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Equation of a Line

GCSE Exam-Style Questions with Worked Solutions

GRADE 8 TARGETED

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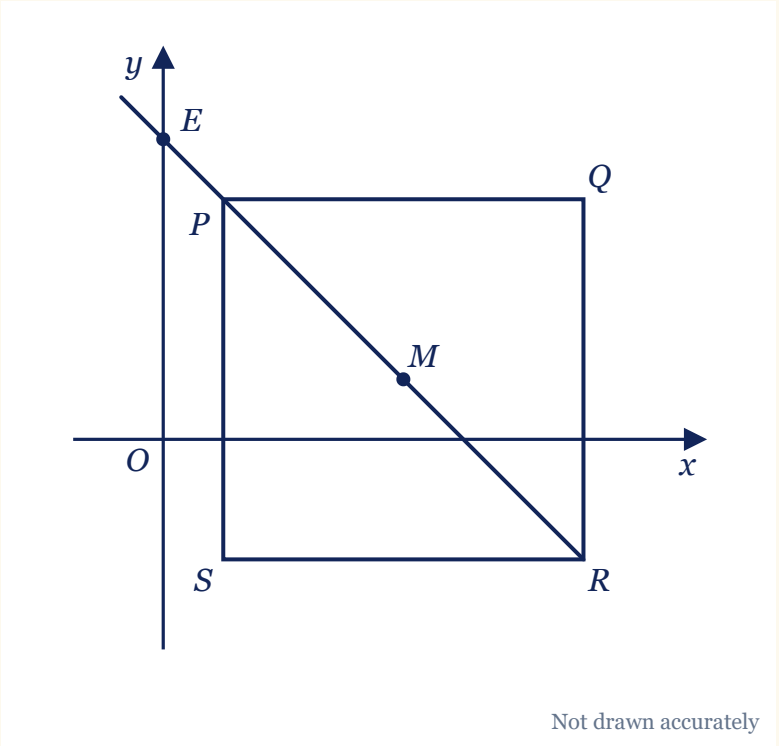
13 exam-style questions, 62 marks, grades 5 to 8. Full worked solutions and mark schemes.

Exam-Style Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1.

GRADE 5



Not drawn accurately

$PQRS$ is a square with side length 6 units. The sides of the square are parallel to the axes.

The coordinates of S are $(1, -2)$.

M is the centre of the square.

Point E has coordinates $(0, 5)$, and E, P, M and R lie on a straight line.

Find the ratio $EM : MR$. Give your answer in its simplest form.
[3 marks]

..... : .....

[Total 3 marks]

2. The lines $y = 4x - 5$ and $y = x + 7$ intersect at the point K.

Line T passes through K and is parallel to the line $2y + 3x = 8$.

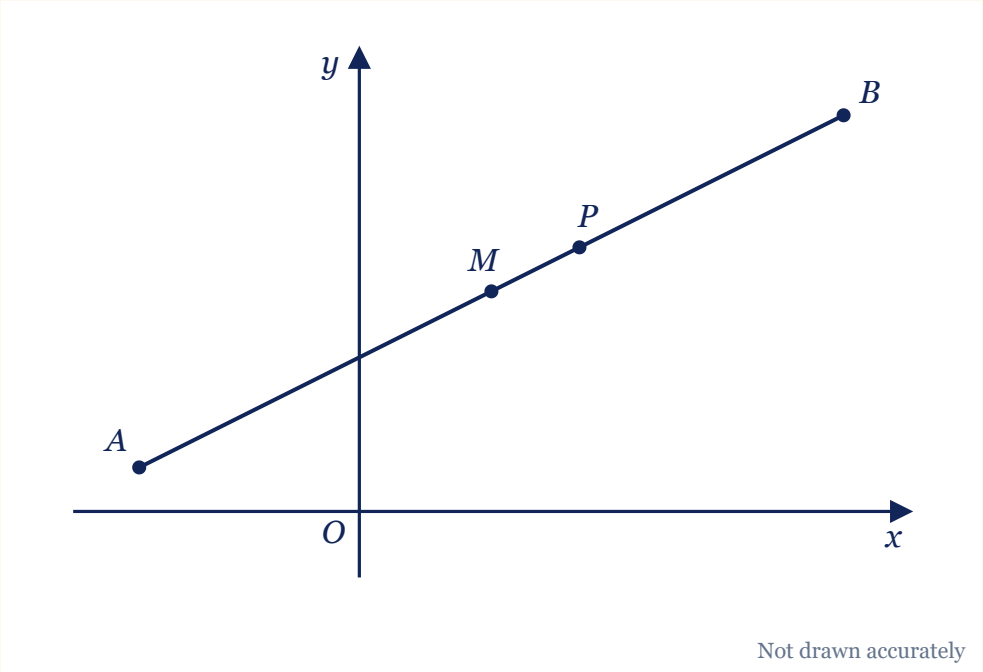
Find the equation of line T. *[5 marks]*

GRADE 6

.....

[Total 5 marks]

3.



Not drawn accurately

M is the midpoint of the line segment AB. A has coordinates $(-5, 1)$ and M has coordinates $(3, 5)$.

(a) Find the coordinates of B. *[2 marks]*

Point P lies on AB such that $MP : PB = 1 : 3$.

(b) Find the ratio $AP : AB$. *[3 marks]*

GRADE 6

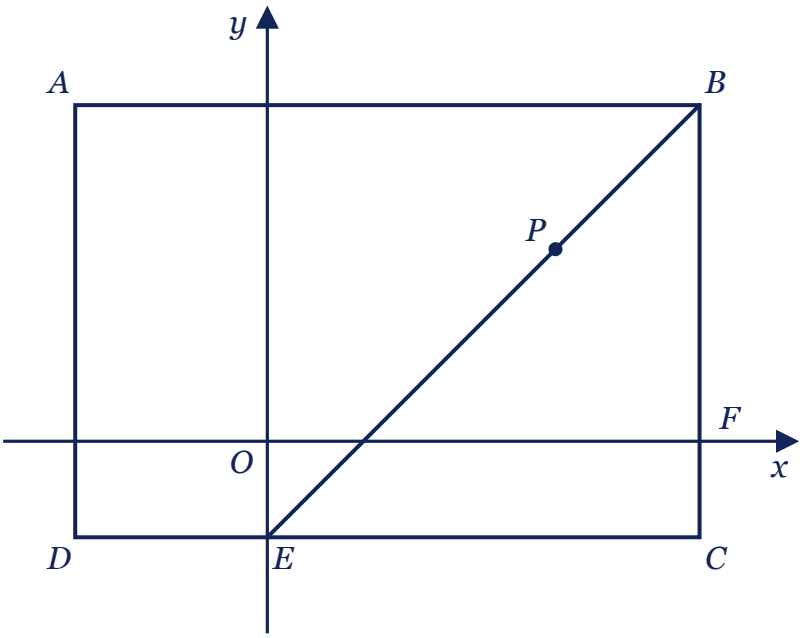
(a) (..... , ...)

(b) ..... : .....

[Total 5 marks]

4.

GRADE 6



Not drawn accurately

The diagram shows rectangle $ABCD$.

Point E lies on DC and has coordinates $(0, -2)$. Point F lies on BC and has coordinates $(9, 0)$. Point B has coordinates $(9, 7)$.

P is the point on the line EB such that $EP : PB = 2 : 1$.

Calculate the length of the line segment PF. *[4 marks]*

..... units

[Total 4 marks]

5. Line L_1 passes through the points $(1, 9)$ and $(7, 5)$.

Line L_2 is perpendicular to L_1 and crosses the x-axis at $(6, 0)$.

Find the equation of line L_2. *[3 marks]*

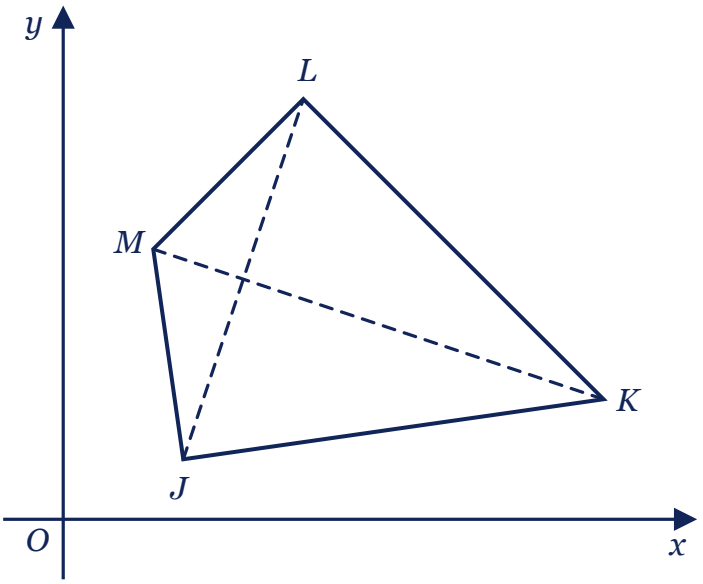
GRADE 6

.....

[Total 3 marks]

6.

GRADE 6



Not drawn accurately

The line JL is one diagonal of the kite $JKLM$ and has equation $y = 3x - 5$.

The point K has coordinates $(9, 2)$.

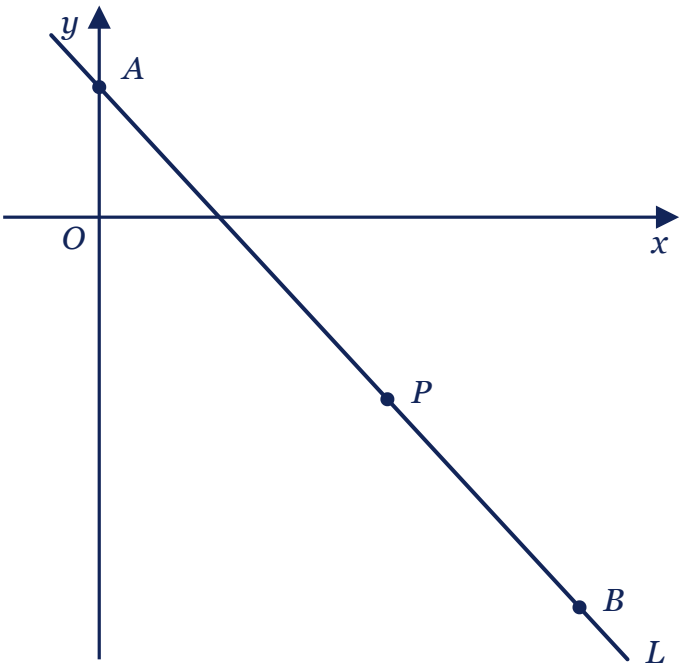
Find the equation of the other diagonal, KM. *[3 marks]*

.....

