and denominators without cancelling, eg $\frac{9}{4} \times \frac{12}{7} = \frac{108}{28}$ or equivalent, eg $\frac{63}{28} \times \frac{48}{28} = \frac{3024}{784}$. Cancelling the 4 into the 12 before multiplying scores this mark just as the multiplication does.	✓
Conclusion reached from correct working	A1	Dependent on M2, for a conclusion to $3\frac{6}{7}$ from correct working - either sight of the result of the	✓

Step	Mark	Description	Got it?
		multiplication, eg $\frac{108}{28}$ or equivalent, must be seen, or correct cancelling prior to the multiplication to $\frac{27}{7}$. Working is required.	
Decimals	**Note**	Use of decimals scores no marks unless as a check.	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

Orange 0.12, pink 0.20, brown 0.38, black $4x$, white x
One spin lands on exactly one colour, so these five outcomes are all of them.
The spinner is spun once in part (a), and 350 times in part (b).

Find

(a) The probability of orange or pink or brown (b) An estimate of how many of the 350 spins land on black

Plan the Solution

- Part (a) needs no algebra. The three probabilities are printed and the outcomes cannot happen together, so add them.
- For part (b), all five probabilities total 1, so $4x$ and x share whatever the first three leave over.
- That leftover is 5 equal shares of x. Work out one share, then take 4 of them.
- Multiply the probability of black by 350 to estimate the number of spins.

Worked Solution [5 marks]

> Rule - for one spin the probabilities of all the possible outcomes add up to 1, and an estimate for the number of times an outcome happens is $\text{probability} \times \text{number of trials}$.

Step 1: add the three probabilities that are given

$0.12 + 0.20 + 0.38 = 0.7$

(Reason: One spin cannot land on two colours, so orange, pink and brown cannot happen together. For outcomes that cannot happen together, the probability of one or another of them is the sum.)

Step 2: find how much probability is left for black and white

$1 - 0.7 = 0.3$

(Reason: The five probabilities must total 1, so whatever the first three do not use belongs to $4x$ and x together.)

Step 3: work out the value of x

$5x = 0.3$

$\frac{0.3}{5} = 0.06$

(Reason: Black is $4x$ and white is x, which is 5 equal shares of x, so the 0.3 is divided by 5.)

Step 4: work out the probability of black

$4 \times 0.06 = 0.24$

(Reason: Black is $4x$, so it is worth four of those shares.)

Step 5: estimate the number of spins that land on black

$0.24 \times 350 = 84$

(Reason: An estimate for the number of times an outcome happens is its probability multiplied by the number of trials. It is an estimate, not a promise: 350 real spins would rarely give exactly this.)

(a) 0.7

(b) 84

Verification

✓ **Check 1:** Put both answers back into the table and add all five probabilities. With $4x = 0.24$ and $x = 0.06$ the total must be 1. $0.12 + 0.20 + 0.38 + 0.24 + 0.06 = 1$

✓ **Check 2:** Count the spins instead. Work out the expected number for each of the three printed colours, subtract them all from 350, then split what is left in the ratio $4 : 1$.

$350 - 42 - 70 - 133 = 105$ and $\frac{105}{5} \times 4 = 84$

✓ **Check 3:** Turn the answer back into a probability. If 84 of the 350 spins are black, the probability of black should come back. $\frac{84}{350} = 0.24$, which is $4x$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	for 0.7 oe, eg $\frac{7}{10}$ oe or 70% or $\frac{0.7}{1}$. If probabilities are given as percentages then the % sign must be seen.	✓
(b)	**M1ft**	for $1-(0.12+0.20+0.38)=0.3$ oe, or $1-0.7=0.3$ oe using their 0.7, or $0.12+0.20+0.38+4x+x=1$ oe, or their $0.7\times350=245$ oe, or $0.12\times350=42$, or $0.38\times350=133$. Follow through their 0.7 from part (a). If probabilities are given as percentages then the % sign must be seen.	✓
(b)	**M1**	for their $\frac{0.3}{5}=0.06$ or their $\frac{0.3}{5}\times4=0.24$ or 0.24, or $x=0.06$ or $4x=0.24$, or their $0.3\times350=105$ oe, or $350-245=105$ oe using their 245, or $350-42-0.20\times350-133=105$ oe.	✓
(b)	**M1**	for their $0.06\times350=21$ oe, or their $\frac{105}{5}=21$ oe, or their $0.06\times4\times350$ oe, or their 0.24×350. Or for $\frac{21}{350}$ or $\frac{84}{350}$.	✓
(b)	**A1**	for 84 cao. A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 5/5

Question 5 - Exam Solution

Understanding the Question

Given

$\mathscr{E}=\{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\}$, the universal set, with 12 members

$A=\{2,\ 4,\ 6,\ 8,\ 10,\ 12\}$, the even members of $\mathscr{E}$

$B=\{3,\ 6,\ 9,\ 12\}$, the multiples of 3 in $\mathscr{E}$

Find

(a) the members of $A\cup B$ and the members of B'; (b) the symbol that makes $A\cap C=\ldots$ true, and the symbol that makes $13\ldots\mathscr{E}$ true.

$C = \{1,\ 3,\ 5,\ 7,\ 9,\ 11\}$, the odd members of $\mathscr{E}$
The box offers six symbols: $\mathscr{E}$, $\cap$, $\cup$, $\varnothing$, $\in$ and $\notin$

Plan the Solution

- $\cup$ means union, so for (a)(i) take everything that is in A, in B, or in both, and write each member once, in order.
- The dash in B' means complement, so for (a)(ii) keep everything in $\mathscr{E}$ that is not in B.
- $\cap$ means intersection, so for (b)(i) look for numbers that appear in A and in C.
- For part (b), first ask what each blank sits between. $\varnothing$ and $\mathscr{E}$ are sets, $\cap$ and $\cup$ join two sets, and $\in$ and $\notin$ go between a number and a set.

Worked Solution [4 marks]

> Rule - Set notation: $A \cup B$ is everything in either set, $A \cap B$ is only what is in both, B' is everything in $\mathscr{E}$ that is not in B, and $\varnothing$ is the set with no members.

Step 1: List $A \cup B$

$A = \{2,\ 4,\ 6,\ 8,\ 10,\ 12\}$
$B = \{3,\ 6,\ 9,\ 12\}$
$A \cup B = \{2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12\}$

(Reason: Union collects every member of A together with every member of B. Only 3 and 9 are new, because 6 and 12 are already in A, and a member is never written twice.)

Step 2: List B'

$\mathscr{E} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\}$
$B' = \{1,\ 2,\ 4,\ 5,\ 7,\ 8,\ 10,\ 11\}$

(Reason: The complement is everything left in $\mathscr{E}$ once the members of B are crossed out, so 3, 6, 9 and 12 go, and the eight numbers that are left stay.)

Step 3: Work out $A \cap C$

$A = \{2,\ 4,\ 6,\ 8,\ 10,\ 12\}$
$C = \{1,\ 3,\ 5,\ 7,\ 9,\ 11\}$
$A \cap C = \varnothing$

(Reason: Every member of A is even and every member of C is odd, so no number can belong to both. An intersection with no members is the empty set, written $\varnothing$.)

Step 4: Compare 13 with $\mathscr{E}$

$\mathscr{E} = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12\}$

$13 \notin \mathscr{E}$

(Reason: This blank sits between a number and a set, so it takes $\in$ or $\notin$. The largest member of $\mathscr{E}$ is 12, so 13 is not one of its members and the statement is made true by $\notin$.)

(a)(i) $A \cup B = \{2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12\}$

(a)(ii) $B' = \{1,\ 2,\ 4,\ 5,\ 7,\ 8,\ 10,\ 11\}$

(b)(i) $A \cap C = \varnothing$

(b)(ii) $13 \notin \mathscr{E}$

Verification

✓ **Check 1 - count $A \cup B$ a different way:** A has 6 members and B has 4, and the two share 6 and 12, so the union should be 2 members shorter than $6+4$. $6+4-2=8$, and the listed union has 8 members.

✓ **Check 2 - B and B' must fill $\mathscr{E}$ between them:** Every member of $\mathscr{E}$ is either in B or in B' and never in both, so the two sizes must add to 12 and the two lists must share nothing. $4+8=12$, and $\{3,\ 6,\ 9,\ 12\}$ and $\{1,\ 2,\ 4,\ 5,\ 7,\ 8,\ 10,\ 11\}$ have no member in common.

✓ **Check 3 - rule out the other symbols in the box:** In (b)(i) the blank has to name a set, and only $\varnothing$ and $\mathscr{E}$ are sets, so $\cap$, $\cup$, $\in$ and $\notin$ cannot go there. In (b)(ii) the blank joins a number to a set, so only $\in$ and $\notin$ can go there. $\mathscr{E}$ has 12 members while $A \cap C$ has none, which leaves $\varnothing$; and 13 is greater than 12, so $\in$ would be false and $\notin$ is left.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)(i) $A \cup B$	**B1**	$2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12$	✓
(a)(ii) B'	**B1**	$1,\ 2,\ 4,\ 5,\ 7,\ 8,\ 10,\ 11$	✓
(b)(i) $A \cap C = \ldots$	**B1**	$\varnothing$	✓
(b)(ii) $13 \ldots \mathscr{E}$	**B1**	$\notin$	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

A number line carrying an open circle above -2 and a solid circle above 1, with a thick line joining them.
The inequality $7a - 5 \leq 3a + 28$.

Find

(a) the inequality the number line shows
(b) the solution of $7a - 5 \leq 3a + 28$, with the algebra shown

Plan the Solution

- Read the two ends off the number line first: the shading starts at one tick and stops at another.
- Then let each circle choose its own sign. An open circle leaves its value out, so it gives $<$ or $>$. A solid circle takes its value in, so it gives $\leq$ or $\geq$.
- For part (b), work exactly as you would with an equation: gather the a terms on one side, the numbers on the other, then divide.
- Watch the one thing that is different from an equation. Multiplying or dividing by a negative number turns the sign round. Here you divide by 4, which is positive, so nothing turns.

Worked Solution [4 marks]

> Rule - an inequality is solved like an equation, except that multiplying or dividing both sides by a negative number reverses the sign. On a number line an open circle leaves its value out and a solid circle takes it in.

(a) Step 1 - read the two ends off the line

$$\text{open circle above } -2$$
$$\text{solid circle above } 1$$

(Reason: The thick line runs from the circle above -2 to the circle above 1, so those two numbers are the ends of the region.)

(a) Step 2 - let each circle choose its sign

$$x > -2$$
$$x \leq 1$$

(Reason: The left circle is hollow, so -2 itself is not included and the sign is strict. The right circle is filled, so 1 itself is included.)

(a) Step 3 - write the two conditions as one inequality

$-2 < x \leq 1$

(Reason: Both conditions hold at once, so x sits between the two ends, with only the right-hand end taken in.)

(b) Step 1 - collect the a terms on one side

$7a - 5 \leq 3a + 28$

$7a - 3a \leq 28 + 5$

(Reason: Subtract $3a$ from both sides and add 5 to both sides. Adding and subtracting never turn the sign round, so it stays as it is.)

(b) Step 2 - simplify each side

$4a \leq 33$

(Reason: On the left $7a - 3a$ leaves $4a$, and on the right $28 + 5$ makes 33.)

(b) Step 3 - divide both sides by 4

$a \leq \dfrac{33}{4}$

$a \leq 8.25$

(Reason: 4 is positive, so dividing by it leaves the sign pointing the same way. The paper allows a calculator, but the working still has to be shown.)

(a) $-2 < x \leq 1$

(b) $a \leq 8.25$

Verification

✓ **Check 1 - test each end of the number line, and a point between them:** Take $x = 0$, which sits under the thick line: $-2 < 0$ and $0 \leq 1$ are both true. Take $x = -2$, where the circle is hollow: $-2 < -2$ is false, so -2 is left out, as the open circle asks. Take $x = 1$, where the circle is filled: $1 \leq 1$ is true, so 1 is taken in. The inequality agrees with the line at both ends and in between.

✓ **Check 2 - put the boundary value back into both sides:** At the boundary the two sides must be equal, because 8.25 is exactly where the inequality turns into an equation. $7 \times 8.25 - 5 = 52.75$ and $3 \times 8.25 + 28 = 52.75$

✓ **Check 3 - test a value on each side of the boundary:** At $a = 8$: $7 \times 8 - 5 = 51$ and $3 \times 8 + 28 = 52$, so the inequality holds. At $a = 9$: $7 \times 9 - 5 = 58$ and $3 \times 9 + 28 = 55$, so it fails. Values below the boundary work and values above it do not, so the sign points the right way.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $-2 < x \leq 1$	**B2**	accept $1 \geq x > -2$ or $x > -2, x \leq 1$	✓
(a) $-2 < x$ or $x \leq 1$ or $-2 \leq x < 1$ or $-2 \leq x \leq 1$ or $-2 < x < 1$	**B1**	if not B2 then B1 for any one of these	✓
(a) a letter other than x	**Note**	Condone use of a variable other than x but not 0.	✓
$7a - 3a \leq 28 + 5$ or $4a \leq 33$ or $-5 - 28 \leq 3a - 7a$ or $-33 \leq -4a$	**M1**	for a terms on one side and numbers on the other. Condone $=$ rather than $\leq$ or any other sign for this mark.	✓
$a \leq 8.25$ (working required)	**A1**	(dep on M1) oe eg $a \leq \frac{33}{4}$ or $a \leq 8\frac{1}{4}$ or $8.25 \geq a$. Must have correct sign on answer line.	✓
(b) the correct answer seen in the working space only	**Note**	Sight of the correct answer in the working space and just 8.25 on the answer line gains M1 only.	✓

Full marks: 4/4

Question 7 - Exam Solution

Understanding the Question

Given

A speed of $50x$ metres per second.
Two facts about the units: 1 hour is 3600 seconds, and 1 kilometre is 1000 metres.

Find

The same speed written in kilometres per hour. No value is given for x, so it stays in the answer.

Plan the Solution

- Change the time first: a speed given per second covers 3600 times as far in one hour, so multiply by 3600.
- Change the distance next: 1000 metres make 1 kilometre, so divide by 1000.
- Carry the x through every line. It is part of the speed, not something to be solved for.
- Finish by doing both moves at once, as a single multiplier, and check the two agree.

Worked Solution [3 marks]

> Rule - metres per second to kilometres per hour: multiply by the seconds in an hour, $60 \times 60 = 3600$, then divide by the metres in a kilometre, 1000. The two together are one multiplier, $\dfrac{3600}{1000} = 3.6$.

Step 1: change the seconds into hours

$$50x \times 3600 = 180\,000x$$

(Reason: An hour is 3600 seconds, so whatever the speed covers in one second it covers 3600 times over in one hour. The distance is still in metres, so this is a speed in metres per hour.)

Step 2: change the metres into kilometres

$$\frac{180\,000x}{1000} = 180x$$

(Reason: There are 1000 metres in a kilometre, so dividing by 1000 turns metres per hour into kilometres per hour. Both units have now been changed, so the conversion is complete.)

