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Edexcel IGCSE 4MA1

4MA1/2F (Foundation Tier) – Wednesday 4 June 2025

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/2F (Foundation Tier), June 2025 series, sat Wednesday 4 June 2025 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Mei is finding out the areas, in square kilometres, of five countries in South America.
The table gives this information.

Country	Area (square kilometres)
Paraguay	406 752
Suriname	163 800
Uruguay	173 626
Venezuela	926 690
Chile	756 100

(a) Which of these countries has the largest area? *[1 mark]*

(b) Write the number 163 800 in words. *[1 mark]*

(c) Circle the word that describes the value of the 7 in 406 752
7 tenths 7 tens 7 hundreds 7 hundredths 7 thous
[1 mark]

(d) Write the number 173 626 correct to the nearest thousand. *[1 mark]*

(e) Work out the difference between the area of Chile and the area of Uruguay. *[1 mark]*

(a) .....

(b) .....

(d) .....

(e) square kilometres .....

[Total 5 marks]

2. Nadia asks 20 adults which garden bird they most like to see.

Here are her results.

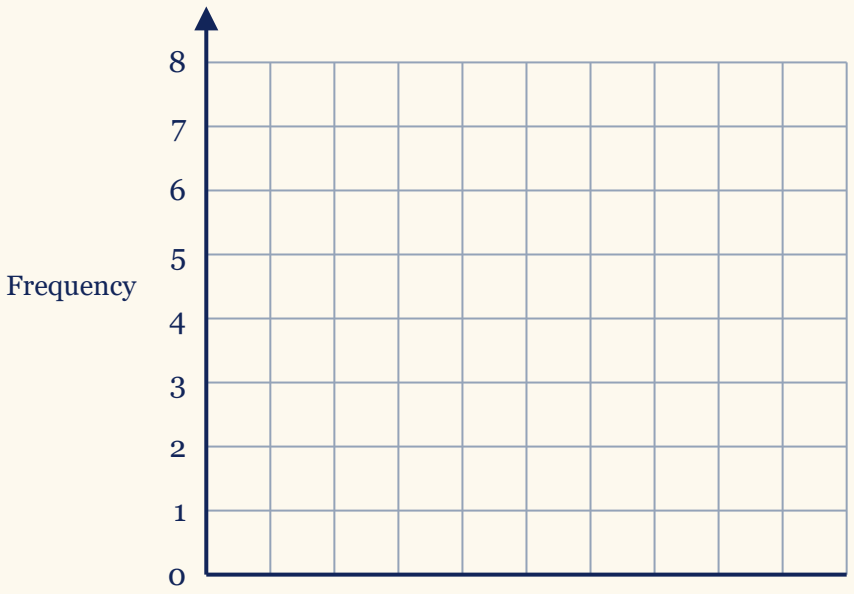
robin	blackbird	sparrow	wren	robin
blackbird	magpie	robin	sparrow	blackbird
wren	robin	blackbird	robin	sparrow
magpie	blackbird	wren	robin	sparrow

(a) Complete the frequency table to show this information.

Bird	Tally	Frequency
robin		
sparrow		
wren		
blackbird		
magpie		

[2 marks]

(b) Complete the bar chart for the information in your table. *[3 marks]*



[Total 5 marks]

3. (a) Write $d + d + d + d$ in a simpler form. *[1 mark]*

(b) Simplify $w \times w \times w \times w \times w$ *[1 mark]*

(c) Solve the equation $6x = 42$ *[1 mark]*

(d) Simplify $7r + 9y + 3r - 4y$ *[2 marks]*

(a) .....

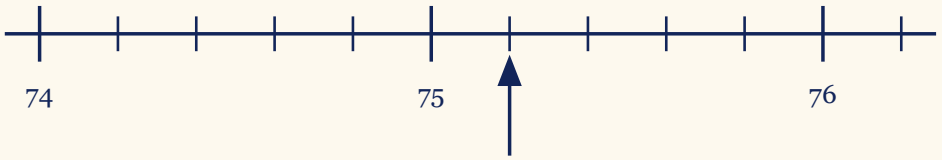
(b) .....

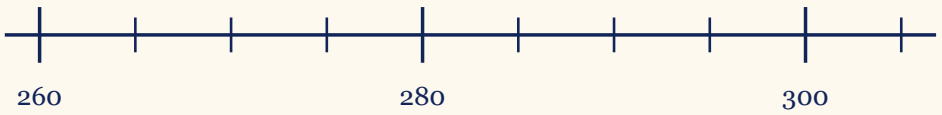
(c) x = ...

(d) .....

[Total 5 marks]

4. The diagram below shows a number line.





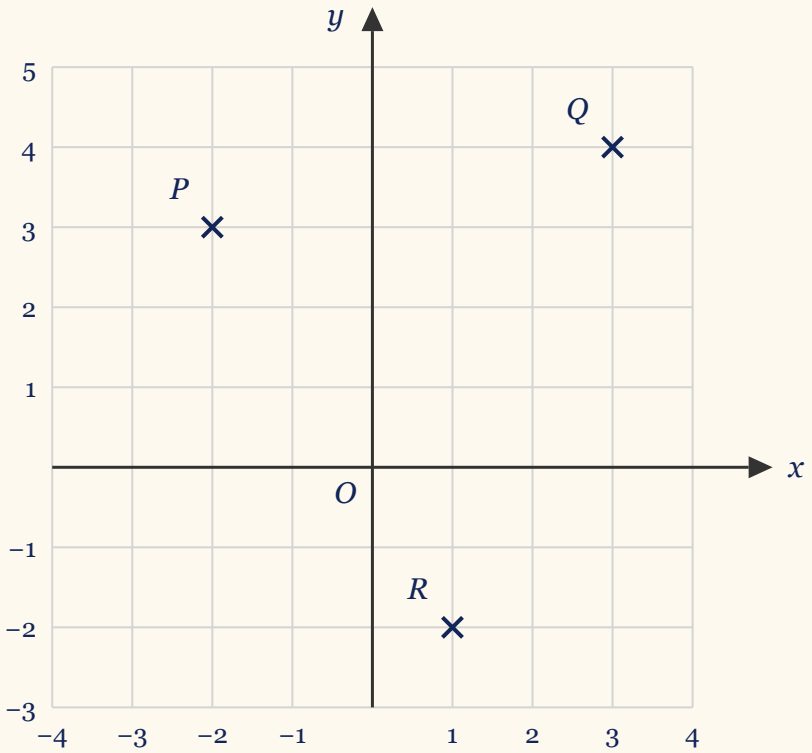
(a) Write down the number that the arrow is pointing to. *[1 mark]*

(b) On the number line below, mark the number 265 with an arrow (↑). *[1 mark]*

(a) .....

[Total 2 marks]

5. The grid below shows three points, P, Q and R.



(a) Write down the coordinates of the point P *[1 mark]*

(b) On the grid, mark with a cross (×) the point with coordinates $(3, -1)$
Label this point T *[1 mark]*

(c) Work out the coordinates of the midpoint of QR *[2 marks]*

(a) .....

(c) .....

[Total 4 marks]

6. (a) Change $\frac{19}{5}$ into a mixed number. *[1 mark]*

(b) Circle the two fractions in the list below that are equivalent.

$\frac{19}{20}$ $\frac{4}{5}$ $\frac{20}{24}$ $\frac{28}{35}$ $\frac{5}{9}$ *[1 mark]*

(c) Change 0.3 into a fraction. *[1 mark]*

(a) .....

(c) .....

[Total 3 marks]

7. 2 muffins and 5 biscuits cost \$4.50
3 muffins cost \$2.25

Work out the cost of 1 biscuit. *[4 marks]*

\$ .....

[Total 4 marks]

8. In the diagram, ABC and EBD are straight lines.
Triangle BCD is isosceles, with $BD = BC$

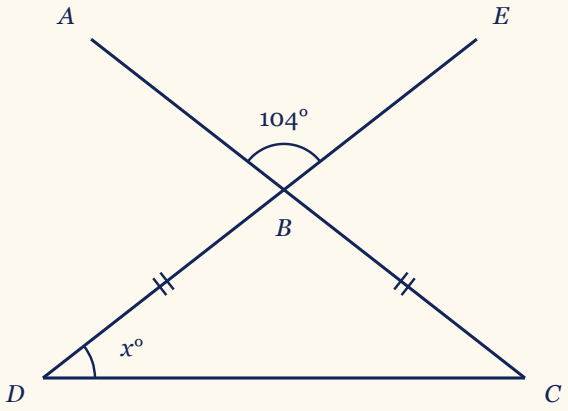


Diagram NOT accurately drawn

Angle $ABE = 104°$

Work out the value of x *[3 marks]*

x = ...

[Total 3 marks]

9. Neil has 2 litres of lemonade in a large bottle and some empty glasses.
He pours lemonade from the bottle to completely fill as many glasses as possible.
He pours 180 millilitres of lemonade into each glass.

Work out how much lemonade Neil has left in the bottle after he has completely filled as many glasses as possible.
Give your answer in millilitres. *[4 marks]*

..... millilitres

[Total 4 marks]

10. Wei is asked to work out the value of $30 - 4x$ when $x = -5$
Here is his working and his answer.

$30 - 4x = 30 - 4 \times -5$
$= 30 - 20$
$= 10$

Wei's answer is wrong.

(a) Explain the mistake Wei has made in his working. *[1 mark]*

Amara is thinking of a number.
She calls her number Y
Y is a positive, odd number and $Y \leqslant 9$

(b) Write down all the possible values of Y *[2 marks]*

(a) .....

(b) .....

[Total 3 marks]

11. The members of a summer activity camp take part in one session, either on Thursday or on Friday.
They each pick one activity from badminton or swimming or archery.
The two-way table gives some information about their choices.

	badminton	swimming	archery	Total
Thursday	29		58	110
Friday				
Total	62		105	240

(a) Complete the two-way table. *[3 marks]*

One of the members who picks archery is chosen at random.

(b) Write down the probability that this member takes part in archery on Thursday. *[1 mark]*

(b) .....

[Total 4 marks]

12. (a) Rearrange $w = 3y - d$ to make y the subject. *[2 marks]*

Marco buys b bags of marbles and p pouches of marbles.

There are 12 marbles in each bag.
There are 3 marbles in each pouch.

In total, Marco buys T marbles.

(b) Write down a formula for T in terms of b and p *[3 marks]*

(a) .....

(b) .....

[Total 5 marks]

13. Find the value of $\dfrac{9.75 - 1.4^2}{9.2 + \sqrt{6.3}}$

Write down all the figures on your calculator display. *[2 marks]*

.....

[Total 2 marks]

14. A factory makes bicycles.

The factory makes 125 bicycles each hour for 9 hours a day.
The factory makes bicycles for 5 days each week.

24% of the bicycles the factory makes are blue.

Work out the number of blue bicycles that the factory makes each week. *[3 marks]*

.....

[Total 3 marks]

15. A circle has a radius of 16 cm.

Work out the circumference of the circle.
Give your answer correct to 3 significant figures.
[2 marks]

..... cm

[Total 2 marks]

16. The diagram shows the positions of two lighthouses, A and B

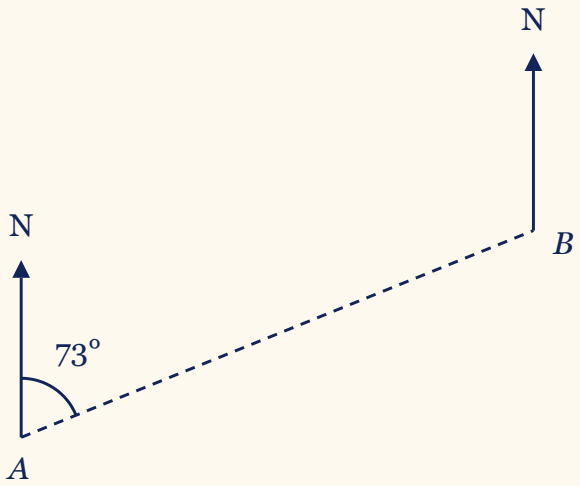


Diagram NOT accurately drawn

The bearing of B from A is 073°

Work out the bearing of A from B *[2 marks]*

..... °

[Total 2 marks]

17. Here is a list of ingredients to make a mushroom pasta bake for 4 people.

Mushroom pasta bake **Ingredients for 4 people**
100 g mushrooms
50 g grated Cheddar
300 g penne
40 g butter
2 onions

Owen is going to make the mushroom pasta bake for 12 people.

(a) Work out how much grated Cheddar he needs. *[2 marks]*

Nadia makes the mushroom pasta bake.
She uses 180 g of butter.

(b) Work out how many people Nadia makes the mushroom pasta bake for.
[2 marks]

(a) .....

(b) .....

[Total 4 marks]

18. Callum has six tiles.
He writes a number on each tile so that

the range of the numbers is 10
the median of the numbers is 7.5
the mode of the numbers is 6

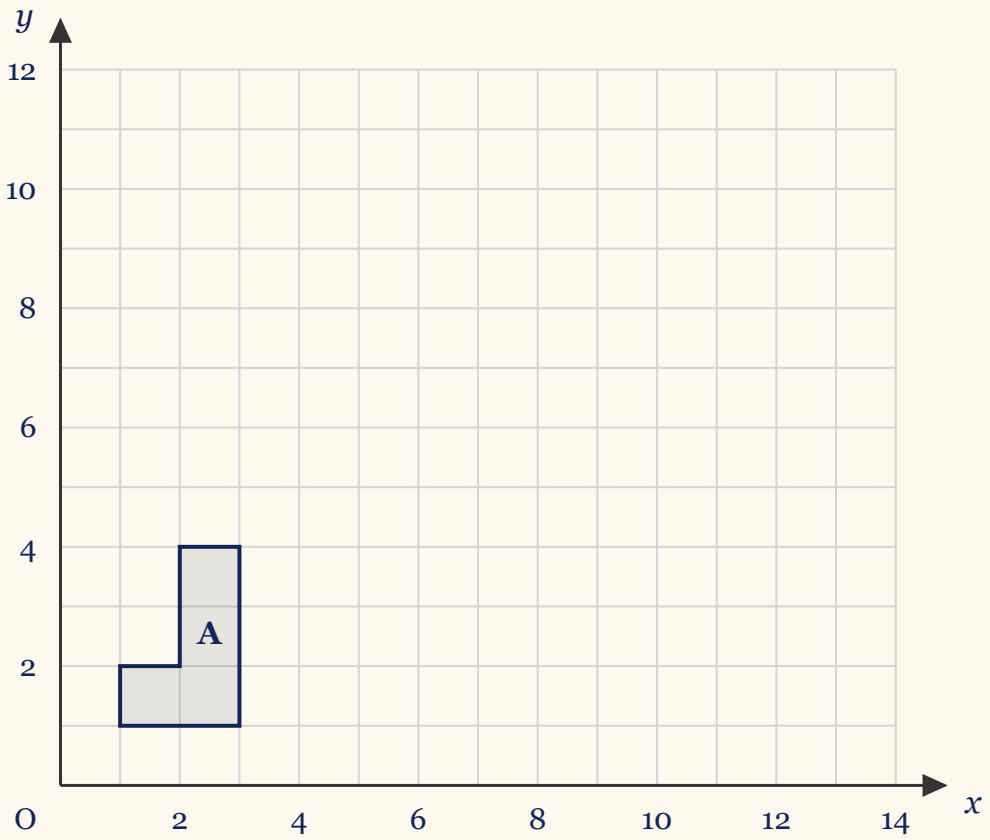
Callum arranges the tiles so that the numbers are in order of size.

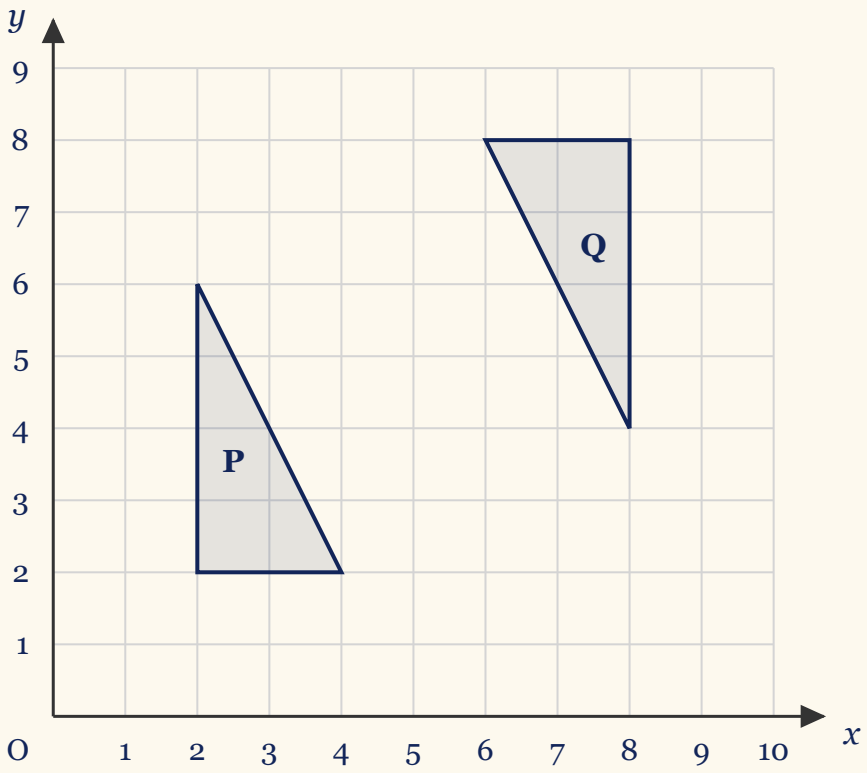
Three of the numbers are hidden.

Complete the tiles above to show the three numbers that are hidden. *[3 marks]*

[Total 3 marks]

19. (a) On the grid, draw the enlargement of shape A with scale factor 3 and centre $(0, 1)$. *[2 marks]*





(b) Give a full description of the single transformation that maps triangle P onto triangle Q. *[3 marks]*

(b) .....

[Total 5 marks]

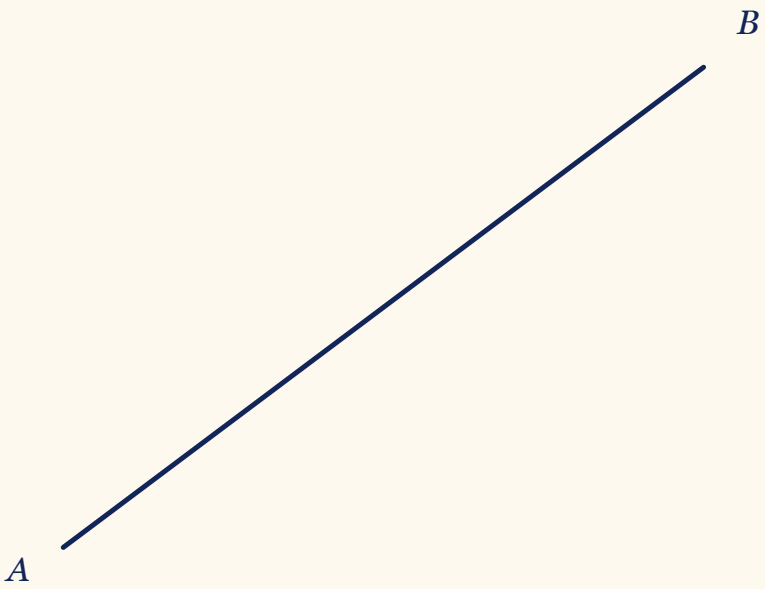
20. Show that $7\frac{1}{3} - 3\frac{4}{7} = 3\frac{16}{21}$

You must show all your working. *[3 marks]*

[Total 3 marks]

21. Using a ruler and a pair of compasses only, construct the perpendicular bisector of the line AB

Show all your construction lines. *[2 marks]*



[Total 2 marks]

22. $ABCD$ and $EFGH$ are similar quadrilaterals.
The diagram shows both of them.

