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Edexcel IGCSE 4MA1

4MA1/2H (Higher Tier) – Wednesday 4 June 2025

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/2H (Higher Tier), June 2025 series, sat Wednesday 4 June 2025 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Theo has six tiles.
He writes one number on each tile so that

		6		10	14

the range of the numbers is 10
the median of the numbers is 7.5
the mode of the numbers is 6

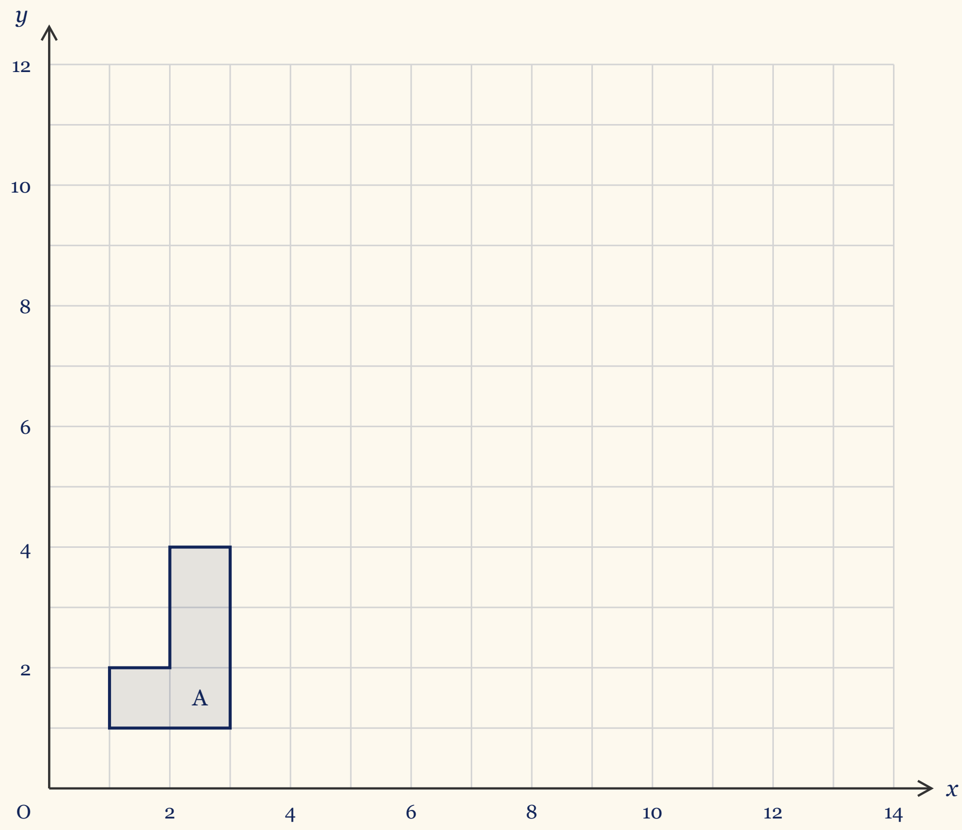
Theo lays the tiles in a row so that the numbers are in order of size.

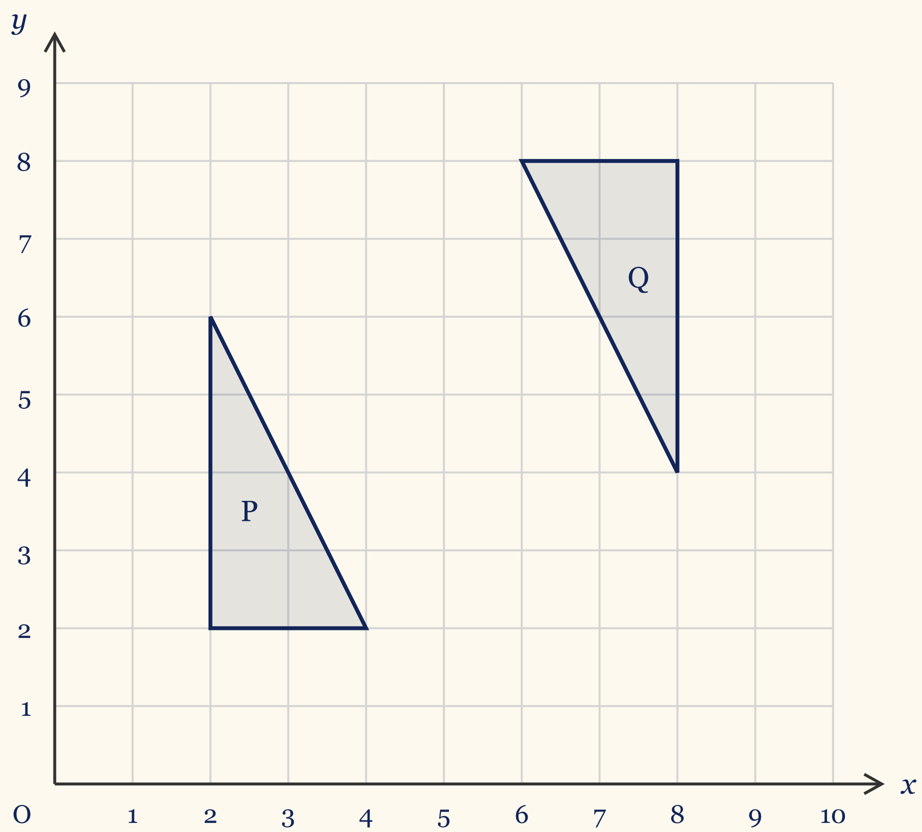
The numbers on three of the tiles are hidden.

Write the three hidden numbers on the tiles above. *[3 marks]*

[Total 3 marks]

2. (a) On the grid, draw an enlargement of shape A with scale factor 3 and centre $(0, 1)$. *[2 marks]*





(b) Describe fully the single transformation that takes triangle P to triangle Q. *[3 marks]*

(b) ....

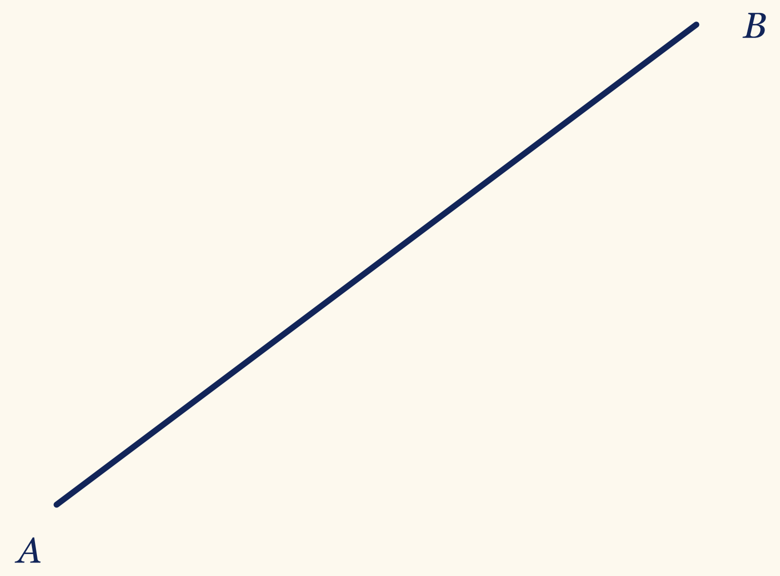
[Total 5 marks]

3. Show that $7\frac{1}{3} - 3\frac{4}{7} = 3\frac{16}{21}$

You must show your working. *[3 marks]*

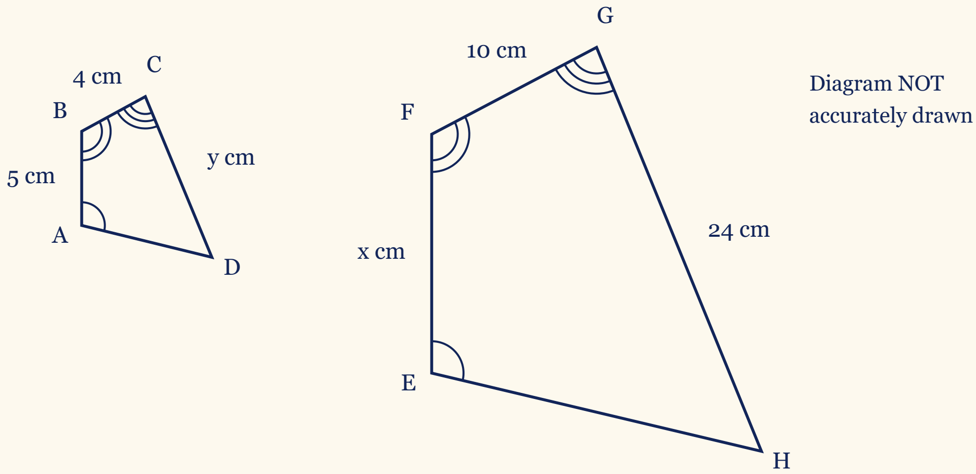
[Total 3 marks]

4. Use a ruler and a pair of compasses only to construct the perpendicular bisector of the line AB
You must leave all of your construction lines on the diagram. *[2 marks]*



[Total 2 marks]

5. The diagram shows quadrilateral $ABCD$ and quadrilateral $EFGH$.
The two quadrilaterals are similar.



$AB = 5$ cm, $BC = 4$ cm, $CD = y$ cm
$EF = x$ cm, $FG = 10$ cm, $GH = 24$ cm

(a) Find the value of x *[2 marks]*

(b) Find the value of y *[2 marks]*

x = ...

y = ...

[Total 4 marks]

6. Marta, Leo and Freya are paid £240 for clearing a garden.
They share the money in the ratios $3 : 4 : 5$

Leo and Freya each give £10 of their share to Marta.

Work out the ratio of the amounts of money that Marta, Leo and Freya now have.
Give your answer in its simplest form. *[4 marks]*

....

[Total 4 marks]

7. Bruno buys a painting for $4\,000$ Swiss francs.
The value of the painting increases by 7% each year.

Work out the value of the painting at the end of 3 years.
Give your answer correct to the nearest Swiss franc. *[3 marks]*

.... Swiss francs

[Total 3 marks]

8. Find the values of x and y that satisfy both of these equations.

$$3x + 5y = 8$$
$$4x + y = -3.5$$

You must show clear algebraic working. *[3 marks]*

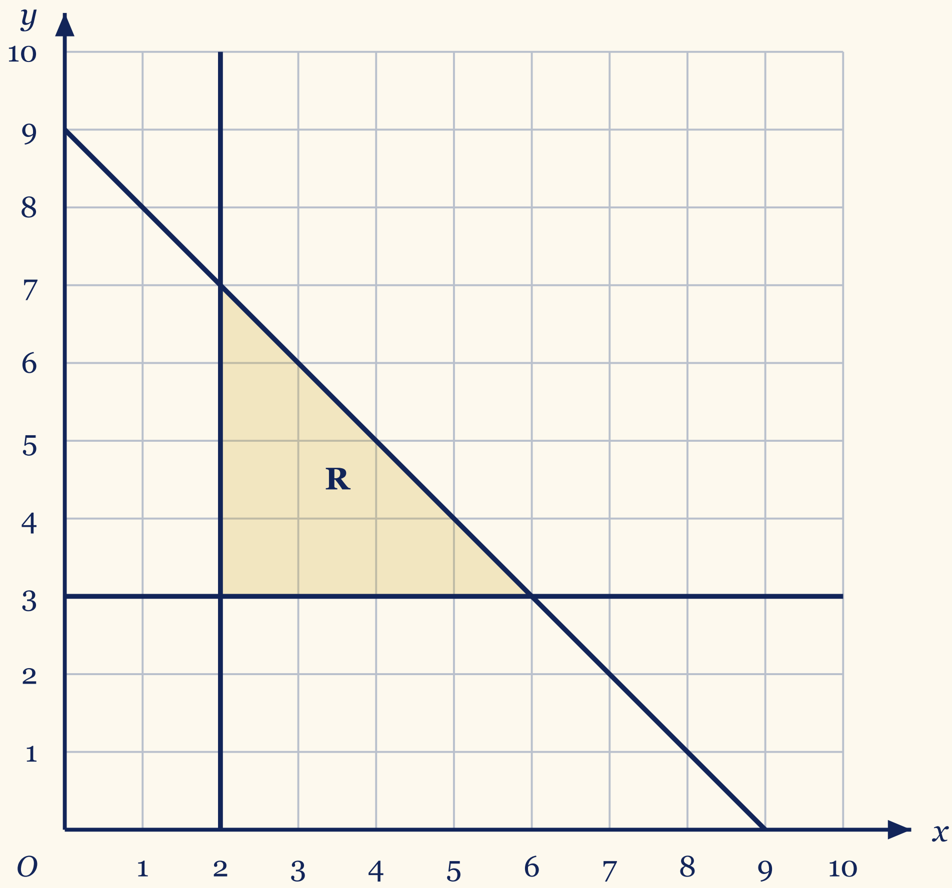
x = ...

y = ...

[Total 3 marks]

9. (a) Solve the inequality

$7 - 3t < 2t + 15$ *[2 marks]*



The region R, shown shaded on the grid below, is bounded by three straight lines.

(b) Write down three inequalities that describe the region R. *[3 marks]*

(a) ....

(b) ....

(b) ....

(b) ....

[Total 5 marks]

10. ABD and ABC are right-angled triangles.

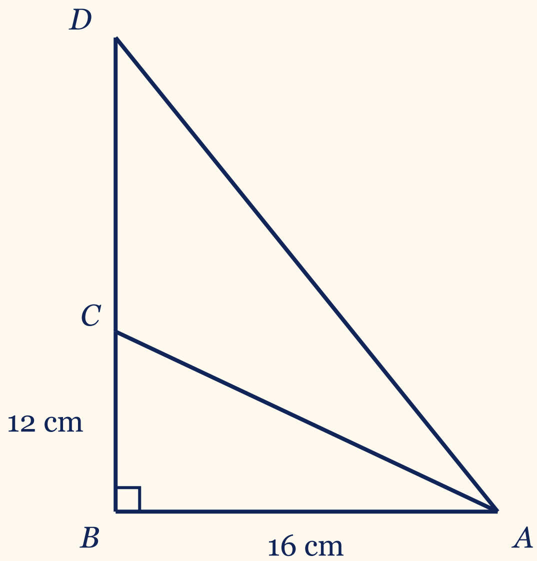


Diagram NOT
accurately drawn

$AB = 16$ cm $\quad BC = 12$ cm $\quad AD = 1.5 \times AC$

Work out the length of CD.
Give your answer correct to 3 significant figures. *[5 marks]*

.... cm

[Total 5 marks]

11. Yusuf has a jar of glass beads.

17 of the beads are green
28 of the beads are yellow
the rest of the beads are purple

Yusuf is going to take at random a bead from the jar.

The probability that Yusuf will take a purple bead is $\frac{4}{9}$

Work out the number of purple beads that are in the jar. *[3 marks]*

....

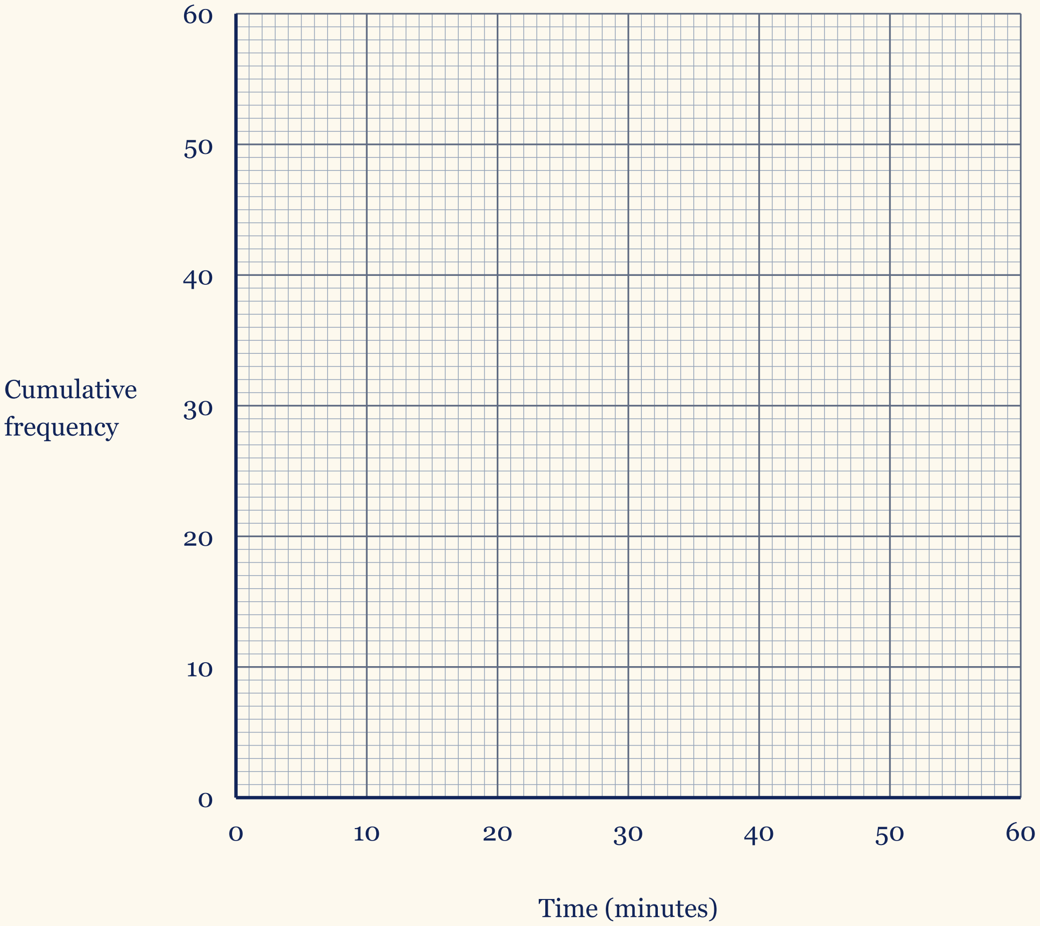
[Total 3 marks]

12. Multiply out and simplify $3x(2x+5)(7x-4)$ *[3 marks]*

....

[Total 3 marks]

13. The frequency table gives information about the times, in minutes, that 60 visitors took to find their way through a hedge maze.



Time (t minutes)	**Frequency**
$0 < t \leq 10$	8
$10 < t \leq 20$	13
$20 < t \leq 30$	12
$30 < t \leq 40$	17
$40 < t \leq 50$	7
$50 < t \leq 60$	3

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$0 < t \leq 10$	
$0 < t \leq 20$	
$0 < t \leq 30$	
$0 < t \leq 40$	
$0 < t \leq 50$	
$0 < t \leq 60$	

[1 mark]

(b) On the grid below, draw a cumulative frequency graph for your table. *[2 marks]*

(c) Use your graph to find an estimate for the median time. *[1 mark]*

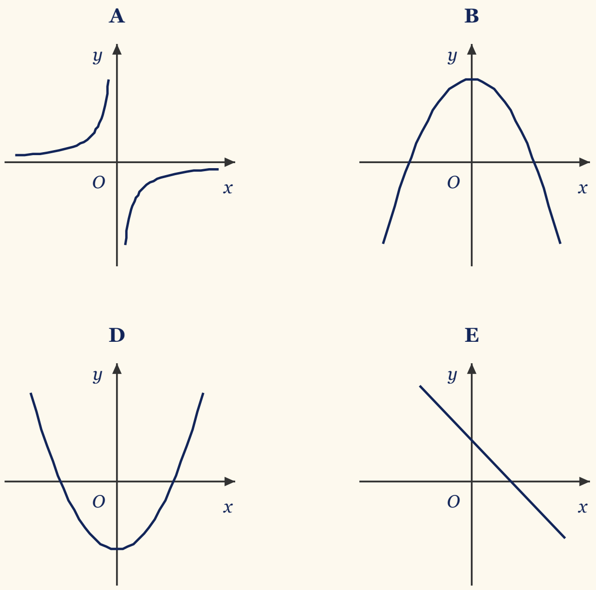
(d) Use your graph to find an estimate for the number of these visitors who took more than 45 minutes to find their way through the maze. *[2 marks]*

(c) minutes ....

(d) ....

[Total 6 marks]

14. The five graphs below are labelled A to E.



(a) State the letter of the graph that could have the equation $y = 5 - x^2$ *[1 mark]*

(b) State the letter of the graph that could have the equation $y = 2x^3$ *[1 mark]*

(a) ....

(b) ....

[Total 2 marks]

15. Simplify fully

$$\left(\frac{2a^3}{10a^7c^2}\right)^{-3}$$ *[3 marks]*

....

[Total 3 marks]

16. Rearrange the formula

$$c = \frac{t^2 + 3}{7 - 8t^2}$$

to make t the subject. *[4 marks]*

....

[Total 4 marks]

17. $y = 4x^3 + 5x^2 + 2x$

(a) Find $\frac{dy}{dx}$ *[2 marks]*

(b) Work out the coordinates of the turning points on the curve with equation $y = 4x^3 + 5x^2 + 2x$

You must show clear algebraic working. *[4 marks]*

(a) ....

(b) ....

[Total 6 marks]

18. Show, using algebra, that $0.\dot{7}0\dot{2} = \frac{26}{37}$ *[2 marks]*

[Total 2 marks]

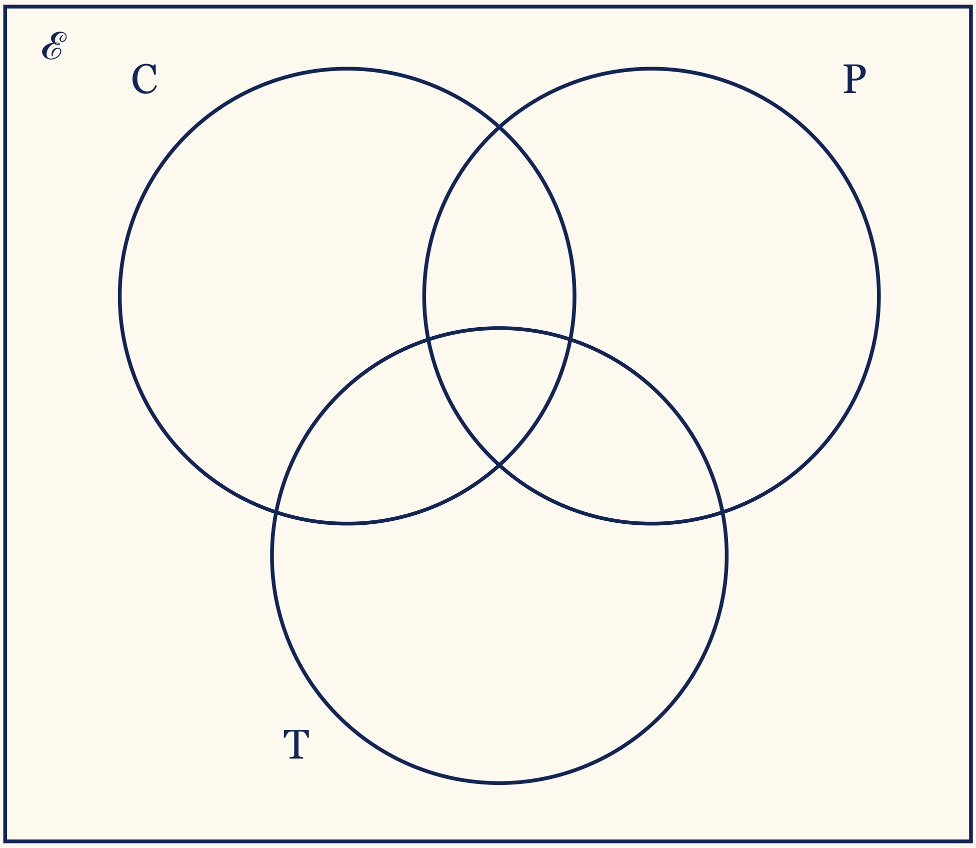
19. Solve the inequality $4x^2 + 4x - 15 < 0$

You must show clear algebraic working. *[3 marks]*

....

[Total 3 marks]

20. 120 members of a sports club were asked whether they play cricket (C) or padel (P) or tennis (T)



Of these members

43 play cricket
12 play cricket and padel and tennis
18 play cricket and padel
27 play cricket and tennis
32 play padel and tennis
29 do not play cricket or padel or tennis

The number of these members who play only padel is equal to the number of these members who play only tennis.

(a) Complete the Venn diagram to show this information.