[Total 3 marks]

7.

GRADE 7



Not drawn accurately

Line L passes through the points $A(0, 5)$ and $B(10, -15)$, as shown in the diagram.

(a) Find the equation of line L. *[3 marks]*

(b) Find the equation of the line that is parallel to line L and passes through the point $(4, 1)$. *[2 marks]*

(c) Point P lies on the line segment AB such that $AP : PB = 3 : 2$. Find the coordinates of P. *[3 marks]*

(a) .....

(b) .....

(c) (..... , ...)

[Total 8 marks]

8. A straight line, W, passes through the points (p, q) and (r, s).

It is given that $3p = 3r + 6$ and $q + 8 = s$.

(a) Find the gradient of line W. *[3 marks]*

(b) Line V is perpendicular to line W and passes through the point $(8, 1)$. Find the equation of line V. *[2 marks]*

GRADE 7



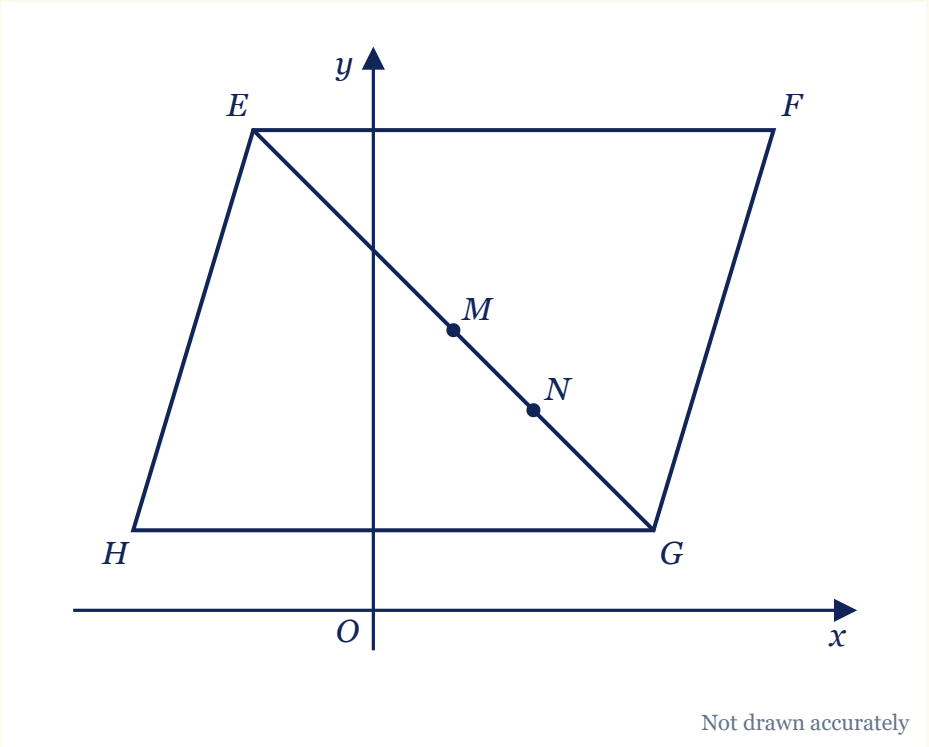
Problem solving

(a) .....

(b) .....

[Total 5 marks]

9.



Not drawn accurately

GRADE 7

Problem solving

$EFGH$ is a parallelogram.

The coordinates of E are $(-3, 12)$ and the coordinates of H are $(-6, 2)$.

M is the midpoint of the diagonal EG and has coordinates $(2, 7)$.

Point N lies on EG such that $EM : MN : NG = 5 : 2 : 3$.

Find the exact length of NF. *[6 marks]*

..... units

[Total 6 marks]

10. Lines L_1 and L_2 are perpendicular and meet at the point T.

L_1 has equation $x + 4y = 58$ and L_2 passes through the point $(4, 5)$.

Find the coordinates of point T. *[5 marks]*

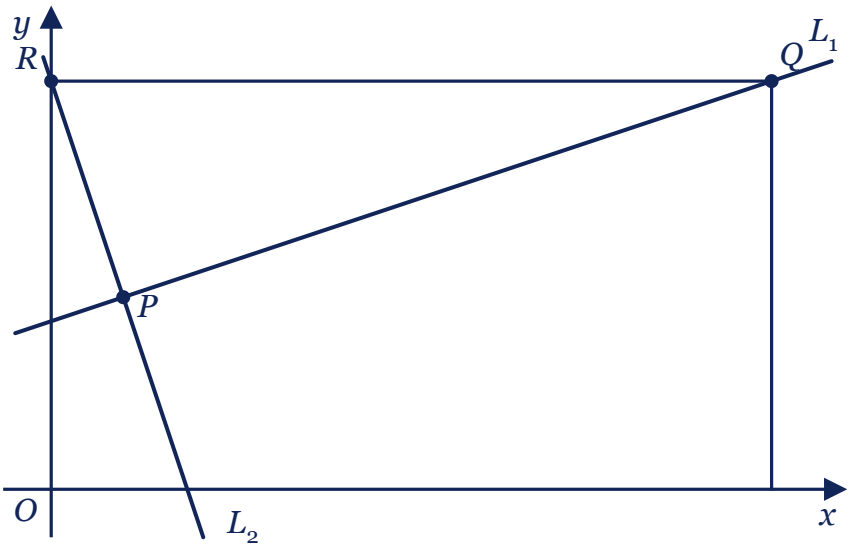
GRADE 7

(..... , ...)

[Total 5 marks]

11.

GRADE 7



Not drawn accurately

The line L_1 has equation $3y - x = 21$ and passes through the points $P(3, 8)$ and Q.

L_2 is the line that is perpendicular to L_1 and passes through P.

L_2 crosses the y-axis at R. RQ is horizontal.

Find the coordinates of Q. *[5 marks]*

(..... , ...)

[Total 5 marks]

12. The points $A(2, 9)$, $B(6, 1)$, $C(11, 2)$ and $D(5, 10)$ are plotted on a grid.

GRADE 8

A student says that the line perpendicular to AB through the midpoint of AB also passes through the midpoint of CD.

Is the student correct? Show how you decide. *[4 marks]*

.....

[Total 4 marks]

13. Lines L_1 and L_2 are parallel.

L_1 has equation $3x + 4y = 24$ and L_2 passes through the point $(3, 10)$.

Line L_3 is perpendicular to L_1 and L_2, and meets L_1 at the point $(4, 3)$.

Find the coordinates of the point where L_2 and L_3 intersect.
[6 marks]

GRADE 8

Problem solving

(..... , ...)

[Total 6 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy, B for an independent mark) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

Square $PQRS$ with side length 6 units, sides parallel to the axes.

$S(1, -2)$ and $E(0, 5)$, with M the centre of the square.

E, P, M and R all lie on one straight line.

Find

The ratio $EM : MR$ in its simplest form.

Plan the Solution

- Use the side length to find the coordinates of R and Q from S.
- The centre M is the midpoint of the diagonal SQ.
- E, M and R are on one straight line, so compare the x-steps from E to M and from M to R, then simplify the ratio.

Worked Solution [3 marks]

> **Rule - Ratios along a straight line:** when points lie on one straight line, the distances between them are in the same ratio as their x-steps (or their y-steps), so you can compare the steps instead of working out each distance.

Step 1: Use the side length to find R and Q.

$R = (1 + 6,\ -2) = (7, -2)$

$Q = (7,\ -2 + 6) = (7, 4)$

(Reason: the sides are parallel to the axes, so R is 6 units to the right of S, and Q is 6 units above R.)

Step 2: Find the centre M, the midpoint of the diagonal SQ.

$$M = \left(\frac{1 + 7}{2},\ \frac{-2 + 4}{2}\right) = (4, 1)$$

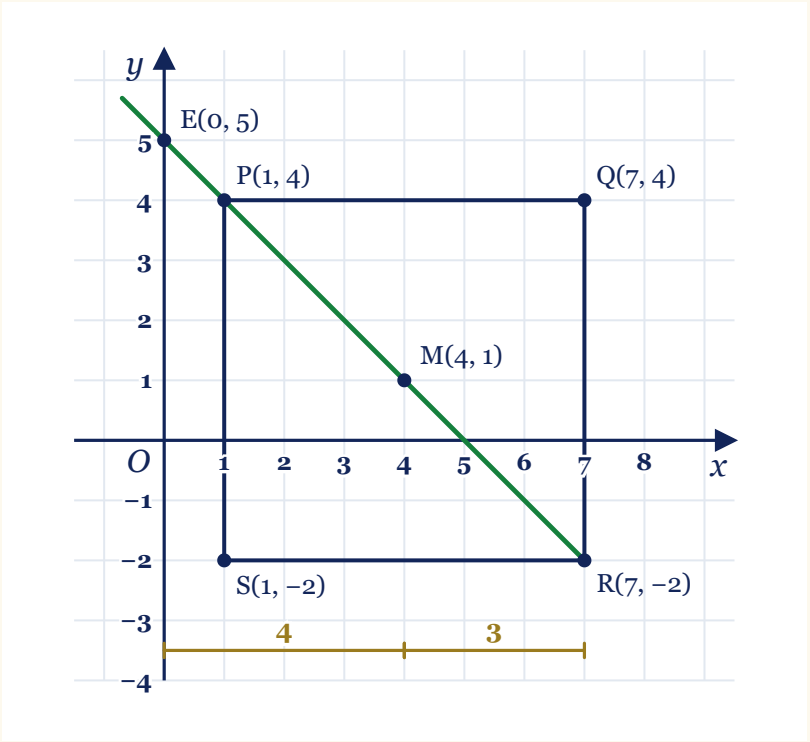
(Reason: the diagonals of a square cross at its centre, so M is halfway along the diagonal from S to Q.)