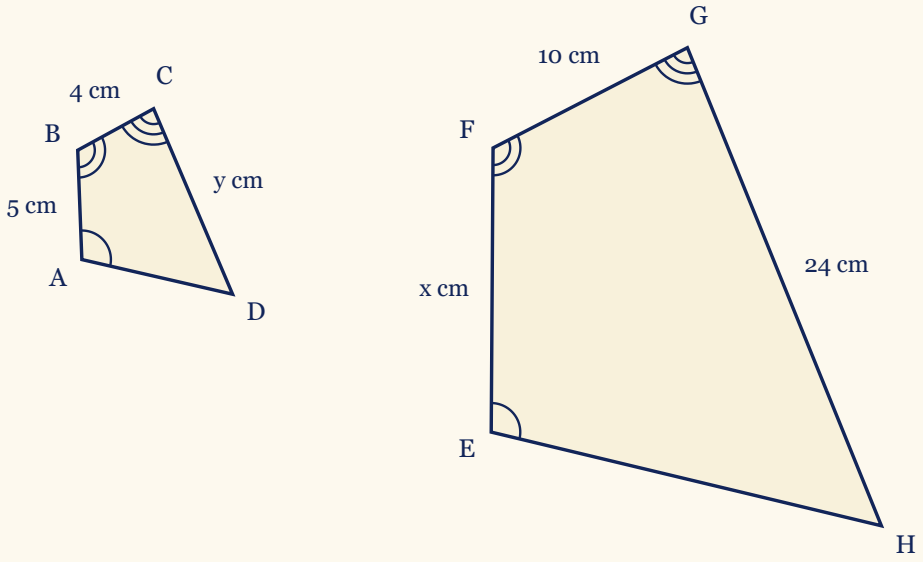


Diagram NOT accurately drawn

$AB = 5$ cm, $BC = 4$ cm, $CD = y$ cm
$EF = x$ cm, $FG = 10$ cm, $GH = 24$ cm

(a) Find the value of x. *[2 marks]*

(b) Find the value of y. *[2 marks]*

x = ...

y = ...

[Total 4 marks]

23. Nico, Ella and Rosa run a craft stall together.
One Saturday the stall takes £240, which they share in the ratios $3 : 4 : 5$

Ella and Rosa each give £10 of their share to Nico.

Work out the ratio of the amounts of money that Nico, Ella and Rosa now have.
Give your answer in its simplest form. *[4 marks]*

.....

[Total 4 marks]

24. Lukas buys a vintage wristwatch for 4000 Swiss francs.
The value of the wristwatch increases by 7% each year.

Work out the value of the wristwatch at the end of 3 years.
Give your answer correct to the nearest Swiss franc. *[3 marks]*

..... Swiss francs

[Total 3 marks]

25. Here is a pair of simultaneous equations.

$3x + 5y = 8$
$4x + y = -3.5$

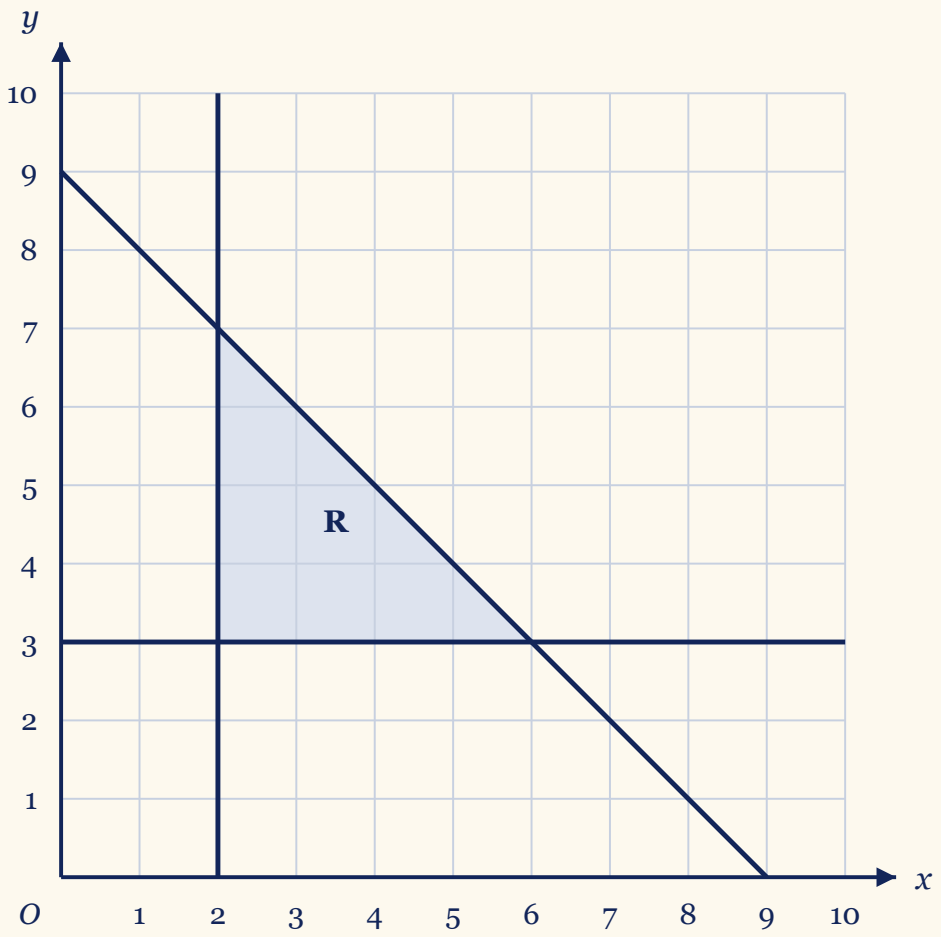
Work out the value of x and the value of y.
You must show clear algebraic working. *[3 marks]*

x = ...

y = ...

[Total 3 marks]

26. (a) Solve the inequality $7 - 3t < 2t + 15$ *[2 marks]*



The region R, shaded on the grid below, is bounded by three straight lines.

(b) Write down three inequalities that together describe the region R. *[3 marks]*

(a) .....

(b) .....

(b) .....

(b) .....

[Total 5 marks]

27. In the diagram, ABD and ABC are right-angled triangles.

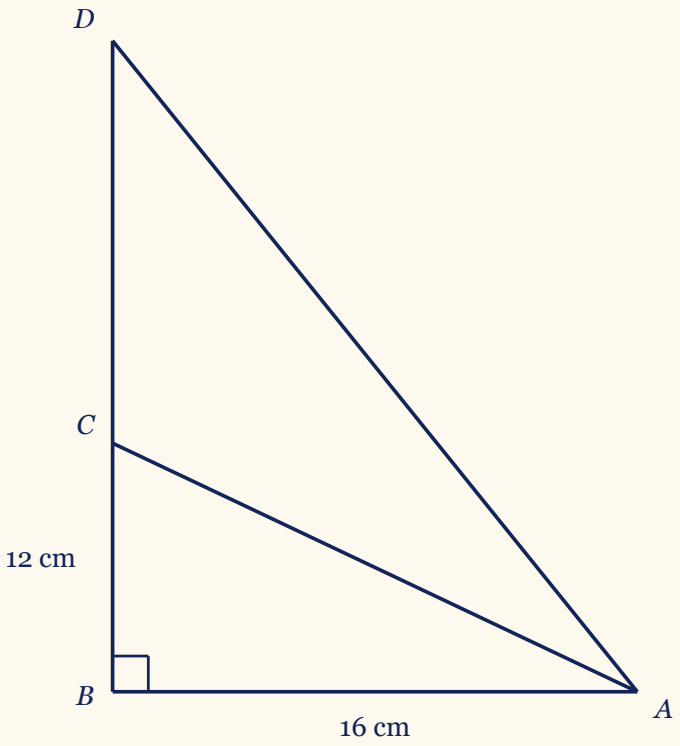


Diagram NOT accurately drawn

$AB = 16$ cm, $BC = 12$ cm, $AD = 1.5 \times AC$

Work out the length of CD.
Give your answer correct to 3 significant figures. *[5 marks]*

..... cm

[Total 5 marks]

28. Bilal has a jar of glass beads.

17 of the beads are green
28 of the beads are yellow
the rest of the beads are purple

Bilal is going to take at random a bead from the jar.

The probability that Bilal will take a purple bead is $\frac{4}{9}$

Work out the number of purple beads that are in the jar. *[3 marks]*

.....

[Total 3 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Paraguay, 406 752 square kilometres
Suriname, 163 800 square kilometres
Uruguay, 173 626 square kilometres
Venezuela, 926 690 square kilometres
Chile, 756 100 square kilometres
Every area is a whole number with six digits, so the same place-value columns line up across all five.

Find

(a) the country with the largest area (b) 163 800 written out in words (c) what the 7 in 406 752 is worth (d) 173 626 to the nearest thousand (e) the difference between the areas of Chile and Uruguay

Plan the Solution

- Compare the areas from the left. The first column where they differ settles which is largest.
- Split 163 800 at the thousands, say each piece, then join them.
- Count the columns from the right in 406 752 until you reach the 7.
- For the nearest thousand, find the two multiples of 1 000 the number sits between and see which one it is closer to.
- Difference means subtract, so take the smaller area away from the larger one.

Worked Solution [5 marks]

> Rule - Place value: reading a whole number from the right, the columns are units, tens, hundreds, thousands, ten thousands and hundred thousands. Line two numbers up by those columns and you can compare them, round them and subtract them.

Step 1: (a) compare the five areas

$926\,690 > 756\,100 > 406\,752 > 173\,626 > 163\,800$

(Reason: All five areas have six digits, so the hundred thousands column decides it straight away: 9 beats 7, 4, 1 and 1. Nothing below that column can change the order.)

Step 2: (b) split the number at the thousands

$$163\,800 = 163\,000 + 800$$

(Reason: The digits above the thousands column read 163, so that part is one hundred and sixty three thousand. What is left is 800, which is eight hundred.)

Step 3: (c) write the number in expanded form

$$406\,752 = 400\,000 + 6\,000 + 700 + 50 + 2$$

$$7 \times 100 = 700$$

(Reason: Counting from the right, the 2 is units, the 5 is tens and the 7 is the third column, the hundreds. So the 7 stands for seven hundreds.)

Step 4: (d) find the nearest multiple of one thousand

$$173\,000 < 173\,626 < 174\,000$$

$$173\,626 - 173\,000 = 626$$

$$174\,000 - 173\,626 = 374$$

(Reason: The number sits between those two multiples of 1 000. It is 374 from the one above and 626 from the one below, so the one above is nearer.)

Step 5: (e) subtract the smaller area from the larger

$$756\,100 - 173\,626 = 582\,474$$

(Reason: Chile has the larger area, so Chile goes on top. Take the area of Uruguay away from it, lining the two numbers up by their place-value columns.)

(a) Venezuela

(b) one hundred and sixty three thousand and eight hundred

(c) 7 hundreds

(d) 174 000

(e) 582 474 square kilometres

Verification

- ✓ **Check 1:** Read only the hundred thousands column of the five areas: 4, 1, 1, 9, 7. The 9 belongs to Venezuela, and no other area even reaches nine hundred thousand square kilometres.
- ✓ **Check 2:** Count the columns of 406 752 from the right instead of from the left: 2 units, 5 tens, then the 7. The 7 lands in the third column, which is the hundreds column, worth 700.

✓ **Check 3:** Test it against the halfway point instead of the two gaps. Halfway between 173 000 and 174 000 is 173 500. 173 626 is above the halfway point, so it rounds up to 174 000.

✓ **Check 4:** Reverse the subtraction. Add the difference back on to the area of Uruguay as the table gives it, and the area of Chile should come back. $173\,626 + 582\,474 = 756\,100$, which is exactly Chile's area.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Venezuela	**B1**	Allow incorrect spelling if the meaning is clear. Allow V. Accept the numerical answer 926 690, and accept it written with a comma separating the thousands from the hundreds.	✓
(b) one hundred (and) sixty three thousand (and) eight hundred	**B1**	Allow incorrect spelling if the meaning is clear, eg hunder or hudred for hundred, sixety or sisty for sixty etc.	✓
(c) (7) hundreds circled	**B1**	(7) hundreds clearly indicated with no other words indicated.	✓
(d) 174 000	**B1**	Allow 'one hundred and seventy-four thousand' or 174 thousand.	✓
(e) 582 474	**B1**	The printed scheme carries no notes beside this row.	✓

Full marks: 5/5

Question 2 - Exam Solution

Understanding the Question

Given

A list of 20 results, one for each adult Nadia asked.
Five birds appear in the list: robin, sparrow, wren, blackbird and magpie.

Find

(a) The frequency of each bird, written into the table. (b) A bar chart of those frequencies, with every bar labelled.

An empty frequency table, and a grid whose vertical axis is numbered from 0 to 8 in ones.

Plan the Solution

- Go along the list in the order it is written and put one tally mark against a bird each time it appears.
- Total each row of tally marks. That total is the frequency.
- Add the five frequencies. They must come to 20, because every adult is counted exactly once.
- Draw a bar for each bird up to its frequency, keeping the widths equal and the gaps equal, and write the bird's name under its own bar.

Worked Solution [5 marks]

> Rule - Frequency: a frequency is a count of how many times something appears, so the frequencies must add up to the number of results. On a bar chart the height of a bar is that frequency.

Step 1: Tally the results, one mark at a time

Bird	Tally
robin	𝍸 |
sparrow	𝍸
wren	𝍸
blackbird	𝍸 |
magpie	𝍸

(Reason: Working through the list in the order it is written means no result is missed and none is counted twice. Grouping the marks in fives makes them quick to total.)

Step 2: Total each row of tally marks

Bird	Tally	Frequency
robin	𝍸 |	6
sparrow	𝍸	4
wren	𝍸	3
blackbird	𝍸 |	5
magpie	𝍸	2

(Reason: The frequency is simply how many tally marks that bird has, so each row is counted and the total written in the last column.)

Step 3: Check the frequencies against the number of adults asked

$6 + 4 + 3 + 5 + 2 = 20$

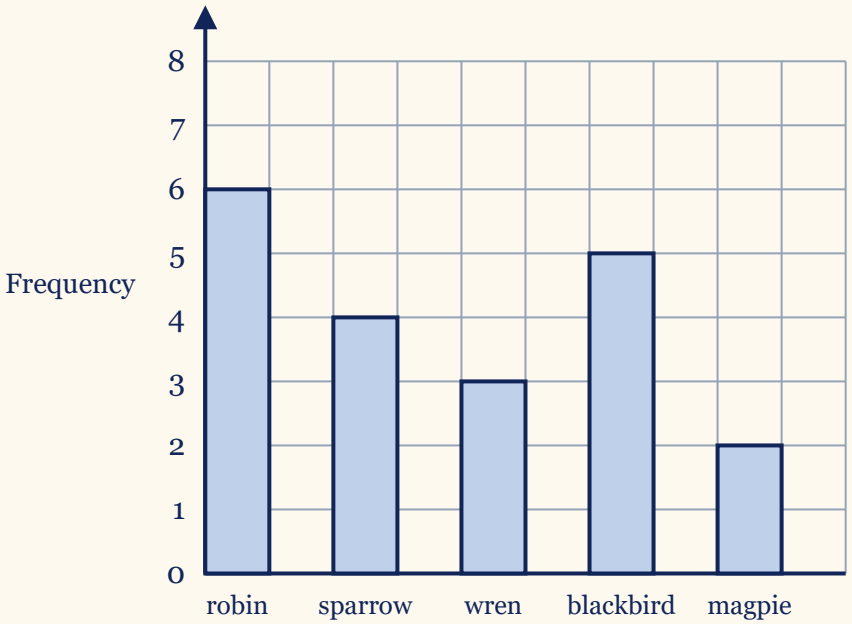
(Reason: Every one of the 20 answers is counted exactly once, so the five frequencies have to add up to 20. They do, so nothing has been missed and nothing counted twice.)

Step 4: Read the bar heights off the table

6 4 3 5 2

(Reason: The height of each bar is that bird's frequency, taken in the same order as the table. The scale is numbered in ones, so every bar top sits on a grid line.)

Step 5: Draw the bars and label them



(Reason: Bars of equal width with equal gaps, each drawn up to its frequency, and the bird's name written under its own bar. The labels earn a mark of their own, so a chart with five correct bars and nothing written underneath does not score full marks.)

(a) robin 6, sparrow 4, wren 3, blackbird 5, magpie 2

(b) bars of height 6, 4, 3, 5 and 2, each labelled with its bird

Verification

✓ **Check 1 - count down the columns instead of along the rows:** Counting the 20 results column by column uses a completely different order, so a result missed the first time is not missed in the same place again. robin 6, sparrow 4, wren 3, blackbird 5, magpie 2 - the same five frequencies

✓ **Check 2 - the frequencies add to the number of adults asked:** Add the five frequencies and compare with the number of adults. $6 + 4 + 3 + 5 + 2 = 20$, and Nadia asked 20 adults

✓ **Check 3 - read the finished bar chart back:** Read each bar height off the grid and compare it with the table it came from. The bars measure 6, 4, 3, 5 and 2, and the tallest bar is the robin, which is the bird that appears most often in the list

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Frequencies 6, 4, 3, 5, 2	**B2**	for all correct frequencies (ignore tally column)	✓
(a) Part marks	**B1**	for 3 or 4 correct frequencies, or for 4 or 5 tallies correct but not totalled, or for frequencies written as probabilities with 3 or 4 or 5 correct numerators	✓
(b) A correctly labelled bar chart	**B2**	ft for all 5 correct bars, follow through their figures (do not follow through 0)	✓
(b) Part marks	**B1ft**	for 3 or 4 correct bars, follow through their figures (do not follow through 0), or for 5 correct indications of heights, eg cross, dot, etc (do not follow through 0)	✓
(b) What is condoned	**Note**	Condone gaps of different widths or no gaps between bars, and also bars of different widths	✓
(b) Labels	**B1**	for all 5 bars labelled - use of initials or a key is acceptable	✓

Full marks: 5/5

Question 3 - Exam Solution

Understanding the Question

Given

(a) $d + d + d + d$, the same letter added four times

Find

(a) the sum written as one term (b) the product written as one power of w (c) the

(b) $w \times w \times w \times w \times w$, the same letter multiplied five times
(c) $6x = 42$, an equation with one operation in it
(d) $7r + 9y + 3r - 4y$, four terms sharing two different letters

value of x (d) the expression with its like terms collected

Plan the Solution

- Parts (a) and (b) look alike and are not. One is repeated addition, the other is repeated multiplication, so settle which before writing anything down.
- Repeated addition of a letter gives a coefficient, so count the terms in (a): four lots of d.
- Repeated multiplication of a letter gives an index, so count the factors in (b): five copies of w.
- Part (c) has one operation on x, so undo it with the opposite operation.
- Part (d) mixes two letters. Collect the r terms and the y terms separately, and carry each sign with the term in front of which it sits.

Worked Solution [5 marks]

> Like terms only: repeated addition gives a coefficient, $d + d + d + d = 4d$; repeated multiplication gives an index, $w \times w \times w \times w \times w = w^5$; and terms in different letters, such as $10r$ and $5y$, can never be added into one term.

(a) Count the terms

$$d + d + d + d = 4 \times d = 4d$$

(Reason: Adding the same letter four times is the same as multiplying it by 4. The number in front records how many there are, so the answer is one term, not four. It is not d^4: nothing here is being multiplied.)

(b) Count the factors

$$w \times w \times w \times w \times w = w^5$$

(Reason: There are 5 copies of w multiplied together, and the index is exactly that count. It is not $5w$, which would mean five copies added.)

(c) Undo the multiplication

$$6x = 42$$

$$x = \frac{42}{6}$$

$$x = 7$$

(Reason: x has been multiplied by 6, so dividing both sides by 6 leaves x on its own. Whatever is done to one side is done to the other.)

(d) Collect the r terms

$$7r + 3r = 10r$$

(Reason: $7r$ and $3r$ are like terms, because they carry the same letter, so their coefficients add: $7 + 3 = 10$.)

(d) Collect the y terms

$$9y - 4y = 5y$$

(Reason: The same letter again, so the coefficients combine. The sign in front of the $4y$ belongs to that term and travels with it, so read the operation before deciding what to do with the 9 and the 4.)

(d) Write the two collected terms together

$$7r + 9y + 3r - 4y = 10r + 5y$$

(Reason: $10r$ and $5y$ are unlike terms, so there is nothing further to collect. Two terms is the finished answer, and writing $15ry$ would be inventing a multiplication the expression never had.)

(a) $4d$

(b) w^5

(c) $x = 7$

(d) $10r + 5y$

Verification

✓ **Check 1:** Put $d = 3$ into both forms in (a), then $w = 2$ into both forms in (b). A correct simplification agrees with the original at every value. $3 + 3 + 3 + 3 = 12$ and $4 \times 3 = 12$; $2 \times 2 \times 2 \times 2 \times 2 = 32$ and $2^5 = 32$

✓ **Check 2:** Put $x = 7$ back into the equation given in (c) and read the left-hand side. $6 \times 7 = 42$, which is the right-hand side

✓ **Check 3:** Put $r = 2$ and $y = 5$ into the expression in (d) and into the collected answer. $14 + 45 + 6 - 20 = 45$ and $10 \times 2 + 5 \times 5 = 45$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Simplify $d + d + d + d$	**B1**	$4d$. Allow $d4$.	✓

Step	Mark	Description	Got it?
(b) Simplify $w \times w \times w \times w \times w$	**B1**	w^5	✓
(c) Solve $6x = 42$	**B1**	7	✓
(d) Simplify $7r + 9y + 3r - 4y$	**B2**	For $10r + 5y$.	✓
(d) Partial credit	**B1**	For $10r$ or for $(+)5y$.	✓

Full marks: 5/5

Question 4 - Exam Solution

Understanding the Question

Given

A number line with 74, 75 and 76 labelled, and 5 equal small divisions between one labelled number and the next. An arrow points at the first mark to the right of 75.
A second number line with 260, 280 and 300 labelled, and 4 equal small divisions between one labelled number and the next. Nothing is marked on it yet.