Step 3: the same conversion as one multiplier

$$\frac{3600}{1000} = 3.6$$

$$50x \times 3.6 = 180x$$

(Reason: Multiplying by 3600 and dividing by 1000 can be done in one move, because every speed in metres per second is 3.6 times as many kilometres per hour. It reaches the same answer, which is the point of doing it.)

$180x$ km/h

Verification

✓ **Check 1:** Test it on a number. Take $x = 2$, so the speed is 100 metres per second. In one hour it covers $100 \times 3600 = 360\,000$ metres, and that is 360 kilometres. The answer gives $180 \times 2 = 360$, the same speed.

✓ **Check 2:** Run the conversion backwards. $180x$ kilometres per hour is $180\,000x$ metres per hour, and an hour is 3600 seconds. $\dfrac{180\,000x}{3600} = 50x$ metres per second, which is the speed the question gave.

✓ **Check 3:** Use the other standard fact instead: 1 kilometre per hour is $\dfrac{1000}{3600} = \dfrac{5}{18}$ metres per second, so divide the speed by $\dfrac{5}{18}$. $50x \times \dfrac{18}{5} = 180x$, the same answer along a different route.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\dfrac{50x}{1000}$ $(= 0.05x)$ oe or $50x \times 60 \times 60$ $(= 180\,000x)$ oe or $\dfrac{50x}{\frac{1}{3600}}$ $(= 180\,000x)$ oe or $\dfrac{50x}{1000} \times 60$ $(= 3x)$ or $\dfrac{3600}{1000}$ or $\dfrac{18}{5}$ or 3.6 or $\dfrac{1000}{3600}$ or $\dfrac{5}{18}$ or $0.277(77\ldots)$	**M1**	Condone omission of x for this mark	✓
eg $\dfrac{50x \times 60 \times 60}{1000}$ oe or $50x \times 3.6$ oe or $\dfrac{50x}{\frac{1000}{3600}}$ oe or 180	**M1**	For a complete method including x or for an answer of 180	✓
$180x$	**A1**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 3/3

Question 8 - Exam Solution

Understanding the Question

Given

Part (a): the product $a^6 \times a^{10}$, two powers of the same letter multiplied together.

Part (b): the quotient $\frac{c^{30}}{c^{12}}$, two powers of the same letter divided.

Part (c): the quadratic expression $y^2 - 10y + 21$, whose y^2 term has coefficient 1.

Find

Part (a): the product written as a single power of a. Part (b): the quotient written as a single power of c. Part (c)(i): the expression written as a product of two brackets. Part (c)(ii): the values of y that satisfy the equation. It is a quadratic, so expect two solutions.

Plan the Solution

- Parts (a) and (b) are the index laws. Multiplying powers of one letter adds the indices; dividing them subtracts.
- Part (c)(i): the coefficient of y^2 is 1, so look for two numbers whose product is 21 and whose sum is -10.
- The product is positive while the sum is negative, so both numbers are negative. That leaves very few pairs to test.
- Part (c)(ii): the word Hence means carry the brackets down from part (c)(i). A product is zero only when one of its brackets is zero.

Worked Solution [5 marks]

> Rule - Index laws and factorising: $a^m \times a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, and $y^2 + by + c = (y+p)(y+q)$ when $pq = c$ and $p + q = b$.

Step 1 - Part (a): combine the two powers

$$a^6 \times a^{10} = a^{6+10} = a^{16}$$

(Reason: A power of a is a run of as multiplied together, so a product of two powers is one longer run: 6 of them followed by 10 more.)

Step 2 - Part (b): cancel the common powers

$$\frac{c^{30}}{c^{12}} = c^{30-12} = c^{18}$$

(Reason: Each c on the bottom cancels one c on the top. 12 of the 30 are cancelled and 18 are left.)

Step 3 - Part (c)(i): find the pair of numbers

$$p \times q = 21 \quad \text{and} \quad p + q = -10$$

$(-3) \times (-7) = 21$
$(-3) + (-7) = -10$

(Reason: The factor pairs of 21 are 1 with 21, and 3 with 7. Both numbers must be negative for the sum to be negative, and only -3 with -7 adds to -10.)

Step 4 - Part (c)(i): write down the two brackets

$y^2 - 10y + 21 = (y-3)(y-7)$

(Reason: $p = -3$ gives the bracket $(y-3)$, and $q = -7$ gives $(y-7)$.)

Step 5 - Part (c)(ii): set each bracket to zero

$(y-3)(y-7) = 0$
$y - 3 = 0 \quad \text{or} \quad y - 7 = 0$
$y = 3 \quad \text{or} \quad y = 7$

(Reason: Two numbers multiply to give zero only when at least one of them is zero, so each bracket is taken to be zero in turn. This is why part (c)(ii) says Hence: the brackets are already there from part (c)(i).)

(a) a^{16}

(b) c^{18}

(c)(i) $(y-3)(y-7)$

(c)(ii) $y = 3$ **or** $y = 7$

Verification

✓ **Check 1 - part (a):** Substitute $a = 2$ and work each side out as an ordinary number. $2^6 \times 2^{10} = 2^{16}$, and both sides come to $65\,536$.

✓ **Check 2 - part (b):** A division is checked by multiplying back: the answer times the divisor must return the top of the fraction. $c^{18} \times c^{12} = c^{18+12} = c^{30}$

✓ **Check 3 - part (c)(i):** Expand the two brackets again and collect the y terms. $(y-3)(y-7) = y^2 - 7y - 3y + 21 = y^2 - 10y + 21$

✓ **Check 4 - part (c)(ii):** Substitute each solution into the left-hand side of the equation; both must give zero. $3^2 - 10 \times 3 + 21 = 0$ and $7^2 - 10 \times 7 + 21 = 0$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) a^{16}	**B1**	for a^{16}	✓

Step	Mark	Description	Got it?
(b) c^{18}	**B1**	for c^{18}	✓
(c)(i) $(y \pm 3)(y \pm 7)$	**M1**	for $(y \pm 3)(y \pm 7)$ or for $(y \pm a)(y \pm b)$ with $ab = 21$ or $a + b = -10$	✓
(c)(i) $(y - 3)(y - 7)$	**A1**	for correct factors	✓
(c)(i)	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓
(c)(ii) $3, 7$	**B1ft**	ft dep on factorising in the form $(y \pm p)(y \pm q)$	✓

Full marks: 5/5

Question 9 - Exam Solution

Understanding the Question

Given

Triangle PQR, with the right angle at Q.
The side PQ is 6.5 cm long.
The angle at R, between RQ and RP, is 24°.

Find

The length of QR, the side that joins the right angle to the 24° angle. The answer is wanted correct to 3 significant figures, so the rounding is part of what is being marked.

Plan the Solution

- Stand at R and name the two sides that matter: PQ is opposite the angle and QR is adjacent to it.
- Opposite and adjacent together are the tangent ratio, so the hypotenuse PR is not needed here. A route through the hypotenuse does exist, and it is two steps longer; it is Check 3 below.
- The unknown starts underneath the fraction, so rearrange first and only then reach for the calculator: QR ends up as 6.5 divided by the tangent.
- Keep the calculator's full value to the end and round once, to 3 significant figures.

Worked Solution [3 marks]

Rule - Tangent ratio in a right-angled triangle: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$, so a known angle and a known opposite side give the adjacent side.

Step 1 - Name the sides as the angle at R sees them

$$\tan 24^\circ = \frac{PQ}{QR}$$

$$\tan 24^\circ = \frac{6.5}{QR}$$

(Reason: Seen from R, the side PQ is opposite the angle and QR runs from the angle to the right angle, so QR is the adjacent side. Opposite over adjacent is the tangent, and the hypotenuse PR does not appear in it.)

Step 2 - Rearrange to make QR the subject

$$QR \times \tan 24^\circ = 6.5$$

$$QR = \frac{6.5}{\tan 24^\circ}$$

(Reason: QR is underneath the fraction, so multiply both sides by it first and then divide both sides by $\tan 24^\circ$. Doing the rearranging before any button is pressed is what keeps the two operations the right way round.)

Step 3 - Work the value out on the calculator

$$\tan 24^\circ = 0.4452287$$

$$\frac{6.5}{0.4452287} = 14.5992$$

(Reason: The calculator must be in degree mode - in radian mode the same keystrokes read 24 as radians and return a different number altogether. The size of the answer is the check on the operation: $\tan 24^\circ$ is less than 1, and a small angle at R has to leave QR longer than 6.5 cm, not shorter.)

Step 4 - Round to 3 significant figures

$$14.5992\ldots \to 14.6$$

$$QR = 14.6 \text{ cm}$$

(Reason: The first three significant figures are the 1, the 4 and the 5. The digit after them is a 9, so the 5 rounds up to a 6.)

$$QR = 14.6 \text{ cm}$$

Verification

✓ **Check 1 - multiply back:** A division is checked by multiplying back. If the length is right, the length times $\tan 24°$ must return the side PQ. $0.4452287 \times 14.5992 = 6.5$ to 1 decimal place

✓ **Check 2 - work from the third angle instead:** The three angles add to 180 degrees, so the angle at P is $180 - 90 - 24 = 66$ degrees. Seen from P the side QR is the opposite one, so this time it is 6.5 multiplied by the tangent rather than divided by it. $6.5 \times 2.246037 = 14.5992$, the same length from a different ratio

✓ **Check 3 - Pythagoras through the hypotenuse:** The sine ratio gives the hypotenuse PR from the same two given values, $\dfrac{6.5}{0.4067366} = 15.9809$, and the three sides of a right-angled triangle must then satisfy Pythagoras. $15.9809^2 - 6.5^2 = 213.14$ and $14.5992^2 = 213.14$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $\tan 24 = \dfrac{6.5}{QR}$ or $\dfrac{6.5}{\sin 24} = \dfrac{QR}{\sin(180 - 90 - 24)}$ oe or $\tan(180 - 90 - 24) = \dfrac{QR}{6.5}$ or $(PR =)\dfrac{6.5}{\sin 24} = 15.9\ldots$ and $6.5^2 + QR^2 = 15.9^2$, where the 15.9 may be the candidate's own value for PR	**M1**	for setting up a trig equation in QR or for a complete method to find PR and then setting up Pythagoras or a trig equation for QR	✓
eg $(QR =)\dfrac{6.5}{\tan 24}$ or $(QR =)\dfrac{6.5}{\sin 24} \times \sin 66$ or $(QR =) 6.5 \tan 66$ where $66 = 180 - 90 - 24$ or $(QR =)\sqrt{15.9^2 - 6.5^2}$, again with the candidate's own value for PR	**M1**	for a complete method	✓
14.6	**A1**	accept 14.5 to 14.61	✓
Answer	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 3/3

Question 10 - Exam Solution

Understanding the Question

Given

A cuboid trough measuring 35 cm by 28 cm by 20 cm, holding water to a depth of 9 cm.
A cylindrical drum of radius 10 cm and height 33 cm, completely full of water.
All the water in the drum is poured into the trough, and none of it is spilt.

Find

A comparison rather than a single value: the room the trough still has against the water the drum holds. The demand is show that, so every figure has to be written down. A conclusion with no working earns nothing here, and there is no answer line to fill in.

Plan the Solution

- Work out how much empty space the trough still has. The base is the same rectangle at every height, so it is the base area times the height that is still empty.
- Work out the volume of the drum with $V = \pi r^2 h$. The drum is completely full, so that volume is exactly the water that is poured in.
- Compare the two. If the water poured in is smaller than the space waiting for it, the trough cannot fill up.
- Keep π on the calculator until the last line. Rounding it early is what pushes a volume outside the range the mark scheme accepts.

Worked Solution [3 marks]

> Rule - Volume of a cuboid: $V = l \times w \times h$. Volume of a cylinder: $V = \pi r^2 h$. A container is full when the water in it has the same volume as the container itself.

Step 1 - Work out the space still empty in the trough

$35 \times 28 = 980$

$(20 - 9) \times 980 = 10\,780$

(Reason: Every horizontal slice of the trough is the same rectangle, so one base area serves every depth. The water already stands 9 cm up a 20 cm side, which leaves $20 - 9 = 11$ cm of height empty above it, and that empty part is a cuboid of its own.)

Step 2 - Work out the volume of water in the drum

$V = \pi r^2 h$

$V = \pi \times 10^2 \times 33 = 3300\pi = 10\,367.25\ldots$

(Reason: The radius is 10 cm and the height is 33 cm, so squaring the radius gives 100 and $100 \times 33 = 3300$. The drum is completely full, so this volume is exactly the water that is poured out of it.)

Step 3 - Compare the water poured in with the space available

$10\,367.25\ldots < 10\,780$

$10\,780 - 10\,367.25\ldots = 412.74\ldots$

(Reason: The drum holds less water than the trough has room for, so all of it fits and none of it spills. The gap between the two figures is the volume that never gets filled: about 413 cm³ of the trough is still empty once the pouring is finished, so the trough is not completely full of water.)

Water poured in $3300\pi = 10\,367.25\ldots$ cm³, space waiting for it $10\,780$ cm³

The trough is left $412.74\ldots$ cm³ short of full, so it is not completely full of water

Verification

✓ **Check 1 - compare the total volumes instead:** Add the two lots of water and compare with the trough's own volume. The water already there is $9 \times 980 = 8820$ cm³ and the trough itself holds $20 \times 980 = 19\,600$ cm³. $8820 + 10\,367.25\ldots = 19\,187.25\ldots$, which is less than $19\,600$

✓ **Check 2 - work out the depth the water would stand at:** Divide the total volume of water by the area of the base. If the trough were completely full the depth would read 20 cm exactly. $\dfrac{19\,187.25\ldots}{980} = 19.57\ldots$ cm, which is below the 20 cm rim

✓ **Check 3 - reach the shortfall by two different subtractions:** The volume left empty can be taken from the space above the water or from the trough's full capacity. Two different subtractions have to land on the same figure. $10\,780 - 10\,367.25\ldots = 412.74\ldots$ and $19\,600 - 19\,187.25\ldots = 412.74\ldots$

Mark Scheme Breakdown

Step	Mark	Description
(volume of water =) $9 \times 35 \times 28$ $(= 8820)$ or (total volume of cuboid =) $20 \times 35 \times 28$ $(= 19\,600)$ or (volume of space =) $(20 - 9) \times 35 \times 28$ $(= 10\,780)$	**M1**	for a method to find a relevant volume for the cuboid
$\pi \times 10^2 \times 33$ $(= 3300\pi$ or $10\,367(.25\ldots))$ oe	**M1**	(indep) for a method to find the volume of the cylinder, ac to $10\,368.6$. Allow $3.14\ldots$ or $\frac{22}{7}$ for π
(total volume of water =) $8820 + 10\,367(.25\ldots)$ $(= 19\,187(.25\ldots))$ (difference between volumes of both solids =) $19\,600 - 10\,367(.25\ldots)$ $(= 9232(.74\ldots))$ (volume not filled =) $19\,600 - 8820 - 10\,367(.25\ldots)$ $(= 412(.74\ldots))$ Answer column: Shown	**A1**	correct workings with accurate figures, eg <table><tr><th>Value 1</th><th></th></tr><tr><td>10 780</td><td>10 367(.25</td></tr><tr><td>19 600</td><td>19 187(.25</td></tr><tr><td>8820</td><td>9232(.7</td></tr><tr><td>412(.74...) or 413 accept 408 to 418</td><td></td></tr></table>
Working required	**Note**	printed in italic in the working column of the A1 row: the n the page, so a bare statement that the trough will not fill sc

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

Meridian Bank: $\$2500$ invested, with amount invested to total interest after 2

Find

How much more interest Meridian Bank pays than Halewood Bank, in dollars.

years in the ratio $20:3$
Halewood Bank: $\$3000$ invested at 4% per year compound interest for 2 years
One panel states its interest as a ratio, the other as a compound percentage - two different pieces of work before anything can be compared.

