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Edexcel IGCSE 4MA1

4MA1/2HR (Higher Tier) – Wednesday 4 June 2025

100 marks · 2 hours · Calculator allowed

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Full original worked solutions and mark schemes for every question on this paper.

Original worked solutions for Edexcel International GCSE Mathematics A, Paper 4MA1/2HR (Higher Tier), June 2025 series, sat Wednesday 4 June 2025 –100 marks, 2 hours, calculator allowed. The questions have been reworded; all numerical values match the original paper. The official question paper and mark scheme are published by Pearson Edexcel. This resource reproduces neither the exam paper nor the official mark scheme.

Both are PDF files hosted by Pearson: **official question paper (PDF)** and **official mark scheme (PDF)**.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. (a) Fully factorise $18c - 45cd$ *[2 marks]*

(b) Solve the equation $\dfrac{5 - 2x}{6} = 3x - 4$

You must show clear algebraic working. *[3 marks]*

(a) ....

(b) x = ...

[Total 5 marks]

2. Express 1400 as a product of powers of its prime factors.
You must show your working clearly. *[3 marks]*

....

[Total 3 marks]

3. Solve the simultaneous equations

$$3x + 2y = 10$$
$$3x - 4y = 16$$

You must show clear algebraic working. *[3 marks]*

x = ...

y = ...

[Total 3 marks]

4. Nathan is going to cover a studio wall with acoustic panels for a customer.
The area of the wall is $45\ \text{m}^2$

Nathan buys one pack of panels for each $1.5\ \text{m}^2$ of wall area.
Each pack of panels costs £64

Nathan also buys 5 tubes of panel adhesive.
Each tube of panel adhesive costs £12

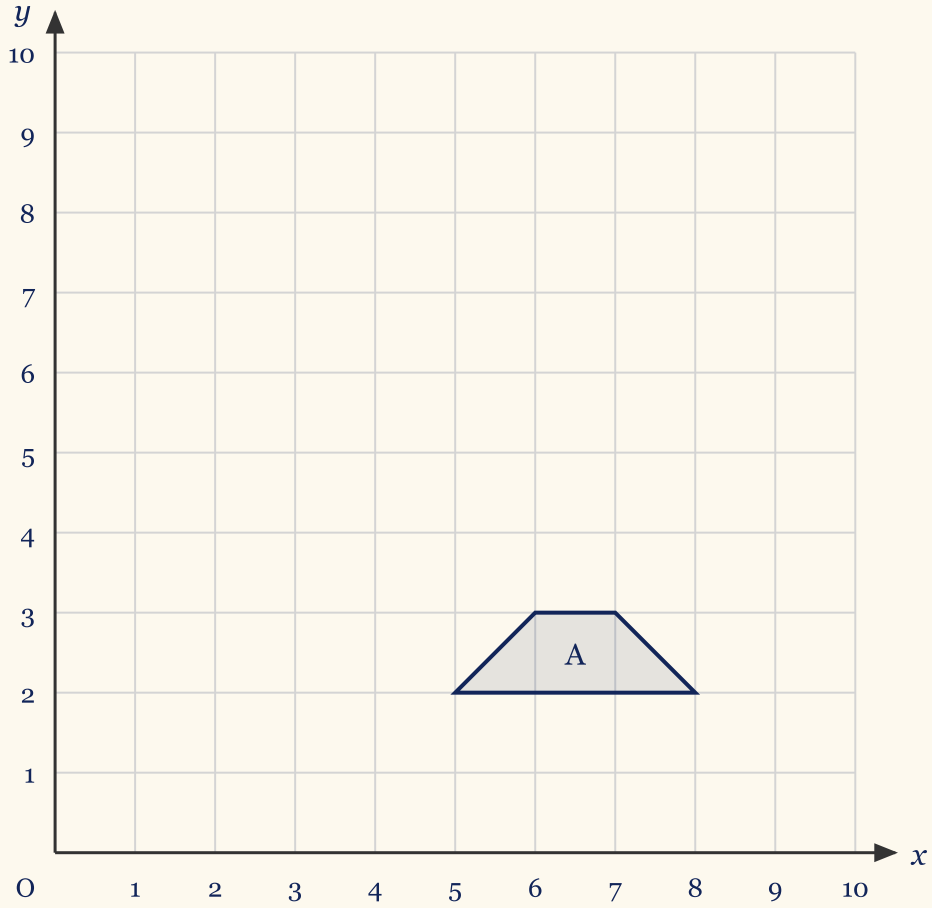
Nathan charges the customer £3000

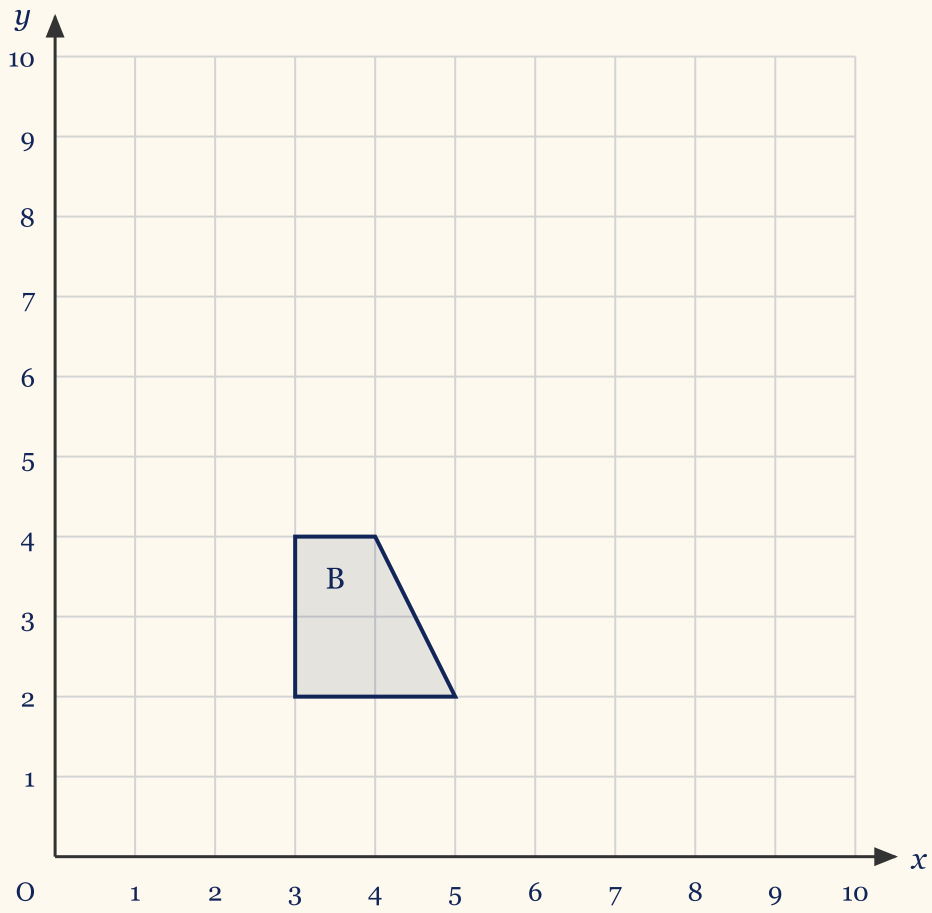
Work out his percentage profit.
Give your answer correct to one decimal place. *[5 marks]*

.... %

[Total 5 marks]

5. (a) On the grid above, draw the reflection of shape A in the line $y = x$
[2 marks]





(b) On the grid above, draw the enlargement of shape B with scale factor 2 and centre $(1, 1)$ *[2 marks]*

[Total 4 marks]

6. (a) State the value of 5^0 *[1 mark]*

$$\frac{5^9 \times 5^{-3}}{5^{-2}} = 5^k$$

(b) Work out the value of k *[2 marks]*

(c) Simplify fully $\left(2d^4e^5\right)^3$ *[2 marks]*

(a) ....

(b) k = ...

(c) ....

[Total 5 marks]

7. The mass of a solid silver pendant is 48.3 g
The density of silver is 10.5 g/cm³

Work out the volume of the silver pendant. *[2 marks]*

.... cm³

[Total 2 marks]

8. A list of 7 numbers has a mean of 60

The mean of 3 of the numbers in the list is 46

Work out the mean of the remaining 4 numbers. *[3 marks]*

....

[Total 3 marks]

9. An electrical store reduces all of its normal prices by 15% in a clearance sale.
The sale price of an espresso machine is 612 Swiss francs.
Work out the normal price of the espresso machine. *[3 marks]*

.... Swiss francs

[Total 3 marks]

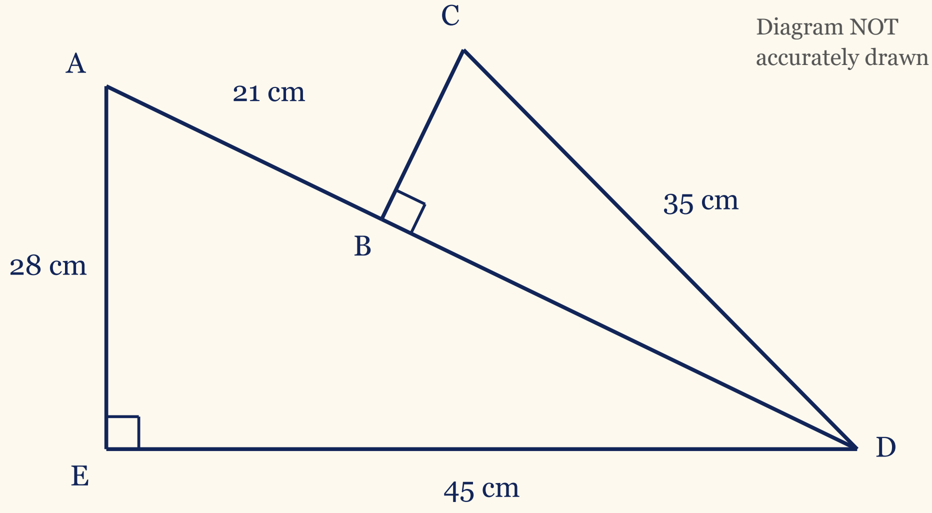
10. The straight line L is parallel to the line with equation $y = 2 - 5x$
The line L passes through the point $(0, 6)$

Work out an equation of the line L *[2 marks]*

....

[Total 2 marks]

11. The diagram shows two triangles, ADE and CDB.



ABD is a straight line.
$AE = 28$ cm, $ED = 45$ cm, $AB = 21$ cm and $CD = 35$ cm
angle AED = angle $CBD = 90°$

Find the area of triangle CDB.
Give your answer correct to 3 significant figures. *[5 marks]*

.... cm²

[Total 5 marks]

12. Zubair buys a motorboat for $16 000

The motorboat depreciates at a rate of 12% each year for the first 2 years.
In the third year, the motorboat depreciates at a rate of x%

At the end of 3 years, the value of the motorboat is $11 461.12

Work out the value of x *[3 marks]*

....

[Total 3 marks]

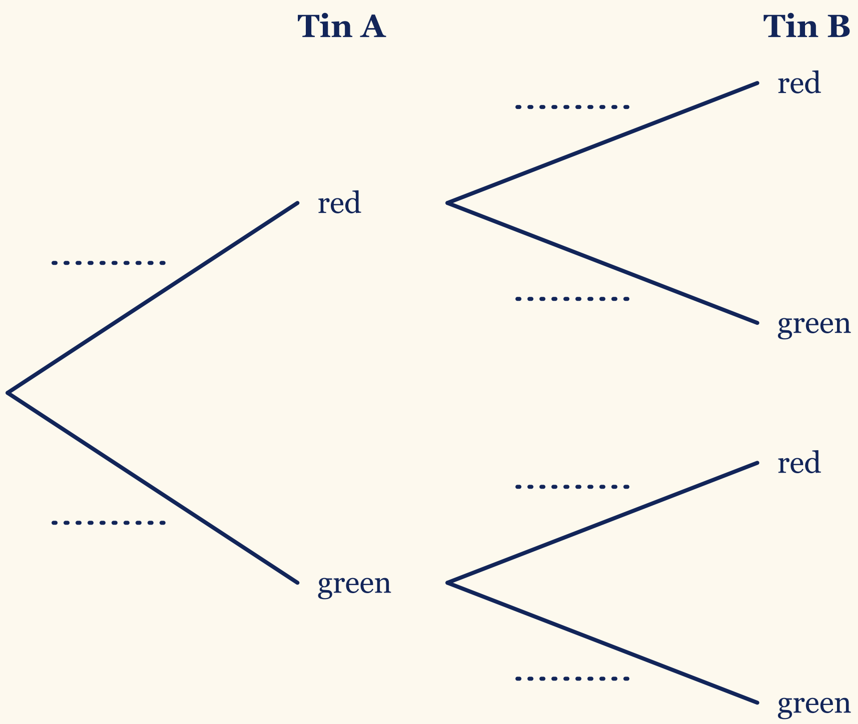
13. (a) Factorise the expression $4x^2 - 25y^2$ *[2 marks]*

(b) Show that $4x(x + 3)(2x - 5)$ can be written in the form
$ax^3 + bx^2 + cx$,
where a, b and c are integers whose values you must find. *[3 marks]*

....

[Total 5 marks]

14. Hana has two tins of counters, tin A and tin B.
In tin A there are only 5 red counters and 4 green counters.
In tin B there are only 7 red counters and 3 green counters.



Hana takes at random a counter from tin A
She then takes at random a counter from tin B

(a) Use this information to complete the probability tree diagram.
[2 marks]

(b) Work out the probability that Hana takes two red counters. *[2 marks]*

Hana puts the counters back into the tins they came from.

Hana also has a jar of counters.
In the jar, there are only red counters and green counters.
When a counter is taken at random from the jar, the probability that it is a green counter is $\dfrac{2}{11}$

Hana takes at random a counter from tin A
She then takes at random a counter from tin B
She then takes at random a counter from the jar.

(c) Work out the probability that Hana takes more red counters than green counters. *[3 marks]*

(b) ....

(c) ....

[Total 7 marks]

15. A, B and C are points on a circle with centre O.

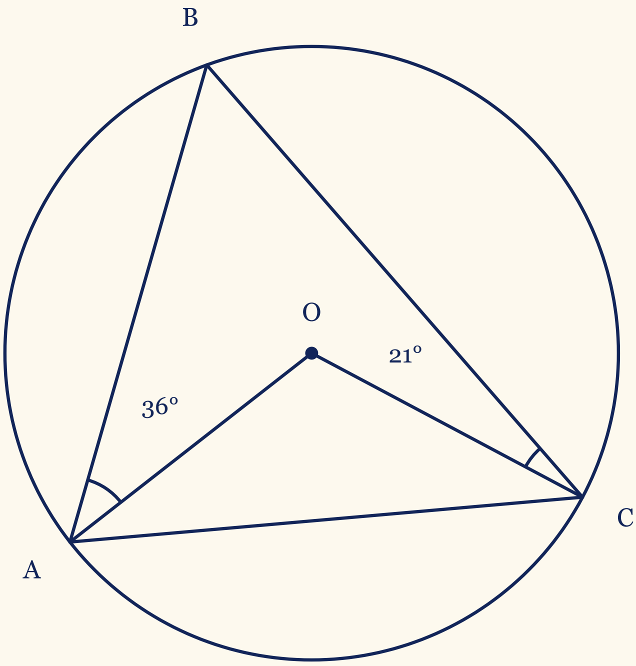


Diagram NOT accurately drawn

Angle $BAO = 36°$
Angle $BCO = 21°$

Find the size of angle ACO. *[3 marks]*

.... °

[Total 3 marks]

16. Show that $\dfrac{4}{3\sqrt{5}+7}$ can be written in the form $a-\sqrt{b}$, where a and b are integers.
Show every stage of your working. *[3 marks]*

[Total 3 marks]

17. Rearrange the formula

$$p = \frac{8k^2+5}{7-3k^2}$$

to make k the subject. *[4 marks]*

....

[Total 4 marks]

18. The diagram shows triangle OAB.

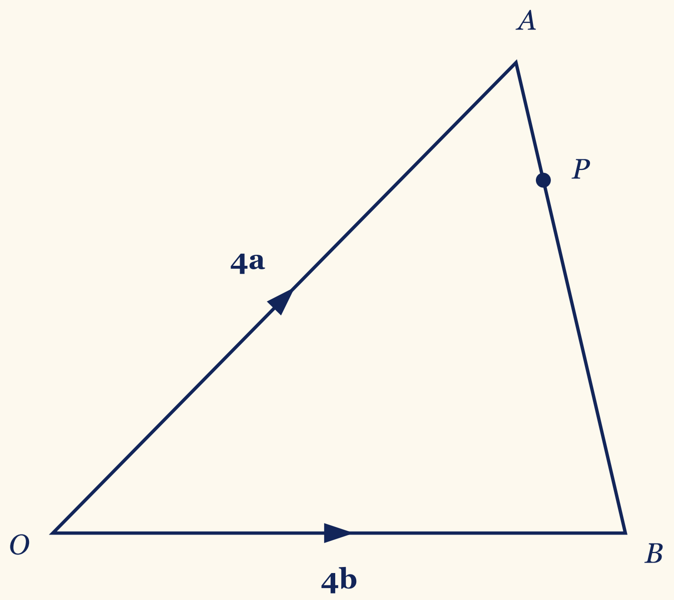


Diagram NOT accurately drawn

$\overrightarrow{OA} = 4\mathbf{a}$
$\overrightarrow{OB} = 4\mathbf{b}$

The point P lies on AB so that $AP : PB = 1 : 3$

(a) Write down an expression for $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ *[1 mark]*

(b) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form. *[2 marks]*

(a) ....

(b) ....

[Total 3 marks]

19. Two functions f and g are defined as follows.

$\mathrm{f}(x) = 3x - 4$ where $x > 2$

$\mathrm{g}(x) = \dfrac{x}{2x + 1}$

(a) Write down the value of x that must be left out of every domain of g
[1 mark]

(b) Work out $\mathrm{gf}(x)$

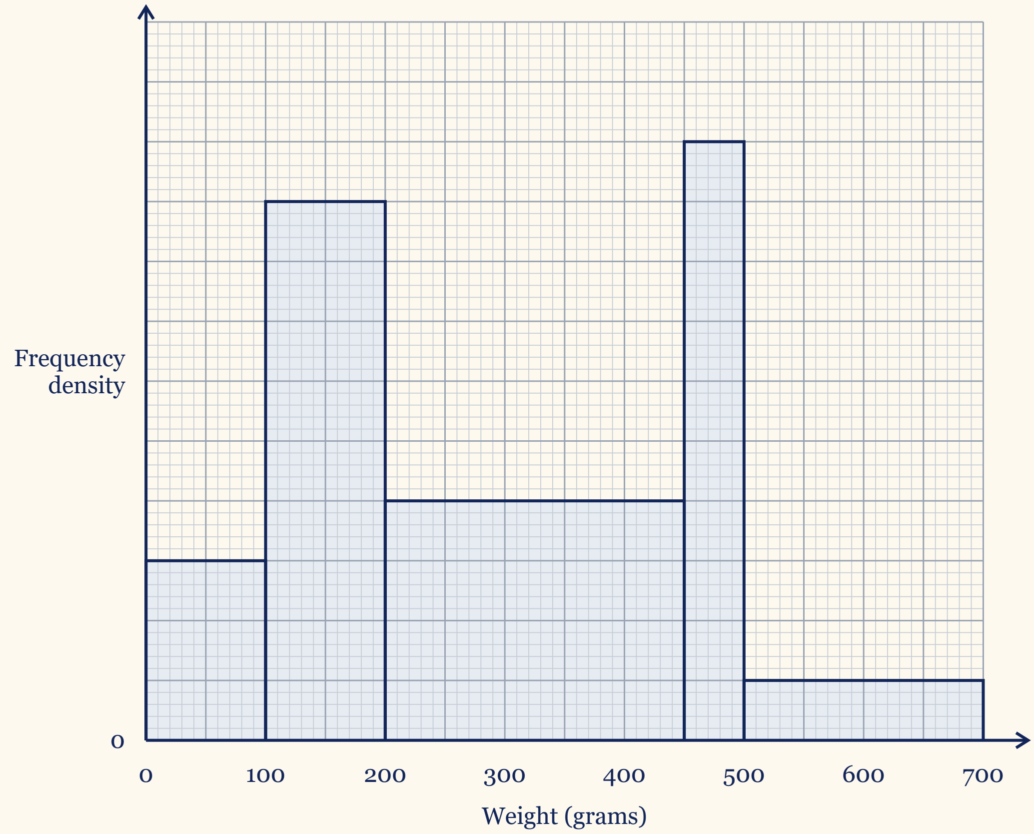
Give your answer in its simplest form. *[2 marks]*

(a) ....

(b) gf(x) = ...