Find

(a) The number the arrow is pointing to.
(b) Which mark to draw the arrow on, so that it shows 265.

Plan the Solution

- Work out what one small division is worth on each line. Both parts turn on that single value, and the two lines do not have the same one.
- Share the gap between two labelled numbers equally between the SPACES on the line. The marks and the spaces are different things, and there is always one fewer mark than there are spaces.
- For part (a), start at the labelled number just below the arrow and count on. For part (b), work out how many small divisions the target is above the labelled number just below it, then draw the arrow on that mark.

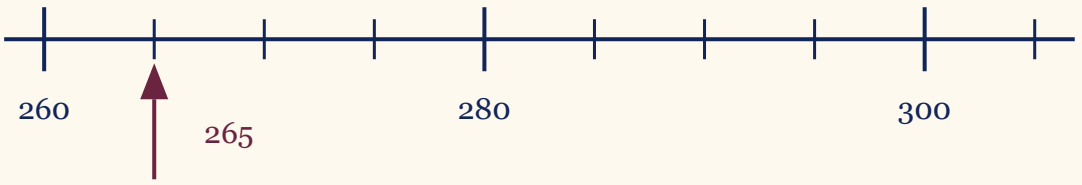
Worked Solution [2 marks]

> Rule - one small division: share the gap between two labelled numbers equally between the spaces on the line. Count the spaces between the marks, never the marks themselves.

Part (a): find what one small division is worth

$$75 - 74 = 1$$

$$\frac{1}{5} = 0.2$$



(Reason: The gap from 74 to 75 is 1, and it is split into 5 equal spaces, so one small division is one fifth of 1. There are only 4 marks between the two labels, which is exactly why counting marks instead of spaces goes wrong.)

Part (a): count on from 75

$$75 + 0.2 = 75.2$$

(Reason: The arrow stands on the first mark to the right of 75, which is one small division past it, so add one small division to 75.)

Part (b): find what one small division is worth on the second line

$$280 - 260 = 20$$

$$\frac{20}{4} = 5$$

(Reason: This line is scaled differently. The gap from 260 to 280 is 20, split into 4 equal spaces, so here each small division is worth 5.)

Part (b): count how many divisions past 260 the arrow goes

$$265 - 260 = 5$$

$$\frac{5}{5} = 1$$

(Reason: 265 is 5 more than 260, and one small division is worth 5, so the arrow goes on the first mark to the right of 260. Draw it pointing up at that mark.)

(a) 75.2

(b) The arrow on the first mark to the right of 260, the mark that stands for 265

Verification

✓ **Check 1 - part (a), read the line from the other end:** From 74 to the arrow is 6 small divisions, and from 74 to 76 is 10 small divisions covering 2. This never uses the value of one division. $74 + \frac{6}{10} \times 2 = 75.2$

✓ **Check 2 - part (a), as a mixed number:** One small division is a fifth, so the first mark after 75 is 75 and a fifth. The mark scheme accepts that form as well as the decimal, so the two must agree. $75 + \frac{1}{5} = 75.2$

✓ **Check 3 - part (b), count back from the far end:** 300 is 8 small divisions above 260. Counting back 7 of them must land on the same mark the arrow was drawn on. $300 - 7 \times 5 = 265$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	75.2. Allow $75\frac{1}{5}$ or equivalent.	✓
(b)	**B1**	A clear indication of marking the first notch after 260. Correct answer only.	✓

Full marks: 2/2

Question 5 - Exam Solution

Understanding the Question

Given

Three points marked with crosses on a square grid: P, Q and R.

Find

(a) The coordinates of P. (b) Where a cross for $(3, -1)$ goes, labelled T. (c) The coordinates of the midpoint of QR.

The grid is numbered from -4 to 4 across and from -3 to 5 up.

Plan the Solution

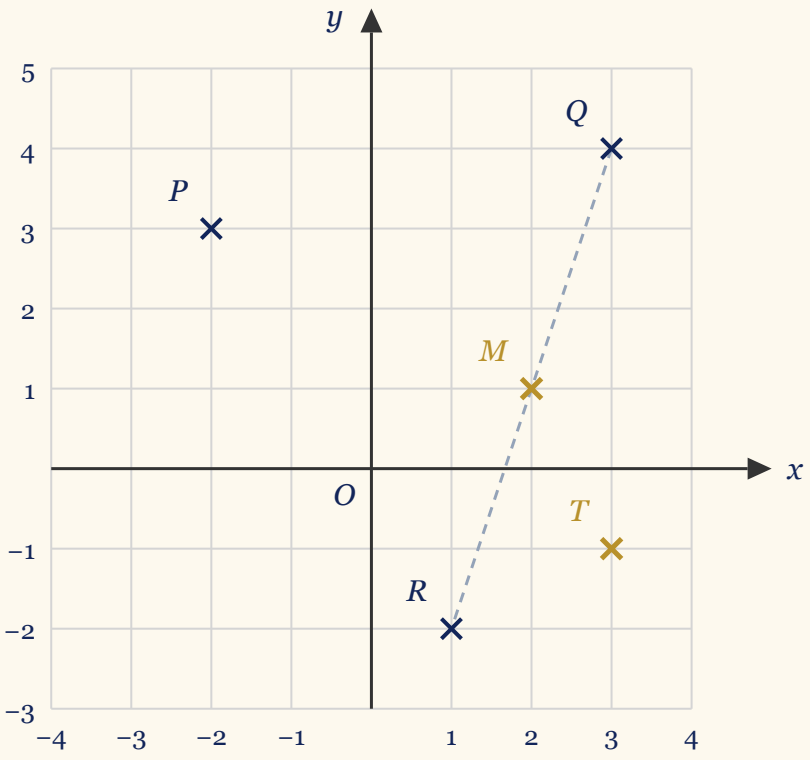
- Read every coordinate as a pair, across first and then up. A point left of the vertical axis has a negative first number, and a point below the horizontal axis has a negative second number.
- For (b), start at the origin, count 3 squares right and 1 square down, and put the cross where those two gridlines meet.
- For (c), a midpoint sits halfway between the two ends, so average the two across-numbers and then average the two up-numbers.

Worked Solution [4 marks]

Rule - Midpoint: the midpoint of the segment joining $(x_1,\ y_1)$ and $(x_2,\ y_2)$ is $\left(\dfrac{x_1 + x_2}{2},\ \dfrac{y_1 + y_2}{2}\right)$.

Step 1: read P off the grid

$P = (-2,\ 3)$



(Reason: From the origin, P is 2 squares to the left, so the across-number is -2, and 3 squares up, so the up-number is 3. A coordinate pair is always written across first, then up.)

Step 2: put the cross for T at $(3, -1)$

$T = (3,\ -1)$

(Reason: Start at O. Count 3 squares right, then 1 square down, and mark that grid point with a cross. Write T beside it so the examiner can see which point is meant.)

Step 3: average the across-numbers of Q and R

$Q = (3,\ 4) \qquad R = (1,\ -2)$

$$\frac{3 + 1}{2} = \frac{4}{2} = 2$$

(Reason: The midpoint is halfway along QR, so its across-number is halfway between 3 and 1.)

Step 4: average the up-numbers of Q and R

$$\frac{4 + (-2)}{2} = \frac{2}{2} = 1$$

(Reason: Adding -2 is the same as taking away 2, so work out the top of the fraction first and then halve it.)

Step 5: write the midpoint as a coordinate pair

midpoint of $QR = (2,\ 1)$

(Reason: Put the two averages together, across first. On the grid, $(2, 1)$ sits exactly halfway along the line from Q to R.)

(a) $(-2,\ 3)$

(b) T marked at $(3,\ -1)$ on the grid

(c) $(2,\ 1)$

Verification

- ✓ **Check 1:** From $Q(3, 4)$ to $(2, 1)$ is 1 square left and 3 squares down. Take that same step once more from $(2, 1)$. It lands on $(1, -2)$, which is R, so $(2, 1)$ is exactly halfway.
- ✓ **Check 2:** A midpoint is the same distance from both ends. From $(2, 1)$ to Q is 1 across and 3 up; from $(2, 1)$ to R is 1 across and 3 down. $1^2 + 3^2 = 10$ both times, so the two distances are equal.
- ✓ **Check 3:** P is left of the vertical axis and above the horizontal axis, so its pair must read negative then positive. T is right of the vertical axis and below the horizontal axis, so its pair must read positive then negative. $(-2, 3)$ and $(3, -1)$ carry exactly those signs.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	$(-2, 3)$	✓
(b)	**B1**	T clearly marked. A point clearly marked at $(3, -1)$ need not be labelled if meaning is clear.	✓
(c)	**B2**	$(2, 1)$	✓
(c)	**B1**	for $(2, X)$ or $(Y, 1)$ or the midpoint unambiguously marked in the correct place on the diagram	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

The improper fraction $\frac{19}{5}$.

A list of five fractions: $\frac{19}{20}$, $\frac{4}{5}$, $\frac{20}{24}$, $\frac{28}{35}$ and $\frac{5}{9}$.

The decimal 0.3.

Find

(a) $\frac{19}{5}$ written as a mixed number. (b) The two fractions in the list that are equivalent. (c) 0.3 written as a fraction.

Plan the Solution

- For (a), find how many whole ones are hidden inside $\frac{19}{5}$. Every 5 fifths make one whole, so divide the top by the bottom and keep the remainder in fifths.
- For (b), put every fraction in the list into its simplest form. Two fractions are equivalent exactly when they simplify to the same thing, so once they are all cancelled down the pair stands out.
- For (c), read the decimal off its place value. The first place after the point is tenths, so the denominator is 10.

Worked Solution [3 marks]

> Rule - Equivalent fractions: multiplying or dividing the top and the bottom by the same number leaves the value unchanged, so $\frac{a}{b} = \frac{a \times k}{b \times k}$. A mixed number splits an improper fraction into the whole ones it contains plus the part left over.

Step 1: count the whole ones inside $\frac{19}{5}$

$19 = 3 \times 5 + 4$

(Reason: Every 5 fifths make one whole one. $3 \times 5 = 15$, so three whole ones use up 15 of the fifths and 4 fifths are left over.)

Step 2: keep the leftover over the same denominator

$\frac{19}{5} = \frac{15}{5} + \frac{4}{5}$

$\frac{19}{5} = 3\frac{4}{5}$

(Reason: The 4 that is left over is still being counted in fifths, so it keeps the denominator 5. A mixed number is written with the whole number and the fraction side by side, with no plus sign between them.)

Step 3: put each fraction in the list into its simplest form

$\frac{20}{24} = \frac{5}{6}$

$\frac{28}{35} = \frac{4}{5}$

(Reason: Cancel wherever the top and the bottom share a factor: 20 and 24 share a factor of 4, and 28 and 35 share a factor of 7. The other three will not cancel at all - 19 is prime, and $\frac{4}{5}$ and $\frac{5}{9}$ are already as simple as they go.)

Step 4: pick out the two that match

$\frac{4}{5} = \frac{4 \times 7}{5 \times 7} = \frac{28}{35}$

(Reason: Two fractions in the list simplify to the same thing, and those are the two to circle: $\frac{4}{5}$ and $\frac{28}{35}$. Multiplying the top and the bottom of $\frac{4}{5}$ by 7 is the same statement read the other way round.)

Step 5: read 0.3 off its place value

$0.3 = \frac{3}{10}$

(Reason: The first place after the decimal point is the tenths place, so 0.3 is 3 tenths, which is 3 over 10. Nothing divides into both 3 and 10, so it will not cancel any further.)

(a) $3\frac{4}{5}$

(b) $\frac{4}{5}$ **and** $\frac{28}{35}$

(c) $\frac{3}{10}$

Verification

✓ **Check 1:** Turn the mixed number in (a) back into an improper fraction. Three whole ones are 15 fifths, and the 4 fifths are added on. $3 \times 5 + 4 = 19$, which is the numerator the question started with.

✓ **Check 2:** Test the pair in (b) without cancelling anything. Two fractions are equal exactly when the cross products match, so multiply the top of each by the bottom of the other. $4 \times 35 = 140$ and $5 \times 28 = 140$, so the two are equal. No other pair in the list gives a match.

✓ **Check 3:** Turn the answer to (c) back into a decimal by writing it in hundredths. $\frac{3}{10} = \frac{30}{100} = 0.3$, which is the decimal the question gave.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	$3\frac{4}{5}$	✓
(b)	**B1**	$\frac{4}{5}$, $\frac{28}{35}$. Both fractions and no other fraction circled or clearly indicated in the list	✓
(c)	**B1**	$\frac{3}{10}$ oe fraction	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

One order of 2 muffins and 5 biscuits costs $4.50 altogether.
A second order of 3 muffins costs $2.25 with no biscuits in it at all.
Every muffin costs the same as every other muffin, and every biscuit costs the same as every other biscuit.

Find

The cost of one biscuit.

Plan the Solution

- The second order is the way in, because it holds only one kind of item. An order of muffins and nothing else prices a muffin.
- Once one muffin has a price, the 2 muffins hiding inside the first order can be costed and taken off the $4.50, and what is left is the price of the 5 biscuits between them.
- Share that remainder between the 5 biscuits to reach one biscuit.
- Work in dollars all the way through, and keep the two orders apart until the subtraction.

Worked Solution [4 marks]

> Rule - one kind of item at a time: an order holding a single kind of item gives the price of that item by division, that price then lifts the same item out of a mixed order by subtraction, and the remainder is shared out by division again.

Step 1: price one muffin from the muffins-only order

$$\frac{2.25}{3} = 0.75$$

(Reason: (Reason: the second order is 3 muffins and nothing else, so its $2.25 is shared equally between 3 identical muffins. Dividing by 3 gives $0.75 for one of them.))

Step 2: cost the muffins that sit inside the first order

$$2 \times 0.75 = 1.50$$

(Reason: (Reason: the first order contains 2 of those same muffins, and a muffin costs the same wherever it is bought, so the muffin part of that order comes to $1.50 in all.))

Step 3: take the muffins off the first order

$4.50 - 1.50 = 3.00$

(Reason: (Reason: the first order is the 2 muffins and the 5 biscuits together. Remove everything the muffins account for and what remains is exactly what the 5 biscuits cost between them.))

Step 4: share that remainder between the five biscuits

$$\frac{3.00}{5} = 0.60$$

(Reason: (Reason: the 5 biscuits are identical, so the money left over splits equally between them. One biscuit costs $0.60, which is 60 cents.))

$0.60

Verification

- ✓ **Check 1:** Rebuild both orders from the two prices. A muffin at $0.75 and a biscuit at $0.60 have to reproduce the two totals the question gives, not just one of them. $3 \times 0.75 = 2.25$ for the muffins-only order, and $2 \times 0.75 + 5 \times 0.60 = 4.50$ for the mixed one. Both are the printed totals.
- ✓ **Check 2:** Reach the biscuit price without ever pricing a muffin. Take 3 copies of the first order, which is 6 muffins and 15 biscuits, and 2 copies of the second, which is 6 muffins on their own. Subtracting removes the muffins outright. $3 \times 4.50 - 2 \times 2.25 = 9$ for 15 biscuits, so one biscuit is $\frac{9}{15} = 0.60$, and the muffin price was never used at any point.
- ✓ **Check 3:** Is the size sensible? There are more biscuits than muffins in the first order, so the biscuits should take the larger share of the $4.50, and a biscuit should come out cheaper than a muffin rather than dearer. The biscuits account for 3.00 of the 4.50 and the muffins 1.50, and a biscuit at $0.60 sits just under a muffin at $0.75 - both of which are what a bakery counter should look like.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method to find the cost of 1 muffin	**M1**	For $\frac{2.25}{3}$ $(= 0.75)$ or $\frac{225}{3}$ $(= 75)$ or $\frac{2}{3} \times 2.25$ $(= 1.50)$ or $\frac{2}{3} \times 225$ $(= 150)$. Allow $\frac{1}{3}$ as $0.33\ldots$ or $\frac{2}{3}$ as $0.66\ldots$, truncated or rounded. Or for setting up two equations in which the coefficients of m	✓

Step	Mark	Description	Got it?
		are equal: $6m + 15b = 13.50$ and $6m = 4.50$ oe, or $6m + 15b = 1350$ and $6m = 450$ oe, with m for muffins and b for biscuits, or the use of 2 different letters (which may not be defined).	
A correct method to find the cost of the 5 biscuits	**M1**	For $4.50 - 2 \times 0.75$ $(= 3)$ or $450 - 2 \times 75$ $(= 300)$, where the printed scheme puts the 0.75 and the 75 in quotation marks, so the candidate's own earlier value for one muffin may be used in their place. Or for finding an equation for b: $15b = 9$ oe or $15b = 900$ oe.	✓
A correct method to find the cost of 1 biscuit	**M1**	For $\frac{3}{5}$ or $\frac{300}{5}$ $(= 60)$, where the printed scheme again puts the 3 and the 300 in quotation marks, so the candidate's own value for the 5 biscuits may be used. Or for $\frac{9}{15}$ or $\frac{900}{15}$ $(= 60)$.	✓
The cost of 1 biscuit	**A1**	For 0.6 or 0.60, or for the dollar sign crossed out and 60 cents given instead.	✓
An answer given with no working	**Note**	Working is not required here, so a correct answer scores full marks unless it has come from obviously incorrect working.	✓
Special case - the muffin division turned upside down, carried through both steps	**SC B2**	Awarded only when no other marks are earned, for $4.50 - 2 \times \frac{3}{2.25}$, which comes to $1.83\ldots$, and then that value divided by 5, which comes to $0.36\ldots$ dollars. Or, in cents, for $450 - 2 \times \frac{3}{225}$, which comes to $449.9\ldots$, and then that value divided by 5, which comes to $89.9\ldots$ cents. The error behind it is the first division taken the wrong way round, $\frac{3}{2.25}$ in place of $\frac{2.25}{3}$, after which the method is followed correctly.	✓
Special case - the same upside-down division, first step only	**SC B1**	Awarded only when no other marks are earned, for $4.50 - 2 \times \frac{3}{2.25}$ on its own, which comes to $1.83\ldots$, or for $450 - 2 \times \frac{3}{225}$ on its own, which comes to $449.9\ldots$ cents.	✓

Full marks: 4/4

Question 8 - Exam Solution

Understanding the Question

Given

ABC and EBD are straight lines, crossing at B
Triangle BCD is isosceles, with $BD = BC$
Angle $ABE = 104°$
The angle marked $x°$ is angle BDC, the angle of the triangle at D

Find

The value of x, which is the size of angle BDC in degrees

Plan the Solution

- The two straight lines cross at B, so the angle of the triangle at B is vertically opposite the $104°$ that is marked above it. That carries the given angle down into the triangle.
- The equal sides BD and BC make the angles at D and at C equal, so those two share whatever the angle at B leaves.
- Take the angle at B away from $180°$, then halve what is left.

Worked Solution [3 marks]

> Rule - Vertically opposite angles are equal; the three angles of a triangle add up to $180°$; and in an isosceles triangle the two angles opposite the equal sides are equal.

Step 1: Carry the $104°$ down into the triangle

angle DBC = angle $ABE = 104°$

(Reason: ABC and EBD are straight lines that cross at B, so angle DBC and angle ABE are vertically opposite, and vertically opposite angles are equal.)

Step 2: Find what the angles at D and C share

angle BDC + angle $BCD = 180° - 104° = 76°$

(Reason: The three angles of triangle BCD add up to $180°$, so taking the $104°$ at B away leaves $76°$ for the other two angles together.)