[3 marks]

(b) Find $\mathrm{n}(T' \cap C)$ *[1 mark]*

One of the members who plays cricket is chosen at random.

(c) Calculate the probability that this member also plays padel. *[2 marks]*

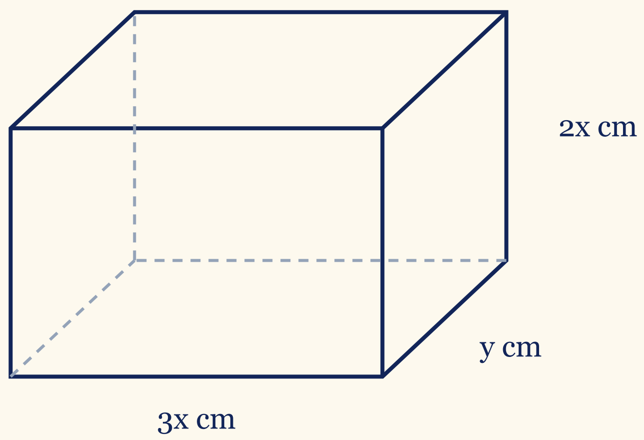
(b) ....

(c) ....

[Total 6 marks]

21. The diagram shows a cuboid.

Diagram NOT accurately drawn

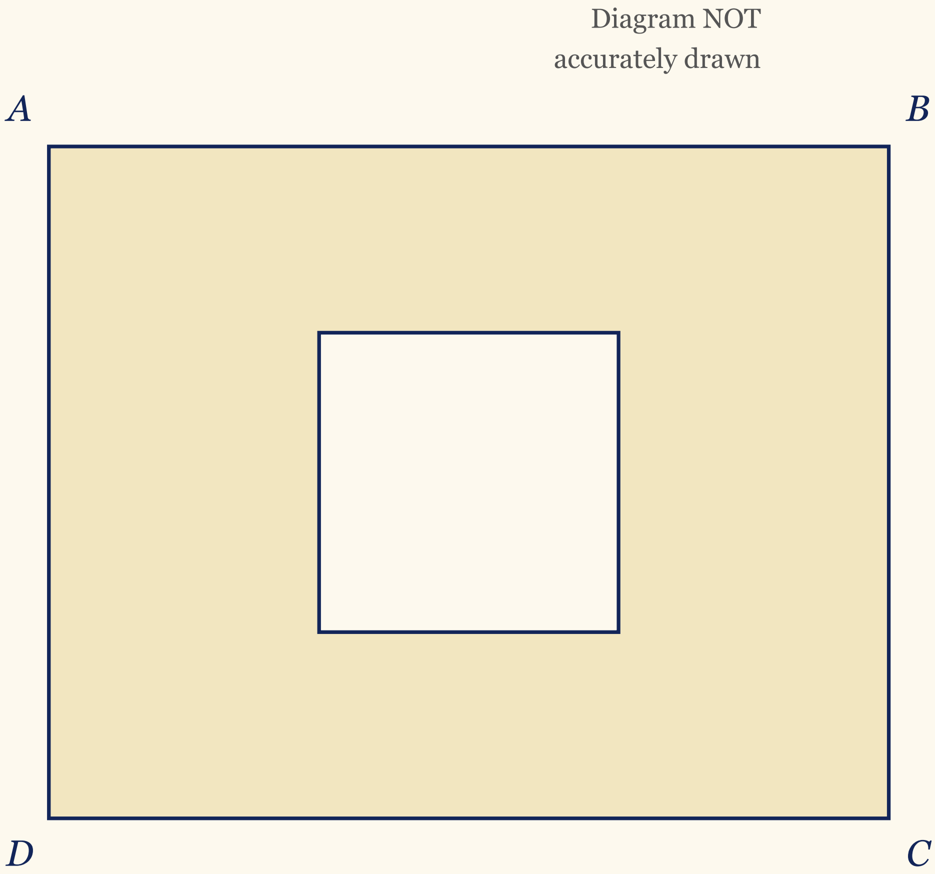


The edges of the cuboid are $3x$ cm, $2x$ cm and y cm
This cuboid has a volume of 1014 cm³
Its total surface area is A cm²

Show that $A = 12x^2 + \dfrac{1690}{x}$
You must show every stage of your working. *[3 marks]*

[Total 3 marks]

22. Rectangle $ABCD$ has a square drawn inside it, as shown in the diagram.



The part of the rectangle that lies outside the square is shaded.
The total area of the shaded region is X cm^2.

$AB = 11.5$ cm correct to the nearest 0.5 cm
$BC = 9.2$ cm correct to 2 significant figures
side of the square $= 4.1$ cm correct to 2 significant figures

By considering bounds, work out the value of X to a suitable degree of accuracy.
You must show your working clearly. *[4 marks]*

X = ...

[Total 4 marks]

23. $$\frac{\dfrac{4x^2 - 4x - 120}{5x^2 - 180}}{\dfrac{x^2 + 5x}{10x^2 + 60x}} = p$$ where p is an integer.

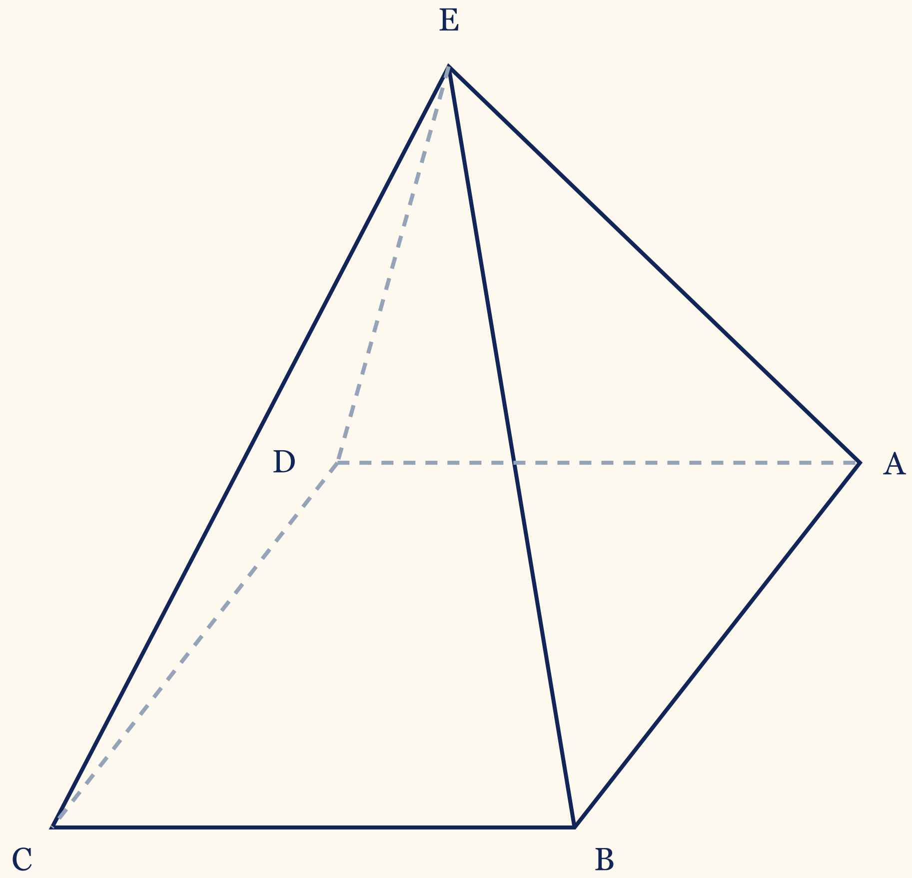
Work out the value of p

Show clear algebraic working. *[4 marks]*

p = ...

[Total 4 marks]

24. The diagram shows a square-based pyramid $ABCDE$.



Not drawn accurately

$EA = EB = EC = ED$

M is the centre of the horizontal square base $ABCD$
Q is the midpoint of AB
Angle $EQM = 80°$

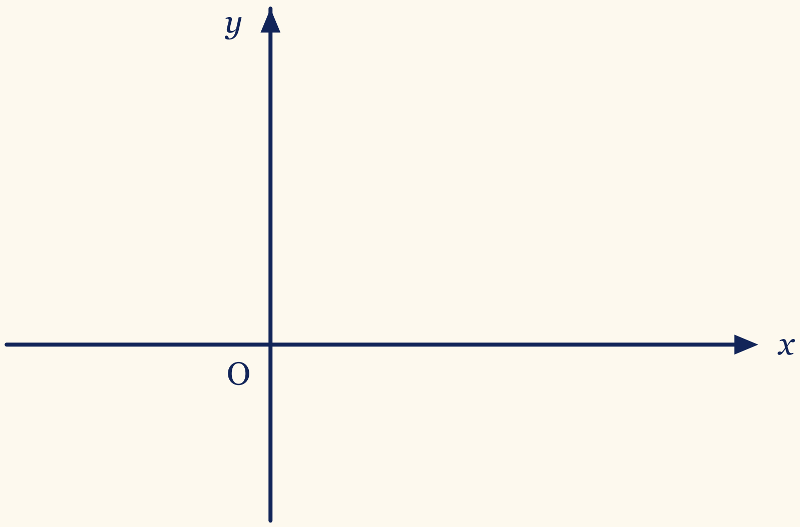
$EA : AB = n : 1$

Work out the value of n
Give your answer correct to 3 significant figures. *[4 marks]*

n = ...

[Total 4 marks]

25. (a) Express $28 + 24x - 6x^2$ in the form $a - b(x - c)^2$, where a, b and c are integers. *[3 marks]*



(b) On the axes below, sketch the curve with equation
$y = 28 + 24x - 6x^2$
Show clearly the coordinates of the turning point and the coordinates of the point where the curve meets the y-axis. *[3 marks]*

....

[Total 6 marks]

26. Two solid candles, A and B, are mathematically similar.

The height of candle A is 31 cm
The height of candle B is 18.6 cm

Given that
volume of candle A $-$ volume of candle B $= 735$ cm^3

work out the volume of candle A *[4 marks]*

.... cm^3

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Six numbers, one on each tile, laid out in order of size: a, b, 6, c, 10, 14
The range of the numbers is 10
The median of the numbers is 7.5
The mode of the numbers is 6

Find

The three hidden numbers a, b and c - the first, second and fourth tiles.

Plan the Solution

- The tiles are in order of size, so 14 is the largest number and a is the smallest. The range gives a straight away.
- Six numbers have two middle ones, the third and the fourth. The median gives c.
- Once a and c are known, every settled number appears just once, so the mode is what forces a second 6. That gives b.
- Finish by testing all three statistics on the completed row.

Worked Solution [3 marks]

> Rule - Range $=$ largest $-$ smallest. For six numbers in order of size the median is $\dfrac{3\text{rd} + 4\text{th}}{2}$. The mode is the value that appears most often.

Step 1: Name the three hidden numbers

$a, b, 6, c, 10, 14$

4	6	6	9	10	14

(Reason: The tiles are already in order of size, so a is the smallest number and 14 is the largest.)

Step 2: Use the range to find the first tile

$$14 - a = 10$$

$$a = 14 - 10 = 4$$

(Reason: The range is the largest number take away the smallest, and the row is in order, so those two are 14 and a.)

Step 3: Use the median to find the fourth tile

$$\frac{6 + c}{2} = 7.5$$

$$c = 2 \times 7.5 - 6 = 9$$

(Reason: Six numbers have two middle ones. Doubling the median and taking away the other middle number leaves c.)

Step 4: Use the mode to find the second tile

$$4, b, 6, 9, 10, 14$$

$$b = 6$$

(Reason: 4, 9, 10 and 14 each appear once, so the mode can only be 6 if a second 6 is written. The row is in order of size, so b lies between 4 and 6, and only 6 itself makes 6 the most common number.)

Step 5: Write the three numbers on the tiles

$$4, 6, 6, 9, 10, 14$$

(Reason: The hidden numbers are 4, 6 and 9, on the first, second and fourth tiles.)

First tile: 4

Second tile: 6

Fourth tile: 9

Completed row: 4, 6, 6, 9, 10, 14

Verification

✓ **Check 1:** Take the smallest number of the completed row away from the largest. $14 - 4 = 10$, which is the range given.

✓ **Check 2:** The two middle numbers of the completed row are 6 and 9. Work out their mean. $\frac{6+9}{2} = 7.5$, which is the median given.

✓ **Check 3:** Count how many times each number appears in the completed row, and read the row from left to right. 6 appears twice and every other number appears once, so the mode is 6, and the row is still in order of size.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
First tile	**B1**	4	✓
Second tile	**B1**	6, or a list of 6 numbers with a mode of 6	✓
Fourth tile	**B1**	9	✓
Special case	**SC B2**	for 4, 6 and 9 in the incorrect order	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

(a) Shape A has vertices $(1, 1)$, $(3, 1)$, $(3, 4)$, $(2, 4)$, $(2, 2)$ and $(1, 2)$.
(a) Scale factor 3, centre $(0, 1)$.
(b) Triangle P has vertices $(2, 2)$, $(2, 6)$ and $(4, 2)$.
(b) Triangle Q has vertices $(6, 8)$, $(8, 8)$ and $(8, 4)$.

Find

(a) The image of shape A, drawn on the grid. (b) The single transformation that takes triangle P to triangle Q, described fully.

Plan the Solution

- (a) Take one vertex at a time. Count the step from the centre to the vertex, multiply that step by 3, then count that far out from the centre again.
- (a) Join the six images in the same order as the original, so the image comes out the same shape the right way round.
- (b) Check the size first. P and Q are congruent, so nothing that changes the size can do it.
- (b) Join each vertex of P to the matching vertex of Q. A half turn puts the centre at the midpoint of every join, so all three midpoints must agree.

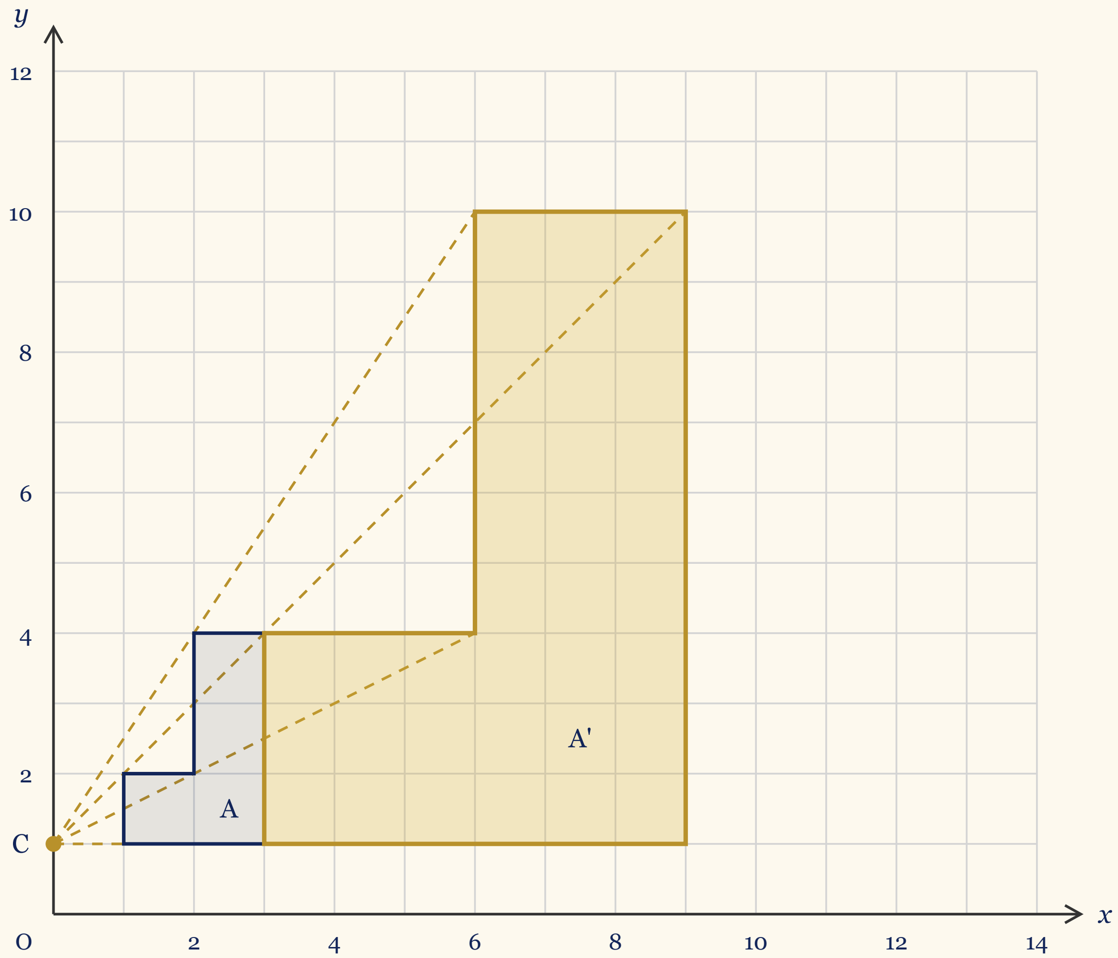
- (b) A full description of a rotation needs three things: the word rotation, the angle, and the centre.

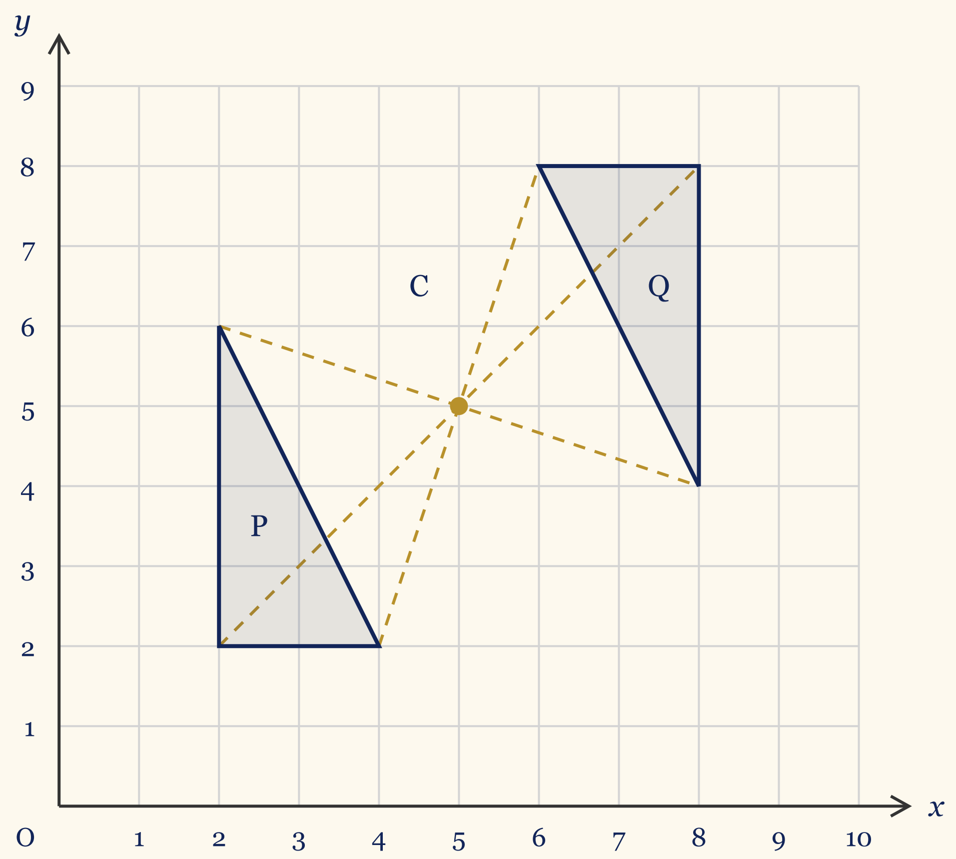
Worked Solution [5 marks]

Rule - Enlargement with scale factor k about a centre C: count the step from C to the point, multiply that step by k, and count out from C again. Rule - A half turn about (a, b) sends (x, y) to $(2a - x, 2b - y)$, so the centre is the midpoint of every point and its image.

Step 1: Read the vertices of shape A off the grid

$$(1, 1),\ (3, 1),\ (3, 4),\ (2, 4),\ (2, 2),\ (1, 2)$$





(Reason: (Reason: the enlargement is worked out corner by corner, so the six vertices have to be read off before anything is multiplied. Going round the outline in order is what keeps the L the right way round later.))

Step 2: Enlarge each vertex from the centre $(0, 1)$

$(1,1) \rightarrow (0,1) + 3(1,0) = (3,1)$
$(3,1) \rightarrow (0,1) + 3(3,0) = (9,1)$
$(3,4) \rightarrow (0,1) + 3(3,3) = (9,10)$
$(2,4) \rightarrow (0,1) + 3(2,3) = (6,10)$
$(2,2) \rightarrow (0,1) + 3(2,1) = (6,4)$
$(1,2) \rightarrow (0,1) + 3(1,1) = (3,4)$

(Reason: (Reason: the bracket is the step from the centre to the vertex, not the vertex itself. Multiplying that step by 3 and adding it back on to the centre lands on the image.))

Step 3: Plot the six images and join them in the same order

$(3,1),\ (9,1),\ (9,10),\ (6,10),\ (6,4),\ (3,4)$

(Reason: (Reason: joining them in the original order keeps the L the right way round. The image lands directly to the right of A and shares the whole edge from $(3,1)$ to $(3,4)$ with it, which is a quick way to see the drawing is in the right place.))

Step 4: Compare triangle P with triangle Q

$(2,2) \to (8,8)$
$(2,6) \to (8,4)$
$(4,2) \to (6,8)$

(Reason: (Reason: both triangles have a short side of 2 and a long side of 4, so they are congruent and nothing that changes the size can be involved. The two legs of P point up and right; the matching legs of Q point down and left, which is a turn of $180°$.))

Step 5: Join matching vertices and take the midpoint of each join

$$\left(\frac{2+8}{2}, \frac{2+8}{2}\right) = (5,5)$$
$$\left(\frac{2+8}{2}, \frac{6+4}{2}\right) = (5,5)$$
$$\left(\frac{4+6}{2}, \frac{2+8}{2}\right) = (5,5)$$

(Reason: (Reason: a half turn carries every point to the far side of the centre and the same distance away, so the centre is the midpoint of every join. All three midpoints agree, and that agreement is the check that the centre is right.))

Step 6: Test the rule about $(5,5)$

$(x,y) \to (2 \times 5 - x, 2 \times 5 - y) = (10 - x, 10 - y)$
$(2,2) \to (10-2, 10-2) = (8,8)$
$(2,6) \to (10-2, 10-6) = (8,4)$
$(4,2) \to (10-4, 10-2) = (6,8)$

(Reason: (Reason: the rule sends every vertex of P to a vertex of Q, so the description is complete - a rotation, $180°$, about $(5,5)$. No direction is needed: a half turn clockwise and a half turn anticlockwise land in the same place.))

(a) Image with vertices $(3,1)$, $(9,1)$, $(9,10)$, $(6,10)$, $(6,4)$ and $(3,4)$

(b) Rotation of $180°$ about $(5,5)$

Verification

✓ **Check 1:** Every length in the image should be 3 times the matching length in shape A. The bottom edge of A runs from $(1,1)$ to $(3,1)$, so it is 2 squares long. $3 \times 2 = 6$, and the image edge from $(3,1)$ to $(9,1)$ is 6 squares long.

✓ **Check 2:** Area multiplies by the square of the scale factor, which is a test the side lengths cannot give on their own. Shape A covers 4 squares. $3^2 \times 4 = 36$, and the image covers

36 squares.

✓ **Check 3:** Apply the half-turn rule $(x, y) \to (10 - x, 10 - y)$ to all three vertices of P and see whether triangle Q comes out. It gives $(8, 8)$, $(8, 4)$ and $(6, 8)$, which is triangle Q exactly.