Both interest amounts have to be found first; neither panel states one directly.

Plan the Solution

- Read the Meridian panel as a ratio: 20 parts is the amount invested and 3 parts is the interest. Divide $\$2500$ by 20 to get one part, then take 3 of them.
- Read the Halewood panel as compound interest: multiply $\$3000$ by 1.04 once for each year, which grows the balance to a total.
- That total still contains the money invested, so take the amount invested off it to leave the interest on its own.
- Subtract the smaller interest from the larger one. Compare interest with interest - never the two totals, because the amounts invested are different.

Worked Solution [5 marks]

Rule - Compound interest: the total after n years is $P \times (1+r)^n$, and the interest is that total minus the amount invested P. Rule - Ratio: one part is the whole divided by the number of parts.

Step 1: Find one part of the Meridian ratio

$$\frac{2500}{20} = 125$$

(Reason: the ratio $20:3$ sets the amount invested against 20 parts, and $\$2500$ was invested, so one part is worth $\$125$.)

Step 2: Work out the interest from Meridian Bank

$$125 \times 3 = 375$$

(Reason: the second number in the ratio is the interest, and it is 3 parts, so the interest is 3 lots of $\$125$.)

Step 3: Grow the Halewood investment for two years

$$3000 \times 1.04^2 = 3244.80$$

(Reason: adding 4% multiplies a balance by 1.04, and that happens once for each of the 2 years, so the multiplier is 1.04^2.)

Step 4: Take off the amount invested to leave the interest

$3244.80 - 3000 = 244.80$

(Reason: the $\$3244.80$ is the whole balance, and $\$3000$ of it is the money that was put in, so the rest is what the account earned.)

Step 5: Compare the two amounts of interest

$375 - 244.80 = 130.20$

(Reason: both amounts of interest are now known, so subtracting the smaller from the larger says how much more the first bank pays.)

$\$130.20$

Verification

✓ **Check 1 - rebuild the Meridian account a different way:** The ratio $20 : 3$ means the account returns 23 parts altogether. One part is $\$125$, so the balance after 2 years can be built first and the amount invested taken off afterwards. $23 \times 125 = 2875$, and $2875 - 2500 = 375$, which is the interest found in Step 2.

✓ **Check 2 - add the Halewood interest year by year:** Year 1 earns 4% of $\$3000$. That interest is added on, so year 2 earns 4% of the larger balance. The two years together should give the amount found in Step 4. $0.04 \times 3000 = 120$, then $0.04 \times 3120 = 124.8$, and $120 + 124.8 = 244.8$.

✓ **Check 3 - is the size sensible?** Meridian pays $\frac{3}{20}$ of $\$2500$, and $\frac{3}{20}$ is 15%. Halewood pays 4% twice on a growing balance, so a little over 8% of $\$3000$. The two amounts of interest should therefore be close to $\$375$ and $\$245$, leaving a gap of roughly $\$130$. The difference found, $\$130.20$, is the size that estimate predicts, and it is Meridian that is ahead.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Interest from Meridian Bank	M1	For a method to find the interest for Meridian Bank: $\frac{2500}{20} \times 3$ $(= 375)$ oe, or 125×3 $(= 375)$, or $\frac{7500}{20}$ $(= 375)$. The same method on the other amount also earns it: $\frac{3000}{20} \times 3$ $(= 450)$ oe, or	✓

Step	Mark	Description	Got it?
		150×3 $(= 450)$, or $\dfrac{9000}{20}$ $(= 450)$. An answer of 2875 or 3450 implies this method mark.	
Four per cent, or one hundred and four per cent, of an amount	M1	For finding 4% or 104% of 3000 or 2500: 0.04×3000 $(= 120)$ oe, or 0.04×2500 $(= 100)$, or 1.04×3000 $(= 3120)$ oe, or 1.04×2500 $(= 2600)$. Or award M2 here for 3000×1.04^2 $(= 3244.8)$ or 2500×1.04^2 $(= 2704)$.	✓
Total in the Halewood account	M1	For completing the method to find the total amount for Halewood Bank: 1.04×3120 $(= 3244.8)$ oe, or 1.04×2600 $(= 2704)$, where the candidate's own value from the row above may be used in place of 3120 or 2600. Or 3000×1.04^3 $(= 3374.59)$ or 2500×1.04^3 $(= 2812.16)$.	✓
Interest from Halewood Bank	M1	For a complete method to find the interest for Halewood Bank, eg $3244.8 - 3000$ $(= 244.8)$ or $2704 - 2500$ $(= 204)$, where the candidate's own total may be used in place of 3244.8 or 2704.	✓
How much more	A1	130.2, with or without the trailing zero (130.20). A correct answer scores full marks unless it comes from obviously incorrect working.	✓
Special case	SC	If neither the second nor the third method mark is gained, award one special-case method mark for 0.08×3000 $(= 240)$ oe, or 240, or 1.08×3000 $(= 3240)$, or 0.08×2500 $(= 200)$ oe, or 200, or 1.08×2500 $(= 2700)$, or $3000 \times (1 - 0.04)^2$ $(= 2764.8)$, or $2500 \times (1 - 0.04)^2$ $(= 2304)$. These are the values of two named slips: simple interest over the two years, and a 4% decrease each year in place of an increase.	✓
Notation	Note	Accept $(1 + 0.04)$ or $\left(1 + \dfrac{4}{100}\right)$ as equivalent to 1.04 throughout.	✓

Full marks: 5/5

Question 12 - Exam Solution

Understanding the Question

Given

A grouped frequency table for the ages of the 80 visitors, in six classes of width 10 years running from 10 to 70.
The frequencies are 12, 15, 20, 18, 9 and 6, which add to 80.

Find

(a) the six cumulative frequencies. (b) the cumulative frequency graph, drawn on the grid. (c) the percentage of the 80 visitors who are more than 46, to the nearest whole number.

Plan the Solution

- Add the frequencies downwards. Each cumulative frequency is a running total, so it counts everyone up to the TOP of that class.
- Plot every cumulative frequency against the UPPER end of its class, and start at $(10,\ 0)$ because nobody is aged 10 or under.
- Draw a line up from 46 to the graph and across to the cumulative frequency axis. That reading estimates how many people are 46 or under.
- Take the reading away from 80 to count the people OVER 46, then write that count as a percentage of 80.

Worked Solution [6 marks]

> Rule - Cumulative frequency: each entry is the running total of the frequencies so far, and every point is plotted at the UPPER end of its class. Reading up from a value on the horizontal axis estimates how many are at or below it.

Step 1: add the frequencies downwards

12

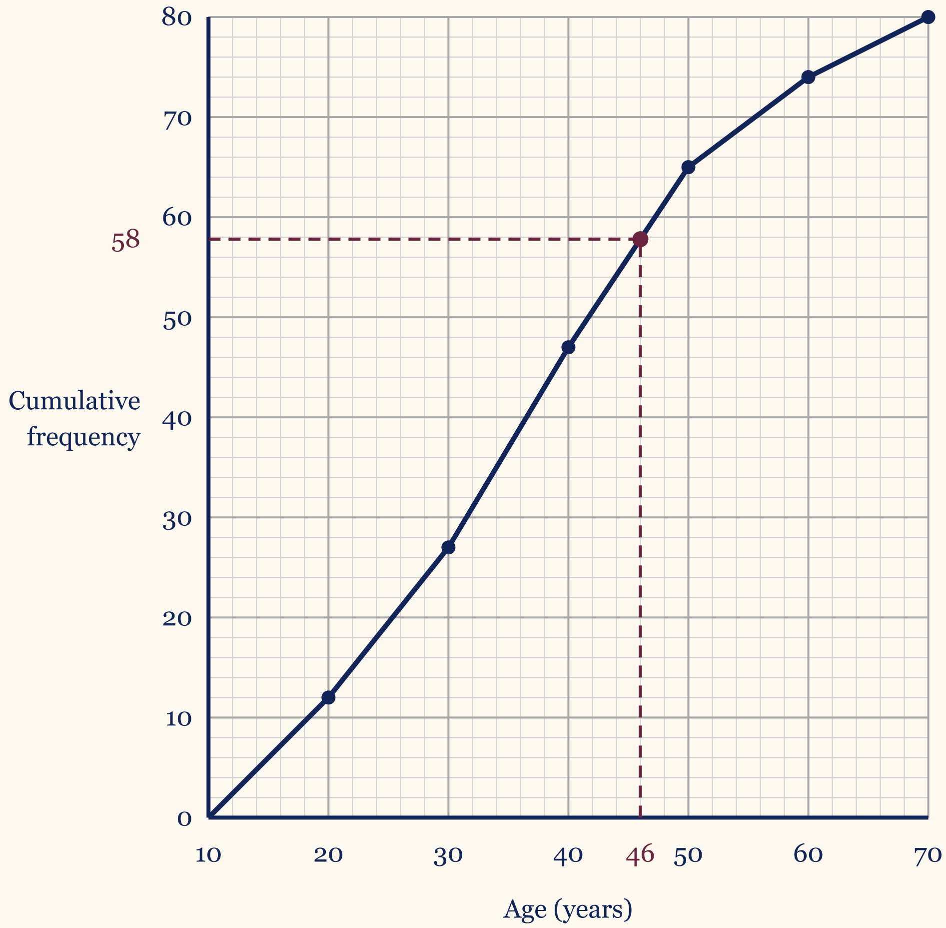
$12 + 15 = 27$

$27 + 20 = 47$

$47 + 18 = 65$

$65 + 9 = 74$

$74 + 6 = 80$



(Reason: Each row of the cumulative frequency table counts everyone up to the top of that class, so every row is the row above it plus the next frequency. The last row must come out as 80, because by the top class every visitor has been counted.)

Step 2: plot each total at the upper end of its class

$(10,\ 0)$

$(20,\ 12)$

$(30,\ 27)$

$(40,\ 47)$

$(50,\ 65)$

$(60,\ 74)$

$(70,\ 80)$

(Reason: A cumulative frequency is only complete at the TOP of its class, so the 12 is plotted at 20 and not at the midpoint 15. The graph starts at $(10,\ 0)$ because nobody is aged 10 or under. Join the points with a smooth curve or with straight line segments.)

Step 3: read the graph at 46

$$47 + \frac{6}{10} \times (65 - 47) = 57.8$$

$57.8 \approx 58$

(Reason: The value 46 lies between 40 and 50, where the graph climbs from 47 to 65. Six tenths of the way across gives 57.8, so the graph is read as about 58 people aged 46 or under. The mark scheme accepts any reading from 57 to 59.)

Step 4: count the people over 46

$80 - 58 = 22$

(Reason: Every visitor is either 46 or under, or over 46, so taking the reading away from the total 80 leaves the number who are over 46.)

Step 5: write that count as a percentage of 80

$$\frac{22}{80} \times 100 = 27.5$$

$27.5 \approx 28$

(Reason: A percentage is the part over the whole, multiplied by 100. Here the part is the 22 people over 46 and the whole is 80. The question asks for the nearest whole number, so 27.5 is written as 28.)

(a) $12, 27, 47, 65, 74, 80$

(b) the seven points joined in order, rising from $(10,\ 0)$ **to** $(70,\ 80)$

(c) 28%

Verification

✓ **Check 1:** Add the six frequencies straight down the table. The last cumulative frequency has to equal that total, because by the top class everybody has been counted. $12 + 15 + 20 + 18 + 9 + 6 = 80$, and the last entry of the completed table is 80.

✓ **Check 2:** Work out the percentage aged 46 or under instead. Everybody sits on one side or the other, so the two percentages must add to 100. $\frac{58}{80} \times 100 = 72.5$ and $27.5 + 72.5 = 100$.

✓ **Check 3:** Sense-check the reading against the table. Because 46 sits inside the class $40 < n \leq 50$, the reading must lie between the cumulative frequencies at the two ends of that class. $47 < 57.8 < 65$. Also 22 out of 80 is a little over a quarter, and 28% is a little over 25%.

✓ **Check 4:** Test whether rounding the reading up to 58 changed the answer, by redoing the percentage from the unrounded 57.8. $\frac{80 - 57.8}{80} \times 100 = 27.75$, which is still 28 to the nearest whole number.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	$12, 27, 47, 65, 74, 80$	✓
(b)	**M1**	ft from table for at least 5 points plotted correctly at end of interval or ft from sensible table (ft from a table with only one arithmetic error that may be continued through table) for all 6 points plotted consistently within each interval in the freq table at the correct height	✓
(b)	**A1**	correct cf graph. Accept curve or line segments. Accept curve that is not joined at $(10,\ 0)$	✓
(b)	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓
(c)	**M1ft**	a line up from 46 to their graph and a line across to the vertical axis or a mark on the curve at the correct point and a mark on the vertical axis at the correct point or a reading of 57 to 59 from their cf graph or a value of 21 to 23 or a correct value for their graph. Must be ascending (could be a line of best fit).	✓
(c)	**M1ft**	method to find the fraction or percentage of people aged over or under 46, ft from their graph, or a value in the range 0.26 to 0.29 or 0.71 to 0.74 or 71% to 74%. For example, over 46: $\dfrac{80-58}{80} = 0.275$, or under 46: $\dfrac{58}{80} = 0.725$. The printed scheme sets the 58 in quotation marks, so the candidate's own reading may be used in place of it.	✓
(c)	**A1ft**	28. Accept 26 to 29, ft their cf graph.	✓
(c)	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 6/6

Question 13 - Exam Solution

Understanding the Question

Given

The numbers of bicycles hired on each of 11 days, listed as the kiosk recorded them.
The eleven values are 9, 10, 12, 15, 17, 18, 19, 19, 20, 25 and 26, already written from smallest to largest.

Find

The interquartile range of the 11 values.

Plan the Solution

- Check the list is in order and count how many values there are. A quartile is a position in an ordered list, so the order and the count decide everything that follows.
- Work out the position of the lower quartile and the position of the upper quartile. With 11 values both positions land on a whole number, so no in-between value is needed.
- Count along the ordered list to read the value sitting at each of those two positions.
- Take the lower quartile away from the upper quartile.

