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Equation of a Line

IGCSE Exam-Style Questions with Worked Solutions

GRADE 8 TARGETED

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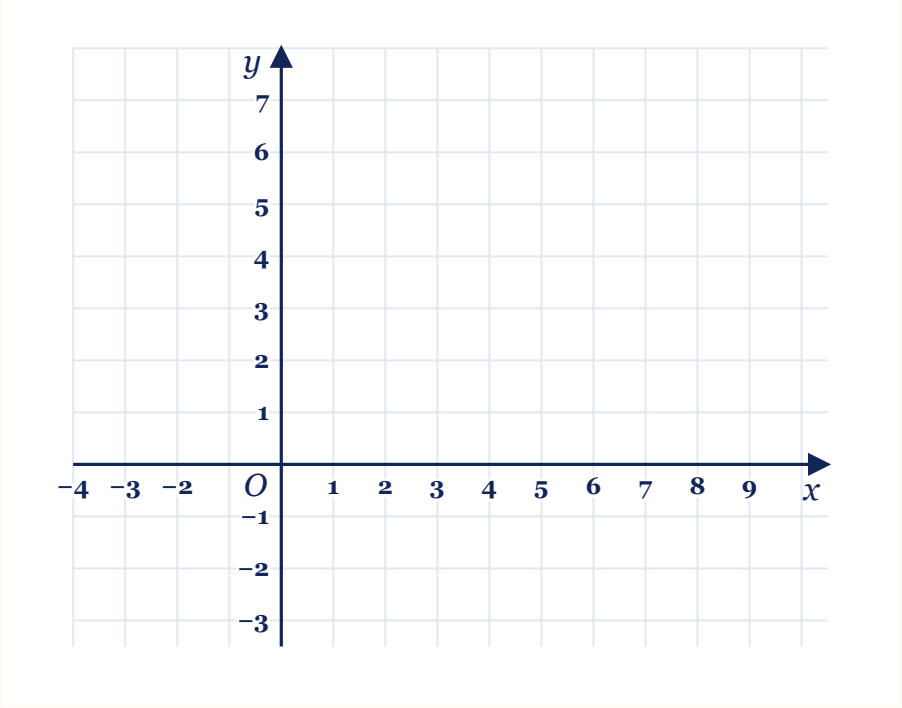
17 exam-style questions, 85 marks, grades 3 to 8. Full worked solutions and mark schemes.

Exam-Style Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Draw the graph of $3y + 2x = 12$ for values of x from -3 to 9. *[3 marks]*

GRADE 3



[Total 3 marks]

2. Point A has coordinates $(7, 3)$ and point B has coordinates $(-5, 8)$.

(a) Find the coordinates of the midpoint of AB. *[2 marks]*

(b) Point C has coordinates (p, q). The midpoint of AC is $(2, -1)$. Find the values of p and q. *[3 marks]*

GRADE 3

(a) (..... , ...)

(b) $p =$ ..... $q =$ .....

[Total 5 marks]

3. A straight line has equation $4x + 2y = 9$.

GRADE 5



(a) Find the gradient of the line. *[2 marks]*

(b) Find the equation of the line that is parallel to this line and passes through the point $(3, -1)$. *[3 marks]*

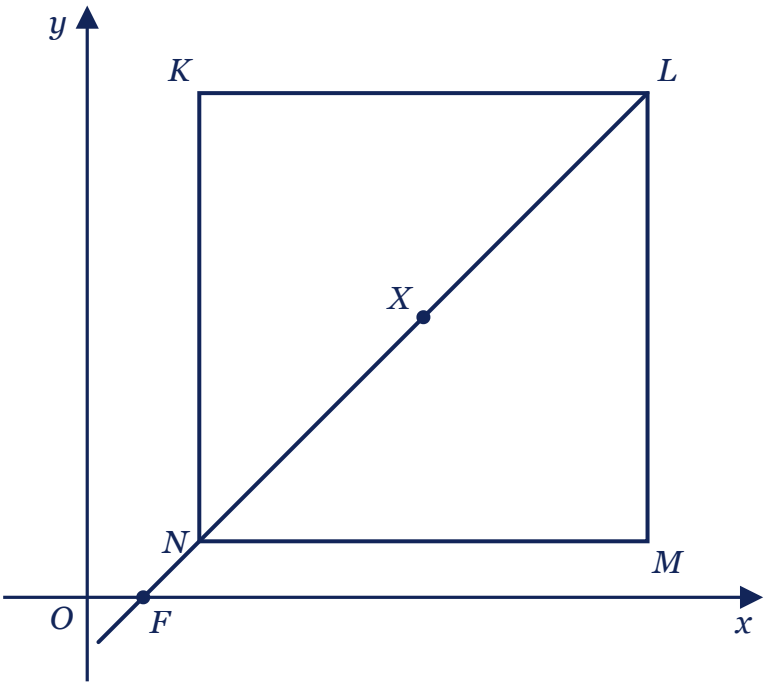
(a) .....

(b) .....

[Total 5 marks]

4.

GRADE 5



Not drawn accurately

$KLMN$ is a square with side length 8 units. The sides of the square are parallel to the axes.

The coordinates of N are $(2, 1)$.

X is the centre of the square.

Point F has coordinates $(1, 0)$, and F, N, X and L lie on a straight line.

Find the ratio $FX : XL$. Give your answer in its simplest form.
[3 marks]

..... : .....

[Total 3 marks]

5. P has coordinates $(-4, 1)$ and Q has coordinates $(8, 6)$.

GRADE 6



(a) Calculate the length of PQ. *[3 marks]*

(b) PQ is a diameter of a circle. Write down the coordinates of the centre of the circle and the radius of the circle. *[2 marks]*

(a) .....

(b) centre (..... , ...)

radius .....

[Total 5 marks]

6. Line L_1 passes through the points $(2, 1)$ and $(5, 13)$.

GRADE 6



Line L_2 is perpendicular to L_1 and crosses the x-axis at $(12, 0)$.