✓ **Check 4:** A rotation has only one centre, so every join must have the same midpoint. Working the three out separately tests $(5, 5)$ a second way. All three joins have midpoint $(5, 5)$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Correct shape drawn in correct position	**B2**	Shape drawn with coordinates $(3, 1)$, $(9, 1)$, $(9, 10)$, $(6, 10)$, $(6, 4)$, $(3, 4)$.	✓
(a) A shape of the correct size but in the wrong position	**B1**	B1 for a shape of the correct size but in the wrong position.	✓
(a) Marking aid	**Note**	NB Overlay is available.	✓
(a) The error behind the B1	**Note**	A shape of the correct size in the wrong position is what enlarging from the origin gives instead of from $(0, 1)$: every image lands 2 squares too high.	✓
(b) Rotation	**B1**	With no mention of any other transformation words or move, flip, transform, up, right etc. Turn on its own is not sufficient.	✓
(b) $180°$	**B1**	Allow half turn.	✓
(b) (centre) $(5, 5)$	**B1**	Must be a coordinate and not a vector.	✓
(b) Alternative: enlargement, scale factor -1	**B2**	B2 for enlargement SF -1 in place of the first two marks. Ignore any reference to clockwise or anticlockwise.	✓

Full marks: 5/5

Question 3 - Exam Solution

Understanding the Question

Given

A subtraction of two mixed numbers:

$7\frac{1}{3} - 3\frac{4}{7}$

The denominators 3 and 7 are different.

The fraction being taken away, $\frac{4}{7}$, is the bigger of the two fractional parts.

Find

Show that the difference comes to exactly $3\frac{16}{21}$.

Plan the Solution

- Turn each mixed number into an improper fraction, so the whole numbers and the fractions travel together instead of being handled separately.
- Rewrite both fractions over the common denominator 21, the lowest common multiple of 3 and 7.
- Subtract the numerators, then turn the improper fraction back into a mixed number and compare it with the result the question states.

Worked Solution [3 marks]

> Rule - Subtracting mixed numbers: write each mixed number as an improper fraction, put both fractions over a common denominator, subtract the numerators, then write the result back as a mixed number.

Step 1: Write each mixed number as an improper fraction

$$7\frac{1}{3} = \frac{7 \times 3 + 1}{3} = \frac{22}{3}$$

$$3\frac{4}{7} = \frac{3 \times 7 + 4}{7} = \frac{25}{7}$$

(Reason: Multiply the whole number by the denominator and add the numerator: $7 \times 3 + 1 = 22$ and $3 \times 7 + 4 = 25$. The whole numbers are now inside the fractions, so nothing has to be borrowed later.)

Step 2: Put both fractions over the denominator 21

$$\frac{22}{3} = \frac{22 \times 7}{3 \times 7} = \frac{154}{21}$$

$$\frac{25}{7} = \frac{25 \times 3}{7 \times 3} = \frac{75}{21}$$

(Reason: The lowest common multiple of 3 and 7 is 21, so the first fraction is multiplied top and bottom by 7 and the second top and bottom by 3. Multiplying top and bottom by the same number does not change the value.)

Step 3: Subtract the numerators

$$\frac{154}{21} - \frac{75}{21} = \frac{154 - 75}{21} = \frac{79}{21}$$

(Reason: Once both fractions share a denominator the denominator is left alone and only the numerators are subtracted, because the two fractions are now counted in the same sized pieces.)

Step 4: Write the improper fraction back as a mixed number

$$79 = 3 \times 21 + 16$$
$$\frac{79}{21} = 3\frac{16}{21}$$

(Reason: There are 3 whole lots of 21 inside 79, with 16 twenty-firsts left over. That is the result the question gives, so the statement is shown.)

$$7\frac{1}{3} - 3\frac{4}{7} = \frac{79}{21} = 3\frac{16}{21} \text{ as required}$$

Verification

✓ **Check 1:** Add $3\frac{4}{7}$ back on to the result. If the subtraction is right the total must return to $7\frac{1}{3}$. $\frac{79}{21} + \frac{75}{21} = \frac{154}{21} = \frac{22}{3}$

✓ **Check 2:** Work both sides out as decimals on the calculator and compare them. $\frac{22}{3} - \frac{25}{7} = 3.7619$ and $3 + \frac{16}{21} = 3.7619$

✓ **Check 3:** Do the subtraction the other way the mark scheme allows, keeping the whole numbers apart: $7\frac{7}{21} - 3\frac{12}{21}$ leaves $4 - \frac{5}{21}$. $4 - \frac{5}{21} = \frac{84 - 5}{21} = \frac{79}{21}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Improper fractions, or the fractional parts over a common denominator	**M1**	for correct improper fractions or fractional part of numbers written correctly over a common denominator: $\frac{22}{3}$ $(-)$ $\frac{25}{7}$ or $(7)\frac{7}{21}$ $(-)$ $(3)\frac{12}{21}$ or $(7)\frac{7a}{21a}$ $(-)$ $(3)\frac{12a}{21a}$	✓
Correct fractions over a common denominator, with the subtraction shown	**M1**	for correct fractions with a common denominator with minus sign or mixed numbers to the stage shown: $\frac{154}{21} - \frac{75}{21}$ or $\frac{22\times7}{21} - \frac{25\times3}{21}$ or $\frac{22\times7-25\times3}{21}$ or $\frac{154a}{21a} - \frac{75a}{21a}$ or $7\frac{7}{21} - 3\frac{12}{21} = 4 - \frac{5}{21}$ oe or $7\frac{7}{21} - 3\frac{12}{21} = 6\frac{28}{21} - 3\frac{12}{21}$. $\frac{154}{21} - \frac{75}{21}$ or $\frac{22\times7}{21} - \frac{25\times3}{21}$ implies the first M1	✓
A fully correct solution reaching the given result	**A1**	Dep on M2 for a correct answer from fully correct working: $\frac{154}{21} - \frac{75}{21} = \frac{79}{21} = 3\frac{16}{21}$ or $4 - \frac{5}{21} = 3\frac{16}{21}$ or $7\frac{7}{21} - 3\frac{12}{21} = 6\frac{28}{21} - 3\frac{12}{21} = 3\frac{16}{21}$. If a student shows that $3\frac{16}{21} = \frac{79}{21}$ then they must show correct working to $\frac{79}{21}$ and can gain full marks for this	✓
Working required	**Note**	The result is printed in the question, so it is the working that is being marked. A solution that writes the given result down again, with nothing between the two mixed numbers and it, earns none of the three marks.	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

A straight line segment AB, already drawn on the page.
A ruler and a pair of compasses, and nothing else.

Find

The perpendicular bisector of AB: the line that cuts AB into two equal halves and meets it at a right angle. Every construction line must still be on the page at the end. Nothing is rubbed out.

Plan the Solution

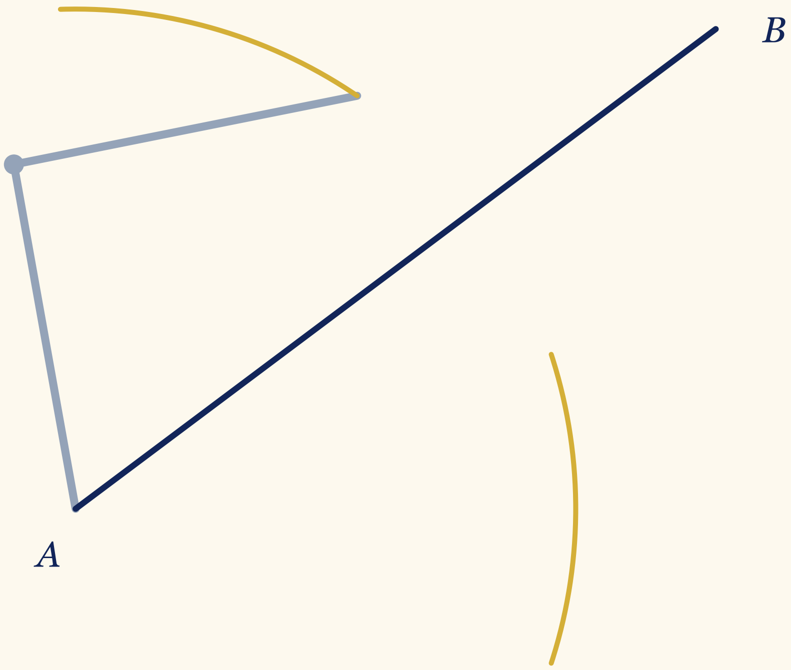
- Open the compasses to a radius bigger than half of AB, so that arcs swung from the two ends can reach each other.
- Swing a pair of arcs from A, one either side of the line, then a matching pair from B without touching the compass setting in between.
- Rule the straight line through the two points where the arcs cross, and take it a little way past the line on both sides.

Worked Solution [2 marks]

> Rule - Perpendicular bisector: a point is the same distance from A as it is from B exactly when it lies on the perpendicular bisector of AB, so any two such points fix that line.

Step 1: Set the compasses and swing two arcs from A

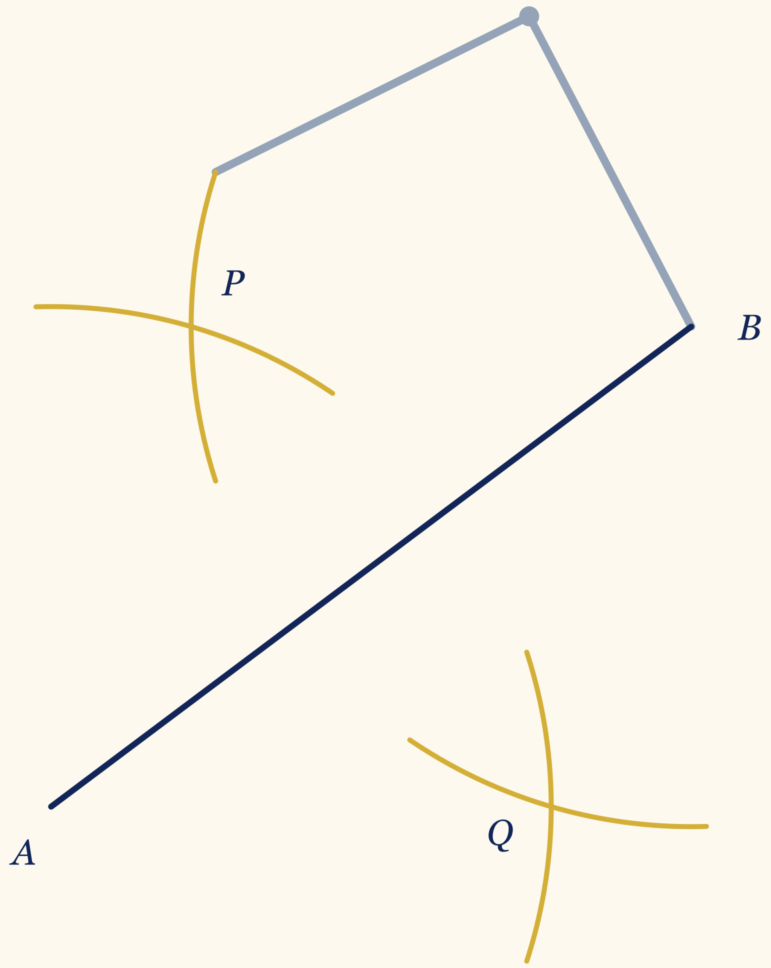
$$r > \frac{1}{2}AB$$



(Reason: The radius r is chosen once and then left alone for the whole construction. If r were smaller than half of AB, arcs swung from the two ends could never reach each other and there would be nothing to cross. Swing one arc into the space above the line and one into the space below it.)

Step 2: Swing two matching arcs from B

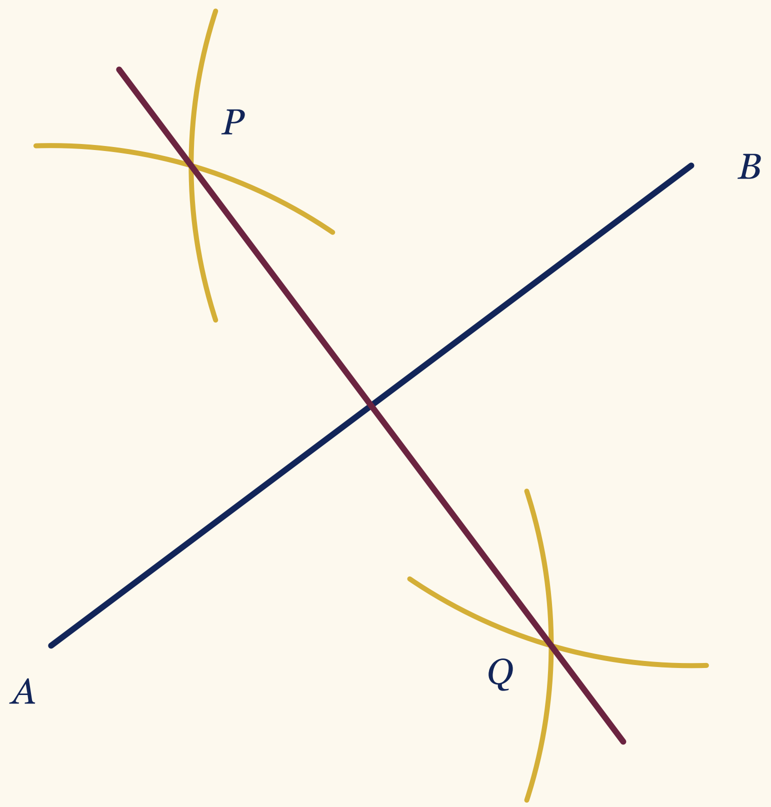
$$AP = BP = AQ = BQ = r$$



(Reason: The compasses are not adjusted between the two pairs, so all four of these lengths are the one radius. That is what makes P and Q each exactly as far from A as they are from B.)

Step 3: Rule the line through P and Q

$PQ \perp AB$

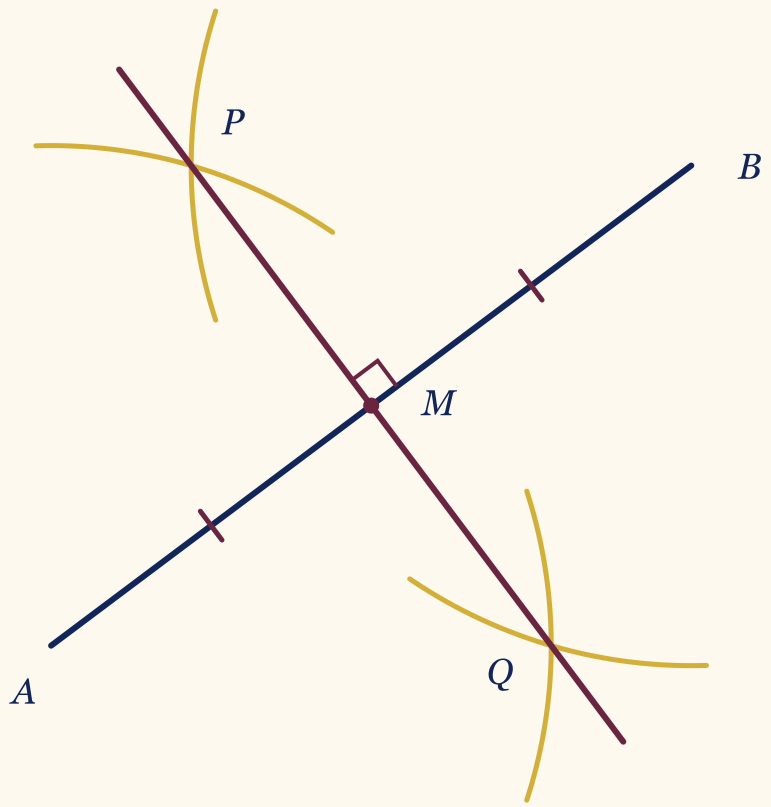


(Reason: Two points fix a straight line, and both of these are equidistant from A and B, so the line through them is the perpendicular bisector. Take it past AB on both sides so the crossing is clear, and leave all four arcs on the page, because the mark scheme asks to see them.)

Step 4: Why the ruled line is the perpendicular bisector

$$AP = PB = BQ = QA$$

$$PQ \perp AB \quad \text{and} \quad AM = MB$$



(Reason: Four equal sides make $APBQ$ a rhombus, and the diagonals of a rhombus cross at right angles and cut each other in half. The diagonals here are PQ and AB, so PQ meets AB at 90° and at its midpoint M. That is both halves of what perpendicular bisector means.)

The perpendicular bisector of AB is the line ruled through the two arc crossings P and Q, with both pairs of arcs left showing on the diagram.

Verification

✓ **Check 1:** Look at the four radii. The compasses were opened once and never touched again, so AP, BP, AQ and BQ are all the same length. Both arc crossings are the same distance from A as from B, so both of them lie on the bisector.

✓ **Check 2:** Measure the finished figure where the ruled line cuts AB. A protractor should read 90°, and a ruler should give the same length either side of the crossing. The line meets AB at a right angle and $AM = MB$, which is exactly what the question asked for.

✓ **Check 3:** Fold the paper along the line that has just been ruled. A lands on top of B, which only happens when the fold is the perpendicular bisector of AB.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Construct the perpendicular bisector of AB	**B2**	B2 for a fully correct perpendicular bisector with 2 pairs of intersecting arcs shown (the line and the arcs can intersect on or within the overlay guidelines).	✓
A partly correct construction	**B1**	B1 for 2 pairs of intersecting arcs and no perpendicular bisector drawn, or for a correct perpendicular bisector drawn within or on guidelines but no arcs or insufficient arcs, or for one pair of intersecting arcs and the perpendicular bisector drawn on just one side of AB.	✓
Marking note	**Note**	An overlay is available.	✓

Full marks: 2/2

Question 5 - Exam Solution

Understanding the Question

Given

Two similar quadrilaterals, $ABCD$ and $EFGH$, with the letters matching in order.
$AB = 5$ cm, $BC = 4$ cm,
$CD = y$ cm
$EF = x$ cm, $FG = 10$ cm,
$GH = 24$ cm

Find

(a) the value of x, which is the length EF
(b) the value of y, which is the length CD

Plan the Solution

- Read the correspondence off the letters: A goes with E, B with F, C with G and D with H. So AB matches EF, BC matches FG, and CD matches GH.

- Only one matching pair has both of its lengths given, BC and FG, so that pair is what fixes the scale factor.
- Multiply by the scale factor to go from the smaller quadrilateral to the larger one, and divide by it to come back the other way.
- Finish by checking the size of each answer: x is a side of the larger quadrilateral, so it must be bigger than 5, and y is a side of the smaller one, so it must be smaller than 24.

Worked Solution [4 marks]

> Rule - Similar shapes: every pair of matching sides is in the same ratio. Writing k for the scale factor from the smaller shape to the larger one,
> $\text{large side} = k \times \text{matching small side}$ and $\text{small side} = \dfrac{\text{matching large side}}{k}$.

Step 1 (a): pick the matching pair whose lengths are both known

$BC = 4$ cm
$FG = 10$ cm

(Reason: The letters match in order, so BC in the smaller quadrilateral corresponds to FG in the larger one. Every other pair has an unknown in it, so this is the only pair that can give the scale factor.)

Step 2 (a): work out the scale factor

$$k = \frac{FG}{BC} = \frac{10}{4} = 2.5$$

(Reason: Dividing a large side by its matching small side gives the scale factor from $ABCD$ to $EFGH$. A value bigger than 1 is the sign that $EFGH$ is the enlargement.)

Step 3 (a): scale AB up to EF

$$x = EF = k \times AB = 2.5 \times 5 = 12.5$$

(Reason: Going from the smaller quadrilateral to the larger one multiplies every length by k, and EF is the side that matches AB.)

Step 4 (b): scale GH back down to CD

$$y = CD = \frac{GH}{k} = \frac{24}{2.5} = 9.6$$

(Reason: Coming back the other way undoes the enlargement, so divide by the same scale factor, and CD is the side that matches GH.)

(a) $x = 12.5$

(b) $y = 9.6$

Verification

✓ **Check 1:** Every matching pair must give the same scale factor. Test the other two pairs, AB with EF and CD with GH. $\frac{12.5}{5} = 2.5$ and $\frac{24}{9.6} = 2.5$

✓ **Check 2:** Work inside each quadrilateral instead of across the pair. The ratio $AB : BC$ in $ABCD$ must equal the ratio $EF : FG$ in $EFGH$. $\frac{5}{4} = 1.25$ and $\frac{12.5}{10} = 1.25$

✓ **Check 3:** Check that each answer has landed on the right shape. x is a side of the larger quadrilateral and y is a side of the smaller one, so swapping the two answers over is the slip to rule out. $12.5 > 5$ and $9.6 < 24$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\frac{10}{4}$ $(= \frac{5}{2} = 2.5)$ or $\frac{4}{10}$ $(= \frac{2}{5} = 0.4)$ or $\frac{x}{5} = \frac{10}{4}$ oe or $\frac{x}{10} = \frac{5}{4}$ oe	**M1**	For a correct scale factor, which can be expressed as a fraction, a decimal or a ratio (it may or may not be used), or for a correct equation in x. Allow any letter for x.	✓
(a) Working not required, so a correct answer scores full marks (unless it comes from obviously incorrect working)	**A1**	12.5 oe, eg $\frac{50}{4}$ or $\frac{25}{2}$ or $12\frac{1}{2}$ or $12\frac{2}{4}$	✓
(b) $\frac{24}{[2.5]}$ oe or $\frac{y}{24} = \frac{4}{10}$ oe or $\frac{y}{24} = \frac{5}{[12.5]}$ oe or $\frac{y}{4} = \frac{24}{10}$ oe or $\frac{y}{5} = \frac{24}{[12.5]}$ oe	**M1ft**	Follow through, ie $[2.5]$ is their scale factor from (a); or for a correct equation in y. Allow any letter for y. Follow through their answer to (a), ie $[12.5]$ is their answer to (a).	✓

Step	Mark	Description	Got it?
(b) Working not required, so a correct answer scores full marks (unless it comes from obviously incorrect working)	A1	9.6 oe, eg $\frac{48}{5}$ or $9\frac{3}{5}$. If (a) $x = 9.6$ and (b) $y = 12.5$ then M1A0M1A0.	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

Marta, Leo and Freya share $£240$ in the ratios $3 : 4 : 5$, so the money is cut into parts and Marta takes 3 of them, Leo takes 4 and Freya takes 5.
Leo hands Marta $£10$ and Freya hands Marta $£10$, so two separate gifts arrive with Marta and each of the other two makes one.

Find

The ratio $\text{Marta} : \text{Leo} : \text{Freya}$ of what the three of them are holding once the gifts have been made, written in its simplest form.

Plan the Solution

- Add the three parts of the ratio to see how many equal parts the money is cut into, then find what one part is worth.
- Multiply to get the three starting amounts, and add them up: they have to come back to $£240$ before going any further.
- Move the gifts. Marta receives twice, Leo and Freya each give once.
- Set the three new amounts side by side as a ratio, then cancel by their highest common factor.

Worked Solution [4 marks]

Rule - Sharing in a ratio: one part $= \dfrac{\text{total}}{\text{the parts added together}}$, and each person takes their own number of parts. A ratio is in its simplest form once the only whole number that divides every part is 1.

Step 1: find what one part of the ratio is worth

$$3 + 4 + 5 = 12$$
$$\frac{240}{12} = 20$$

(Reason: It is the parts added together that the total is divided by, never one part on its own, because $3 : 4 : 5$ cuts the money into 12 equal parts. One part is worth £20.)

Step 2: work out what each of them starts with

Marta: $3 \times 20 = 60$

Leo: $4 \times 20 = 80$

Freya: $5 \times 20 = 100$

(Reason: Each of them takes their own number of parts, one part being £20. The three amounts add back to 240, which is the sign that the sharing has been done correctly.)

Step 3: hand over the two gifts of £10

Marta: $60 + 10 + 10 = 80$

Leo: $80 - 10 = 70$

Freya: $100 - 10 = 90$

(Reason: Marta is given £10 by Leo and another £10 by Freya, so she gains £20 altogether, while Leo and Freya are each £10 worse off. Nothing leaves the three of them, so the amounts still add to 240.)