Worked Solution [2 marks]

> Rule - Interquartile range: for n values written in order, the lower quartile is the value in position $\dfrac{n+1}{4}$, the upper quartile is the value in position $\dfrac{3(n+1)}{4}$, and the interquartile range is the upper quartile minus the lower quartile.

Step 1: check the order, and count the values

$9 \quad 10 \quad 12 \quad 15 \quad 17 \quad 18 \quad 19 \quad 19 \quad 20 \quad 25 \quad 26$

(Reason: Quartiles are read from a list that runs from smallest to largest, so the first job is always to check the order. Here the values are already in order, so nothing moves. Counting along gives 11 values, so $n = 11$.)

Step 2: find the lower quartile

$$\frac{11+1}{4} = 3$$

$$3\text{rd value} = 12$$

(Reason: The lower quartile sits at position $\dfrac{n+1}{4}$, which here is position 3. Counting three along the ordered list gives 9, then 10, then 12, so the third value is 12. Count the first value as position one, not as position zero.)

Step 3: find the upper quartile

$$\frac{3 \times (11+1)}{4} = 9$$

$9\text{th value} = 20$

(Reason: The upper quartile sits at position $\frac{3(n+1)}{4}$, which here is position 9. Counting nine along the ordered list lands on 20. A quick way to check the position is that it is three times the lower quartile's position, $3 \times 3 = 9$.)

Step 4: subtract

$20 - 12 = 8$

(Reason: The interquartile range is the upper quartile minus the lower quartile. It measures how spread out the middle half of the data is, so it is a subtraction and not an average. This is the line the mark scheme wants to see: the two quartiles 20 and 12 identified without any doubt about which is which.)

8

Verification

- ✓ **Check 1:** Find the quartiles a different way, without any position formula. Split the ordered list at the median and take the middle value of each half. With 11 values the median is the sixth one, and it belongs to neither half. The median is 18. The lower half is 9, 10, 12, 15, 17, whose middle value is 12. The upper half is 19, 19, 20, 25, 26, whose middle value is 20. Again $20 - 12 = 8$.
- ✓ **Check 2:** Count the values outside the two quartiles. A quarter of the data should sit below the lower quartile and a quarter above the upper quartile, so the two tails should hold about the same number of values. Below 12 there are two values, 9 and 10. Above 20 there are two values, 25 and 26. The tails match, so the quartiles are sitting where a quarter of the way in and a quarter of the way back should put them.
- ✓ **Check 3:** Compare the answer with the full range. The interquartile range covers only the middle half of the values, so it has to come out smaller than the range of all 11. The range is $26 - 9 = 17$, and $8 < 17$. The interquartile range is a little under half the range, which is what a list with one low value and two high ones should give.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Working	M1	$20\ (-)\ 12$ for both values unambiguously identified. The printed scheme brackets the minus sign, so the subtraction itself need not be written down.	✓
Answer	A1	8	✓
Note	Note	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 2/2

Question 14 - Exam Solution

Understanding the Question

Given

Two numbers written in standard form, 6.7×10^{135} and 3×10^{134}, to be added. The two powers of ten are not the same: one term carries 10^{135} and the other carries 10^{134}, so they are one power apart.

Find

The total, written in standard form.

Plan the Solution

- Compare the two powers of ten. They are different, so the front numbers cannot be added straight away.
- Rewrite one term so that both terms carry the same power of ten. Moving up one power divides the front number by 10; moving down one power multiplies it by 10.
- Add the two front numbers, keeping the shared power of ten as a common factor.
- Check the total is in standard form: the front number must be at least 1 and less than 10.

Worked Solution [2 marks]

Rule - Adding in standard form: two terms can be added only when they carry the same power of ten. Match the powers first, then add the front numbers and keep the shared power. Moving up one power divides the front number by 10; moving down one power multiplies it by 10. The answer is written as $a \times 10^n$ with $1 \leq a < 10$ and n a whole number.

Step 1: make the two powers of ten match

$3 \times 10^{134} = 0.3 \times 10^{135}$

(Reason: The two terms carry different powers of ten, so nothing can be added yet. Writing the smaller term at the larger power raises the power by one, from 10^{134} to 10^{135}, and the front number must then be divided by 10 so that the value itself does not change. Dividing 3 by 10 gives 0.3, so the second term becomes 0.3×10^{135} and is ready to be added to the first.)

Step 2: add the front numbers

$6.7 \times 10^{135} + 0.3 \times 10^{135} = (6.7 + 0.3) \times 10^{135}$
$(6.7 + 0.3) \times 10^{135} = 7 \times 10^{135}$

(Reason: Both terms now carry 10^{135}, so it is a common factor and only the front numbers are added. This is the working the method mark is for: the mark scheme awards it for any line that gets the two terms on to one power of ten, and $(6.7 + 0.3) \times 10^{135}$ written on its own is enough.)

Step 3: check the answer is in standard form

7×10^{135}

(Reason: Standard form needs a front number of at least 1 and less than 10, multiplied by a whole-number power of ten. The front number here is 7, which is inside that range, so the answer is already in standard form and nothing has to be adjusted. Had the powers been matched at 10^{134} instead, the total would have come out as 70×10^{134} - the same value, but with a front number too big for standard form, so one more step would still be needed.)

$$7 \times 10^{135}$$

Verification

✓ **Check 1:** Match the powers the other way round. Carry both terms at 10^{134} instead of 10^{135}. A correct total cannot depend on which power was chosen. Moving down one power multiplies the front number by 10, so $6.7 \times 10^{135} = 67 \times 10^{134}$. Adding gives $67 \times 10^{134} + 3 \times 10^{134} = 70 \times 10^{134}$, and converting that back to standard form gives $70 \times 10^{134} = 7 \times 10^{135}$. The same total, reached without ever writing 0.3.

✓ **Check 2:** Undo the addition. Take the second term back off the answer; the first term should reappear exactly, because addition and subtraction are inverses. $7 \times 10^{135} - 3 \times 10^{134} = 6.7 \times 10^{135}$, which is the term the question started with, so nothing has been gained or lost in the rewriting.

✓ **Check 3:** Sanity-check the size. The second term is far smaller than the first, so the total must be a little more than the first term and nowhere near double it. Doubling the first term gives $2 \times 6.7 \times 10^{135} = 1.34 \times 10^{136}$, and the answer 7×10^{135} sits just above the first term and well below that. The first term is a little over 22 times the second, so a small rise is exactly what adding it should produce.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Working	**M1**	eg 0.3×10^{135} or 67×10^{134} or $(6.7 + 0.3) \times 10^{135}$ or 70×10^{134} or 0.7×10^{136} or 7×10^{n} with $n \neq 135$	✓
Answer	**A1**	7×10^{135}	✓
Note	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 2/2

Question 15 - Exam Solution

Understanding the Question

Given

An equation made of two algebraic fractions: $\dfrac{5a+8}{3} - \dfrac{2a+5}{4} = 23$

An expression raised to the power -1: $\left(\dfrac{\sqrt{y}}{3}\right)^{-1}$

Find

(a) The value of a, with clear algebraic working shown. (b) The numbers c and n that turn the expression into cy^n.

Plan the Solution

- (a) Multiply every term by 12, the lowest common multiple of 3 and 4, so both denominators disappear.
- (a) Expand the two brackets, collect the a terms on one side and the numbers on the other, then divide.
- (b) A power of -1 means the reciprocal, so start by turning the fraction upside down.
- (b) Replace the square root by a half power, move it to the numerator, then read off c and n.

Worked Solution [6 marks]

Rule - Clear fractions first, then use the index laws: multiply every term by the lowest common multiple of the denominators, and use $\left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$ with $\sqrt{y} = y^{\frac{1}{2}}$ and $\frac{1}{y^m} = y^{-m}$.

Step 1: Multiply every term by 12

$$12 \times \frac{5a+8}{3} - 12 \times \frac{2a+5}{4} = 12 \times 23$$

$$4(5a+8) - 3(2a+5) = 276$$

(Reason: The lowest common multiple of 3 and 4 is 12. Multiplying every term by it cancels both denominators, leaving 12 divided by 3, which is 4, in front of the first bracket and 3 in front of the second. On the right, $12 \times 23 = 276$.)

Step 2: Expand the brackets

$$20a + 32 - 6a - 15 = 276$$

(Reason: Multiply out each bracket: $4(5a+8)$ gives $20a + 32$, and $3(2a+5)$ gives $6a + 15$. The minus sign in front of the second bracket applies to both of its terms, so both are taken away.)

Step 3: Collect like terms

$$14a + 17 = 276$$

(Reason: The two terms in a give $20a - 6a = 14a$, and the two numbers on the left give $32 - 15 = 17$.)

Step 4: Solve for a

$$14a = 276 - 17 = 259$$

$$a = \frac{259}{14} = \frac{37}{2} = 18.5$$

(Reason: Subtract 17 from both sides, then divide both sides by 14. Both 259 and 14 have a factor of 7, so the fraction cancels to $\frac{37}{2}$, which is 18.5. The mark scheme accepts any of these three forms.)

Step 5: Deal with the power of -1

$$\left(\frac{\sqrt{y}}{3}\right)^{-1} = \frac{3}{\sqrt{y}}$$

(Reason: A power of -1 means the reciprocal, and the reciprocal of a fraction is that fraction turned upside down: $\left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$.)

Step 6: Write the square root as an index

$$\frac{3}{\sqrt{y}} = \frac{3}{y^{\frac{1}{2}}} = 3y^{-\frac{1}{2}}$$

(Reason: A square root is a half power, so $\sqrt{y} = y^{\frac{1}{2}}$. Moving a power up from the denominator changes the sign of the index, which gives $c = 3$ and $n = -\frac{1}{2}$, that is $n = -0.5$.)

(a) $a = 18.5$

(b) $3y^{-0.5}$**, so** $c = 3$ **and** $n = -0.5$

Verification

✓ **Check 1:** Put $a = 18.5$ back into the left-hand side of the original equation. The numerators become 100.5 and 42. $\frac{100.5}{3} - \frac{42}{4} = 33.5 - 10.5 = 23$

✓ **Check 2:** Check the fraction forms the mark scheme also accepts really are the same number: cancel $\frac{259}{14}$ by 7, since $259 = 7 \times 37$ and $14 = 7 \times 2$. $\frac{259}{14} = \frac{37}{2} = 18.5$

✓ **Check 3:** Test part (b) with $y = 9$. Then $\sqrt{9} = 3$, so the original expression is the reciprocal of 1, which is 1. $\frac{3}{\sqrt{9}} = \frac{3}{3} = 1$

✓ **Check 4:** Test part (b) with $y = 36$. Then $\sqrt{36} = 6$, so the original expression is the reciprocal of 2, which is 0.5. $\frac{3}{\sqrt{36}} = \frac{3}{6} = 0.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) For example $12 \times \frac{5a+8}{3} - 12 \times \frac{2a+5}{4} = 12 \times 23$ or $4(5a+8) - 3(2a+5) = 12 \times 23\,(= 276)$ or $\frac{4(5a+8)}{12} - \frac{3(2a+5)}{12}\,(= 23)$ or $\frac{4(5a+8) - 3(2a+5)}{12}\,(= 23)$	**M1**	for clear intention to multiply all terms by 12 or a multiple of 12, or to express the left-hand side as two fractions over 12 or a multiple of 12, or as a single fraction with a denominator of 12 or a multiple of 12. If the numerator is expanded, allow one sign error or one numerical error but not both. Accept $\frac{20a+32}{12} - \frac{6a+15}{12}\,(= 23)$ or $\frac{20a+32-6a+15}{12}\,(= 23)$.	✓
(a) For example $20a + 32 - 6a - 15 = 12 \times 23\,(= 276)$ or $14a + 17 = 276$	**M1**	follow through, for expanding the brackets and multiplying both sides by the denominator with no more than one error in total, leading to a linear equation. Accept a linear equation leading to $14a - 17 = 276$ oe or $14a + 47 = 276$ oe or $26a + 17 = 276$. This mark implies the previous M mark if that has not already been awarded.	✓
(a) For example $20a - 6a = 276 - 32 + 15$ oe or $14a = 259$	**M1**	follow through, dependent on the previous M1, for correctly rearranging so that the terms in a are on one side and the number terms are on the other side.	✓
(a) $a = 18.5$ - working required	**A1**	or equivalent, dependent on both method marks, for example $\frac{259}{14}$ or $\frac{37}{2}$.	✓
(b) For example $\frac{3}{\sqrt{y}}\left(= \frac{3\sqrt{y}}{y}\right)$ or $\frac{3}{y^{0.5}}$ or $\frac{3}{y^{\frac{1}{2}}}$ or $\left(\frac{y^{0.5}}{3}\right)^{-1}$ or	**M1**	for a correct first step by applying one of the following index rules: $\sqrt{x} = x^{\frac{1}{2}} = x^{0.5}$ or $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.	✓

Step	Mark	Description	Got it?
$\left(\dfrac{y^{\frac{1}{2}}}{3}\right)^{-1}$ oe			
(b) $3y^{-0.5}$	A1	or equivalent, for example $3y^{-\frac{1}{2}}$; accept $c = 3$ and $n = -0.5$ oe. A correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 6/6

Question 16 - Exam Solution

Understanding the Question

Given

The recurring decimal $0.6\dot{1}\dot{2}$, with a dot over the 1 and a dot over the 2. Written out in full that is $0.6121212\ldots$, so the 6 does not recur and the block 12 repeats forever.

Find

Show that this decimal is exactly $\dfrac{101}{165}$.

The working is the answer here, and the second mark is only given if algebra is used, so a calculator display proves nothing on its own.

Plan the Solution

- Give the decimal a letter, x, so that what follows is algebra and not arithmetic.
- Multiply by 10 to move the non-recurring 6 in front of the decimal point.
- The block is 2 digits long, so multiply by a further 100 to get a second number carrying exactly the same tail.
- Subtract the two. The endless tails are identical, so they cancel and leave a whole number.
- Divide by the coefficient, then cancel the fraction down to $\dfrac{101}{165}$.

Worked Solution [2 marks]

Recurring decimals: multiply x by two powers of ten that leave the same recurring tail immediately after the decimal point, then subtract to remove that tail.

Step 1: Give the decimal a letter

$x = 0.6\dot{1}\dot{2} = 0.6121212\ldots$

(Reason: Naming the decimal is what turns this into algebra, and the final mark depends on it. Everything after this line is ordinary equation solving.)

Step 2: Multiply by two powers of ten

$10x = 6.121212\ldots$

$1\,000x = 612.121212\ldots$

(Reason: One digit sits between the point and the block, so $10x$ starts the block immediately after the point. The block is 2 digits long, so a further 100 gives $1\,000x$ with exactly the same tail.)

Step 3: Subtract to remove the recurring tail

$1\,000x - 10x = 612.121212\ldots - 6.121212\ldots$

$990x = 606$

(Reason: Both numbers end in the same endless $0.121212\ldots$, so subtracting takes it away exactly and leaves the whole number 606.)