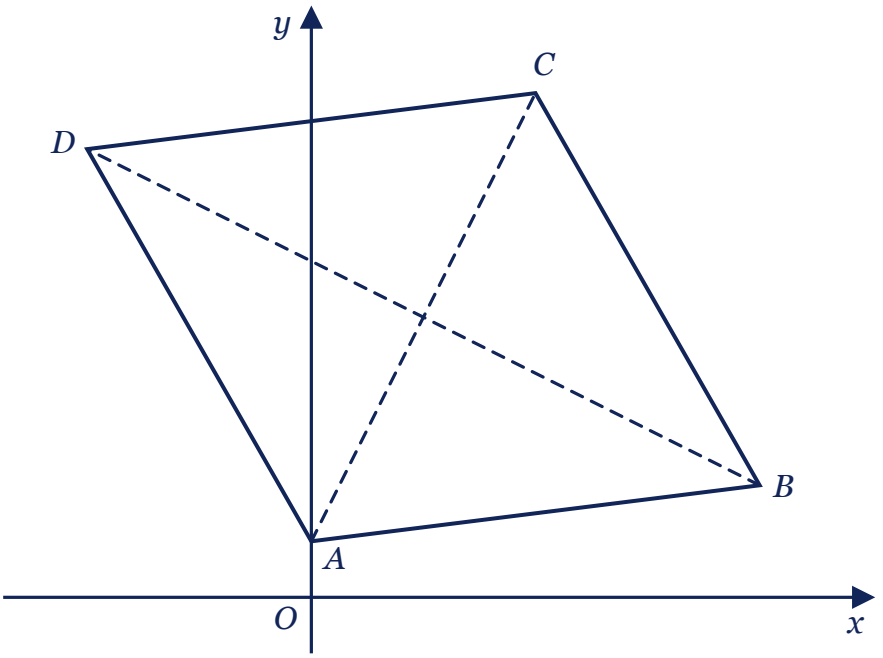
Find the equation of line L_2. *[3 marks]*

.....

[Total 3 marks]

7.

GRADE 6



Not drawn accurately

$ABCD$ is a rhombus. The diagonal AC has equation $y = 2x + 1$.

The point B has coordinates $(8, 2)$.

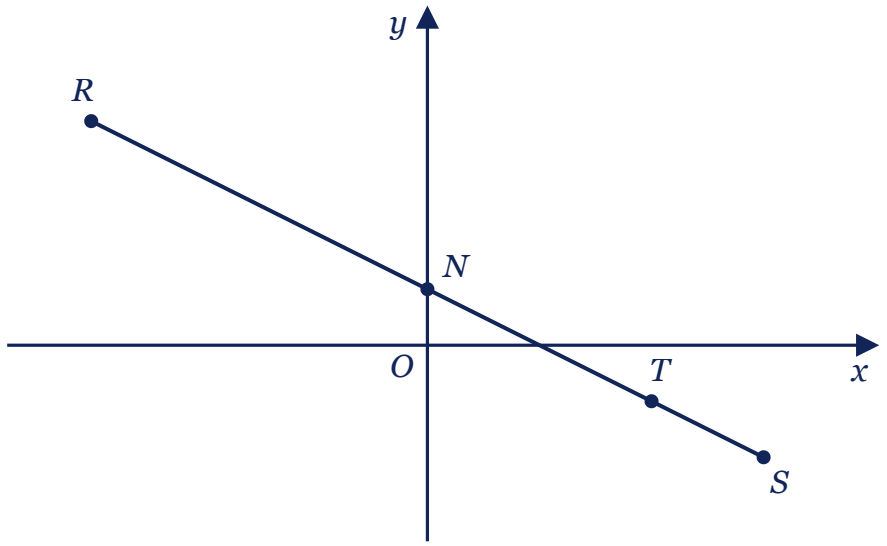
Find the equation of the other diagonal, BD. *[3 marks]*

.....

[Total 3 marks]

8.

GRADE 6



Not drawn accurately

N is the midpoint of the line segment RS. R has coordinates $(-6, 4)$ and N has coordinates $(0, 1)$.

(a) Find the coordinates of S. *[2 marks]*

Point T lies on RS such that $NT : TS = 2 : 1$.

(b) Find the ratio $RT : RS$. *[3 marks]*

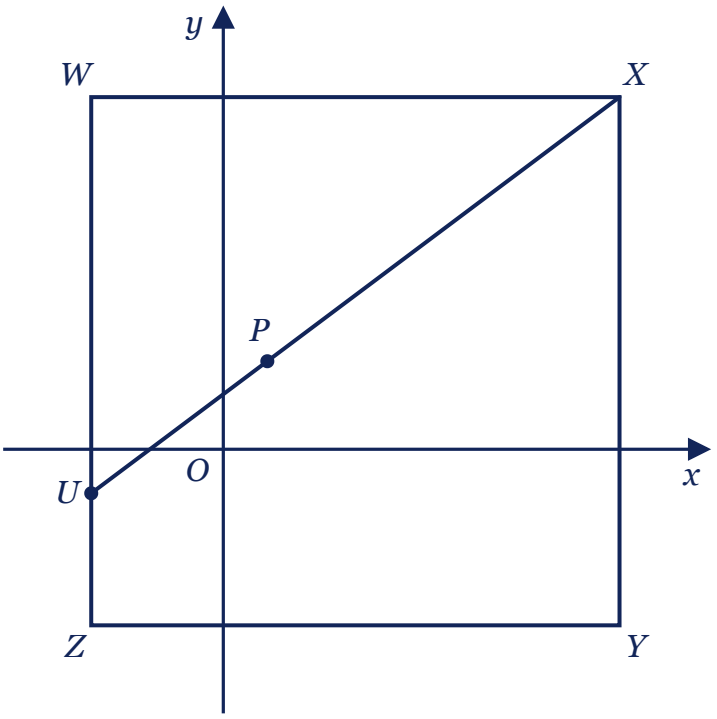
(a) (..... , ...)

(b) ..... : .....

[Total 5 marks]

9.

GRADE 6



Not drawn accurately

The diagram shows rectangle $WXYZ$.

Point U lies on WZ and has coordinates $(-3, -1)$. Point X has coordinates $(9, 8)$ and point Y has coordinates $(9, -4)$.

P is the point on the line UX such that $UP : PX = 1 : 2$.