Step 4: write the new amounts as a ratio and cancel

$$80 : 70 : 90$$
$$\frac{80}{10} : \frac{70}{10} : \frac{90}{10} = 8 : 7 : 9$$

(Reason: The highest common factor of 80, 70 and 90 is 10. Cancelling by a smaller common factor, 2 or 5, does divide all three but leaves a ratio that can still be cancelled, so it is not yet in its simplest form.)

$$8 : 7 : 9$$

Verification

✓ **Check 1 - the money has only moved:** Add the three new amounts. The £20 that leaves Leo and Freya arrives with Marta, so the total cannot have changed.
$80 + 70 + 90 = 240$

✓ **Check 2 - undo the two gifts:** Give the two gifts back. The three of them should be holding the amounts the ratio the question starts from hands out. $80 - 10 - 10 = 60$, $70 + 10 = 80$, $90 + 10 = 100$, and $60 : 80 : 100 = 3 : 4 : 5$

✓ **Check 3 - test the answer against the total:** A ratio of $8 : 7 : 9$ has 24 parts in it, so it claims Marta ends with $\frac{8}{24}$ of the $£240$, Leo with $\frac{7}{24}$ and Freya with $\frac{9}{24}$. Those three fractions of the total have to give the three amounts back. $\frac{8}{24} \times 240 = 80$, $\frac{7}{24} \times 240 = 70$, $\frac{9}{24} \times 240 = 90$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A correct method to find the value of one share	**M1**	For a correct method to find the value of one share: $\frac{240}{3+4+5}$ $(= 20)$ or $240 \times \frac{3}{3+4+5}$ $(= 60)$ oe or $240 \times \frac{4}{3+4+5}$ $(= 80)$ oe or $240 \times \frac{5}{3+4+5}$ $(= 100)$ oe. NB $\frac{240}{3}$ $(= 80)$, $\frac{240}{4}$ $(= 60)$ and $\frac{240}{5}$ $(= 48)$ scores M0.	✓
The correct values for 2 of the people after the two gifts	**M1**	For the correct values for 2 of the people after Leo and Freya give Marta $£10$. For two of: (Marta) $3 \times [20] + 10 + 10$ $(= 80)$ or $[60] + 10 + 10$ $(= 80)$; (Leo) $4 \times [20] - 10$ $(= 70)$ or $[80] - 10$ $(= 70)$; (Freya) $5 \times [20] - 10$ $(= 90)$ or $[100] - 10$ $(= 90)$. A value in square brackets is one the printed scheme puts in quotation marks, so the candidate's own value from the first method mark may be used in its place.	✓
All three amounts, or the final ratio in the wrong order, or the final ratio unsimplified	**M1**	For all 3 of 80, 70 and 90 correct (ignore units), or for the correct values for the final ratio in the wrong order (ignore units), eg $9 : 7 : 8$ oe, or for the correct values for the final ratio unsimplified (ignore units), eg $4 : 3.5 : 4.5$ oe.	✓
The final ratio in its simplest form	**A1**	$8 : 7 : 9$. Other orders are acceptable if they are labelled correctly. Working is not required, so a correct	✓

Step	Mark	Description	Got it?
		answer scores full marks unless it comes from obvious incorrect working.	

Full marks: 4/4

Question 7 - Exam Solution

Understanding the Question

Given

Price paid for the painting: $4\,000$ Swiss francs

Rise in value: 7% each year, worked out on the value the year starts with

Time: 3 years

Find

The value of the painting at the end of the 3 years, correct to the nearest Swiss franc

Plan the Solution

- Write the 7% rise as a decimal and add it to 1 to get the multiplier for one year.
- Multiply by that number once for each year, always starting from the value the year opened with.
- Collect the three multiplications into a single power, 1.07^3, which is the same calculation in one line.
- Round only at the very end, so no part of a franc is lost part way through.

Worked Solution [3 marks]

> Rule - Repeated percentage increase: a rise of $p\%$ multiplies a value by $1 + \dfrac{p}{100}$, and the same rise repeated over n years multiplies it by that number n times, so a starting value P growing at a rate r (as a decimal) is worth $P \times (1 + r)^n$.

Step 1: Turn the 7% rise into a one-year multiplier

$$\frac{7}{100} = 0.07$$

$$1 + 0.07 = 1.07$$

(Reason: the painting keeps all of the value it already has and gains 7% on top, so one year of growth is a single multiplication by 1.07. Multiplying by 0.07 on its own gives only the rise, 280 francs, which is what the first method mark accepts and is not the new value)

Step 2: Multiply once for each of the three years

$$4\,000 \times 1.07 = 4\,280$$
$$4\,280 \times 1.07 = 4\,579.6$$
$$4\,579.6 \times 1.07 = 4\,900.172$$

(Reason: each year's 7% is worked out on the value that year started with, not on the original $4\,000$, so the three rises are 280, 299.6 and 320.572 and they grow as the painting does)

Step 3: Do the same three multiplications in one line

$$1.07^3 = 1.225043$$
$$4\,000 \times 1.225043 = 4\,900.172$$

(Reason: multiplying by 1.07 three times is multiplying by 1.07 to the power 3, so a calculator reaches the same value in a single press. Adding 7% of the original three times over instead, which is the multiplier $1 + 0.07 \times 3 = 1.21$, is simple interest and earns nothing here)

Step 4: Round to the nearest Swiss franc

$$4\,900.172 \approx 4\,900$$

(Reason: the first digit after the decimal point is 1, which is below 5, so the whole number of francs is unchanged and nothing is carried)

$4\,900$ Swiss francs

Verification

✓ **Check 1 - undo the three years:** Divide the final value by the multiplier once for each year. If the three years were applied correctly, the price paid comes back exactly.
$$\frac{4\,900.172}{1.07^3} = 4\,000$$

✓ **Check 2 - add the three rises separately:** Work out each year's rise on its own and add all three to the price. This never forms a multiplier or a power, so it reaches the value a different way. $280 + 299.6 + 320.572 = 900.172$, and $4\,000 + 900.172 = 4\,900.172$

✓ **Check 3 - measure it against simple interest:** Simple interest would add 7% of $4\,000$, that is 280 francs, in each of the 3 years, giving 840 of growth and a value of $4\,840$. Compound growth has to come out above that, and over only 3 years not far above it. $900.172 - 840 = 60.172$ of extra growth, so the value is $4\,900.172$, which is above $4\,840$ and only a little above it

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$0.07 \times 4\,000\ (= 280)$	**M1**	For finding 7% of $4\,000$ or 107% of $4\,000$: $0.07 \times 4\,000\ (= 280)$ or $1.07 \times 4\,000\ (= 4\,280)$.	✓
$0.07 \times 4\,579.6\ (= 320.572)$	**M1**	$4\,000 + 280\ (= 4\,280)$ oe and $0.07 \times 4\,280\ (= 299.6)$ and $4\,280 + 299.6\ (= 4\,579.6)$ and $0.07 \times 4\,579.6\ (= 320.572)$, or $280 + 299.6 + 320.572\ (= 900.172)$. Every value carried forward in this row is one the printed scheme puts in quotation marks, so the candidate's own earlier value may be used in its place, and the printed scheme writes the last total as $900(.172)$. The scheme also awards M2 across this row and the one above it for $1.07^3 \times 4\,000$ or $1.07^4 \times 4\,000\ (= 5\,243\ldots)$.	✓
$4\,900$	**A1**	Allow answers in the range $4\,900$ to $4\,901$.	✓
One of the recognised wrong methods, and no other mark awarded	**SC B1**	If no other mark is awarded, SC B1 for $4\,000 \times 0.07 \times 3\ (= 840)$ or $4\,000 \times 0.21\ (= 840)$ or $4\,000 + 4\,000 \times 0.07 \times 3\ (= 4\,840)$ or $4\,000 \times 1.21\ (= 4\,840)$ or $0.93 \times 4\,000\ (= 3\,720)$ or $0.79 \times 4\,000\ (= 3\,160)$ or $0.93^3 \times 4\,000\ (= 3\,217\ldots)$ or $4\,000 \times 1.07^2\ (= 4\,579\ldots)$.	✓
A correct answer with no working shown	**Note**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working. This row earns nothing on its own.	✓

Full marks: 3/3

Question 8 - Exam Solution

Understanding the Question

Given

The first equation: $3x + 5y = 8$

The second equation: $4x + y = -3.5$

Find

The value of x and the value of y, each one supported by algebraic working

Two straight-line equations in the same two unknowns, so a single pair of values fits both at once.

Plan the Solution

- Look for the variable whose coefficients are cheapest to match. The y terms are $5y$ and y, so only the second equation has to be multiplied, and only by 5.
- Multiply every term of that equation, on both sides, so the new line says exactly what the old one said.
- Subtract one equation from the other. The matching y terms cancel and a single equation in x is left.
- Put that value of x back into the original equation with the smaller numbers and read off y.
- Test the pair in both original equations, because a pair that fits only one of them is not a solution.

Worked Solution [3 marks]

> Rule - Elimination: multiply one equation, or both, until the same variable has the same coefficient in each. Then subtract the equations when those matching terms have the same sign, and add them when the signs are opposite. That variable disappears and the other one is left on its own, ready to be found.

Step 1: Number the two equations

$3x + 5y = 8 \quad (1)$

$4x + y = -3.5 \quad (2)$

(Reason: numbering them lets every later line say which equation it came from, and working a marker can follow is part of what showing clear algebraic working means. Neither line has been changed: $4x + y = -3.5$ is written with the coefficient 1 left unwritten in front of the y, exactly as the question prints it)

Step 2: Multiply equation (2) by 5 so that both equations carry $5y$

$5 \times (4x + y) = 5 \times (-3.5)$

$20x + 5y = -17.5 \quad (3)$

(Reason: multiplying both sides by the same number leaves the equation saying the same thing, and it turns the lone y into the $5y$ that equation (1) already has. Every term must be multiplied, the -3.5 included: multiplying only the left-hand side would give a different equation and the method mark would be lost)

Step 3: Subtract equation (1) from equation (3) to remove the y terms

$(20x + 5y) - (3x + 5y) = -17.5 - 8$

$17x = -25.5$

$x = \dfrac{-25.5}{17} = -1.5$

(Reason: both equations carry $+5y$, the same term with the same sign, so subtracting is what removes it; had the signs been opposite the equations would have been added instead. On the right, taking 8 away from -17.5 moves further below zero, which is why the value of x comes out negative)

Step 4: Put $x = -1.5$ back into equation (2) to find y

$4 \times (-1.5) + y = -3.5$

$-6 + y = -3.5$

$y = -3.5 + 6 = 2.5$

(Reason: equation (2) is the one with the smaller numbers, so it is the safer of the two to substitute into. The term that has to cross the equals sign is negative, and moving a negative term across is a subtraction of a negative number, which is the point in this question where the working has to be read most carefully)

Step 5: State the pair the question asks for

$x = -1.5$

$y = 2.5$

(Reason: the question asks for two values and the answer line prints a blank for each, so both are written out. A pair counts as a solution only when it fits both of the original equations, which is what the checks below confirm)

$$x = -1.5$$

$$y = 2.5$$

Verification

✓ **Check 1 - test the pair in the first equation:** Equation (1) was used to eliminate y and never used to work y out, so putting both values into it is a genuine test rather than a repeat of the working. $3 \times (-1.5) + 5 \times 2.5 = -4.5 + 12.5 = 8$, which is the right-hand side of the first equation

✓ **Check 2 - test the pair in the second equation:** The second equation has to hold as well. A pair that satisfies one equation and not the other is not a solution, and a slip made while substituting would show up here. $4 \times (-1.5) + 2.5 = -6 + 2.5 = -3.5$, which is the right-hand side of the second equation

✓ **Check 3 - reach the same pair by eliminating the other variable:** Multiply the first equation by 4 and the second by 3, so that both carry $12x$, then subtract. This route never multiplies by 5 and never substitutes a found value, so it can go wrong in different places from the working above. $12x + 20y = 32$ and $12x + 3y = -10.5$, so $17y = 42.5$ and $y = 2.5$, and the second equation then gives $x = -1.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $3x + 5y = 8$ and $20x + 5y = -17.5$, subtracting: $3x - 20x = 8 - -17.5$ or $-17x = 25.5$, or $3x + 5(-3.5 - 4x) = 8$, or $4x + \frac{8 - 3x}{5} = -3.5$, or eg $12x + 20y = 32$ and $12x + 3y = -10.5$, subtracting: $20y - 3y = 32 - -10.5$ or $17y = 42.5$, or $3\left(\frac{-3.5 - y}{4}\right) + 5y = 8$, or $4\left(\frac{8 - 5y}{3}\right) + y = -3.5$	M1	For a correct method to eliminate x or y: coefficients of x or y the same and correct operator to eliminate the selected variable (condone any one arithmetic error in multiplication), or writing x or y in terms of the other variable and correctly substituting (condone missing brackets). NB the mark is for the method and not for the result of the method. However, if the correct result of the method is seen, the mark can be awarded.	✓
eg $3 \times (-1.5) + 5y = 8$ or $4 \times (-1.5) + y = -3.5$ or $y = -3.5 - 4 \times (-1.5)$ or $y = \frac{8 - 3 \times (-1.5)}{5}$, or $3x + 5 \times 2.5 = 8$ or $4x + 2.5 = -3.5$ or $x = \frac{-3.5 - 2.5}{4}$ or $x = \frac{8 - 5 \times 2.5}{3}$	M1	Dep on the first M1, for a correct method to find the other variable by substitution of the found variable into one equation, or for repeating the above method to find the second variable. The printed scheme puts every -1.5 and every 2.5 in this row inside quotation marks, so a candidate may use their own earlier value in place of the correct one and still earn the mark.	✓
$x = -1.5$ and $y = 2.5$	A1	oe, dep on M1. The printed scheme writes 'Working required' in the working column of this row, so the pair has to be supported by algebraic working rather than stated on its own.	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given

(a) The inequality $7 - 3t < 2t + 15$
(b) The region R on the grid, bounded by a vertical line through 2 on the x-axis, a horizontal line through 3 on the y-axis, and a sloping line joining $(0, 9)$ to $(9, 0)$ All three boundary lines are drawn solid, and the shading runs right up to each of them.

Find

(a) Every value of t that makes the inequality true, written as an inequality in t (b) Three inequalities which are all true at every point of R and not all true anywhere else

Plan the Solution

- (a) Treat the inequality like an equation: collect the terms in t on one side and the numbers on the other. Send the t terms to whichever side leaves their coefficient positive, and the sign never has to be reversed.
- (a) Divide, then rewrite the statement with t first, because that is the form the answer line asks for.
- (b) Take the three boundary lines one at a time and write down the equation of each: two of them are parallel to an axis, and the third needs its gradient and its intercept.
- (b) An equation names a line, not a side of it, so fix the direction of each inequality by testing one point that is plainly inside R.
- (b) Decide between a strict inequality and one that includes its boundary by looking at how the lines are drawn and how far the shading reaches.

Worked Solution [5 marks]

> Rule - Inequalities: every move that is allowed on an equation is allowed here, with one exception. Multiplying or dividing both sides by a NEGATIVE number reverses the inequality sign, and so does writing the whole statement the other way round. Rule - A region on a grid: each boundary line gives one inequality, the equation of the line fixes the $=$ part of it, and the side the region lies on fixes which way it points.

Step 1: Collect the terms in t on one side and the numbers on the other

$7 - 3t < 2t + 15$

$7 - 15 < 2t + 3t$

$-8 < 5t$

(Reason: adding $3t$ to both sides and taking 15 from both sides sends the letters one way and the numbers the other. The terms in t are sent to the right-hand side on purpose, because $2t + 3t = 5t$ is positive there, and a positive coefficient is what lets the next step divide without touching the inequality sign)

Step 2: Divide both sides by 5

$-\frac{8}{5} < t$

$-\frac{8}{5} = -1.6$

(Reason: 5 is positive, so dividing by it leaves the inequality sign exactly as it was. As a decimal $-\frac{8}{5}$ is -1.6, and the mark scheme accepts either form)

Step 3: Turn the statement round so that t is written first

$t > -1.6$

(Reason: the answer line asks for t, so the statement is rewritten with t on the left. Reading $-1.6 < t$ from right to left says that t is the greater of the two, so the symbol turns round with the statement. The accuracy mark is for the symbol pointing the correct way: an answer line reading $t = -1.6$ scores the method mark and nothing more)

Step 4: Read off the two boundaries that run parallel to the axes

$x = 2$

$y = 3$

(Reason: one boundary is vertical, and every point on a vertical line has the same x-coordinate. It meets the x-axis at 2, so its equation is $x = 2$. The other is horizontal, so every point on it has the same y-coordinate, and it meets the y-axis at 3)

Step 5: Find the equation of the sloping boundary

$\frac{0 - 9}{9 - 0} = -1$

$y = 9 - x$

$x + y = 9$

(Reason: the sloping line passes through $(0, 9)$ and $(9, 0)$, so it drops one square for every square it moves across and its gradient is -1. It cuts the y-axis at 9, which gives $y = 9 - x$. Adding x to both sides puts it in the form the mark scheme uses, and that form says something worth remembering: the two coordinates of every point on this line add up to 9)

Step 6: Test one point inside R to fix the direction of each inequality

$3 \geq 2$
$4 \geq 3$
$3 + 4 = 7$

(Reason: a line cuts the grid in two, and its equation says nothing about which half is wanted. The point $(3, 4)$ is clearly inside the shaded region, so all three inequalities have to be true there. Its x-coordinate is at least 2, its y-coordinate is at least 3, and its coordinates add to 7, which is not more than 9. One interior point settles all three directions at once)

Step 7: Write down the three inequalities

$x \geq 2$
$y \geq 3$
$x + y \leq 9$

(Reason: each boundary line contributes one inequality and step 6 has fixed which way each one points. The lines are drawn solid and the shading reaches them, so points on the boundaries belong to R and each inequality is written with the bar underneath. The printed mark scheme also accepts the strict forms $x > 2$, $y > 3$ and $x + y < 9$, so a student who reads the boundaries as excluded is not penalised)

(a) $t > -1.6$

(b) $x \geq 2$, $y \geq 3$, $x + y \leq 9$

Verification

✓ **Check 1 - test a value from each side of the boundary:** Take one value from inside the answer and one from outside it, and put each into $7 - 3t$ and $2t + 15$ separately. This tests the answer against the inequality as the question prints it, rather than against any line of the working. $7 - 3 \times 0 = 7$ and $2 \times 0 + 15 = 15$, and 7 is less than 15, so $t = 0$ belongs. $7 - 3 \times (-2) = 13$ and $2 \times (-2) + 15 = 11$, and 13 is not less than 11, so $t = -2$ does not

✓ **Check 2 - the boundary value itself:** Put -1.6 into both sides. If it really is the end of the solution then the two sides meet there, and meeting is also the reason the value itself is left out. $7 - 3 \times (-1.6) = 11.8$ and $2 \times (-1.6) + 15 = 11.8$, which are equal, so the two sides swap over at -1.6 and the inequality there is strict

✓ **Check 3 - the three corners of the region:** The corners of R are where the boundary lines cross: $(2, 7)$, $(2, 3)$ and $(6, 3)$. Every corner must satisfy all three inequalities, and each should sit exactly on two of the three lines. $2 + 7 = 9$, $2 + 3 = 5$ and $6 + 3 = 9$, so no corner has coordinates adding to more than 9, and every corner has x at least 2 and y at least 3

✓ **Check 4 - step outside across one boundary at a time:** Each of these points lies just outside R across a different edge, so each should break exactly one of the three inequalities. A point that broke none would mean the region is larger than the shading, and a point that broke two would mean one inequality is doing another one's work. $(1, 4)$ fails only $x \geq 2$; $(3, 2)$ fails only $y \geq 3$; and $(5, 5)$ gives $5 + 5 = 10$, which fails only $x + y \leq 9$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) eg $-3t - 2t < 15 - 7$ or $-5t < 8$ oe, or $7 - 15 < 2t + 3t$ or $-8 < 5t$ oe, or $t = -1.6$ or $t < -1.6$	M1	For correctly isolating terms in t on one side and number terms on the other side (use of $=$ or any inequality symbol or variable is permitted).	✓
(a) Working not required, so a correct answer scores full marks (unless from obvious incorrect working)	A1	$t > -1.6$, oe eg $-1.6 < t$ or $t > -\frac{8}{5}$ or $-\frac{8}{5} < t$ oe. Must have correct inequality symbol on answer line. NB Sight of correct answer in the working space and just $(t =) - 1.6$ oe on the answer line gains M1 only.	✓
(b) $x \geq 2$	B1	oe, allow $x > 2$ or $2 < x$	✓
(b) $y \geq 3$	B1	oe, allow $y > 3$ or $3 < y$	✓
(b) $x + y \leq 9$	B1	oe, allow $x + y < 9$ or $y < 9 - x$ or $9 > x + y$	✓
(b) All of $x \leq 2$, $y \leq 3$, $x + y \geq 9$ oe, or all of $x < 2$, $y < 3$, $x + y > 9$	SC B2	for all three	✓
(b) All of $x = 2$, $y = 3$, $x + y = 9$ oe	SC B1	for all three	✓

Full marks: 5/5

Question 10 - Exam Solution

Understanding the Question

Given

ABD and ABC are right-angled triangles, and in both of them the right angle is at B

$AB = 16$ cm

$BC = 12$ cm

$AD = 1.5 \times AC$

C lies on BD, so BC and CD together make BD

Find

The length of CD, correct to 3 significant figures

Plan the Solution

- Use Pythagoras in triangle ABC to find AC. Both short sides are known, so AC is the hypotenuse.
- Scale that answer by 1.5 to get AD.
- Use Pythagoras again in triangle ABD to find BD. This time the hypotenuse AD is known, so the squares are subtracted.
- Take BC away from BD, because C sits on BD.
- Round once, at the very end. Rounding BD first would push the final answer out.

Worked Solution [5 marks]

> Pythagoras: in a right-angled triangle the square on the hypotenuse equals the sum of the squares on the other two sides, $a^2 + b^2 = c^2$. Rearranged for a shorter side, $a^2 = c^2 - b^2$. Which form to use is decided by which side is the hypotenuse, never by which numbers are given.

Step 1: find AC from triangle ABC

$$AC^2 = 12^2 + 16^2 = 144 + 256 = 400$$

$$AC = \sqrt{400} = 20 \text{ cm}$$

(Reason: The right angle is at B, so AC is the side facing it and is the hypotenuse of triangle ABC. The two known squares are added.)

Step 2: scale up to get AD

$AD = 1.5 \times 20 = 30$ cm

(Reason: The question states that AD is 1.5 times AC, and AC has just been found.)

Step 3: find BD from triangle ABD

$BD^2 = 30^2 - 16^2 = 900 - 256 = 644$

$BD = \sqrt{644} = 25.3771\ldots$ cm

(Reason: In triangle ABD the right angle is again at B, so AD is the hypotenuse and BD is one of the shorter sides. A shorter side is found by subtracting, not by adding.)

Step 4: subtract BC to reach CD

$CD = BD - BC = 25.3771\ldots - 12 = 13.3771\ldots$

(Reason: C lies on BD, so BC and CD make up the whole of BD. Keep the unrounded value until the last line.)

Step 5: round to 3 significant figures

$CD = 13.4$ cm

(Reason: The unrounded length is $13.3771\ldots$ and the fourth significant figure is 7, so the third rounds up from 3 to 4.)