Step 3: Halve it, because the two angles are equal

$$x = \frac{76}{2} = 38$$

(Reason: $BD = BC$, so the angles opposite those two sides, at D and at C, are equal. Each one is therefore half of the $76°$ they share.)

$$x = 38$$

Verification

✓ **Check 1:** Put the three angles of triangle BCD back together: 104 at B, and 38 at each of D and C. $104 + 38 + 38 = 180$

✓ **Check 2:** Work along the straight line ABC instead. Angle ABD is $180 - 104 = 76$, and it is the exterior angle of the triangle at B, so it must equal the two interior angles opposite it added together. $38 + 38 = 76$

✓ **Check 3:** Split the isosceles triangle down the middle. The line from B to the midpoint of DC is perpendicular to DC and halves the angle at B, giving a right-angled triangle with $52°$ at B. $90 - 52 = 38$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
angle $DBC = 104$ or $180 - 104\ (= 76)$	**M1**	for correctly finding DBC or EBC or ABD. May be seen on the diagram. This is not awarded if the angles are incorrectly assigned. (Ignore incorrect angles on the diagram if a student shows 38 on the answer line)	✓
$\frac{180 - 104}{2}$ oe or $\frac{76}{2}$ oe	**M1**	The printed scheme gives no further notes against this mark. oe means any equivalent working is accepted.	✓
Working not required, so correct answer scores full marks (unless from obvious incorrect working). Answer 38	**A1**	cao - correct answer only.	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given

2 litres of lemonade in the bottle at the start
Every completely filled glass takes 180 millilitres
A conversion is needed: 1 litre is 1000 millilitres

Find

The amount of lemonade left in the bottle, in millilitres, once no further glass can be completely filled.

Plan the Solution

- Put both amounts into the same unit by changing the 2 litres into millilitres.
- Divide to see how many 180 millilitre glasses fit into that amount.
- The division does not come out as a whole number, so keep the whole-number part only - a part-filled glass does not count.
- Multiply that number of glasses by 180 to find how much lemonade leaves the bottle, then subtract from the starting amount.

Worked Solution [4 marks]

> Rule - Convert first, then count whole glasses: 1 litre $= 1000$ millilitres, and lemonade left $=$ start $-$ (number of full glasses $\times 180$).

Step 1: Put both amounts into the same unit

$$2 \times 1000 = 2000$$

(Reason: The glasses are measured in millilitres, so the bottle must be measured in millilitres too. There are 1000 millilitres in 1 litre, so the bottle holds 2000 millilitres.)

Step 2: Divide to see how many glasses can be completely filled

$$\frac{2000}{180} = \frac{100}{9}$$

(Reason: Each full glass takes 180 millilitres, so dividing counts the glasses. Now $\frac{100}{9}$ is a little more than 11 but less than 12, so eleven glasses can be filled completely and a twelfth cannot.)

Step 3: Work out how much lemonade is poured out

$11 \times 180 = 1980$

(Reason: Only completely filled glasses are poured, and there are 11 of them, each taking 180 millilitres.)

Step 4: Subtract to find what is left in the bottle

$2000 - 1980 = 20$

(Reason: The bottle started with 2000 millilitres and 1980 millilitres went into the glasses, so what stays behind is the difference. The question asks for millilitres, so no converting back is needed.)

20 millilitres

Verification

- ✓ **Check 1:** Add the glasses up instead of multiplying: 180, 360, 540, 720 and so on reaches 1800 after ten glasses, and one more glass makes 1980. $1980 + 20 = 2000$, so every millilitre of lemonade is accounted for
- ✓ **Check 2:** Test that a twelfth glass really is impossible. Filling one needs another 180 millilitres, and only 20 millilitres are left in the bottle. $20 < 180$, so no further glass can be completely filled and the pouring stops here
- ✓ **Check 3:** Work in litres all the way through instead. Each glass is 0.18 litres, so eleven glasses take 1.98 litres, leaving 0.02 litres in the bottle. Multiplying by 1000 turns that into 20 millilitres, the same answer by a different unit

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Writes 2 litres as 2000 (millilitres), or 180 millilitres as 0.18 (litres)	**B1**	Can be implied from correct working	✓
eg $\frac{2000}{180}$ oe or $\frac{2}{0.18}$ oe or $11(.111....)$ or $\frac{100}{9}$ or $11\frac{1}{9}$ or $180, 360, 540, 720, ..., 1800, 1980$ or $2000, 1820, 1640, 1460, ..., 200, 20$	**M1**	ft their millilitres over 180, or 2 over their litres, for this mark. NB Repeated subtraction must continue to a number less than 180. eg $\frac{200}{180}$ or $200 - 180$ gains this mark only. eg $\frac{500}{180}$ or	✓

Step	Mark	Description	Got it?
		$500 - 180 - 180$ gains this mark only. eg $\frac{2}{1.8}$ or $2 - 1.8$ gains this mark only. Allow one arithmetic error for repeated addition or repeated subtraction for this mark	
$11 \times 180 (= 1980)$ or $11 \times 0.18 (= 1.98)$ or $180, 360, 540, 720, ..., 1800, 1980$ or $2000, 1820, 1640, 1460, ..., 200, 20$	**M1**	dep on B1, or for an answer of 0.02. No errors allowed for repeated addition or repeated subtraction for this mark	✓
Answer: 20	**A1**	Working not required, so correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 4/4

Question 10 - Exam Solution

Understanding the Question

Given

Wei must work out $30 - 4x$ when $x = -5$, and his working ends at 10

Y is positive, Y is odd, and $Y \leqslant 9$

Find

(a) what Wei has done wrong, said in words (b) every value Y can take

Plan the Solution

- (a) Substitute $x = -5$, then follow the order of operations: the multiplication is done before the subtraction.

- (a) Work the correct third line out, compare it with Wei's third line, and name the difference.
- (b) Take the three conditions one at a time. Positive removes 0 and the negative numbers, odd removes the even numbers, and $Y \leqslant 9$ sets the ceiling.

Worked Solution [3 marks]

> Rule - a negative multiplied by a negative gives a positive, so $-4 \times -5 = +20$, and a term worth -20 that is being subtracted adds to the total.

Part (a), Step 1: substitute $x = -5$

$$30 - 4x = 30 - 4 \times -5$$

(Reason: $4x$ means $4 \times x$, so x is replaced by -5 and nothing else on the line changes. Wei's first line is right.)

Part (a), Step 2: multiply before subtracting

$$-4 \times -5 = +20$$

(Reason: The term being taken away is 4×-5, which is -20. Reading the expression as $30 + (-4) \times (-5)$ makes the sign rule visible: two negatives multiply to a positive, so this is $+20$.)

Part (a), Step 3: finish the line

$$30 - 4 \times -5 = 30 + 20$$
$$30 + 20 = 50$$

(Reason: Taking away a negative amount adds to the total, so the third line should read $30 + 20$ and the value of the expression is 50.)

Part (a), Step 4: say what went wrong

$$30 - 20 = 10$$

(Reason: Wei's own third line is correct arithmetic, but it starts from the wrong sign: he treated -4×-5 as -20 instead of $+20$, so he subtracted where he should have added. Naming that one sign is what the mark is for.)

Part (b), Step 5: take the conditions one at a time

$$Y > 0, \quad Y \text{ odd}, \quad Y \leqslant 9$$

(Reason: Positive rules out 0 and every negative number. Odd rules out 2, 4, 6 and 8. The third condition sets the ceiling, and $Y \leqslant 9$ lets 9 itself in.)

Part (b), Step 6: write the list

$$Y = 1, 3, 5, 7, 9$$

(Reason: Start at 1 and count on in twos. The next odd number after 9 is 11, which breaks $Y \leqslant 9$, so the list stops at 9.)

(a) $-4 \times -5 = +20$, not -20, so the third line should be $30 + 20$ and the answer should be 50

(b) $Y = 1, 3, 5, 7, 9$

Verification

- ✓ **Check 1:** Work the value out the other way round: build the term $4x$ first, then subtract it from 30. $4 \times -5 = -20$ and $30 - (-20) = 50$, which is the value Step 3 reached
- ✓ **Check 2:** Find where Wei's 10 really comes from by substituting the positive value $x = 5$ instead. $30 - 4 \times 5 = 10$, so 10 is the value at $x = 5$, not at $x = -5$
- ✓ **Check 3:** Count what the conditions allow: from 1 to 9 every other whole number is odd. Five values are allowed, and the list holds five
- ✓ **Check 4:** Test the three numbers just outside the list, one for each condition. 0 is not positive, 2 is even, and 11 is greater than 9, so nothing is missing and nothing extra belongs

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A correct reason	**B1**	He should have got $30 + 20$ oe, or the answer should be 50 oe, or $-4 \times -5 = (+)20$ oe, or minus and minus gives a positive oe, etc	✓
(b) $1, 3, 5, 7, 9$	**B2**	For $1, 3, 5, 7, 9$ with no extras (in any order)	✓
(b) part marks	**B1**	For four correct values with no more than one incorrect, or for five correct values with no more than one incorrect	✓

Full marks: 3/3

Question 11 - Exam Solution

Understanding the Question

Given

Find

Every member takes part on exactly one of the two days, and picks exactly one of the three activities.
The Thursday row reads 29 for badminton and 58 for archery, and adds to 110.
The Total row reads 62 for badminton and 105 for archery, and the grand total is 240.
Six cells of the table are empty: the whole Friday row, one cell in the Thursday row and one cell in the Total row.

(a) The six missing entries in the two-way table. (b) The probability that a member picked at random from those who chose archery is one of the Thursday members.

Plan the Solution

- Start with a row or a column that has only ONE empty cell, because that cell is forced: it is whatever is left when the entries already there are taken from the total.
- Each entry filled in opens up another row or another column with a single gap, so keep working outwards until every cell is full.
- Part (b) is a conditional probability. Only the 105 members who picked archery are in the running, so the archery total is the denominator, not the grand total of 240.

Worked Solution [4 marks]

> Rule - two-way table: every row adds to its own row total, every column adds to its own column total, and the row totals and the column totals both add to the same grand total of 240.

Step 1: close the Thursday row

$$110 - 29 - 58 = 23$$

(Reason: The Thursday row has only one empty cell, so that cell is forced. Take the two Thursday entries already printed from the row total of 110, which leaves 23 for swimming on Thursday.)

Step 2: the Friday total

$$240 - 110 = 130$$

(Reason: The Total column holds only the two day totals, and they add to the grand total, so the Friday total is 240 take away the Thursday total of 110.)

Step 3: the Friday badminton and archery entries

$$62 - 29 = 33$$

$$105 - 58 = 47$$

(Reason: Each of those two columns has a single gap, so take the Thursday entry from the column total. Badminton leaves 33 and archery leaves 47.)

Step 4: close the Friday row

$$130 - 33 - 47 = 50$$

(Reason: The Friday row now has one empty cell left, so take the two Friday entries just worked out in step 3 from the Friday total of 130.)

Step 5: the swimming column total

$$240 - 62 - 105 = 73$$

(Reason: The three column totals add to the grand total, so the swimming total is what is left when the badminton total of 62 and the archery total of 105 are taken from 240. Reading it down the swimming column instead gives the same thing: $23 + 50 = 73$.)

Step 6: the completed table

	badminton	swimming	archery	Total
Thursday	29	23	58	110
Friday	33	50	47	130
Total	62	73	105	240

(Reason: Every row now adds to the total at the end of it and every column adds to the total at the foot of it, which is what part (a) is marked on. The marks are for the entries, so a table that is right in only some cells still scores.)

Step 7: the probability for part (b)

$$\mathrm{P}(\text{Thursday}) = \frac{58}{105}$$

(Reason: The member is picked from those who chose archery, so the archery total of 105 is the denominator. Of those 105, the 58 sitting in the Thursday row are the ones that count. The fraction will not cancel, because $58 = 2 \times 29$ and $105 = 3 \times 5 \times 7$ share no factor.)

(a) Thursday swimming 23; Friday badminton 33, swimming 50, archery 47, total 130; swimming total 73

(b) $\dfrac{58}{105}$

Verification

✓ **Check 1:** Add each row across and compare it with the total printed at the end of that row: $29 + 23 + 58$, $33 + 50 + 47$ and $62 + 73 + 105$. 110, 130 and 240, which are the three row totals the table already carries.

✓ **Check 2:** Now add down the columns instead, which uses each entry a second time in a different pairing: $29 + 33$, $23 + 50$, $58 + 47$ and $110 + 130$. 62, 73, 105 and 240, which are the four column totals the table already carries.

✓ **Check 3:** The archery column has only two entries, so the Thursday share and the Friday share of it must add to 1. Work out the Friday share on its own, from the cell part (b) never touches. $\frac{58}{105} + \frac{47}{105} = \frac{105}{105} = 1$, so nobody who picked archery has been counted twice or left out.

✓ **Check 4:** Test the denominator by picking the wrong one on purpose. Using the grand total would give $\frac{58}{240}$, which is the chance that a member picked from the WHOLE camp does archery on Thursday. The two fractions are nowhere near each other: $\frac{58}{240}$ is under a quarter, while $\frac{58}{105}$ is over a half. The two questions have different answers, and the archery total is the denominator this part needs.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A correctly completed table	**B3**	B3 for a correctly completed table. (B2 for 4 or 5 correct entries. B1 for 2 or 3 correct entries.)	✓
(b) $\frac{58}{105}$	**B1**	B1 for $\frac{58}{105}$ oe, or $0.55(238\ldots)$ or $55(.238\ldots)\%$, truncated or rounded.	✓

Full marks: 4/4

Question 12 - Exam Solution

Understanding the Question

Given

(a) The formula $w = 3y - d$

(b) b bags holding 12 marbles each, and p pouches holding 3 marbles each

Find

(a) y on its own, written in terms of w and d (b) A formula for T in terms of b and p

(b) T is the total number of marbles bought

Plan the Solution

- (a) y is buried inside $3y - d$. Undo the operations around it in reverse order: the subtraction first, then the multiplication.
- (a) Whatever is done to one side is done to the other, so the formula stays true on every line.
- (b) Work out the marbles that come from the bags and the marbles that come from the pouches separately, then add the two amounts.
- (b) There is no number to find here. The answer is a formula, so the letters stay in it.

Worked Solution [5 marks]

> Rule - Changing the subject: undo each operation around the wanted letter, doing the same thing to both sides, until that letter stands alone. Rule - Building a formula: multiply the number in one item by how many of those items there are, then add the separate amounts together.

Part (a), Step 1 - undo the subtraction

$$w = 3y - d$$

$$w + d = 3y$$

(Reason: Reason: on the right-hand side d is being taken away, so adding d to both sides cancels it and leaves $3y$ standing alone.)

Part (a), Step 2 - undo the multiplication

$$3y = w + d$$

$$y = \frac{w + d}{3}$$

(Reason: Reason: y is multiplied by 3, so dividing both sides by 3 leaves y by itself. The formula is then written with y on the left, which is what making y the subject means.)

Part (b), Step 1 - the marbles in the bags

$$12 \times b = 12b$$

(Reason: Reason: one bag holds 12 marbles and there are b bags, so the bags account for $12b$ marbles.)

Part (b), Step 2 - the marbles in the pouches

$$3 \times p = 3p$$

(Reason: Reason: one pouch holds 3 marbles and there are p pouches, so the pouches account for $3p$ marbles.)

Part (b), Step 3 - add the two amounts

$T = 12b + 3p$

(Reason: Reason: every marble comes either from a bag or from a pouch, so the total T is the two amounts added together. This cannot be tidied any further, because $12b$ and $3p$ are not like terms.)

(a) $y = \dfrac{w + d}{3}$

(b) $T = 12b + 3p$

Verification

✓ **Check 1 - put numbers through part (a):** Choose $y = 4$ and $d = 5$. The formula in the question gives $3 \times 4 - 5 = 7$, so $w = 7$. Now feed $w = 7$ and $d = 5$ into the rearranged formula. $\dfrac{7 + 5}{3} = \dfrac{12}{3} = 4$, which is the value of y that was started with.

✓ **Check 2 - rearrange the answer back again:** Multiply both sides of the answer by 3 to get $3y = w + d$, then take d away from both sides. $3y - d = w$, which is the formula the question started from, so nothing has been lost.

✓ **Check 3 - count part (b) a different way:** Take $b = 5$ bags and $p = 4$ pouches. Counting them separately gives $5 \times 12 = 60$ marbles from the bags and $4 \times 3 = 12$ marbles from the pouches, so 72 marbles altogether. The formula gives $12 \times 5 + 3 \times 4 = 60 + 12 = 72$, the same total.

✓ **Check 4 - test the two numbers in front of the letters:** One bag and no pouches must come to 12 marbles, and no bags and one pouch must come to 3 marbles. Put those two cases into the formula. $12 \times 1 + 3 \times 0 = 12$ and $12 \times 0 + 3 \times 1 = 3$, so neither number in front of a letter has been swapped.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A correct first step	**M1**	For a correct first step: $w + d = 3y$ oe or $-3y = -w - d$ oe or $\dfrac{w}{3} = y - \dfrac{d}{3}$ oe or $\dfrac{w + d}{3}$ oe or $y = w + d$ divided by 3.	✓

Step	Mark	Description	Got it?
(a) The rearranged formula	**A1**	$y = \frac{w+d}{3}$ oe, eg $y = \frac{-d-w}{-3}$ or $y = \frac{w}{3} + \frac{d}{3}$ oe, or $y = (w+d)$ divided by 3. Must see $y = \ldots$ on the answer line or in the working.	✓
(a) Working	**Note**	Working is not required, so a correct answer scores full marks, unless it comes from obviously incorrect working.	✓
(b) The formula	**B3**	For $T = 12b + 3p$ oe. Accept $T = 12 \times b + 3 \times p$.	✓
(b) Almost the full formula	**B2**	For $12b + 3p$ or $T = 12b + xp$ or $T = yb + 3p$ or a correct equation with other letters, eg $T = 12m + 3n$.	✓
(b) One correct part of the formula	**B1**	For $12b + xp$ or $12b$ or $yb + 3p$ or $3p$ or $12p + 3b$ or $T = kb + cp$ where $k \neq 0$ or $k \neq 12$ and $c \neq 0$ or $c \neq 3$.	✓
(b) Letters	**Note**	Accept upper or lower case for T, b and p, including a mixture of these, for B3, B2 and B1.	✓

Full marks: 5/5

Question 13 - Exam Solution

Understanding the Question

Given

One fraction, with $9.75 - 1.4^2$ on the top line and $9.2 + \sqrt{6.3}$ on the bottom line.

A calculator is allowed, and the answer is to be copied from its display.

Find

The value of the fraction, written with every figure the display shows.

Plan the Solution

- Work out the top line on its own, squaring before subtracting.
- Work out the bottom line on its own, adding $\sqrt{6.3}$ to 9.2 without rounding the root.

- Divide the top line by the bottom line, then copy the display exactly as it stands.

Worked Solution [2 marks]

> Rule - the fraction bar groups: work out the whole of the top line and the whole of the bottom line first, and only then divide the one by the other.

Step 1: work out the top line

$9.75 - 1.4^2 = 9.75 - 1.96 = 7.79$

(Reason: The power is worked out before the subtraction, so square the 1.4 first and only then take it away from 9.75.)

Step 2: work out the bottom line

$\sqrt{6.3} = 2.50998\ldots$

$9.2 + 2.50998\ldots = 11.70998\ldots$

(Reason: The root does not stop, so keep it as the calculator holds it. Every figure of the bottom line is needed, because the question asks for every figure of the answer.)

Step 3: divide, and copy the whole display

$$\frac{7.79}{11.70998\ldots} = 0.6652445134$$

(Reason: Do the division in one calculation. Retyping a rounded 11.70998 agrees to seven figures and then drifts, so the end of the display would be wrong; and nothing is rounded at the finish, because all the figures are what is asked for.)

$$0.6652445134$$

Verification

✓ **Check 1:** Estimate with easier numbers. The top line is about 7.8 and the bottom line about 11.7, and that ratio is exactly two thirds. $\frac{7.8}{11.7} = \frac{2}{3}$, so the answer must sit just below 0.6667, and 0.6652445134 does.

✓ **Check 2:** Work backwards. Multiplying the displayed value by the bottom line must return the top line, 7.79. $0.6652445134 \times 11.70998008 = 7.79$, the top line back again.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
7.79 or $11.709\ldots$ or 0.67 or 0.665 or 0.6652	**M1**	Any one of these values seen scores the method mark.	✓
Working not required, so correct answer scores full marks (unless from obvious incorrect working)	**A1**	Answer $0.66524(4\ldots)$, to at least 5 significant figures. The calculator value is 0.6652445134.	✓

Full marks: 2/2

Question 14 - Exam Solution

Understanding the Question

Given

The factory makes 125 bicycles each hour.
It makes bicycles for 9 hours a day, on 5 days each week.
24% of the bicycles it makes are blue.

Find

The number of blue bicycles the factory makes in one week.

Plan the Solution

- Build the weekly total up in stages: one hour, then one day, then one week.
- Write 24% as a decimal, then multiply it by that weekly total.
- A calculator is allowed, but write the multiplications down: the method marks are for the chain, not for the final number.

Worked Solution [3 marks]

> Percentage of an amount: write the percentage as a decimal, then multiply by the amount. 24% of an amount is 0.24 times that amount.