Calculate the length of the line segment PY. *[4 marks]*

..... units

[Total 4 marks]

10. A has coordinates $(-2, 7)$ and B has coordinates $(6, 3)$.

Find the equation of the perpendicular bisector of AB. *[5 marks]*

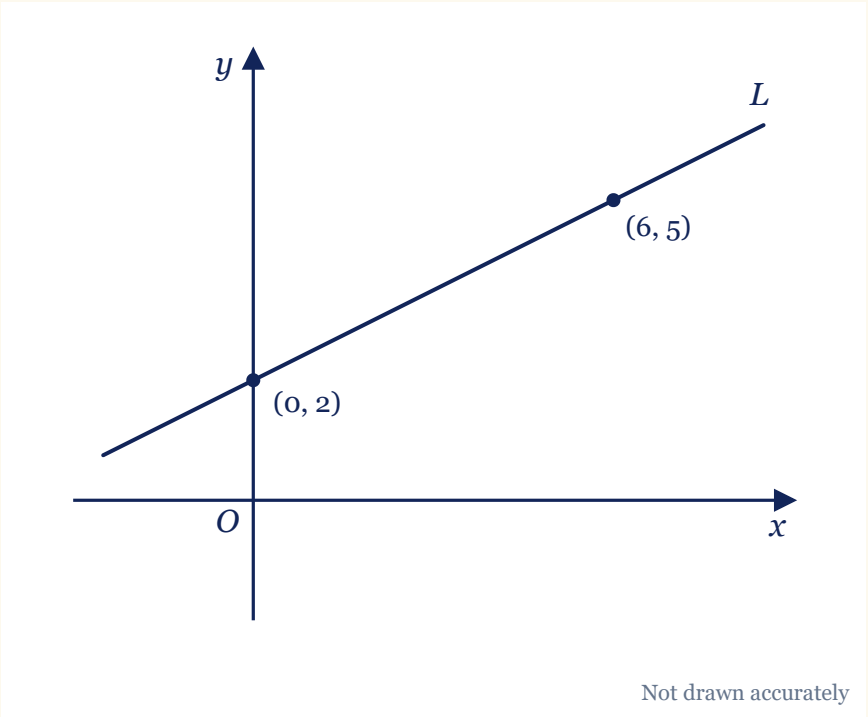
GRADE 7

.....

[Total 5 marks]

11.

GRADE 7



Not drawn accurately

Line L passes through the points $(0, 2)$ and $(6, 5)$, as shown in the diagram.

(a) Find the equation of line L. *[3 marks]*

(b) Find the equation of the line that is parallel to line L and passes through the point $(4, 9)$. *[2 marks]*

(c) Find the equation of the line that is perpendicular to line L and passes through the point $(-4, 6)$. *[3 marks]*

(a) .....

(b) .....

(c) .....

[Total 8 marks]

12. The lines with equations $3y - 4x = 6$ and $2y - x = 14$ intersect at the point T.

GRADE 7



(a) Find the gradient of the line with equation $3y - 4x = 6$. *[3 marks]*

(b) Find the coordinates of point T. *[5 marks]*

(a) .....

(b) (..... , ...)

[Total 8 marks]

13. Lines L_1 and L_2 are perpendicular and intersect at the point H.

GRADE 7



L_1 has equation $x + 3y = 30$ and L_2 passes through the point $(2, -4)$.

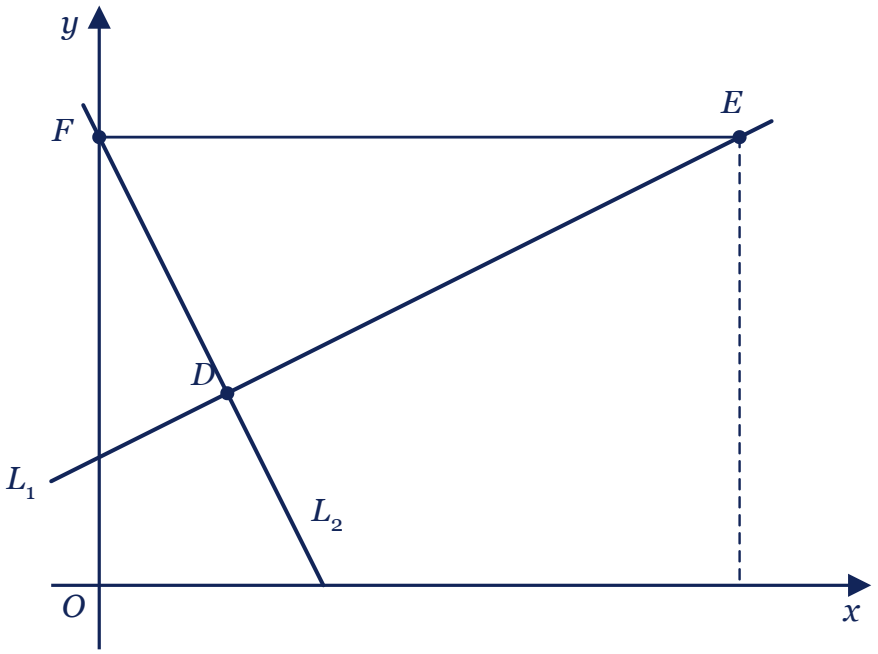
Find the coordinates of H. *[5 marks]*

(..... , ...)

[Total 5 marks]

14.

GRADE 7



Not drawn accurately

The line L_1 has equation $2y - x = 8$ and passes through the points $D(4, 6)$ and E.

L_2 is the line that is perpendicular to L_1 and passes through D.

L_2 intercepts the y-axis at F. FE is horizontal.

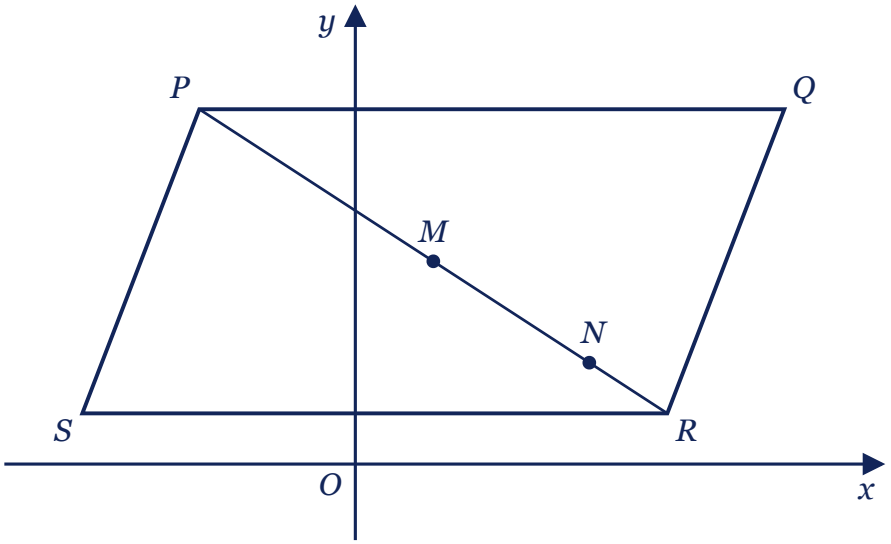
Find the coordinates of E. *[5 marks]*

(..... , ...)