Step 3: Compare the x-steps from E to M and from M to R.

$E \to M:\ 4 - 0 = 4$

$M \to R:\ 7 - 4 = 3$

$EM : MR = 4 : 3$



(Reason: E, M and R lie on one straight line, so EM and MR are in the same ratio as their x-steps. 4 and 3 have no common factor, so the ratio is already in its simplest form.)

$$EM : MR = 4 : 3$$

Verification

✓ **Check 1:** Work out both lengths with Pythagoras:
$EM = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$ and $MR = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$, so $EM : MR = 4\sqrt{2} : 3\sqrt{2} = 4 : 3$.

✓ **Check 2:** The line through $P(1, 4)$ and $R(7, -2)$ has gradient $\dfrac{-2 - 4}{7 - 1} = -1$, so its equation is $y = -x + 5$. At $x = 0$, $y = 5$, so $E(0, 5)$ lies on it, and so does $M(4, 1)$, because $-4 + 5 = 1$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Coordinates of R and M: $R(7, -2)$ and $M(4, 1)$, both needed	**B1**	Independent - coordinates of R and M	✓
Differences from E to M and from M to R: 4 and 3	**M1**	Method - compare the two steps	✓
$EM : MR = 4 : 3$	**A1**	Accuracy - ratio in its simplest form	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

$y = 4x - 5$ and $y = x + 7$ meet at K.

Line T passes through K and is parallel to $2y + 3x = 8$.

Two lines that cross, and a third line that is not written in the form y = mx + c.

Find

The equation of line T.

Plan the Solution

- Find K by setting the two expressions for y equal to each other.
- Rearrange $2y + 3x = 8$ into the form $y = mx + c$ to read off its gradient.
- Line T is parallel, so it has the same gradient. Substitute the coordinates of K to find its value of c.

Worked Solution [5 marks]

> **Rule - Parallel lines:** parallel lines have the same gradient. Rearrange the given line into the form $y = mx + c$ to read off m, then substitute a point on the new line to find its own value of c.

Step 1: Find the x-coordinate of K.

$4x - 5 = x + 7$

$$3x = 12$$

$$x = 4$$

(Reason: where the two lines cross, both equations give the same value of y, so their right-hand sides are equal.)

Step 2: Find the y-coordinate of K.

$$y = 4 + 7 = 11$$

$$K(4, 11)$$

(Reason: substitute x = 4 into either equation; y = x + 7 is the simpler one.)

Step 3: Rearrange 2y + 3x = 8 to find its gradient.

$$2y = -3x + 8$$

$$y = -\frac{3}{2}x + 4$$

$$m = -\frac{3}{2}$$

(Reason: the gradient can be read off only when y is the subject; it is then the coefficient of x.)

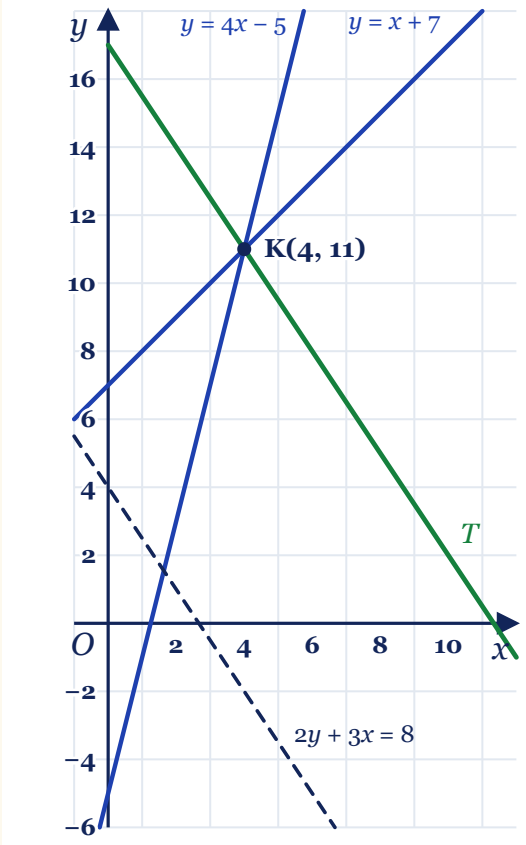
Step 4: Use the same gradient and substitute K to find c.

$$y = -\frac{3}{2}x + c$$

$$11 = -\frac{3}{2}(4) + c$$

$$11 = -6 + c$$

$$c = 17$$



(Reason: T is parallel to $2y + 3x = 8$, so it has the same gradient, and K lies on T, so its coordinates satisfy the equation of T.)

$$y = -\frac{3}{2}x + 17$$

Verification

✓ **Check 1:** Write T through $K(4, 11)$ in point-gradient form: $y - 11 = -\frac{3}{2}(x - 4)$, which expands to $y = -\frac{3}{2}x + 6 + 11 = -\frac{3}{2}x + 17$, the same equation.

✓ **Check 2:** $K(4, 11)$ lies on both given lines: $4(4) - 5 = 11$ and $4 + 7 = 11$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Equate the two lines: $4x - 5 = x + 7$	**M1**	Method - equate to find K	✓
$x = 4$	**A1**	Accuracy - x-coordinate of K	✓

Step	Mark	Description	Got it?
$K(4, 11)$	A1	Accuracy - coordinates of K	✓
Gradient $-\frac{3}{2}$ from $2y + 3x = 8$, and substitute $K(4, 11)$ into $y = -\frac{3}{2}x + c$	M1	Method - parallel gradient and substitution	✓
$y = -\frac{3}{2}x + 17$ oe	A1	Accuracy - equation of T	✓

Full marks: 5/5

Question 3 - Exam Solution

Understanding the Question

Given

$A(-5, 1)$ and $M(3, 5)$, where M is the midpoint of AB.

P lies on AB with $MP : PB = 1 : 3$.

A midpoint, then a point that divides MB in a given ratio.

Find

The coordinates of B, then the ratio $AP : AB$.

Plan the Solution

- For part (a), M is halfway from A to B, so the step from M to B is the same as the step from A to M.
- For part (b), $MP : PB = 1 : 3$ splits MB into 4 equal parts, so P is one quarter of the way from M to B.
- Find P, compare the x-steps from A to P and from A to B, and simplify.

Worked Solution [5 marks]

> **Rule - Midpoints and ratios on a line segment:** a midpoint is halfway, so the step from A to M is repeated from M to B. A point that divides a segment in the ratio $p : q$ is $\frac{p}{p+q}$ of the way along it, so add that fraction of each step to the starting point.

Step 1 (part a): Find the step from A to M, then repeat it from M.

$A \to M:\ (3-(-5),\ 5-1) = (8, 4)$

$B = (3+8,\ 5+4) = (11, 9)$

(Reason: M is the midpoint, so B is as far beyond M as M is beyond A.)

Step 2 (part b): Take one quarter of the step from M to B.

$M \to B:\ (11-3,\ 9-5) = (8, 4)$

$\frac{1}{4}$ of $(8, 4) = (2, 1)$

$P = (3+2,\ 5+1) = (5, 6)$

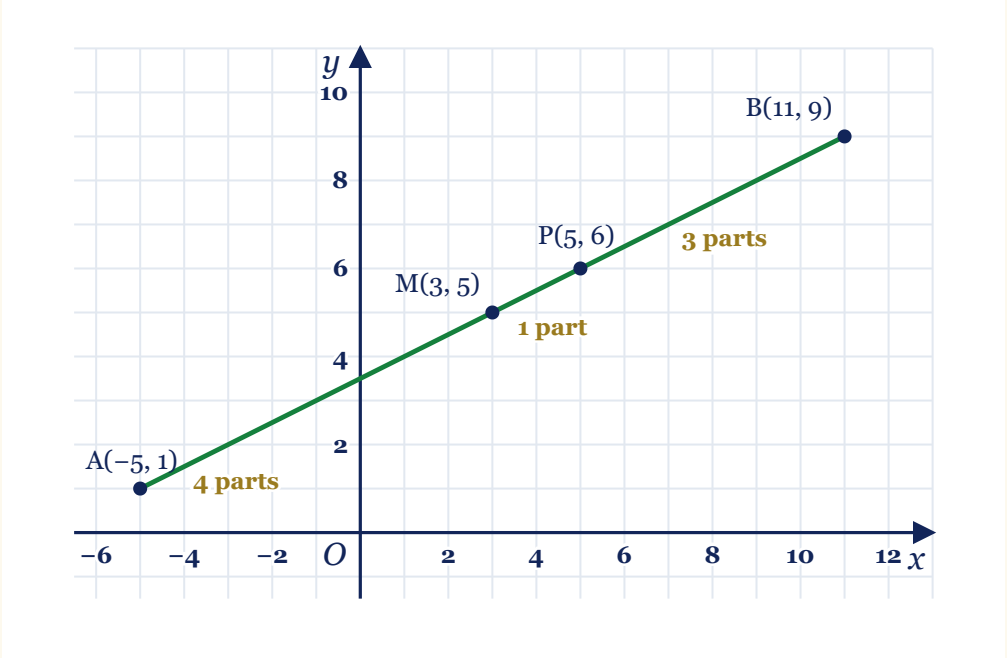
(Reason: MP : PB = 1 : 3 has 1 + 3 = 4 parts, and P is 1 part along from M, so P is one quarter of the way from M to B.)

Step 3 (part b): Compare the x-steps from A to P and from A to B.

$A \to P:\ 5-(-5) = 10$

$A \to B:\ 11-(-5) = 16$

$AP : AB = 10 : 16 = 5 : 8$



(Reason: A, P and B lie on one straight line, so AP and AB are in the same ratio as their x-steps; dividing both by 2 gives the simplest form.)

(a) $B(11, 9)$

(b) $AP : AB = 5 : 8$

Verification

✓ **Check 1 (part b):** Count equal parts. $AM = MB$ and MB is $1 + 3 = 4$ parts, so $AM = 4$, $MP = 1$ and $PB = 3$. Then $AP = 4 + 1 = 5$ parts and $AB = 4 + 1 + 3 = 8$ parts, giving $5 : 8$.