Step 1: Find the bicycles made in one day

$$125 \times 9 = 1125$$

(Reason: Each hour produces 125 bicycles and the factory runs for 9 hours, so a day is nine lots of 125.)

Step 2: Find the bicycles made in one week

$1125 \times 5 = 5625$

(Reason: A week is 5 of those days, so multiply the daily total by 5.)

Step 3: Write the percentage as a decimal

$$\frac{24}{100} = 0.24$$

(Reason: Per cent means out of 100, so 24% is 24 parts out of 100.)

Step 4: Take that share of the weekly total

$0.24 \times 5625 = 1350$

(Reason: The blue bicycles are 0.24 of every bicycle made that week, and the answer comes out as a whole number, which is what a count of bicycles must be.)

1350 blue bicycles

Verification

✓ **Check 1:** Take the percentage first instead of last. 24% of 125 is 30 blue bicycles an hour, and the factory runs $9 \times 5 = 45$ hours a week. $30 \times 45 = 1350$

✓ **Check 2:** Count the bicycles that are not blue instead. They are the other 76% of the week, and the two groups must add back to the weekly total. $0.76 \times 5625 = 4275$ and $4275 + 1350 = 5625$

✓ **Check 3:** Avoid decimals altogether. $\frac{24}{100}$ cancels to $\frac{6}{25}$, and 5625 splits into 25 equal parts of 225. $\frac{6}{25} \times 5625 = 6 \times 225 = 1350$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One correct product	**M1**	$125 \times 9\ (= 1125)$ or $125 \times 5\ (= 625)$ or $9 \times 5\ (= 45)$ or $125 \times 9 \times 5\ (= 5625)$ or $0.24 \times 125\ (= 30)$ or $0.24 \times 9\ (= 2.16)$ or $0.24 \times 5\ (= 1.2)$	✓
A complete method	**M1**	For a complete method: $0.24 \times (125 \times 9 \times 5)$ oe or $0.24 \times$ ”5625” oe or ”30” $\times 9 \times 5$ oe or $125 \times$ ”2.16” $\times 5$ oe or $125 \times 9 \times$ ”1.2” oe. A value in quotation marks is the candidate's own earlier value, so this mark follows through from it.	✓

Step	Mark	Description	Got it?
The answer	**A1**	1350	✓
Unsupported answers	**Note**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
Special case	**SC B2**	SCB2 for 4275	✓
Where that special case comes from	**Note**	4275 is 76% of 5625, so it is the count of the bicycles that are not blue. The whole weekly chain has been built correctly and the wrong share taken at the last step, which is why the special case is worth two marks and not none.	✓

Full marks: 3/3

Question 15 - Exam Solution

Understanding the Question

Given

A circle whose radius is $r = 16$ cm. The length given is the radius, so the distance right across the circle is twice it.

Find

The circumference of the circle - the distance all the way round the edge. The answer is wanted correct to 3 significant figures.

Plan the Solution

- Double the radius to get the diameter, because $C = \pi d$ is built on the distance right across the circle.
- Multiply that diameter by π, using the calculator's own value of π and not a rounded one.
- Round at the very end, and only then, to 3 significant figures.

Worked Solution [2 marks]

> Rule - Circumference of a circle: $C = \pi d$, where d is the diameter. The diameter is twice the radius, so this is the same rule as $C = 2\pi r$.

Step 1: Double the radius to get the diameter

$d = 2 \times 16 = 32$

(Reason: The circumference formula multiplies π by the distance right across the circle, and that distance is two radii, so 32 cm.)

Step 2: Multiply the diameter by π

$C = \pi \times 32 = 100.5309649\ldots$

(Reason: The calculator's own value for π is used, so every figure is still there when the rounding comes. Nothing is written down to 2 or 3 figures at this stage.)

Step 3: Round to 3 significant figures

$C = 101$ cm

(Reason: The first three significant figures of $100.5309649\ldots$ are 1, 0 and 0, and the digit after them is 5, so the last of them rounds up and 100 becomes 101.)

101 cm

Verification

✓ **Check 1:** Work the calculation backwards. Dividing the circumference by π must give the diameter back, and halving that must return the radius the question started with. $\frac{100.5309649\ldots}{\pi} = 32$, and half of 32 is 16 cm, which is the radius given.

✓ **Check 2:** Squeeze it. π lies between 3 and 3.2, so the circumference has to lie between those two multiples of the diameter. $3 \times 32 = 96$ and $3.2 \times 32 = 102.4$, so the circumference is somewhere between 96 cm and 102.4 cm, and 101 cm sits inside that.

✓ **Check 3:** Redo it with $\frac{22}{7}$ in place of π, one of the approximations the mark scheme allows. $\frac{22}{7} \times 32 = 100.5714\ldots$, which still rounds to 101 to 3 significant figures.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $2 \times \pi \times 16$ or $\pi \times 32$	M1	allow $3.14\ldots$ or $\frac{22}{7}$ for π. The printed row writes the 32 in quotation marks, so the candidate's own value for 2×16 may be used here.	✓

Step	Mark	Description	Got it?
101 Working not required, so correct answer scores full marks (unless from obvious incorrect working)	**A1**	accept 100 to 101	✓
NB, printed in the A1 row's notes	**Note**	An answer of 101 reached from working written as $\pi \times 16^2$ scores M0A0. The printed row glosses that working as 32π, the value such a candidate has evidently keyed in. This is the 'unless from obvious incorrect working' exception in the row above doing its job: the answer on the line matches, but the method written beside it is the area formula, so neither mark is earned. Separately, the range that row accepts is what the approximations allowed by the M1 need - $3.14 \times 32 = 100.48$ gives 100, and $\frac{22}{7}$ gives 101.	✓

Full marks: 2/2

Question 16 - Exam Solution

Understanding the Question

Given

Two lighthouses, A and B, with a north line drawn at each
The bearing of B from A is $073°$
The two north lines point the same way, so they are parallel

Find

The bearing of A from B, measured clockwise from the north line at B

Plan the Solution

- A bearing is the clockwise turn from north, and it is always written with three figures.

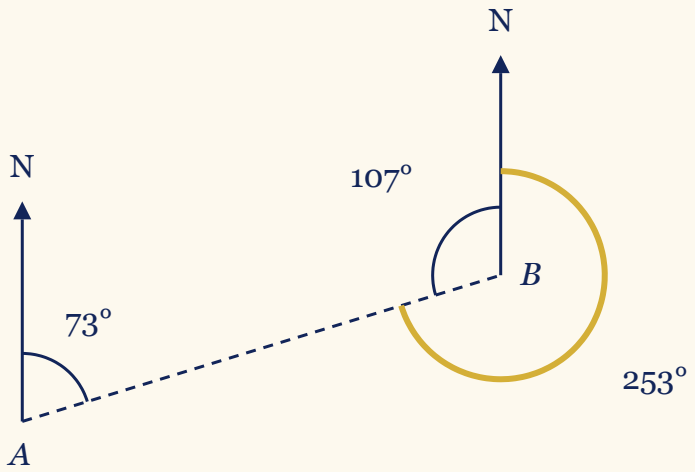
- Use the parallel north lines to find the angle at B between north and the line BA.
- That angle opens the wrong way, so take it off a full turn of $360°$ to get the clockwise bearing.

Worked Solution [2 marks]

> Rule - Back bearing: the north lines at A and B are parallel, so the bearing of A from B and the bearing of B from A always differ by $180°$.

Step 1: Mark the angle the diagram gives you at A

$\angle NAB = 73°$



(Reason: The bearing $073°$ is the clockwise turn from the north line at A round to the line AB, which is exactly the angle printed on the diagram.)

Step 2: Use the parallel north lines to get the angle at B

$180° - 73° = 107°$

(Reason: The north lines are parallel and AB cuts across both of them, so the angle at A and the angle at B between north and BA are allied (co-interior) angles and add to $180°$.)

Step 3: Turn clockwise from north at B

$360° - 107° = 253°$

(Reason: The $107°$ opens anticlockwise from the north line at B, and a bearing must be measured clockwise, so take that angle off the full turn of $360°$.)

$253°$

Verification

✓ **Check 1:** Reverse it. Taking $180°$ off a back bearing must return the bearing you started with. $253° - 180° = 073°$

✓ **Check 2:** Test the parallel-line step on its own: allied angles across a transversal must add to $180°$. $73° + 107° = 180°$

✓ **Check 3:** Sense check the direction. B is north-east of A, so A must be south-west of B, and every south-west bearing lies between $180°$ and $270°$. $180° < 253° < 270°$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$180 + 73$ or $360 - (180 - 73)$ or $360 - 107$ or $270 - (90 - 73)$ or $270 - 17$ or $270 - (180 - 90 - 73)$	M1	or for 73 or for 107 or for 253 seen in the correct place on the diagram by point B or correctly identified by labelling	✓
253	A1	Working not required, so correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 2/2

Question 17 - Exam Solution

Understanding the Question

Given

The recipe makes enough for 4 people.
It uses 50 g of grated Cheddar and 40 g of butter.
Owen is cooking for 12 people.
Nadia uses 180 g of butter.

Find

(a) The mass of grated Cheddar needed for 12 people. (b) The number of people 180 g of butter is enough for.

Plan the Solution

- Both parts are the same idea. A recipe scales up and down in one ratio, so every ingredient is multiplied by the same number.

- (a) Compare the two numbers of people to get that multiplier, then apply it to the 50 g of Cheddar.
- (b) Compare the two masses of butter to get the multiplier, then apply it to the 4 people the recipe serves.
- The unitary method - working out one person's share first - reaches the same answers and is used in the checks.

Worked Solution [4 marks]

> Rule - Scaling a recipe: every ingredient, and the number of people, is multiplied by the same scale factor, and $\text{scale factor} = \dfrac{\text{new amount}}{\text{recipe amount}}$.

Step 1: part (a) - find the scale factor

$$\frac{12}{4} = 3$$

(Reason: the recipe serves 4 people and Owen is cooking for 12, so every ingredient is multiplied by 3.)

Step 2: part (a) - scale the grated Cheddar

$$50 \times 3 = 150$$

(Reason: the recipe uses 50 g of Cheddar for 4 people, so three times the recipe needs three times the Cheddar, in grams.)

Step 3: part (b) - find the scale factor from the butter

$$\frac{180}{40} = 4.5$$

(Reason: here the multiplier has to come from the butter, because that is the ingredient Nadia measured out.)

Step 4: part (b) - scale the number of people

$$4 \times 4.5 = 18$$

(Reason: the recipe serves 4 people, and the number of people is multiplied by the same scale factor as every ingredient.)

(a) 150 g

(b) 18 people

Verification

✓ **Check 1:** Part (a) by the unitary method. One person needs $\frac{50}{4} = 12.5$ g of grated Cheddar, so 12 people need 12.5×12 g. 150 g, which is the answer to part (a).

✓ **Check 2:** Part (b) by the unitary method. One person needs $\frac{40}{4} = 10$ g of butter, so 180 g of butter is enough for $\frac{180}{10}$ people. 18 people, which is the answer to part (b).

✓ **Check 3:** Work backwards from part (b). Serving 18 people is $\frac{18}{4} = 4.5$ recipes, and that much butter is $4.5 \times 40 = 180$ g. The butter comes back to the 180 g Nadia used, and 18 is larger than 12, as it must be, because 12 people would use only 120 g.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\frac{12}{4}$ $(= 3)$ or $\frac{50}{4}$ $(= 12.5)$ oe	M1	For a correct method to find the scale factor, or a correct calculation to find the amount of grated Cheddar for one person.	✓
(a) 150	A1	Working not required, so a correct answer scores full marks (unless from obvious incorrect working).	✓
(b) $\frac{180}{40}$ $(= 4.5)$ oe or $\frac{40}{4}$ $(= 10)$	M1	For a correct method to find the scale factor, or a correct calculation to find the amount of butter for one person.	✓
(b) 18	A1	Working not required, so a correct answer scores full marks (unless from obvious incorrect working).	✓

Full marks: 4/4

Question 18 - Exam Solution

Understanding the Question

Given

Six numbers, one on each tile, laid out in order of size.

Find

The number on the first tile, the number on the second tile and the number on the

The tiles that can be read are the third, 6, the fifth, 10, and the sixth, 14.
The range is 10, the median is 7.5 and the mode is 6.

fourth tile.

Plan the Solution

- The tiles are in order of size, so the smallest number is on the first tile and the largest is on the sixth. That turns the range into one subtraction.
- Six numbers have no single middle value, so the median is the mean of the third and the fourth. The third is already on the page, so the fourth follows.
- The mode is the only condition left, and the second tile is the only tile left, so the mode fixes it.
- Finish by testing all three conditions on the completed row, and by checking the row is still in order of size.

Worked Solution [3 marks]

Rule - Range: $\text{largest} - \text{smallest}$. Median of six values in order: the mean of the third and the fourth, $\frac{3\text{rd} + 4\text{th}}{2}$. Mode: the value that occurs most often.

Step 1: use the range to find the first tile

$14 - 10 = 4$

4	**6**	6	**9**	10	14

(Reason: The numbers are in order of size, so the largest is the 14 on the last tile and the smallest is on the first. The range is the largest take away the smallest, so the smallest sits 10 below 14.)

Step 2: use the median to find the fourth tile

$2 \times 7.5 = 15$

$15 - 6 = 9$

(Reason: Six numbers have no single middle value, so the median is the mean of the third and the fourth. Those two must therefore total twice 7.5, and the third tile already reads 6, so take that away from the total.)

Step 3: use the mode to find the second tile

$4, 6, 6, 9, 10, 14$

(Reason: The second tile lies between 4 and 6 in the order. A 5 would leave every value appearing once, so there would be no mode at all, and a 4 would make 4 the most common value instead. Only a second 6 makes 6 the mode.)

First tile: 4

Second tile: 6

Fourth tile: 9

The completed row: 4, 6, 6, 9, 10, 14

Verification

✓ **Check 1 - the range:** Take the smallest number on the completed row away from the largest. $14 - 4 = 10$

✓ **Check 2 - the median:** Six numbers, so average the third and the fourth. $\frac{6+9}{2} = 7.5$

✓ **Check 3 - the mode:** Count how many times each value appears on the completed row. 6 appears twice and every other value appears once, so the mode is 6.

✓ **Check 4 - the order:** Read the completed row from left to right. Every number is at least as large as the one before it, so the tiles are still in order of size.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
First tile	**B1**	First tile 4	✓
Second tile	**B1**	Second tile 6, or a list of 6 numbers with a mode of 6	✓
Fourth tile	**B1**	Fourth tile 9	✓
Special case	**SC B2**	for 4, 6 and 9 in the incorrect order	✓

Full marks: 3/3

Question 19 - Exam Solution

Understanding the Question

Given

Shape A on the grid has vertices $(1, 1)$, $(3, 1)$, $(3, 4)$, $(2, 4)$, $(2, 2)$ and $(1, 2)$.
The enlargement has scale factor 3 and centre $(0, 1)$.
Triangle P has vertices $(2, 2)$, $(4, 2)$ and $(2, 6)$.
Triangle Q has vertices $(6, 8)$, $(8, 8)$ and $(8, 4)$.

Find

(a) the six vertices of the enlarged shape, so that it can be drawn on the grid (b) the single transformation that maps P onto Q, described fully

Plan the Solution

- (a) Work from the centre $(0, 1)$. For each vertex of shape A, count how far across and how far up it is from that centre.
- (a) Multiply both counts by 3, then count out again from the centre to reach the image vertex.
- (a) Join the six image points in the same order as shape A.
- (b) The two triangles are the same size, so look for a rotation, a reflection or a translation rather than an enlargement that changes the size.
- (b) Match each vertex of P with its image on Q, join the pairs, and use the joins to find the centre.

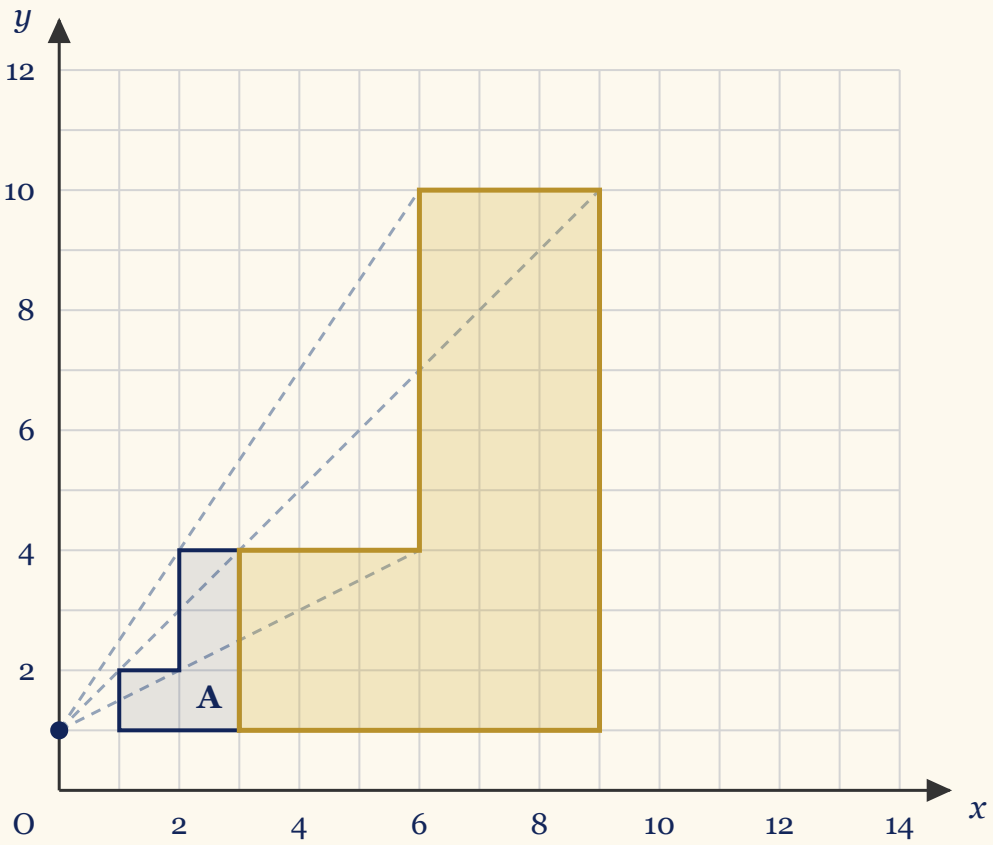
Worked Solution [5 marks]

> Rule - Enlargement: if a point is a across and b up from the centre, its image is $3a$ across and $3b$ up from that same centre. Rule - Describing a rotation: give the word rotation, the angle, the direction (not needed for $180°$) and the centre.

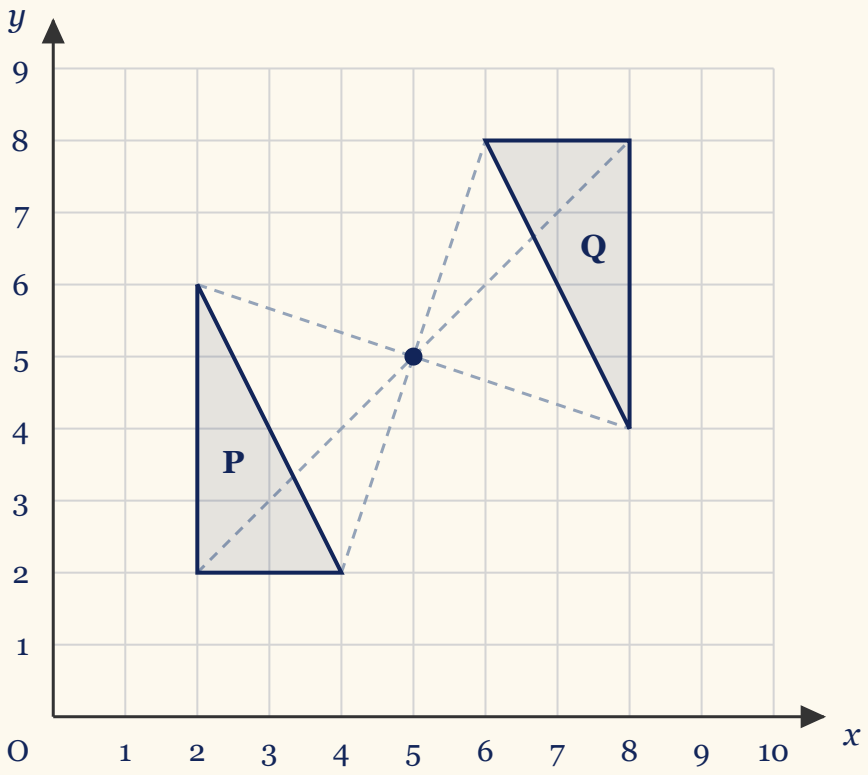
Step 1: Count from the centre $(0, 1)$ to the first vertex

$1 - 0 = 1$

$1 - 1 = 0$



centre of enlargement (0, 1)



centre of rotation (5, 5)

(Reason: The vertex $(1, 1)$ is 1 across and 0 up from the centre. In an enlargement every distance is measured from the centre, never from the origin, and here the centre is not the origin.)