[Total 5 marks]

15.

GRADE 7



Not drawn accurately

$PQRS$ is a parallelogram. The coordinates of P are $(-4, 7)$ and the coordinates of S are $(-7, 1)$.

M is the midpoint of the line PR and has coordinates $(2, 4)$.

Point N lies on the line PR such that $PM : MN : NR = 3 : 2 : 1$.

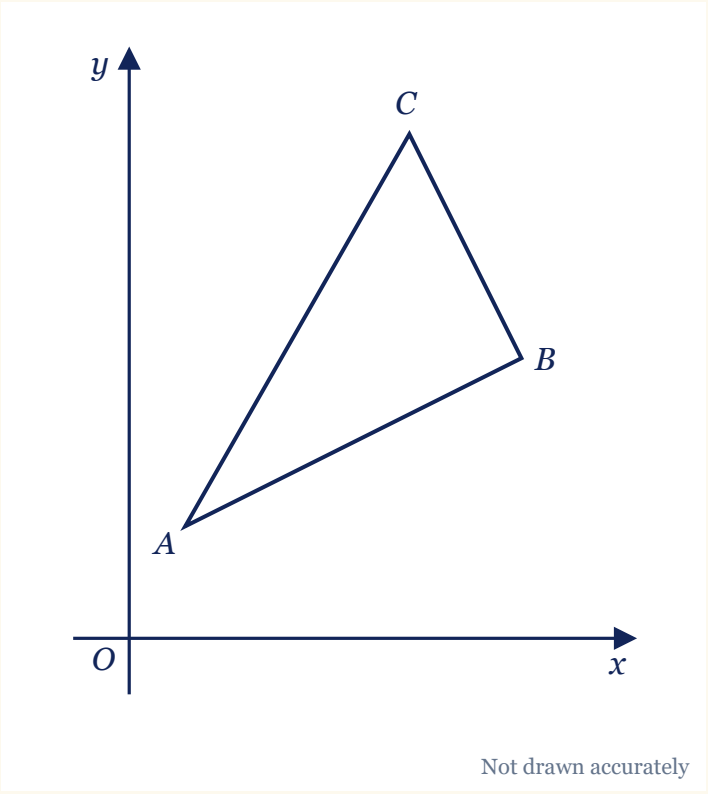
Find the exact length of NQ. *[6 marks]*

..... units

[Total 6 marks]

16.

GRADE 8



Not drawn accurately

Triangle ABC has vertices $A(1, 2)$, $B(7, 5)$ and $C(5, 9)$.

(a) Show that angle ABC is a right angle. *[3 marks]*

(b) Find the area of triangle ABC. *[3 marks]*

(b) ..... square units

[Total 6 marks]

17. Lines L_1 and L_2 are parallel.

L_1 has equation $3x + 2y = 30$ and L_2 passes through the point $(10, 13)$.

Line L_3 is perpendicular to L_1 and L_2 and intersects L_1 at $(6, 6)$.

Find the coordinates of the point of intersection of L_2 and L_3.
[6 marks]

GRADE 8

(..... , ...)

[Total 6 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy, B for an independent mark) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

The equation $3y + 2x = 12$.

Values of x from -3 to 9.

The equation is not in the form y = mx + c, so find points by substituting.

Find

The graph of the line for x from -3 to 9.

Plan the Solution

- Find where the line crosses each axis: put $x = 0$, then put $y = 0$.
- Find the points at the two ends of the range, $x = -3$ and $x = 9$.
- Plot the points, check that they lie in a straight line, and join them with a ruler from $x = -3$ to $x = 9$.

Worked Solution [3 marks]

> **Rule - Drawing a straight-line graph:** two points fix a straight line, and a third point checks it. Substitute a value of x (or of y) into the equation to find each point, plot the points and join them with a ruler.

Step 1: Put x = 0 to find where the line crosses the y-axis.

$3y + 2(0) = 12$

$3y = 12$

$y = 4$

$(0, 4)$

(Reason: every point on the y-axis has x = 0.)

Step 2: Put y = 0 to find where the line crosses the x-axis.

$3(0) + 2x = 12$

$2x = 12$

$x = 6$

$(6, 0)$

(Reason: every point on the x-axis has y = 0.)

Step 3: Find the points at the two ends, x = −3 and x = 9.

$x = -3: \ 3y + 2(-3) = 12$

$3y = 18, \ y = 6$

$x = 9: \ 3y + 2(9) = 12$

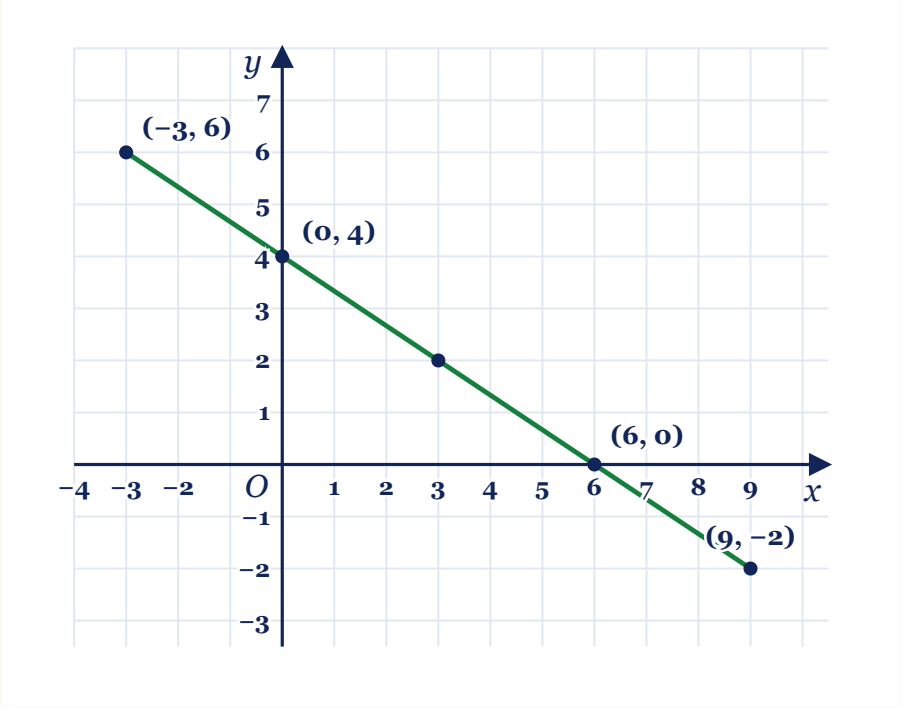
$3y = -6, \ y = -2$

$(-3, 6)$ and $(9, -2)$

(Reason: the graph is needed only for x from −3 to 9, so these two points are the ends of the line.)