$CD = 13.4$ cm **(correct to 3 significant figures)**

Verification

- ✓ **Check 1:** Rebuild BD from the answer and test triangle ABD the other way round.
  $BD = 12 + 13.3771\ldots = 25.3771\ldots$, and squaring that gives 644.
  $644 + 256 = 900 = 30^2$, which is AD^2, so the rebuilt triangle really is right-angled at B.
- ✓ **Check 2:** Redo the middle of the question with trigonometry instead of Pythagoras. In triangle ABD, $\cos(BAD) = \dfrac{16}{30}$, so angle $BAD = 57.7690\ldots°$.
  $30 \times \sin(57.7690\ldots°) = 25.3771\ldots$, and taking 12 away leaves $13.3771\ldots$, the same length as before.
- ✓ **Check 3:** Check the size is sensible. CD is only part of BD, and BD is a shorter side of a right-angled triangle whose hypotenuse AD measures 30 cm, so CD must be well under 30 cm. $0 < 13.4 < 25.4 < 30$, so CD is shorter than BD, which is shorter than AD.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
A method in triangle ABC	**M1**	For a correct method using triangle ABC: $(AC^2 =)\ 12^2 + 16^2\ (= 144 + 256 = 400)$ or $(BAC =)\ \tan^{-1}\left(\frac{12}{16}\right)\ (= 36.8(698\ldots))$ or 36.9 or $(BCA =)\ \tan^{-1}\left(\frac{16}{12}\right)\ (= 53.1(301\ldots))$.	✓
Find AC	**M1**	For a correct method to find AC: $(AC =)\sqrt{12^2 + 16^2}\ (= \sqrt{144 + 256} = \sqrt{400} = 20)$ or $(AC =)\frac{16}{\cos 36.8^\circ}(= 20)$ or $(AC =)\frac{12}{\sin 36.8^\circ}(= 20)$ or $(AC =)\frac{16}{\sin 53.1^\circ}(= 20)$ or $(AC =)\frac{12}{\cos 53.1^\circ}(= 20)$. The printed scheme puts 36.8 and 53.1 inside quotation marks, which means an angle found earlier in the answer may be used in place of the correct one.	✓
A method in triangle ABD	**M1**	For a correct method using a triangle to find BD^2 or angle BAD or angle BDA, or for a correct equation for side CD: $(BD^2 =)(1.5 \times 20)^2 - 16^2\ (= 644)$ or $(BD^2 =)30^2 - 16^2\ (= 900 - 256 = 644)$ or $(BAD =)\ \cos^{-1}\left(\frac{16}{30}\right)\ (= 57.7(690\ldots))$ or 57.8 or $(BDA =)\ \sin^{-1}\left(\frac{16}{30}\right)\ (= 32.2(309\ldots))$ or $(BCA =)\ \sin^{-1}\left(\frac{16}{20}\right)\ (= 53.1(301\ldots))$ together with $\frac{CD}{\sin(180 - 126.9 - 32.2)} = \frac{30}{\sin(180 - 53.1)}$, or equivalent. The printed scheme puts 20 and 30 inside quotation marks, so a value found earlier in the answer may be used in place of the correct one.	✓
Find BD or CD	**M1**	For a correct method to find BD or CD: $(BD =)\sqrt{(1.5 \times 20)^2 - 16^2}\ (= 25.3(771)\ldots)$ or $(BD =)\sqrt{30^2 - 16^2}\ (= \sqrt{900 - 256} = \sqrt{644} = 2\sqrt{161} = 25.3(771\ldots))$ or $(BD =)16 \times \tan 57.7^\circ\ (= 25.3(771\ldots))$ or $(BD =)30 \times \sin 57.7^\circ\ (= 25.3(771\ldots))$ or	✓

Step	Mark	Description	Got it?
		$(BD =)\sqrt{16^2 + 30^2 - 2 \times 16 \times 30 \times \cos 57.7°}$ $(= 25.3(771\ldots))$ or $(BD =)30 \times \cos 32.2°$ $(= 25.3(771\ldots))$ or $(BD =)\dfrac{16}{\tan 32.2°}(= 25.3(771\ldots))$ or $(BD =)\dfrac{16}{\sin 32.2°} \times \sin 57.7°$ $(= 25.3(771\ldots))$ or $(CD =)\dfrac{30}{\sin 126.9°} \times \sin 20.9°$, or equivalent. Here too the printed scheme quotes 20 and 30, so an earlier value may be used.	
The answer	A1	awrt 13.4. Working is not required, so a correct answer scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

The jar holds 17 green beads, 28 yellow beads, and some purple beads.
One bead is taken at random, and the probability that it is purple is $\frac{4}{9}$.

Find

The number of purple beads in the jar.
The total is unknown as well, so expect to find the whole jar on the way to the 4 ninths that are purple.

Plan the Solution

- The green and the yellow beads are exactly the beads that are not purple, so add them: that is the only part of the jar the question actually counts.
- Purple and not purple are the only two outcomes, so their probabilities add to 1. That turns the counted beads into a known fraction of the jar.
- Split that fraction into its 5 equal ninths to find one ninth, then build the whole jar back up and take the purple share.

Worked Solution [3 marks]

> Rule - the probability of taking a purple bead is the number of purple beads over the total number of beads, and the probabilities of all the outcomes add to 1.

Step 1: count the beads that are not purple

$$17 + 28 = 45$$

(Reason: Every bead is green, yellow or purple, so the 17 green and the 28 yellow beads together are all the beads that are not purple.)

Step 2: find the probability of not purple

$$1 - \frac{4}{9} = \frac{5}{9}$$

(Reason: Purple and not purple are the only two outcomes, so their probabilities add to 1. Those 45 beads therefore make up $\frac{5}{9}$ of the jar.)

Step 3: work out one ninth of the jar

$$\frac{45}{5} = 9$$

(Reason: Those 45 beads are 5 of the 9 equal ninths, so one ninth is 45 shared into 5 equal groups.)

Step 4: build the whole jar, then take the purple share

$$9 \times 9 = 81$$

$$4 \times 9 = 36$$

(Reason: The whole jar is 9 ninths, so it holds 81 beads, and the purple beads are the 4 ninths that are left once the counted beads are taken out.)

36 purple beads

Verification

- ✓ **Check 1:** Put the answer back in. The jar would then hold 17 green, 28 yellow and 36 purple beads, so work out the probability of purple from those counts. $$\frac{36}{17 + 28 + 36} = \frac{36}{81} = \frac{4}{9}$$
- ✓ **Check 2:** Cross-multiply instead of dividing: nine lots of the purple beads should equal four lots of all the beads. $9 \times 36 = 324$ and $4 \times 81 = 324$
- ✓ **Check 3:** A different quantity altogether. The beads that are not purple should outnumber the purple ones in the ratio five to four, because their probabilities are five ninths and four ninths. $\frac{45}{36} = \frac{5}{4}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{17+28}{9-4}$ or $\frac{45}{5}$ or $1-\frac{4}{9}\left(=\frac{5}{9}\right)$ or $(17+28=)\frac{5}{9}$ oe or $\frac{p}{p+28+17}=\frac{4}{9}$ oe or $\frac{p}{p+45}=\frac{4}{9}$ or $\frac{m-45}{m}=\frac{4}{9}$	**M1**	Allow $0.55(555\ldots)$ or $55(.555\ldots)\%$ truncated or rounded	✓
$\frac{17+28}{5}\times 4$ or $\frac{45}{5}\times 4$ or $\frac{17+28}{\text{their }\frac{5}{9}}(=81)$ or $\frac{45}{\text{their }\frac{5}{9}}(=81)$ or $(17+28)\times\text{their }\frac{9}{5}(=81)$ or $45\times\text{their }\frac{9}{5}(=81)$ or $\text{their }\frac{5}{9}=\frac{17+28}{n}$ oe or $n=81$ or $9p=4(p+28+17)$ or $9p-4p=180$ oe or $5p=180$ oe or $9(m-45)=4m$ or $9m-4m=405$ oe or $5m=405$ oe or $m=81$	**M1**	for the correct calculation for the total number of beads or for the correct calculation for the number of purple beads or for the correct equation for the total number of beads (removing the denominators) or for the correct equation for the number of purple beads (removing the denominators). A value in quotation marks in the printed scheme is written here as their value, meaning the candidate's own earlier value may be used.	✓
36	**A1**	cao. Working not required, so correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 3/3

Question 12 - Exam Solution

Understanding the Question

Given

The expression $3x(2x+5)(7x-4)$
Three factors multiplied together: the single term $3x$, the bracket $(2x+5)$ and the bracket $(7x-4)$.

Find

The same expression written without brackets, with every pair of like terms collected.

Plan the Solution

- Multiplication can be done in any order, so take the factors two at a time.
- Expand $(2x+5)(7x-4)$ first: every term in one bracket multiplies every term in the other, which gives four products.
- Collect the two x terms, so the bracket becomes a quadratic with three terms.
- Multiply that quadratic by $3x$, one term at a time.
- Check the result by expanding in a different order, and by substituting a number.

Worked Solution [3 marks]

> Rule - Expanding brackets: multiply every term in one bracket by every term in the other, then collect like terms. When two single terms are multiplied, multiply the numbers and add the indices: $x^a \times x^b = x^{a+b}$.

Step 1: expand $(2x+5)(7x-4)$

$(2x+5)(7x-4) = 2x \times 7x + 2x \times (-4) + 5 \times 7x + 5 \times (-4)$
$= 14x^2 - 8x + 35x - 20$

(Reason: Each term in the first bracket multiplies each term in the second, so there are four products. The signs travel with the terms: $2x \times (-4) = -8x$ and $5 \times (-4) = -20$.)

Step 2: collect the like terms inside the bracket

$-8x + 35x = 27x$
$(2x+5)(7x-4) = 14x^2 + 27x - 20$

(Reason: Only the two x terms are alike. $14x^2$ is the only squared term and -20 is the only number, so neither of them changes.)

Step 3: multiply the quadratic by $3x$

$3x(14x^2+27x-20) = 3x \times 14x^2 + 3x \times 27x - 3x \times 20$
$= 42x^3 + 81x^2 - 60x$

(Reason: Multiply the numbers and add the indices each time: $3 \times 14 = 42$ and $x \times x^2 = x^3$, so $3x \times 14x^2 = 42x^3$. Nothing can be collected now, because the three terms carry different powers of x.)

$$42x^3 + 81x^2 - 60x$$

Verification

- ✓ **Check 1 - expand in a different order:** Take the single term into the first bracket instead: $3x(2x+5) = 6x^2 + 15x$, then multiply that by $(7x-4)$. $(6x^2+15x)(7x-4) = 42x^3 - 24x^2 + 105x^2 - 60x$, which collects to $42x^3 + 81x^2 - 60x$.
- ✓ **Check 2 - substitute a value for x:** Put $x = 2$ into the original expression. Then $3x = 6$, $2x+5 = 9$ and $7x-4 = 10$. The original gives $6 \times 9 \times 10 = 540$, and the answer gives $42 \times 8 + 81 \times 4 - 60 \times 2 = 540$.
- ✓ **Check 3 - is the answer the right shape?** Each factor has degree 1, so the product has degree 3. Every term carries a factor of x, so there is no number on its own, and the highest coefficient is $3 \times 2 \times 7 = 42$. The answer is a cubic, its highest term is $42x^3$, and it has no constant term.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One pair of factors expanded: $3x(2x+5) = 6x^2+15x$ or $3x(7x-4) = 21x^2-12x$ or $(2x+5)(7x-4) = 14x^2-8x+35x-20$ $(14x^2+27x-20)$	**M1**	An expansion with only one error. Do not award this mark for $6x^2+15x+21x^2-12x$ or $(6x^2+15x)(21x^2-12x)$.	✓
$(6x^2+15x)(7x-4) = 42x^3 - 24x^2 + 105x^2 - 60x$ or $(21x^2-12x)(2x+5) = 42x^3 + 105x^2 - 24x^2 - 60x$ or $3x(14x^2-8x+35x-20) = 42x^3 - 24x^2 + 105x^2 - 60x$	**M1**	ft dep on M1. Allow one further error.	✓

Step	Mark	Description	Got it?
or $3x(14x^2 + 27x - 20) = 42x^3 + 81x^2 - 60x$			
$42x^3 + 105x^2 - 24x^2 - 60x$ with 3 of the terms correct	**M2**	An alternative to the two method marks above, not an addition to them. M2 for 3 (out of a maximum of 4) of $42x^3 + 105x^2 - 24x^2 - 60x$. M1 for 2 correct out of a maximum of 4.	✓
$42x^3 + 81x^2 - 60x$	**A1**	cao (terms may be in any order but must be simplified) dep on M1. ISW correct factorisation, eg $3(14x^3 + 27x^2 - 20x)$. Do not ISW incorrect simplification, eg $14x^3 + 27x^2 - 20x$.	✓
Working not required, so a correct answer scores full marks (unless from obvious incorrect working).	**Note**	Guidance for the whole question. This row awards nothing of its own.	✓

Full marks: 3/3

Question 13 - Exam Solution

Understanding the Question

Given

The times, in minutes, that 60 visitors took, grouped into six classes each of width 10 minutes.
The frequencies, in order, are 8, 13, 12, 17, 7 and 3.

Find

(a) the cumulative frequency for each class (b) the cumulative frequency graph (c) an estimate of the median time (d) an estimate of how many visitors took more than 45 minutes

Plan the Solution

- Add the frequencies down the table. Each running total counts every visitor who had finished by the END of that class.
- Plot each running total against the UPPER end of its class, so the first point sits at 10 minutes and not in the middle of the class, then join the points.
- The median is read at half of 60, so go across from 30 on the cumulative frequency axis and down to the time axis.
- For part (d), read UP from 45 minutes to find how many had already finished, then take that away from 60.

Worked Solution [6 marks]

> Rule - Cumulative frequency: plot each running total against the UPPER end of its class, and estimate the median by reading across from $\frac{n}{2}$ on the cumulative frequency axis.

Step 1: add the frequencies down the table

8

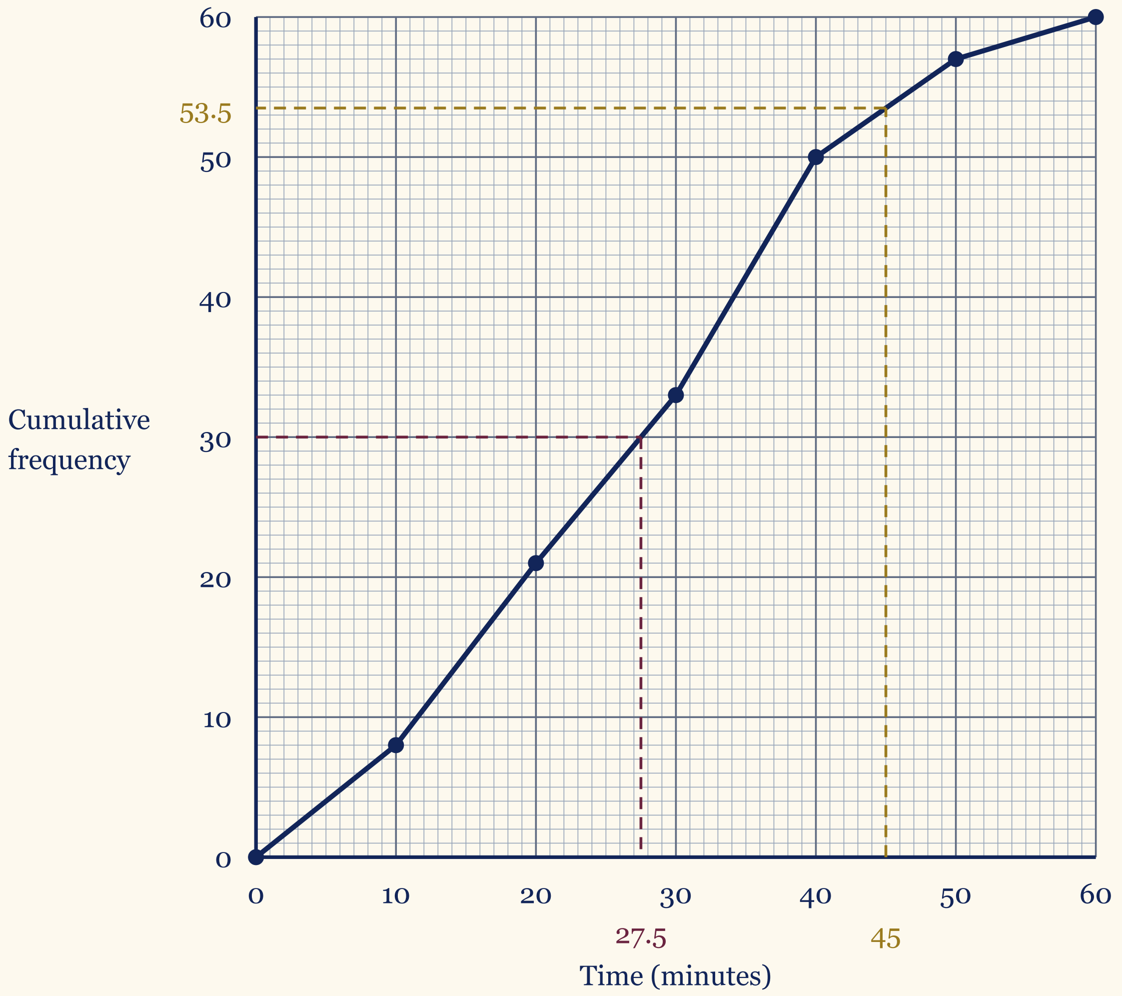
$8 + 13 = 21$

$21 + 12 = 33$

$33 + 17 = 50$

$50 + 7 = 57$

$57 + 3 = 60$



(Reason: Each cumulative frequency counts every visitor who had finished by the END of that class, so it is the running total of the frequency column. The last entry has to come out as 60, the number of visitors, and that is the check that no frequency has been dropped.)

Step 2: plot each total at the end of its class

$(10,\ 8)\qquad(20,\ 21)\qquad(30,\ 33)$

$(40,\ 50)\qquad(50,\ 57)\qquad(60,\ 60)$

(Reason: Every visitor in the first class had finished by 10 minutes, so that point goes at 10 and not at the middle of the class. Joining the points gives the graph, and it may be started at $(0,\ 0)$ because nobody had finished after 0 minutes.)

Step 3: read the median off the graph

$$\frac{60}{2} = 30$$

$$20 + \frac{30 - 21}{33 - 21} \times 10 = 27.5$$

(Reason: Half of 60 is 30, so go across from 30 on the cumulative frequency axis. The graph passes that height on the segment from $(20,\ 21)$ to $(30,\ 33)$, and the working finds how far along that

segment it happens.)

Step 4: read up from 45 minutes, then subtract

$$50 + \frac{45 - 40}{50 - 40} \times (57 - 50) = 53.5$$

$$60 - 54 = 6$$

(Reason: Reading up from 45 minutes to the graph and across to the cumulative frequency axis gives 53.5. A cumulative frequency counts whole visitors, so about 54 had finished within 45 minutes, and everyone else out of the 60 took longer than that.)

(a) 8, 21, 33, 50, 57, 60

(b) the six totals plotted at the ends of the classes and joined

(c) 27.5 **minutes**

(d) 6 **visitors**

Verification

- ✓ **Check 1:** Add the six frequencies straight up. The last cumulative frequency has to be the number of visitors, because by 60 minutes every visitor has finished. $8 + 13 + 12 + 17 + 7 + 3 = 60$
- ✓ **Check 2:** Find the median from the other end of its class. The graph reaches 33 at 30 minutes, which overshoots by 3 visitors out of the 12 in that class, so come back that fraction of the 10 minutes. $30 - \frac{33 - 30}{33 - 21} \times 10 = 27.5$
- ✓ **Check 3:** Count the visitors still going instead of subtracting from 60. Of the 7 in the class $40 < t \leq 50$ the graph gives $54 - 50 = 4$ finished by 45 minutes, and all 3 in the last class were still going. $(57 - 54) + 3 = 6$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	for 8, 21, 33, 50, 57, 60	✓
(b)	**M1ft**	for at least 5 points plotted correctly at the end of the interval, or follow through from an ascending table (follow through from a table with only one arithmetic error that may be continued through the table) for all 6 points plotted consistently within each interval in the frequency table at the correct height	✓

Step	Mark	Description	Got it?
(b)	**A1**	for fully correct plotting with points joined; accept a curve or line segments; accept a curve that is not joined at $(0, 0)$	✓
(b)	**Note**	A histogram or bar chart type graph scores zero marks unless a cumulative frequency diagram is drawn over the histogram or bar chart. Ignore any part of the graph before $(10, 8)$. Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓
(c)	**B1**	accept an answer in the range 27 to 28, or follow through an ascending graph	✓
(d)	**M1ft**	follow through for a line going up from the x-axis at 45 to the line and across to the y-axis, or for a mark on the line at the correct point, or for a correct reading from the vertical scale, for example 54 or 53.5	✓
(d)	**A1**	accept integer value 5 or 6 or 7, or follow through from their ascending graph for an integer value	✓
(d)	**Note**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 6/6

Question 14 - Exam Solution

Understanding the Question

Given

Five sketch graphs on separate axes, labelled A to E. No scales are marked, so only the shape of each curve can be read. Two equations to match to a sketch: $y = 5 - x^2$ and $y = 2x^3$.

Find

(a) The letter of the graph that could have the equation $y = 5 - x^2$. (b) The letter of the graph that could have the equation $y = 2x^3$.

Plan the Solution

- Read the highest power of x in each equation. That alone decides which family of curve it belongs to.
- Read the sign in front of that highest power. It decides which way up a parabola sits, and which way a cubic runs.
- Sort the five sketches into families first. Each family appears exactly once, so once the family is named the letter is forced.

Worked Solution [2 marks]

> The highest power of x fixes the shape. A term in x^2 gives a parabola, opening upwards when its coefficient is positive and downwards when it is negative. A term in x^3 gives a cubic, running from bottom left to top right when its coefficient is positive. A term in x alone gives a straight line, and $\frac{1}{x}$ gives a curve in two branches with a break at $x = 0$.

Step 1: read the shape of $y = 5 - x^2$

$$y = 5 - x^2 = -x^2 + 5$$

(Reason: The highest power is 2, so the curve is a parabola. The number in front of x^2 is -1, which is negative, so the parabola opens downwards: it turns at a maximum, not a minimum.)

Step 2: test three values, then match

$$5 - (-3)^2 = -4$$
$$5 - 0^2 = 5$$
$$5 - 3^2 = -4$$

(Reason: The curve is below the x-axis at $x = -3$, above it at $x = 0$ and below it again at $x = 3$ - up in the middle and down at both ends. Graph B is the only sketch that does that. Graph D is a parabola as well, but it opens upwards, which needs a positive x^2 term.)

Step 3: read the shape of $y = 2x^3$

$$2 \times (-2)^3 = -16$$
$$2 \times 2^3 = 16$$

(Reason: The highest power is 3, so the curve is a cubic. The two values either side of the y-axis are the same size but opposite in sign, so the curve sits below the x-axis on the left and above it on the right. The coefficient 2 is positive, so it runs upwards from left to right.)

Step 4: match the cubic to a sketch

$$2 \times 0^3 = 0$$

(Reason: The curve must pass through the origin, which rules out graph A - it has a break at $x = 0$ and never meets either axis - and rules out graph E, which is straight and crosses the y-axis above the origin. Both parabolas are out too, because a parabola gives the same value at x and at $-x$ while a

cubic changes sign. Graph C is the one left: it climbs from the bottom left, flattens at the origin and rises steeply to the top right.)

(a) Graph B

(b) Graph C

Verification

✓ **Check 1:** Test the symmetry for part (a). Put $x = 1$ and $x = -1$ into $y = 5 - x^2$. $5 - 1^2 = 4$ and $5 - (-1)^2 = 4$ - the same height either side of the y-axis. Only B and D have that symmetry, and of those two only B turns at a maximum.

✓ **Check 2:** Test part (b) the same way. Put $x = 1$ and $x = -1$ into $y = 2x^3$. $2 \times 1^3 = 2$ and $2 \times (-1)^3 = -2$ - equal in size but opposite in sign, so the curve crosses from below the x-axis to above it as it passes through the origin. Graph C is the only sketch shaped that way.

✓ **Check 3:** Count the families. The five sketches are five different shapes: a two-branch curve, a downward parabola, a cubic, an upward parabola and a straight line. A downward parabola and a cubic each appear exactly once, so the two answers are forced and no letter is used twice.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B1**	B - cao	✓
(b)	**B1**	C - cao	✓

Full marks: 2/2

Question 15 - Exam Solution

Understanding the Question

Given

The expression $\left(\dfrac{2a^3}{10a^7c^2}\right)^{-3}$

Find

The same expression written as simply as possible. Expect one number multiplied by one power of a and one power of c.

One fraction in a and c, raised to the power -3.

Plan the Solution

- Tidy the inside of the bracket first: cancel the numbers, then subtract the indices of a with $\frac{x^m}{x^n} = x^{m-n}$.
- Leave the index outside the bracket until last. It is negative, so it turns the fraction over: raising to -3 is the same as flipping the fraction and raising to 3.
- Cube each factor separately. The number in front is a factor too, so it is cubed as well as the letters.

Worked Solution [3 marks]

> Rule - Index laws: $\frac{x^m}{x^n} = x^{m-n}$, $(x^m)^n = x^{mn}$ and $x^{-n} = \frac{1}{x^n}$.

Step 1: Simplify inside the bracket

$$\frac{2a^3}{10a^7c^2} = \frac{2}{10} \times \frac{a^3}{a^7} \times \frac{1}{c^2}$$
$$\frac{2}{10} = \frac{1}{5}$$
$$\frac{a^3}{a^7} = a^{3-7} = a^{-4} = \frac{1}{a^4}$$
$$\frac{2a^3}{10a^7c^2} = \frac{1}{5a^4c^2}$$

(Reason: Dividing powers of the same letter subtracts the indices, and a^{-4} is another way of writing $\frac{1}{a^4}$, so both letters end up in the denominator.)

Step 2: Turn the fraction over to deal with the negative index

$$\left(\frac{1}{5a^4c^2}\right)^{-3} = \left(5a^4c^2\right)^3$$

(Reason: A negative index means the reciprocal: $y^{-3} = \frac{1}{y^3}$, so a fraction raised to -3 is that fraction turned upside down and raised to 3. Flipping $\frac{1}{5a^4c^2}$ leaves $5a^4c^2$.)