Step 4: Divide, then cancel

$$x = \frac{606}{990}$$

$$\frac{606}{990} = \frac{101 \times 6}{165 \times 6} = \frac{101}{165}$$

(Reason: Both 606 and 990 are multiples of 6. Nothing is left to cancel afterwards, because 101 is prime and is not a factor of 165.)

$$0.6\dot{1}\dot{2} = \frac{101}{165}\ \textbf{as required}$$

Verification

✓ **Check 1:** Go the other way. Divide 101 by 165 and read the digits off the display. $\frac{101}{165} = 0.6121212\ldots$, which is the decimal the question started from, dots and all.

✓ **Check 2:** Use a different pair of multipliers. Subtract x from $100x$ instead: $61.212121\ldots - 0.612121\ldots$. $99x = 60.6$, and $\frac{60.6}{99} = \frac{606}{990} = \frac{101}{165}$ again. This pair leaves a terminating decimal rather than a whole number, which the mark scheme allows just as readily.

✓ **Check 3:** Split the decimal instead of shifting it: $0.6\dot{1}\dot{2} = 0.6 + 0.0\dot{1}\dot{2}$, and the recurring part on its own is $\frac{12}{990}$. $\frac{6}{10} + \frac{12}{990} = \frac{594+12}{990} = \frac{606}{990} = \frac{101}{165}$, reached without any subtraction at all.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two multiples of x carrying the same recurring tail, written down so that one can be subtracted from the other	**M1**	For 2 recurring decimals that when subtracted give a whole number or terminating decimal, with intention to subtract (ie give 60.6 or 606 or $6\,060$ etc), eg $(1\,000x =)\ 612.12\ldots$ and $(10x =)\ 6.12\ldots$, or $(100\,000x =)\ 61\,212.12\ldots$ and $(1\,000x =)\ 612\ldots$, or $(100x =)\ 61.212\ldots$ and $(x =)\ 0.612\ldots$, with intention to subtract. x is not required to award this mark. If recurring dots are not shown in both numbers, then at least one of the numbers must be shown to at least 5 significant figures. Or $\frac{6}{10} + 1\,000x\,(12.12\ldots) - 10x\,(0.12\ldots)$.	✓
Complete the algebra to $\frac{101}{165}$	**A1**	For completion to $\frac{101}{165}$, dep on M1, and algebra must be used for this final mark to be awarded. Eg $10\,000x - 100x = 6\,121.21\ldots - 61.21\ldots = 6\,060$, $(9\,900x = 6\,060)$ and $\frac{6\,060}{9\,900} = \frac{101}{165}$, or $1\,000x - 10x = 612.12\ldots - 6.12\ldots = 606$, $(990x = 606)$ and $\frac{606}{990} = \frac{101}{165}$, or $100x - x = 61.212\ldots - 0.612\ldots = 60.6$, $(99x = 60.6)$ and $\frac{60.6}{99} = \frac{101}{165}$ oe, or $0.6 + \ldots$ and $(1\,000x - 10x = 990x = 12)$ and $0.6 + \frac{12}{990} = \frac{0.6 \times 990 + 12}{990} = \frac{101}{165}$ oe. Allow for instance $99x = 60.6$ and then $\frac{606}{990} = \frac{101}{165}$.	✓
Working required	**Note**	No algebra used gets a maximum of 1 mark.	✓

Full marks: 2/2

Question 17 - Exam Solution

Understanding the Question

Given

Three numbers that are consecutive multiples of 4
Consecutive multiples of 4 rise in steps of 4, so once the smallest is named the other two follow from it

Find

A proof that the square of the largest number minus the square of the smallest number is always a multiple of 64 The proof has to be algebraic: it must cover every such set of three numbers at once, not a few chosen examples.

Plan the Solution

- Name the three numbers with one letter: $4n$, $4n + 4$ and $4n + 8$, where n is any integer.
- Square the largest and square the smallest, then subtract one from the other.
- Factorise what is left and show that 64 comes out as a factor, with an integer beside it.

Worked Solution [3 marks]

> Rule - Consecutive multiples of 4: every multiple of 4 is $4n$ for some integer n, and the next two are $4n + 4$ and $4n + 8$. A number is a multiple of 64 exactly when it can be written as $64 \times (\text{an integer})$.

Step 1: name the three numbers algebraically

$$4n,\ \ 4n + 4,\ \ 4n + 8$$

(Reason: n stands for any integer, so $4n$ stands for any multiple of 4. Adding 4 each time gives the next two consecutive multiples, so the smallest of the three is $4n$ and the largest is $4n + 8$.)

Step 2: square the largest and square the smallest

$$(4n + 8)^2 = 16n^2 + 64n + 64$$
$$(4n)^2 = 16n^2$$

(Reason: Expand the bracket in full - square the first term, double the product of the two terms, then square the second term. Only the largest and the smallest are squared, so the middle number $4n + 4$ plays no part in the proof.)

Step 3: subtract, then take out the common factor

$(4n+8)^2 - (4n)^2 = 16n^2 + 64n + 64 - 16n^2$
$= 64n + 64$
$= 64(n+1)$

(Reason: The $16n^2$ terms cancel, which is what makes this difference so tidy. Both terms left over have 64 as a factor, since $64n = 64 \times n$ and $64 = 64 \times 1$, so 64 can be taken outside a bracket.)

Step 4: read the conclusion off the factorised form

$64(n+1)$

(Reason: n is an integer, so $n+1$ is an integer as well. The difference is therefore 64 multiplied by an integer, which is precisely what it means to be a multiple of 64. Nothing in the working depended on which integer n is, so this holds for any three consecutive multiples of 4.)

$$(4n+8)^2 - (4n)^2 = 64n + 64 = 64(n+1)$$

$n+1$ is an integer, so the difference is always a multiple of 64

Verification

✓ **Check 1:** Test the result on real numbers. Take $n = 3$, which gives the three consecutive multiples 12, 16 and 20, then square the largest and the smallest and subtract. $20^2 - 12^2 = 400 - 144 = 256$, and $256 = 64 \times 4$, which is the $64(n+1)$ above with $n = 3$

✓ **Check 2:** Test a second, larger set so the check is not a coincidence. Take $n = 7$, giving 28, 32 and 36. $36^2 - 28^2 = 1296 - 784 = 512$, and $512 = 64 \times 8$, again matching $64(n+1)$ with $n = 7$

✓ **Check 3:** Redo the algebra a different way, so no expansion is repeated. Use the difference of two squares, $a^2 - b^2 = (a+b)(a-b)$, with $a = 4n+8$ and $b = 4n$. $(4n+8+4n)(4n+8-4n) = (8n+8) \times 8 = 64n + 64$, the same expression the expansion gave

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $4n,\ 4n+4,\ 4n+8$ or $4n,\ 4(n+1),\ 4(n+2)$ or $4n-4,\ 4n,\ 4n+4$ or $4(n-1),\ 4n,\ 4(n+1)$	M1	for correct expressions for 3 consecutive multiples of 4 (any letter can be used) may just see the first and third multiple for this mark	✓

Step	Mark	Description	Got it?
eg $(4n+8)^2 - (4n)^2$ $(= 16n^2 + 64n + 64 - 16n^2)$ or $(4(n+2))^2 - (4n)^2$ $(= 16n^2 + 64n + 64 - 16n^2)$ or $(4n+4)^2 - (4n-4)^2$ $(= 16n^2 + 32n + 16 - 16n^2 + 32n - 16)$ or $(4(n+1))^2 - (4(n-1))^2$ $(= 16n^2 + 32n + 16 - 16n^2 + 32n - 16)$	M1	for squaring the largest and smallest multiple of 4 and subtracting (no need to expand or simplify for this mark)	✓
eg $(4n+8)^2 - (4n)^2 = 16n^2 + 64n + 64 - 16n^2 = 64n + 64$ or $(4n+8)^2 - (4n)^2 = (4n+8+4n)(4n+8-4n) = 8(8n+8) = 64n + 64$ or $(4n+4)^2 - (4n-4)^2 = 16n^2 + 32n + 16 - 16n^2 + 32n - 16 = 64n$ or $(4(n+1))^2 - (4(n-1))^2 = (4(n+1) + 4(n-1))(4(n+1) - 4(n-1)) = 8n \times 8 = 64n$ Answer given as correctly shown	A1	dep on M2, for use of algebra to show correct conclusion	✓
Working required	Note	algebraic working must be seen. Trying particular multiples of 4, however many of them, shows the statement in those cases and does not prove it in every case	✓

Full marks: 3/3

Question 18 - Exam Solution

Understanding the Question

Given

F varies inversely with the cube of r

One pair of values: $F = 6$ when $r = 2$

Find

(a) A formula for F in terms of r (b) The value of r when $F = 3072$

Plan the Solution

- Turn the words into an equation. Inverse proportion to the cube of r means $F = \frac{k}{r^3}$ for some constant k.
- Put the one pair of values the question gives into that equation to pin k down, then write the formula out in full. That is part (a).
- For part (b), put $F = 3072$ into the finished formula, rearrange to leave r^3 on its own, and take the cube root.

Worked Solution [5 marks]

> Rule - Inverse proportion: $F \propto \frac{1}{r^3}$ means $F = \frac{k}{r^3}$, where k is a constant. One pair of values fixes k, and that same k then holds for every other pair.

Step 1: Write the statement as an equation

$$F \propto \frac{1}{r^3}$$

$$F = \frac{k}{r^3}$$

(Reason: Inversely proportional to the cube means F is a constant divided by r^3, never multiplied by it, and the cube is on the r alone. The constant has to be carried as a letter such as k until a pair of values fixes it; the mark scheme allows any letter here but not 1.)

Step 2: Substitute $F = 6$ and $r = 2$

$$6 = \frac{k}{2^3}$$

$$6 = \frac{k}{8}$$

(Reason: The pair of values is the only thing that can pin k down, so it goes into the equation just written. Cubing the 2 gives 8, so the equation now has one unknown in it instead of three.)

Step 3: Work out k, then write the formula

$k = 6 \times 8 = 48$

$F = \dfrac{48}{r^3}$

(Reason: Multiplying both sides by 8 leaves k on its own. Putting $k = 48$ back into $F = \dfrac{k}{r^3}$ gives the formula part (a) asks for, with F as the subject.)

Step 4: Put $F = 3072$ into the formula

$3072 = \dfrac{48}{r^3}$

$r^3 = \dfrac{48}{3072} = \dfrac{1}{64}$

(Reason: Multiply both sides by r^3 and then divide by 3072, so that r^3 is left on its own. Watch which number ends up on top: F has gone up a long way from 6, so r must come out smaller than the 2 that produced it.)

Step 5: Take the cube root

$r = \sqrt[3]{\dfrac{1}{64}}$

$r = \dfrac{1}{4} = 0.25$

(Reason: Cube rooting undoes the cube. Since $4^3 = 64$, the cube root of $\dfrac{1}{64}$ is $\dfrac{1}{4}$. The mark scheme prints $\dfrac{1}{4}$ and accepts any equivalent form, so 0.25 scores the mark just as well.)

(a) $F = \dfrac{48}{r^3}$

(b) $r = \dfrac{1}{4}$**, or** 0.25

Verification

✓ **Check 1:** Put $r = 2$ back into the formula from part (a). Cubing 2 gives 8, and the F that comes out has to be the 6 the question states. $\dfrac{48}{8} = 6$

✓ **Check 2:** Put $r = \dfrac{1}{4}$ into the same formula. Cubing $\dfrac{1}{4}$ gives $\dfrac{1}{64}$, and dividing by $\dfrac{1}{64}$ is the same as multiplying by 64. $48 \times 64 = 3072$

✓ **Check 3:** Work in multipliers and never use the constant at all. F is multiplied by $\frac{3072}{6} = 512$, so r^3 is divided by 512 and r is divided by the cube root of 512. $8^3 = 512$ and $\frac{2}{8} = \frac{1}{4}$

✓ **Check 4:** For inverse proportion to the cube, the product $F \times r^3$ is the same for every pair, so the pair the question gives and the answer pair must produce the same number. $6 \times 2^3 = 48$ and $3072 \times \left(\frac{1}{4}\right)^3 = 48$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$F = \frac{k}{r^3}$ or $Fr^3 = k$ or $kF = \frac{1}{r^3}$	M1	oe. k can be any letter (must be a letter and not 1)	✓
$6 = \frac{k}{2^3}$ oe or $k = 48$ or $6k = \frac{1}{2^3}$ oe or $k = \frac{1}{48}$	M1	for substitution of F and r into a correct formula, implies the first M1 if you see this stage. Condone use of $\propto$ for method marks	✓
$F = \frac{48}{r^3}$	A1	oe with F the subject eg $F = 48 \times \frac{1}{r^3}$ or $F = 48 \times r^{-3}$. Award 3 marks if answer is $F = \frac{k}{r^3}$ and $k = 48$ clearly given in the body of the script. M2A0 for $Fr^3 = 48$ or $r = \sqrt[3]{\frac{48}{F}}$ or $r^3 = \frac{48}{F}$	✓
$(r^3 =)\frac{48}{3072}$ oe eg $\frac{1}{64}$ or $0.01(5625)$ rounded or truncated	M1ft	allow use of their 48 as long as M2 was gained in part (a). The printed scheme puts that 48 in quotation marks, which is what grants the follow-through from a candidate's own constant	✓
$\frac{1}{4}$	A1	oe	✓

Step	Mark	Description	Got it?
On the answer row of part (a) and of part (b)	Note	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 5/5

Question 19 - Exam Solution

Understanding the Question

Given

Nine tiles, carrying the numbers 1, 1, 2, 3, 3, 3, 5, 6 and 6.
Two tiles are taken at random, and the first one is NOT put back before the second is taken.

Find

The probability that the two numbers taken add up to less than 5.

Plan the Solution

- Work out which pairs of numbers actually add to less than 5. Only the small numbers can manage it, so that list is short.
- Count the outcomes. There are 9 tiles for the first draw but only 8 for the second, because the first tile is not put back.
- Take each possible FIRST tile in turn, write down the probability of that branch, and add the branches up. Two different first tiles cannot both happen, so adding is allowed.

Worked Solution [3 marks]

> Rule - Without replacement: the second fraction has a denominator one smaller than the first, because one tile has gone. For two draws in a row, $P(A \text{ then } B) = P(A) \times P(B \text{ after } A)$, and outcomes that cannot happen together are added.

Step 1: which pairs add to less than 5?

$1 + 1 = 2$

$1 + 2 = 3$

$1 + 3 = 4$

$2 + 3 = 5$

$1 + 5 = 6$

(Reason: Start with the smallest numbers on the tiles and work upwards. The first three sums are under 5, the fourth is exactly 5 and so does not count, and every remaining pair is larger still. So one of the two tiles has to be a 1, and the other has to be a 1, a 2 or a 3.)