Step 4: Plot the points and join them with a ruler.

$(-3, 6), \ (0, 4), \ (6, 0), \ (9, -2)$



(Reason: the four points lie in one straight line, which confirms that none of them is wrong. Draw the line from x = −3 to x = 9.)

The straight line through $(-3, 6), (0, 4), (6, 0)$ and $(9, -2)$

Verification

✓ **Check 1:** Rearranged, the equation is $y = 4 - \frac{2}{3}x$, which gives the same four points: $4 - \frac{2}{3}(-3) = 6, 4 - \frac{2}{3}(0) = 4, 4 - \frac{2}{3}(6) = 0$ and $4 - \frac{2}{3}(9) = -2$.

✓ **Check 2:** $(3, 2)$ is also on the line, because $3(2) + 2(3) = 12$, and it sits on the line drawn.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
One correct point plotted, for example $(0, 4)$	**B1**	Independent - one correct point plotted	✓
A second correct point plotted, for example $(6, 0)$	**B1**	Independent - a second correct point plotted	✓
Correct straight line from $x = -3$ to $x = 9$	**B1**	Independent - correct straight line over the full range	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

$A(7, 3)$ and $B(-5, 8)$.

$C(p, q)$, and the midpoint of AC is $(2, -1)$.

Part (b) works backwards, from a midpoint to an end point.

Find

The midpoint of AB, then the values of p and q.

Plan the Solution

- For part (a), add the two x-coordinates and halve, then do the same with the two y-coordinates.
- For part (b), write the midpoint of AC in terms of p and q.
- Set each coordinate equal to the given midpoint $(2, -1)$ and solve the two equations.

Worked Solution [5 marks]

Rule - Midpoint of a line segment: the midpoint of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$: the mean of the two x-coordinates and the mean of the two y-coordinates.

Step 1 (part a): Add the coordinates and halve.

$$\left(\frac{7 + (-5)}{2}, \frac{3 + 8}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{11}{2}\right)$$

$$= (1, 5.5)$$

(Reason: the midpoint is halfway between the two points, so each coordinate is the mean of the two values.)

Step 2 (part b): Use the x-coordinate of the midpoint of AC to find p.

$$\frac{7 + p}{2} = 2$$

$$7 + p = 4$$

$$p = -3$$

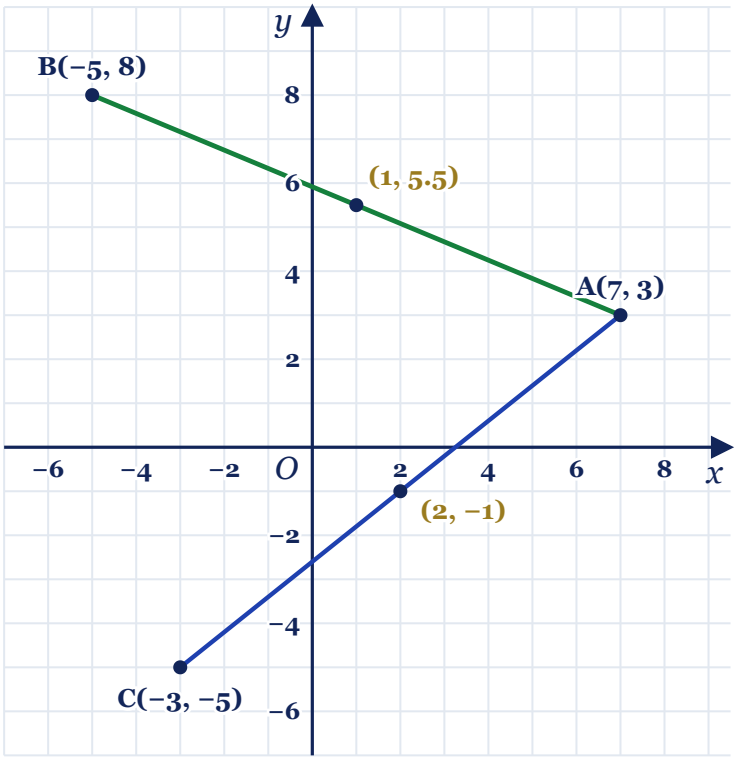
(Reason: the x-coordinate of the midpoint is the mean of 7 and p, and it is given as 2.)

Step 3 (part b): Use the y-coordinate of the midpoint of AC to find q.

$$\frac{3 + q}{2} = -1$$

$$3 + q = -2$$

$$q = -5$$



(Reason: the y-coordinate of the midpoint is the mean of 3 and q, and it is given as –1.)

(a) $(1, 5.5)$

(b) $p = -3,\ q = -5$

Verification

✓ **Check 1 (part a):** Go half of the way from A to B. The step from A to B is $(-12, 5)$, and half of it is $(-6, 2.5)$, so the midpoint is $(7 - 6,\ 3 + 2.5) = (1, 5.5)$.

✓ **Check 2 (part b):** The midpoint of $(7, 3)$ and $(-3, -5)$ is $\left(\frac{7 + (-3)}{2}, \frac{3 + (-5)}{2}\right) = (2, -1)$, as given.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): adds the coordinates and halves, $\frac{7 + (-5)}{2}$ or $\frac{3 + 8}{2}$	M1	Method - adds the coordinates and halves	✓

Step	Mark	Description	Got it?
Part (a): $(1, 5.5)$	A1	Accuracy - midpoint of AB	✓
Part (b): a correct method, $\frac{7+p}{2} = 2$ or $\frac{3+q}{2} = -1$	M1	Method - correct method for p or q	✓
Part (b): $p = -3$	A1	Accuracy - value of p	✓
Part (b): $q = -5$	A1	Accuracy - value of q	✓

Full marks: 5/5

Question 3 - Exam Solution

Understanding the Question

Given

A straight line with equation $4x + 2y = 9$.