✓ **Check 2 (part b):** The y-steps give the same ratio: $A \to P:\ 6 - 1 = 5$ and $A \to B:\ 9 - 1 = 8$, so $AP : AB = 5 : 8$.

✓ **Check 3 (part a):** The midpoint of $A(-5, 1)$ and $B(11, 9)$ is $\left(\frac{-5 + 11}{2},\ \frac{1 + 9}{2}\right) = (3, 5)$, which is M.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): $x = 11$	**B1**	Independent - x-coordinate of B	✓
Part (a): $y = 9$	**B1**	Independent - y-coordinate of B	✓
Part (b): a correct method for P, for example $(3, 5) + \frac{1}{4}(8, 4)$	**M1**	Method - find P	✓
Part (b): $P(5, 6)$	**A1**	Accuracy - coordinates of P	✓
Part (b): $AP : AB = 5 : 8$	**A1**	Accuracy - ratio in its simplest form	✓

Full marks: 5/5

Question 4 - Exam Solution

Understanding the Question

Given

Rectangle $ABCD$, with $E(0, -2)$ on DC, $F(9, 0)$ on BC and $B(9, 7)$.
P lies on EB with $EP : PB = 2 : 1$.

Find

The length of PF.

A point that divides a line segment in a given ratio, then a distance.

Plan the Solution

- $EP : PB = 2 : 1$ has 3 parts, so P is two thirds of the way from E to B.
- Find the step from E to B, take two thirds of it, and add it to E.
- Use Pythagoras with the horizontal and vertical distances from P to F.

Worked Solution [4 marks]

> **Rule - Dividing a line segment, then Pythagoras:** for $EP : PB = 2 : 1$, P is $\frac{2}{3}$ of the way from E to B. The distance between two points is $\sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}$.

Step 1: Find the step from E to B.

$$E \to B: \ (9 - 0,\ 7 - (-2)) = (9, 9)$$

(Reason: the differences in x and in y describe the move from E to B along the line.)

Step 2: Take two thirds of the step and add it to E.

$$\frac{2}{3} \text{ of } (9, 9) = (6, 6)$$

$$P = (0 + 6,\ -2 + 6) = (6, 4)$$

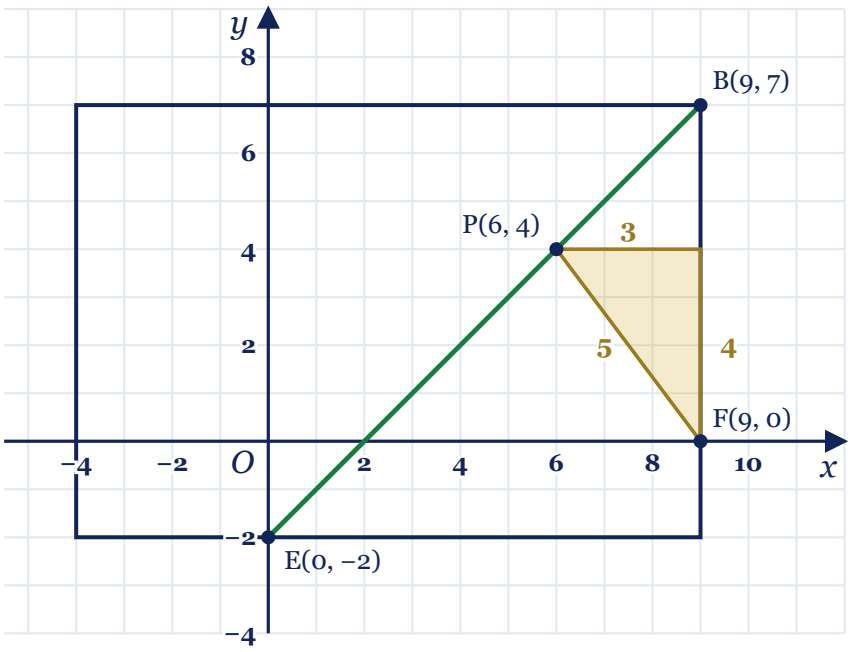
(Reason: EP : PB = 2 : 1 has 2 + 1 = 3 parts, and P is 2 parts along from E, so P is two thirds of the way from E to B.)

Step 3: Use Pythagoras to find PF.

$$PF^2 = (9 - 6)^2 + (0 - 4)^2$$

$$PF^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$PF = \sqrt{25} = 5$$



(Reason: the horizontal distance 3 and the vertical distance 4 are the two shorter sides of a right-angled triangle, and PF is its hypotenuse.)

$$PF = 5 \text{ units}$$

Verification

✓ **Check 1:** The section formula gives the same point:

$$P = \left(\frac{0 + 2(9)}{3}, \frac{-2 + 2(7)}{3}\right) = (6, 4).$$

✓ **Check 2:** P lies on EB: the gradient of EP is $\frac{4 - (-2)}{6 - 0} = 1$ and the gradient of EB is $\frac{7 - (-2)}{9 - 0} = 1$, the same.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Differences in x and in y from E to B: 9 and 9	M1	Method - steps from E to B	✓
$P(6, 4)$	A1	Accuracy - coordinates of P	✓
Pythagoras: $PF^2 = 3^2 + 4^2$	M1	Method - Pythagoras for PF	✓

Step	Mark	Description	Got it?
$PF = 5$	A1	Accuracy - length of PF	✓

Full marks: 4/4

Question 5 - Exam Solution

Understanding the Question

Given

L_1 passes through $(1, 9)$ and $(7, 5)$.

L_2 is perpendicular to L_1 and passes through $(6, 0)$.

The point (6, 0) is where the second line crosses the x-axis.

Find

The equation of L_2.

Plan the Solution

- Find the gradient of L_1 from its two points.
- Perpendicular gradients multiply to give -1, so take the negative reciprocal for the gradient of L_2.
- Substitute $(6, 0)$ into $y = mx + c$ to find c.

Worked Solution [3 marks]

> **Rule - Perpendicular lines:** the gradients of two perpendicular lines multiply to give -1, so each gradient is the negative reciprocal of the other: $m_2 = -\dfrac{1}{m_1}$.

Step 1: Find the gradient of the first line.

$$m_1 = \frac{5 - 9}{7 - 1} = \frac{-4}{6} = -\frac{2}{3}$$

(Reason: the gradient is the change in y divided by the change in x, with the two points taken in the same order on the top and on the bottom.)

Step 2: Find the gradient of the perpendicular line.

$$-\frac{2}{3} \times m_2 = -1$$

$$m_2 = \frac{3}{2}$$

(Reason: perpendicular gradients multiply to give −1, so m_2 is the negative reciprocal of $-\frac{2}{3}$: turn the fraction upside down and change its sign.)

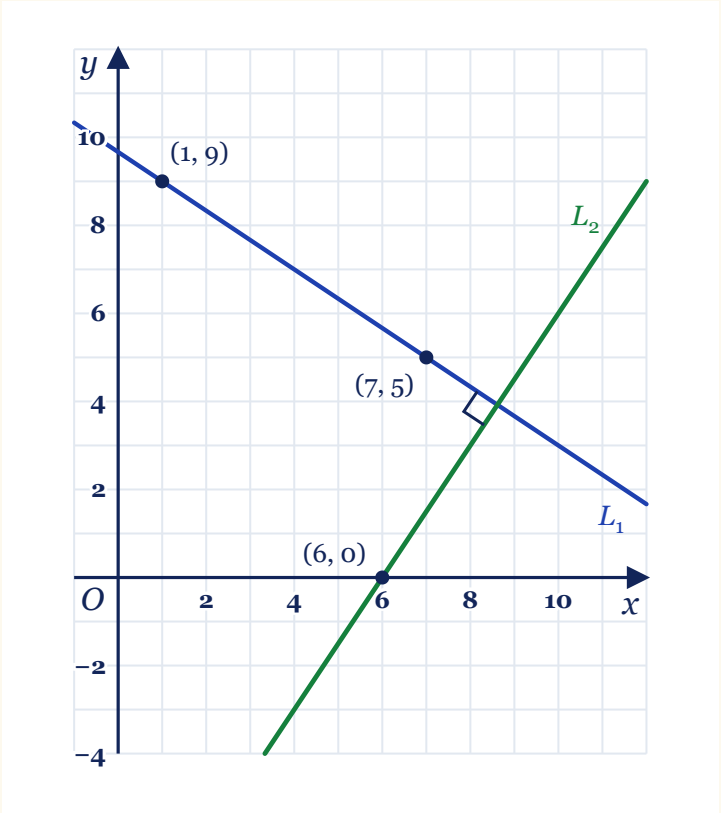
Step 3: Substitute (6, 0) to find c.

$$y = \frac{3}{2}x + c$$

$$0 = \frac{3}{2}(6) + c$$

$$0 = 9 + c$$

$$c = -9$$



(Reason: the second line crosses the x-axis at (6, 0), so this point lies on it and its coordinates satisfy the equation.)

$$y = \frac{3}{2}x - 9$$

Verification

✓ **Check 1:** $m_1 \times m_2 = -\frac{2}{3} \times \frac{3}{2} = -1$, so the two lines are perpendicular.

✓ **Check 2:** Through $(6, 0)$ with gradient $\frac{3}{2}$, the line is $y = \frac{3}{2}(x - 6)$, which expands to $y = \frac{3}{2}x - 9$, the same equation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $\frac{5-9}{7-1} = -\frac{2}{3}$	M1	Method - gradient of L_1	✓
Gradient of L_2: $\frac{3}{2}$	M1	Method - perpendicular gradient	✓
$y = \frac{3}{2}x - 9$ oe	A1	Accuracy - equation of L_2	✓

Full marks: 3/3

Question 6 - Exam Solution

Understanding the Question

Given

Kite $JKLM$, with the diagonal JL on the line $y = 3x - 5$.

The other diagonal, KM, passes through $K(9, 2)$.

Only one point on KM is given, so its gradient has to come from the other diagonal.

Find

The equation of the diagonal KM.

Plan the Solution

- Read off the gradient of JL from $y = 3x - 5$.
- The diagonals of a kite are perpendicular, so the gradient of KM is the negative reciprocal of the gradient of JL.
- Substitute $K(9, 2)$ into $y = mx + c$ to find c.