Step 3: Cube every factor inside the bracket

$\left(5a^4c^2\right)^3 = 5^3 \times a^{4\times3} \times c^{2\times3}$
$5^3 \times a^{4\times3} \times c^{2\times3} = 125a^{12}c^6$

(Reason: A power outside a bracket multiplies every index inside it, so a^4 becomes a^{12} and c^2 becomes c^6. The number in front is a factor of the bracket like the letters are, so it is raised to the power too.)

$$125a^{12}c^6$$

Verification

✓ **Check 1:** Put $a = 2$ and $c = 3$ into the original expression, and into the answer, and compare the two values. Inside the bracket $\dfrac{2 \times 8}{10 \times 128 \times 9} = \dfrac{16}{11\,520} = \dfrac{1}{720}$, so the original is $720^3 = 373\,248\,000$, and the answer gives $125 \times 4096 \times 729 = 373\,248\,000$.

✓ **Check 2:** Work backwards. Take the cube root of the answer and check it is the reciprocal of the bracket that Step 1 produced. $\sqrt[3]{125a^{12}c^6} = 5a^4c^2$, and $\dfrac{1}{5a^4c^2}$ is exactly the simplified bracket, so the negative index has been undone correctly.

✓ **Check 3:** Take a different order of working: turn the fraction over first, cube the top and the bottom without simplifying anything, and only then cancel. $\dfrac{(10a^7c^2)^3}{(2a^3)^3} = \dfrac{1000a^{21}c^6}{8a^9} = 125a^{12}c^6$, since $\dfrac{1000}{8} = 125$ and $a^{21-9} = a^{12}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Fully simplified: $125a^{12}c^6$	**B3**	for $125a^{12}c^6$ oe, eg $\dfrac{125a^{12}}{c^{-6}}$ or $\dfrac{125c^6}{a^{-12}}$ or $\dfrac{125}{a^{-12}c^{-6}}$ or $\dfrac{a^{12}}{125^{-1}c^{-6}}$ or $\dfrac{1}{125^{-1}a^{-12}c^{-6}}$ or $\dfrac{a^{12}c^6}{125^{-1}}$	✓
Two correct terms	**B2**	for 2 correct terms (Allow eg $125a^{12}$ or $125c^6$ or $ka^{12}c^6$ as long as not added to any other terms)	✓
One correct term	**B1**	for one correct term (allow eg a^{12} or $a^{-12}c^6$ or $\dfrac{1}{125^{-1}}$ or $\dfrac{1000a^{21}c^6}{8a^9}$ as long as not added to any other terms)	✓

Full marks: 3/3

Question 16 - Exam Solution

Understanding the Question

Given

The formula $c = \dfrac{t^2 + 3}{7 - 8t^2}$.

The letter t appears in the numerator and in the denominator, and only ever as t^2.

Find

A formula for t in terms of c.

Plan the Solution

- Multiply both sides by $7 - 8t^2$ so that no t is left underneath a fraction bar.
- Gather every term containing t^2 on one side and everything else on the other.
- Take t^2 out as a common factor, then divide by the bracket that is left.
- Square root last, and keep both signs.

Worked Solution [4 marks]

> Rule - Changing the subject: clear the fraction, collect every term containing the new subject on one side, factorise it out, then undo what is left around it. Here the new subject appears as t^2, so the last step is a square root and both signs are kept.

Step 1: multiply both sides by the denominator

$$c(7 - 8t^2) = t^2 + 3$$
$$7c - 8ct^2 = t^2 + 3$$

(Reason: (Reason: while t sits underneath a fraction bar nothing can be collected, so the fraction is cleared first. Multiplying out the bracket turns the left-hand side into the two terms $7c$ and $-8ct^2$.))

Step 2: collect the t^2 terms on one side

$$7c - 3 = t^2 + 8ct^2$$

(Reason: (Reason: add $8ct^2$ to both sides and take the 3 across, so that the two t^2 terms end up together on the right and the terms with no t in them end up on the left.))

Step 3: factorise t^2 out

$$7c - 3 = t^2(1 + 8c)$$

(Reason: (Reason: t^2 is a common factor of t^2 and $8ct^2$, and taking it out is what turns two t^2 terms into one.))

Step 4: divide, then square root

$$t^2 = \frac{7c-3}{1+8c}$$

$$t = \pm\sqrt{\frac{7c-3}{1+8c}}$$

(Reason: (Reason: dividing by the bracket leaves t^2 on its own. Squaring hides a sign, so undoing it gives two values: both t and $-t$ lead back to the same c.))

$$t = \pm\sqrt{\frac{7c-3}{1+8c}}$$

Verification

✓ **Check 1 - put $t = 1$through both formulas:** The formula on the paper gives $\frac{1+3}{7-8} = \frac{4}{-1} = -4$, so $c = -4$. Feeding $c = -4$ into the rearranged formula gives $\frac{7\times(-4)-3}{1+8\times(-4)} = \frac{-31}{-31} = 1$, and the square root of that is 1 again.

✓ **Check 2 - work the other way, from c back to t:** Put $c = 1$ into the rearranged formula: $\frac{7-3}{1+8} = \frac{4}{9}$, so $t = \pm\frac{2}{3}$. Putting $t^2 = \frac{4}{9}$ back into the formula on the paper, and multiplying the top and the bottom by 9, gives $\frac{4+27}{63-32} = \frac{31}{31} = 1$, the value of c we started with.

✓ **Check 3 - why both signs are kept:** The formula uses t only as t^2, so t and $-t$ have to give the same c. $t = -1$ gives $\frac{1+3}{7-8} = -4$, the same value as $t = 1$, so an answer carrying one sign only would lose half of it.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$7c - 8ct^2 = t^2 + 3$ oe	M1	For multiplying both sides by the denominator and expanding the brackets.	✓

Step	Mark	Description	Got it?
$7c - 3 = t^2 + 8ct^2$ oe or $-8ct^2 - t^2 = 3 - 7c$ oe	**M1**	Follow through, dependent on 2 terms in t^2 and 2 other terms. For collecting the t^2 terms on one side and the other terms on the other side.	✓
$7c - 3 = t^2(1 + 8c)$ oe or $t^2(-8c - 1) = 3 - 7c$ oe	**M1**	Follow through, dependent on the previous M1. For factorising for t^2.	✓
$t = (\pm)\sqrt{\dfrac{7c-3}{1+8c}}$	**A1**	Or equivalent, for example $t = (\pm)\sqrt{\dfrac{3-7c}{-8c-1}}$ or $t = (\pm)\left(\dfrac{7c-3}{1+8c}\right)^{\frac{1}{2}}$ or $t = (\pm)\left(\dfrac{7c-3}{1+8c}\right)^{0.5}$.	✓
NB	**Note**	To award the A1 we must see $t = (\pm)\sqrt{\dfrac{7c-3}{1+8c}}$ in the working if $(\pm)\sqrt{\dfrac{7c-3}{1+8c}}$ alone is given as the answer.	✓
Working	**Note**	Working is not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 17 - Exam Solution

Understanding the Question

Given

$y = 4x^3 + 5x^2 + 2x$

A cubic, so the derivative is a quadratic and the curve can have up to two turning

Find

(a) $\dfrac{dy}{dx}$ (b) the coordinates of the turning points, so an x and a y for each one points.

Plan the Solution

- Differentiate term by term: multiply each term by its power, then take one off the power.
- A turning point is where the curve is momentarily flat, so set $\frac{dy}{dx} = 0$ and solve.
- Every coefficient of the derivative is even, so take the factor 2 out before factorising the quadratic.
- Put each root back into the original cubic to get its y value. The derivative gives gradients, not heights.

Worked Solution [6 marks]

> Rule - Turning points: differentiate, set $\frac{dy}{dx} = 0$, solve for x, then substitute each x back into the original equation to get y.

Step 1: Differentiate term by term

$y = 4x^3 + 5x^2 + 2x$

$\frac{dy}{dx} = 3 \times 4x^2 + 2 \times 5x + 2$

$\frac{dy}{dx} = 12x^2 + 10x + 2$

(Reason: The power rule: multiply by the power, then take one off it. So $4x^3$ gives $12x^2$, $5x^2$ gives $10x$, and $2x$ gives 2. That is part (a) answered.)

Step 2: Set the gradient equal to zero

$12x^2 + 10x + 2 = 0$

$2(6x^2 + 5x + 1) = 0$

$6x^2 + 5x + 1 = 0$

(Reason: At a turning point the tangent is horizontal, so the gradient is 0. Every coefficient is even, so dividing through by 2 leaves smaller numbers to factorise.)

Step 3: Factorise and solve for x

$6x^2 + 5x + 1 = (3x + 1)(2x + 1)$

$(3x + 1)(2x + 1) = 0$

$3x + 1 = 0 \implies x = -\frac{1}{3}$

$$2x + 1 = 0 \implies x = -\frac{1}{2}$$

(Reason: Split the 6 as 3×2 and the 1 as 1×1; the cross terms $3x + 2x$ rebuild the middle term $5x$. A product is zero only when one of its brackets is zero.)

Step 4: Substitute $x = -\frac{1}{2}$ into the original equation

$$y = 4\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$$

$$y = -\frac{1}{2} + \frac{5}{4} - 1 = -\frac{1}{4}$$

(Reason: The height comes from the original cubic, never from the derivative. Cubing a negative keeps it negative, while squaring it makes it positive, so the first term is $-\frac{1}{2}$ and the second is $+\frac{5}{4}$.)

Step 5: Substitute $x = -\frac{1}{3}$ into the original equation

$$y = 4\left(-\frac{1}{3}\right)^3 + 5\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right)$$

$$y = -\frac{4}{27} + \frac{5}{9} - \frac{2}{3} = -\frac{7}{27}$$

(Reason: The same substitution again. Writing all three terms over a denominator of 27 keeps the fraction arithmetic exact.)

(a) $\dfrac{dy}{dx} = 12x^2 + 10x + 2$

(b) $\left(-\frac{1}{2}, -\frac{1}{4}\right)$ **and** $\left(-\frac{1}{3}, -\frac{7}{27}\right)$

Verification

✓ **Check 1:** Expand the factorisation back out. $(3x+1)(2x+1)$ should rebuild $6x^2 + 5x + 1$, and doubling every term should rebuild part (a). $2(6x^2 + 5x + 1) = 12x^2 + 10x + 2$

✓ **Check 2:** Put each root back into the derivative. The gradient must come out as exactly 0 at a turning point. $12\left(-\frac{1}{2}\right)^2 + 10\left(-\frac{1}{2}\right) + 2 = 0$ and

$$12\left(-\frac{1}{3}\right)^2 + 10\left(-\frac{1}{3}\right) + 2 = 0$$

✓ **Check 3:** The two roots of $6x^2 + 5x + 1 = 0$ must add to $-\frac{5}{6}$ and multiply to $\frac{1}{6}$.

$$-\frac{1}{2} - \frac{1}{3} = -\frac{5}{6} \text{ and } \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right) = \frac{1}{6}$$

✓ **Check 4:** Redo the second turning point in decimals on the calculator. $x \approx -0.3333$ gives a height just below the first turning point, which is what a local minimum sitting to the right of a local maximum should do. $-\frac{7}{27} \approx -0.2593$, lower than $-\frac{1}{4} = -0.25$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Two of $12x^2 + 10x + 2$	M1	for differentiating 2 or 3 terms correctly	✓
(a) $12x^2 + 10x + 2$	A1	for all 3 terms correct	✓
(b) $(3x+1)(4x+2)$ $(=0)$ or $(6x+2)(2x+1)(=0)$ or $2(3x+1)(2x+1)$ $(=0)$ or $(3x+1)(2x+1)(=0)$ or $\frac{-10 \pm \sqrt{10^2 - 4 \times 12 \times 2}}{2 \times 12}$ or $\frac{-5 \pm \sqrt{5^2 - 4 \times 6 \times 1}}{2 \times 6}$ or $12\left[\left(x + \frac{10}{24}\right)^2 - \left(\frac{10}{24}\right)^2\right] + 2\ (=0)$ oe or $6\left[\left(x + \frac{5}{12}\right)^2 - \left(\frac{5}{12}\right)^2\right] + 1\ (=0)$ oe	M1	ft dep on M1 for a correct method to solve their 3 term quadratic equation (with at least 2 correct coefficients) using any correct method (if factorising, allow brackets which expanded give 2 out of 3 terms correct) (if using formula allow one sign error and some simplification - allow as far as $\frac{-10 \pm \sqrt{100 - 96}}{24}$) Derivative must be a 3 term quadratic for this M mark NB Can be implied by answers of $(x =) -\frac{1}{2}$ and $(x =) -\frac{1}{3}$	✓
(b) $-\frac{1}{2}, -\frac{1}{3}$	A1	oe dep on previous M1. Allow $-0.33(333)$ or $-0.\dot{3}$ for correct x values	✓

Step	Mark	Description	Got it?
(b) $(y=)4\times\left(-\frac{1}{2}\right)^3+5\times\left(-\frac{1}{2}\right)^2+2\left(-\frac{1}{2}\right)\left(=-\frac{1}{4}\right)$ or $(y=)4\times\left(-\frac{1}{3}\right)^3+5\times\left(-\frac{1}{3}\right)^2+2\left(-\frac{1}{3}\right)\left(=-\frac{7}{27}\right)$	M1	ft dep on previous M1 for substituting at least one x value into y NB Can be implied by one correct value of y	✓
(b) Working required. $\left(-\frac{1}{2},\ -\frac{1}{4}\right)$, $\left(-\frac{1}{3},\ -\frac{7}{27}\right)$	A1	oe dep on M1 for correct coordinates $(-0.5,\ -0.25)$, $(-0.33,\ -0.25(9\ldots))$	✓

Full marks: 6/6

Question 18 - Exam Solution

Understanding the Question

Given

The recurring decimal $0.\dot{7}0\dot{2}$. The dots sit on the 7 and on the 2, so the block 702 repeats for ever: $0.702702702\ldots$

The fraction it is claimed to equal: $\frac{26}{37}$

Find

Show that $0.\dot{7}0\dot{2}$ is exactly $\frac{26}{37}$. The working is the answer here, so there is nothing to write on an answer line, and the working must be algebra - a solution with no algebra is capped at 1 mark.

Plan the Solution

- Give the recurring decimal a letter, so that there is something to do algebra with.
- Count the digits in the repeating block: 702 is 3 digits long, so the multiplier is $10^3 = 1000$.
- Write the two lines under each other and subtract. The endless tails are identical, so they cancel and leave a whole number.

- Divide to reach a fraction, then cancel that fraction to its lowest terms.

Worked Solution [2 marks]

> Rule - Recurring decimal to fraction: call the decimal x. If the repeating block is n digits long, multiply by 10^n and subtract x. The identical tails cancel, and $(10^n - 1)x$ is left equal to a whole number.

Step 1: give the decimal a letter

$x = 0.\dot{7}0\dot{2} = 0.702702702\ldots$

(Reason: Algebra needs something to hold on to, so the decimal is called x. The dots say the block 702 repeats without end, so the digits after the point never stop and never change pattern.)

Step 2: multiply by 1000

$1000x = 702.\dot{7}0\dot{2} = 702.702702\ldots$

(Reason: The repeating block is 3 digits long, so multiplying by $10^3 = 1000$ slides the point past exactly one whole block. The tail left after the point is then the same tail as before, digit for digit.)

Step 3: subtract the two lines

$1000x - x = 702.702702\ldots - 0.702702\ldots$

$999x = 702$

(Reason: Both lines carry exactly the same endless tail, so subtracting wipes it out and leaves a whole number. Choosing the multiplier to match the block length is what makes that happen, and it is what earns the first mark.)

Step 4: divide, then cancel

$x = \dfrac{702}{999}$

$702 = 27 \times 26$

$999 = 27 \times 37$

$\dfrac{702}{999} = \dfrac{26}{37}$

(Reason: Divide both sides by 999. The highest common factor of 702 and 999 is 27, so dividing the top and the bottom by 27 puts the fraction in its lowest terms. Cancelling by anything smaller leaves the fraction unfinished, and the second mark is for the completion.)

$$0.\dot{7}0\dot{2} = \frac{26}{37} \textbf{ as required}$$

Verification

✓ **Check 1:** Two fractions are equal exactly when their cross products match. Multiply 26 by 999, then multiply 37 by 702. $26 \times 999 = 25\,974$ and $37 \times 702 = 25\,974$

✓ **Check 2:** Start again with a different pair of lines, $10000x$ and $10x$. Their tails match too, so subtracting gives $9990x = 7020$. $\dfrac{7020}{9990} = \dfrac{26}{37}$ - the same fraction, from different multipliers

✓ **Check 3:** Go the other way and divide 26 by 37 by hand. The remainders come out 1, 10, 26 - and 26 is where the division started. Once a remainder repeats the digits must repeat too, so the quotient is $0.702702702\ldots$, which is $0.\dot{7}0\dot{2}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $(1000x =)702.702\ldots$ and $(x =)0.702\ldots$	**M1**	for 2 recurring decimals that when subtracted give a whole number or terminating decimal with intention to subtract (ie give 702 or 7020 or 70200 etc) eg $(1000x =)702.702\ldots$ and $(x =)0.702\ldots$ or $(10000x =)7027.02\ldots$ and $(10x =)7.02\ldots$ or $(100000x =)70270.20\ldots$ and $(100x =)70.20\ldots$ with intention to subtract x is not required to award this mark (if recurring dots not shown in both numbers then showing at least one of the numbers to at least 6sf) NB Accept bar notation for dot notation to indicate recurring decimals	✓
eg $1000x - x = 702.702\ldots - 0.702\ldots = 702$ $(999x = 702)$ and $\dfrac{702}{999} = \dfrac{26}{37}$ or $10000x - 10x = 7027.02\ldots - 7.02\ldots = 7020$	**A1**	for completion to $\dfrac{26}{37}$ dep on M1 and must use algebra for this final mark to be awarded No algebra used gets a maximum of 1 mark	✓

Step	Mark	Description	Got it?
$(9990x = 7020)$ and $\dfrac{7020}{9990} = \dfrac{26}{37}$ or $100000x - 100x = 70270.20\ldots - 70.20\ldots = 70200$ $(99900x = 70200)$ and $\dfrac{70200}{99900} = \dfrac{26}{37}$ Answer column: shown			
Working required	**Note**	The fraction is printed in the question, so a solution that only restates it earns nothing: the working is what is being marked.	✓

Full marks: 2/2

Question 19 - Exam Solution

Understanding the Question

Given

The quadratic inequality
$4x^2 + 4x - 15 < 0$.
The coefficient of x^2 is 4, which is positive, so $y = 4x^2 + 4x - 15$ is a parabola opening upwards: it dips below the axis once and comes back up.
The sign is strict. < 0 means negative, so a value of x that makes the expression exactly 0 is not part of the solution.

Find

Every value of x for which
$4x^2 + 4x - 15$ is negative, written as one inequality. Clear algebraic working is asked for, so the two critical values must be reached by factorising, by the formula or by completing the square. Both answer marks depend on that method mark, so critical values written down with nothing behind them score nothing at all.

Plan the Solution

- Solve the equation $4x^2 + 4x - 15 = 0$ first. Its two solutions are the critical values, where the curve crosses the x-axis and where the expression can change sign.

- Factorise by splitting the middle term: look for two numbers with product $4 \times (-15) = -60$ and sum 4.
- Decide which side of the critical values the curve sits below the axis. With a positive x^2 coefficient that is the region between them, and one test value confirms it.
- Write the answer as a single double inequality, with strict signs at both ends.

Worked Solution [3 marks]

> Rule - Quadratic inequality: solve the equation to find the critical values, then choose the region. For $ax^2 + bx + c < 0$ with $a > 0$ the expression is negative between the two roots and positive outside them.

Step 1: solve the equation $4x^2 + 4x - 15 = 0$

$4x^2 + 4x - 15 = 0$
$4 \times (-15) = -60$
$10 \times (-6) = -60$
$10 + (-6) = 4$
$4x^2 + 10x - 6x - 15 = 0$
$2x(2x + 5) - 3(2x + 5) = 0$
$(2x + 5)(2x - 3) = 0$

(Reason: The expression can only change sign where it is zero, so the equation comes first. To factorise it, look for two numbers whose product is $4 \times (-15) = -60$ and whose sum is 4: they are 10 and -6. Splitting the middle term with them leaves the common bracket $(2x + 5)$ to take out. This is the method mark, and the quadratic formula or completing the square would earn it just as well.)

Step 2: read the critical values off the brackets

$$2x + 5 = 0 \implies x = -\frac{5}{2} = -2.5$$

$$2x - 3 = 0 \implies x = \frac{3}{2} = 1.5$$

(Reason: A product is zero only when one of its factors is zero, so each bracket gives one critical value. These are the two places where the curve $y = 4x^2 + 4x - 15$ meets the x-axis, and they are what the first answer mark is for.)

Step 3: decide which side of the critical values is negative

$4 \times 0^2 + 4 \times 0 - 15 = -15$
$4 \times 2^2 + 4 \times 2 - 15 = 9$
$4 \times (-3)^2 + 4 \times (-3) - 15 = 9$

(Reason: The critical values -2.5 and 1.5 cut the number line into three pieces, and the expression keeps one sign right across each piece. Testing $x = 0$ from between them gives -15, which is

negative. Testing $x = 2$ and $x = -3$ from outside gives 9 both times, which is positive. That is exactly the shape a positive x^2 coefficient forces: below the axis between the roots, above it either side.)

Step 4: write the solution as one inequality

$-2.5 < x < 1.5$

(Reason: The negative piece is the one between the critical values, so x has to be greater than -2.5 and less than 1.5 at the same time, and those two conditions are written as one double inequality. Both signs stay strict, because at -2.5 and at 1.5 the expression is 0, and the question asks for less than 0. Turning the answer inside out, as $x < -2.5$ or $x > 1.5$, would describe where the expression is positive instead.)

$$-2.5 < x < 1.5 \textbf{ (equivalently } -\frac{5}{2} < x < \frac{3}{2}\textbf{)}$$

Verification

✓ **Check 1:** Take two values from inside the interval, $x = 1$ and $x = -2$, and work the expression out for each. $4 + 4 - 15 = -7$ and $16 - 8 - 15 = -7$, both negative, so both values belong

✓ **Check 2:** Take one value from beyond each end, $x = 2$ and $x = -3$, so that both sides outside the interval are tested. $16 + 8 - 15 = 9$ and $36 - 12 - 15 = 9$, both positive, so neither value belongs

✓ **Check 3:** Multiply the brackets out again and compare the result with the expression the question prints. $(2x + 5)(2x - 3) = 4x^2 - 6x + 10x - 15 = 4x^2 + 4x - 15$

✓ **Check 4:** Reach the two critical values a completely different way, with the quadratic formula, and see whether they come back the same. $\dfrac{-4 \pm \sqrt{16 + 240}}{8} = \dfrac{-4 \pm 16}{8}$, giving 1.5 and -2.5 again

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(2x - 3)(2x + 5)$ or $\dfrac{-4 \pm \sqrt{4^2 - 4 \times 4 \times -15}}{2 \times 4}$ or	**M1**	for a correct method to solve the quadratic equation $4x^2 + 4x - 15 = 0$ Allow $(4x - 6)(x + 2.5)$ or $(4x + 10)(x - 1.5)$ or $(4x - 6)(4x + 10)$ leading to $(x - 1.5)(x + 2.5)$ or $(4x - 6)(4x + 10)$ leading to correct values of x	✓

Step	Mark	Description	Got it?
$4\left[\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2\right]-15\,(=0)$ or $4\left(x+\frac{4}{2\times 4}\right)^2-\frac{4^2}{4\times 4}+-15\,(=0)$		Do not allow $(x-1.5)(x+2.5)$ without previous working (If using formula allow some simplification - allow as far as $\frac{-4\pm\sqrt{16+240}}{8}$)	
$1.5, -2.5$ oe	A1	oe dep on M1	✓
Answer column: $-2.5 < x < 1.5$	A1	oe dep on M1 Allow $x > -2.5$ (and) $x < 1.5$ oe Allow any variable as long as used all the way through	✓
Where the three marks fall	**Note**	The two answer marks sit in separate rows and both depend on the method mark, so a solution that states the critical values with no method behind them - the $(x-1.5)(x+2.5)$ case the first row rules out - scores nothing rather than one. The second answer mark is for the region and not for the numbers, so a candidate who reaches 1.5 and -2.5 and then writes $x < -2.5$ or $x > 1.5$ keeps the first answer mark and loses the second: that is the region where the expression is positive.	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

A sports club with 120 members, each asked whether they play cricket C, padel

Find

(a) The number in each of the eight regions of the Venn diagram. (b)

P or tennis T.
$\mathrm{n}(C) = 43, \mathrm{n}(C \cap P) = 18,$
$\mathrm{n}(C \cap T) = 27, \mathrm{n}(P \cap T) = 32,$
$\mathrm{n}(C \cap P \cap T) = 12.$
29 members play none of the three, and the number who play only padel equals the number who play only tennis.