A parallel line passes through $(3, -1)$.

The equation is not written in the form y = mx + c.

Find

The gradient of the line, then the equation of the parallel line.

Plan the Solution

- Rearrange $4x + 2y = 9$ to make y the subject, then read off the gradient.
- For part (b), a parallel line has the same gradient.
- Substitute $(3, -1)$ into $y = mx + c$ to find c.

Worked Solution [5 marks]

> **Rule - Parallel lines:** parallel lines have equal gradients. The gradient can be read off only when the equation is in the form $y = mx + c$, so make y the subject first.

Step 1 (part a): Make y the subject.

$4x + 2y = 9$

$2y = -4x + 9$

$$y = -2x + \frac{9}{2}$$

(Reason: subtract 4x from both sides, then divide every term by 2.)

Step 2 (part a): Read off the gradient.

$$m = -2$$

(Reason: in the form y = mx + c, the gradient is the number multiplying x.)

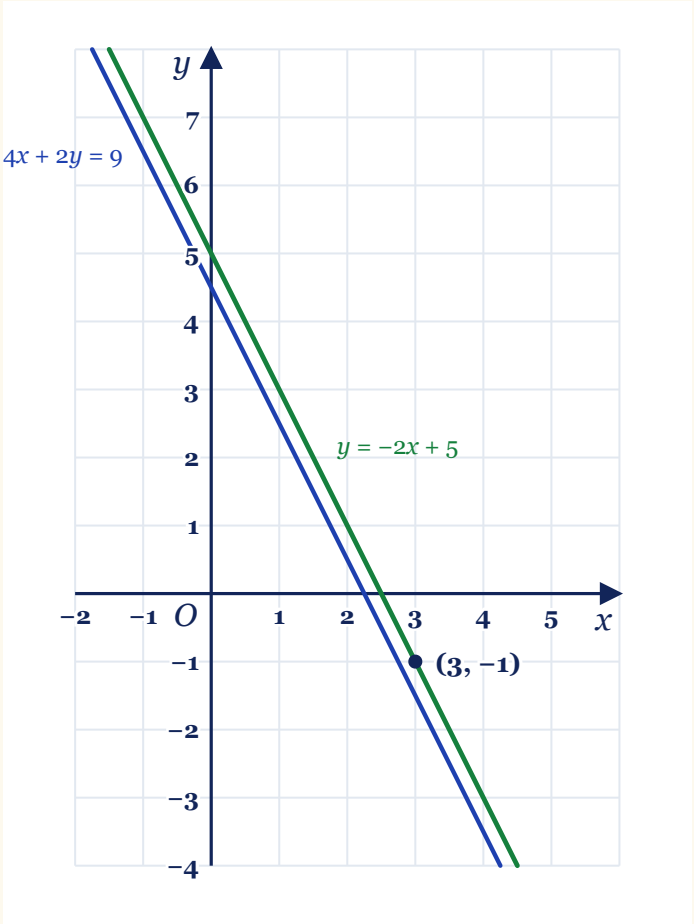
Step 3 (part b): Use the same gradient for the parallel line.

$$y = -2x + c$$

(Reason: parallel lines have equal gradients.)

Step 4 (part b): Substitute (3, −1) to find c.

$$-1 = -2(3) + c$$
$$-1 = -6 + c$$
$$c = -1 + 6 = 5$$



(Reason: (3, −1) lies on the new line, so its coordinates satisfy the equation.)

(a) $m = -2$

(b) $y = -2x + 5$

Verification

✓ **Check (part b):** A parallel line has the form $4x + 2y = k$. At $(3, -1)$, $k = 4(3) + 2(-1) = 10$, and $4x + 2y = 10$ rearranges to $y = -2x + 5$, the same equation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): makes y the subject, $2y = -4x + 9$ or $y = -2x + \frac{9}{2}$	M1	Method - makes y the subject	✓
Part (a): -2	A1	Accuracy - gradient of the line	✓
Part (b): uses gradient -2, $y = -2x + c$	M1	Method - uses the same gradient	✓
Part (b): substitutes $(3, -1)$, $-1 = -2(3) + c$	M1	Method - substitutes the point	✓
Part (b): $y = -2x + 5$ oe	A1	Accuracy - equation of the parallel line	✓

Full marks: 5/5

Question 4 - Exam Solution

Understanding the Question

Given

Square $KLMN$ with side length 8 units, sides parallel to the axes.

$N(2, 1)$ and $F(1, 0)$, with X the centre of the square.

Find

The ratio $FX : XL$ in its simplest form.

F, N, X and L all lie on one straight line.

Plan the Solution

- Use the side length to find the coordinates of L, the corner opposite N.
- The centre X is the midpoint of the diagonal NL.
- F, X and L are on one straight line, so compare the x-steps from F to X and from X to L, then simplify the ratio.

Worked Solution [3 marks]

> **Rule - Ratios along a straight line:** when points lie on one straight line, the distances between them are in the same ratio as their x-steps (or their y-steps), so you can compare the steps instead of working out each distance.

Step 1: Use the side length to find L, the corner opposite N.

$$L = (2 + 8,\ 1 + 8) = (10, 9)$$

(Reason: the straight line from N through the centre X reaches the opposite corner, so L is opposite N. The sides are parallel to the axes, so L is 8 units to the right of N and 8 units above it.)

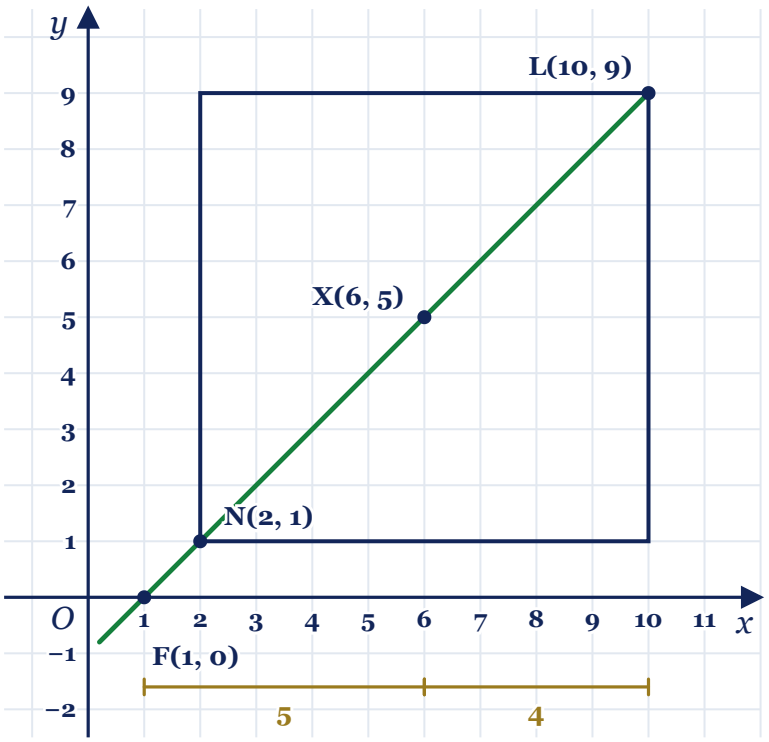
Step 2: Find the centre X, the midpoint of the diagonal NL.