[Total 3 marks]

20. The histogram gives some information about the weights, in grams, of some pebbles.



75 pebbles weigh less than 100 grams.

A pebble is chosen at random.

Find an estimate for the probability that this pebble weighs between 400 grams and 600 grams. *[4 marks]*

....

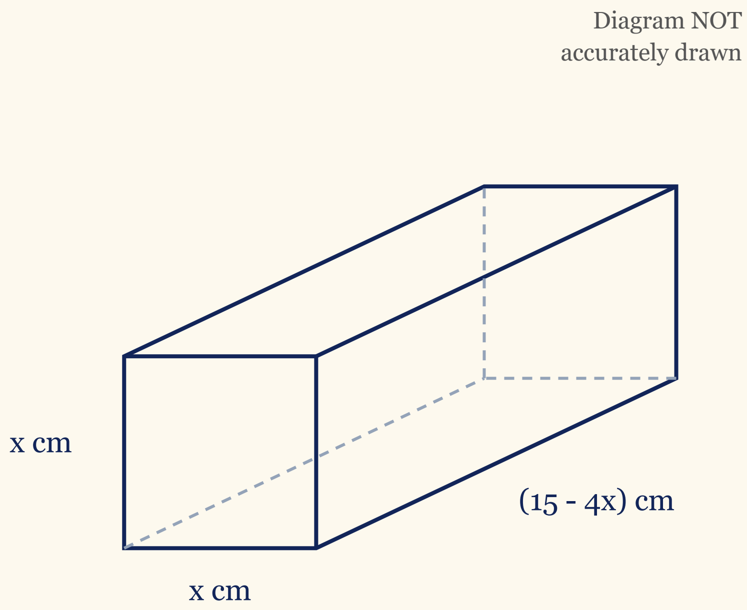
[Total 4 marks]

21. Solve the inequality $2x^2 - 7x - 15 > 0$
You must show clear algebraic working. *[3 marks]*

....

[Total 3 marks]

22. The diagram shows a solid wooden block in the shape of a cuboid.



The volume of the block is V cm³

Work out the maximum value of V *[5 marks]*

....

[Total 5 marks]

23. The diagram shows a triangular prism $ABCDEF$.
The base $ABCD$ is a horizontal rectangle.

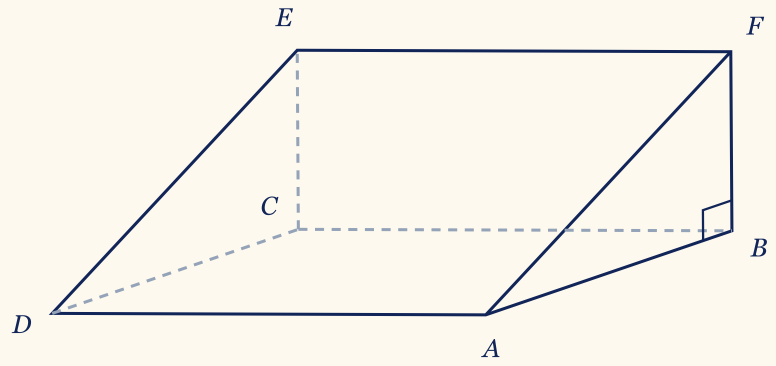


Diagram NOT accurately drawn

M is the midpoint of the line AC

$AC = 40$ cm
angle $CAE = 35°$
angle $ABF = 90°$

Find the size of angle CME
Give your answer correct to 3 significant figures. *[3 marks]*

.... °

[Total 3 marks]

24. PQ is a straight line drawn on a square grid, where 1 unit on each axis represents 1 cm.

The point P has coordinates $(-5,\ a)$ and the point Q has coordinates $(7,\ 3a)$, where $a > 0$

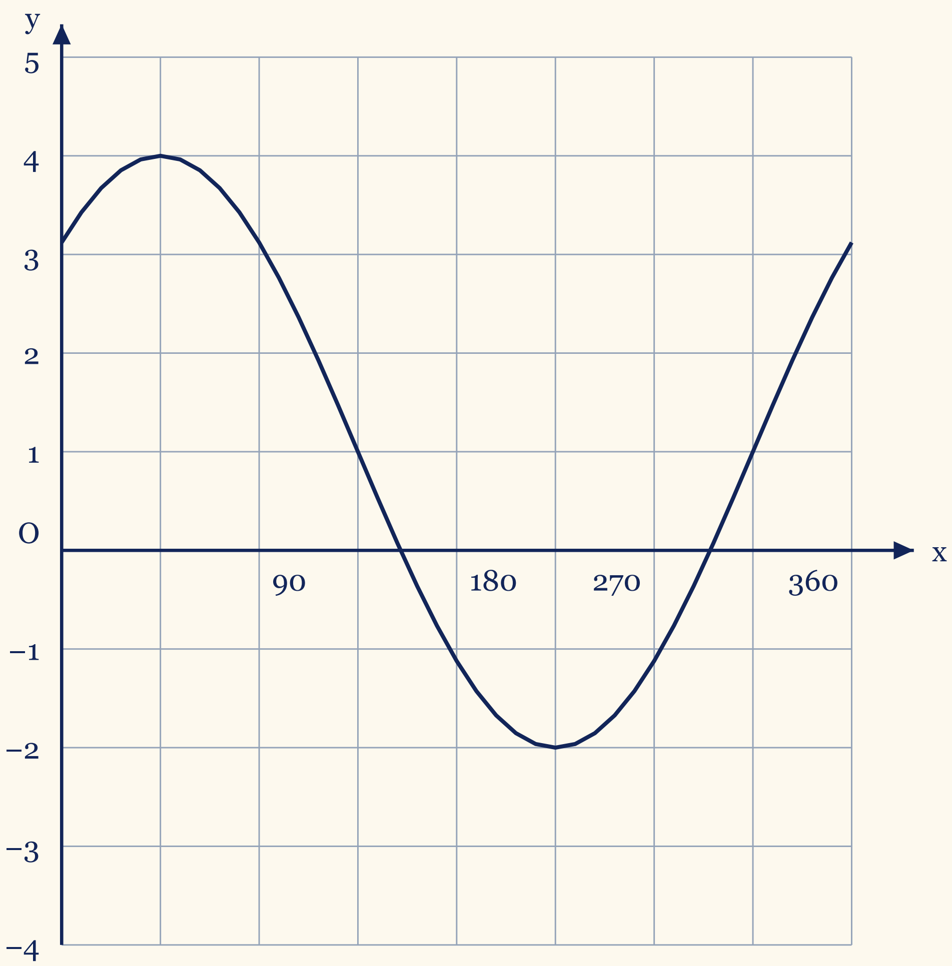
The length of PQ is $4\sqrt{10}$ cm.

Work out an equation of the perpendicular bisector of PQ.
Give your answer in the form $y = mx + c$ where m and c are integers.
[6 marks]

....

[Total 6 marks]

25. The grid below shows the graph of $y = a\sin(x+b)^\circ + c$ for $0 \le x \le 360$



Work out a suitable value for a, a suitable value for b and a suitable value for c *[3 marks]*

a = ...

b = ...

c = ...

[Total 3 marks]

26. The diagram shows a solid hemisphere, H

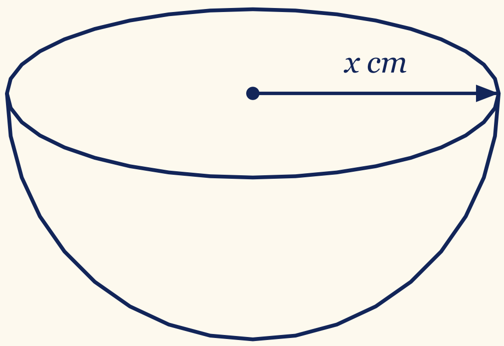


Diagram NOT accurately drawn

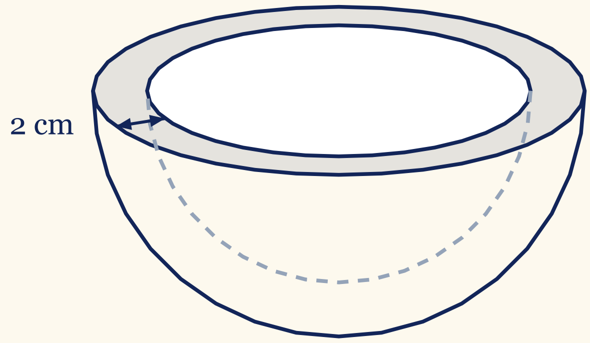


Diagram NOT accurately drawn

The radius of H is x cm
The volume of H is 6174π cm^3

A dish is made by removing a solid hemisphere from H, so that the dish has a uniform thickness of 2 cm

Work out the total surface area of the dish.
Give your answer in terms of π *[5 marks]*

.... cm^2

[Total 5 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

(a) The expression $18c - 45cd$

(b) The equation $\dfrac{5-2x}{6} = 3x - 4$

Find

(a) $18c - 45cd$ written as a product, with the highest common factor outside a single bracket. (b) The value of x, with clear algebraic working shown.

Plan the Solution

- (a) Split each term into its number and its letters. The highest common factor is the largest number that divides both numbers, together with every letter that appears in both terms.
- (a) Divide each term by that factor, put what is left inside one bracket, then expand to check that nothing is still waiting to come out.
- (b) Multiply both sides by 6 to clear the fraction. Every term on the right is multiplied, not only the first one.
- (b) Gather the x terms on one side and the numbers on the other, then divide by the coefficient of x.

Worked Solution [5 marks]

> Rule - Factorise fully: $ab - ac = a(b - c)$, where a is the highest common factor, so the bracket has nothing left to take out. Rule - Solve: an equation stays balanced only when whatever is done to one side is done to the whole of the other side.

Step 1: (a) Find the highest common factor of the two terms

$18 = 2 \times 3^2$

$45 = 3^2 \times 5$

(Reason: The largest number dividing both 18 and 45 is 9. The letter c is in both terms and d is in the second term only, so the highest common factor is $9c$.)

Step 2: (a) Divide each term by 9c and write the bracket

$$\frac{18c}{9c} = 2$$

$$\frac{45cd}{9c} = 5d$$

$$18c - 45cd = 9c(2 - 5d)$$

(Reason: Taking $9c$ out of each term leaves 2 and $5d$, and the minus sign between the terms is kept. Inside the bracket 2 and 5 share no factor and d is not in both terms, so the factorisation is full.)

Step 3: (b) Multiply both sides by 6 to clear the fraction

$$\frac{5 - 2x}{6} \times 6 = (3x - 4) \times 6$$

$$5 - 2x = 18x - 24$$

(Reason: The fraction has denominator 6, so multiplying both sides by 6 clears it. On the right, every term must be multiplied, not only the $3x$.)

Step 4: (b) Collect the x terms on one side and the numbers on the other

$$5 + 24 = 18x + 2x$$

$$29 = 20x$$

(Reason: Adding $2x$ to both sides moves the x terms to the right, and adding 24 to both sides moves the numbers to the left. That leaves four terms rearranged into two.)

Step 5: (b) Divide by the coefficient of x

$$x = \frac{29}{20}$$

$$\frac{29}{20} = 1.45$$

(Reason: Dividing both sides by 20 leaves x on its own. The scheme accepts the fraction $\frac{29}{20}$ and the mixed number $1\frac{9}{20}$ as well as the decimal 1.45.)

(a) $9c(2 - 5d)$

(b) $x = 1.45$

Verification

✓ **Check 1:** Multiply $9c$ by each term inside the bracket and compare with the expression the question prints. $9c \times 2 = 18c$ and $9c \times 5d = 45cd$, so the bracket expands back to $18c - 45cd$.

✓ **Check 2:** Look inside the bracket for anything else that could come out: a number dividing both terms, or a letter appearing in both terms. 2 and 5 share no factor above 1, and d is missing from the first term, so nothing is left to take out.

✓ **Check 3:** Put $x = 1.45$ into the left-hand side and the right-hand side of the original equation separately. $\dfrac{5 - 2 \times 1.45}{6} = 0.35$ and $3 \times 1.45 - 4 = 0.35$, so the two sides agree.

✓ **Check 4:** Turn the fraction answer and the mixed number into decimals and compare all three forms. $\dfrac{29}{20} = 1.45$ and $1 + \dfrac{9}{20} = 1.45$, so $\dfrac{29}{20}, 1\dfrac{9}{20}$ and 1.45 are the same number.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a)	**B2**	for $9c(2 - 5d)$ or $-9c(5d - 2)$	✓
(a)	**(B1)**	for $9(2c - 5cd)$ or $c(18 - 45d)$ or $3c(6 - 15d)$ or $3(6c - 15cd)$ or $9c(p + qd)$ where p and q are non-zero integers or $(2 - 5d)$ as a factor	✓
(b)	**M1**	eg $5 - 2x = 18x - 24$ or $\dfrac{5}{6} - \dfrac{2}{6}x = 3x - 4$. For removal of the fraction and correctly multiplying out RHS by 6 in an equation, or separating fractions on the LHS in an equation.	✓
(b)	**M1ft**	$5 + 24 = 18x + 2x$ oe or $29 = 20x$ oe or $\dfrac{5}{6} + 4 = \dfrac{2}{6}x + 3x$ oe. Dep on 4 terms, for correctly rearranging their 4 term equation for terms in x on one side of the equation and number terms on the other.	✓
(b)	**A1**	Working required. 1.45, dep on M1, oe eg $\dfrac{29}{20}$ or $1\dfrac{9}{20}$	✓

Full marks: 5/5

Question 2 - Exam Solution

Understanding the Question

Given

The number 1400.
Nothing else is given, so every prime factor has to be found by dividing.

Find

1400 written as a product of powers of its prime factors, with the working shown.

Plan the Solution

- Divide by the smallest prime that goes into the number, then keep dividing each quotient the same way.
- Stop as soon as the quotient left is itself a prime number.
- Count how many times each prime was used, and write that count as an index.

Worked Solution [3 marks]

> Rule - Prime factorisation: divide by the smallest prime that goes in, repeat on each quotient until what is left is prime, then write a prime used n times as that prime to the power n.

Step 1: Divide by 2 while the number stays even

$$\frac{1400}{2} = 700$$
$$\frac{700}{2} = 350$$
$$\frac{350}{2} = 175$$

(Reason: 1400 is even, and so are 700 and 350, so 2 divides three times. 175 is odd, so 2 will not go again.)

Step 2: Move on to the next prime that divides, which is 5

$$\frac{175}{5} = 35$$
$$\frac{35}{5} = 7$$

(Reason: The digits of 175 add to 13, so 3 does not divide it, but it ends in 5 so 5 does. 35 ends in 5 as well, so 5 divides a second time.)

Step 3: Stop at a prime, and list every factor used

$$1400 = 2 \times 2 \times 2 \times 5 \times 5 \times 7$$

(Reason: 7 is prime, so the dividing stops there. The factors used are the three 2s, the two 5s and the 7 that is left, which is the list of six prime factors the question is asking to be written more neatly.)

Step 4: Collect equal primes into powers

$2 \times 2 \times 2 = 2^3$

$5 \times 5 = 5^2$

$1400 = 2^3 \times 5^2 \times 7$

(Reason: A repeated prime is written as a power, so the number of times a prime was used becomes its index. A prime used once keeps no index, which is why the 7 is written on its own.)

$$2^3 \times 5^2 \times 7$$

Verification

✓ **Check 1:** Multiply the powers back together: $2^3 = 8$ and $5^2 = 25$, so $8 \times 25 = 200$ and $200 \times 7 = 1400$. 1400, the number the question started with.

✓ **Check 2:** Start the factorising somewhere completely different: $1400 = 14 \times 100$, with $14 = 2 \times 7$ and $100 = 2^2 \times 5^2$. The same three primes and the same three indices, $2^3 \times 5^2 \times 7$, so the starting split does not change the answer.

✓ **Check 3:** Confirm every base really is prime: 2, 5 and 7 each have no factors apart from 1 and themselves. All three bases are prime, so this is a product of powers of prime factors and not just any factorisation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Two prime factors reached from the start of the factorisation	M1	For finding 2 prime factors after at least 2 stages of prime factorisation with 0 incorrect stages, or for finding 2 prime factors after at least 3 stages of prime factorisation with no more than 1 incorrect stage. Each stage gives 2 factors and may be in a factor tree, in a table or listed; examples of the amount of work needed for the award of this mark are $2 \times 2 \times 350$, $2 \times 7 \times 100$, $2 \times 5 \times 140$, $5 \times 5 \times 56$, $7 \times 5 \times 40$ and $2 \times 7 \times 25 \times 4$, but we want to see 2 prime factors. Example of finding 2 prime factors after at least 3 stages with 1 incorrect stage: $1400 = 10 \times 14 = 2 \times 5 \times 2 \times 7$.	✓
Every prime factor identified, and nothing else	M1	Dependent on the first M1. For factors $2,\ 2,\ 2,\ 5,\ 5,\ 7$ identified with no others, in any form, for example listed, multiplied or added, such as $2 \times 2 \times 2 \times 5 \times 5 \times 7$. Ignore 1s. It may be seen in a fully correct factor tree or ladder.	✓

Step	Mark	Description	Got it?
The product written in index form	A1	Dependent on both method marks. $2^3 \times 5^2 \times 7$. It may be in any order, and the multiplication signs may be written as dots. Working is required.	✓

Full marks: 3/3

Question 3 - Exam Solution

Understanding the Question

Given

Two equations that must both be true at the same time:
$3x + 2y = 10$
$3x - 4y = 16$
The coefficient of x is 3 in each of them.

Find

The value of x and the value of y that satisfy both equations at once. Both equations are linear, so expect exactly one solution pair - not two.

Plan the Solution

- Write the two equations one under the other and compare the coefficients.
- The x terms are already identical, so subtracting one equation from the other removes x with no multiplying needed first.
- Solve the single equation in y that is left.
- Substitute that value back into one of the original equations to find x.
- Test the pair in both original equations, because a pair that fits only one of them is not a solution.

Worked Solution [3 marks]

> Rule - Elimination: when a variable has the same coefficient in both equations, subtracting one equation from the other removes that variable and leaves a single equation in the other variable.

Step 1: line the equations up and compare the x terms

$$3x + 2y = 10 \quad (1)$$
$$3x - 4y = 16 \quad (2)$$

(Reason: The coefficient of x is 3 in both equations, so neither equation has to be multiplied to make the coefficients match.)

Step 2: subtract equation (2) from equation (1) to eliminate x

$(3x + 2y) - (3x - 4y) = 10 - 16$

$6y = -6$

$y = -1$

(Reason: Subtracting reverses the sign of every term of equation (2), so the $3x$ terms cancel and an equation in y alone is left.)

Step 3: substitute $y = -1$ into equation (1) to find x

$3x + 2 \times (-1) = 10$

$3x - 2 = 10$

$3x = 12$

$$x = \frac{12}{3} = 4$$

(Reason: With y known, equation (1) holds one unknown only. Equation (2) would give the same value, and it is used as the second check below.)

$$x = 4$$
$$y = -1$$

Verification

- ✓ **Check 1:** Put $x = 4$ and $y = -1$ into equation (1). $3 \times 4 + 2 \times (-1) = 12 - 2 = 10$
- ✓ **Check 2:** Put the same pair into equation (2), which was not the equation used to find x. $3 \times 4 - 4 \times (-1) = 12 + 4 = 16$
- ✓ **Check 3:** Solve again by the other elimination: double equation (1) to get $6x + 4y = 20$, then add equation (2) so that the y terms cancel. $9x = 36$, so $x = 4$, and equation (1) then gives $y = -1$ again.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $3x + 2y = 10$ and $3x - 4y = 16$ subtracted, giving $(6y = -6)$ or eg $6x + 4y = 20$ and $3x - 4y = 16$ added, giving	M1	a correct method to eliminate x or y: coefficients of x or y are the same and the correct operation to eliminate is selected; if operator not written, the correct operation can be implied by 2	✓

Step	Mark	Description	Got it?
$(9x = 36)$ or eg $3\left(\dfrac{10-2y}{3}\right) - 4y = 16$ or $3\left(\dfrac{16+4y}{3}\right) + 2y = 10$ or eg $3x - 4\left(\dfrac{10-3x}{2}\right) = 16$ or $3x + 2\left(\dfrac{3x-16}{4}\right) = 10$ or $6y = -6$ oe or $9x = 36$ oe		out of 3 terms correct. Allow one arithmetic error if multiplying to equate coefficients. Or for a correct substitution of one variable into the other equation. NB: the mark is for the method and not for the result of the method. However, if the correct result of this method is seen, the mark can be awarded.	
eg $3x + 2 \times (-1) = 10$ or $3x - 4 \times (-1) = 16$ or $x = \dfrac{10 - 2 \times (-1)}{3}$ or $x = \dfrac{16 + 4 \times (-1)}{3}$ or eg $3 \times 4 + 2y = 10$ or $3 \times 4 - 4y = 16$ or $y = \dfrac{10 - 3 \times 4}{2}$ or $y = \dfrac{3 \times 4 - 16}{4}$ The printed scheme sets each of these values in quotation marks, so the candidate's own earlier value may be used in place of -1 or 4.	M1	dep on M1 a correct substitution to find the value of the second variable using their value, or for starting again with elimination or substitution (as above)	✓
$x = 4$ $y = -1$ Working required	A1	dep on M1. The working column of the printed scheme states that working is required.	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

The wall has area 45 m²

One pack of acoustic panels covers 1.5 m² and costs £64

5 tubes of panel adhesive are bought, at £ 12 each

The customer is charged £3000

Find

The percentage profit, correct to one decimal place.