Step 2: count the outcomes

$9 \times 8 = 72$

(Reason: There are 9 tiles to choose from first and only 8 left for the second draw, so there are 72 outcomes in order. If the tile were put back there would be $9 \times 9 = 81$ outcomes instead and the answer would come out as $\frac{21}{81}$ - that is exactly the wrong answer the mark scheme carries a special case for, so read the words does not put the tile back carefully.)

Step 3: take each possible first tile in turn

$$\frac{2}{9} \times \frac{5}{8} = \frac{10}{72}$$
$$\frac{1}{9} \times \frac{2}{8} = \frac{2}{72}$$
$$\frac{3}{9} \times \frac{2}{8} = \frac{6}{72}$$

(Reason: The first tile is a 1 with probability $\frac{2}{9}$, and then the other 1, the 2 and all three 3s still finish the job, which is 5 of the 8 tiles left. The first tile is the 2 with probability $\frac{1}{9}$, and then only a 1 works, which is 2 of the 8 left. The first tile is a 3 with probability $\frac{3}{9}$, and again only a 1 works. A first tile of 5 or 6 can never work, so there is no fourth line.)

Step 4: add the branches

$$\frac{10}{72} + \frac{2}{72} + \frac{6}{72} = \frac{18}{72}$$
$$\frac{18}{72} = \frac{1}{4}$$

(Reason: The three branches start with different tiles, so they cannot happen together and their probabilities simply add. Dividing top and bottom by 18 turns $\frac{18}{72}$ into $\frac{1}{4}$. The mark scheme prints $\frac{18}{72}$ and accepts any equivalent form, so $\frac{9}{36}$, $\frac{1}{4}$, 0.25 and 25% all score the accuracy mark.)

$$\frac{18}{72} = \frac{1}{4}, \text{ or } 0.25$$

Verification

✓ **Check 1:** Count pairs instead of ordered draws. Order does not change a sum, so there are ${}^9C_2 = 36$ pairs altogether. The pairs that work are the two 1s together, which is 1 pair, a 1 with the 2, which is 2 pairs, and a 1 with a 3, which is 6 pairs. $1 + 2 + 6 = 9$ and $\frac{9}{36} = \frac{1}{4}$

✓ **Check 2:** Count the draws that FAIL instead. Going through the value pairs the other way - a 1 with a 5 or a 6, the 2 with a 3, a 5 or a 6, two 3s together, and so on - gives 54 ordered draws whose sum is 5 or more. The two counts have to fill the 72 between them. $\frac{54}{72} = \frac{3}{4}$ and $\frac{18}{72} + \frac{54}{72} = 1$

✓ **Check 3:** Is the size sensible? Divide 18 by 72. One draw in four working is a believable size for an event that needs one of only two 1s to turn up among the two tiles taken. $\frac{18}{72} = 0.25$, which is 25%

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{2}{9} \times \frac{1}{8}\left(= \frac{2}{72}\right)$ oe or $\frac{1}{9} \times \frac{2}{8}\left(= \frac{2}{72}\right)$ oe or $\frac{2}{9} \times \frac{3}{8}\left(= \frac{6}{72}\right)$ oe or $\frac{3}{9} \times \frac{2}{8}\left(= \frac{6}{72}\right)$ oe or $\frac{2}{9} \times \frac{5}{8}\left(= \frac{10}{72}\right)$ oe or $\frac{5}{9} \times \frac{2}{8}\left(= \frac{10}{72}\right)$ oe, or ${}^9C_2$ or $\frac{9!}{2!7!}$ or $\frac{9 \times 8}{2}$ or 36 or $1 + 2 + 6\ (= 9)$	M1	for finding one correct product, or for the correct number of total outcomes, or for the correct number of outcomes when the sum < 5. NB if using decimals allow 2 decimal places truncated or rounded	✓

Step	Mark	Description	Got it?
$3 \times \frac{2}{72} + 2 \times \frac{6}{72}$ oe or $\frac{2}{72} + \frac{6}{72} + \frac{10}{72}$ oe, or ${}^{9}C_2$ or $\frac{9!}{2!7!}$ or $\frac{9 \times 8}{2}$ or 36 and $1 + 2 + 6\ (= 9)$	**M1**	for a complete correct method, or for the correct number of total outcomes AND for the correct number of outcomes when the sum < 5. The printed scheme puts each of those fractions in quotation marks, which is what allows a candidate's own products from the first M1 to be used here	✓
$\frac{18}{72}$	**A1**	oe eg $\frac{9}{36}$ or 0.25 or 25%	✓
$\frac{21}{81}$	**SC B1**	oe eg $\frac{7}{27}$ or $0.259(25\ldots)$ or $25.9(25\ldots)\%$ truncated or rounded	✓
ALT method - $\frac{2}{9} \times \frac{1}{8} \left(= \frac{2}{72}\right)$ oe or $\frac{1}{9} \times \frac{2}{8} \left(= \frac{2}{72}\right)$ or $\frac{1}{9} \times \frac{1}{8} \left(= \frac{1}{72}\right)$ oe or $\frac{2}{9} \times \frac{2}{8} \left(= \frac{4}{72}\right)$ or $\frac{3}{9} \times \frac{1}{8} \left(= \frac{3}{72}\right)$ oe or $\frac{1}{9} \times \frac{3}{8} \left(= \frac{3}{72}\right)$ oe or	**Note**	The mark scheme prints a full ALT method on its next page, carrying the same three marks and	✓

Step	Mark	Description	Got it?
$\frac{2}{9}\times\frac{3}{8}\left(=\frac{6}{72}\right)$ oe or $\frac{3}{9}\times\frac{2}{8}\left(=\frac{6}{72}\right)$ oe or $\frac{1}{9}\times\frac{6}{8}\left(=\frac{6}{72}\right)$ oe or $\frac{6}{9}\times\frac{1}{8}\left(=\frac{6}{72}\right)$ oe or $\frac{3}{9}\times\frac{6}{8}\left(=\frac{18}{72}\right)$ oe or $\frac{6}{9}\times\frac{3}{8}\left(=\frac{18}{72}\right)$ oe or $\frac{1}{9}\times\frac{8}{8}\left(=\frac{8}{72}\right)$ oe or $\frac{8}{9}\times\frac{1}{8}\left(=\frac{8}{72}\right)$ or $\frac{2}{9}\times\frac{8}{8}\left(=\frac{16}{72}\right)$ oe or $\frac{8}{9}\times\frac{2}{8}\left(=\frac{16}{72}\right)$ oe		working from the draws that fail instead. This is that method's first method mark: for finding one correct product. NB if using decimals allow 2 decimal places truncated or rounded	
ALT method - $1-\left(2\times\frac{1}{72}+7\times\frac{2}{72}+4\times\frac{3}{72}+2\times\frac{4}{72}+3\times\frac{6}{72}\right)$ or $1-\left(\frac{6}{72}+\frac{6}{72}+\frac{18}{72}+\frac{8}{72}+\frac{16}{72}\right)$ or $1-\frac{54}{72}$ oe	**Note**	The ALT method's second method mark: for a complete correct method. The printed scheme puts each of those fractions in quotation marks, which is what allows a candidate's own products from the ALT first mark to be used here	✓
On the answer row	**Note**	Correct answer scores full marks (unless from obvious incorrect working)	✓

Step	Mark	Description	Got it?
On the answer row	**Note**	Do not allow $\frac{6}{9} \times \frac{3}{8} = \frac{18}{72}$ or $\frac{3}{9} \times \frac{6}{8} = \frac{18}{72}$ as this an incorrect method (M1M0A0)	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

A cuboid $ABCDEFGH$, with the rectangle $ABGF$ as the end face nearest the reader.
$AB = 9$ cm and $AF = 7$ cm, the two edges of that end face.
$FC = 18$ cm, the diagonal running from F through the inside of the solid to C.

Find

The length of BC, the depth of the cuboid, correct to 3 significant figures.

Plan the Solution

- Pythagoras only works inside one right-angled triangle, and no single triangle here holds AB, AF and FC together. So use it twice.
- First across the end face. F and B are opposite corners of the rectangle $ABGF$, so FB is its diagonal.
- Then in triangle FBC. The edge BC leaves the end face at right angles, which puts the right angle at B and makes FC the hypotenuse.
- Carry FB forward as a squared value. Taking the root and squaring it again only undoes the work, and rounding halfway through would move the third significant figure.

Worked Solution [3 marks]

> Rule - Pythagoras, used twice: in any right-angled triangle $h^2 = a^2 + b^2$, where h is the hypotenuse. Across a face it gives the face diagonal; in the triangle standing on that diagonal it gives the edge that leaves the face.

Step 1: The diagonal FB across the end face

$FB^2 = AF^2 + AB^2$

$7^2 + 9^2 = 49 + 81 = 130$

(Reason: Angle FAB is a right angle, because AF is a vertical edge of the cuboid and AB is a horizontal one. Stop at 130: the next step wants FB^2, not FB.)

Step 2: Triangle FBC, right-angled at B

$FC^2 = FB^2 + BC^2$

$18^2 - 130 = 324 - 130 = 194$

(Reason: The edge BC is perpendicular to the whole end face $ABGF$, so it is perpendicular to FB as well. That places the right angle at B and makes FC the hypotenuse, so it is the two shorter squares that add to give the longest one.)

Step 3: Square root, then round

$BC = \sqrt{194} = 13.9284\ldots$

$BC = 13.9$

(Reason: Take the positive root: a length cannot be negative. The fourth significant figure is a 2, so the third one stays as it is and $13.9284\ldots$ rounds to 13.9.)

$$BC = 13.9 \text{ cm}$$

Verification

✓ **Check 1:** Put the answer back into three-dimensional Pythagoras, $FC^2 = AB^2 + AF^2 + BC^2$. The three edge squares must rebuild 18^2.
$81 + 49 + 194 = 324$, and $18^2 = 324$

✓ **Check 2:** Test the rounding rather than the arithmetic. Anything that rounds to 13.9 to 3 significant figures lies between 13.85 and 13.95, so 194 has to sit between their squares.
$13.85^2 = 191.8225$ and $13.95^2 = 194.6025$, and 194 lies between the two

✓ **Check 3:** Reach the same place along a different pair of triangles, through the base diagonal AC. Triangle FAC is right-angled at A, so $AC^2 = FC^2 - AF^2$; then triangle ABC is right-angled at B, so $BC^2 = AC^2 - AB^2$. $324 - 49 = 275$, then $275 - 81 = 194$, the same square as before

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $(AC^2 =)\ 18^2 - 7^2\ (= 275)$ or $(AC =)\ \sqrt{18^2 - 7^2}\ (= \sqrt{275}$ or $5\sqrt{11}$ or $16.5(831\ldots))$ or $(FB^2 =)\ 9^2 + 7^2\ (= 130)$ or $(FB =)\ \sqrt{9^2 + 7^2}\ (= \sqrt{130}$ or $11.4(017\ldots))$ or $(GC^2 =)\ 18^2 - 9^2\ (= 243)$ or $(GC =)\ \sqrt{18^2 - 9^2}\ (= \sqrt{243}$ or $9\sqrt{3}$ or $15.5(884\ldots))$ or $18^2 = (BC)^2 + 7^2 + 9^2$ oe	M1	for method to find AC^2 or AC or FB^2 or FB or GC^2 or GC or for a correct equation using BC^2 and 18 and 7 and 9 Other longer ways to find AC, FB, GC may be used but must be a complete method, eg $\angle FCA = \sin^{-1}\left(\frac{7}{18}\right)\ (= 22.88\ldots)$ and $AC = \frac{7}{\tan 22.88\ldots}$, where the printed scheme's quotation marks round $22.88\ldots$ mean the candidate's own angle may be used.	✓
eg $275 - 9^2\ (= 194)$ or $16.5\ldots^2 - 9^2\ (= 194)$ or $18^2 - 130\ (= 194)$ or $18^2 - 11.4\ldots^2\ (= 194)$ or $243 - 7^2\ (= 194)$ or $15.5\ldots^2 - 7^2\ (= 194)$ or $18^2 - 7^2 - 9^2\ (= 194)$ or $\angle FCB = \sin^{-1}\left(\frac{11.4\ldots}{18}\right)\ (= 39.3(036\ldots))$ and $\cos 39.3\ldots = \frac{BC}{18}$ or $\tan 39.3\ldots = \frac{11.4\ldots}{BC}$ oe Every value the printed scheme sets in quotation marks - 275, 130, 243, $16.5\ldots$, $11.4\ldots$, $15.5\ldots$ and $39.3\ldots$ - may be	M1	for a complete method to find BC^2 Other longer ways to find BC may be used but must be a complete method, leading to a trigonometric equation in BC.	✓

Step	Mark	Description	Got it?
the candidate's own value carried through from the first mark.			
13.9 A correct answer scores full marks, unless it comes from obviously incorrect working.	A1	accept 13.8 to 14	✓

Full marks: 3/3

Question 21 - Exam Solution

Understanding the Question

Given

$y = f(x)$ has exactly one turning point, and it is at $(-6, 9)$
The curve C, $y = g(x)$, is transformed into the curve S, $y = g(x + a) + b$
The point $(4, -5)$ on C is mapped to the point $(1, -16)$ on S

Find

(a) the coordinates of the turning point on $y = f(3x)$ (b) the value of a and the value of b

Plan the Solution

- Read $f(3x)$ as a change to the INPUT of f. The curve turns when that input is -6, so solve $3x = -6$.
- A change inside the bracket moves the curve sideways only, so the y-coordinate of the turning point is untouched.
- For part (b), write the image of (x, y) under $y = g(x + a) + b$ as $(x - a, y + b)$, then match it against the two given points one coordinate at a time.
- Both parts say "write down", so no method is demanded - but knowing which way each transformation goes is what keeps the signs right.

Worked Solution [3 marks]

> Rule - Function transformations: $y = f(ax)$ divides every x-coordinate by a and leaves the y-coordinate unchanged; $y = g(x + a) + b$ moves every point a to the left and b up.

Step 1: (a) Find the new x-coordinate

$3x = -6$

$$x = \frac{-6}{3} = -2$$

(Reason: The curve $y = f(3x)$ takes at x the value that $y = f(x)$ takes at $3x$, so it turns where the input to f is -6. Divide by 3.)

Step 2: (a) The y-coordinate does not move

$y = 9$

$(-2, 9)$

(Reason: Replacing x by $3x$ changes only the input to f, never its output, so the height of the turning point stays at 9. This is a horizontal stretch of scale factor $\frac{1}{3}$.)

Step 3: (b) Say what the transformation does to one point

$x_S = x_C - a$

$y_S = y_C + b$

(Reason: $g(x + a)$ slides the curve a units in the negative x-direction, and the $+b$ slides it b units up. So a point at (x, y) on C lands at $(x - a, y + b)$ on S.)

Step 4: (b) Compare the x-coordinates

$4 - a = 1$

$a = 4 - 1 = 3$

(Reason: The point $(4, -5)$ on C is mapped to $(1, -16)$ on S, so the 4 has to become a 1.)

Step 5: (b) Compare the y-coordinates

$-5 + b = -16$

$b = -16 + 5 = -11$

(Reason: The same point drops from -5 to -16, so b comes out negative: the curve has moved 11 down as well as 3 to the left.)