Worked Solution [3 marks]

> **Rule - The diagonals of a kite:** the diagonals of a kite cross at right angles, so their gradients multiply to give -1: $m_{KM} = -\dfrac{1}{m_{JL}}$.

Step 1: Read off the gradient of JL.

$y = 3x - 5$

$m_{JL} = 3$

(Reason: the equation is already in the form y = mx + c, so the gradient is the coefficient of x.)

Step 2: Use the perpendicular diagonals to find the gradient of KM.

$3 \times m_{KM} = -1$

$m_{KM} = -\dfrac{1}{3}$

(Reason: the diagonals of a kite are perpendicular, and perpendicular gradients multiply to give –1, so m_{KM} is the negative reciprocal of 3.)

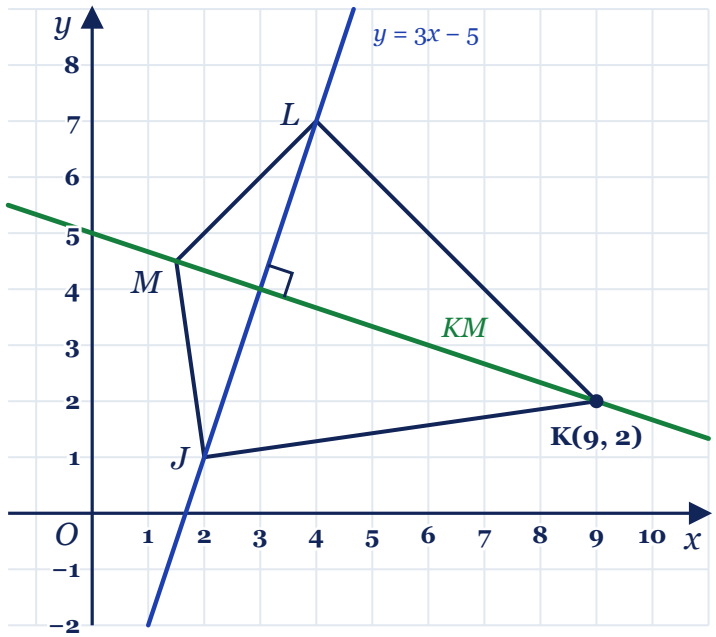
Step 3: Substitute K(9, 2) to find c.

$y = -\dfrac{1}{3}x + c$

$2 = -\dfrac{1}{3}(9) + c$

$2 = -3 + c$

$c = 5$



(Reason: K is one end of the diagonal KM, so its coordinates satisfy the equation of KM.)

$$y = -\frac{1}{3}x + 5$$

Verification

✓ **Check 1:** $m_{JL} \times m_{KM} = 3 \times \left(-\frac{1}{3}\right) = -1$, so the two diagonals are perpendicular.

✓ **Check 2:** Through $K(9, 2)$ with gradient $-\frac{1}{3}$, the line is $y - 2 = -\frac{1}{3}(x - 9)$, which expands to $y = -\frac{1}{3}x + 3 + 2 = -\frac{1}{3}x + 5$, the same equation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
States that the diagonals of a kite are perpendicular	**B1**	Independent - kite property	✓
Gradient $-\frac{1}{3}$, and substitute $K(9, 2)$ into $y = -\frac{1}{3}x + c$	**M1**	Method - perpendicular gradient and substitution	✓

Step	Mark	Description	Got it?
$y = -\frac{1}{3}x + 5$ oe	A1	Accuracy - equation of KM	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

Line L through $A(0, 5)$ and $B(10, -15)$.

A parallel line through $(4, 1)$, and P on AB with $AP : PB = 3 : 2$.

A sits on the y-axis, so it gives the y-intercept of L directly.

Find

The equation of L, the equation of the parallel line, and the coordinates of P.

Plan the Solution

- For part (a), find the gradient from A and B. Because A is on the y-axis, it gives c directly.
- For part (b), a parallel line has the same gradient, so substitute $(4, 1)$ to find its own value of c.
- For part (c), $AP : PB = 3 : 2$ splits AB into 5 equal parts, so P is three fifths of the way from A to B.

Worked Solution [8 marks]

Rule - Gradients, parallel lines and dividing a segment: the gradient is $\frac{\text{change in } y}{\text{change in } x}$, and parallel lines have the same gradient. A point that divides AB in the ratio $p : q$ is $\frac{p}{p+q}$ of the way from A to B.

Step 1 (part a): Find the gradient of L from A and B.

$$m = \frac{-15 - 5}{10 - 0} = \frac{-20}{10} = -2$$

(Reason: the gradient is the change in y divided by the change in x, with the two points taken in the same order on the top and on the bottom.)

Step 2 (part a): Read off c from A, then write the equation.

$$c = 5$$

$$y = -2x + 5$$

(Reason: A has x-coordinate 0, so it is the point where L crosses the y-axis, and its y-coordinate is the value of c.)

Step 3 (part b): Use the same gradient and substitute (4, 1).

$$y = -2x + c$$
$$1 = -2(4) + c$$
$$1 = -8 + c$$
$$c = 9$$
$$y = -2x + 9$$

(Reason: parallel lines have the same gradient, and (4, 1) lies on the new line, so its coordinates satisfy the equation.)

Step 4 (part c): Find the step from A to B.

$$A \to B: \ (10 - 0, \ -15 - 5) = (10, -20)$$

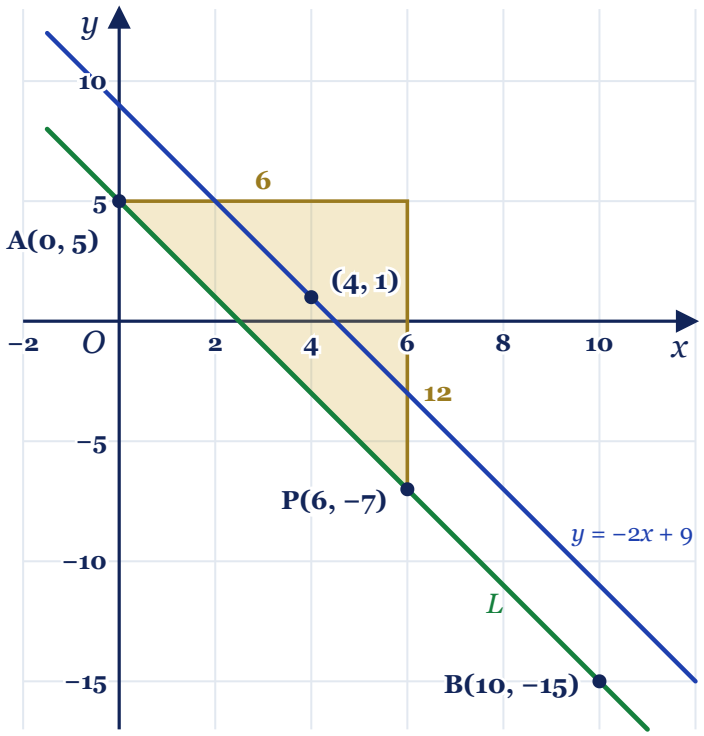
(Reason: the differences in x and in y describe the move from A to B along the line.)

Step 5 (part c): Take three fifths of the step and add it to A.

$$\frac{3}{5} \text{ of } (10, -20) = (6, -12)$$

$$P = \left(0 + \frac{3}{5}(10), \ 5 + \frac{3}{5}(-20)\right)$$

$$P = (0 + 6, \ 5 - 12) = (6, -7)$$



(Reason: AP : PB = 3 : 2 has 3 + 2 = 5 parts, and P is 3 parts along from A, so P is three fifths of the way from A to B.)

(a) $y = -2x + 5$

(b) $y = -2x + 9$

(c) $P(6, -7)$

Verification

✓ **Check 1 (part a):** B lies on L, because $-2(10) + 5 = -15$.

✓ **Check 2 (part b):** Through $(4, 1)$ with gradient -2, the line is $y - 1 = -2(x - 4)$, which expands to $y = -2x + 8 + 1 = -2x + 9$, the same equation.

✓ **Check 3 (part c):** The section formula gives the same point:
$P = \left(\frac{2(0) + 3(10)}{5}, \frac{2(5) + 3(-15)}{5} \right) = (6, -7)$, and P lies on L, because $-2(6) + 5 = -7$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): gradient $\frac{-15-5}{10-0}=-2$	M1	Method - gradient of L	✓
Part (a): $c=5$	B1	Independent - y-intercept from A	✓
Part (a): $y=-2x+5$	A1	Accuracy - equation of L	✓
Part (b): substitute $(4,1)$ into $y=-2x+c$	M1	Method - parallel gradient and substitution	✓
Part (b): $y=-2x+9$	A1	Accuracy - equation of the parallel line	✓
Part (c): differences in x and in y from A to B: 10 and -20	M1	Method - steps from A to B	✓
Part (c): multiply by $\frac{3}{5}$ to get $(6,-12)$	M1	Method - three fifths of the step	✓
Part (c): $P(6,-7)$	A1	Accuracy - coordinates of P	✓

Full marks: 8/8

Question 8 - Exam Solution

Understanding the Question

Given

Line W through (p,q) and (r,s), where $3p=3r+6$ and $q+8=s$.

Line V is perpendicular to W and passes through $(8,1)$.

No point on W is given in numbers, only how its two points are related.

Find

The gradient of W, then the equation of V.

Plan the Solution

- Rearrange the two conditions to write r and s in terms of p and q.

- Put both points into the gradient formula. The letters p and q cancel, leaving a number.
- For part (b), take the negative reciprocal for the gradient of V, then substitute $(8, 1)$ to find c.

Worked Solution [5 marks]

> **Rule - A gradient from algebraic points:** the gradient needs only the change in y and the change in x between the two points, $m = \dfrac{s-q}{r-p}$, so it can be found even when the points themselves are not known. Perpendicular gradients multiply to give -1.

Step 1 (part a): Write r and s in terms of p and q.

$3p = 3r + 6$

$p = r + 2$

$r = p - 2$

$s = q + 8$

(Reason: dividing every term of $3p = 3r + 6$ by 3 gives $p = r + 2$, and $q + 8 = s$ already gives s in terms of q.)