Plan the Solution

- Divide the wall area by the area one pack covers, to get the number of packs.
- Cost the packs, cost the adhesive, and add the two together to get what the job cost Nathan.
- Take the total cost away from the amount charged, to get the profit.
- Write the profit as a percentage of the COST. Percentage profit is always measured against what was spent, never against what was charged.

Worked Solution [5 marks]

> Percentage profit: $\text{percentage profit} = \dfrac{\text{profit}}{\text{total cost}} \times 100$, where $\text{profit} = \text{amount charged} - \text{total cost}.$

Step 1: Work out how many packs of panels are needed

$$\frac{45}{1.5} = 30$$

(Reason: One pack covers 1.5 m², so the number of packs is the wall area divided by 1.5. It comes out whole, and it fits the wall exactly: $30 \times 1.5 = 45$.)

Step 2: Work out the cost of the panels

$$30 \times 64 = 1920$$

(Reason: Every pack costs the same £64, so the cost of the panels is the number of packs multiplied by the price of one pack.)

Step 3: Work out the total cost of the job

$$5 \times 12 = 60$$

$$1920 + 60 = 1980$$

(Reason: The adhesive is bought as well as the panels, so its cost is added on. The job costs Nathan £ 1980 altogether.)

Step 4: Work out the profit

$3000 - 1980 = 1020$

(Reason: Profit is what the customer is charged minus what the job cost.)

Step 5: Write the profit as a percentage of the cost

$$\frac{1020}{1980} \times 100 = \frac{1700}{33}$$
$$\frac{1700}{33} \approx 51.5$$

(Reason: A calculator gives $51.5151\ldots$, and one decimal place is asked for, so it rounds to 51.5. Notice what the profit is divided by: the cost 1980, not the 3000 charged.)

51.5%

Verification

✓ **Check 1:** Put the profit back on to the cost. The total cost and the profit must rebuild the amount charged. $1980 + 1020 = 3000$, which is exactly what the customer pays.

✓ **Check 2:** Reach the total cost by the other route the mark scheme allows: the cost per square metre first, then the whole wall. $\frac{64}{1.5} \times 45 = 1920$, and $1920 + 60 = 1980$, the same total cost by a different road.

✓ **Check 3:** A size check. Half of the cost is 990, so a fifty per cent profit would be a profit of £990. The actual profit of £1020 is a little above £990, so the percentage must be a little above fifty per cent, and 51.5% is.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{45}{1.5}$ (= 30) or 5×12 (= 60) or $\frac{64}{1.5}$ (= $\frac{128}{3}$, that is $42.6(6\ldots)$)	**M1**	for a method to find the number of packs needed or the cost of adhesive or cost of panels per m²	✓
30×64 (= 1920) or $42.6(6\ldots) \times 45$ (= 1920)	**M1**	for a method to find the cost of the packs of panels. The printed scheme puts 30 and $42.6(6\ldots)$ in quotation marks, so the candidate's own value from the row above may be used.	✓

Step	Mark	Description	Got it?
$1920 + 60$ (= 1980) or $3000 - 1920 - 60$ (= 1020)	**M1**	for a method to find the total cost or the profit. The printed scheme puts 1920 and 60 in quotation marks on the addition route, so the candidate's own earlier values may be used there; the subtraction alternative is printed unquoted.	✓
eg $\frac{3000 - 1980}{1980}$ $(= 0.515\ldots)$ or $\frac{3000 - 1980}{1980} \times 100$ or $\frac{3000}{1980}$ $(= 1.515\ldots)$ or $\frac{3000}{1980} \times 100$ $(= 151.5\ldots)$ or $\frac{3000}{1980} \times 100 - 100$	**M1**	for a method to find the percentage profit or be one step away. The printed scheme puts 1980 in quotation marks, so the candidate's own total cost may be used.	✓
Correct answer scores full marks (unless from obvious incorrect working). Answer 51.5	**A1**	awrt 51.5	✓
answer 56.3 or 56.25 (from use of 1920 instead of 1980 as total cost)	**SC B3**	a special case in the printed scheme: this answer scores 3 marks	✓

Full marks: 5/5

Question 5 - Exam Solution

Understanding the Question

Given

Shape A is a trapezium with vertices $(5, 2)$, $(8, 2)$, $(7, 3)$ and $(6, 3)$. Shape B is a trapezium with vertices $(3, 2)$, $(3, 4)$, $(4, 4)$ and $(5, 2)$.

Find

The image of shape A after the reflection, drawn on the first grid. The image of shape B after the enlargement, drawn on the second grid.

Part (a) reflects shape A in the line $y = x$.
Part (b) enlarges shape B by scale factor 2 with centre $(1, 1)$.

Plan the Solution

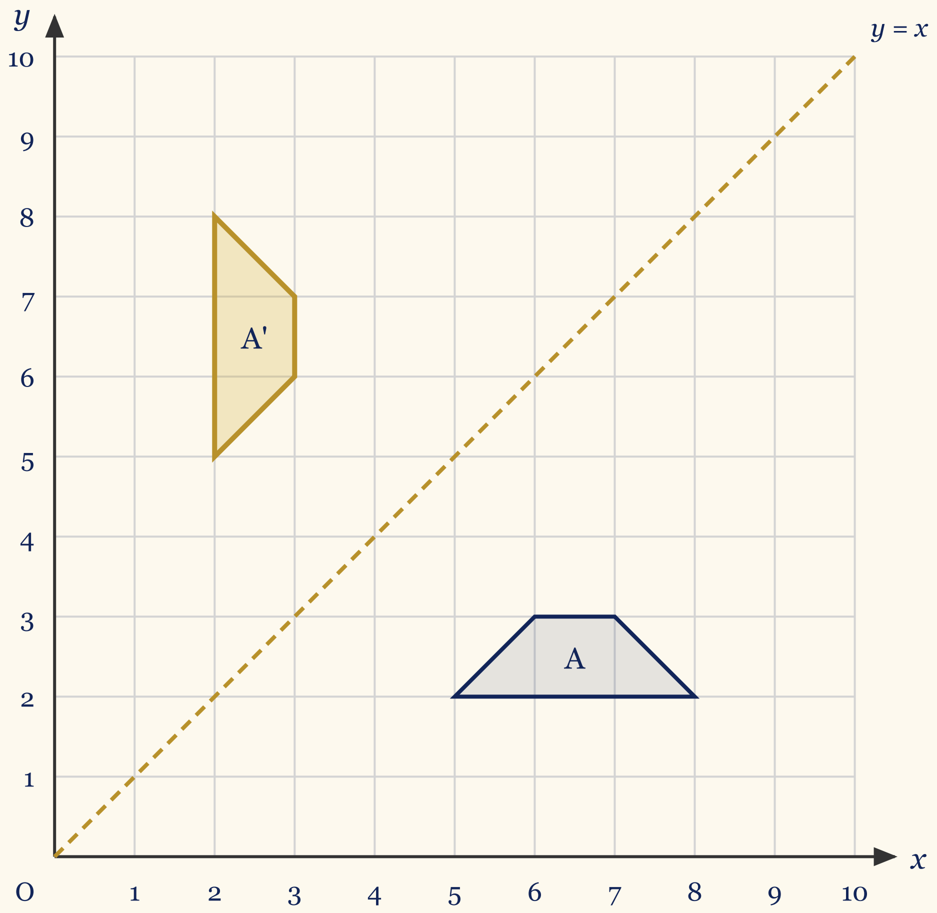
- Read the four vertices of each shape off the grid before doing anything else. Both parts are then done one vertex at a time.
- For part (a), use the rule for a reflection in $y = x$: the two coordinates of a point change places. Drawing the mirror line first is worth doing, and the mark scheme gives a mark for it on its own.
- For part (b), measure each vertex FROM the centre $(1, 1)$, double that step, then step out again from the centre. Doubling the coordinates themselves would enlarge from the origin, which gives a shape of the right size in the wrong place.
- Plot the four image points, join them in the same order as the original, and label the image.

Worked Solution [4 marks]

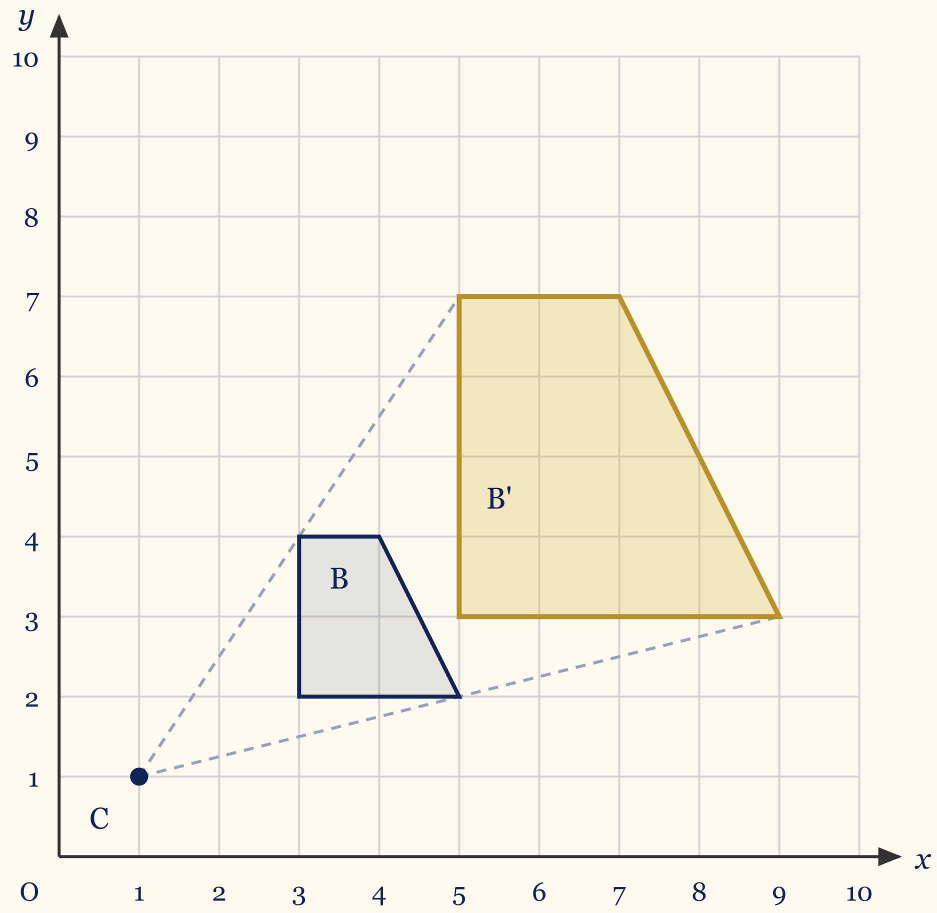
> Reflection in the line $y = x$ sends $(x, y) \to (y, x)$. Enlargement with centre (a, b) and scale factor k sends $(x, y) \to (a + k(x - a),\ b + k(y - b))$.

Step 1: Read the vertices of shape A off the grid

$(5, 2)$, $(8, 2)$, $(7, 3)$, $(6, 3)$



y
x
y = x
O
1
2
3
4
5
6
7
8
9
10
A
A'



(Reason: Shape A is a trapezium: its base is 3 squares long, its top is 1 square long, and both sloping sides rise one square as they move one square across. Everything in part (a) is done to these four points.)

Step 2: Reflect each vertex by swapping its coordinates

$(5, 2) \to (2, 5)$

$(8, 2) \to (2, 8)$

$(7, 3) \to (3, 7)$

$(6, 3) \to (3, 6)$

(Reason: The mirror line $y = x$ is the line through $(0, 0)$ and $(10, 10)$ where the two coordinates are equal, and reflecting in it exchanges the roles of x and y. So $(5, 2)$ goes to $(2, 5)$: the point that was 5 across and 2 up becomes 2 across and 5 up.)

Step 3: Plot the four image points and join them up

$(2, 5), (2, 8), (3, 7), (3, 6)$

(Reason: Join them in the same order as the original, so the 3-square side still faces the 1-square side. The image is congruent to shape A: a reflection moves a shape and turns it over, and never changes its size.)

Step 4: Read the vertices of shape B off the grid

$(3, 2), (3, 4), (4, 4), (5, 2)$

(Reason: Shape B is a trapezium with a vertical left-hand side 2 squares tall, a horizontal top 1 square long and a horizontal base 2 squares long.)

Step 5: Work out the image of the first vertex

$1 + 2 \times (3 - 1) = 5$

$1 + 2 \times (2 - 1) = 3$

$(3, 2) \rightarrow (5, 3)$

(Reason: The vertex $(3, 2)$ is 2 squares right of the centre and 1 square above it. Scale factor 2 doubles those steps to 4 and 2, and they are stepped out from the centre $(1, 1)$, which is why the centre's own coordinate opens each line. Measuring from the origin instead is the classic slip: it gives a shape of exactly the right size and the right way round, sitting in the wrong place.)

Step 6: Do the same for the other three vertices

$(3, 4) \rightarrow (5, 7)$

$(4, 4) \rightarrow (7, 7)$

$(5, 2) \rightarrow (9, 3)$

(Reason: Each one is treated the same way: double the step from the centre, then step that doubled amount out from $(1, 1)$. For $(4, 4)$ the step is 3 right and 3 up, which doubles to 6 and 6, giving $(7, 7)$.)

Step 7: Plot the four image points and join them up

$(5, 3), (5, 7), (7, 7), (9, 3)$

(Reason: Join them in the original order. Every side has doubled in length and stays parallel to the side it came from, which is what an enlargement of scale factor 2 looks like. The centre $(1, 1)$ itself does not move, and a ray drawn from it through any vertex passes straight through that vertex's image.)

(a) $(2, 5), (2, 8), (3, 6), (3, 7)$

(b) $(5, 3), (5, 7), (7, 7), (9, 3)$

Verification

✓ **Check 1:** Measure the sides of the image in part (a). A reflection is a congruence, so every side must keep the length it had. Both trapeziums have sides of 3, $\sqrt{2}$, 1 and $\sqrt{2}$ squares, so the image really is congruent to shape A.

✓ **Check 2:** Take a vertex and its image and find the midpoint of the line joining them. If the reflection is right, that midpoint sits on the mirror line. For $(5, 2)$ and $(2, 5)$ the midpoint is $\dfrac{5+2}{2} = 3.5$ across and $\dfrac{2+5}{2} = 3.5$ up, and $(3.5, 3.5)$ has equal coordinates, so it lies on $y = x$.

✓ **Check 3:** Step from the centre $(1, 1)$ to a vertex, and from the centre to that vertex's image. The second step must be exactly 2 times the first, or the image is not on the ray. To $(3, 2)$ the step is 2 right and 1 up; to $(5, 3)$ it is $2 \times 2 = 4$ right and $2 \times 1 = 2$ up, so the image vertex sits on the same ray, twice as far out.

✓ **Check 4:** Count the area of shape B and the area of its image. An enlargement of scale factor 2 multiplies area by 2^2, not by 2. Shape B covers 3 squares and its image covers 12 squares, and $12 = 4 \times 3$, which is the area scale factor it should be.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Vertices at $(2, 5)$ $(2, 8)$ $(3, 6)$ $(3, 7)$	**B2**	for correct shape in correct position (B1 for correct orientation of shape but wrong position or for 3 out of 4 vertices correct or for $y = x$ drawn)	✓
(b) Vertices at $(5, 3)$ $(5, 7)$ $(7, 7)$ $(9, 3)$	**B2**	for correct shape in correct position (B1 for correct size and orientation of shape but wrong position or for 3 out of 4 vertices correct)	✓

Full marks: 4/4

Question 6 - Exam Solution

Understanding the Question

Given

(a) 5^0

(b) $\dfrac{5^9 \times 5^{-3}}{5^{-2}} = 5^k$

(c) $\left(2d^4e^5\right)^3$

Three separate demands on the index laws. They share no working, so each is finished on its own.

Find

The value of 5^0. The value of k, which is an index and not a power of 5. The simplified form of $\left(2d^4e^5\right)^3$, with no bracket left in it.

Plan the Solution

- Part (a) is the zero index read straight off. Any non-zero number raised to the power 0 is 1, so there is nothing to calculate.
- Part (b): every term is a power of 5, so leave the base alone and work in the indices only. Multiplying adds them, dividing subtracts them.
- The index on the bottom is -2, and subtracting a negative adds, so that division makes the index bigger rather than smaller. That is the whole difficulty of the part.
- Part (c): the bracket holds a product, so the outside power 3 lands on all three factors, the 2 included. Then a power of a power multiplies the indices.

Worked Solution [5 marks]

> Rule - Index laws: $a^m \times a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$, and $a^0 = 1$ for every non-zero a.

Step 1: part (a), read off the zero index

$5^0 = 1$

(Reason: Any non-zero number raised to the power 0 is 1. The division law forces it: $\dfrac{5^3}{5^3} = 5^{3-3} = 5^0$, and that fraction is $\dfrac{125}{125} = 1$.)

Step 2: part (b), collapse the multiplication on the top

$5^9 \times 5^{-3} = 5^{9+(-3)} = 5^6$

(Reason: The base is the same in both, so multiplying adds the indices. $9 + (-3) = 6$, so the whole top is a single power, 5^6.)

Step 3: part (b), divide by the power on the bottom

$$\frac{5^6}{5^{-2}} = 5^{6-(-2)} = 5^8$$

(Reason: The base is the same again, so dividing subtracts the indices. The index on the bottom is -2, so it is that whole negative number that gets subtracted, and subtracting a negative adds.)

Step 4: part (b), match the indices on the two sides

$5^8 = 5^k$

$k = 8$

(Reason: Two equal powers of the same base must have equal indices. The question asks for k, so the answer is 8 and not 5^8.)

Step 5: part (c), put the outside power on to every factor

$(2d^4e^5)^3 = 2^3 \times (d^4)^3 \times (e^5)^3$
$= 8 \times d^{4\times3} \times e^{5\times3} = 8d^{12}e^{15}$

(Reason: A power outside a bracket applies to each factor inside it, so the 2 is cubed as well and becomes 8, not 6. A power of a power then multiplies the indices: $4 \times 3 = 12$ and $5 \times 3 = 15$.)

(a) $5^0 = 1$

(b) $k = 8$

(c) $8d^{12}e^{15}$

Verification

- ✓ **Check 1 - part (a) from the pattern of powers:** Go down the powers of 5, dividing by 5 each time: $5^3 = 125$, $5^2 = 25$, $5^1 = 5$. One more division continues the pattern to 5^0. $\frac{5}{5} = 1$, so the pattern lands on $5^0 = 1$
- ✓ **Check 2 - part (b) by working both sides out as numbers:** Evaluate the left-hand side without any index law. $5^9 \times 5^{-3} = \frac{1953125}{125} = 15625$, and dividing by 5^{-2} multiplies by 25. $15625 \times 25 = 390625$ and $5^8 = 390625$, so the index is $k = 8$
- ✓ **Check 3 - part (c) by substituting numbers:** Put $d = 2$ and $e = 1$ into the original bracket and into the simplified answer, and compare the two numbers. $(2 \times 2^4 \times 1^5)^3 = 32^3 = 32768$ and $8 \times 2^{12} \times 1^{15} = 8 \times 4096 = 32768$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) 1	**B1**	cao	✓
(b) eg $(5^9 \times 5^{-3} =)\ 5^6$ or $\left(\frac{5^9}{5^{-2}} =\right) 5^{11}$ or $\left(\frac{5^{-3}}{5^{-2}} =\right) 5^{-1}$ or $(5^k \times 5^{-2} =)\ 5^{k-2}$ or $9 - 3 = k - 2$ oe or $9 - 3 - -2$ or $9 - 3 + 2$	**M1**	for one correct application of an index rule (must be seen in powers of 5) this could be after an initial mistake - working will need to be clearly seen or for forming a correct equation in the indices alone or for a complete method for the value of k	✓

Step	Mark	Description	Got it?
8 Correct answer scores full marks (unless from obvious incorrect working)	**A1**	condone 5^8	✓
(c) $8d^{12}e^{15}$	**B2**	for a correct answer (B1 for answer of the form $kd^m e^n$ where at least two of $k = 8, m = 12$ and $n = 15$ are correct)	✓

Full marks: 5/5

Question 7 - Exam Solution

Understanding the Question

Given

Mass of the pendant: 48.3 g
Density of silver: 10.5 g/cm³
The pendant is solid silver right through, so that one density applies to the whole of it.