Step 2: Multiply both counts by the scale factor 3

$3 \times 1 = 3$

$3 \times 0 = 0$

$(1, 1) \rightarrow (3, 1)$

(Reason: The image is 3 across and 0 up from $(0, 1)$, which lands on $(3, 1)$.)

Step 3: Do the same for the other five vertices

$(3, 1) \rightarrow (9, 1)$

$(3, 4) \rightarrow (9, 10)$

$(2, 4) \rightarrow (6, 10)$

$(2, 2) \rightarrow (6, 4)$

$(1, 2) \rightarrow (3, 4)$

(Reason: Take $(3, 4)$ as the pattern: it is 3 across and 3 up from the centre, so its image is 9 across and 9 up, and 9 up from $y = 1$ is $y = 10$.)

Step 4: Plot the six image points and join them in order

$3 \times 2 = 6$

$3 \times 3 = 9$

(Reason: Joining the images in the same order as shape A keeps the shape itself and changes only its size. Shape A is 2 squares wide and 3 squares tall, so the image is 6 wide and 9 tall.)

Step 5: Compare triangle P with triangle Q

$6 - 2 = 4$

$8 - 4 = 4$

$4 - 2 = 2$

$8 - 6 = 2$

(Reason: Triangle P has a vertical side of 4 and a horizontal side of 2, and so does triangle Q. The triangles are congruent, so the transformation does not change the size.)

Step 6: Match each vertex of P with its image on Q

$(2, 2) \rightarrow (8, 8)$

$(4, 2) \rightarrow (6, 8)$

$(2, 6) \rightarrow (8, 4)$

(Reason: The right angle of P is at $(2, 2)$ and the right angle of Q is at $(8, 8)$, so those two corners match. The short side and the long side then fix the other two.)

Step 7: Join each vertex to its image and find the midpoint

$$\frac{2+8}{2} = 5$$

$$\frac{4+6}{2} = 5$$

$$\frac{6+4}{2} = 5$$

(Reason: All three joins have the same midpoint $(5, 5)$: halfway between 2 and 8 is 5, and halfway between 4 and 6 is 5. A transformation that sends every point to the opposite side of one fixed point, the same distance away, is a half turn about that point.)

Step 8: Write the description in full

$180°$

(Reason: A half turn is a rotation of $180°$. A full description of a rotation needs the word rotation, the angle and the centre; at $180°$ no direction is needed, because clockwise and anticlockwise give the same image.)

(a) Enlargement drawn with vertices $(3, 1)$, $(9, 1)$, $(9, 10)$, $(6, 10)$, $(6, 4)$ and $(3, 4)$

(b) Rotation of $180°$ about $(5, 5)$

Verification

✓ **Check 1:** (a) Every image vertex must be 3 times as far from the centre as its object vertex. The vertex $(3, 4)$ is 3 across and 3 up from $(0, 1)$, and its image $(9, 10)$ is 9 across and 9 up. $9 = 3 \times 3$, so the image vertex is on the ray and three times as far out

✓ **Check 2:** (a) An enlargement of scale factor 3 makes every length three times as long. Shape A is 2 squares wide and 3 squares tall; the shape drawn is 6 wide and 9 tall. $\frac{6}{2} = 3$ and $\frac{9}{3} = 3$

✓ **Check 3:** (b) A half turn about $(5, 5)$ sends (x, y) to $(10 - x, 10 - y)$. Test it on the vertex $(2, 6)$ of P. $10 - 2 = 8$ and $10 - 6 = 4$, which is the vertex $(8, 4)$ of Q

✓ **Check 4:** (b) A translation would leave every side pointing the same way. In P the short side points right from $(2, 2)$; in Q it points left from $(8, 8)$. The sides point opposite ways, so the transformation is a turn and not a slide

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Correct shape drawn in correct position	**B2**	Shape drawn with coordinates $(3, 1)$, $(9, 1)$, $(9, 10)$, $(6, 10)$, $(6, 4)$, $(3, 4)$	✓
(a) Shape of the correct size, wrong position	**B1**	for a shape of the correct size but in the wrong position	✓
(a) Marking note	**Note**	NB Overlay is available	✓
(b) The transformation is named	**B1**	Rotation, with no mention of any other transformation words or move, flip, transform, up, right etc	✓
(b) The angle	**B1**	$180°$, allow half turn	✓
(b) The centre	**B1**	(centre) $(5, 5)$, must be a coordinate and not a vector	✓
(b) Alternative full description	**B2**	for enlargement scale factor -1 (Ignore any reference to clockwise or anticlockwise)	✓
(b) Marking note	**Note**	Turn is not sufficient	✓

Full marks: 5/5

Question 20 - Exam Solution

Understanding the Question

Given

The subtraction of two mixed numbers, $7\frac{1}{3} - 3\frac{4}{7}$.

The result that has to be proved: $7\frac{1}{3} - 3\frac{4}{7} = 3\frac{16}{21}$.

Find

A complete piece of working that ends at $3\frac{16}{21}$. The answer is printed in the question, so the marks are for the working, not for the number.

Plan the Solution

- Write each mixed number as an improper fraction: multiply the whole number by the denominator, then add the numerator.

- The denominators 3 and 7 share no factor, so the lowest common denominator is $3 \times 7 = 21$.
- Rewrite both fractions over that denominator, then subtract the numerators.
- Turn the improper fraction back into a mixed number and compare it with the printed result.
- A second route avoids improper fractions altogether: keep the whole numbers, borrow one whole from the 7, and subtract in two parts. The mark scheme awards full marks for either route.

Worked Solution [3 marks]

> Rule - Mixed numbers, then a common denominator: $n\frac{p}{q} = \frac{n \times q + p}{q}$, and $\frac{a}{b} - \frac{c}{d} = \frac{a \times d - c \times b}{b \times d}$.

Step 1: Write each mixed number as an improper fraction

$$7\frac{1}{3} = \frac{7 \times 3 + 1}{3} = \frac{22}{3}$$
$$3\frac{4}{7} = \frac{3 \times 7 + 4}{7} = \frac{25}{7}$$

(Reason: Multiply the whole number by the denominator and then add the numerator on top. The denominator itself never changes, so the 3 and the 7 stay where they are. This alone earns the first method mark.)

Step 2: Write both fractions over the common denominator 21

$$\frac{22}{3} = \frac{22 \times 7}{21} = \frac{154}{21}$$
$$\frac{25}{7} = \frac{25 \times 3}{21} = \frac{75}{21}$$

(Reason: The denominators 3 and 7 have no common factor, so the lowest common denominator is $3 \times 7 = 21$. Multiply the top and the bottom of the first fraction by 7, and the top and the bottom of the second fraction by 3. Whatever is done underneath must be done on top, or the value of the fraction changes.)

Step 3: Subtract the numerators

$$\frac{154}{21} - \frac{75}{21} = \frac{154 - 75}{21} = \frac{79}{21}$$

(Reason: Once both fractions are over the same denominator, only the numerators are subtracted. The denominator 21 is carried straight through, and $154 - 75 = 79$.)

Step 4: Write the improper fraction back as a mixed number

$79 = 3 \times 21 + 16$

$\frac{79}{21} = 3\frac{16}{21}$

(Reason: 21 goes into 79 three times with 16 left over, so the whole-number part is 3 and the remainder 16 sits over 21. That is the printed result, as required.)

Step 5: The same result without improper fractions

$7\frac{1}{3} = 7\frac{7}{21}$

$3\frac{4}{7} = 3\frac{12}{21}$

$7\frac{7}{21} - 3\frac{12}{21} = 6\frac{28}{21} - 3\frac{12}{21} = 3\frac{16}{21}$

(Reason: This time only the fraction parts are rewritten. Since $\frac{7}{21}$ is smaller than $\frac{12}{21}$, take one whole from the 7 and add it to the fraction as $\frac{21}{21}$, which turns $7\frac{7}{21}$ into $6\frac{28}{21}$. The whole numbers and the fractions then subtract separately, and this route is worth full marks too.)

$$7\frac{1}{3} - 3\frac{4}{7} = \frac{22}{3} - \frac{25}{7} = \frac{79}{21} = 3\frac{16}{21} \textbf{ as required}$$

Verification

✓ **Check 1:** Add the answer back on. If the difference really is $3\frac{16}{21}$, then adding $3\frac{4}{7}$ to it must return the number the question started from, $7\frac{1}{3}$.

$3\frac{16}{21} + 3\frac{12}{21} = 6\frac{28}{21} = 7\frac{7}{21} = 7\frac{1}{3}$

✓ **Check 2:** Take the whole numbers apart first, which is the mark scheme's other route. $\frac{7}{21}$ is $\frac{5}{21}$ short of $\frac{12}{21}$, so the answer sits $\frac{5}{21}$ below 4. $4 - \frac{5}{21} = \frac{84-5}{21} = \frac{79}{21} = 3\frac{16}{21}$

✓ **Check 3:** A decimal check on the calculator, which is allowed on this paper:

$7\frac{1}{3} \approx 7.333333333$ and $3\frac{4}{7} \approx 3.571428571$.

$7.333333333 - 3.571428571 \approx 3.761904762$ and $3\frac{16}{21} \approx 3.761904762$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Correct improper fractions, $\frac{22}{3}$ and $\frac{25}{7}$, or the fractional parts written over a common denominator, $(7)\frac{7}{21}$ and $(3)\frac{12}{21}$, or $(7)\frac{7a}{21a}$ and $(3)\frac{12a}{21a}$.	**M1**	for correct improper fractions or fractional part of numbers written correctly over a common denominator	✓
$\frac{154}{21}-\frac{75}{21}$ or $\frac{22\times7}{21}-\frac{25\times3}{21}$ or $\frac{22\times7-25\times3}{21}$ or $\frac{154a}{21a}-\frac{75a}{21a}$ or $7\frac{7}{21}-3\frac{12}{21}=4-\frac{5}{21}$ oe or $7\frac{7}{21}-3\frac{12}{21}=6\frac{28}{21}-3\frac{12}{21}$.	**M1**	for correct fractions with a common denominator with minus sign or mixed numbers to the stage shown. $\frac{154}{21}-\frac{75}{21}$ or $\frac{22\times7}{21}-\frac{25\times3}{21}$ implies the first M1	✓
$\frac{154}{21}-\frac{75}{21}=\frac{79}{21}=3\frac{16}{21}$ or $4-\frac{5}{21}=3\frac{16}{21}$ or $7\frac{7}{21}-3\frac{12}{21}=6\frac{28}{21}-3\frac{12}{21}=3\frac{16}{21}$. Working required.	**A1**	Dep on M2 for a correct answer from fully correct working. If a student shows that $3\frac{16}{21}=\frac{79}{21}$ then they must show correct working to $\frac{79}{21}$ and can gain full marks for this	✓
What the answer column asks for	**Note**	The answer column reads: a fully correct solution shown. The result $3\frac{16}{21}$ is printed in the question, so there is no separate answer to write on an answer line and every mark here is for the working.	✓

Full marks: 3/3

Question 21 - Exam Solution

Understanding the Question

Given

A line segment AB, drawn on the page. A ruler and a pair of compasses, and nothing else.

Find

The perpendicular bisector of AB: the straight line that cuts AB in half and crosses it at 90°. Every construction arc left showing on the diagram.

Plan the Solution

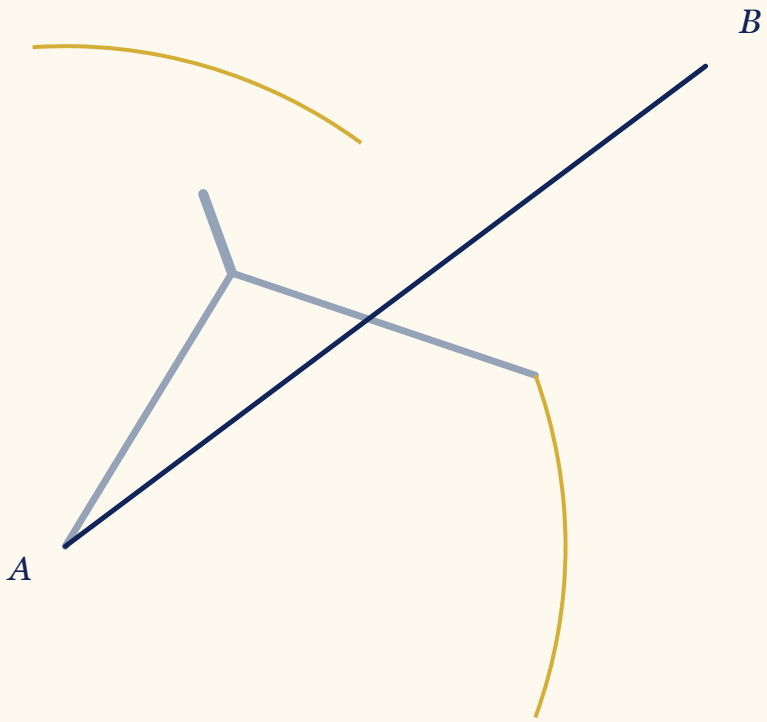
- Open the compasses to a radius bigger than half of AB, and do not change that setting again.
- Stand the point on A and draw an arc on each side of the line, then move the point to B and draw two more.
- The two pairs of arcs cross at two points. Rule the straight line through them.
- Leave every arc on the page: the arcs are half of what is being marked.

Worked Solution [2 marks]

> Rule - Perpendicular bisector: with the same radius from A and from B, the arcs meet at two points P and Q with $AP = BP = AQ = BQ$. A point that is the same distance from A as from B lies on the perpendicular bisector, so the line PQ is that bisector.

Step 1: Open the compasses to more than half of AB, then draw an arc from A on each side of the line

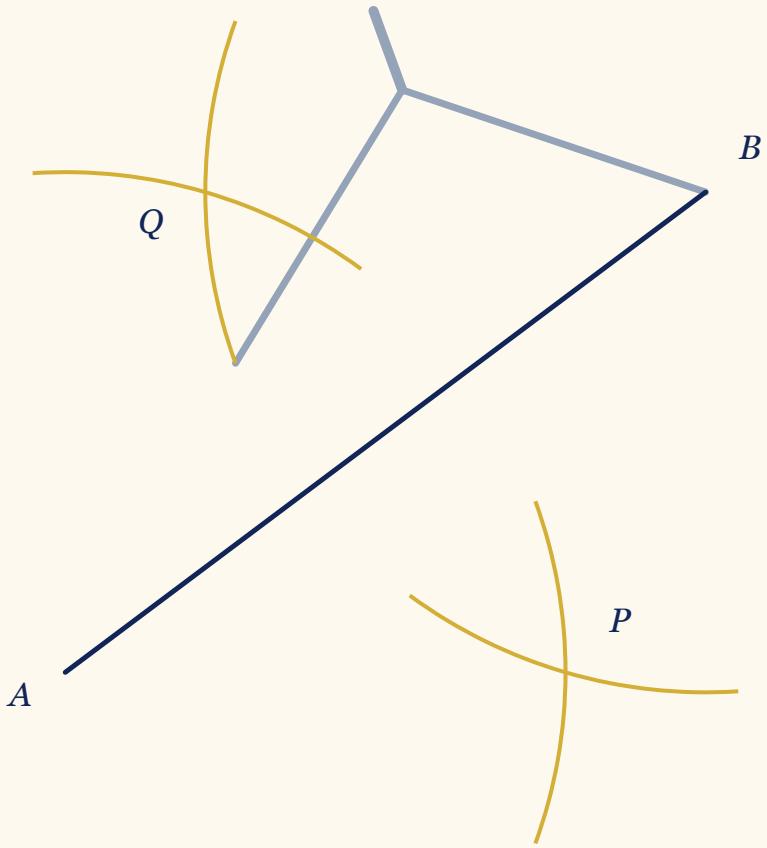
$$r > \frac{1}{2}AB$$



(Reason: If the radius r is smaller than half of AB, an arc drawn from A and an arc drawn from B never reach each other and nothing crosses. Any setting past halfway works, and it must not be touched again.)

Step 2: Keep the same radius and draw two arcs from B

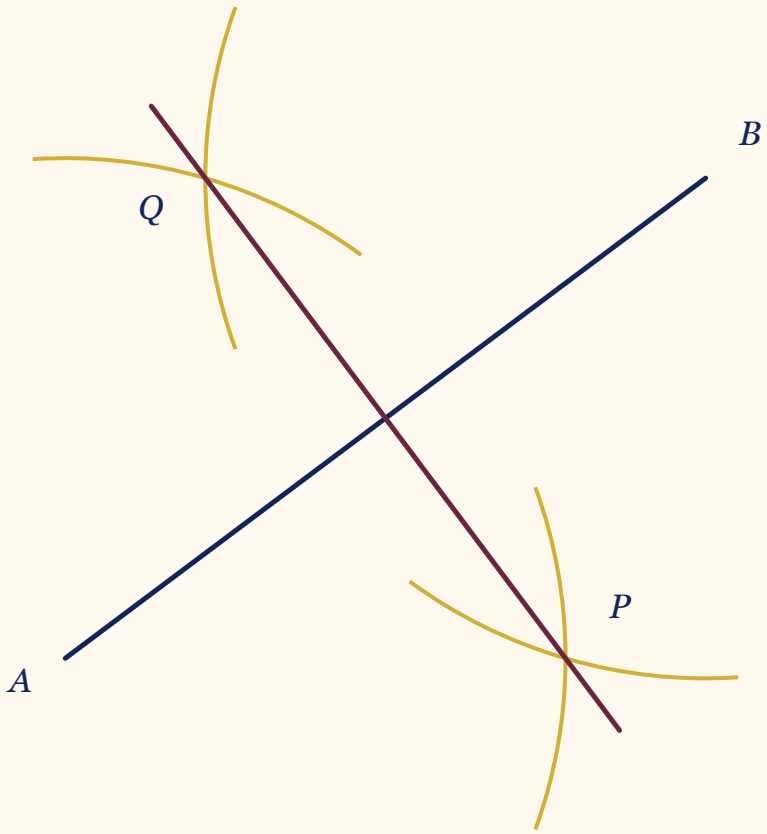
$AP = BP = AQ = BQ = r$



(Reason: The radius has not changed, so all four of those lengths are the same r. The arcs cross at two points: P on one side of AB and Q on the other.)

Step 3: Rule the straight line through P and Q

PQ is the perpendicular bisector of AB

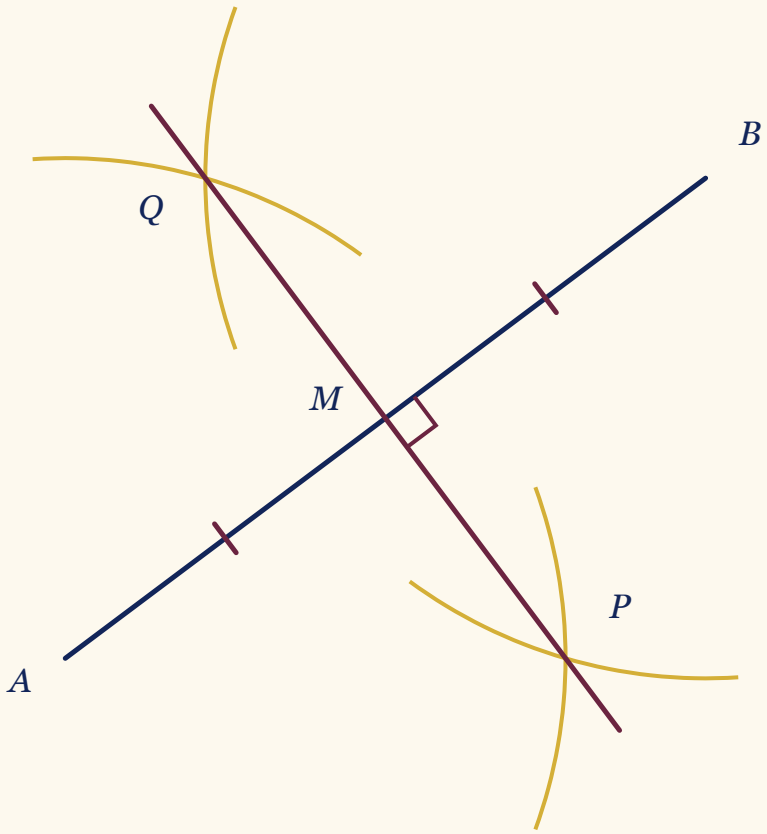


(Reason: P and Q are each the same distance from A as from B, and every point with that property lies on the perpendicular bisector. Two points fix a line, so ruling PQ draws it.)