Step 2 (part a): Substitute both points into the gradient formula.

$$m = \frac{s-q}{r-p} = \frac{(q+8)-q}{(p-2)-p}$$

$$m = \frac{8}{-2} = -4$$

(Reason: the change in y is 8 and the change in x is -2 whatever the values of p and q, so the gradient is a number.)

Step 3 (part b): Find the gradient of V.

$-4 \times m_V = -1$

$$m_V = \frac{1}{4}$$

(Reason: V is perpendicular to W, so m_V is the negative reciprocal of -4: turn it upside down and change its sign.)

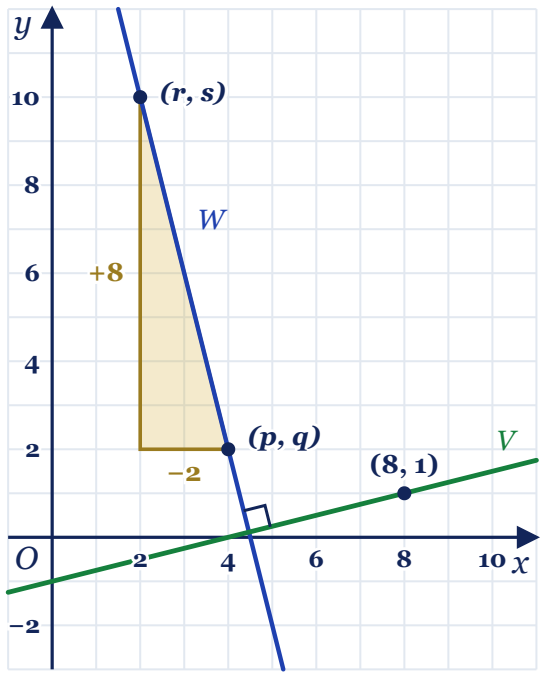
Step 4 (part b): Substitute (8, 1) to find c.

$$y = \frac{1}{4}x + c$$

$$1 = \frac{1}{4}(8) + c$$

$1 = 2 + c$

$c = -1$



One possible position of line W

(Reason: (8, 1) lies on V, so its coordinates satisfy the equation of V.)

(a) $m = -4$

(b) $y = \frac{1}{4}x - 1$

Verification

✓ **Check 1 (part a):** Choose values that fit the conditions, for example $p = 4$ and $q = 2$. Then $r = 2$ and $s = 10$, so the points are $(4, 2)$ and $(2, 10)$, and the gradient is $\frac{10 - 2}{2 - 4} = -4$.

✓ **Check 2 (part b):** $-4 \times \frac{1}{4} = -1$, so V is perpendicular to W.

✓ **Check 3 (part b):** Through $(8, 1)$ with gradient $\frac{1}{4}$, the line is $y - 1 = \frac{1}{4}(x - 8)$, which expands to $y = \frac{1}{4}x - 2 + 1 = \frac{1}{4}x - 1$, the same equation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): $r = p - 2$ and $s = q + 8$	M1	Method - (r, s) in terms of p and q	✓
Part (a): change in y over change in x, $\dfrac{(q+8)-q}{(p-2)-p}$	M1	Method - gradient formula	✓
Part (a): -4	A1	Accuracy - gradient of W	✓
Part (b): gradient $\dfrac{1}{4}$, and substitute $(8, 1)$ into $y = \dfrac{1}{4}x + c$	M1	Method - perpendicular gradient and substitution	✓
Part (b): $y = \dfrac{1}{4}x - 1$ oe	A1	Accuracy - equation of V	✓

Full marks: 5/5

Question 9 - Exam Solution

Understanding the Question

Given

Parallelogram $EFGH$ with $E(-3, 12)$ and $H(-6, 2)$.

$M(2, 7)$ is the midpoint of EG, and N lies on EG with $EM : MN : NG = 5 : 2 : 3$.

Neither G nor F is given, so both have to be found before NF can be worked out.

Find

The exact length of NF.

Plan the Solution

- M is the midpoint of EG, so the step from M to G repeats the step from E to M.
- $MN : NG = 2 : 3$, so N is two fifths of the way from M to G.
- Opposite sides of a parallelogram are parallel and equal, so move from E by the step from H to G to find F. Then use Pythagoras for NF.

Worked Solution [6 marks]

> **Rule - Opposite sides of a parallelogram:** they are parallel and equal, so the step from H to G is the same as the step from E to F, which gives $F = E + (G - H)$. An exact length is left as a surd.

Step 1: Find G from the midpoint M.

$$G = (2(2) - (-3),\ 2(7) - 12) = (7, 2)$$

(Reason: M is halfway from E to G, so G is as far beyond M as M is beyond E: double each coordinate of M and take away the matching coordinate of E.)

Step 2: Take two fifths of the step from M to G.

$$M \to G:\ (7 - 2,\ 2 - 7) = (5, -5)$$

$$\frac{2}{5} \text{ of } (5, -5) = (2, -2)$$

$$N = (2 + 2,\ 7 - 2) = (4, 5)$$

(Reason: MN : NG = 2 : 3 has 2 + 3 = 5 parts, and N is 2 parts along from M, so N is two fifths of the way from M to G.)

Step 3: Find F from the step from H to G.

$$H \to G:\ (7 - (-6),\ 2 - 2) = (13, 0)$$

$$F = (-3 + 13,\ 12 + 0) = (10, 12)$$

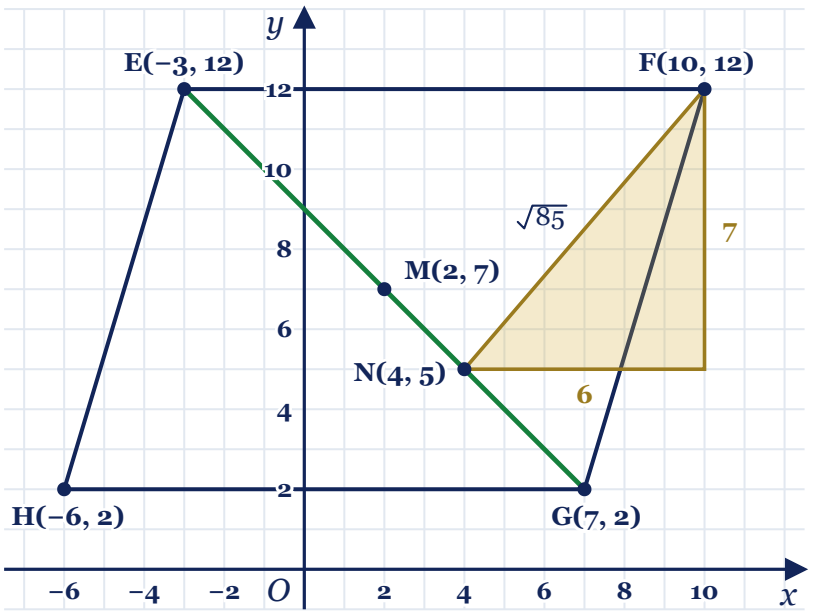
(Reason: EF is parallel and equal to HG, so the move from E to F is the same as the move from H to G.)

Step 4: Use Pythagoras to find NF.

$$NF^2 = (10 - 4)^2 + (12 - 5)^2$$

$$NF^2 = 6^2 + 7^2 = 36 + 49 = 85$$

$$NF = \sqrt{85}$$



(Reason: the horizontal distance 6 and the vertical distance 7 are the two shorter sides of a right-angled triangle, and NF is its hypotenuse. $85 = 5 \times 17$ has no square factor, so the surd does not simplify.)

$$NF = \sqrt{85} \text{ units}$$

Verification

✓ **Check 1:** Find N from E instead. $EN : NG = (5+2) : 3 = 7 : 3$, so $N = (-3, 12) + \frac{7}{10}(10, -10) = (4, 5)$, the same point.

✓ **Check 2:** The diagonals of a parallelogram bisect each other, so M is also the midpoint of FH: $\left(\frac{10 + (-6)}{2}, \frac{12 + 2}{2}\right) = (2, 7)$, which is M.

✓ **Check 3:** The lengths along EG are in the given ratio: $EM = \sqrt{5^2 + 5^2} = 5\sqrt{2}$, $MN = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ and $NG = \sqrt{3^2 + 3^2} = 3\sqrt{2}$, so $EM : MN : NG = 5 : 2 : 3$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$G(7, 2)$	B1	Independent - coordinates of G	✓
A correct method for N, for example $(2, 7) + \frac{2}{5}(5, -5)$	M1	Method - find N	✓
$N(4, 5)$	A1	Accuracy - coordinates of N	✓
$F(10, 12)$	B1	Independent - coordinates of F	✓
Pythagoras: $NF^2 = 6^2 + 7^2$	M1	Method - Pythagoras for NF	✓
$NF = \sqrt{85}$	A1	Accuracy - exact length of NF	✓

Full marks: 6/6

Question 10 - Exam Solution

Understanding the Question

Given

L_1 has equation $x + 4y = 58$.

L_2 is perpendicular to L_1, passes through $(4, 5)$ and meets L_1 at T.

The first line is not written in the form y = mx + c, and the second is known only by one point and its direction.

Find

The coordinates of T.

Plan the Solution

- Rearrange $x + 4y = 58$ into the form $y = mx + c$ to read off the gradient of L_1.
- Take the negative reciprocal for the gradient of L_2, then substitute $(4, 5)$ to find its equation.
- T lies on both lines, so solve the two equations simultaneously.

Worked Solution [5 marks]

> **Rule - Where two lines meet:** the point where two lines meet lies on both of them, so its coordinates satisfy both equations at once. Substitute one equation into the other and solve.

Step 1: Rearrange x + 4y = 58 to find the gradient of the first line.

$$4y = -x + 58$$
$$y = -\frac{1}{4}x + \frac{29}{2}$$
$$m_1 = -\frac{1}{4}$$

(Reason: the gradient can be read off only when y is the subject; it is then the coefficient of x.)

Step 2: Find the gradient of the perpendicular line.

$$-\frac{1}{4} \times m_2 = -1$$
$$m_2 = 4$$

(Reason: perpendicular gradients multiply to give –1, so m_2 is the negative reciprocal of $-\frac{1}{4}$.)