Find

The volume of the pendant, in cm³.
Density ties mass and volume together, so this is the third quantity of a set of three with the other two given.

Plan the Solution

- Density is a compound measure: it says how much mass sits in each cm³ of the material. One formula ties the three quantities together, so any two of them give the third.
- Write that formula down first and put the two given values into it. The volume is the unknown, and in the formula it starts underneath, so it cannot simply be read off.
- Rearrange to make the volume the subject, then divide. Grams divided by grams per cm³ leaves cm³, so the unit looks after itself and nothing needs converting.
- A calculator is allowed, so the division is done directly. The numbers are chosen so that it terminates, and an answer that runs on and on is the sign that the mass and the density went into the fraction the wrong way up.

Worked Solution [2 marks]

Rule - Density: $\text{density} = \dfrac{\text{mass}}{\text{volume}}$, which rearranges to $\text{volume} = \dfrac{\text{mass}}{\text{density}}$.

Step 1: write the density formula and substitute the two given values

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$10.5 = \frac{48.3}{V}$$

(Reason: Density is mass per unit volume, so the mass goes on top and the volume underneath. Writing V for the volume in cm³ puts the unknown exactly where the formula puts it, and the letter itself does not matter.)

Step 2: rearrange to make the volume the subject

$$10.5 \times V = 48.3$$

$$V = \frac{48.3}{10.5}$$

(Reason: Multiplying both sides by V lifts it out of the denominator, and dividing both sides by 10.5 then leaves it alone. The mass finishes on top, which is the sensible way round: a heavier pendant of the same metal takes up more room.)

Step 3: work out the division

$$V = \frac{48.3}{10.5} = 4.6$$

(Reason: Grams divided by grams per cm³ leaves cm³, so the answer is already a volume with no conversion needed. Silver is dense, so a mass of nearly fifty grams is packed into only a few cm³.)

$$4.6\ \text{cm}^3$$

Verification

✓ **Check 1 - put the answer back into the density formula:** Density multiplied by volume must return the mass the question gives. Multiply 10.5 by the answer and compare. $10.5 \times 4.6 = 48.3$, which is the mass in the question

✓ **Check 2 - do the same division as a fraction instead:** Clear both decimals by multiplying top and bottom by 10, then cancel by 21. This is the form the mark scheme also accepts. $\dfrac{48.3}{10.5} = \dfrac{483}{105} = \dfrac{23}{5}$, and $\dfrac{23}{5} = 4.6$

✓ **Check 3 - trap the answer between two whole numbers:** One cm³ of silver has a mass of 10.5 g, so work out the mass of 4 cm³ and of 5 cm³ and see where the pendant falls between them. $10.5 \times 4 = 42$ and $10.5 \times 5 = 52.5$, and $42 < 48.3 < 52.5$, so the volume lies between 4 and 5 cm³

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$10.5 = \dfrac{48.3}{v}$ or $10.5v = 48.3$ or $(v =)\dfrac{48.3}{10.5}$	M1	oe for substituting 10.5 and 48.3 correctly into a correct formula for density; may use any letter for the volume	✓
4.6 Correct answer scores full marks (unless from obvious incorrect working)	A1	allow $\dfrac{23}{5}$ or $4\dfrac{3}{5}$ oe	✓

Full marks: 2/2

Question 8 - Exam Solution

Understanding the Question

Given

A list of 7 numbers whose mean is 60
A group of 3 of those numbers whose mean is 46
The other 4 numbers are whatever is left of the list once that group of 3 is taken out, so the two groups do not overlap and between them they are the whole list.
Not one individual number is given, and none is needed.

Find

The mean of the remaining 4 numbers. A mean is asked for, not a total, so whatever else happens the last move is a division by 4.

Plan the Solution

- A mean on its own says nothing about how many numbers went into it, so the two means given here cannot be combined directly. Totals can be combined, and a mean turns into a total in one multiplication.

- Multiply each mean by how many numbers it covers. The whole list gives one total; the group of 3 gives another.
- The group and the rest make up the list between them and share no numbers, so subtracting the smaller total from the larger leaves exactly the total of the remaining 4 numbers.
- Divide that total by 4, because that is how many numbers it covers. Nothing here forces a whole-number answer, and this one is not a whole number.

Worked Solution [3 marks]

> Rule - Mean: $\text{mean} = \dfrac{\text{total}}{\text{how many}}$, which rearranges to $\text{total} = \text{mean} \times \text{how many}$.
> Totals may be added and subtracted; means may not.

Step 1: turn the overall mean into the total of all 7 numbers

$\text{total} = \text{mean} \times \text{how many}$

$60 \times 7 = 420$

(Reason: A mean of 60 spread over 7 numbers carries exactly the same information as a total of 420 shared equally between them, so nothing is lost by multiplying. The numbers themselves stay unknown and the question never needs them.)

Step 2: turn the mean of the smaller group into that group's total

$46 \times 3 = 138$

(Reason: The same rule applied to the group on its own: its mean multiplied by how many numbers are in that group gives the total those numbers contribute to the list. Which count belongs to which mean is the thing to be careful about here, because the question hands over two of them.)

Step 3: subtract to leave the total of the remaining 4 numbers

$420 - 138 = 282$

(Reason: The group of 3 and the remaining 4 together account for all 7 numbers and share none of them, so their two totals must add up to 420. Taking one total away therefore leaves the other.)

Step 4: divide that total by how many numbers it covers

$$\frac{282}{4} = 70.5$$

(Reason: The total 282 belongs to 4 numbers, so dividing by 4 turns it back into a mean. It lands above 60, which is what has to happen: the group taken out averaged below 60, so what is left has to sit above 60 to pull the whole list back up to it.)

70.5

Verification

✓ **Check 1 - rebuild a list that fits both facts and take its mean directly:** Nothing says the numbers are all different, so take the 3 as 46 each and the other 4 as 70.5 each. Both groups then have the right mean, so the mean of all 7 must come back to 60.

$3 \times 46 + 4 \times 70.5 = 138 + 282 = 420$, and $\dfrac{420}{7} = 60$, which is the mean the question gives

✓ **Check 2 - balance the distances from the overall mean:** Measure every number against the overall mean of 60 instead of totalling anything. The 3 numbers average 14 below it, so they are short by 42 between them, and the other 4 have to be 42 above it between them to balance that out. $(60 - 46) \times 3 = 42$, then $\dfrac{42}{4} = 10.5$ above the mean each, so $60 + 10.5 = 70.5$

✓ **Check 3 - read the last division as a fraction:** Leave the final division uncalculated and cancel it instead, which also gives the answer in the other forms the mark scheme accepts. $\dfrac{282}{4} = \dfrac{141}{2}$, which is 70.5, also written $70\frac{1}{2}$, and $46 < 60 < 70.5$ as it must be

Mark Scheme Breakdown

Step	Mark	Description	Got it?
60×7 $(= 420)$ or 46×3 $(= 138)$	M1	may be embedded within an equation	✓
$420 - 138$ $(= 282)$	M1	for a method to find the sum of the 4 numbers. Allow this mark if they do further incorrect work using 282. The printed scheme puts 420 and 138 in quotation marks, so the candidate's own earlier values may be used.	✓
70.5 Correct answer scores full marks (unless from obvious incorrect working)	A1	allow $\dfrac{141}{2}$ oe eg $\dfrac{282}{4}$ or $70\frac{1}{2}$	✓

Full marks: 3/3

Question 9 - Exam Solution

Understanding the Question

Given

Every normal price in the store is reduced by 15% in the sale.
The sale price of the espresso machine is 612 Swiss francs.

Find

The normal price of the espresso machine, in Swiss francs, before the reduction.

Plan the Solution

- The 15% comes off the NORMAL price, so the ticket in the sale shows 85% of it.
- That makes the sale price the normal price multiplied by 0.85, and the sale price is the value we already know.
- So reverse the multiplication: divide 612 by 0.85.
- Finish by taking 15% off the answer, which must bring it back to 612.

Worked Solution [3 marks]

Reverse percentage: $\text{sale price} = \text{normal price} \times \text{multiplier}$, so $\text{normal price} = \dfrac{\text{sale price}}{\text{multiplier}}$. Dividing by the multiplier is what undoes the reduction.

Step 1: find the multiplier for a sale price

$$100\% - 15\% = 85\%$$

$$\frac{85}{100} = 0.85$$

(Reason: Taking 15% off leaves 85% of the normal price, and a percentage becomes a decimal multiplier when it is divided by 100.)

Step 2: write the sale price in terms of the normal price

$$0.85 \times n = 612$$

(Reason: Let n be the normal price. The store multiplies every normal price by 0.85, and for this espresso machine that gives the 612 Swiss francs on the sale ticket.)

Step 3: reverse the multiplication

$$n = \frac{612}{0.85} = 720$$

(Reason: Multiplying by 0.85 made the price smaller, so dividing by 0.85 undoes the reduction and returns the price before the sale.)

720 Swiss francs

Verification

✓ **Check 1:** Take the reduction off the answer. 15% of 720 is $0.15 \times 720 = 108$, and that must leave the price on the sale ticket. $720 - 108 = 612$, the sale price the question gives

✓ **Check 2:** Work it out a second way, by the unitary method. If 85% is 612 Swiss francs, then 1% is $\frac{612}{85} = 7.2$ Swiss francs. $7.2 \times 100 = 720$, the same normal price by a different method

✓ **Check 3:** Sanity check the size and the share. A sale price must be smaller than the normal price, and the amount taken off must be 15% of the normal price, not of the sale price. $720 > 612$ and $\frac{108}{720} = 0.15$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$1 - 0.15$ $(= 0.85)$ or $100(\%) - 15(\%)$ $(= 85(\%))$ or $\frac{612}{85}$ $(= 7.2)$ oe	M1	May be seen embedded. Do not allow $(1 - 15\%)$ unless it is processed correctly.	✓
$\frac{612}{0.85}$ oe, where the 0.85 may be the candidate's own value from the row above or $\frac{612}{85} \times 100$ oe, again with their own 85 or 7.2×100, with their own 7.2	M1	For a complete method.	✓
Correct answer scores full marks (unless from obvious incorrect working)	A1	The answer is 720.	✓

Full marks: 3/3

Question 10 - Exam Solution

Understanding the Question

Given

A line with equation $y = 2 - 5x$
The line L is parallel to that line
The line L passes through $(0, 6)$

Find

An equation of the line L

Plan the Solution

- Write $y = 2 - 5x$ in the form $y = mx + c$ so its gradient can be read straight off.
- Parallel means equal gradients, so give L the same m and leave c unknown.
- Substitute the coordinates of $(0, 6)$ to pin down c, then write the equation out.

Worked Solution [2 marks]

> Rule - Parallel lines: two straight lines are parallel exactly when their gradients are equal.
> So a line parallel to $y = mx + c$ keeps the same m and only its c changes.

Step 1: Read the gradient of the given line

$y = 2 - 5x$

$y = -5x + 2$

(Reason: In the form $y = mx + c$ the number multiplying x is the gradient, so putting the x-term first shows the given line has gradient -5. The 2 is that line's intercept and plays no part in the gradient.)

Step 2: Give L the same gradient

$y = -5x + c$

(Reason: Parallel lines have equal gradients, so L also has gradient -5. Only the intercept c is still unknown, and that is what the point is for.)

Step 3: Use the point $(0, 6)$ to find c

$6 = -5 \times 0 + c$

$c = 6$

(Reason: The point lies on L, so its coordinates satisfy $y = -5x + c$: substitute $x = 0$ and $y = 6$. The x-term vanishes, which is why a point on the y-axis hands you the intercept directly.)

Step 4: Write the equation of L

$y = -5x + 6$

(Reason: Putting the gradient -5 and the intercept 6 back into $y = mx + c$ gives the equation of L.)

$$y = -5x + 6$$

Verification

✓ **Check 1 - the line really passes through the point:** Put $x = 0$ into $y = -5x + 6$. $-5 \times 0 + 6 = 6$, which is the y-coordinate of $(0, 6)$.

✓ **Check 2 - the gradient read back from two points:** Take $x = 1$ and $x = 3$ on L: $-5 \times 1 + 6 = 1$ and $-5 \times 3 + 6 = -9$. The gradient between those two points is $\dfrac{-9 - 1}{3 - 1} = -5$, the same gradient as $y = 2 - 5x$.

✓ **Check 3 - the gap between the two lines:** The given line at those same two values gives $2 - 5 \times 1 = -3$ and $2 - 5 \times 3 = -13$. The vertical gap is $1 - (-3) = 4$ and $-9 - (-13) = 4$ - the same at both values, so the two lines stay a constant distance apart and never meet.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $y = -5x\ (+k)$ or $y - a = -5(x - b)$ or $y = mx + 6$ or $y - 6 = m(x - 0)$ or $-5x + 6$ or $L = -5x + 6$	M1	for the equation of any line with gradient -5 other than $y = 2 - 5x$ or for the equation of any line passing through the point $(0, 6)$ or the correct line with $y =$ missing or with the wrong subject	✓
$y = -5x + 6$	A1	oe equation eg $y = 6 - 5x$ or $y - 6 = -5(x - 0)$ or $y + 5x = 6$	✓
Guidance printed beside the answer row	Note	Correct answer scores full marks (unless from obvious incorrect working)	✓

Full marks: 2/2

Question 11 - Exam Solution

Understanding the Question

Given	Find

Two triangles, ADE and CDB, with ABD a straight line $AE = 28$ cm, $ED = 45$ cm, $AB = 21$ cm, $CD = 35$ cm angle AED = angle $CBD = 90°$, so both triangles are right-angled	The area of triangle CDB, correct to 3 significant figures

Plan the Solution

- Triangle AED is right-angled at E and AD is its hypotenuse, so Pythagoras gives AD.
- B lies on AD, so BD is what is left of AD once AB is taken off.
- In triangle CBD the right angle is at B, so this time CD is the hypotenuse and Pythagoras gives BC.
- BC meets BD at a right angle, so BD is a base of triangle CDB and BC is the height that belongs to it.

Worked Solution [5 marks]

> Rule - Pythagoras: in a right-angled triangle $h^2 = a^2 + b^2$, where h is the hypotenuse, so a shorter side is found by subtracting: $a^2 = h^2 - b^2$. Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$, with the height measured perpendicular to that base.

Step 1: find AD from triangle AED

$$AD^2 = AE^2 + ED^2$$

$$28^2 + 45^2 = 784 + 2025 = 2809$$

$$AD = \sqrt{2809} = 53$$

(Reason: Angle $AED = 90°$, so AD is the hypotenuse of triangle AED and the two given sides are the shorter pair. 2809 is a perfect square, so AD comes out whole.)

Step 2: find BD

$$BD = AD - AB$$

$$53 - 21 = 32$$

(Reason: ABD is a straight line, so B sits on AD and the two pieces AB and BD add to AD. That one length is what carries the first triangle into the second.)

Step 3: find BC from triangle CBD

$$BC^2 = CD^2 - BD^2$$

$$35^2 - 32^2 = 1225 - 1024 = 201$$

$BC = \sqrt{201} \approx 14.1774$

(Reason: Angle $CBD = 90°$, so here it is CD that is the hypotenuse and BC is one of the shorter sides. Its square is therefore the DIFFERENCE of the other two squares, not the sum. Leaving it as $\sqrt{201}$ keeps the working exact.)

Step 4: find the area of triangle CDB

$\text{Area} = \frac{1}{2} \times BD \times BC$

$\frac{1}{2} \times 32 \times \sqrt{201} = 16\sqrt{201} = 226.8391...$

(Reason: The base BD and the height BC are perpendicular, which is exactly what the area formula asks for. Halving the 32 first turns $\frac{1}{2} \times 32$ into 16 and leaves the exact answer $16\sqrt{201}$.)

Step 5: round to 3 significant figures

$226.8391... = 227$ (to 3 significant figures)

(Reason: The fourth significant figure of $226.8391...$ is 8, which is 5 or more, so the third figure rounds up and 226 becomes 227. The unit is cm^2 because an area is being measured.)

227 cm² (exact value $16\sqrt{201}$ cm²)

Verification

✓ **Check 1:** Put both results back through Pythagoras. In triangle AED, $28^2 + 45^2 = 2809$ and $53^2 = 2809$. In triangle CBD, $32^2 + 201 = 1225$ and $35^2 = 1225$. $2809 = 2809$ and $1225 = 1225$, so both right angles are consistent with the lengths used

✓ **Check 2:** Work the area out from the angle at D instead, so no height is used at all. $\cos BDC = \frac{32}{35}$ gives angle $BDC = 23.8955...°$, and the area is $\frac{1}{2} \times 35 \times 32 \times \sin BDC$. This gives $226.8391...$ as well, from a formula that never mentions BC

✓ **Check 3:** Is the size sensible? $\sqrt{201}$ lies between $\sqrt{196} = 14$ and $\sqrt{225} = 15$, so the area $16\sqrt{201}$ lies between $16 \times 14 = 224$ and $16 \times 15 = 240$. 227 sits inside that range, near the lower end because 201 is much closer to 196 than to 225

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(AD^2 =)\ 28^2 + 45^2$ ($784 + 2025 = 2809$) or $(EDA =) \tan^{-1}\left(\frac{28}{45}\right)$ (31.8(9...)) or $(EAD =) \tan^{-1}\left(\frac{45}{28}\right)$ (58.1(09...))	M1	for a correct method to find AD^2 or angle EDA or angle EAD	✓
eg $(AD =) \sqrt{28^2 + 45^2}$ ($\sqrt{784 + 2025} = \sqrt{2809} = 53$) oe or $(AD =) \frac{28}{\sin 31.8(9...)}$ (53...) or ($AD =) \frac{45}{\cos 31.8(9...)}$ (53...) oe or $(AD =) \frac{45}{\sin 58.1...}$ (53...) or $(AD =)$ $\frac{28}{\cos 58.1...}$ (53...) oe	M1	for a correct method to find AD	✓
eg $(BC =) \sqrt{35^2 - (53 - 21)^2}$ ($\sqrt{1225 - 1024} = \sqrt{201} = 14.1(7...)$) or $(BDC =) \cos^{-1}\left(\frac{53 - 21}{35}\right)$ (23.8(9...)) or $(BCD =) \sin^{-1}\left(\frac{53 - 21}{35}\right)$ (66.1...) The printed scheme puts the 53 in quotation marks, so the candidate's own value for AD may be used here.	M1	for correct method to find BC or angle BDC or angle BCD	✓
eg $\frac{1}{2} \times 14.1(7...) \times (53 - 21)$ $(16\sqrt{201})$ or $\frac{1}{2} \times 35 \times (53 - 21) \times \sin 23.8(9...)$ or $\frac{1}{2} \times 35 \times 14.1(7...) \times \sin 66.1...$ The 53, the 14.1(7...), the 23.8(9...) and the 66.1... are all in quotation marks in the printed scheme, so the candidate's own earlier values may be used.	M1	for a correct method to find the area of triangle CDB	✓

Step	Mark	Description	Got it?
227	**A1**	awrt 227; accept $16\sqrt{201}$	✓
Correct answer scores full marks (unless from obvious incorrect working)	**Note**	Guidance printed alongside the scheme. It awards nothing of its own and does not change the total of 5 marks.	✓

Full marks: 5/5

Question 12 - Exam Solution

Understanding the Question

Given

Value when new: $\$16\,000$
12% depreciation in each of the first 2 years
$x\%$ depreciation in the third year
Value after 3 years: $\$11\,461.12$

Find

The value of x

Plan the Solution

- A loss of 12% leaves 88%, so multiply by 0.88 once for each of the first two years.
- Divide the value after three years by the value after two years. What is left is the multiplier for the third year on its own.
- Turn that multiplier into a percentage loss to read off x.

Worked Solution [3 marks]

> Rule - a decrease of $r\%$ multiplies a value by $1 - \dfrac{r}{100}$, and a run of yearly changes multiplies those multipliers together.

Step 1: write the 12% loss as a multiplier

$$1 - \frac{12}{100} = 0.88$$

(Reason: Losing 12% leaves 88% of the value standing, and 88% as a decimal is 0.88.)

Step 2: apply that multiplier for the first 2 years

$0.88^2 = 0.7744$

$16\,000 \times 0.7744 = 12\,390.4$

(Reason: Two years of the same loss multiply together, so the value after two years is $16\,000 \times 0.88^2$. Squaring the multiplier is not the same as doubling the loss.)

Step 3: find the multiplier for the third year

$$\frac{11\,461.12}{12\,390.4} = 0.925$$

(Reason: The third year acts on the value at the start of that year, not on the price when new. Dividing the final value by $12\,390.4$ leaves the third year's multiplier on its own.)