Step 4: Read off what the ruled line has done to AB

$APBQ$ is a rhombus

$AM = MB$ and $\angle AMQ = 90°$



(Reason: All four sides of $APBQ$ are the compass radius r, so it is a rhombus, and the diagonals of a rhombus cut each other in half at right angles. Those diagonals are AB and PQ, so M is the midpoint of AB and the angle there is a right angle.)

The perpendicular bisector of AB: the straight line ruled through the two points where the pairs of arcs cross, with all four arcs left on the diagram.

Verification

- ✓ **Check 1:** Measure AM and MB along AB with the ruler. They come out equal, so the ruled line has cut AB in half.
- ✓ **Check 2:** Fold the paper along the ruled line. A lands exactly on B, which happens only along the perpendicular bisector.
- ✓ **Check 3:** Look at the quadrilateral $APBQ$: every side is one compass radius, because the setting never changed. A rhombus, and the diagonals of a rhombus bisect each other at right angles, which is exactly what this construction claims.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A fully correct perpendicular bisector of AB, with 2 pairs of intersecting arcs shown	**B2**	B2 for a fully correct perpendicular bisector with 2 pairs of intersecting arcs shown (the line and the arcs can intersect on or within the overlay guidelines)	✓
Partial credit	**(B1)**	B1 for 2 pairs of intersecting arcs and no perpendicular bisector drawn, or for a correct perpendicular bisector drawn within or on guidelines but no arcs or insufficient arcs, or one pair of intersecting arcs and perpendicular bisector drawn on just one side of AB	✓
Marking note	**Note**	NB Overlay is available	✓

Full marks: 2/2

Question 22 - Exam Solution

Understanding the Question

Given

$ABCD$ and $EFGH$ are similar, so one is an enlargement of the other.
$AB = 5$ cm, $BC = 4$ cm, $CD = y$ cm
$EF = x$ cm, $FG = 10$ cm, $GH = 24$ cm
The two names are written in matching order, so the sides pair off in that order.

Find

(a) the value of x, which is the length of EF (b) the value of y, which is the length of CD

Plan the Solution

- Pair the sides off by their letters: AB with EF, BC with FG, and CD with GH.
- Only BC and FG have a number at both ends, so that is the pair the scale factor has to come from.
- Part (a) asks for a side of the larger quadrilateral, so multiply by the scale factor.
- Part (b) asks for a side of the smaller one, so divide by the scale factor instead.

Worked Solution [4 marks]

> Rule - Similar shapes: matching sides are in the same ratio, so $\frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD}$.

Step 1: pair off the sides that match

$AB \to EF, \quad BC \to FG, \quad CD \to GH$

(Reason: The names are written in matching order, so the side named by the first two letters of one is the side named by the first two letters of the other.)

Step 2: work out the scale factor

$$\text{scale factor} = \frac{FG}{BC} = \frac{10}{4} = 2.5$$

(Reason: BC and FG are the only matching pair with a number on both quadrilaterals, so they are the only pair that can give the scale factor.)

Step 3: (a) multiply, to cross to the larger quadrilateral

$x = 5 \times 2.5 = 12.5$

(Reason: EF is on the larger quadrilateral and the side that matches it is $AB = 5$ cm, so 5 is multiplied by the scale factor.)

Step 4: (b) divide, to cross back to the smaller quadrilateral

$$y = \frac{24}{2.5} = 9.6$$

(Reason: CD is on the smaller quadrilateral and the side that matches it is $GH = 24$ cm, so 24 is divided by the scale factor.)

(a) $x = 12.5$

(b) $y = 9.6$

Verification

✓ **Check 1:** Divide each side of $EFGH$ by the side of $ABCD$ that matches it. Similar shapes have to give the same ratio every time. $\frac{12.5}{5} = 2.5$, $\frac{10}{4} = 2.5$, $\frac{24}{9.6} = 2.5$

✓ **Check 2:** Work the other way round. Shrinking the larger quadrilateral by the scale factor has to give the smaller one's own numbers back. $\frac{12.5}{2.5} = 5$ and $9.6 \times 2.5 = 24$

✓ **Check 3:** Size sense. $EFGH$ is the larger quadrilateral, so every side of it must be longer than the side of $ABCD$ it matches. $12.5 > 5$ and $24 > 9.6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) A correct scale factor, or a correct equation in x	M1	For a correct scale factor, which may be expressed as a fraction, a decimal or a ratio, and may or may not be used: $\frac{10}{4} = \frac{5}{2} = 2.5$ or $\frac{4}{10} = \frac{2}{5} = 0.4$, or for a correct equation in x: $\frac{x}{5} = \frac{10}{4}$ oe or $\frac{x}{10} = \frac{5}{4}$ oe. Allow any letter for x.	✓
(a) The value of x	A1	12.5 oe, eg $\frac{50}{4}$ or $\frac{25}{2}$ or $12\frac{1}{2}$ or $12\frac{2}{4}$. Working is not required, so a correct answer scores full marks unless it comes from obvious incorrect working.	✓
(b) A correct method, or a correct equation in y	M1	$\frac{24}{[2.5]}$ oe, or a correct equation in y: $\frac{y}{24} = \frac{4}{10}$ oe or $\frac{y}{24} = \frac{5}{[12.5]}$ oe or $\frac{y}{4} = \frac{24}{10}$ oe or $\frac{y}{5} = \frac{24}{[12.5]}$ oe. Follow through: $[2.5]$ is their scale factor from (a), and $[12.5]$ is their answer to (a). Allow any letter for y.	✓
(b) The value of y	A1	9.6 oe, eg $\frac{48}{5}$ or $9\frac{3}{5}$. Working is not required, so a correct answer scores full marks unless it comes from obvious incorrect working. If (a) gives $x = 9.6$ and (b) gives $y = 12.5$, the marks are M1 A0 M1 A0.	✓

Full marks: 4/4

Question 23 - Exam Solution

Understanding the Question

Given

Nico, Ella and Rosa share £240 in the ratios $3 : 4 : 5$, so Nico takes 3 parts, Ella

Find

The ratio Nico : Ella : Rosa of the amounts of money they hold after the two

takes 4 parts and Rosa takes 5 parts.
Ella and Rosa each give £10 of their share to Nico, so Nico is given £10 twice.

gifts, in its simplest form.

Plan the Solution

- Add the three parts of the ratio to find how many equal shares the £240 is split into.
- Divide by that number to find one share, then multiply to find what each of them starts with.
- Move the money: Nico gains £10 from Ella and £10 from Rosa, and each of those two is £10 worse off.
- Write the three new amounts as a ratio, then divide every part by their highest common factor.

Worked Solution [4 marks]

> Rule - Sharing in a given ratio: $\text{one share} = \dfrac{\text{total}}{\text{sum of the parts}}$, and each person receives their own number of parts lots of one share.

Step 1: count the shares the money is split into

$3 + 4 + 5 = 12$

(Reason: The ratio $3 : 4 : 5$ splits the money into 12 equal shares. It is the parts added together that the total is divided by, never one part on its own.)

Step 2: work out one share

$$\frac{240}{12} = 20$$

(Reason: The £240 is split into 12 equal shares, so one share is worth £20.)

Step 3: work out what each of them starts with

Nico: $3 \times 20 = 60$

Ella: $4 \times 20 = 80$

Rosa: $5 \times 20 = 100$

(Reason: Each of them takes their own number of parts lots of one share, and the three amounts add back to £240.)

Step 4: move the two £10 gifts

Nico: $60 + 10 + 10 = 80$

Ella: $80 - 10 = 70$

Rosa: $100 - 10 = 90$

(Reason: Nico is given £10 by Ella and another £10 by Rosa, so he gains £20 altogether, while Ella and Rosa are each £10 worse off.)

Step 5: write the new amounts as a ratio and simplify

$80 : 70 : 90$

$$\frac{80}{10} : \frac{70}{10} : \frac{90}{10} = 8 : 7 : 9$$

(Reason: The highest common factor of 80, 70 and 90 is 10, so dividing every part by 10 leaves the ratio in its simplest form.)

$$8 : 7 : 9$$

Verification

- ✓ **Check 1:** Add the three new amounts. The £20 that leaves Ella and Rosa arrives with Nico, so the total has to still be £240. $80 + 70 + 90 = 240$
- ✓ **Check 2:** Work backwards. Share £240 in the ratio $8 : 7 : 9$, whose parts add to 24, and the three amounts must come back. $\frac{240}{24} = 10$, then $8 \times 10 = 80$, $7 \times 10 = 70$, $9 \times 10 = 90$
- ✓ **Check 3:** Test that nothing is left to cancel, because the answer has to be in its simplest form. 7 is prime and divides neither 8 nor 9. The only whole number that divides all three parts is 1, so $8 : 7 : 9$ cannot be cancelled any further.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method to find the value of one share	M1	For a correct method to find the value of one share: $\frac{240}{3+4+5} = 20$ or $240 \times \frac{3}{3+4+5} = 60$ oe or $240 \times \frac{4}{3+4+5} = 80$ oe or $240 \times \frac{5}{3+4+5} = 100$ oe. NB $\frac{240}{3} = 80$, $\frac{240}{4} = 60$ and $\frac{240}{5} = 48$ scores Mo.	✓

Step	Mark	Description	Got it?
The correct values for two of the three people after the gifts	M1	For the correct values for 2 of the people after Ella and Rosa give Nico £10. For two of: (Nico) $3 \times [20] + 10 + 10 = 80$ or $[60] + 10 + 10 = 80$; (Ella) $4 \times [20] - 10 = 70$ or $[80] - 10 = 70$; (Rosa) $5 \times [20] - 10 = 90$ or $[100] - 10 = 90$. A value in square brackets is the candidate's own value from the first method mark, so this mark follows through on it.	✓
All three new amounts, or the final ratio in the wrong order, or the final ratio unsimplified	M1	For all 3 of 80, 70 and 90 correct (ignore units), or for the correct values for the final ratio in the wrong order (ignore units), eg $9 : 7 : 8$ oe, or for the correct values for the final ratio unsimplified (ignore units), eg $4 : 3.5 : 4.5$ oe.	✓
The final ratio in its simplest form	A1	$8 : 7 : 9$. Working is not required, so a correct answer scores full marks unless it comes from obvious incorrect working. Other orders are acceptable if they are labelled correctly on the answer line or in the working.	✓

Full marks: 4/4

Question 24 - Exam Solution

Understanding the Question

Given

Lukas pays 4000 Swiss francs for the wristwatch.
The value of the wristwatch increases by 7% each year, and each year's increase is worked out on the value at the start of that year, not on the price he paid.

Find

The value of the wristwatch at the end of 3 years, correct to the nearest Swiss franc.

Plan the Solution

- Turn the 7% rise into a single decimal multiplier, so that one multiplication does a whole year.

- Use that multiplier once for each of the 3 years, which means raising it to the power 3.
- Multiply the 4000 Swiss francs by it to get the value at the end of the third year.
- Round only at the very end, so that no accuracy is lost part-way through.

Worked Solution [3 marks]

> Rule - Repeated percentage change: $\text{final value} = \text{starting value} \times (\text{multiplier})^{n}$, where the multiplier for a rise of $p\%$ is $1 + \dfrac{p}{100}$ and n is the number of years.

Step 1: write the increase as one multiplier

$$100\% + 7\% = 107\%$$

$$\frac{107}{100} = 1.07$$

(Reason: Adding 7% to a value leaves 107% of it, and 107% written as a decimal is 1.07. Multiplying by 1.07 does the whole year in one move, so there is no separate addition to make.)

Step 2: use the multiplier once for each year

$$4000 \times 1.07 \times 1.07 \times 1.07 = 4000 \times 1.07^{3}$$

(Reason: The value is multiplied by 1.07 at the end of every year, so over 3 years it is multiplied by 1.07 three times. Three of the same factor is written as a power, and the power is the number of years.)

Step 3: work out the value at the end of the third year

$$4000 \times 1.07^{3} = 4900.172$$

(Reason: A calculator gives $1.07^{3} = 1.225043$, and the 4000 Swiss francs multiplied by that is the value at the end of the third year, before any rounding.)

Step 4: round to the nearest Swiss franc

$$4900.172 \approx 4900$$

(Reason: The first digit after the decimal point is 1, which is less than 5, so the whole-number part does not change. The rounding is done once, at the end, so the 4579.6 and the other yearly values are never rounded on the way through.)

4900 Swiss francs

Verification

✓ **Check 1:** Do the three rises one at a time instead, working each year's 7% out on the value at the start of that year. $4000 + 280 = 4280$, then $4280 + 299.6 = 4579.6$, then

$4579.6 + 320.572 = 4900.172$

✓ **Check 2:** Undo the three rises. Dividing by 1.07 three times has to give back the price Lukas paid. $\frac{4900.172}{1.07} = 4579.6$, $\frac{4579.6}{1.07} = 4280$, $\frac{4280}{1.07} = 4000$

✓ **Check 3:** Test the size against simple interest. Taking 7% of the original 4000 three times gives $3 \times 280 = 840$, and compound growth must beat that, because the second and third rises are worked out on larger amounts. The rise here is $4900.172 - 4000 = 900.172$, which is $900.172 - 840 = 60.172$ more than the flat 840. Bigger, as compounding requires, and only a little bigger over 3 years.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method to find 7% of 4000 or 107% of 4000	**M1**	For finding 7% of 4000 or 107% of 4000: 0.07×4000 $(= 280)$ or 1.07×4000 $(= 4280)$.	✓
The three yearly rises taken in turn, or the three rises added together	**M1**	For $4000 + [280]$ $(= 4280)$ oe and $0.07 \times [4280]$ $(= 299.6)$ and $[4280] + [299.6]$ $(= 4579.6)$ and $0.07 \times [4579.6]$ $(= 320.572)$, or for $[280] + [299.6] + [320.572]$ $(= 900.172)$. A value in square brackets is the candidate's own value from earlier in the question, so this mark follows through on it. The printed row gives that last total as 900 with its decimal part in brackets, so either form is accepted.	✓
The power method, worth both method marks on its own	**M2**	For $1.07^3 \times 4000$ or $1.07^4 \times 4000$ $(= 5243.18404)$.	✓
The value at the end of 3 years, to the nearest Swiss franc	**A1**	4900. Allow answers in the range 4900 to 4901. Working is not required, so a correct answer scores full marks unless it comes from obvious incorrect working.	✓

Step	Mark	Description	Got it?
Special case, if no other mark is awarded	**SCB1**	If no other mark is awarded, SCB1 for $4000 \times 0.07 \times 3\ (= 840)$ or $4000 \times 0.21\ (= 840)$ or $4000 + 4000 \times 0.07 \times 3\ (= 4840)$ or $4000 \times 1.21\ (= 4840)$ or $0.93 \times 4000\ (= 3720)$ or $0.79 \times 4000\ (= 3160)$ or $0.93^3 \times 4000\ (= 3217.428)$ or $4000 \times 1.07^2\ (= 4579.6)$.	✓
Where the special cases come from, and how the M2 row is used	**Note**	This row awards nothing. Each special case above is one nameable slip. $4000 \times 0.07 \times 3$, 4000×0.21 and 4000×1.21 all take 7% of the original 4000 three times, which is simple interest rather than a value that grows each year. 0.93×4000, 0.79×4000 and $0.93^3 \times 4000$ treat the change as a fall instead of a rise. 4000×1.07^2 uses the multiplier one year too few. On the printed scheme the M2 row sits beside both M1 rows and replaces them, so a candidate who writes it has the two method marks outright.	✓

Full marks: 3/3

Question 25 - Exam Solution

Understanding the Question

Given

The first equation, $3x + 5y = 8$.
The second equation, $4x + y = -3.5$.
Both are linear, so exactly one pair of values fits them both.

Find

The value of x and the value of y that satisfy both equations at once. Algebraic working is required, so the pair has to be reached by elimination or substitution, never by trial.

Plan the Solution

- The y term in the second equation is just y, so multiplying that equation by 5 is enough to make both equations carry $5y$.
- Subtract one equation from the other to remove y and leave a single equation in x.
- Solve that for x, then substitute the value back into the simpler equation to find y.
- Test the pair in both original equations. A sign slip often satisfies one equation and fails the other, so one substitution is not a check.

Worked Solution [3 marks]

> Rule - Elimination: multiply one or both equations so that the coefficients of the same letter match, then add or subtract the equations to remove that letter.

Step 1: label the two equations

$3x + 5y = 8 \qquad (1)$

$4x + y = -3.5 \qquad (2)$

(Reason: Numbering them keeps the working clear, so every line below can say which equation it came from. That is part of what showing clear algebraic working means.)

Step 2: match the y terms

$5 \times (4x + y) = 5 \times (-3.5)$

$20x + 5y = -17.5 \qquad (3)$

(Reason: Equation (2) carries just y, so multiplying every term of it by 5 makes both equations carry $5y$. The right-hand side is multiplied as well: $5 \times (-3.5) = -17.5$.)

Step 3: subtract to eliminate y, then solve for x

$(3x + 5y) - (20x + 5y) = 8 - (-17.5)$

$-17x = 25.5$

$$x = \frac{25.5}{-17} = -1.5$$

(Reason: Both equations now contain $5y$, so subtracting removes y completely and leaves $3x - 20x = -17x$. On the right, subtracting a negative adds: $8 - (-17.5) = 8 + 17.5 = 25.5$. Dividing a positive by a negative then gives a negative value.)

Step 4: substitute back to find y

$4 \times (-1.5) + y = -3.5$

$-6 + y = -3.5$

$y = -3.5 + 6 = 2.5$

(Reason: Equation (2) is the simpler one to put x back into, and $4 \times (-1.5) = -6$. Moving that -6 across the equals sign changes its sign, so 6 is added to -3.5.)

$$x = -1.5$$

$$y = 2.5$$

Verification

✓ **Check 1:** Put both values into equation (1), which was not the equation used to find y.
$3 \times (-1.5) + 5 \times 2.5 = -4.5 + 12.5 = 8$

✓ **Check 2:** Put both values into equation (2) as well. A sign slip usually satisfies one equation and fails the other, so both have to be tested.
$4 \times (-1.5) + 2.5 = -6 + 2.5 = -3.5$

✓ **Check 3:** Solve the pair a second way, eliminating x instead of y: 4 times equation (1) and 3 times equation (2) both give $12x$. $12x + 20y = 32$ minus $12x + 3y = -10.5$ gives $17y = 42.5$, so $y = \dfrac{42.5}{17} = 2.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method to eliminate x or y	**M1**	For a correct method to eliminate x or y: coefficients of x or y the same and correct operator to eliminate the selected variable (condone any one arithmetic error in multiplication), or writing x or y in terms of the other variable and correctly substituting (condone missing brackets). For example $3x + 5y = 8$ with $20x + 5y = -17.5$ and subtracting ($3x - 20x = 8 - (-17.5)$ or $-17x = 25.5$), or $12x + 20y = 32$ with $12x + 3y = -10.5$ and subtracting ($20y - 3y = 32 - (-10.5)$ or $17y = 42.5$), or $3x + 5(-3.5 - 4x) = 8$, or $4x + \dfrac{8 - 3x}{5} = -3.5$, or $3\left(\dfrac{-3.5 - y}{4}\right) + 5y = 8$, or $4\left(\dfrac{8 - 5y}{3}\right) + y = -3.5$. NB The mark is for the method and not for the result of the method. However, if the correct result of the method is seen, the mark can be awarded.	✓
A correct method to find the other variable	**M1**	Dependent on the first M1. For a correct method to find the other variable by substitution of the found variable into one equation, or for repeating the above method to find the second variable. For example $3 \times [-1.5] + 5y = 8$, or $4 \times [-1.5] + y = -3.5$, or $y = -3.5 - 4 \times [-1.5]$, or $y = \dfrac{8 - 3 \times [-1.5]}{5}$, or $3x + 5 \times [2.5] = 8$, or $4x + [2.5] = -3.5$, or $x = \dfrac{-3.5 - [2.5]}{4}$, or $x = \dfrac{8 - 5 \times [2.5]}{3}$. A value in square brackets is the	✓

Step	Mark	Description	Got it?
		candidate's own value from the first method mark, so this mark follows through on it.	
Both values, with working shown	**A1**	$x = -1.5$ and $y = 2.5$, oe, dependent on the first M1. The printed working column says that working is required, so an answer given with no algebraic working scores nothing here.	✓
Marking note	**Note**	oe means or equivalent: the same two values written another way, such as $x = -\frac{3}{2}$ and $y = \frac{5}{2}$, are the same answer. The second method mark depends on the first, so a candidate who never reaches a correct elimination or substitution cannot pick it up by finding a second value from a wrong first one.	✓

Full marks: 3/3

Question 26 - Exam Solution

Understanding the Question

Given

Part (a): the inequality $7 - 3t < 2t + 15$, with t on both sides. Part (b): a shaded region R on a grid, bounded by three straight lines. Read off the grid: one line is upright through 2 on the x axis, one is flat through 3 on the y axis, and the sloping one joins $(0, 9)$ to $(9, 0)$.