(a) $(-2, 9)$

(b) $a = 3$ **and** $b = -11$

Verification

✓ **Check 1:** Put $x = -2$ into $f(3x)$. The input to f becomes $3 \times (-2) = -6$, which is exactly where f turns. $f(3 \times (-2)) = f(-6) = 9$, so the turning point is $(-2, 9)$

✓ **Check 2:** Reach the same point a different way: read $y = f(3x)$ as a horizontal stretch of scale factor $\frac{1}{3}$ and stretch the x-coordinate directly. A stretch of $\frac{1}{3}$ must pull the point towards the y-axis, so the answer has to be closer to 0 than -6 is. $-6 \times \frac{1}{3} = -2$, and the y-coordinate is unchanged at 9

✓ **Check 3:** Put $a = 3$ and $b = -11$ back into $y = g(x + a) + b$ and test it at $x = 1$. The point $(4, -5)$ lying on C tells us $g(4) = -5$. $g(1 + 3) - 11 = -5 - 11 = -16$, which is the y-coordinate on S

✓ **Check 4:** Read the answer as a translation - 3 to the left and 11 down - and move the point on the grid instead of using algebra. $(4 - 3, -5 - 11) = (1, -16)$, the point given on S

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	$(-2, 9)$	✓
(b)	**B1**	$a = 3$	✓
(b)	**B1**	$b = -11$	✓

Full marks: 3/3

Question 22 - Exam Solution

Understanding the Question

Given

The circle $x^2 + y^2 = 41$
The straight line $2x + y = 3$
One equation is quadratic and the other is linear, so this is a straight line meeting a circle.

Find

The pairs of values of x and y that satisfy both equations at the same time.
Substituting the line into the circle leaves a quadratic, so expect 2 solution pairs.

Plan the Solution

- Rearrange the linear equation to make y the subject, because y already has a coefficient of 1.

- Substitute that expression for y into $x^2 + y^2 = 41$ so that only x is left.
- Expand the bracket, collect everything on one side, and solve the three term quadratic.
- Put each value of x back into the linear equation to get its partner value of y.

Worked Solution [5 marks]

> Rule - Substitution (line into curve): make one letter the subject of the linear equation, substitute it into the quadratic equation, solve the quadratic that results, then find the partner value for each root.

Step 1: make y the subject of the linear equation

$$2x + y = 3$$
$$y = 3 - 2x$$

(Reason: y has a coefficient of 1, so rearranging costs no fractions and keeps the substitution clean.)

Step 2: substitute into $x^2 + y^2 = 41$ and expand

$$x^2 + (3 - 2x)^2 = 41$$
$$x^2 + 9 - 12x + 4x^2 = 41$$

(Reason: Squaring a bracket needs every term. $(3 - 2x)^2$ has a middle term of $-12x$, and that is the term most often lost.)

Step 3: collect the terms into a three term quadratic

$$5x^2 - 12x + 9 = 41$$
$$5x^2 - 12x - 32 = 0$$

(Reason: A quadratic can only be factorised, or put through the formula, once every term is on one side and the other side is 0.)

Step 4: solve the quadratic

$$(5x + 8)(x - 4) = 0$$
$$x = -\frac{8}{5} \text{ or } x = 4$$

(Reason: The brackets multiply back out to $5x^2 - 12x - 32$, and a product is zero only when one of its factors is zero.)

Step 5: find the value of y that goes with $x = -\frac{8}{5}$

$$y = 3 - 2 \times \left(-\frac{8}{5}\right) = 3 + \frac{16}{5} = \frac{31}{5}$$

(Reason: Substituting into the linear equation is quicker than substituting into the circle, and it gives one value of y rather than two.)

Step 6: find the value of y that goes with $x = 4$

$y = 3 - 2 \times 4 = 3 - 8 = -5$

(Reason: Each root of the quadratic carries its own partner value, so the two roots give two separate solution pairs.)

$$x = -\frac{8}{5}, y = \frac{31}{5}$$
$$x = 4, y = -5$$

Verification

✓ **Check 1 - the first pair:** Put $x = -\frac{8}{5}$ and $y = \frac{31}{5}$ into both of the original equations.
$\frac{64}{25} + \frac{961}{25} = \frac{1025}{25} = 41$ and $-\frac{16}{5} + \frac{31}{5} = \frac{15}{5} = 3$

✓ **Check 2 - the second pair:** Put $x = 4$ and $y = -5$ into both of the original equations.
$4^2 + (-5)^2 = 16 + 25 = 41$ and $2 \times 4 + (-5) = 8 - 5 = 3$

✓ **Check 3 - the roots of the quadratic:** For $5x^2 - 12x - 32 = 0$ the two roots must add to $\frac{12}{5}$ and multiply to $-\frac{32}{5}$. $-\frac{8}{5} + 4 = \frac{12}{5}$ and $-\frac{8}{5} \times 4 = -\frac{32}{5}$

✓ **Check 4 - eliminate the other letter instead:** Substitute $x = \frac{3-y}{2}$ instead. The quadratic in y must give the same two values of y. $5y^2 - 6y - 155 = 0$ factorises as $(5y - 31)(y + 5) = 0$, giving $y = \frac{31}{5}$ and $y = -5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$x^2 + (-2x+3)^2 = 41$ or $\left(\frac{-y+3}{2}\right)^2 + y^2 = 41$	M1	Substitution of $y = \pm 2x \pm 3$ (or $x = \frac{\pm y \pm 3}{2}$) into $x^2 + y^2 = 41$ to obtain an equation in x only (or y only).	✓
eg $5x^2 - 12x - 32\ (= 0)$ oe or $5x^2 - 12x = 32$; or eg	M1ft	Dep on previous M1 for multiplying out and collecting terms, forming a	✓

Step	Mark	Description	Got it?
$5y^2-6y-155\,(=0)$ or $5y^2-6y=155$		three term quadratic in any form of $ax^2+bx+c\,(=0)$ where at least 2 coefficients (a or b or c) are correct.	
$(5x+8)(x-4)\,(=0)$ or $(x=)\dfrac{12\pm\sqrt{(-12)^2-4\times5\times(-32)}}{2\times5}$ or $5\left[\left(x-\dfrac{6}{5}\right)^2-\left(\dfrac{6}{5}\right)^2\right]-32\,(=0)$, which should give $(x=)-\dfrac{8}{5}$, 4; or the same three methods on the y quadratic, eg $(5y-31)(y+5)\,(=0)$, which should give $(y=)\dfrac{31}{5}$, -5	**M1ft**	Dep on M1 method to solve their three term quadratic using any correct method (allow one sign error and some simplification - allow as far as eg $\dfrac{12\pm\sqrt{144+640}}{10}$ or $\dfrac{6\pm\sqrt{36+3100}}{10}$, or if factorising allow brackets which expanded give 2 out of 3 terms correct) or correct values for x or correct values for y.	✓
eg $2\times4+y=3$ and $2\times-\dfrac{8}{5}+y=3$; or eg $2x+\dfrac{31}{5}=3$ and $2x-5=3$	**M1ft**	Dep on previous M1 for substituting their 2 found values of x or y into one of the two given equations or their rearranged equation used in the substitution, or for one correct pair of values.	✓
$x=-\dfrac{8}{5}, y=\dfrac{31}{5}, x=4, y=-5$. Working required: if the correct answers come from incorrectly using $y=2x-3$ oe, award M4A0.	**A1**	oe, dep on M2, for all 4 values (allow coordinates).	✓
If they find the values of y but think they are the values of x, then the maximum mark is 3 .	**Note**	A guidance row printed with the scheme. It carries no mark of its own.	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

Two chords of the same circle, PTR and QTS, crossing at T inside the circle. $PT = (3y - 5)$ cm and $RT = 4x$ cm are the two parts of PTR. $QT = (y + 3)$ cm and $ST = (x + 2)$ cm are the two parts of QTS.

Find

An expression for y in terms of x. Expect an algebraic fraction: y sits inside two of the four lengths, so it can only be freed by factorising and then dividing.

Plan the Solution

- Turn the four lengths into one equation using the intersecting chords rule.
- Expand both sides so that every term is written out on its own.
- Gather every term containing y on one side and every term without it on the other.
- Factorise so that y appears exactly once, then divide by the bracket beside it.

Worked Solution [5 marks]

> Rule - Intersecting chords: when two chords of a circle cross at a point inside it, the two parts of one chord have the same product as the two parts of the other, so $PT \times RT = QT \times ST$.

Step 1: use the intersecting chords rule

$PT \times RT = QT \times ST$

$(3y - 5) \times 4x = (y + 3)(x + 2)$

(Reason: The two chords cross at T, so the two parts of PTR multiply to give the same value as the two parts of QTS. That single equation is what carries both unknowns.)

Step 2: expand both sides

$12xy - 20x = xy + 2y + 3x + 6$

(Reason: Multiply $4x$ through the first bracket, and expand the pair of brackets on the right in the usual way. Nothing is collected yet - every term is simply written out.)

Step 3: collect the y terms on one side

$12xy - xy - 2y = 3x + 20x + 6$

$11xy - 2y = 23x + 6$

(Reason: Every term carrying a y goes to the left; every term without one goes to the right. Note that $12xy$ and xy can be subtracted because they are like terms.)

Step 4: factorise so that y appears once

$y(11x - 2) = 23x + 6$

(Reason: y is a common factor of $11xy$ and $-2y$, so it comes outside a bracket. This is the step the whole question is built around: until y appears once, it cannot be made the subject.)

Step 5: divide by the bracket

$$y = \frac{23x + 6}{11x - 2}$$

(Reason: Dividing both sides by $(11x - 2)$ leaves y on its own, and the answer is a single algebraic fraction in x.)

$$y = \frac{23x + 6}{11x - 2}$$

Verification

✓ **Check 1:** Put $x = 2$ into the answer, which gives $\frac{46+6}{22-2} = \frac{13}{5}$, then work out the four lengths and compare the two products. $PT \times RT = \frac{14}{5} \times 8 = \frac{112}{5}$ and $QT \times ST = \frac{28}{5} \times 4 = \frac{112}{5}$

✓ **Check 2:** Work backwards. Multiply the answer by its own denominator and put every term back where it came from; the expansion in Step 2 should reappear. $y(11x - 2) = 23x + 6$ opens out to $11xy - 2y = 23x + 6$, and moving the terms back gives $12xy - 20x = xy + 2y + 3x + 6$, which is exactly $4x(3y - 5) = (y + 3)(x + 2)$ expanded.

✓ **Check 3:** Test a second value, in case $x = 2$ was lucky. At $x = 1$ the answer gives $y = \frac{29}{9}$, and all four lengths come out positive, as lengths must. $\frac{14}{3} \times 4 = \frac{56}{3}$ and $\frac{56}{9} \times 3 = \frac{56}{3}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $(y+3)(x+2) = 4x(3y-5)$ or $\dfrac{3y-5}{x+2} = \dfrac{y+3}{4x}$ or $\dfrac{x+2}{3y-5} = \dfrac{4x}{y+3}$ or $\dfrac{3y-5}{y+3} = \dfrac{x+2}{4x}$ or $\dfrac{4x}{x+2} = \dfrac{y+3}{3y-5}$	M1	for correct use of intersecting chords theorem to form an equation	✓
eg $xy + 2y + 3x + 6 = 12xy - 20x$ oe	M1	for expanding the brackets or for removing the fractions and expanding the brackets, allow one error in one term. We can ft $4x(y+3) = (x+2)(3y-5)$ oe for the 2nd, 3rd and 4th method marks	✓
eg $20x + 3x + 6 = 12xy - xy - 2y$ oe or $23x + 6 = 11xy - 2y$	M1ft	dep on previous M1 for correctly collecting all the y terms on one side and non-y terms on the other side	✓
eg $23x + 6 = y(11x - 2)$ or $20x + 3x + 6 = y(12x - x - 2)$	M1ft	dep on 2nd M mark for factorising, for y, an equation in the form $ax + b = cxy + dy$ (may not be simplified). The factorisation must be correct	✓
$\dfrac{23x+6}{11x-2}$. Correct answer scores full marks (unless from obvious incorrect working)	A1	oe	✓

Full marks: 5/5

Question 24 - Exam Solution

Understanding the Question

Given

Find

The first 3 terms of an arithmetic series are $(2x+5)$, $(3y-4)$ and $(4x-2)$. The series is arithmetic, so consecutive terms differ by the same amount d. The first 9 terms have a sum of 216.

The value of x. The value of y.

Plan the Solution

- Two unknowns need two equations. The first comes from the definition of an arithmetic series: the gap from term 1 to term 2 equals the gap from term 2 to term 3.
- Terms 1 and 3 are two common differences apart, and both are written in x only. So d can be found with no y in it at all, which is what keeps the second equation short.
- Put that a and that d into the sum formula with $n=9$. Everything in it is now an x, so it solves on its own.
- Finish by putting x back into the first equation to get y.

Worked Solution [6 marks]

> Rule - Arithmetic series: consecutive terms differ by a constant d, and the sum of the first n terms is $S_n = \frac{n}{2}[2a+(n-1)d]$, where a is the first term.

Step 1: Use the common difference to link the two letters

$(3y-4)-(2x+5) = (4x-2)-(3y-4)$

$3y-2x-9 = 4x-3y+2$

$6y-6x = 11$

(Reason: (Reason: in an arithmetic series every term is the one before it plus the same d, so term 2 minus term 1 must equal term 3 minus term 2. Collecting the letters on the left gives the first of the two equations.))

Step 2: Write d in terms of x alone

$2d = (4x-2)-(2x+5)$

$2d = 2x-7$

$d = x-3.5$

(Reason: (Reason: term 3 is two steps along from term 1, so the difference between them is $2d$ and not d. Both of those terms are written in x, so the y never enters and d comes out in x on its own.))

Step 3: Build an equation from the sum of the first 9 terms

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$216 = \frac{9}{2}\left[2(2x+5) + 8(x-3.5)\right]$$

(Reason: (Reason: here $a = 2x+5$ is the first term, d is the common difference found in Step 2, and $n = 9$. The bracket now holds nothing but x, so this is one equation in one unknown.))

Step 4: Solve for x

$$216 = \frac{9}{2}\left[4x + 10 + 8x - 28\right]$$

$$216 = \frac{9}{2}(12x - 18)$$

$$48 = 12x - 18$$

$$12x = 66$$

$$x = \frac{66}{12} = \frac{11}{2}$$

(Reason: (Reason: multiply out both brackets, tidy the bracket to $12x - 18$, then undo the $\frac{9}{2}$ by multiplying both sides by $\frac{2}{9}$. The answer is left as a fraction because $\frac{11}{2}$ is exact.))