Step 4: turn that multiplier into a percentage loss

$1 - 0.925 = 0.075$

$0.075 \times 100 = 7.5$

(Reason: A multiplier of 0.925 is a decrease of 0.075, which is 7.5%. Subtracting the other way round, $0.925 - 1$, gives -0.075 and the answer -7.5 - the size is right but the sign says the boat gained value.)

$$x = 7.5$$

Verification

✓ **Check 1:** Work all three years forward, using 0.925 for the third year, and compare with the value the question gives. $16\,000 \times 0.88 \times 0.88 \times 0.925 = 11\,461.12$

✓ **Check 2:** Take the multiplier for all three years together and divide out the two years that are known. This is the mark scheme's own alternative and it uses the price when new, so it does not reuse Step 2's value. $\dfrac{11\,461.12}{16\,000} = 0.71632$ and $\dfrac{0.71632}{0.7744} = 0.925$

✓ **Check 3:** Work out the money lost in the third year and write it as a percentage of the value at the start of that year. $12\,390.4 - 11\,461.12 = 929.28$ and $\dfrac{929.28}{12\,390.4} \times 100 = 7.5$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$16\,000 \times \left(1 - \frac{12}{100}\right)^2$ $(= 12\,390.4)$ or $16\,000 \times 0.7744$ $(= 12\,390.4)$ oe or $16\,000 \times \left(1 - \frac{12}{100}\right)$ $(= 14\,080)$ and $14\,080 \times \left(1 - \frac{12}{100}\right)$ $(= 12\,390.4)$ oe or $\left(1 - \frac{12}{100}\right)^2$ $(= 0.7744)$ and $\frac{11\,461.12}{16\,000}\left(= \frac{4477}{6250} = 0.71632\right)$	**M1**	for a method to find the value of the motorboat after two years or a method to find the overall percentage multiplier after two years and the overall percentage multiplier for the three years May be seen embedded, eg in an equation Do not allow $(1 - 12\%)$ unless processed correctly The printed scheme quotes $14\,080$ in the second alternative, so the candidate's own value may be used there	✓
eg $\frac{12\,390.4 - 11\,461.12}{12\,390.4}$ $(\times 100)$ $(= 0.075)$ or $1 - \frac{11\,461.12}{12\,390.4}$ $(\times 100)$ or $1 - 0.925$ $(= 0.075)$ or $\frac{11\,461.12}{12\,390.4}$ $(\times 100)$ $(= 0.925)$ or $\frac{0.71632}{0.7744}$ $(\times 100)$ $(= 0.925)$	**M1**	for a complete method to find the value of the decimal equivalent of $x\%$ or for a complete method to find the percentage multiplier for the third year May be seen embedded, eg within a correct equation rearranged to one of these equivalent forms The printed scheme quotes $12\,390.4$, 0.71632 and 0.7744 here, so the candidate's own values from the first mark may be used	✓
7.5	**A1**	oe	✓

Step	Mark	Description	Got it?
Correct answer only	**Note**	Correct answer only scores full marks (unless from obviously incorrect working)	✓
for an answer of -7.5	**SC B2**	SCB2 for answer -7.5	✓

Full marks: 3/3

Question 13 - Exam Solution

Understanding the Question

Given

(a) The expression $4x^2 - 25y^2$ - two terms with a minus sign between them. (b) The product $4x(x+3)(2x-5)$ - a single term multiplied by two brackets.

Find

(a) $4x^2 - 25y^2$ written as a product. (b) The expanded, simplified form, and with it the integers a, b and c.

Plan the Solution

- (a) Test each term for being a perfect square: $4x^2 = (2x)^2$ and $25y^2 = (5y)^2$, with a subtraction between them.
- (a) Apply the difference of two squares with $A = 2x$ and $B = 5y$.
- (b) Expand two of the three factors, then multiply that result by the factor left over. Any pair may go first.
- (b) Collect like terms, then compare with $ax^3 + bx^2 + cx$ to read the three integers off.

Worked Solution [5 marks]

> Rule - Difference of two squares: $A^2 - B^2 = (A+B)(A-B)$. Rule - Expanding a triple product: expand any two of the factors first, then multiply that answer by the third, and collect like terms at the end.

Step 1: write each term of part (a) as a square

$4x^2 = (2x)^2$

$25y^2 = (5y)^2$

(Reason: Both 4 and 25 are square numbers, and x^2 and y^2 are already squares, so each term is a perfect square and the subtraction between them makes it a difference of two squares.)

Step 2: use the difference of two squares

$4x^2 - 25y^2 = (2x)^2 - (5y)^2$

$(2x)^2 - (5y)^2 = (2x + 5y)(2x - 5y)$

(Reason: With $A = 2x$ and $B = 5y$, the rule $A^2 - B^2 = (A + B)(A - B)$ gives the two brackets. One sign is a plus and the other a minus: two plus signs would give $4x^2 + 20xy + 25y^2$ instead.)

Step 3: part (b) - expand the first two factors

$4x(x + 3) = 4x^2 + 12x$

(Reason: Multiply $4x$ by each term inside the bracket: $4x \times x = 4x^2$ and $4x \times 3 = 12x$.)

Step 4: multiply that by the remaining bracket

$(4x^2 + 12x)(2x - 5) = 8x^3 - 20x^2 + 24x^2 - 60x$

(Reason: Every term in the first bracket multiplies every term in the second: $4x^2 \times 2x = 8x^3$, $4x^2 \times (-5) = -20x^2$, $12x \times 2x = 24x^2$ and $12x \times (-5) = -60x$.)

Step 5: collect the two x^2 terms

$8x^3 - 20x^2 + 24x^2 - 60x = 8x^3 + 4x^2 - 60x$

(Reason: Only the two x^2 terms are like terms. The x^3 term and the x term each stand alone, so they are carried down unchanged.)

Step 6: read off a, b and c

$a = 8$

$b = 4$

$c = -60$

(Reason: Comparing $8x^3 + 4x^2 - 60x$ with $ax^3 + bx^2 + cx$ matches the coefficients term by term. All three are integers, which is what the question asks for. There is no constant term in either, so nothing is left over.)

(a) $(2x + 5y)(2x - 5y)$

(b) $8x^3 + 4x^2 - 60x$

(b) $a = 8, b = 4, c = -60$

Verification

✓ **Check 1:** Expand $(2x + 5y)(2x - 5y)$ term by term and watch what happens to the two middle terms. $4x^2 - 10xy + 10xy - 25y^2 = 4x^2 - 25y^2$, which is the expression the question printed.

✓ **Check 2:** Put $x = 3$ and $y = 1$ into the printed expression, and into the factorised form. $4 \times 3^2 - 25 \times 1^2 = 36 - 25 = 11$ and $(2 \times 3 + 5) \times (2 \times 3 - 5) = 11 \times 1 = 11$

✓ **Check 3:** Put $x = 4$ into the original product $4x(x + 3)(2x - 5)$, and into the cubic. $4 \times 4 \times 7 \times 3 = 336$ and $8 \times 64 + 4 \times 16 - 60 \times 4 = 336$

✓ **Check 4:** Run part (b) backwards: take out the common factor $4x$, then factorise the quadratic that is left inside. $8x^3 + 4x^2 - 60x = 4x(2x^2 + x - 15) = 4x(2x - 5)(x + 3)$, the product the question started from.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) Write it as a difference of two squares	M1	$(2x \pm 5y)(2x \pm 5y)$ or $(2x)^2 - (5y)^2$	✓
(a) The factorised answer	A1	$(2x + 5y)(2x - 5y)$	✓
(a)	Note	Correct answer only scores full marks, unless it comes from obviously incorrect working.	✓
(b) Expand one pair of factors	M1	$4x(x + 3) = 4x^2 + 12x$ or $4x(2x - 5) = 8x^2 - 20x$ or $(x + 3)(2x - 5) = 2x^2 - 5x + 6x - 15$ $(= 2x^2 + x - 15)$. An expansion with only one error. Do not award this mark for $4x^2 + 12x + 8x^2 - 20x$.	✓
(b) Multiply by the factor that is left	M1	$(4x^2 + 12x)(2x - 5) = 8x^3 - 20x^2 + 24x^2 - 60x$ or $(8x^2 - 20x)(x + 3) = 8x^3 + 24x^2 - 20x^2 - 60x$ or $4x(2x^2 - 5x + 6x - 15) = 8x^3 - 20x^2 + 24x^2 - 60x$ or $4x(2x^2 + x - 15) = 8x^3 + 4x^2 - 60x$. Follow through, dependent on the first M1, allowing one further error.	✓
(b) The simplified cubic	A1	$8x^3 + 4x^2 - 60x$. Working required. Correct answer only, dependent on the first M1. Terms may be in any order but must be simplified.	✓

Step	Mark	Description	Got it?
(b) Alternative to the two method marks above	**M2**	For 3 terms, out of a maximum of 4 terms, from $8x^3 - 20x^2 + 24x^2 - 60x$. If not M2, then M1 for 2 correct out of a maximum of 4.	✓
(b)	**Note**	Ignore subsequent working after a correct factorisation: $8x^3 + 4x^2 - 60x$ must be seen previously to award 3 marks, for example $4(2x^3 + x^2 - 15x)$ or $x(8x^2 + 4x - 60)$. Do not ignore subsequent working after an incorrect simplification, or after further incorrect work following $8x^3 + 4x^2 - 60x$: for example $8x^3 + 4x^2 - 60x = 2x^3 + x^2 - 15x$ gets M2A0.	✓

Full marks: 5/5

Question 14 - Exam Solution

Understanding the Question

Given

Tin A: 5 red counters and 4 green counters
Tin B: 7 red counters and 3 green counters
The jar: the probability of taking a green counter is $\frac{2}{11}$
One counter is taken at random from each container, so the three choices are independent

Find

(a) the six missing probabilities on the tree diagram (b) the probability of two red counters (c) the probability of more red counters than green counters when one counter is taken from tin A, one from tin B and one from the jar

Plan the Solution

- Write each colour as a fraction of the counters in that tin, then fill the six branches in.
- For two red counters, multiply along the pair of red branches.
- For the jar, take the green probability away from 1 to get the red probability.
- More red than green out of three counters means two red or three red, so list those four outcomes, work each one out, and add them.

Worked Solution [7 marks]

> Rule - Tree diagrams: multiply the probabilities along a path to find the probability of that path, then add the probabilities of the separate paths that make up the event. The two branches leaving any one point always add to 1.

Step 1: The two branches for tin A

$5+4=9$

$\frac{5}{9}$ red $\qquad \frac{4}{9}$ green

(Reason: Tin A holds 9 counters altogether, so 5 of the 9 are red and 4 of the 9 are green. These two go on the first pair of branches.)

Step 2: The four branches for tin B

$7+3=10$

$\frac{7}{10}$ red $\qquad \frac{3}{10}$ green

(Reason: Tin B is untouched by what came out of tin A, so the same pair of probabilities goes on both of the second-stage pairs. That completes part (a).)

Step 3: Part (b), two red counters

$$\frac{5}{9}\times\frac{7}{10}=\frac{35}{90}=\frac{7}{18}$$

(Reason: Two red counters is the single path red then red, so multiply along it. Cancelling $\frac{35}{90}$ by 5 gives $\frac{7}{18}$.)

Step 4: The jar's red probability

$$1-\frac{2}{11}=\frac{9}{11}$$

(Reason: The jar holds red counters and green counters only, so the two probabilities add to 1. The question gives the green one, so the red one is what is left.)

Step 5: Which outcomes have more red than green

RRR $\qquad$ RRG $\qquad$ RGR $\qquad$ GRR

(Reason: Three counters are taken, so more red than green means either three red or exactly two red. Writing the colours in the order tin A, tin B, jar, that is one all-red outcome and three outcomes with a single green, which is 4 paths in all.)

Step 6: Work out each of the four paths

$$\frac{5}{9} \times \frac{7}{10} \times \frac{9}{11} = \frac{315}{990}$$
$$\frac{5}{9} \times \frac{7}{10} \times \frac{2}{11} = \frac{70}{990}$$
$$\frac{5}{9} \times \frac{3}{10} \times \frac{9}{11} = \frac{135}{990}$$
$$\frac{4}{9} \times \frac{7}{10} \times \frac{9}{11} = \frac{252}{990}$$

(Reason: Each path is multiplied straight through, in the order RRR, RRG, RGR, GRR. The denominators are always $9 \times 10 \times 11$, so writing every path over 990 makes the next step a single addition.)

Step 7: Add the four paths

$$\frac{315}{990} + \frac{70}{990} + \frac{135}{990} + \frac{252}{990} = \frac{772}{990}$$
$$\frac{772}{990} = \frac{386}{495}$$

(Reason: The four paths cannot happen together, so their probabilities add. Both 772 and 990 are even, so the fraction cancels by 2.)

(a) $\frac{5}{9}$ **and** $\frac{4}{9}$ **for tin** A**, then** $\frac{7}{10}$ **and** $\frac{3}{10}$ **on each tin** B **pair**

(b) $\frac{7}{18}$

(c) $\frac{386}{495}$

Verification

✓ **Check 1:** The two branches leaving any one point must add to 1, so test each pair on the completed diagram. $\frac{5}{9} + \frac{4}{9} = 1$ and $\frac{7}{10} + \frac{3}{10} = 1$

✓ **Check 2:** Do part (b) the other way round. The three paths that are not two reds come to $\frac{15}{90}$, $\frac{28}{90}$ and $\frac{12}{90}$, which total $\frac{55}{90}$. $1 - \frac{55}{90} = \frac{35}{90} = \frac{7}{18}$

✓ **Check 3:** Do part (c) the other way round. The four outcomes with more green than red are RGG, GRG, GGR and GGG, and over 990 they come to $30 + 56 + 108 + 24 = 218$. $1 - \frac{218}{990} = \frac{772}{990} = \frac{386}{495}$

✓ **Check 4:** All eight outcomes of the three-stage tree must total 1. Over 990 their numerators are $315+70+135+30+252+56+108+24=990$. $\frac{990}{990}=1$, and as a decimal the answer to part (c) is $\frac{386}{495}=0.7798$ to 4 decimal places, which is a sensible size for an event that is more likely than not.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $\frac{5}{9}$ and $\frac{4}{9}$ on the tin A pair; $\frac{7}{10}$ and $\frac{3}{10}$ on each tin B pair	**B2**	for all 3 correct pairs of probabilities on the correct branches	✓
(a) 1 or 2 of the three pairs completed correctly	**(B1)**	for 1 or 2 correct pairs of probabilities on the correct branches	✓
(a) the forms accepted on a branch	**Note**	Accept decimals or percentages rounded or truncated to at least 2sf. NB $\frac{5}{9}=0.55(55\ldots)$ and $\frac{4}{9}=0.44(44\ldots)$	✓
(b) $\frac{5}{9}\times\frac{7}{10}$ or $1-\left(\frac{5}{9}\times\frac{3}{10}+\frac{4}{9}\times\frac{7}{10}+\frac{4}{9}\times\frac{3}{10}\right)$	**M1ft**	ft diagram, oe. Allow ft their tree diagram provided the relevant probabilities are less than 1 in each case	✓
(b) $\frac{7}{18}$	**A1ft**	ft diagram, oe fraction, decimal or percentage. NB $\frac{7}{18}=\frac{35}{90}=0.38(88\ldots)$. For A1, allow decimals or percentages that round or truncate correctly to at least 2sf. ISW any attempt to convert to other form once correct probability seen	✓
(b) an answer written down with no working	**Note**	Correct answer only scores full marks (unless from obviously	✓

Step	Mark	Description	Got it?
		incorrect working)	
(c) (RRR =) $\frac{5}{9} \times \frac{7}{10} \times \left(1 - \frac{2}{11}\right)$ $\left(= \frac{315}{990} = \frac{7}{22}\right)$ or (RRG =) $\frac{5}{9} \times \frac{7}{10} \times \frac{2}{11}$ $\left(= \frac{70}{990} = \frac{7}{99}\right)$ or (RGR =) $\frac{5}{9} \times \frac{3}{10} \times \left(1 - \frac{2}{11}\right)$ $\left(= \frac{135}{990} = \frac{3}{22}\right)$ or (GRR =) $\frac{4}{9} \times \frac{7}{10} \times \left(1 - \frac{2}{11}\right)$ $\left(= \frac{252}{990} = \frac{14}{55}\right)$ OR (GGG =) $\frac{4}{9} \times \frac{3}{10} \times \frac{2}{11}$ $\left(= \frac{24}{990} = \frac{4}{165}\right)$ or (GGR =) $\frac{4}{9} \times \frac{3}{10} \times \left(1 - \frac{2}{11}\right)$ $\left(= \frac{108}{990} = \frac{6}{55}\right)$ or (GRG =) $\frac{4}{9} \times \frac{7}{10} \times \frac{2}{11}$ $\left(= \frac{56}{990} = \frac{28}{495}\right)$ or (RGG =) $\frac{5}{9} \times \frac{3}{10} \times \frac{2}{11}$ $\left(= \frac{30}{990} = \frac{1}{33}\right)$	M1ft	ft diagram, for a correct calculation to find the probability of one relevant outcome, eg RRR or RRG or RGR or GRR, OR eg GGG or GGR or GRG or RGG. Allow ft their tree diagram provided the relevant probabilities are less than 1 in each case	✓

Step	Mark	Description	Got it?
(c) (RRR =) $\left[\frac{7}{18}\right] \times \left(1 - \frac{2}{11}\right)$ and (RRG =) $\left[\frac{7}{18}\right] \times \frac{2}{11}$	**Note**	May see RRR or RRG found using their answer to part (b), where $\left[\frac{7}{18}\right]$ stands for their answer to part (b) and must be less than 1	✓
(c) $"\frac{7}{22}" + "\frac{7}{99}" + "\frac{3}{22}" + "\frac{14}{55}"$ oe OR $1 - ("\frac{4}{165}" + "\frac{6}{55}" + "\frac{28}{495}" + "\frac{1}{33}")$	**M1ft**	ft diagram, for a method to find the probability required. Condone one error in one of the four relevant outcomes or omission of one outcome. The quotation marks mean the candidate's own values, taken from their own tree diagram, may be used	✓
(c) $\frac{5}{9} \times \frac{7}{10} \left(= \frac{7}{18}\right)$	**Note**	Note that P(RR) is P(RRR) added to P(RRG), so $\frac{5}{9} \times \frac{7}{10}$ may be seen in place of $"\frac{7}{22}" + "\frac{7}{99}"$ for this mark, and similarly for P(GG)	✓
(c) $\frac{386}{495}$	**A1ft**	ft diagram, correct probability, oe fraction, decimal or percentage. NB $\frac{386}{495} = \frac{772}{990} = 0.77(979\ldots)$. For A1, allow decimals or percentages that round or truncate correctly to at least 2sf. ISW any attempt to convert to other form once correct probability seen	✓
(c) an answer written down with no working	**Note**	Correct answer only scores full marks (unless from obviously incorrect working)	✓

Full marks: 7/7

Question 15 - Exam Solution

Understanding the Question

Given

A, B and C lie on a circle with centre O
Angle $BAO = 36^\circ$
Angle $BCO = 21^\circ$
$OA = OB = OC$, because all three are radii of the same circle

Find

The size of angle ACO

Plan the Solution

- OB splits the figure into three triangles that meet at O. Every one of them has two radii for sides, so every one of them is isosceles.
- Use the equal base angles to write angle ABO and angle CBO, then add them to get angle ABC.
- Double angle ABC to get angle AOC, the angle at the centre standing on the same arc.
- Finish inside triangle OAC, where the two base angles share whatever is left of 180°.

Worked Solution [3 marks]

> Rule - Circle angles: two radii are equal, so the triangle they make is isosceles and its base angles are equal; and the angle at the centre is twice the angle at the circumference standing on the same arc, $AOC = 2 \times ABC$.

Step 1: Name the equal sides

$OA = OB = OC$

(Reason: Every radius of a circle is the same length, so triangle OAB, triangle OBC and triangle OAC are all isosceles. That single fact is what the two marked angles are there to be used with, and nothing else in the question is needed.)

Step 2: The base angles of the two triangles the question marks

$ABO = BAO = 36^\circ$
$CBO = BCO = 21^\circ$

(Reason: In triangle OAB the equal sides are OA and OB, so the angles opposite them, at A and at B, are equal. Triangle OBC works the same way, which turns the 21° at C into an angle at B.)

Step 3: Add them to get angle ABC

$ABC = ABO + CBO$
$36 + 21 = 57$

(Reason: O lies inside triangle ABC, so OB runs across the inside of angle ABC and cuts it into exactly the two pieces found in step 2.)

Step 4: Turn it into the angle at the centre on arc AC

$AOC = 2 \times ABC$

$2 \times 57 = 114$

(Reason: Angle ABC stands at the circumference and angle AOC stands at the centre, and both stand on the same arc AC, so the one at the centre is twice the one at the circumference.)