Step 3: Substitute (4, 5) to find the equation of the second line.

$$y = 4x + c$$
$$5 = 4(4) + c$$
$$5 = 16 + c$$
$$c = -11$$
$$y = 4x - 11$$

(Reason: the second line passes through (4, 5), so this point satisfies its equation.)

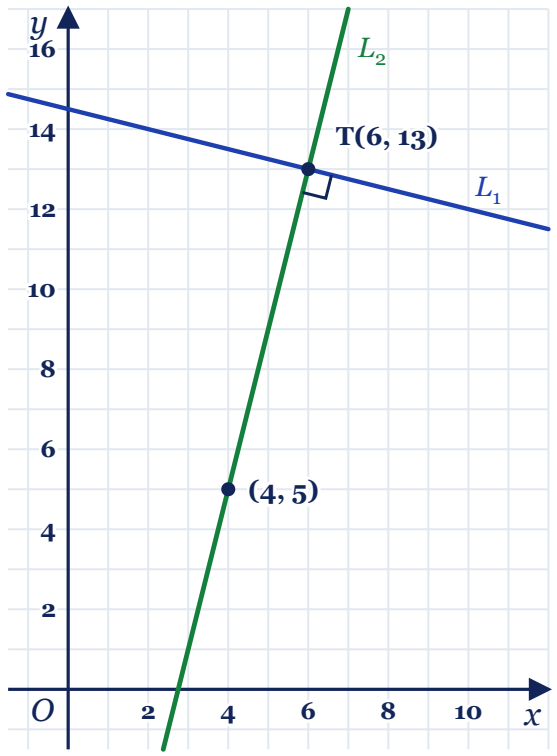
Step 4: Substitute y = 4x – 11 into x + 4y = 58.

$$x + 4(4x - 11) = 58$$
$$x + 16x - 44 = 58$$
$$17x = 102$$
$$x = 6$$

(Reason: T lies on both lines, so the expression for y from the second line can replace y in the first equation.)

Step 5: Find the y-coordinate of T.

$$y = 4(6) - 11 = 13$$
$$T(6, 13)$$



(Reason: substitute x = 6 into either equation; y = 4x − 11 is the simpler one.)

$$T(6, 13)$$

Verification

✓ **Check 1:** $T(6, 13)$ lies on both lines: $6 + 4(13) = 58$ and $4(6) - 11 = 13$.

✓ **Check 2:** Equating the two lines in the form $y = mx + c$ gives the same value of x: $-\frac{1}{4}x + \frac{29}{2} = 4x - 11$, so $\frac{17}{4}x = \frac{51}{2}$ and $x = 6$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $-\frac{1}{4}$	**M1**	Method - gradient of L_1	✓
Gradient of L_2: 4	**M1**	Method - perpendicular gradient	✓
$y = 4x - 11$	**A1**	Accuracy - equation of L_2	✓

Step	Mark	Description	Got it?
Solve $x + 4y = 58$ and $y = 4x - 11$ simultaneously	**M1**	Method - solve simultaneously	✓
$T(6, 13)$	**A1**	Accuracy - coordinates of T	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

L_1 has equation $3y - x = 21$ and passes through $P(3, 8)$ and Q.

L_2 is perpendicular to L_1, passes through P and crosses the y-axis at R. RQ is horizontal.

Q is given by name only: it is fixed by lying on L1 and on the horizontal line through R.

Find

The coordinates of Q.

Plan the Solution

- Rearrange $3y - x = 21$ into the form $y = mx + c$ to read off the gradient of L_1.
- Take the negative reciprocal for the gradient of L_2, then substitute $P(3, 8)$ to find its equation. Its value of c gives R.
- RQ is horizontal, so Q has the same y-coordinate as R. Substitute it into the equation of L_1 to find the x-coordinate of Q.

Worked Solution [5 marks]

> **Rule - A horizontal line:** every point on a horizontal line has the same y-coordinate, so the horizontal line through $(0, c)$ has equation $y = c$. A line crosses the y-axis where $x = 0$, at the point $(0, c)$.

Step 1: Rearrange 3y – x = 21 to find the gradient of L1.

$3y = x + 21$

$$y = \frac{1}{3}x + 7$$

$$m_1 = \frac{1}{3}$$

(Reason: adding x to both sides and dividing every term by 3 makes y the subject; the gradient is then the coefficient of x.)

Step 2: Find the gradient of L2.

$$\frac{1}{3} \times m_2 = -1$$

$$m_2 = -3$$

(Reason: perpendicular gradients multiply to give –1, so m_2 is the negative reciprocal of $\frac{1}{3}$.)

Step 3: Substitute P(3, 8) to find the equation of L2.

$$y = -3x + c$$

$$8 = -3(3) + c$$

$$8 = -9 + c$$

$$c = 17$$

$$y = -3x + 17$$

(Reason: P lies on L2, so its coordinates satisfy the equation of L2.)

Step 4: Find R, then the y-coordinate of Q.

$$R(0, 17)$$

$$y_Q = 17$$

(Reason: R is on the y-axis, so its x-coordinate is 0 and its y-coordinate is the value of c for L2, 17. RQ is horizontal, so Q has the same y-coordinate as R.)

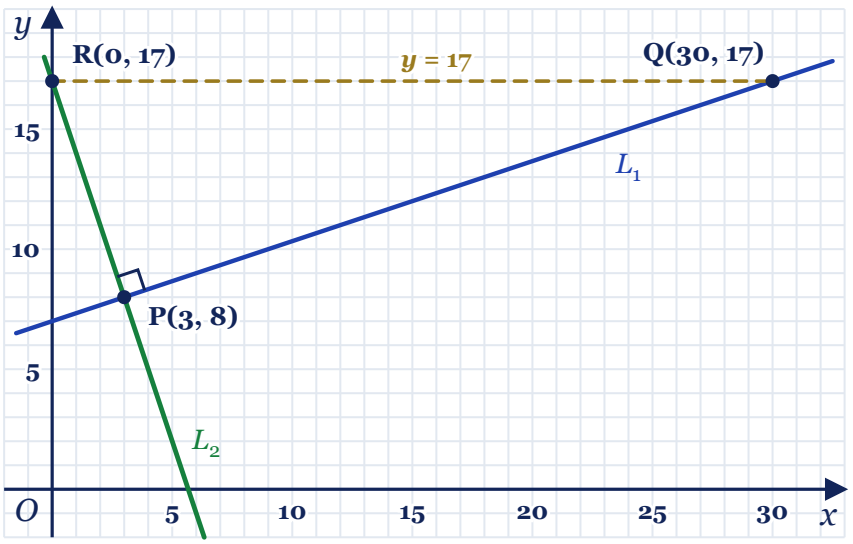
Step 5: Substitute y = 17 into the equation of L1.

$$3(17) - x = 21$$

$$51 - x = 21$$

$$x = 30$$

$$Q(30, 17)$$



(Reason: Q lies on L1, so its coordinates satisfy 3y − x = 21.)

$$Q(30, 17)$$

Verification

✓ **Check 1:** $Q(30, 17)$ lies on L_1: $3(17) - 30 = 21$. It also lies on the horizontal line through $R(0, 17)$, because both points have $y = 17$.

✓ **Check 2:** Build L_2 another way. The line $3(x - 3) + (y - 8) = 0$ passes through $P(3, 8)$, because both brackets are 0 there, and it rearranges to $y = -3x + 17$: the line through P with gradient -3, the same equation as before.

✓ **Check 3:** $\frac{1}{3} \times (-3) = -1$, so L_1 and L_2 are perpendicular.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $\frac{1}{3}$	M1	Method - gradient of L_1	✓
Gradient of L_2: -3	M1	Method - perpendicular gradient	✓
$y = -3x + 17$	A1	Accuracy - equation of L_2	✓

Step	Mark	Description	Got it?
From $R(0, 17)$ and RQ horizontal, Q has $y = 17$	B1	Independent - y-coordinate of Q	✓
$Q(30, 17)$	A1	Accuracy - coordinates of Q	✓

Full marks: 5/5

Question 12 - Exam Solution

Understanding the Question

Given

$A(2, 9)$, $B(6, 1)$, $C(11, 2)$ and $D(5, 10)$.

The claim: the line perpendicular to AB through the midpoint of AB passes through the midpoint of CD.

The line in the claim is the perpendicular bisector of AB. The claim has to be tested, not assumed.

Find

Whether the student is correct, with a reason.

Plan the Solution

- Find the midpoint and the gradient of AB.
- Take the negative reciprocal for the gradient of the perpendicular line, then substitute the midpoint of AB to find its equation.
- Find the midpoint of CD and substitute its x-coordinate into the equation. The student is correct only if the line gives the same y-coordinate.

Worked Solution [4 marks]

> **Rule - Testing a point on a line:** a point lies on a line only if its coordinates satisfy the equation of the line. Substitute the x-coordinate of the point and compare the value of y the line gives with the y-coordinate of the point.

Step 1: Find the midpoint and the gradient of AB.

$$M_{AB} = \left(\frac{2+6}{2},\ \frac{9+1}{2}\right) = (4, 5)$$

$$m_{AB} = \frac{1-9}{6-2} = \frac{-8}{4} = -2$$

(Reason: the midpoint is halfway between A and B in each direction, so take the mean of each pair of coordinates; the gradient is the change in y divided by the change in x.)