Find

Part (a): every value of t that makes the inequality true. Part (b): three inequalities that between them describe R and nothing outside it.

Plan the Solution

- Part (a): rearrange it exactly as you would an equation. Move the t terms to one side and the numbers to the other, then divide.
- Keep an eye on the symbol. It turns round only when both sides are multiplied or divided by a negative number, or when the two sides are swapped over.

- Part (b): write each boundary line as an equation first. Two of them are read straight off the axes; the sloping one comes from the two points where it meets them.
- Then take one point well inside R and see which way each symbol has to point.
- Last, look at whether the edges belong to R. The lines are drawn solid and the corners are part of the region, so each symbol carries an equals sign as well.

Worked Solution [5 marks]

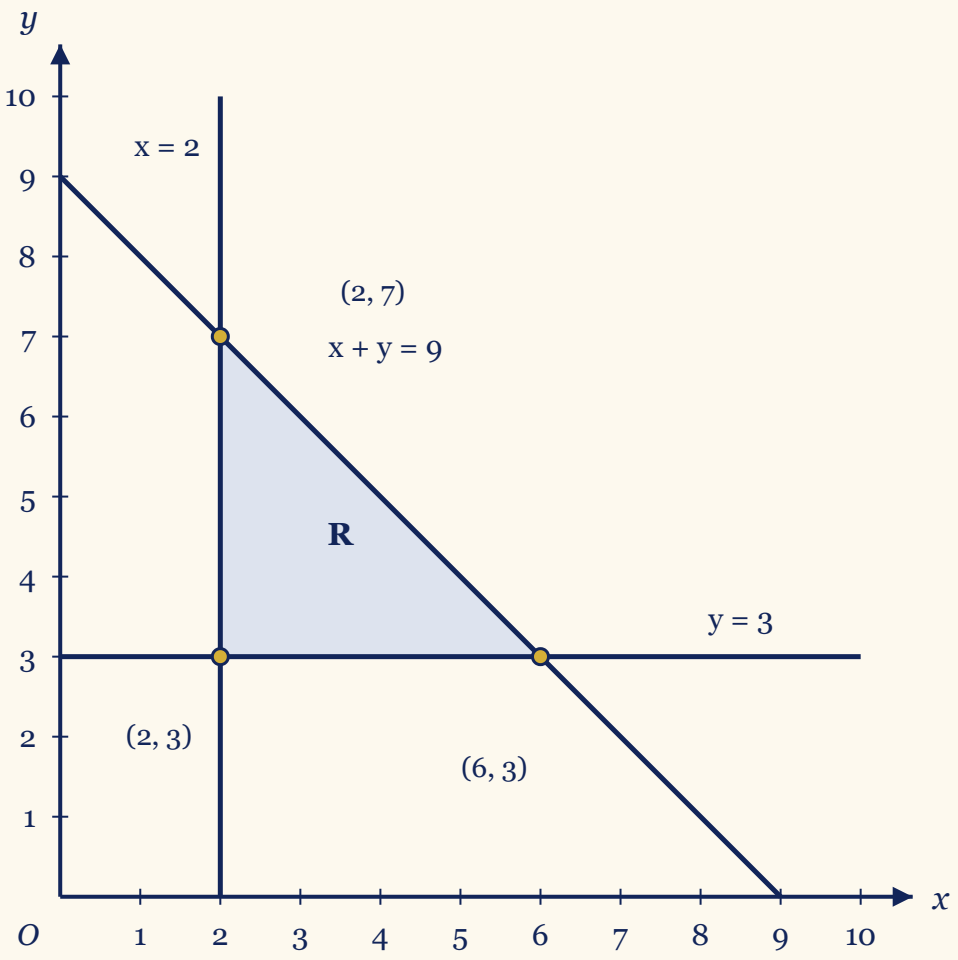
> Rule - Inequalities and regions: rearrange an inequality exactly as you would an equation, remembering that multiplying or dividing both sides by a negative number turns the symbol round; and describe a region by writing each boundary line as an equation first, then fixing the direction of every symbol by testing one point inside the region.

(a) Get the t terms on one side

$7 - 3t < 2t + 15$

$7 - 3t + 3t < 2t + 15 + 3t$

$7 < 5t + 15$



(Reason: Adding $3t$ to both sides clears the $-3t$ on the left and leaves $5t$ on the right. Adding the same amount to both sides never changes which side is the bigger, so the symbol is untouched.)

(a) Get the number terms on the other side

$$7 - 15 < 5t + 15 - 15$$

$$-8 < 5t$$

(Reason: Taking 15 from both sides leaves the $5t$ on its own, and on the left $7 - 15 = -8$. Subtracting the same amount from both sides leaves the symbol alone too.)

(a) Divide both sides by 5

$$\frac{-8}{5} < \frac{5t}{5}$$

$$-1.6 < t$$

(Reason: The divisor is 5, which is positive, so the symbol stays exactly as it is. It is only dividing (or multiplying) by a negative number that turns it round. And $-\frac{8}{5} = -1.6$.)

(a) Write the answer with t first

$$t > -1.6$$

(Reason: Reading $-1.6 < t$ from the other end says that t is the greater of the two. Swapping the two sides over is the other move that turns the symbol round, and the two statements say the same thing.)

(b) Write down the equation of each boundary line

$$x = 2$$

$$y = 3$$

$$x + y = 9$$

(Reason: The upright line crosses the x axis at 2 and never moves left or right, so every point on it has $x = 2$. The flat line has $y = 3$ all the way along. The sloping line joins $(0, 9)$ to $(9, 0)$, and the two coordinates of every point on it add up to 9.)

(b) Test one point inside R to fix each direction

$$3 \geq 2$$

$$4 \geq 3$$

$$3 + 4 \leq 9$$

(Reason: The point $(3, 4)$ sits well inside the shaded triangle. Its x is to the right of $x = 2$, its y is above $y = 3$, and $3 + 4 = 7$, which is below 9. One point inside settles which way all three symbols point.)

(b) Decide whether the edges belong to the region

$$2 + 7 = 9$$

$$6 + 3 = 9$$

(Reason: The corners $(2, 7)$ and $(6, 3)$ lie on the sloping line, and the corner $(2, 3)$ lies on both of the other two. The lines are drawn solid, so the edges count as part of R and each symbol carries an equals sign as well.)

(b) Write the three inequalities

$x \geq 2$

$y \geq 3$

$x + y \leq 9$

(Reason: Each boundary line has become an inequality pointing towards R. Together the three of them describe the shaded triangle and nothing outside it.)

(a) $t > -1.6$

(b) $x \geq 2, y \geq 3, x + y \leq 9$

Verification

✓ **Check 1:** Part (a). Try a value on each side of the boundary. Put $t = 0$ into both sides, then put $t = -2$ into both sides. $7 - 0 = 7$ and $2 \times 0 + 15 = 15$, and $7 < 15$ is true. But $7 - 3 \times (-2) = 13$ and $2 \times (-2) + 15 = 11$, and $13 < 11$ is false. So the values above -1.6 work and the values below it do not.

✓ **Check 2:** Part (a). Put the boundary value $t = -1.6$ into both sides. If it really is the boundary, the two sides must come out equal there. $7 - 3 \times (-1.6) = 11.8$ and $2 \times (-1.6) + 15 = 11.8$, so the two sides meet exactly at that value and nowhere else.

✓ **Check 3:** Part (b). Test the three corners of R against all three inequalities. At $(2, 3)$ the two coordinates add to 5, at $(2, 7)$ they add to 9, and at $(6, 3)$ they add to 9. Every corner has $x \geq 2$ and $y \geq 3$ and a total of at most 9, so all three corners belong to R.

✓ **Check 4:** Part (b). Take one point just outside each edge and confirm that exactly one inequality fails each time. That is what shows no inequality is missing and none of them is cutting into the region. $(1, 5)$ fails $x \geq 2$ and nothing else, $(4, 1)$ fails $y \geq 3$ and nothing else, and $(7, 5)$ fails $x + y \leq 9$ and nothing else.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$-3t - 2t < 15 - 7$ or $-5t < 8$ oe, or $7 - 15 < 2t + 3t$ or $-8 < 5t$ oe, or	M1	for correctly isolating terms in t on one side and number terms on the other side (use of $=$ or any inequality symbol or variable is permitted)	✓

Step	Mark	Description	Got it?
$t = -1.6$ or $t < -1.6$			
(a) $t > -1.6$	**A1**	oe eg $-1.6 < t$ or $t > -\frac{8}{5}$ or $-\frac{8}{5} < t$ oe. Must have correct inequality symbol on answer line. Working not required, so a correct answer scores full marks (unless it comes from obvious incorrect working). NB sight of the correct answer in the working space and just $(t =) -1.6$ oe on the answer line gains M1 only.	✓
(b) $x \geq 2$	**B1**	oe, allow $x > 2$ or $2 < x$	✓
(b) $y \geq 3$	**B1**	oe, allow $y > 3$ or $3 < y$	✓
(b) $x + y \leq 9$	**B1**	oe, allow $x + y < 9$ or $y < 9 - x$ or $9 > x + y$	✓
(b) special case	**SC B2**	for all of $x \leq 2, y \leq 3, x + y \geq 9$ oe, or $x < 2$, $y < 3, x + y > 9$	✓
(b) special case	**SC B1**	for all of $x = 2, y = 3, x + y = 9$ oe	✓
(b) what the two special cases are	**Note**	The first is the script of a student who found the right three lines but pointed every symbol the wrong way, so the answer describes the region outside the triangle. The second is the script of a student who wrote the three boundary lines as equations and never turned them into inequalities at all.	✓

Full marks: 5/5

Question 27 - Exam Solution

Understanding the Question

Given

Triangles ABD and ABC are both right-angled at B

Find

The length of CD The answer correct to 3 significant figures

$AB = 16$ cm and $BC = 12$ cm
$AD = 1.5 \times AC$
B, C and D lie on one straight line, with C between B and D

Plan the Solution

- Triangle ABC has both short sides given, so Pythagoras gives its hypotenuse AC.
- Scale that by 1.5 to get AD, which is the hypotenuse of triangle ABD.
- In triangle ABD the hypotenuse and one short side are then known, so Pythagoras gives BD.
- C sits on BD, so CD is what is left when BC is taken off BD.
- Keep the full decimal for BD and round only at the very end.

Worked Solution [5 marks]

> Pythagoras' theorem. In a right-angled triangle the square on the hypotenuse equals the sum of the squares on the other two sides, so $h^2 = a^2 + b^2$ when the hypotenuse is the unknown, and $a^2 = h^2 - b^2$ when the hypotenuse is the side already known.

Step 1: Pythagoras in triangle ABC

$$AC^2 = BC^2 + AB^2$$
$$12^2 + 16^2 = 144 + 256 = 400$$
$$AC = \sqrt{400} = 20 \text{ cm}$$

(Reason: The right angle is at B, so AC is the hypotenuse of this triangle and the two squares are added.)

Step 2: Scale AC up to get AD

$$AD = 1.5 \times AC$$
$$1.5 \times 20 = 30$$

(Reason: The question states that AD is 1.5 times AC, and AC has just been found to be 20 cm.)

Step 3: Pythagoras in triangle ABD

$$BD^2 = AD^2 - AB^2$$
$$30^2 - 16^2 = 900 - 256 = 644$$
$$BD = \sqrt{644} = 25.3771\ldots$$

(Reason: This time the hypotenuse AD is the side that is known, so AB^2 is taken away rather than added. Keep the full decimal for the next step.)

Step 4: Subtract to reach CD

$CD = BD - BC$

$25.3771\ldots - 12 = 13.3771\ldots$

(Reason: B, C and D lie on one straight line with C between the other two, so the two lengths simply subtract.)

Step 5: Round to 3 significant figures

$CD = 13.3771\ldots \approx 13.4$ cm

(Reason: The first three significant figures of $13.3771\ldots$ are 1, 3 and 3. The next digit is 7, so the third one rounds up to 4.)

$$CD = 13.4 \text{ cm}$$

Verification

✓ **Check 1:** Work backwards. If $BD = 25.3771\ldots$ and $AB = 16$, then Pythagoras must rebuild the hypotenuse AD. $644 + 256 = 900$ and $\sqrt{900} = 30$, which is exactly 1.5×20

✓ **Check 2:** Reach BD by trigonometry instead. In triangle ABD, $\cos BAD = \dfrac{16}{30}$, and then $BD = 30 \sin BAD$. $BAD = 57.769\ldots^\circ$ and $30 \sin 57.769\ldots^\circ = 25.377\ldots$, giving $CD = 13.4$ cm again

✓ **Check 3:** Do it in surd form, with no decimal until the last line.
$BD^2 = 2.25 \times 400 - 256$, so $BD = 2\sqrt{161}$.
$2.25 \times 400 - 256 = 900 - 256 = 644$ and $644 = 4 \times 161$, so
$CD = 2\sqrt{161} - 12 = 13.377\ldots$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(AC^2 =)\ 12^2 + 16^2\ (= 144 + 256 = 400)$ or $(BAC =) \tan^{-1}\left(\dfrac{12}{16}\right)\ (= 36.8(698\ldots))$ or 36.9 or $(BCA =) \tan^{-1}\left(\dfrac{16}{12}\right)\ (= 53.1(301\ldots))$	M1	for a correct method using triangle ABC	✓

Step	Mark	Description	Got it?
$(AC =)\sqrt{12^2+16^2}\ (=\sqrt{144+256}=\sqrt{400}=20)$ or $(AC =)\dfrac{16}{\cos 36.8^\circ}\ (=20)$ or $(AC =)\dfrac{12}{\sin 36.8^\circ}\ (=20)$ or $(AC =)\dfrac{16}{\sin 53.1^\circ}\ (=20)$ or $(AC =)\dfrac{12}{\cos 53.1^\circ}\ (=20)$	M1	for a correct method to find AC. The printed scheme puts 36.8 and 53.1 in quotation marks, so a candidate's own angle from the first row may be used in their place.	✓
$(BD^2 =)(1.5\times 20)^2-16^2\ (=644)$ or $(BD^2 =)30^2-16^2\ (=900-256=644)$ or $(BAD =)\cos^{-1}\left(\dfrac{16}{30}\right)\ (=57.7(690\ldots))$ or 57.8 or $(BDA =)\sin^{-1}\left(\dfrac{16}{30}\right)\ (=32.2(309\ldots))$ or $(BCA =)\sin^{-1}\left(\dfrac{16}{20}\right)\ (=53.1(301\ldots))$ and $\dfrac{CD}{\sin(180-126.9-32.2)}=\dfrac{30}{\sin(180-53.1)}$ or equivalent	M1	for a correct method using a triangle to find BD^2 or angle BAD or angle BDA, or for a correct equation for side CD. The printed scheme puts 20 and 30 in quotation marks, so the candidate's own earlier values may be used.	✓
$(BD =)\sqrt{(1.5\times 20)^2-16^2}\ (=25.3(771\ldots))$ or $(BD =)\sqrt{30^2-16^2}\ (=\sqrt{900-256}=\sqrt{644}=2\sqrt{161}=25.3(771\ldots))$ or $(BD =)16\tan 57.7^\circ\ (=25.3(771\ldots))$ or $(BD =)30\sin 57.7^\circ\ (=25.3(771\ldots))$ or $(BD =)\sqrt{16^2+30^2-2\times 16\times 30\cos 57.7^\circ}\ (=25.3(771\ldots))$ or $(BD =)30\cos 32.2^\circ\ (=25.3(771\ldots))$ or $(BD =)\dfrac{16}{\tan 32.2^\circ}\ (=25.3(771\ldots))$ or	M1	for a correct method to find BD or CD. Here too the printed scheme quotes 20, 30 and the angles, so the candidate's own earlier values are allowed in their place.	✓

Step	Mark	Description	Got it?
$(BD =)\dfrac{16}{\sin 32.2^\circ} \times \sin 57.7^\circ \ (= 25.3(771\ldots))$ or $(CD =)\dfrac{30}{\sin 126.9^\circ} \times \sin 20.9^\circ$ or equivalent			
the final answer	**A1**	awrt 13.4	✓
Working not required, so a correct answer scores full marks	**Note**	unless it comes from obviously incorrect working	✓

Full marks: 5/5

Question 28 - Exam Solution

Understanding the Question

Given

A jar of glass beads: 17 green, 28 yellow, and the rest purple.
One bead is taken at random, and the probability that it is purple is $\dfrac{4}{9}$.

Find

The number of purple beads in the jar.
The jar's total is not given, so it has to come out of the $\dfrac{4}{9}$ as well.

Plan the Solution

- Green and yellow are the only other colours, so adding them gives the number of beads that are not purple.
- Take $\dfrac{4}{9}$ away from 1 to get the probability that a bead is not purple.
- Those beads make up $\dfrac{5}{9}$ of the jar, so dividing by 5 gives one ninth of it.
- Purple takes four of the nine ninths, so multiply one ninth by 4.

Worked Solution [3 marks]

> Rule - The probabilities of all the outcomes add to 1, so the beads that are not purple make up $1 - \dfrac{4}{9}$ of the jar. Ninths are equal shares, so every ninth holds the same number

of beads.

Step 1: Count the beads that are not purple

$17 + 28 = 45$

(Reason: Green and yellow are the only other colours in the jar, so these 45 beads are exactly the ones that are not purple.)

Step 2: Work out the probability that a bead is not purple

$$1 - \frac{4}{9} = \frac{5}{9}$$

(Reason: A bead taken at random is either purple or it is not, so the two probabilities add to 1.)

Step 3: Find how many beads make one ninth of the jar

$$\frac{45}{5} = 9$$

(Reason: The 45 beads that are not purple are $\frac{5}{9}$ of the jar, which is five equal ninths of it.)

Step 4: Scale one ninth up to the purple share

$4 \times 9 = 36$

(Reason: Purple takes four of the nine equal ninths, and each ninth holds 9 beads.)

36 purple beads

Verification

✓ **Check 1:** Put the purple beads back with the others: $45 + 36 = 81$ beads in the jar altogether. Now read the probability straight off that jar. $\frac{36}{81} = \frac{4}{9}$

✓ **Check 2:** Test the other colours the same way. Out of 81 beads, 45 are green or yellow, and that share should be the $\frac{5}{9}$ from Step 2. $\frac{45}{81} = \frac{5}{9}$, and $\frac{4}{9} + \frac{5}{9} = 1$

✓ **Check 3:** Reach it by algebra instead of by ninths. With p purple beads the jar holds $p + 45$ beads, so $\frac{p}{p+45} = \frac{4}{9}$. Multiplying out the denominators gives $9p = 4p + 180$. $5p = 180$, so $p = 36$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{17+28}{9-4}$ or $\frac{45}{5}$ or $1-\frac{4}{9}\left(=\frac{5}{9}\right)$ or $17+28$ identified as $\frac{5}{9}$ of the beads, oe, or $\frac{p}{p+28+17}=\frac{4}{9}$ oe or $\frac{p}{p+45}=\frac{4}{9}$ or $\frac{m-45}{m}=\frac{4}{9}$	**M1**	Allow $0.55(555\ldots)$ or $55(.555\ldots)\%$ truncated or rounded	✓
$\frac{17+28}{5}\times 4$ or $\frac{45}{5}\times 4$ or $17+28$ divided by their $\frac{5}{9}$ to give 81 or 45 divided by their $\frac{5}{9}$ to give 81 or $17+28$ multiplied by their $\frac{9}{5}$ to give 81 or 45 multiplied by their $\frac{9}{5}$ to give 81 or their $\frac{5}{9}=\frac{17+28}{n}$ oe or $n=81$ or $9p=4(p+28+17)$ or $9p-4p=180$ oe or $5p=180$ oe or $9(m-45)=4m$ or $9m-4m=405$ oe or $5m=405$ oe or $m=81$	**M1**	for the correct calculation for the total number of beads or for the correct calculation for the number of purple beads or for the correct equation for the total number of beads (removing the denominators) or for the correct equation for the number of purple beads (removing the denominators). A value written as the candidate's own, shown here as "their", may be taken from their earlier working.	✓
Working not required, so correct answer scores full marks (unless from obvious incorrect working)	**A1**	36 cao	✓

Full marks: 3/3

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