Step 5: Substitute back to find y

$$6y - 6\left(\frac{11}{2}\right) = 11$$

$$6y - 33 = 11$$

$$6y = 44$$

$$y = \frac{44}{6} = \frac{22}{3}$$

(Reason: (Reason: the equation from Step 1 is the only one still carrying a y, so it is the one to substitute into. $\frac{22}{3}$ does not terminate as a decimal, which is why the fraction is the answer to give.))

$$x = \frac{11}{2}$$

$$y = \frac{22}{3}$$

Verification

✓ **Check 1:** Put $x = \frac{11}{2}$ and $y = \frac{22}{3}$ back into the three printed terms and look at the gaps between them. The terms are 16, 18 and 20, so the common difference is 2 both times, and the series really is arithmetic.

✓ **Check 2:** Add the 9 terms without the formula, by pairing the first with the last. The ninth term is $16 + 8 \times 2 = 32$, so the average term is $\frac{16+32}{2} = 24$. $9 \times 24 = 216$, which is the total the question states.

✓ **Check 3:** An arithmetic series with an odd number of terms has its middle term equal to the mean, so the fifth term must be $\frac{216}{9} = 24$. Counting up from the first term, the fifth term is $16 + 4 \times 2 = 24$, so the two agree.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $(d =)(3y-4)-(2x+5)\ (= 3y-2x-9)$ or $(2x+5)+d = 3y-4$ oe or $(d =)(4x-2)-(3y-4)\ (= 4x-3y+2)$ or $(3y-4)+d = (4x-2)$ or $(2d =)(4x-2)-(2x+5)\ (= 2x-7)$ or $(2x+5)+2d = (4x-2)$ oe	**M1**	For a correct expression or equation using the common difference. It may be in terms of d for this mark, and an expression for $-d$ or $-2d$ is allowed.	✓
eg $216 = \frac{9}{2}[2(2x+5)+(9-1)d]$ or the same equation with the candidate's own $3y-2x-9$, $4x-3y+2$ or $x-3.5$ written in place of d or $216 = \frac{9}{2}[2(2(d+3.5)+5)+8d]$	**M1**	For a correct equation for the sum of 9 terms in x and d, or in terms of x and y, or in terms of x, or in terms of d. For $3y-2x-9$ allow $(3y-4)-(2x+5)$, or allow the candidate's own incorrect simplification of it where that simplification is shown. For $4x-3y+2$ allow $(4x-2)-(3y-4)$, or allow the candidate's own incorrect simplification of it where that simplification is shown. Similarly for the candidate's	✓

Step	Mark	Description	Got it?
		own $x - 3.5$ and the candidate's own $d + 3.5$.	
Left-hand column: eg $6x - 6y = -11$ oe and $12y - 6x = 55$ oe or $6x - 6y = -11$ oe and $18x - 12y = 11$ oe or $2x - 2d = 7$ oe and $8d + 4x = 38$ oe Right-hand column: eg $d = x - 3.5$ oe and $48 = 12x - 18$ oe or $x = d + 3.5$ oe and $48 = 12d + 24$ oe or $d = 4.75 - 0.5x$ oe and $3x - 2 = 14.50$ oe One equation must come from the differences and one equation must come from the sum of the terms.	**M2**	Left-hand column: 2 correct equations in terms of x and y in the form $px + qy = r$ oe, or 2 correct equations in terms of x and d in the form $px + qd = r$ oe. Right-hand column: 2 correct equations in terms of x and y where one is substituted into the other to get a correct equation in the form $px + q = r$ or $py + q = r$, or 2 correct equations in terms of x and d where one is substituted into the other to get a correct equation in the form $px + q = r$ or $pd + q = r$. If not M2 then M1 for one correct equation in any of the required forms from the left-hand or the right-hand column.	✓
$x = \dfrac{11}{2}$	**A1 dep on M2**	Working is required. Or equivalent, for example 5.5. The printed scheme states this as A2 for both values together, with A1 for either value on its own, so this row and the next carry one of those two marks each.	✓
$y = \dfrac{22}{3}$	**A1 dep on M2**	Working is required. Or equivalent; allow $7.3(33\ldots)$. The printed scheme states this as A2 for both values together, with A1 for either value on its own, so this row and the one above carry one of those two marks each.	✓

Step	Mark	Description	Got it?
A correct pair of values with no algebra shown scores nothing here.	**Note**	The question demands clear algebraic working, and the printed scheme marks the final row "Working required". Trial and improvement that lands on $x = 5.5$ and $y = \dfrac{22}{3}$ earns no accuracy mark without the two equations behind it.	✓

Full marks: 6/6

Question 25 - Exam Solution

Understanding the Question

Given

BAC is a sector of a circle, centre A, with angle $BAC = 40°$
BCD is a sector of a circle, centre C, with angle $BCD = 130°$
The shaded segment, between the chord BC and the arc BC, has area 28 cm^2

Find

The length of the arc BD, correct to 3 significant figures

Plan the Solution

- The shaded piece is a segment: it is what is left when triangle ABC is cut out of sector BAC.
- Call the radius of the first circle r. The sector and the triangle are both multiples of r^2, so the area 28 gives one equation in r^2.
- The second sector has centre C, so its radius is CB and not r. Get CB out of triangle ABC.
- The arc is then $\dfrac{130}{360}$ of the circumference of a circle of radius CB.

Worked Solution [6 marks]

Rule - Sector, segment and arc: a sector of radius r with angle θ has arc length $\dfrac{\theta}{360} \times 2\pi r$ and area $\dfrac{\theta}{360} \times \pi r^2$. A triangle with two sides r and the angle θ between them has area $\dfrac{1}{2} r^2 \sin\theta$. A segment is the sector minus that triangle.

Step 1: Write the shaded segment in terms of the radius

$$\frac{40}{360} \times \pi \times r^2 - \frac{1}{2} \times r^2 \times \sin 40^\circ = 28$$

(Reason: Both AB and AC are radii of the circle with centre A, so write each of them as r. Cutting triangle ABC away from sector BAC leaves exactly the shaded segment, so sector minus triangle is 28.)

Step 2: Take r^2 out as a factor and solve

$$r^2\left(\frac{40}{360} \times \pi - \frac{1}{2} \times \sin 40^\circ\right) = 28$$

$$\frac{40}{360} \times \pi - \frac{1}{2} \times \sin 40^\circ = 0.0276720\ldots$$

$$r^2 = \frac{28}{0.0276720\ldots} = 1011.85\ldots$$

$$r = \sqrt{1011.85\ldots} = 31.8096\ldots$$

(Reason: Every term carries r^2, so the bracket is a plain number the calculator works out once. Keep the unrounded value on the calculator - rounding r here would drift into the final answer.)

Step 3: Find BC, the radius of the second sector

$$BC^2 = 2 \times 1011.85\ldots - 2 \times 1011.85\ldots \times \cos 40^\circ$$

$$BC^2 = 473.4565\ldots$$

$$BC = \sqrt{473.4565\ldots} = 21.7590\ldots$$

(Reason: The second sector has centre C, so its radius is CB and not r. In triangle ABC the two sides from A are both r and the angle between them is 40°, so the cosine rule gives BC. Splitting the isosceles triangle in half gives the same value, $BC = 2r\sin 20^\circ$.)

Step 4: Take $\dfrac{130}{360}$ of the circumference

$$\text{arc } BD = \frac{130}{360} \times 2 \times \pi \times 21.759 = 49.3696\ldots$$

(Reason: The arc BD is $\dfrac{130}{360}$ of the whole circumference of the circle with centre C, and that circle has radius CB.)

Step 5: Round to 3 significant figures

arc $BD = 49.4$ cm (3 s.f.)

(Reason: The first three significant figures are 4, 9 and 3, and the digit after them is 6, so the 3 rounds up.)

49.4 cm

Verification

✓ **Check 1:** Put the unrounded $r^2 = 1011.85\ldots$ back into the segment: work out the sector and the triangle separately, then subtract. $\frac{40}{360} \times \pi \times r^2 = 353.203$, $\frac{1}{2} \times r^2 \times \sin 40^\circ = 325.203$, and $353.203 - 325.203 = 28$, the area the question gives.

✓ **Check 2:** Find BC a completely different way. Triangle ABC is isosceles, so its base angles are $\frac{180-40}{2} = 70$ degrees each and the sine rule applies. $BC = \frac{r \times \sin 40^\circ}{\sin 70^\circ} = 21.7590\ldots$, the same value the cosine rule gave.

✓ **Check 3:** Check the size a third way. A full circle of radius $21.7590\ldots$ has circumference 136.72, and 130° is a little over a third of a full turn. $\frac{130}{360} \times 136.72 = 49.37$, which agrees with the answer to the accuracy shown.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
e.g. $\frac{40}{360}\pi r^2 - \frac{1}{2}r^2 \sin 40 (= 28)$ oe or $\frac{40}{360}\pi r^2 = 28 + \frac{1}{2}r^2 \sin 40$ oe	M1	For a correct expression for the area of the shaded region. Allow $3.14\ldots$ or $\frac{22}{7}$ for π. $\sin 40 = 0.64\ldots$	✓
$(r^2 =)992$ to 1024, $(r =)31.8(096\ldots)$	A1	Allow answers in the range 31.5 to 32.0	✓
e.g. $(BC^2 =)2 \times 31.8^2 - 2 \times 31.8^2 \cos 40 (= 473.4\ldots)$	M1	For a correct first step to find BC using their clearly identified radius, e.g.	✓

Step	Mark	Description	Got it?
or $\frac{0.5BC}{31.8} = \sin 20$ or $\frac{BC}{\sin 40} = \frac{31.8}{\sin(70)}$		$r = \ldots$ or seen on the diagram. The printed scheme puts that 31.8 in quotation marks, so the candidate may use their own radius here. NB $\frac{180-40}{2} = 70$. $\sin 20 = 0.34\ldots$, $\sin 70 = 0.93\ldots$ or 0.94	
e.g. $(BC =)\sqrt{2 \times 31.8^2 - 2 \times 31.8^2 \cos 40}(= 21.7\ldots)$ or $(BC =)2 \times 31.8 \sin 20 (= 21.7\ldots)$ or $BC = \frac{31.8 \sin 40}{\sin(70)}(= 21.7\ldots)$	**M1 dep**	Dep on the previous M1. For a complete method to find BC. $\cos 40 = 0.76\ldots$ or 0.77	✓
e.g. $\frac{130}{360} \times 2 \times \pi \times 21.7$	**M1 dep**	Dep on the previous M1. For a complete method to find the length of the arc BD. The printed scheme puts that 21.7 in quotation marks, so the candidate may use their own BC.	✓
49.4	**A1**	Accept 48.9 to 49.7. A correct answer scores full marks unless it comes from obvious incorrect working.	✓

Full marks: 6/6

Question 26 - Exam Solution

Understanding the Question

Given

Find

Three mathematically similar jars R, S and T.
The volume of S is 72.8% more than the volume of R.
Height of R is h cm, height of T is $6h$ cm.
Surface area of S is A cm^2, surface area of T is kA cm^2.

The value of k. Since kA compares with A, the value of k is the area scale factor from S to T - not the one from R to T.

Plan the Solution

- Turn 72.8% more into a volume scale factor from R to S.
- Cube root it, because lengths are the cube root of volumes for similar solids.
- Read the length scale factor from R to T straight off the heights.
- Chain the two to get S to T, then square it, because areas scale by the square.

Worked Solution [4 marks]

> Rule - similar solids: if the lengths are in the ratio $1 : n$, then the areas are in the ratio $1 : n^2$ and the volumes are in the ratio $1 : n^3$.

Step 1: the volume scale factor from R to S

$$1 + \frac{72.8}{100} = 1.728$$

(Reason: (Reason: a volume that is 72.8% more is 172.8% of the original, so every volume is multiplied by 1.728.))

Step 2: cube root it for the length scale factor from R to S

$$\sqrt[3]{1.728} = 1.2$$

(Reason: (Reason: volumes scale by the cube of the length scale factor, so the length scale factor is the cube root of 1.728. The heights of R and S are therefore in the ratio $1 : 1.2$.))

Step 3: the length scale factor from S to T

$$\frac{6}{1.2} = 5$$

(Reason: (Reason: the heights put R, S and T in the ratio $1 : 1.2 : 6$, so going from S to T multiplies every length by 5.))

Step 4: square it, because k compares areas

$$5^2 = 25$$

(Reason: (Reason: areas scale by the square of the length scale factor, so the surface area of T is 25 times the surface area of S, giving $kA = 25A$.))

$$k = 25$$

Verification

✓ **Check 1 - cube the length factor back:** If the lengths of R and S really are in the ratio $1 : 1.2$, then cubing 1.2 must return the volume factor the percentage gave. $1.2^3 = 1.728$

✓ **Check 2 - do it entirely in volumes:** The heights put R and T in the ratio $1 : 6$, so their volumes are in the ratio $1 : 216$. Dividing by the volume factor from R to S leaves the volume factor from S to T, whose cube root is the length factor. $\frac{216}{1.728} = 125$ and $5^3 = 125$

✓ **Check 3 - go through the areas instead:** Areas from R to S scale by 1.2^2, and areas from R to T scale by 6^2. Dividing the second by the first jumps straight from S to T without ever naming a length factor. $\frac{36}{1.44} = 25$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
e.g. $\sqrt[3]{1.728}$ giving 1.2 oe or lengths $R : S$ as $1 : 1.2$ oe, e.g. $5 : 6$ or lengths $R : S : T$ as $1 : 1.2 : 6$ oe, e.g. $5 : 6 : 30$ or volumes $S : T$ as $1.728 : 216$, or $\frac{216}{1.728}$ giving 125	**M1**	for method to find the scale factor between the heights of R and S or for a correct ratio for the lengths $R : S$ or for a correct ratio for the lengths $R : S : T$ or for a correct ratio or scale factor for the volumes $S : T$	✓
e.g. $\frac{6}{1.2}$ giving 5, or $\sqrt[3]{\frac{216}{1.728}}$ giving 5 or lengths $S : T$ as $1 : 5$, or as $\sqrt[3]{1.728} : \sqrt[3]{216}$	**M1**	for method to find the scale factor between the heights of S and T or for a correct ratio of the heights of S and T in the form $1 : n$ or for a correct method to find the ratio for the areas $S : T$ follow through their ratio of lengths or for a correct method to find the ratio for the	✓

Step	Mark	Description	Got it?
or areas $S : T$ as $1.2^2 : 6^2$ oe, or as $6^2 : 30^2$ oe or areas $R : S : T$ as $1 : 1.2^2 : 6^2$ oe, or as $5^2 : 6^2 : 30^2$ oe		areas $R : S : T$ follow through their ratio of lengths (maybe seen as the two separate ratios of $R : S$ and $R : T$)	
e.g. 5^2 or areas $S : T$ as $1^2 : 5^2$, or as $1 : 25$ or k from $\dfrac{36}{1.44}$ oe	M1	for squaring the scale factor of the heights of S and T or for a correct ratio in the form $1 : n$ for the areas $S : T$ follow through their ratio of areas, may be implied by their final answer or for a correct calculation using the area ratio of $S : T$ to find the value of k	✓
Correct answer scores full marks (unless from obvious incorrect working) $k = 25$	A1	cao	✓

Full marks: 4/4

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