Step 2: Find the equation of the perpendicular line.

$$m = \frac{1}{2}$$

$$y = \frac{1}{2}x + c$$

$$5 = \frac{1}{2}(4) + c$$

$$c = 3$$

$$y = \frac{1}{2}x + 3$$

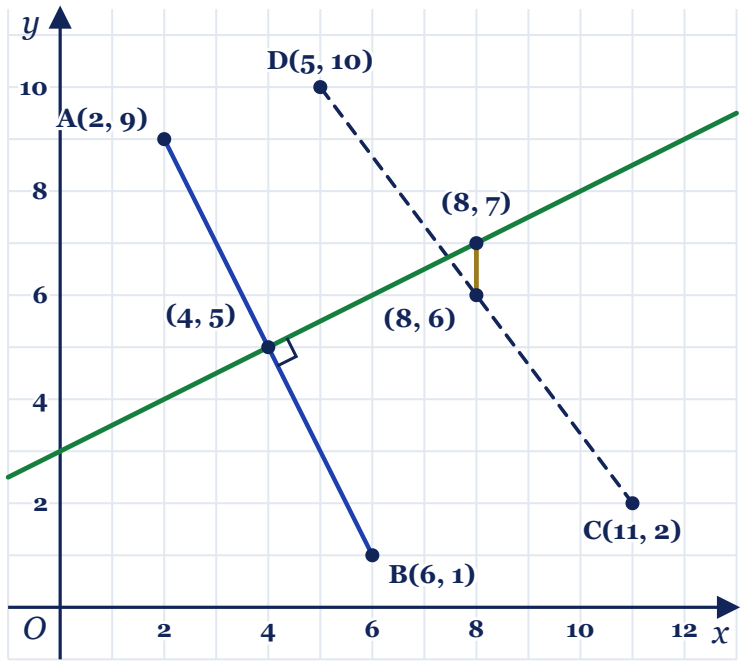
(Reason: the line is perpendicular to AB, so its gradient is the negative reciprocal of -2, and it passes through the midpoint (4, 5), so this point satisfies its equation.)

Step 3: Find the midpoint of CD and test it.

$$M_{CD} = \left(\frac{11+5}{2},\ \frac{2+10}{2}\right) = (8, 6)$$

$$x = 8:\ y = \frac{1}{2}(8) + 3 = 7$$

$$7 \neq 6$$



(Reason: the claim holds only if the line passes through (8, 6). At x = 8 the line is at y = 7, one unit above the midpoint of CD, so the line misses it.)

The student is not correct. The line passes through $(8, 7)$, but the midpoint of CD is $(8, 6)$.

Verification

✓ **Check 1:** Every point on the perpendicular bisector of AB is the same distance from A as from B. From $(8, 6)$, the squared distances are $(8-2)^2 + (6-9)^2 = 45$ to A and $(8-6)^2 + (6-1)^2 = 29$ to B. They are not equal, so $(8, 6)$ is not on the line.

✓ **Check 2:** The point the line does reach at $x = 8$, $(8, 7)$, passes the same test: $(8-2)^2 + (7-9)^2 = 40$ and $(8-6)^2 + (7-1)^2 = 40$. So the line is right, and it is the midpoint of CD that lies off it.

✓ **Check 3:** From the midpoint of AB, $(4, 5)$, to the midpoint of CD, $(8, 6)$, the gradient is $\dfrac{6-5}{8-4} = \dfrac{1}{4}$, not $\dfrac{1}{2}$, so the two midpoints are not joined by the perpendicular line.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Midpoint of AB: $(4, 5)$, and gradient of AB: -2	M1	Method - midpoint and gradient of AB	✓
Perpendicular gradient $\frac{1}{2}$, and $y = \frac{1}{2}x + 3$	M1	Method - perpendicular gradient and equation	✓
Midpoint of CD: $(8, 6)$, and substitutes $x = 8$ into the line to get $y = 7$	M1	Method - midpoint of CD and substitution	✓
Not correct, with a reason: the line passes through $(8, 7)$, not $(8, 6)$	A1	Accuracy - decision with a reason	✓

Full marks: 4/4

Question 13 - Exam Solution

Understanding the Question

Given

L_1 has equation $3x + 4y = 24$, and L_2 is parallel to it through $(3, 10)$.

L_3 is perpendicular to both and meets L_1 at $(4, 3)$.

Only L1 is given as an equation. L2 and L3 have to be built first, then solved together.

Find

The coordinates of the point where L_2 and L_3 intersect.

Plan the Solution

- Rearrange $3x + 4y = 24$ to read off the gradient of L_1. L_2 is parallel, so it has the same gradient: substitute $(3, 10)$ to find its equation.
- Take the negative reciprocal for the gradient of L_3, then substitute $(4, 3)$ to find its equation.
- Set the two expressions for y equal and solve. Multiply through by 12 first to clear the fractions.

Worked Solution [6 marks]

> **Rule - Parallel and perpendicular lines:** parallel lines have equal gradients, and the gradients of perpendicular lines multiply to give -1. The point where two lines meet satisfies both equations at once.

Step 1: Rearrange 3x + 4y = 24 to find the gradient of L1.

$$4y = -3x + 24$$

$$y = -\frac{3}{4}x + 6$$

$$m_1 = -\frac{3}{4}$$

(Reason: with y as the subject, the gradient is the coefficient of x.)

Step 2: Substitute (3, 10) to find the equation of L2.

$$y = -\frac{3}{4}x + c_2$$

$$10 = -\frac{3}{4}(3) + c_2$$

$$10 = -\frac{9}{4} + c_2$$

$$c_2 = \frac{49}{4}$$

$$y = -\frac{3}{4}x + \frac{49}{4}$$

(Reason: L2 is parallel to L1, so it has the same gradient, and (3, 10) lies on L2, so its coordinates satisfy the equation of L2.)

Step 3: Find the gradient of L3.

$$-\frac{3}{4} \times m_3 = -1$$

$$m_3 = \frac{4}{3}$$

(Reason: perpendicular gradients multiply to give –1, so m_3 is the negative reciprocal of $-\frac{3}{4}$.)

Step 4: Substitute (4, 3) to find the equation of L3.

$$y = \frac{4}{3}x + c_3$$

$$3 = \frac{4}{3}(4) + c_3$$

$$3 = \frac{16}{3} + c_3$$

$$c_3 = -\frac{7}{3}$$

$$y = \frac{4}{3}x - \frac{7}{3}$$

(Reason: L3 meets L1 at (4, 3), so this point lies on L3 and satisfies its equation.)

Step 5: Set the equations of L2 and L3 equal and solve for x.

$$-\frac{3}{4}x + \frac{49}{4} = \frac{4}{3}x - \frac{7}{3}$$

$$-9x + 147 = 16x - 28$$

$$175 = 25x$$

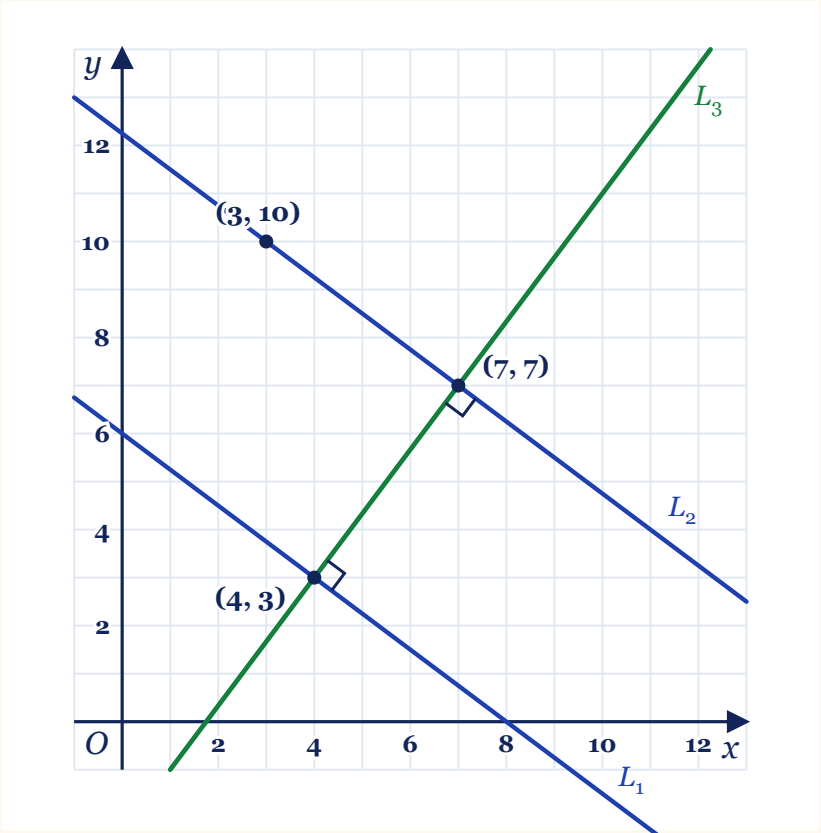
$$x = 7$$

(Reason: the meeting point lies on both lines, so both equations give the same y there. Multiplying every term by 12, the lowest common multiple of 4 and 3, clears the fractions.)

Step 6: Find the y-coordinate of the meeting point.

$$y = \frac{4}{3}(7) - \frac{7}{3} = \frac{28}{3} - \frac{7}{3} = 7$$

$$(7, 7)$$



(Reason: substitute x = 7 into either equation; here it goes into the equation of L3.)

$(7, 7)$

Verification

✓ **Check 1:** $(7, 7)$ lies on L_2. L_2 is parallel to L_1, so its equation has the form $3x + 4y = k$, and $3(7) + 4(7) = 49 = 3(3) + 4(10)$. It lies on L_3 too: $\frac{4}{3}(7) - \frac{7}{3} = 7$.

✓ **Check 2:** Write L_3 another way, as $4(x - 4) - 3(y - 3) = 0$, which passes through $(4, 3)$ and simplifies to $4x - 3y = 7$. Solving $3x + 4y = 49$ and $4x - 3y = 7$ by elimination gives $25x = 175$, so $x = 7$ and $y = 7$, the same point.

✓ **Check 3:** The step from $(4, 3)$ to $(7, 7)$ is 3 across and 4 up, a gradient of $\frac{4}{3}$, and $\frac{4}{3} \times \left(-\frac{3}{4}\right) = -1$, so the step crosses L_1 and L_2 at right angles.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $-\frac{3}{4}$	**M1**	Method - gradient of L_1	✓
$y = -\frac{3}{4}x + \frac{49}{4}$ oe	**M1**	Method - equation of L_2	✓
Gradient of L_3: $\frac{4}{3}$	**M1**	Method - perpendicular gradient	✓
$y = \frac{4}{3}x - \frac{7}{3}$ oe	**A1**	Accuracy - equation of L_3	✓
Equate L_2 and L_3 and solve, for example $-9x + 147 = 16x - 28$	**M1**	Method - equate and solve	✓
$(7, 7)$	**A1**	Accuracy - coordinates of the meeting point	✓

Full marks: 6/6

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