$\mathrm{n}(T' \cap C)$. (c) The probability that a member chosen at random from the cricket players also plays padel.

Plan the Solution

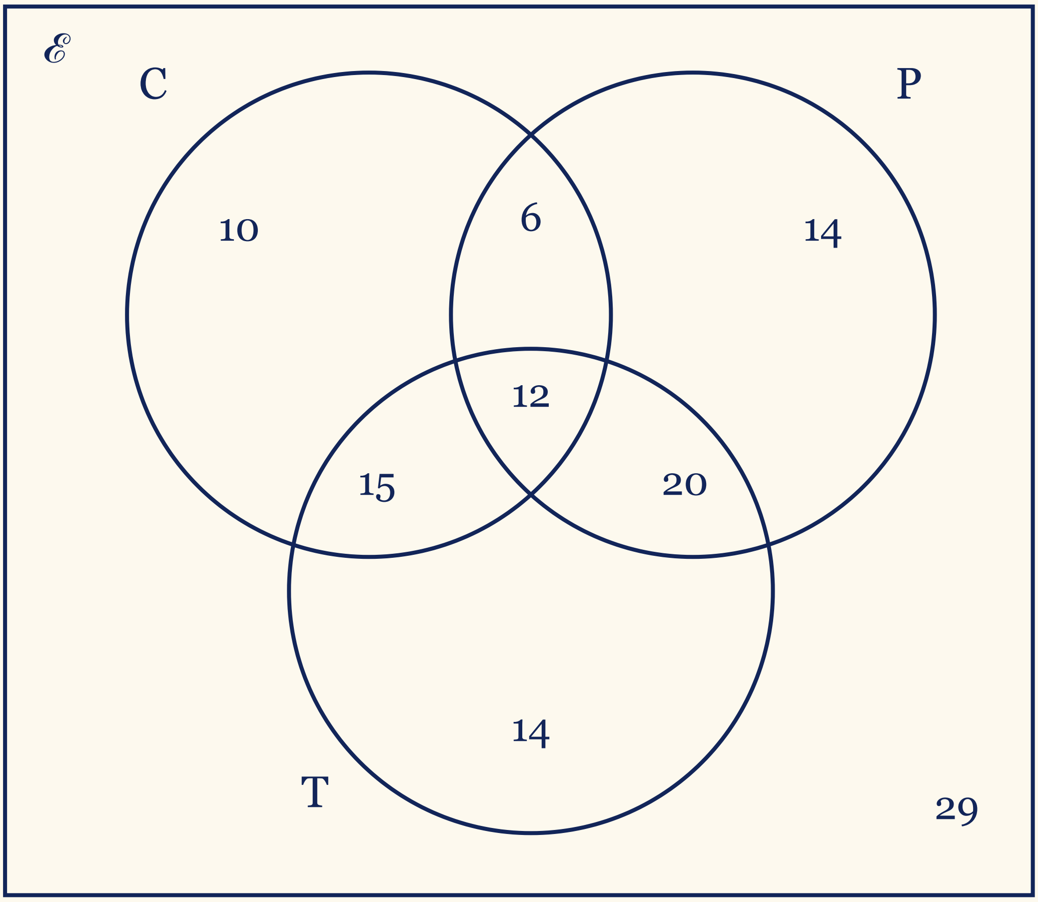
- Start in the middle, where all three circles overlap, because that number is given outright.
- Take the middle away from each pair total to get the three regions where exactly two sports are played.
- Subtract the three known cricket regions from 43 to leave cricket only.
- Everything inside the circles comes to $120 - 29$, so the two equal regions left over can be found and halved.
- Read part (b) off the finished diagram, then write part (c) as a fraction of the 43 cricket players and not of all 120 members.

Worked Solution [6 marks]

> Rule - Fill a Venn diagram from the centre outwards. Every overlap total already contains the region in the middle, so subtract it: $\mathrm{n}(C \cap P) - \mathrm{n}(C \cap P \cap T)$ is the number in both C and P but not in T. When the diagram is complete, all eight regions must add up to 120.

Step 1: Write in the middle region

$\mathrm{n}(C \cap P \cap T) = 12$



(Reason: The question gives the number who play all three sports outright, and every other overlap total contains it, so nothing else can be worked out until it is in place.)

Step 2: Take the middle out of each pair total

$18 - 12 = 6$

$27 - 12 = 15$

$32 - 12 = 20$

(Reason: The 18 who play cricket and padel includes the 12 who play all three, so only 6 play cricket and padel but not tennis. The same subtraction leaves 15 for cricket and tennis only and 20 for padel and tennis only.)

Step 3: Finish the cricket circle

$43 - (6 + 12 + 15) = 43 - 33 = 10$

(Reason: The cricket circle holds 43 members altogether. Three of its four regions are now known, so taking those off leaves the members who play cricket and nothing else.)

Step 4: Use the total to split the last two regions

$120 - 29 = 91$

$91 - (10 + 6 + 15 + 12 + 20) = 28$

$$\frac{28}{2} = 14$$

(Reason: 29 members play none of the three sports, so 91 are somewhere inside the circles. Taking away the five regions already filled leaves 28 to share between padel only and tennis only, and the question says those two are equal, so each of them is 14.)

Step 5: Read $\mathrm{n}(T' \cap C)$ off the diagram

$$\mathrm{n}(T' \cap C) = 10 + 6 = 16$$

(Reason: T' is everything outside the tennis circle, so $T' \cap C$ is the part of the cricket circle that the tennis circle does not reach. That is cricket only together with cricket and padel only, $10 + 6$.)

Step 6: Write part (c) as a fraction of the cricket players

$$6 + 12 = 18$$

$$\frac{18}{43}$$

(Reason: The member is picked from the cricket players only, so the denominator is 43 and not 120. Of those 43, the ones who also play padel are the 6 in cricket and padel only plus the 12 who play all three.)

(a) C only 10, P only 14, T only 14, all three 12

C and P only 6, C and T only 15, P and T only 20, none 29

(b) 16

(c) $\frac{18}{43}$

Verification

✓ **Check 1:** Add every region on the finished diagram: $10 + 6 + 14 + 15 + 12 + 20 + 14 + 29$. The eight regions come to 120, the number of members surveyed, so nobody has been left out or counted twice.

✓ **Check 2:** Count each circle off the finished diagram - $\mathrm{n}(P) = 6 + 14 + 12 + 20$ and $\mathrm{n}(T) = 15 + 12 + 20 + 14$ - then put the three circle totals through the inclusion and exclusion rule. $43 + 52 + 61 - 18 - 27 - 32 + 12 = 91$, which is exactly $120 - 29$, so the circle totals agree with the overlap figures the question gives.

✓ **Check 3:** The padel players inside the cricket circle should come back to the 18 the question states for cricket and padel, and the probability should sit a little under a half.

$6 + 12 = 18$, and $\dfrac{18}{43} = 0.4186$ to four decimal places, which is just below 0.5 as expected because 18 is a little under half of 43.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) The completed Venn diagram	**B3**	For all numbers in correct regions: $10, 6, 14, 15, 12, 20, 14, 29$. B2 for 5, 6 or 7 correct numbers. B1 for 3 or 4 correct numbers.	✓
(b) $\mathrm{n}(T' \cap C)$	**B1**	16. Follow through for their a plus their b. Do not follow through if there are no values for a and b.	✓
(c) A correct probability method	**M1**	For $\dfrac{n}{43}$ where $n < 43$ or $\dfrac{18}{m}$ where $m > 18$ or follow through their $\dfrac{b+c}{p}$ where $p > b + c$ or follow through their $\dfrac{q}{a+b+c+d}$ where $q < a + b + c + d$. Do not follow through if there are no values for a, b, c and d.	✓
(c) The probability	**A1**	$\dfrac{18}{43}$ or equivalent, for example $0.41(860\ldots)$ or $41(.860\ldots)\%$ truncated or rounded, or follow through their $\dfrac{b+c}{a+b+c+d}$ as a fraction or a decimal or a percentage.	✓
Note	**Note**	The printed scheme shades four regions of the cricket circle and names them on its own diagrams: a is C only, b is C and P only, c is the region all three share and d is C and T only. So $a + b + c + d$ is the whole of C and $b + c$ is $C \cap P$.	✓

Full marks: 6/6

Question 21 - Exam Solution

Understanding the Question

Given

A cuboid with edges $3x$ cm, $2x$ cm and y cm
The volume is 1014 cm³
The total surface area is A cm²

Find

Show that $A = 12x^2 + \dfrac{1690}{x}$, so the surface area is written in terms of x alone

Plan the Solution

- Use the volume to link x and y: multiply the three edges together and set the product equal to 1014.
- Rearrange that equation to make y the subject.
- Add the three pairs of faces to get the surface area in terms of x and y.
- Replace y by its expression in x, so that only x is left.

Worked Solution [3 marks]

> Rule - Cuboid: a cuboid with edges a, b and c has volume abc and total surface area $2ab + 2bc + 2ca$, because the six faces come in three matching pairs.

Step 1: write an equation for the volume

$3x \times 2x \times y = 1014$
$6x^2y = 1014$
$x^2y = 169$

(Reason: (Reason: the volume of a cuboid is the product of its three edges, and $3x \times 2x = 6x^2$. Dividing both sides by 6 gives the tidier form, because $\dfrac{1014}{6} = 169$.))

Step 2: make y the subject

$$y = \frac{169}{x^2}$$

(Reason: (Reason: dividing both sides of $x^2y = 169$ by x^2 leaves y on its own, so from here every y can be written in terms of x.))

Step 3: write the surface area in terms of x and y

$A = 2(3x \times 2x) + 2(3x \times y) + 2(2x \times y)$
$A = 12x^2 + 6xy + 4xy$
$A = 12x^2 + 10xy$

(Reason: (Reason: the faces come in three pairs. The top and bottom are each $3x$ by $2x$, the front and back are each $3x$ by y, and the two ends are each $2x$ by y. The two xy terms then collect into one.))

Step 4: replace y and simplify

$$A = 12x^2 + 10x \times \frac{169}{x^2}$$

$$10x \times \frac{169}{x^2} = \frac{10 \times 169}{x} = \frac{1690}{x}$$

$$A = 12x^2 + \frac{1690}{x}$$

(Reason: (Reason: y has been replaced by $\frac{169}{x^2}$. One factor of x cancels from the top and the bottom, which turns $\frac{x}{x^2}$ into $\frac{1}{x}$, and that is the expression the question asks for.))

Shown: $A = 12x^2 + \frac{1690}{x}$

Verification

✓ **Check 1:** Take $x = 1$. The volume equation gives $y = \frac{169}{1^2} = 169$, so the cuboid measures 3 cm by 2 cm by 169 cm. Its volume is $3 \times 2 \times 169 = 1014$ cm³, and adding the three pairs of faces gives $2 \times 6 + 2 \times 507 + 2 \times 338 = 1702$ cm².
$12 \times 1^2 + \frac{1690}{1} = 1702$, the same total

✓ **Check 2:** Take $x = 13$, a value that makes the third edge short. Then $y = \frac{169}{13^2} = 1$, so the cuboid measures 39 cm by 26 cm by 1 cm. Its volume is $39 \times 26 \times 1 = 1014$ cm³, and adding the three pairs of faces gives $2 \times 1014 + 2 \times 39 + 2 \times 26 = 2158$ cm².
$12 \times 13^2 + \frac{1690}{13} = 2158$, the same total again

✓ **Check 3:** Read the printed expression backwards. Matching $12x^2 + 10xy$ with $12x^2 + \frac{1690}{x}$ needs $10xy = \frac{1690}{x}$, and multiplying both sides by x then dividing by 10 gives $x^2y = \frac{1690}{10}$. $x^2y = 169$, which is exactly the volume equation Step 1 produced, so the two ends of the working meet

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Form an equation for the volume in terms of x and y	**M1**	$3x \times 2x \times y = 1014$ oe or $6x^2y = 1014$ oe or $x^2y = 169$ oe. For an equation for volume in terms of x and y.	✓
Write a correct expression for the total surface area	**M1 indep**	$2 \times 3x \times y + 2 \times 2x \times y + 2 \times 3x \times 2x$ oe or $2 \times (3xy + 2xy + 6x^2)$ oe or $6xy + 4xy + 12x^2$ oe or $10xy + 12x^2$ oe. Independent of the first mark. NB the surface area expression may not be seen explicitly in terms of y, eg $2 \times 3x \times \frac{169}{x^2} + 2 \times 2x \times \frac{169}{x^2} + 2 \times 3x \times 2x$ oe. It may be fully substituted using $y = \frac{169}{x^2}$ oe.	✓
Complete the show that	**A1 dep on M2**	Using $y = \frac{1014}{6x^2}\left(= \frac{169}{x^2}\right)$ in the formula for the surface area to obtain a correct expression, eg (SA =) $2 \times 5x \times \frac{169}{x^2} + 2 \times 6x^2 = 12x^2 + \frac{1690}{x}$, or equating their surface area equations, eg $10xy + 12x^2 = 12x^2 + \frac{1690}{x}$ leading to $x^2y = 169$ oe. Dependent on both method marks. For completing the show that by clearly showing the stages that lead to the given expression for the surface area.	✓
Working required	**Note**	Working required. Total 3 marks.	✓

Full marks: 3/3

Question 22 - Exam Solution

Understanding the Question

Given

The length AB is 11.5 cm, correct to the nearest 0.5 cm
The length BC is 9.2 cm, correct to 2 significant figures
The side of the square, s, is 4.1 cm, correct to 2 significant figures
The shaded region is the rectangle with the square removed, so X is $AB \times BC - s^2$

Find

The value of X to a suitable degree of accuracy: the most accurate value that every possible set of measurements agrees on

Plan the Solution

- Write the lower and upper bound of each of the three lengths. A bound sits half a rounding unit either side of the value given.
- The shaded area is a difference, so it is largest when the rectangle is largest and the square is smallest, and smallest when the rectangle is smallest and the square is largest.
- Work out the upper bound of X and the lower bound of X.
- Round both bounds to the same accuracy, starting with the most accurate, and stop at the first degree of accuracy where they agree. That shared value is X.

Worked Solution [4 marks]

> Rule - Bounds of a difference: to make $P - Q$ as large as possible, take the upper bound of P with the lower bound of Q. To make it as small as possible, take the lower bound of P with the upper bound of Q.

Step 1: Write down the bounds of the three lengths

$$\frac{0.5}{2} = 0.25$$
$$\frac{0.1}{2} = 0.05$$
$$11.25 \le AB < 11.75$$
$$9.15 \le BC < 9.25$$
$$4.05 \le s < 4.15$$

(Reason: AB is given to the nearest 0.5 cm, so its bounds are 0.25 either side of 11.5. Giving 9.2 and 4.1 to 2 significant figures is rounding to the nearest 0.1 cm, so those bounds are 0.05 either side.)

Step 2: The upper bound of X

$11.75 \times 9.25 - 4.05^2 = 108.6875 - 16.4025 = 92.285$

(Reason: The shaded area is as large as it can be when the rectangle is as large as it can be and the square taken out of it is as small as it can be, so this uses 11.75, 9.25 and 4.05.)

Step 3: The lower bound of X

$11.25 \times 9.15 - 4.15^2 = 102.9375 - 17.2225 = 85.715$

(Reason: The shaded area is as small as it can be when the rectangle is as small as it can be and the square taken out of it is as large as it can be, so this uses 11.25, 9.15 and 4.15.)

Step 4: Round both bounds until they agree

$92.285 \approx 92$ (2 s.f.)

$85.715 \approx 86$ (2 s.f.)

$92.285 \approx 90$ (1 s.f.)

$85.715 \approx 90$ (1 s.f.)

(Reason: To 2 significant figures the two bounds give 92 and 86, which are different, so that is more accuracy than the measurements support. To 1 significant figure both give 90, so every value the shaded area could take rounds to the same number, and that number is the answer.)

$X = 90$ **(to 1 significant figure)**

Verification

✓ **Check 1 - round both bounds together:** Round the upper bound and the lower bound to the same accuracy and compare them. To 2 significant figures they are 92 and 86. They disagree at 2 significant figures and agree at 1, which gives 90.

✓ **Check 2 - use the measurements as they are stated:** Work out the shaded area from the stated lengths themselves: $11.5 \times 9.2 - 4.1^2$. $105.8 - 16.81 = 88.99$, which lies between the two bounds and also rounds to 90.

✓ **Check 3 - pair the bounds the other way round:** Take the largest rectangle with the largest square, and the smallest rectangle with the smallest square: $11.75 \times 9.25 - 4.15^2 = 91.465$ and $11.25 \times 9.15 - 4.05^2 = 86.535$. Both sit inside 85.715 to 92.285, so the pairings used above really are the extreme ones.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
11.25, 11.75, 9.15, 9.25, 4.05, 4.15	B1	for a correct bound. Accept $11.74\dot{9}$ or $11.74\overline{9}$ for 11.75, $9.24\dot{9}$ or $9.24\overline{9}$ for 9.25, $4.14\dot{9}$ or $4.14\overline{9}$ for 4.15	✓

Step	Mark	Description	Got it?
$11.75 \times 9.25 - 4.05^2$ $(= 92.285)$	**M1**	for a correct method to find the upper bound of X, allow $(11.5 < AB \leq 11.75) \times (9.2 < BC \leq 9.25) - \left((4.05 \leq s < 4.1)^2\right)$	✓
$11.25 \times 9.15 - 4.15^2$ $(= 85.715)$	**M1**	for a correct method to find the lower bound of X, allow $(11.25 \leq AB < 11.5) \times (9.15 \leq BC < 9.2) - \left((4.1 < s \leq 4.15)^2\right)$	✓
Working required	**A1**	dep on M2. 90, and both the upper bound and the lower bound correct using the correct values 11.25, 11.75, 9.15, 9.25, 4.05 and 4.15	✓

Full marks: 4/4

Question 23 - Exam Solution

Understanding the Question

Given

$$\frac{\dfrac{4x^2 - 4x - 120}{5x^2 - 180}}{\dfrac{x^2 + 5x}{10x^2 + 60x}} = p$$

p is an integer, so every x has to cancel out.

Find

The value of p

Plan the Solution

- Factorise all four expressions fully, top and bottom of both fractions.
- Dividing by a fraction is multiplying by its reciprocal, so turn the second fraction upside down.
- Cancel every bracket that appears on the top and on the bottom, then work out the numbers that are left.

Worked Solution [4 marks]

> Rule - Dividing algebraic fractions: factorise every part, turn the second fraction upside down, multiply, then cancel.

Step 1: Factorise the first fraction

$4x^2 - 4x - 120 = 4(x^2 - x - 30) = 4(x-6)(x+5)$

$5x^2 - 180 = 5(x^2 - 36) = 5(x-6)(x+6)$

(Reason: (Reason: take the common number out first. Then $x^2 - x - 30$ needs two numbers with product -30 and sum -1, which are -6 and 5, while $x^2 - 36$ is the difference of two squares.))

Step 2: Factorise the second fraction

$x^2 + 5x = x(x+5)$

$10x^2 + 60x = 10x(x+6)$

(Reason: (Reason: both terms of each expression contain an x, and the bottom one also has a factor of 10, so $10x$ comes out. The marks here are for factorising fully, not part way.))

Step 3: Turn the second fraction upside down and multiply

$$\frac{4(x-6)(x+5)}{5(x-6)(x+6)} \times \frac{10x(x+6)}{x(x+5)}$$

(Reason: (Reason: dividing by $\frac{x(x+5)}{10x(x+6)}$ is the same as multiplying by its reciprocal $\frac{10x(x+6)}{x(x+5)}$.))

Step 4: Cancel every factor that appears top and bottom

$$\frac{4(x-6)(x+5) \times 10x(x+6)}{5(x-6)(x+6) \times x(x+5)}$$

$$\frac{4 \times 10}{5} = \frac{40}{5} = 8$$

(Reason: (Reason: $(x-6)$, $(x+5)$, $(x+6)$ and x each appear on the top and on the bottom, so all four cancel and only the numbers are left.))

$$p = 8$$

Verification

✓ **Check 1:** Put $x = 1$ into the original expression. The first fraction becomes $\frac{4 - 4 - 120}{5 - 180} = \frac{-120}{-175} = \frac{24}{35}$ and the second becomes $\frac{1+5}{10+60} = \frac{6}{70} = \frac{3}{35}$, so divide the first by the second. $\frac{24}{35} \times \frac{35}{3} = 8$

✓ **Check 2:** Try a second value, $x = 2$. Now the first fraction is $\dfrac{16-8-120}{20-180} = \dfrac{-112}{-160} = \dfrac{7}{10}$ and the second is $\dfrac{4+10}{40+120} = \dfrac{14}{160} = \dfrac{7}{80}$. This is a check only: the mark scheme says substituting values of x is not an acceptable algebraic method for the marks. $\dfrac{7}{10} \times \dfrac{80}{7} = 8$

✓ **Check 3:** Multiply everything out instead of factorising. One fraction is left, $\dfrac{40x^4 + 200x^3 - 1\,440x^2 - 7\,200x}{5x^4 + 25x^3 - 180x^2 - 900x}$, so each coefficient on the top should be 8 times the one underneath it. $8 \times 5 = 40$, $8 \times 25 = 200$, $8 \times (-180) = -1\,440$, $8 \times (-900) = -7\,200$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\dfrac{\frac{4(x-6)(x+5)}{5(x-6)(x+6)}}{\frac{x(x+5)}{10x(x+6)}}$ $(=p)$ or $\dfrac{4(x-6)(x+5)}{5(x-6)(x+6)} \times \dfrac{10x(x+6)}{x(x+5)}$ $(=p)$ or 2 from $4(x-6)(x+5)$ or $5(x-6)(x+6)$ or $x(x+5)$ or $10x(x+6)$	**M1**	for factorising 2 or 3 of the quadratics fully - could be implied by 2 factors cancelled correctly. NB factors must be in the form $(ax+b)$. NB Substitution of values of x into the given equation is not an acceptable algebraic method	✓
$\dfrac{\frac{4(x-6)(x+5)}{5(x-6)(x+6)}}{\frac{x(x+5)}{10x(x+6)}}$ $(=p)$ or $\dfrac{4(x-6)(x+5)}{5(x-6)(x+6)} \times \dfrac{10x(x+6)}{x(x+5)}$ $(=p)$ or $4(x-6)(x+5)$ and $5(x-6)(x+6)$ and $x(x+5)$ and $10x(x+6)$	**M1**	for factorising all of the quadratics fully - could be implied by 2 factors cancelled correctly. NB factors must be in the form $(ax+b)$	✓

Step	Mark	Description	Got it?
$\dfrac{4x^2-4x-120}{5x^2-180} \times \dfrac{10x^2+60x}{x^2+5x} (= p)$	M1	for inverting the 2nd fraction (this mark can be awarded at any time and may be awarded with incorrect factorisation if meaning is clear)	✓
Working required	A1	8 oe dep on M3	✓
ALT $\dfrac{40x^4+200x^3-1\,440x^2-7\,200x}{5x^4+25x^3-180x^2-900x}$	Note	the printed scheme carries an ALT route: a correct expression with no errors scores M3, and the answer 8 then scores the A1, oe dep on M3	✓

Full marks: 4/4

Question 24 - Exam Solution

Understanding the Question

Given

$ABCDE$ is a square-based pyramid with $EA = EB = EC = ED$.
M is the centre of the square base $ABCD$, and that base is horizontal.
Q is the midpoint of AB.
Angle $EQM = 80^\circ$.
The ratio $EA : AB = n : 1$, so n compares a sloping edge with a base edge.

Find

The value of n, correct to 3 significant figures.