$$X = \left(\frac{2 + 10}{2},\ \frac{1 + 9}{2}\right) = (6, 5)$$

(Reason: the diagonals of a square cross at its centre, so X is halfway along the diagonal from N to L.)

Step 3: Compare the x-steps from F to X and from X to L.

$$F \to X:\ 6 - 1 = 5$$
$$X \to L:\ 10 - 6 = 4$$
$$FX : XL = 5 : 4$$



(Reason: F, X and L lie on one straight line, so FX and XL are in the same ratio as their x-steps. 5 and 4 have no common factor, so the ratio is already in its simplest form.)

$$FX : XL = 5 : 4$$

Verification

✓ **Check 1:** Work out both lengths with Pythagoras: $FX = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$ and $XL = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$, so $FX : XL = 5\sqrt{2} : 4\sqrt{2} = 5 : 4$.

✓ **Check 2:** The line through $N(2, 1)$ and $L(10, 9)$ has gradient $\dfrac{9 - 1}{10 - 2} = 1$, so its equation is $y = x - 1$. At $x = 1$, $y = 0$, so $F(1, 0)$ lies on it, and so does $X(6, 5)$, because $6 - 1 = 5$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Coordinates of L and X: $L(10, 9)$ and $X(6, 5)$, both needed	**B1**	Independent - coordinates of L and X	✓

Step	Mark	Description	Got it?
Differences from F to X and from X to L: 5 and 4	M1	Method - compare the two steps	✓
$FX : XL = 5 : 4$	A1	Accuracy - ratio in its simplest form	✓

Full marks: 3/3

Question 5 - Exam Solution

Understanding the Question

Given

$P(-4, 1)$ and $Q(8, 6)$.

PQ is a diameter of a circle.

A length between two points, then a circle drawn on that length.

Find

The length of PQ, then the centre and the radius of the circle.

Plan the Solution

- Find the difference between the x-coordinates and the difference between the y-coordinates.
- These are the two shorter sides of a right-angled triangle with PQ as its hypotenuse, so use Pythagoras.
- For part (b), the centre is the midpoint of the diameter and the radius is half of the diameter.

Worked Solution [5 marks]

> **Rule - Length of a line segment:** the differences in x and in y are the two shorter sides of a right-angled triangle, so $\text{length}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

Step 1 (part a): Find the differences in x and in y.

$8 - (-4) = 12$

$6 - 1 = 5$

(Reason: going from P to Q is 12 across and 5 up, and these are the two shorter sides of a right-angled triangle.)

Step 2 (part a): Use Pythagoras.

$$PQ^2 = 12^2 + 5^2$$

$$PQ^2 = 144 + 25 = 169$$

$$PQ = \sqrt{169} = 13$$

(Reason: PQ is the hypotenuse of the triangle.)

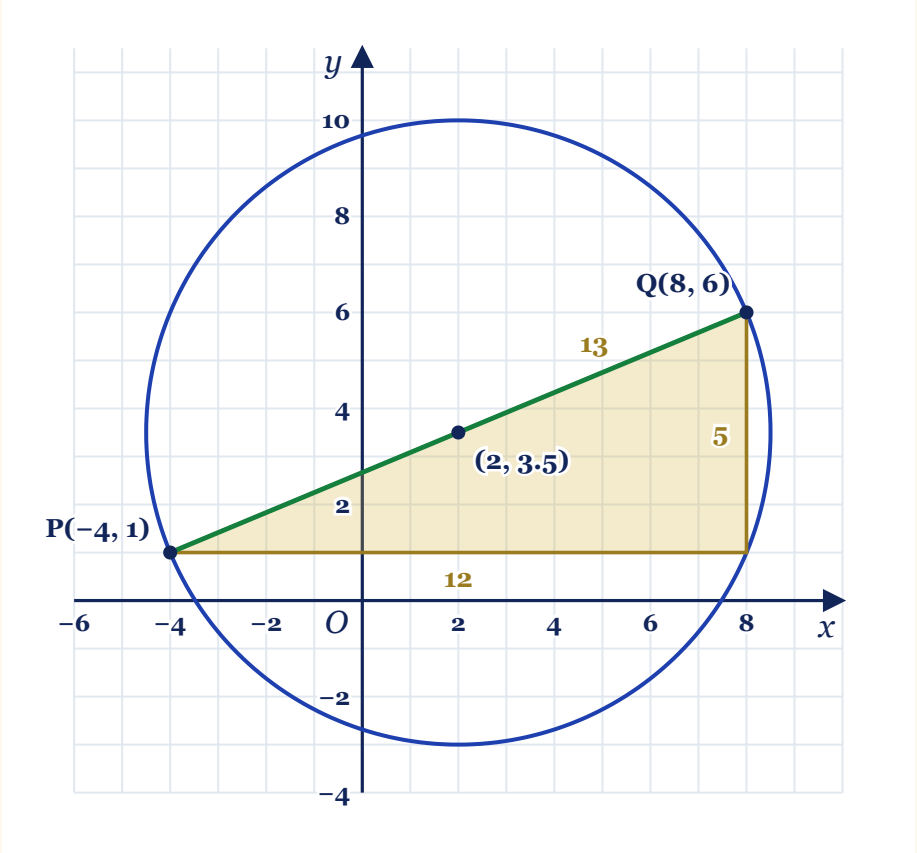
Step 3 (part b): The centre is the midpoint of PQ.

$$\left(\frac{-4+8}{2},\ \frac{1+6}{2}\right) = (2, 3.5)$$

(Reason: the centre of a circle is the midpoint of any diameter.)