Step 5: Finish inside the isosceles triangle OAC

$$ACO = \frac{180 - AOC}{2}$$

$$\frac{180 - 114}{2} = 33$$

(Reason: The three angles of triangle OAC add to $180°$. Taking the angle at the centre away leaves the two base angles together, and $OA = OC$ makes those two equal, so one of them is half of what is left.)

$$ACO = 33°$$

Verification

✓ **Check 1:** Put the answer back into triangle ABC and test its angle sum. Angle BAC is $36 + 33$, angle BCA is $21 + 33$, and angle ABC is 57 from step 3.
$69 + 54 + 57 = 180$

✓ **Check 2:** The three angles at O must fill a full turn. Each one is the apex of its own isosceles triangle, so they are $180 - 2 \times 36$, $180 - 2 \times 21$ and $180 - 2 \times 33$.
$108 + 138 + 114 = 360$

✓ **Check 3:** Reach the answer a completely different way, with the tangent at A. The radius meets a tangent at a right angle, so the angle between that tangent and AB is $90 - 36 = 54$, and the alternate segment theorem makes that equal to angle ACB. Angle ACO is then what is left after angle BCO is taken off. $54 - 21 = 33$, the same answer, from a route that never uses the angle at the centre

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$ABC = 21 + 36$ $(= 57)$ or angle $ABO = 36$ and angle $CBO = 21$ and	M1	for a method to find one of the angles on the scheme; for this mark, values and calculations must be linked to the correct angle by	✓

Step	Mark	Description	Got it?
$21 + 36\ (= 57)$ or (reflex) AOC = $360 - (21 + 21 + 36 + 36)\ (= 246)$ or (obtuse) AOC = $2 \times (21 + 36)\ (= 114)$ or BAX = $90 - 36\ (= 54)$ with the tangent at A drawn oe or BCY = $90 - 21\ (= 69)$ with the tangent at C drawn oe or angle ADB = $180 - 90 - 36\ (= 54)$ OR eg x = angle ACO and y = angle ABC and $x + x + 2y = 180$ oe and $y + (x + 36) + (x + 21) = 180$ oe or eg $180 - 2x = 2(180 - 2x - 21 - 36)$		notation or by being marked on the diagram. May also be awarded for a correct method to set up and solve an equation to find one of these angles; the variable must be clearly defined, where X is a point on the tangent at A, where Y is a point on the tangent at C, where AD is a diameter. OR for any correct pair of simultaneous equations with clearly defined variables one of which must be angle ACO, or any correct equation in terms of ACO only	
$(ACO =) \dfrac{180 - 2 \times 57}{2}$ or $(ACO =)$ $\dfrac{180 - (360 - 246)}{2}$ or $(ACO =)$ $\dfrac{180 - 57 - 21 - 36}{2}$ or $(ACO =)\ 54 - 21$ or $(ACO =)\ 69 - 36$ or $(ACO =)\ 90 - 57$ OR eg $180 - 2x = 2(180 - 2x - 21 - 36) \implies x = \ldots$	**M1**	for a complete method to find angle ACO OR forms a correct equation in terms of ACO only and solves to get a value (condone arithmetic errors). Implies the 1st M mark (provided no incorrect working seen). The printed scheme puts 57, 246, 54 and 69 in quotation marks in this row, which means the candidate's own earlier value may be used in place of each of them	✓

Step	Mark	Description	Got it?
33	A1	Correct answer only scores full marks (unless from obviously incorrect working)	✓

Full marks: 3/3

Question 16 - Exam Solution

Understanding the Question

Given

The fraction $\dfrac{4}{3\sqrt{5}+7}$, which has a surd in its denominator.

The form the answer must take: $a-\sqrt{b}$, with a and b both integers.

Find

The values of a and b, with every stage of the working shown.

Plan the Solution

- Multiply the numerator and the denominator by the conjugate $3\sqrt{5}-7$. That fraction is worth 1, so it changes how the expression looks and not what it is worth.
- Expand the numerator, and expand the denominator as a difference of two squares so that the surd terms cancel and a whole number is left below the line.
- Divide each term of the numerator by that whole number.
- Turn the multiple of a surd into a single surd, so the answer reads $a-\sqrt{b}$.

Worked Solution [3 marks]

> Rationalising a denominator: a denominator of the form $p\sqrt{q}+r$ is cleared by multiplying the numerator and the denominator by its conjugate $p\sqrt{q}-r$. The denominator becomes $(p\sqrt{q})^2-r^2$, and both of those are whole numbers, so no surd is left below the line. A multiple of a surd is then written as one surd using $k\sqrt{m}=\sqrt{k^2m}$.

Step 1: Multiply the numerator and the denominator by the conjugate

$$\frac{4}{3\sqrt{5}+7} \times \frac{3\sqrt{5}-7}{3\sqrt{5}-7}$$

(Reason: The conjugate of $3\sqrt{5}+7$ is $3\sqrt{5}-7$ - the same two terms with the sign between them reversed. The fraction $\frac{3\sqrt{5}-7}{3\sqrt{5}-7}$ is equal to 1, so multiplying by it leaves the value of the expression unchanged.)

Step 2: Expand the numerator

$$4(3\sqrt{5}-7) = 12\sqrt{5}-28$$

(Reason: Multiply each term inside the bracket by 4: $4 \times 3\sqrt{5} = 12\sqrt{5}$ and $4 \times 7 = 28$.)

Step 3: Expand the denominator

$$(3\sqrt{5}+7)(3\sqrt{5}-7) = (3\sqrt{5})^2 - 7^2$$

$$(3\sqrt{5})^2 = 3^2 \times 5 = 45$$

$$45 - 49 = -4$$

(Reason: The two brackets are a difference of two squares, so the two $\sqrt{5}$ terms cancel and no surd survives. Squaring $3\sqrt{5}$ squares the 3 as well as the $\sqrt{5}$, and $7^2 = 49$, so the denominator is left as a whole number. Expanding all four terms instead is equally valid, as long as every one of them is correct.)

Step 4: Divide the numerator by the denominator

$$\frac{12\sqrt{5}-28}{-4} = -3\sqrt{5}+7$$

$$-3\sqrt{5}+7 = 7-3\sqrt{5}$$

(Reason: Divide both terms of the numerator by -4: $12\sqrt{5}$ gives $-3\sqrt{5}$ and -28 gives 7. Writing the whole number first puts the expression the right way round for the form the question asks for.)

Step 5: Write the multiple of a surd as a single surd

$$3\sqrt{5} = \sqrt{3^2 \times 5} = \sqrt{45}$$

$$7 - 3\sqrt{5} = 7 - \sqrt{45}$$

(Reason: Take the 3 inside the square root by squaring it. The expression now reads $a - \sqrt{b}$ with $a = 7$ and $b = 45$, and both of those are integers, which is what the question asked to be shown.)

$$\frac{4}{3\sqrt{5}+7} = 7 - \sqrt{45}$$

so $a = 7$ and $b = 45$, both integers

Verification

✓ **Check 1:** Work both forms out as decimals on the calculator and compare them. The original fraction gives 0.2917960675 and $7 - \sqrt{45}$ gives 0.2917960675, agreeing to ten decimal places, so the two forms are the same number.

✓ **Check 2:** Work backwards. If the answer is right, multiplying it by the denominator we started with must give the numerator we started with: $(7 - 3\sqrt{5})(7 + 3\sqrt{5}) = 49 - 45$. That comes to 4, which is the numerator of the original fraction, so the rationalising was done correctly.

✓ **Check 3:** Check the form itself. The question demands $a - \sqrt{b}$ with a and b integers, and a subtraction sign between them. $7 - \sqrt{45}$ has $a = 7$ and $b = 45$, both whole numbers, and a minus sign between the two terms, so it is in the required form.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\frac{4}{3\sqrt{5}+7} \times \frac{3\sqrt{5}-7}{3\sqrt{5}-7}$ or $\frac{4}{3\sqrt{5}+7} \times \frac{-3\sqrt{5}+7}{-3\sqrt{5}+7}$ oe	M1	for multiplying the numerator and denominator by $3\sqrt{5} - 7$ or $-3\sqrt{5} + 7$ (may be implied)	✓
eg $\frac{4(3\sqrt{5}-7)}{45 - 21\sqrt{5} + 21\sqrt{5} - 49}$ or $\frac{4(3\sqrt{5}-7)}{45 - 7^2}$ or $\frac{12\sqrt{5}-28}{45 - 21\sqrt{5} + 21\sqrt{5} - 49}$ or $\frac{12\sqrt{5}-28}{45-7^2}$ or $\frac{4(3\sqrt{5}-7)}{45-49}$ or	M1	for expanding the denominator in a correct fraction the denominator may be 4 terms which all need to be correct $\frac{4}{3\sqrt{5}+7} \times \frac{3\sqrt{5}-7}{3\sqrt{5}-7} = 7 - 3\sqrt{5}$	✓

Step	Mark	Description	Got it?
$\frac{4(3\sqrt{5}-7)}{-4}$ or $\frac{12\sqrt{5}-28}{45-49}$ or $\frac{12\sqrt{5}-28}{-4}$		scores M1M0 Implies the 1st mark	
Working required $7-\sqrt{45}$	**A1 dep on M2**	dep on M2	✓
Working required	**Note**	The answer column of the scheme is marked $7-\sqrt{45}$ with working required, so the rationalising must be seen for the accuracy mark. The two special cases below say what an unsupported answer is worth instead.	✓
$7-\sqrt{45}$ written down with no method mark earned	**SC B1**	SC B1 for answer $7-\sqrt{45}$ with no method marks awarded	✓
$7-\sqrt{45}$ with the 1st M1 earned but not the 2nd	**SC B2**	SC B2 for $7-\sqrt{45}$ if you would award the 1st M1 but not the 2nd M1 (total 2 marks)	✓

Full marks: 3/3

Question 17 - Exam Solution

Understanding the Question

Given

The formula $p = \frac{8k^2+5}{7-3k^2}$, which gives p in terms of k.

Find

The same formula rearranged so that k stands alone on one side, written in terms of p. Only k^2 can be reached directly, so expect a $\pm$ in the answer.

k appears twice, once above the line and once below it, and every time it appears it is squared.

Plan the Solution

- Multiply both sides by $7 - 3k^2$ to clear the fraction, then expand the bracket.
- Collect every term containing k^2 on one side and every term without it on the other.
- Factorise, so that k^2 appears once only, then divide by the bracket.
- Square-root both sides, keeping both the positive and the negative root.

Worked Solution [4 marks]

> Changing the subject when the new subject appears more than once: clear the fraction first, then gather every term containing the new subject on one side and everything else on the other, factorise so that the new subject appears once only, divide by the bracket, and finally undo the power. Undoing a square gives two roots, so a $\pm$ is kept.

Step 1: Multiply both sides by the denominator

$$p(7 - 3k^2) = 8k^2 + 5$$
$$7p - 3k^2p = 8k^2 + 5$$

(Reason: Multiplying both sides by $7 - 3k^2$ clears the fraction. Expand the left-hand side by multiplying each term inside the bracket by p: $p \times 7 = 7p$ and $p \times (-3k^2) = -3k^2p$. Multiplying the 7 but leaving the $3k^2$ untouched is the commonest slip here, and expanding both terms correctly is exactly what the first method mark is for.)

Step 2: Collect the k^2 terms on one side

$$7p - 5 = 8k^2 + 3k^2p$$

(Reason: Add $3k^2p$ to both sides and subtract 5 from both sides. Every term containing k^2 is now on the right, and every term without it is on the left. A term changes sign as it crosses the equals sign, so the $+5$ on the right becomes -5 on the left.)

Step 3: Factorise, so that k^2 appears once only

$$7p - 5 = k^2(8 + 3p)$$

(Reason: k^2 is a factor of both $8k^2$ and $3k^2p$, so it comes outside a bracket and leaves $8 + 3p$ inside. This is the step the question turns on: while k^2 is written in two separate terms it cannot be made the subject, and once it is written once it can.)

Step 4: Divide, then take the square root

$$k^2 = \frac{7p - 5}{8 + 3p}$$

$k = \pm\sqrt{\dfrac{7p - 5}{8 + 3p}}$

(Reason: Divide both sides by $8 + 3p$ to leave k^2 on its own, then square-root both sides. Squaring hides the sign, so undoing it gives two roots, one positive and one negative, and both of them are genuine values of k.)

$$k = \pm\sqrt{\frac{7p - 5}{8 + 3p}}$$

Verification

✓ **Check 1:** Put one value through both formulas. Take $k = 1$. The formula the question gives makes $p = \dfrac{8 \times 1 + 5}{7 - 3 \times 1} = \dfrac{13}{4} = 3.25$. Now feed that value of p into the rearrangement. $\dfrac{7 \times 3.25 - 5}{8 + 3 \times 3.25} = \dfrac{17.75}{17.75} = 1$, and $\sqrt{1} = 1$, which is the value of k it started from.

✓ **Check 2:** Repeat with a value that makes p negative, so the rearrangement is tested below zero as well as above it. Take $k = 2$, which gives $\dfrac{8 \times 4 + 5}{7 - 3 \times 4} = \dfrac{37}{-5} = -7.4$. $\dfrac{7 \times (-7.4) - 5}{8 + 3 \times (-7.4)} = \dfrac{-56.8}{-14.2} = 4$, and $\sqrt{4} = 2$. Putting $k = -2$ into the original formula gives the same $p = -7.4$, which is why the answer keeps both signs.

✓ **Check 3:** Check it with no numbers at all. Put $k^2 = \dfrac{7p - 5}{8 + 3p}$ back into the top and the bottom of the original formula separately, over the common denominator $8 + 3p$. The top becomes $\dfrac{8(7p - 5) + 5(8 + 3p)}{8 + 3p} = \dfrac{71p}{8 + 3p}$ and the bottom becomes $\dfrac{7(8 + 3p) - 3(7p - 5)}{8 + 3p} = \dfrac{71}{8 + 3p}$. Dividing the top by the bottom leaves $\dfrac{71p}{71} = p$, which is the formula the question started from, so the rearrangement holds for every value of p and not just for the two tested above.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$7p - 3k^2p = 8k^2 + 5$	M1	for correctly multiplying both sides by the denominator and expanding the brackets	✓

Step	Mark	Description	Got it?
$7p - 5 = 8k^2 + 3k^2p$ or $-3k^2p - 8k^2 = 5 - 7p$	M1ft	dep on 2 terms in k^2 and 2 other terms for correctly collecting their k^2 terms on one side and their other terms on the other side Note: eg $8k^2 + 3k^2$ does not count as 2 terms in k^2	✓
eg $7p - 5 = k^2(8 + 3p)$ or $k^2(-3p - 8) = 5 - 7p$	M1ft	dep on previous M1 for correctly factorising for k^2 or $-k^2$ in their equation	✓
$k = (\pm)\sqrt{\dfrac{7p-5}{8+3p}}$	A1	oe eg $k = (\pm)\sqrt{\dfrac{5-7p}{-3p-8}}$ or $k = (\pm)\left(\dfrac{7p-5}{8+3p}\right)^{\frac{1}{2}}$ or $k = (\pm)\left(\dfrac{7p-5}{8+3p}\right)^{0.5}$ (condone omission of $\pm$) NB: to award A1 we must see $k = (\pm)\sqrt{\dfrac{7p-5}{8+3p}}$ in working if $(\pm)\sqrt{\dfrac{7p-5}{8+3p}}$ alone is given as an answer	✓
Working not required	Note	The working column of the printed scheme reads: Working not required, so correct answer scores full marks (unless from obvious incorrect working).	✓

Full marks: 4/4

Question 18 - Exam Solution

Understanding the Question

Given

Find

A triangle OAB with $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 4\mathbf{b}$
P on AB, dividing it so that $AP : PB = 1 : 3$
No lengths and no angles are given, so every step is vector algebra.

(a) $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ (b) $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, simplified

Plan the Solution

- Only $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are known, so travel from A to B the long way, back through O.
- Turn the ratio into a fraction: $1 : 3$ cuts AB into equal parts, and AP is one of them.
- Reach P as $\overrightarrow{OA} + \overrightarrow{AP}$, then collect the $\mathbf{a}$ terms and the $\mathbf{b}$ terms.

Worked Solution [3 marks]

> Rule - Triangle law: $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$. And a point dividing AB in the ratio $m : n$ sits $\dfrac{m}{m+n}$ of the way from A to B.

Step 1: Go from A to B through O

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$\overrightarrow{AB} = -4\mathbf{a} + 4\mathbf{b}$$

(Reason: Reversing a vector changes its sign, so $\overrightarrow{AO} = -\overrightarrow{OA} = -4\mathbf{a}$. That is part (a). Factorised as $4(\mathbf{b} - \mathbf{a})$ it says the same thing.)

Step 2: Turn $AP : PB = 1 : 3$ into a fraction of $\overrightarrow{AB}$

$$1 + 3 = 4$$
$$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$$

(Reason: The two numbers in the ratio count parts, so AB is cut into 4 equal parts and AP is 1 of them. The 3 is the other piece, not the number of parts.)

Step 3: Build $\overrightarrow{OP}$ starting from O

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$
$$\overrightarrow{OP} = 4\mathbf{a} + \frac{1}{4}(-4\mathbf{a} + 4\mathbf{b})$$

(Reason: Travel out along $\overrightarrow{OA}$, then a quarter of the way along $\overrightarrow{AB}$. This substitution alone earns the method mark.)

Step 4: Collect the terms

$\overrightarrow{OP} = 4\mathbf{a} - \mathbf{a} + \mathbf{b}$

$\overrightarrow{OP} = 3\mathbf{a} + \mathbf{b}$

(Reason: A quarter of $-4\mathbf{a}$ is $-\mathbf{a}$, and a quarter of $4\mathbf{b}$ is $\mathbf{b}$. Then $4\mathbf{a} - \mathbf{a} = 3\mathbf{a}$, and nothing else will combine.)

(a) $\overrightarrow{AB} = -4\mathbf{a} + 4\mathbf{b}$

(b) $\overrightarrow{OP} = 3\mathbf{a} + \mathbf{b}$

Verification

✓ **Check 1:** Follow $\overrightarrow{OA}$ and then $\overrightarrow{AB}$. Together they have to land on B.
$4\mathbf{a} + (-4\mathbf{a} + 4\mathbf{b}) = 4\mathbf{b} = \overrightarrow{OB}$

✓ **Check 2:** Reach P from the far end instead, using $\overrightarrow{PB} = \frac{3}{4}\overrightarrow{AB}$.
$\overrightarrow{OB} - \overrightarrow{PB} = 4\mathbf{b} - \frac{3}{4}(-4\mathbf{a} + 4\mathbf{b}) = 3\mathbf{a} + \mathbf{b}$

✓ **Check 3:** Put numbers on the vectors. With **a** one unit right and **b** one unit up, A is at $(4, 0)$ and B is at $(0, 4)$. A quarter of the way from A to B is the point $(3, 1)$. $3\mathbf{a} + \mathbf{b}$ points to $(3, 1)$, the same point

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $-4\mathbf{a} + 4\mathbf{b}$	**B1**	oe eg $4(\mathbf{b} - \mathbf{a})$	✓
(b) $4\mathbf{a} + \frac{1}{4}(-4\mathbf{a} + 4\mathbf{b})$ oe or $4\mathbf{a} + \frac{1}{4}[\overrightarrow{AB}]$ or $4\mathbf{b} - \frac{3}{4}(-4\mathbf{a} + 4\mathbf{b})$ oe or $4\mathbf{b} - \frac{3}{4}[\overrightarrow{AB}]$	**M1ft**	for a correct expression ft their (a), where $[\overrightarrow{AB}]$ is their answer to (a) of the form $m\mathbf{a} + n\mathbf{b}$, $m, n \neq 0$	✓
$3\mathbf{a} + \mathbf{b}$	**A1**	allow $\mathbf{b} + 3\mathbf{a}$	✓

Step	Mark	Description	Got it?
Part (b), from the printed scheme	**Note**	Correct answer only scores full marks (unless from obviously incorrect working).	✓

Full marks: 3/3

Question 19 - Exam Solution

Understanding the Question

Given

$f(x) = 3x - 4$ where $x > 2$

$g(x) = \dfrac{x}{2x+1}$

One linear function and one algebraic fraction.

Find

(a) the value of x that no domain of g may contain (b) $gf(x)$, in its simplest form

Plan the Solution

- For (a), a fraction has no value when its denominator is zero, so solve $2x + 1 = 0$.
- For (b), $gf(x)$ means do f first and then g, so write $3x - 4$ wherever an x appears in the formula for g.
- Expand the bracket in the denominator, collect the constants, then check whether anything cancels.

Worked Solution [3 marks]

> Rule - Composite functions: $gf(x) = g(f(x))$, so the inner function f is applied first. And a fraction has no value wherever its denominator is 0.