Plan the Solution

- A ratio does not care how big the pyramid is, so give the base the side length $AB = 1$. Then n is simply the length EA.
- All four sloping edges are equal, so E sits directly above the centre M. That makes EM vertical, so angle $EMQ = 90^\circ$ and EQM is a right-angled triangle carrying the 80°.
- M is the centre of the square and Q is the midpoint of a side, so MQ is half a side. Cosine in triangle EQM then gives EQ.

- Triangle EAB is isosceles because $EA = EB$, so the line from E to the midpoint Q is perpendicular to AB. Pythagoras in triangle EQA finishes it.

Worked Solution [4 marks]

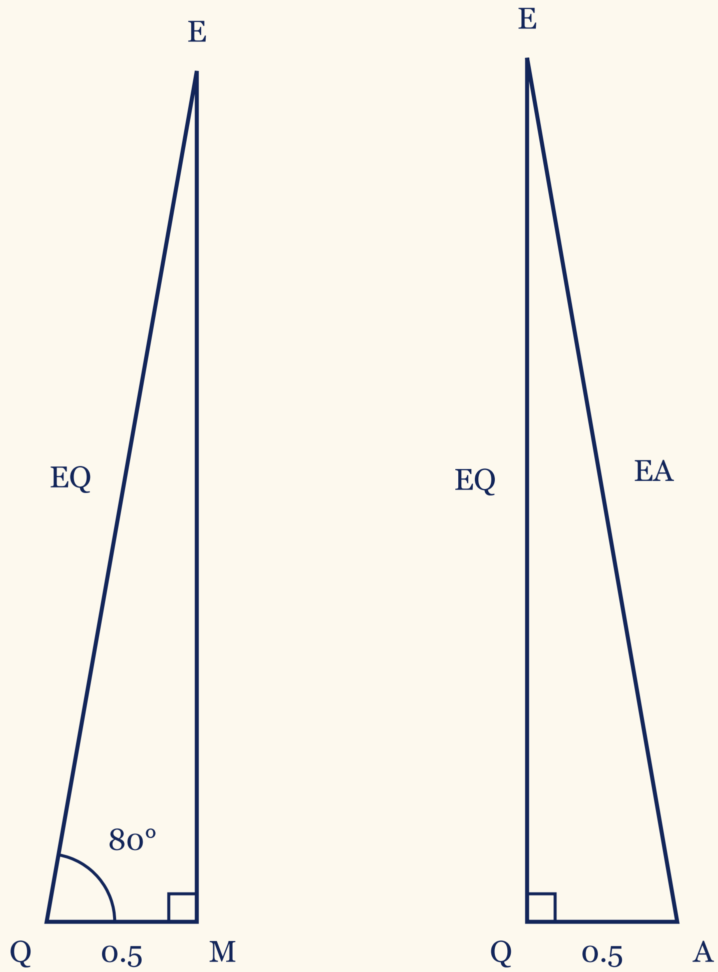
> Rule - Right-angled triangles: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ turns an angle into a side, and Pythagoras $c^2 = a^2 + b^2$ turns two sides into the third.

Step 1 - Give the base a side length, then find MQ and AQ

$AB = 1$

$MQ = \frac{1}{2} \times 1 = 0.5$

$AQ = \frac{1}{2} \times 1 = 0.5$



(Reason: Only the ratio is asked for, so every side length gives the same n. The centre of a square is half a side away from the midpoint of any side, which fixes MQ; and Q is the midpoint of AB, which fixes AQ.)

Step 2 - Use the right angle at M to find EQ

$$\cos 80^\circ = \frac{MQ}{EQ}$$

$$EQ = \frac{0.5}{\cos 80^\circ} = 2.87938\ldots$$

(Reason: Equal sloping edges put E directly above M, so EM is vertical and angle EMQ is a right angle. In triangle EQM the side MQ lies next to the 80° and EQ is the hypotenuse, so cosine is the ratio to use. Keep the unrounded value in the calculator.)

Step 3 - Use the right angle at Q to find EA

$$EA^2 = EQ^2 + AQ^2$$

$$EA^2 = (2.87938\ldots)^2 + 0.5^2 = 8.54085\ldots$$

$$EA = \sqrt{8.54085\ldots} = 2.92247\ldots$$

(Reason: $EA = EB$ makes triangle EAB isosceles, so the line from E to the midpoint Q meets AB at a right angle. Triangle EQA therefore has its right angle at Q, with legs EQ and AQ. Watch which half-length belongs here: AQ is half a side, not half a diagonal.)

Step 4 - Write the ratio, then round

$$EA : AB = 2.92247\ldots : 1$$

$$n = 2.92247\ldots$$

$$n = 2.92$$

(Reason: Because AB was set to 1, the ratio $EA : AB$ is already in the form $n : 1$, so n is the length EA. Rounding to 3 significant figures is the last thing done, not the first.)

$$n = 2.92$$

Verification

✓ **Check 1 - work back to the angle the question gives:** Start from the answer. With $EA^2 = 8.54085\ldots$ and $AQ = 0.5$, Pythagoras run backwards gives $EQ = \sqrt{8.54085\ldots - 0.25} = 2.87938\ldots$, and then $\cos EQM = \dfrac{0.5}{2.87938\ldots} = 0.17364\ldots$. Angle $EQM = 80^\circ$, the angle the question states.

✓ **Check 2 - reach the same edge up the axis instead of along the face:** The vertical height is $EM = 0.5\tan 80^\circ = 2.83564\ldots$, and A is half a diagonal from the centre, so $MA = \sqrt{0.5^2 + 0.5^2} = 0.70710\ldots$. Triangle EMA has its right angle at M, and it shares nothing with the route above except the given angle.

$EA = \sqrt{(2.83564\ldots)^2 + (0.70710\ldots)^2} = 2.92247\ldots$, the same length, so $n = 2.92$ again.

✓ **Check 3 - is the size sensible?** The sloping edge must be longer than the slant height, which must be longer than MQ, and the three values do line up: $2.92247\ldots > 2.87938\ldots > 0.5$. An angle of 80° is a steep face, so a sloping edge close to three times the base edge is what to expect. The three lengths are in the right order and the pyramid comes out tall and narrow, which is what 80° demands.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
1	M1	To find EQ: e.g. $MQ = 0.5\,(= AQ)$, $\dfrac{0.5}{\cos 80}\,(= 2.87(938\ldots))$; e.g. $MQ = x\,(= AQ)$, $\dfrac{x}{\cos 80}\,(= 5.75\ldots x)$. NB $\cos 80 = \sin 10$. Or to find EM: e.g. $MQ = 0.5\,(= AQ)$, $0.5\tan 80\,(= 2.83(564\ldots))$; e.g. $MQ = x\,(= AQ)$, $x\tan 80\,(= 5.67\ldots x)$. NB $\tan 80 = \dfrac{1}{\tan 10}$. Or to find EQ: e.g. $EM = y$, $\dfrac{y}{\sin 80}\,(= 1.01\ldots y)$; or to find $MQ\,(= AQ)$: e.g. $EQ = h$, $h\cos 80\,(= 0.173\ldots h)$. Notes: use of a value for the side AB, e.g. 1 or $2x$ or any value, or let $EM = y$ or any value, or $EQ = h$.	✓
2	M1	To find EA^2: e.g. $AQ = 0.5\,(= MQ)$, $(2.87\ldots)^2 + 0.5^2\,(= 8.54(08\ldots))$; e.g. $AQ = x\,(= MQ)$, $x^2 + \left(\dfrac{x}{\cos 80}\right)^2$. Or to find AM: e.g. $MQ = 0.5\,(= AQ)$, $\sqrt{0.5^2 + 0.5^2}\left(= \dfrac{\sqrt{2}}{2} = 0.707(10\ldots)\right)$; e.g. $MQ = x\,(= AQ)$, $\sqrt{x^2 + x^2}\,(= \sqrt{2x^2} = x\sqrt{2})$. Or to find $MQ\,(= AQ)$: e.g. $EM = y$, $\dfrac{y}{\tan 80}\,(= 0.176\ldots y)$; then to find EA^2: e.g. $EQ = h$, $h^2 + (h\cos 80)^2$. A value the printed scheme shows in quotation marks may be the candidate's own earlier value.	✓

Step	Mark	Description	Got it?
3	M1	To find EA: e.g. $AQ = 0.5\,(= MQ)$, $\sqrt{(2.87\ldots)^2 + 0.5^2}\ \left(= \sqrt{8.54(08\ldots)}\right)$; or e.g. $AQ = x\,(= MQ)$, $\sqrt{x^2 + \left(\frac{x}{\cos 80}\right)^2}\ (= 5.84\ldots x)$. Or e.g. $AQ = 0.5\,(= MQ)$, $\sqrt{(2.83\ldots)^2 + (0.707\ldots)^2}\ \left(= \sqrt{8.54(08\ldots)}\right)$; e.g. $AQ = x\,(= MQ)$, $\sqrt{(x\tan 80)^2 + (x\sqrt{2})^2}\,(= 5.84\ldots x)$. Or e.g. $EM = y$, $\sqrt{\left(\frac{1}{\sin 80}\right)^2 + \left(\frac{1}{\tan 80}\right)^2}\ (= 1.03\ldots y)$, that is $1.03\ldots \times AQ \times \tan 80$; e.g. $EQ = h$, $\sqrt{h^2 + (h\cos 80)^2}\ (= 1.01\ldots h)$, that is $1.01\ldots \times \frac{0.5}{\cos 80}$.	✓
4	A1	Answer 2.92, awrt 2.92. Working not required, so a correct answer scores full marks unless it comes from obviously incorrect working.	✓

Full marks: 4/4

Question 25 - Exam Solution

Understanding the Question

Given

The expression $28 + 24x - 6x^2$
The form it must be written in, $a - b(x - c)^2$, with a, b and c integers
A pair of blank axes for the sketch of $y = 28 + 24x - 6x^2$

Find

(a) The completed square form, so the values of a, b and c (b) The sketch, with the turning point and the point where the curve meets the y-axis both labelled with their coordinates

Plan the Solution

- Write the expression in descending powers, so the x^2 term comes first.
- Take the -6 out of the x^2 and x terms only, and complete the square inside the bracket.
- Multiply the -6 back in and collect the two constants, which gives $a - b(x - c)^2$ straight away.

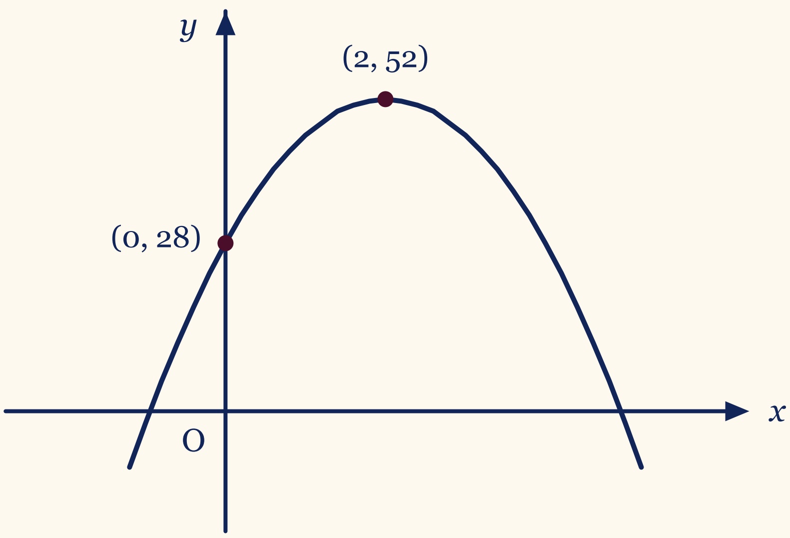
- For part (b), read the turning point off that completed square, put $x = 0$ in to find where the curve meets the y-axis, and use the sign of the x^2 term to decide which way up the curve is.

Worked Solution [6 marks]

> Rule - Completing the square with a negative x^2 term: take that coefficient out of the x^2 and x terms first, complete the square inside the bracket, then multiply back in. In $y = a - b(x-c)^2$ with b positive, the square is never negative, so the curve has a maximum turning point at $(c,\ a)$.

Step 1: Put the terms in descending powers and take out the -6

$$28 + 24x - 6x^2 = -6x^2 + 24x + 28$$
$$-6x^2 + 24x = -6(x^2 - 4x)$$
$$28 + 24x - 6x^2 = -6(x^2 - 4x) + 28$$



(Reason: Dividing 24 by -6 gives -4, so the bracket is $x^2 - 4x$. The 28 has no x in it, so it stays outside the bracket.)

Step 2: Complete the square inside the bracket

$$(x-2)^2 = x^2 - 4x + 4$$
$$x^2 - 4x = (x-2)^2 - 4$$

(Reason: Half of -4 is -2, so the bracket is $(x-2)^2$. Squaring that bracket brings in an extra 4, so 4 is taken away again to keep the value the same.)

Step 3: Multiply the -6 back in and collect the constants

$-6\left[(x-2)^2-4\right]+28=-6(x-2)^2+24+28$

$24+28=52$

$28+24x-6x^2=52-6(x-2)^2$

(Reason: Multiplying the -6 back in changes the sign of the -4 that sits inside the bracket, and the two constants outside are then collected into one.)

Step 4: Read off the three integers

$a-b(x-c)^2=52-6(x-2)^2$

$a=52$

$b=6$

$c=2$

(Reason: All three are integers, which is what part (a) asks for. Note that b is 6 and not -6, because the form already has a minus sign in front of the b.)

Step 5: (b) The turning point, straight from the completed square

$y=52-6(x-2)^2$

$6(x-2)^2\geq 0$

$52-6\times 0=52$

$(2,\ 52)$

(Reason: A square is never negative, so $6(x-2)^2$ only takes away from the 52. The largest value of y therefore happens when the bracket is zero, at $x=2$, and there $y=52$. Because y can never be more than this, the turning point is a maximum.)

Step 6: (b) Where the curve meets the y-axis

$28+24\times 0-6\times 0^2=28$

$(0,\ 28)$

(Reason: Every point on the y-axis has $x=0$, so putting 0 in place of x leaves the constant term, 28.)

Step 7: (b) Draw the sketch

$28+24\times 4-6\times 4^2=28$

$(0,\ 28)\quad(2,\ 52)\quad(4,\ 28)$

(Reason: The coefficient of x^2 is negative, so the curve opens downwards. It is symmetrical about the vertical line through the turning point, so the point level with $(0,\ 28)$ on the other side is $(4,\ 28)$. Draw one smooth curve through the three points, with the maximum at $(2,\ 52)$, and label the turning point and the y-axis crossing with their coordinates.)

(a) $52 - 6(x-2)^2$

(b) a curve opening downwards, turning point $(2,\ 52)$, meeting the y-axis at $(0,\ 28)$

Verification

✓ **Check 1:** Multiply the answer out again and compare it with the expression the question prints. $52 - 6(x^2 - 4x + 4) = 52 - 6x^2 + 24x - 24 = 28 + 24x - 6x^2$

✓ **Check 2:** Put $x = 1$ into both forms. They are the same expression, so they must give the same value. $28 + 24 \times 1 - 6 \times 1^2 = 46$ and $52 - 6 \times (1-2)^2 = 46$

✓ **Check 3:** Test the symmetry the sketch depends on. If the turning point really is at $x = 2$, then $x = 0$ and $x = 4$ are the same distance from it and must give the same y. $28 + 24 \times 0 - 6 \times 0^2 = 28$ and $28 + 24 \times 4 - 6 \times 4^2 = 28$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\pm 6\left(x \pm \frac{4}{2}\right)^2 \ldots$ or $\pm 6(x \pm 2)^2 \ldots$ or $\pm 6\left[(x \pm 2)^2 \ldots\right]$ or $\pm 6\left[\left(x \pm \frac{4}{2}\right)^2 \ldots\right]$ or $\pm 6\left(x \pm \frac{24}{2 \times -6}\right)^2 \ldots$	**M1**	for a start to completing the square, or correct substitution into $a\left(x + \frac{b}{2a}\right)^2 + \ldots$ from the formula $a\left(x + \frac{b}{2a}\right)^2 - \frac{(b)^2}{4a} + c$	✓
(a) $-6\left[\left(x - \frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2\right] \ldots$ or $-6\left[(x-2)^2 - 2^2\right] \ldots$ or $-6\left[\left(x - \frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 \ldots\right]$ or $-6\left[(x-2)^2 - 2^2 \ldots\right]$ or $-6\left(x + \frac{24}{2 \times -6}\right)^2 - \frac{24^2}{4 \times -6} \ldots$	**M1**	for correctly completing the square, but terms do not need to be simplified and 28 may or may not be present, or correct simplification of the first two parts of $a\left(x + \frac{b}{2a}\right)^2 - \frac{(b)^2}{4a}(+c)$. NB: please refer to the ALT mark scheme after (b) for the comparison of coefficients method	✓

Step	Mark	Description	Got it?
(a) $52 - 6(x - 2)^2$ - working not required, so a correct answer scores full marks (unless from obvious incorrect working)	A1	oe eg $-6(x-2)^2 + 52$	✓
(a) ALT $-bx^2 + 2bcx - bc^2 + a$ and $b = 6$ or $b = -6$	M1	for multiplying out $a - b(x-c)^2$ and $b = 6$ or $b = -6$	✓
(a) ALT $2bc = 24$ or $-bc^2 + a = 28$	M1	for equating coefficients	✓
(a) ALT $52 - 6(x - 2)^2$ - working not required, so a correct answer scores full marks (unless from obvious incorrect working)	A1	oe eg $-6(x-2)^2 + 52$	✓
(b) a $\cap$ or $\cup$ shaped symmetrical quadratic curve	B1	for drawing a $\cap$ or $\cup$ shaped symmetrical quadratic curve with the turning point in any quadrant	✓
(b) turning point marked as $(2,\ 52)$	B1	for drawing a $\cap$ shaped symmetrical quadratic curve in the correct quadrant with a turning point at $(2,\ 52)$	✓
(b) intersection with the y-axis marked as $(0,\ 28)$ or crossing at 28 marked	B1	for drawing a $\cap$ shaped symmetrical quadratic curve in the correct quadrant with an intersection on the y-axis marked as $(0,\ 28)$ or marked as 28 on the y-axis	✓

Full marks: 6/6

Question 26 - Exam Solution

Understanding the Question

Given

Find

Two solid candles, A and B, that are mathematically similar.
Height of candle A: 31 cm. Height of candle B: 18.6 cm.
$\text{volume of candle } A - \text{volume of candle } B = 735 \text{ cm}^3$
Neither volume is given on its own, only the gap between them.

The volume of candle A, in cubic centimetres.

Plan the Solution

- Divide the two heights to get the linear scale factor between the candles.
- Cube it. A volume is three-dimensional, so the volume scale factor is the cube of the linear one.
- Use it to write the volume of candle B as a fraction of the volume of candle A, so the given difference holds only one unknown.
- Solve for the volume of candle A, then rebuild both volumes and check the difference comes back to 735 cm^3.

Worked Solution [4 marks]

> Rule for mathematically similar solids: if the linear scale factor is k, the area scale factor is k^2 and the volume scale factor is k^3.

Step 1: Find the linear scale factor

$$\frac{31}{18.6} = \frac{310}{186} = \frac{5}{3}$$

(Reason: Multiply top and bottom by 10 to clear the decimal, then divide both by 62. Candle A is $\frac{5}{3}$ times as tall as candle B.)

Step 2: Cube the scale factor to get the volume scale factor

$$\left(\frac{5}{3}\right)^3 = \frac{125}{27}$$

$$\left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

(Reason: A volume is three-dimensional, so every length being multiplied by $\frac{5}{3}$ multiplies the volume by that factor three times over. The reversed fraction is the one that takes you from candle A down to candle B.)

Step 3: Write candle B's volume in terms of candle A's

$$V_B = \frac{27}{125}V_A$$

(Reason: Candle B is the smaller solid, so its volume is the smaller share. Writing it this way leaves the given difference with only one unknown in it.)

Step 4: Turn the given difference into an equation

$$V_A - \frac{27}{125}V_A = 735$$

$$1 - \frac{27}{125} = \frac{98}{125}$$

$$\frac{98}{125}V_A = 735$$

(Reason: The question gives the gap between the volumes, not either volume, so substitute for V_B and collect the V_A terms. The gap is $\frac{98}{125}$ of candle A.)

Step 5: Solve for the volume of candle A

$$V_A = 735 \times \frac{125}{98}$$

$$\frac{735}{98} = 7.5$$

$$7.5 \times 125 = 937.5$$

(Reason: Read it as parts: candle A is 125 parts and candle B is 27 of them, so the 98 parts in between are worth $735\ \text{cm}^3$. One part is $7.5\ \text{cm}^3$, so all 125 parts are $937.5\ \text{cm}^3$.)

$$937.5\ \text{cm}^3$$

Verification

✓ **Check 1:** Scale the answer back down to candle B with the volume scale factor $\frac{27}{125}$, then take the difference the question gives. $937.5 \times \frac{27}{125} = 202.5$ and $937.5 - 202.5 = 735$

✓ **Check 2:** Redo it without ever simplifying the ratio, working straight from the cubed heights $31^3 = 29791$ and $18.6^3 = 6434.856$. $\frac{29791 \times 735}{29791 - 6434.856} = 937.5$

✓ **Check 3:** Volume divided by height cubed must be the same constant for both candles, since they are mathematically similar. $\frac{937.5}{29791} = \frac{202.5}{6434.856}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$A : B = 31 : 18.6\,(= 5 : 3)$ oe or $A^3 : B^3 = 31^3 : 18.6^3\,(= 5^3 : 3^3)$ oe or $\frac{31}{18.6}\left(=\frac{5}{3}\right)$ oe or $\frac{18.6}{31}\left(=\frac{3}{5}\right)$ oe or $\left(\frac{18.6}{31}\right)^3$ oe or $\frac{27}{125}$ oe or $\left(\frac{31}{18.6}\right)^3$ oe or $\frac{125}{27}$ oe	**M1**	for correct linear SF or volume SF either as a fraction or ratio. Allow $\frac{5}{3} = 1.6(6....)$ truncated or rounded	✓
$V_A - \left(\frac{3}{5}\right)^3 V_A\,(= 735)$ oe or $V_A - \frac{27}{125}V_A\,(= 735)$ oe or $\frac{98}{125}V_A\,(= 735)$ oe or $\frac{V_A}{V_A - 735} = \frac{31^3}{18.6^3}$ oe or $\left(\frac{5}{3}\right)^3 V_B - V_B\,(= 735)$ oe or $\frac{125}{27}V_B - V_B\,(= 735)$ oe or $\frac{98}{27}V_B\,(= 735)$ oe or $\frac{V_B + 735}{V_B} = \frac{31^3}{18.6^3}$ oe or $1 - \left(\frac{3}{5}\right)^3\left(= \frac{98}{125} = 0.784\right)$ oe or $\left(\frac{5}{3}\right)^3 - 1\left(= \frac{98}{27} = 3.62(962...)\right)$ oe or $5^3 - 3^3\,(= 125 - 27 = 98)$	**M1**	Note: 735 is given in the equation. Allow any letter for V_A or for V_B. $V_A - \left(\frac{3}{5}\right)^3 V_A\,(= 735)$ can be written as $V_A - \frac{V_A}{\left(\frac{5}{3}\right)^3}\,(= 735)$, or $\left(\frac{5}{3}\right)^3 V_B - V_B\,(= 735)$ can be written as $\frac{V_B}{\left(\frac{3}{5}\right)^3} - V_B\,(= 735)$	✓
$(V_A =)\,735 \times \frac{125}{98}$ oe or $(V_A =)\,\frac{735}{\frac{98}{125}}$ oe or $(V_A =)\,\frac{31^3 \times 735}{31^3 - 18.6^3}$ oe or $(V_B =)\,735 \times \frac{27}{98}\,(= 202.5)$ oe or $(V_B =)\,\frac{735}{\frac{98}{27}}\,(= 202.5)$ or	**M1**	for a correct method to find V_A or V_B	✓

Step	Mark	Description	Got it?
$(V_B =) \dfrac{18.6^3 \times 735}{31^3 - 18.6^3} (= 202.5)$ oe or $\dfrac{735}{98} \times 5^3$ oe or 7.5×125 oe or $\dfrac{735}{98} \times 3^3 (= 202.5)$ oe or $7.5 \times 27 (= 202.5)$ oe			
937.5 (working not required, so a correct answer scores full marks unless it comes from obvious incorrect working)	A1	oe allow 938 from correct working	✓

Full marks: 4/4

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