Step 1: find where the denominator of g is zero

$2x + 1 = 0$

$2x = -1$

$x = -\dfrac{1}{2}$

(Reason: (Reason: dividing by 0 has no value, so this one value of x has to be left out of every domain of g. Every other value of x is allowed, which is why the answer is a value and not an inequality.))

Step 2: put the whole of $\mathrm{f}(x)$ into g

$$\mathrm{gf}(x) = \mathrm{g}(3x - 4)$$

$$\mathrm{gf}(x) = \frac{3x - 4}{2(3x - 4) + 1}$$

(Reason: (Reason: gf means do f first, so every x in the formula for g is replaced by the whole of $3x - 4$, brackets and all. This unsimplified fraction already earns the method mark.))

Step 3: expand the bracket and collect the denominator

$$2(3x - 4) + 1 = 6x - 8 + 1 = 6x - 7$$

(Reason: (Reason: multiply both terms inside the bracket by 2, then add the 1. Only the denominator changes here; the numerator is left exactly as it is.))

Step 4: check the fraction is already in its simplest form

$$\mathrm{gf}(x) = \frac{3x - 4}{6x - 7}$$

(Reason: (Reason: $3x - 4$ and $6x - 7$ share no common factor, and one is not a multiple of the other, so nothing cancels and the fraction is as simple as it goes. Multiplying top and bottom by -1 gives $\frac{4 - 3x}{7 - 6x}$, which is the same answer written another way.))

(a) $-\dfrac{1}{2}$

(b) $\mathrm{gf}(x) = \dfrac{3x - 4}{6x - 7}$

Verification

✓ **Check 1:** Put $x = -\frac{1}{2}$ into the denominator of g. $2 \times \left(-\frac{1}{2}\right) + 1 = -1 + 1 = 0$, so g has no value there, while every other x gives a denominator that is not zero.

✓ **Check 2:** Take $x = 3$, which is inside the domain of f. Then $3 \times 3 - 4 = 5$, so the composite is $\mathrm{g}(5)$. Working forwards, $\frac{5}{2 \times 5 + 1} = \frac{5}{11}$. The answer gives $\frac{3 \times 3 - 4}{6 \times 3 - 7} = \frac{5}{11}$, the same value.

✓ **Check 3:** The new denominator is zero when $6x - 7 = 0$, that is at $x = \frac{7}{6}$. $\frac{7}{6}$ is smaller than 2, so no value in the given domain $x > 2$ is lost. This agrees with part (a): $\mathrm{f}(x) > 2$

for every $x > 2$, so $\mathrm{f}(x)$ never lands on $-\dfrac{1}{2}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(a) $-\dfrac{1}{2}$	**B1**	oe. Accept $x = -\dfrac{1}{2}$, accept $x \neq -\dfrac{1}{2}$. Do not allow inequalities, eg $x > -\dfrac{1}{2}$ or $x \leq -\dfrac{1}{2}$.	✓
(b) $\dfrac{3x-4}{2(3x-4)+1}$	**M1**	for a correct unsimplified expression for $\mathrm{gf}(x)$	✓
(b) $\dfrac{3x-4}{6x-7}$	**A1**	oe eg $\dfrac{4-3x}{7-6x}$. Correct answer seen followed by incorrect subsequent working scores M1A0. Correct answer only scores full marks (unless from obviously incorrect working).	✓

Full marks: 3/3

Question 20 - Exam Solution

Understanding the Question

Given

A histogram of the weights, in grams, of some pebbles. The frequency density axis carries no numbers, so its scale has to be worked out.
75 pebbles weigh less than 100 grams.
The bars meet at 0, 100, 200, 450, 500 and 700 grams.

Find

An estimate for the probability that a pebble chosen at random weighs between 400 grams and 600 grams.

Plan the Solution

- The first bar is the only one whose frequency is given, so use it to fix the scale: its class width is 100 and its frequency is 75.

- Read the height of every bar off that scale and turn each one into a frequency, to get the total number of pebbles.
- Do the same for the strip between 400 grams and 600 grams, which cuts two of the bars part way.
- The probability is that strip's total over the whole total.

Worked Solution [4 marks]

> On a histogram the frequency of a class is the AREA of its bar, not its height:
> $\text{frequency} = \text{frequency density} \times \text{class width}.$

Step 1: Use the first bar to fix the frequency density scale

$$\frac{75}{100} = 0.75$$

$$\frac{0.75}{3} = 0.25$$

(Reason: The first bar covers 0 to 100 grams, so its class width is 100, and the question gives its frequency as 75. Frequency density is frequency divided by class width, so that bar has a frequency density of 0.75. It stands 3 large squares high, so one large square up the axis is 0.25.)

Step 2: Work out how many pebbles there are altogether

$$0.75 \times 100 = 75$$

$$2.25 \times 100 = 225$$

$$1 \times 250 = 250$$

$$2.5 \times 50 = 125$$

$$0.25 \times 200 = 50$$

$$75 + 225 + 250 + 125 + 50 = 725$$

(Reason: The five bars are 3, 9, 4, 10 and 1 large squares high, so at 0.25 a square their frequency densities are 0.75, 2.25, 1, 2.5 and 0.25. Multiply each one by its own class width to get its frequency, then add. The first bar gives back the 75 the question states, which is the scale confirming itself.)

Step 3: Work out how many pebbles weigh between 400 grams and 600 grams

$$1 \times 50 = 50$$

$$2.5 \times 50 = 125$$

$$0.25 \times 100 = 25$$

$$50 + 125 + 25 = 200$$

(Reason: The strip runs across three bars. It takes the last 50 grams of the third bar (400 to 450), the whole of the fourth bar (450 to 500, also 50 grams wide) and the first 100 grams of the fifth bar (500 to 600). Only the part of a bar inside the strip counts, so use each piece's own width, not the whole class width.)

Step 4: Write the estimate as a probability

$$\frac{200}{725} = \frac{8}{29}$$

(Reason: The probability is the number of pebbles in the strip out of the number of pebbles altogether. Both 200 and 725 divide by 25, which cancels the fraction to $\frac{8}{29}$, or 0.28 to 2 significant figures.)

$$\frac{200}{725} = \frac{8}{29}$$

Verification

✓ **Check 1:** Count large squares instead. One large square is 50 grams wide and 0.25 high, so it stands for $50 \times 0.25 = 12.5$ pebbles. The bars cover $6 + 18 + 20 + 10 + 4 = 58$ large squares, of which $4 + 10 + 2 = 16$ lie between 400 and 600 grams.

$58 \times 12.5 = 725$ and $16 \times 12.5 = 200$, so the probability is $\frac{16}{58} = \frac{8}{29}$

✓ **Check 2:** Count small squares, which is a finer measure of the same areas. One small square is 10 grams wide and 0.05 high, so it stands for $10 \times 0.05 = 0.5$ pebbles. The bars cover $150 + 450 + 500 + 250 + 100 = 1450$ small squares, and $100 + 250 + 50 = 400$ of them lie in the strip. $1450 \times 0.5 = 725$ and $400 \times 0.5 = 200$, giving $\frac{400}{1450} = \frac{8}{29}$

✓ **Check 3:** Is the size sensible? A quarter of the 58 large squares would be $\frac{58}{4} = 14.5$, and the strip covers 16 of them, so the answer must be a little over a quarter. $\frac{8}{29} = 0.28$ to 2 significant figures, which is a little over 0.25

Mark Scheme Breakdown

Step	Mark	Description	Got it?
(frequency density $=$) $\frac{75}{100}$ ($= 0.75$) or any one correct value marked on the frequency density axis or uses their own linear frequency density scale to find	M1	for a correct calculation of frequency density or a correct value on the frequency density axis or use of their own linear frequency density scale to find the area of at least one of the third, fourth or fifth bars OR a correct measure of scale	✓

Step	Mark	Description	Got it?
the area of at least one of the third, fourth or fifth bars OR eg 1 (large) box / 1 (large) square $= 12.5$ pebbles oe or 5 (sml) squares $= 2.5$ pebbles or 1 (sml) square / 1 box $= 0.5$ pebbles		Implied by a correct frequency for the third (250), fourth (125) or fifth (50) bar	
eg using frequency densities $75 + (2.25 \times 100) + (1 \times 250) + (2.5 \times 50) + (0.25 \times 200)$ $(= 75 + 225 + 250 + 125 + 50 = 725)$ OR eg using a measure of scale (eg large squares) $(6 + 18 + 20 + 10 + 4) \times 12.5$ $(= 725)$ oe OR eg total no. of large squares $= 6 + 18 + 20 + 10 + 4$ $(= 58)$ or total no. of sml squares $= 150 + 450 + 500 + 250 + 100$ $(= 1450)$	**M1**	for a method to work out the total number of pebbles OR the total number of squares. Allow one error in a frequency density value, class width value or number of squares but not an omission. Allow ft their own linear frequency density scale. The frequency density values in the row beside this one are the candidate's own values read off their scale, which is what the printed scheme marks by putting them in quotation marks.	✓
eg using frequency densities $(1 \times 50) + (2.5 \times 50) + (0.25 \times 100)$ $(= 50 + 125 + 25 = 200)$ OR eg using a measure of scale (eg large squares) $(4 + 10 + 2) \times 12.5$ $(= 200)$ oe OR eg no. of large squares $= 4 + 10 + 2$ $(= 16)$	**M1**	for a method to estimate the number of pebbles between 400 g and 600 g OR the total number of squares between 400 g and 600 g Allow one error in a frequency density value, class width value or number of squares but not an omission Allow ft their own linear frequency density scale	✓

Step	Mark	Description	Got it?
or no. of sml squares $= 100 + 250 + 50\ (= 400)$			
$\frac{200}{725}$	**A1**	oe eg $\frac{8}{29}$ or any correct decimal or correct percentage rounded or truncated to 2 sf. NB $\frac{200}{725} = 0.2758\ldots$	✓
Correct answer only scores full marks (unless from obviously incorrect working)	**Note**	A correct answer on its own earns all four marks, unless the working shown with it is obviously incorrect.	✓
Counting squares alone, eg counting large squares $\frac{4+10+2}{6+18+20+10+4}$	**Note**	M3 for any complete method that relies only on counting squares. Allow one error in the number of squares in each of the numerator and the denominator, but do not allow any omissions. A consistent method must be used for the award of more than one mark.	✓

Full marks: 4/4

Question 21 - Exam Solution

Understanding the Question

Given

The inequality $2x^2 - 7x - 15 > 0$
The working has to be algebraic, so the critical values must come from factorising, from the formula or from completing the square.

Find

Every value of x for which $2x^2 - 7x - 15$ is greater than 0

Plan the Solution

- Factorise $2x^2 - 7x - 15$ into two brackets.
- Set each bracket equal to 0 to get the two critical values.

- The x^2 coefficient is positive, so the curve is a U-shape. Test one value from each of the three regions to see which ones sit above the axis.
- Write the answer as two separate inequalities.

Worked Solution [3 marks]

> When $a > 0$, the graph of $y = ax^2 + bx + c$ is a U-shape, so $ax^2 + bx + c$ is negative between the two roots and positive outside them.

Step 1: Factorise the quadratic

$2 \times (-15) = -30$

$3 + (-10) = -7$

$2x^2 + 3x - 10x - 15 = x(2x + 3) - 5(2x + 3)$

$2x^2 - 7x - 15 = (2x + 3)(x - 5)$

(Reason: Multiply the x^2 coefficient by the constant term: $2 \times (-15) = -30$. Two numbers with product -30 and sum -7 are 3 and -10, so the middle term splits into $3x - 10x$ and each pair of terms leaves the same bracket.)

Step 2: Find the critical values

$(2x + 3)(x - 5) = 0$

$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

$x - 5 = 0 \Rightarrow x = 5$

(Reason: A product is 0 only when one of its factors is 0, so each bracket is solved on its own. These two values are the critical values: they are where the curve meets the x-axis, and they are the only places the sign of the expression can change.)

Step 3: Test one value in each region

$2(-2)^2 - 7(-2) - 15 = 8 + 14 - 15 = 7$

$2(0)^2 - 7(0) - 15 = 0 - 0 - 15 = -15$

$2(6)^2 - 7(6) - 15 = 72 - 42 - 15 = 15$

(Reason: The critical values $-\frac{3}{2}$ and 5 cut the number line into three regions. Take $x = -2$ from the left region, $x = 0$ from the middle one and $x = 6$ from the right region. Only the two outer regions give a positive value, which is exactly what a U-shaped curve does.)

Step 4: Write the solution as two inequalities

$x < -\frac{3}{2}$

$x > 5$

(Reason: The solution is two separate stretches of the number line, so it is written as two separate inequalities. A single chain such as $-\frac{3}{2} > x > 5$ earns nothing, because no number is both smaller than $-\frac{3}{2}$ and larger than 5 at once. A comma, the word or, or $\cup$ may be used to link them.)

$$x < -\frac{3}{2} \textbf{ or } x > 5$$

Verification

✓ **Check 1:** Expand the brackets again: $(2x+3)(x-5) = 2x^2 - 10x + 3x - 15$. $2x^2 - 7x - 15$, which is the expression the question gives, so the factorisation is right

✓ **Check 2:** Find the critical values a second way, with the quadratic formula: $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times (-15)}}{2 \times 2} = \dfrac{7 \pm \sqrt{169}}{4}$. $\dfrac{7+13}{4} = 5$ and $\dfrac{7-13}{4} = -\dfrac{3}{2}$, the same two critical values

✓ **Check 3:** Substitute one value from each region into $2x^2 - 7x - 15$ and read off the signs. $x = -2$ gives 7, $x = 0$ gives -15 and $x = 6$ gives 15, so only the two outer regions are positive

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $(2x+3)(x-5)$ oe or $\dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times (-15)}}{2 \times 2}$ oe or $2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] - 15$ oe	M1	for a correct method to find the critical values. Minimum evidence for the quadratic formula is a two-term discriminant, eg $\dfrac{7 \pm \sqrt{49+120}}{4}$ (must have the $\pm$). Allow $(x+1.5)(2x-10)$ as a correct factorisation, but do not allow $\left(x + \frac{3}{2}\right)(x-5)$ unless preceded by division of the quadratic by 2. Allow M1A1 for the correct	✓

Step	Mark	Description	Got it?
		critical values and evidence of another algebraic method that has led to these, eg $x(2x+3)-5(2x+3)$ and the correct critical values, or $2x(x-5)+3(x-5)$ and the correct critical values, or $(2x+3)(2x-10)$ and the correct critical values.	
$(x=)-\frac{3}{2}$ and $(x=)5$	**A1**	dep on M1 for correct critical values oe	✓
$x<-\frac{3}{2}, x>5$ Working required	**A1**	dep on M1 for correct inequalities (must be separate inequalities). Allow interval notation, eg $\left(-\infty,-\frac{3}{2}\right)\cup(5,\infty)$ or $\left(-\infty,-\frac{3}{2}\right),(5,\infty)$ or $]-\infty,-\frac{3}{2}[\cup]5,\infty[$ or $]-\infty,-\frac{3}{2}[,]5,\infty[$. Acceptable notation: allow a comma, a space, the word or, the word and, or $\cup$ to link the two regions. Do not allow as a single inequality $-\frac{3}{2}>x>5$.	✓

Full marks: 3/3

Question 22 - Exam Solution

Understanding the Question

Given

Find

A solid cuboid measuring x cm by x cm by $(15 - 4x)$ cm
Its volume is V cm³, so V depends only on x

The maximum value of V

Plan the Solution

- Multiply the three edge lengths together to write V in terms of x.
- Multiply out, so that every term is a power of x and can be differentiated term by term.
- A maximum is a turning point, so differentiate and solve $\frac{dV}{dx} = 0$.
- Two roots come out. Keep the one that gives a real block, confirm it is a maximum, then substitute it back into V.

Worked Solution [5 marks]

> Rule - Turning points: V has a turning point where $\frac{dV}{dx} = 0$, and that turning point is a maximum when $\frac{d^2V}{dx^2}$ is negative there. Differentiate term by term with $\frac{d}{dx}(ax^n) = nax^{n-1}$.

Step 1: Write the volume in terms of x

$V = x \times x \times (15 - 4x)$
$V = x^2(15 - 4x)$

(Reason: The cross-section is a square of side x cm and the block is $(15 - 4x)$ cm long, so the volume is the three edge lengths multiplied together.)

Step 2: Multiply out to get a cubic in x

$V = 15x^2 - 4x^3$

(Reason: $x^2 \times 15 = 15x^2$ and $x^2 \times (-4x) = -4x^3$. Differentiating is only tidy once every term is a single power of x.)

Step 3: Differentiate

$\frac{dV}{dx} = 30x - 12x^2$

(Reason: Bring each power down and reduce it by one: $15x^2$ gives $2 \times 15x = 30x$, and $-4x^3$ gives $3 \times (-4)x^2 = -12x^2$.)

Step 4: Set the gradient to zero and solve

$30x - 12x^2 = 0$

$6x(5 - 2x) = 0$

$x = 0 \quad \text{or} \quad x = 2.5$

(Reason: At a turning point the graph is momentarily flat, so the gradient is 0. Taking out the common factor $6x$ gives the two roots. $x = 0$ would leave a block with no width at all, so the root that matters here is $x = 2.5$.)

Step 5: Confirm that this root gives the maximum

$$\frac{d^2V}{dx^2} = 30 - 24x$$

$30 - 24 \times 2.5 = -30$

(Reason: Differentiating a second time gives $30 - 24x$. At $x = 2.5$ this is negative, so the gradient is falling as it passes through zero and the turning point is a maximum. At the rejected root $x = 0$ the same expression is $+30$, which is a minimum.)

Step 6: Work out V when $x = 2.5$

$V = 15 \times (2.5)^2 - 4 \times (2.5)^3$

$15 \times 6.25 - 4 \times 15.625 = 93.75 - 62.5 = 31.25$

(Reason: Squaring gives $2.5^2 = 6.25$ and cubing gives $2.5^3 = 15.625$. Substituting both into $15x^2 - 4x^3$ gives the largest volume the block can have.)

$$V = 31.25$$

Verification

✓ **Check 1:** Work out V at $x = 2.4$ and at $x = 2.6$, one on each side of 2.5. Both should come out smaller. $2.4^2 \times (15 - 4 \times 2.4) = 31.104$ and $2.6^2 \times (15 - 4 \times 2.6) = 31.096$, both below 31.25

✓ **Check 2:** At $x = 2.5$ the block measures 2.5 cm by 2.5 cm by 5 cm. Multiply those three edges directly, without using the expanded cubic. $2.5 \times 2.5 \times 5 = 31.25$

✓ **Check 3:** Multiply V by 4 to get $4V = (2x)(2x)(15 - 4x)$. Those three brackets add up to 15 whatever x is, and a product of positive numbers with a fixed sum is largest when they are all equal, which needs $2x = 15 - 4x$, so $x = 2.5$ again. Each bracket is then 5, and $5 \times 5 \times 5 = 125$, so $4V$ can never exceed 125 and V can never exceed $\frac{125}{4}$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$(V =)\ x(x)(15 - 4x) \quad (= 15x^2 - 4x^3)$	M1	for a correct expression for the volume (condone missing $V = \ldots$)	✓
$\left(\frac{dV}{dx} =\right)\ 30x - 12x^2$	M1ft	ft dep on a V of the form $ax^3 + bx^2$ where $a, b \neq 0$, for a two-term derivative with at least one term correct for their V, eg $15(2)x$ or $30x$ or $-3(4)x^2$ or $-12x^2$	✓
$30x - 12x^2 = 0 \implies x = \ldots$ or $(x =)\ 2.5$ oe	M1ft	dep on previous M mark, for equating their two-term first derivative to 0 and solving to get a value of x (the value of x obtained must be greater than 0 and must not be where their second derivative is equal to 0) or the correct value of x	✓
$(V =)\ 2.5 \times 2.5 \times (15 - 4 \times 2.5)$ or $(V =)\ 15(2.5)^2 - 4(2.5)^3$	M1	dep on M3, for a full and correct substitution of their value of x. The printed scheme writes that value in quotation marks, so a candidate's own earlier value may be used throughout this row.	✓
31.25	A1	oe eg $\frac{125}{4}$. Ignore any units on the answer line.	✓
Correct answer only scores full marks (unless from obviously incorrect working)	Note	Guidance printed beside the final row of the scheme. It awards no mark of its own.	✓

Full marks: 5/5

Question 23 - Exam Solution

Understanding the Question

Given

A triangular prism $ABCDEF$ whose base $ABCD$ is a horizontal rectangle $AC = 40$ cm, angle $CAE = 35°$, angle $ABF = 90°$

Find

The size of angle CME, correct to 3 significant figures

M is the midpoint of AC, so it sits half way along the base's diagonal

Plan the Solution

- Angle $ABF = 90^\circ$ is the fact that makes BF stand vertically on the base, and CE is the matching edge of the other triangular face, so CE is vertical too.
- A vertical edge is perpendicular to every line of the horizontal base drawn through its foot, so triangle ACE and triangle MCE are both right-angled at C.
- Use the 35° angle with AC to get the height CE, halve AC to get CM, then take the inverse tangent of CE over CM.

Worked Solution [3 marks]

> Rule - Right-angled trigonometry: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$, so a missing opposite side is $\text{adjacent} \times \tan\theta$ and a missing angle is $\tan^{-1}\left(\dfrac{\text{opposite}}{\text{adjacent}}\right)$.

Step 1: Show that CE is perpendicular to the base

$\angle ABF = 90^\circ$

$BF \perp BA$ and $BF \perp BC$

$CE \parallel BF$, so $CE \perp AC$ and $CE \perp CM$

(Reason: The base is a rectangle, so BA and BC already meet at right angles at B. The given angle $ABF = 90^\circ$ makes BF perpendicular to BA as well, and BF is a side of the rectangular face $BCEF$, so it is perpendicular to BC too. Perpendicular to two different directions of the base means perpendicular to the base itself. The two triangular ends are congruent, so CE is parallel and equal to BF and stands vertically as well.)

Step 2: Work out the height CE

$\tan 35^\circ = \dfrac{CE}{40}$

$CE = 40\tan 35^\circ = 28.0083\ldots\ \text{cm}$

(Reason: In triangle ACE the right angle is at C, so AC is the side adjacent to the 35° angle at A and CE is the side opposite it. Any correct route to CE earns the first method mark: $\dfrac{40}{\tan 55^\circ}$ and $\dfrac{40\sin 35^\circ}{\sin 55^\circ}$ both give the same length.)

Step 3: Work out CM

$CM = \frac{40}{2} = 20 \text{ cm}$

(Reason: M is the midpoint of AC, so CM is exactly half of AC. Halving is the whole of what the word midpoint is doing here, and it is the step that is easiest to skip.)

Step 4: Put the two lengths into the tangent ratio

$$\tan(\angle CME) = \frac{CE}{CM}$$

$$\frac{28.0083}{20} = 1.400415$$

(Reason: Triangle MCE has its right angle at C as well, so CE is opposite the angle at M and CM is adjacent to it. Reaching this ratio is the second method mark.)

Step 5: Take the inverse tangent, then round

$$\angle CME = \tan^{-1}(1.400415)$$

$$\angle CME = 54.4703\ldots°$$

$$\angle CME = 54.5° \text{ (3 s.f.)}$$

(Reason: The calculator must be in degree mode. The third significant figure of $54.4703\ldots$ is the 4 in the first decimal place and the digit after it is 7, so it rounds up to 54.5.)

$$\angle CME = 54.5°$$

Verification

✓ **Check 1:** Reach the angle with sine instead of tangent. The hypotenuse of triangle MCE is $ME = \sqrt{20^2 + 28.0083^2} = 34.4161\ldots$, and CE is opposite the angle at M.

$$\sin^{-1}\left(\frac{28.0083}{34.4161}\right) = 54.47°$$

✓ **Check 2:** Do the whole thing in letters first. $CE = AC \tan 35°$ and $CM = \frac{AC}{2}$, so

$\tan(\angle CME) = \dfrac{AC \tan 35°}{\frac{AC}{2}} = 2 \tan 35°$ and the 40 cancels out completely.

$\tan^{-1}(2 \tan 35°) = 54.47°$, the same angle, so the answer never depended on AC

✓ **Check 3:** Is the size sensible? M is only half as far from C as A is, and both look up at the same height CE, so the angle at M has to be the larger of the two. $54.5° > 35°$

Mark Scheme Breakdown

Step	Mark	Description	Got it?
eg $(CE =)\ 40 \times \tan 35\ (= 28(.0\ldots))$ or $(CE =)\ \dfrac{40}{\tan(90-35)}\ (= 28(.0\ldots))$ or $(CE =)\ \dfrac{40 \sin 35}{\sin(90-35)}\ (= 28(.0\ldots))$	**M1**	for a correct method to find CE	✓
eg $(\angle CME =)\ \tan^{-1}\left(\dfrac{[CE]}{\frac{40}{2}}\right)$ or $(\angle CME =)\ \sin^{-1}\left(\dfrac{[CE]}{\sqrt{\left(\frac{40}{2}\right)^2 + [CE]^2}}\right)$ or $(\angle CME =)\ \cos^{-1}\left(\dfrac{\frac{40}{2}}{\sqrt{\left(\frac{40}{2}\right)^2 + [CE]^2}}\right)$	**M1**	for a complete method to find angle CME. The printed scheme puts the height in quotation marks, which means the candidate's own value from the first mark may be used; it is written $[CE]$ here.	✓
54.5	**A1**	awrt 54.5	✓
From the printed scheme, beside the answer	**Note**	Correct answer only scores full marks (unless from obviously incorrect working).	✓

Full marks: 3/3

Question 24 - Exam Solution

Understanding the Question

Given

$P(-5,\ a)$ and $Q(7,\ 3a)$, where $a > 0$

The length $PQ = 4\sqrt{10}$ cm

The grid scale is 1 unit to 1 cm, so a distance in grid units is that same number of centimetres

Find

An equation of the perpendicular bisector of PQ It must come out in the form $y = mx + c$ with m and c integers, so expect whole numbers at the end

Plan the Solution

- The length is given, so square it and use the distance formula to build an equation in a alone.
- Solve that equation, take the positive root, and write down P and Q as ordinary number pairs.
- A perpendicular bisector needs two things: a gradient and a point. The gradient is the negative reciprocal of m_{PQ}.
- The point is the midpoint of PQ, because the bisector cuts PQ in half.
- Finish with the point-gradient form and rearrange into the form asked for.

Worked Solution [6 marks]

Rule - Perpendicular bisector: it passes through the midpoint $\left(\dfrac{x_1 + x_2}{2},\ \dfrac{y_1 + y_2}{2}\right)$, and its gradient $m_\perp$ satisfies $m \times m_\perp = -1$, where m is the gradient of the segment.

Step 1: Turn the given length into an equation in a

$PQ^2 = (7 - (-5))^2 + (3a - a)^2 = 12^2 + (2a)^2$

$(4\sqrt{10})^2 = 4^2 \times 10 = 160$

$144 + 4a^2 = 160$

(Reason: Squaring both sides clears the square root, so a length with a surd in it becomes an ordinary equation. The vertical gap is $3a - a = 2a$, and squaring that gives $4a^2$, not $2a^2$.)

Step 2: Solve for a and write down the two points

$4a^2 = 160 - 144 = 16$

$a^2 = 4$

$a = 2$

$P(-5,\ 2) \qquad Q(7,\ 6)$

(Reason: Both 2 and -2 square to 4, but the question states $a > 0$, so only $a = 2$ is allowed. Then $3a = 6$.)

Step 3: Work out the gradient of PQ

$$m_{PQ} = \frac{6-2}{7-(-5)} = \frac{4}{12} = \frac{1}{3}$$

(Reason: The gradient is the change in y divided by the change in x. Subtracting -5 adds 5, so the horizontal change is 12 and the vertical change is 4.)

Step 4: Turn that into the gradient of the perpendicular bisector

$$m_{PQ} \times m_{\perp} = -1$$
$$\frac{1}{3} \times m_{\perp} = -1$$
$$m_{\perp} = -3$$

(Reason: Perpendicular gradients multiply to -1, so $m_{\perp}$ is the negative reciprocal of $\frac{1}{3}$: turn the fraction upside down and change the sign.)

Step 5: Find the midpoint of PQ

$$M = \left(\frac{-5+7}{2},\ \frac{2+6}{2}\right) = (1,\ 4)$$

(Reason: The bisector cuts PQ in half, so it goes through the midpoint. Average the x coordinates, then average the y coordinates.)

Step 6: Put the gradient and the midpoint into a line

$$y - 4 = -3(x-1)$$
$$y - 4 = -3x + 3$$
$$y = -3x + 7$$

(Reason: Point-gradient form through $(1,\ 4)$ with gradient -3. This gives $m = -3$ and $c = 7$, both integers, which is the form the question asks for.)

$$\boxed{y = -3x + 7}$$

Verification

- ✓ **Check 1 - the midpoint lies on the line:** Put $x = 1$ into $y = -3x + 7$ and compare with the midpoint's y coordinate. $-3(1) + 7 = 4$, which is the y coordinate of $M(1,\ 4)$
- ✓ **Check 2 - the midpoint is the same distance from each end:** Work out MP and MQ with the distance formula, then double one of them and compare with the length the

question gives. $MP = \sqrt{6^2 + 2^2} = \sqrt{40}$ and $MQ = \sqrt{6^2 + 2^2} = \sqrt{40}$, and $2\sqrt{40} = 4\sqrt{10}$, the given length

✓ **Check 3 - the two lines really are perpendicular:** Multiply the gradient of PQ by the gradient of the bisector. $\frac{1}{3} \times (-3) = -1$, so they meet at right angles

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$\sqrt{(3a-a)^2 + (7-(-5))^2} = 4\sqrt{10}$ oe or $(2a)^2 + 12^2 = (4\sqrt{10})^2$ oe or $4a^2 + 144 = 160$ oe	M1	for forming a correct equation in terms of a; brackets must be used correctly, but allow recovery from missing or incorrect brackets to be recovered; condone $4\sqrt{10}^2$ in place of $(4\sqrt{10})^2$	✓
$a = \sqrt{\frac{160-144}{4}}$ $(= \sqrt{4} = 2)$	M1	dep on M1; for a complete method to solve a correct equation for a; condone inclusion of $\pm$. The printed row puts 160 and 144 in quotation marks, so the candidate's own earlier values may be used here	✓
$m_{PQ} = \frac{3a-a}{7-(-5)} \left(= \frac{2a}{12} = \frac{a}{6}\right)$ oe or $m_{PQ} = \frac{3 \times [2] - [2]}{7-(-5)} \left(= \frac{4}{12} = \frac{1}{3}\right)$ oe	M1ft	for a method to find the gradient of PQ, where $[2]$ is what they believe the value of a to be; must be positive and clearly identified	✓
$\left[\frac{a}{6}\right] \times m_{\perp} = -1$ or $m_{\perp} = \frac{-1}{\left[\frac{a}{6}\right]} \left(= -\frac{6}{a}\right)$ oe or	M1ft	for a method to find the gradient of the perpendicular bisector, where $\left[\frac{a}{6}\right]$ or $\left[\frac{1}{3}\right]$ is what they believe to be the gradient of PQ; must be clearly identified	✓

Step	Mark	Description	Got it?
$\left[\dfrac{1}{3}\right] \times m_{\perp} = -1$ or $m_{\perp} = \dfrac{-1}{\left[\dfrac{1}{3}\right]}$ $(= -3)$ oe			
$\dfrac{-5+7}{2}$ $(= 1)$ and $\dfrac{3a+a}{2}$ $(= 2a)$ or $\dfrac{-5+7}{2}$ $(= 1)$ and $\dfrac{3 \times [2] + [2]}{2}$ $(= 4)$	M1ft	for a method to find the x coordinate and y coordinate of the midpoint of PQ; condone if the coordinates are the wrong way around, where $[2]$ is what they believe the value of a to be; must be positive and clearly identified	✓
$y = -3x + 7$. Correct answer only scores full marks (unless from obviously incorrect working)	A1	oe correct equation in required form eg $y = 7 - 3x$	✓

Full marks: 6/6

Question 25 - Exam Solution

Understanding the Question

Given

The graph of $y = a\sin(x + b)^\circ + c$ drawn on a grid for $0 \leq x \leq 360$

A highest point at $(45,\ 4)$ and a lowest point at $(225,\ -2)$

The curve passes through $y = 1$ on the way down at $x = 135$ and on the way back up at $x = 315$

Find

A suitable value for each of a, b and c

Plan the Solution

- Read the highest and the lowest value of y off the grid. Every letter comes out of those two numbers.

- The curve waves about the line halfway between them, and the height of that line is c.
- Half the drop from the highest point to the lowest point is a.
- Get b from where the peak sits, because a sine is at its highest when the angle inside it is $90°$.

Worked Solution [3 marks]

> Sine curves: $y = a\sin(x+b)° + c$ waves about the line $y = c$, reaching $c + a$ at its highest and $c - a$ at its lowest, and it peaks where $x + b = 90$

Step 1: Read the highest and the lowest point off the grid

$y_{\text{max}} = 4$

$y_{\text{min}} = -2$

(Reason: The peak sits on the gridline $y = 4$ and the trough on $y = -2$. These are the only two numbers the graph really has to give, and the rest of the question is what you can build from them.)

Step 2: The value halfway between them is c

$$c = \frac{4 + (-2)}{2} = \frac{2}{2} = 1$$

(Reason: A sine curve is symmetrical about its middle line, so that line sits exactly halfway between the highest and the lowest value. Its height is the number added on at the end, which is c.)

Step 3: Half the drop between them is a

$$a = \frac{4 - (-2)}{2} = \frac{6}{2} = 3$$

(Reason: The peak is a above the middle line and the trough is a below it, so the whole drop from peak to trough is $2a$. Halving it leaves a.)

Step 4: Where the peak sits gives b

$45 + b = 90$

$b = 90 - 45 = 45$

(Reason: A sine reaches its highest value when the angle inside it is $90°$. This peak is at $x = 45$, so the angle inside there is $45 + b$, and that has to be 90.)

Step 5: Write the equation the three values give

$y = 3\sin(x + 45)° + 1$

(Reason: Reading the graph has fixed all three letters. Turning a negative works just as well: $y = -3\sin(x + 225)° + 1$ draws exactly the same curve, which is why the mark scheme accepts that pair too.)

$$a = 3$$
$$b = 45$$
$$c = 1$$

Verification

✓ **Check 1:** Put $x = 225$ into $y = 3\sin(x + 45)^\circ + 1$. The angle inside becomes 270°, and $\sin 270^\circ = -1$. $3 \times (-1) + 1 = -2$, which is the lowest point the grid shows.

✓ **Check 2:** Test the start of the curve instead, which is a reading the working never used. At $x = 0$ the angle inside is 45°, and $\sin 45^\circ = 0.7071$ to four decimal places. $3 \times 0.7071 + 1 = 3.1213$, which is why the curve leaves the y-axis a little above 3 rather than on it.

✓ **Check 3:** Rebuild both turning points from the answer: the highest value should come to $c + a$ and the lowest to $c - a$. $1 + 3 = 4$ and $1 - 3 = -2$, matching the peak and the trough on the grid.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The value of a	**B1**	for $a = 3$ or $a = -3$	✓
The value of b	**B1**	for $a > 0$ and $b = 45$ or for $a < 0$ and $b = 225$ if no answer for a is seen allow this mark for $b = 45$	✓
The value of c	**B1**	for $c = 1$	✓
Alternative angles	**Note**	Allow correct alternative angles for the value of b, eg 45 or -315, 225 or -135	✓

Full marks: 3/3

Question 26 - Exam Solution

Understanding the Question

Given	Find

A solid hemisphere H whose radius is x cm The volume of H is 6174π cm^3 A dish made by removing a solid hemisphere from H, leaving a uniform thickness of 2 cm	The total surface area of the dish, in terms of π

Plan the Solution

- A hemisphere is half a sphere, so its volume is half of $\frac{4}{3}\pi x^3$. Put that equal to 6174π and the radius comes out of it.
- The π sits on both sides of that equation, so it cancels and every number in the working stays whole.
- Taking a hemisphere out of the middle leaves a wall 2 cm thick all the way round, so the hollow has a radius 2 cm smaller than H.
- The dish has three surfaces and no others: the curved outside, the curved inside, and the flat ring round the top. Work out all three, then add them.

Worked Solution [5 marks]

> Hemisphere: volume $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$ and curved surface area $= \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$

Step 1: Turn the volume into an equation

$$\frac{1}{2} \times \frac{4}{3}\pi x^3 = 6174\pi$$

$$\frac{2}{3}\pi x^3 = 6174\pi$$

(Reason: A hemisphere is half a sphere, so its volume is half of $\frac{4}{3}\pi x^3$. Half of $\frac{4}{3}$ is $\frac{2}{3}$, which is where the $\frac{2}{3}\pi x^3$ comes from.)

Step 2: Solve for the radius x

$$x^3 = \frac{6174 \times 3}{2} = 9261$$

$$x = \sqrt[3]{9261} = 21$$

(Reason: Dividing both sides by π clears it completely, so no decimal ever enters the working. Multiplying by $\frac{3}{2}$ undoes the $\frac{2}{3}$, and 9261 is a perfect cube: $21 \times 21 \times 21 = 9261$.)

Step 3: Find the radius of the hollow

$$21 - 2 = 19$$

(Reason: The wall is 2 cm thick all the way round, so the hemisphere that was taken out has the same centre and a radius 2 cm smaller. The dish therefore runs between radius 19 cm on the inside and radius 21 cm on the outside.)

Step 4: The two curved surfaces

$$2\pi \times 21^2 = 2\pi \times 441 = 882\pi$$
$$2\pi \times 19^2 = 2\pi \times 361 = 722\pi$$

(Reason: The curved surface of a hemisphere is half a sphere's $4\pi r^2$, which is $2\pi r^2$. The dish has two of them, because it is curved on the outside and curved on the inside: one at radius 21 and one at radius 19.)

Step 5: The flat ring round the top

$$\pi(21)^2 - \pi(19)^2 = 441\pi - 361\pi = 80\pi$$

(Reason: The top of the dish is a flat ring. Its area is the whole circle of radius 21 with the circle of the opening, radius 19, taken out of it.)

Step 6: Add the three surfaces

$$882\pi + 722\pi + 80\pi = 1684\pi$$

(Reason: The outside, the inside and the ring make up the whole of the dish's surface, with nothing left out and nothing counted twice, so adding them gives the total. The question asks for the answer in terms of π, so it is left exactly as it stands.)

$$1684\pi \text{ cm}^2$$

Verification

✓ **Check 1:** Put $x = 21$ back into the volume of a hemisphere and see whether the figure the paper gives comes back. $\frac{2}{3}\pi \times 21^3 = \frac{2}{3}\pi \times 9261 = 6174\pi$, which is the volume of H.

✓ **Check 2:** Work the ring out a second way. $441 - 361$ is a difference of two squares, so it factorises as $(21 - 19)(21 + 19)$ and neither radius has to be squared at all. $\pi \times 2 \times 40 = 80\pi$, the ring's area again, from arithmetic that shares no step with the first way.

✓ **Check 3:** Regroup the same three pieces. The curved outside and the whole top circle together make $3\pi r^2$ at radius 21; the curved inside with the opening taken off it leaves πr^2 at radius 19. $3\pi \times 441 + \pi \times 361 = 1323\pi + 361\pi = 1684\pi$, the same total by a different grouping, and it is the form the mark scheme's alternative method uses.

✓ **Check 4:** Check which hemisphere is which, because this is the one place the question can be misread. If the 2 cm were added on rather than taken off, the dish would run from radius 21 on the inside out to radius 23. $2\pi \times 529 + 2\pi \times 441 + \pi \times 88 = 2028\pi$, which is the value the mark scheme names as a special case. The question says the hemisphere of volume 6174π cm^3 is H itself and the dish is cut out of it, so 21 is the outside radius and 1684π is the answer.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Form a correct equation from the volume of H	**M1**	oe for forming a correct equation; allow use of any letter eg $\frac{1}{2} \times \frac{4}{3}\pi x^3 = 6174\pi$	✓
Find the radius of the hemisphere	**M1**	for a correct method to find the radius of the hemisphere $(x =)\sqrt[3]{\frac{6174\pi \times 3}{2\pi}}$ $(= \sqrt[3]{9261} = 21)$	✓
The area of the top of the dish, or the two curved surfaces	**M1ft**	oe for a method to find the area of the top of the dish eg $\pi(21)^2 - \pi(21-2)^2$ $(= 441\pi - 361\pi = 80\pi)$ or the total area of the two curved surfaces of the dish eg $2\pi(21)^2 + 2\pi(21-2)^2$ $(= 882\pi + 722\pi = 1604\pi)$ where 21 is what they believe to be the radius of the hemisphere	✓
A complete method for the total	**M1ft**	ft their 21 for a complete method: their 1604π added to their 80π	✓

Step	Mark	Description	Got it?
The total surface area of the dish	**A1**	cao 1684π	✓
Alternative method: the formula for a hemispherical shell	**M2**	oe for use of the formula for the total surface area of a hemispherical shell in a complete method eg $3\pi(21)^2 + \pi(21-2)^2$ This M2 stands in place of the two M1ft marks above. If not M2, allow M1 for this formula used with omission of π eg $3(21)^2 + (21-2)^2$	✓
Special case	**SC**	SC B4 for 2028π (use of 23 as the outer radius)	✓
Correct answer only	**Note**	A correct answer only scores full marks, unless it comes from obviously incorrect working.	✓

Full marks: 5/5

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