Step 4 (part b): The radius is half of the diameter.

$$\frac{13}{2} = 6.5$$



(Reason: the diameter PQ is 13, and a radius is half of a diameter.)

(a) $PQ = 13$

(b) centre $(2, 3.5)$**, radius** 6.5

Verification

✓ **Check (part b):** From the centre $(2, 3.5)$ to $P(-4, 1)$ is 6 across and 2.5 down, and $6^2 + 2.5^2 = 36 + 6.25 = 42.25 = 6.5^2$, so P is 6.5 from the centre.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): differences 12 and 5	M1	Method - differences in x and in y	✓
Part (a): Pythagoras, $12^2 + 5^2$	M1	Method - Pythagoras	✓
Part (a): 13	A1	Accuracy - length of PQ	✓
Part (b): centre $(2, 3.5)$	B1	Independent - centre of the circle	✓
Part (b): radius 6.5	B1	Independent - radius of the circle	✓

Full marks: 5/5

Question 6 - Exam Solution

Understanding the Question

Given

L_1 passes through $(2, 1)$ and $(5, 13)$.

L_2 is perpendicular to L_1 and passes through $(12, 0)$.

The point (12, 0) is where the second line crosses the x-axis.

Find

The equation of L_2.

Plan the Solution

- Find the gradient of L_1 from its two points.
- Perpendicular gradients multiply to give -1, so take the negative reciprocal for the gradient of L_2.
- Substitute $(12, 0)$ into $y = mx + c$ to find c.

Worked Solution [3 marks]

> **Rule - Perpendicular lines:** the gradients of two perpendicular lines multiply to give -1, so each gradient is the negative reciprocal of the other: $m_2 = -\dfrac{1}{m_1}$.

Step 1: Find the gradient of the first line.

$$m_1 = \frac{13 - 1}{5 - 2} = \frac{12}{3} = 4$$

(Reason: the gradient is the change in y divided by the change in x, with the two points taken in the same order on the top and on the bottom.)

Step 2: Find the gradient of the perpendicular line.

$$4 \times m_2 = -1$$

$$m_2 = -\frac{1}{4}$$

(Reason: perpendicular gradients multiply to give –1, so m_2 is the negative reciprocal of 4: write 4 as $\frac{4}{1}$, turn the fraction upside down and change its sign.)

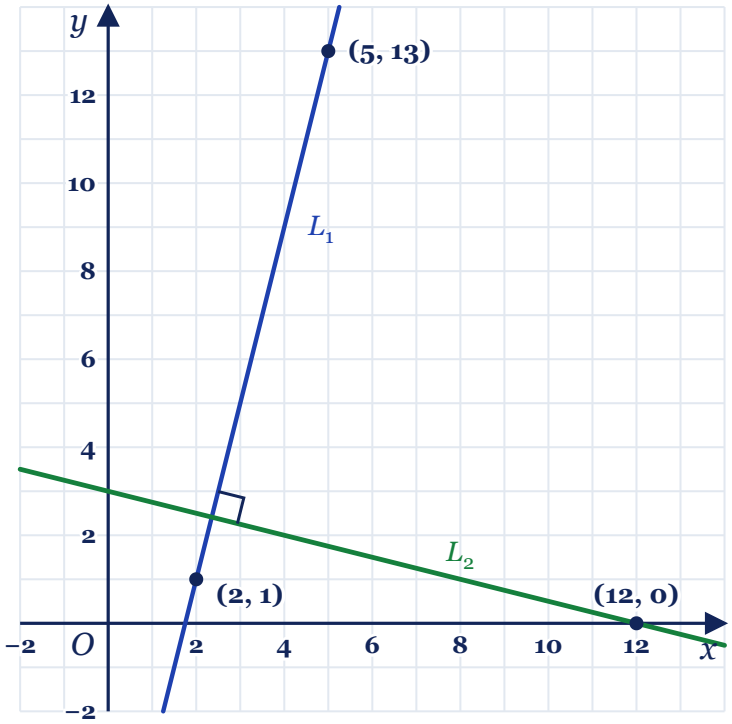
Step 3: Substitute (12, 0) to find c.

$$y = -\frac{1}{4}x + c$$

$$0 = -\frac{1}{4}(12) + c$$

$$0 = -3 + c$$

$$c = 3$$



(Reason: the second line crosses the x-axis at (12, 0), so this point lies on it and its coordinates satisfy the equation.)

$$y = -\frac{1}{4}x + 3$$

Verification

✓ **Check 1:** $m_1 \times m_2 = 4 \times \left(-\frac{1}{4}\right) = -1$, so the two lines are perpendicular.

✓ **Check 2:** Through $(12, 0)$ with gradient $-\frac{1}{4}$, the line is $y = -\frac{1}{4}(x - 12)$, which expands to $y = -\frac{1}{4}x + 3$, the same equation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $\frac{13-1}{5-2} = 4$	M1	Method - gradient of L_1	✓

Step	Mark	Description	Got it?
Gradient of L_2: $-\frac{1}{4}$	M1	Method - perpendicular gradient	✓
$y = -\frac{1}{4}x + 3$ oe	A1	Accuracy - equation of L_2	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

Rhombus $ABCD$, with the diagonal AC on the line $y = 2x + 1$.

The other diagonal, BD, passes through $B(8, 2)$.

Only one point on BD is given, so its gradient has to come from the other diagonal.

Find

The equation of the diagonal BD.

Plan the Solution

- Read off the gradient of AC from $y = 2x + 1$.
- The diagonals of a rhombus are perpendicular, so the gradient of BD is the negative reciprocal of the gradient of AC.
- Substitute $B(8, 2)$ into $y = mx + c$ to find c.

Worked Solution [3 marks]

> **Rule - The diagonals of a rhombus:** the diagonals of a rhombus cross at right angles, so their gradients multiply to give -1: $m_{BD} = -\frac{1}{m_{AC}}$.

Step 1: Read off the gradient of AC.

$y = 2x + 1$

$m_{AC} = 2$

(Reason: the equation is already in the form y = mx + c, so the gradient is the coefficient of x.)

Step 2: Use the perpendicular diagonals to find the gradient of BD.

$$2 \times m_{BD} = -1$$

$$m_{BD} = -\frac{1}{2}$$

(Reason: the diagonals of a rhombus are perpendicular, and perpendicular gradients multiply to give −1, so m_{BD} is the negative reciprocal of 2.)

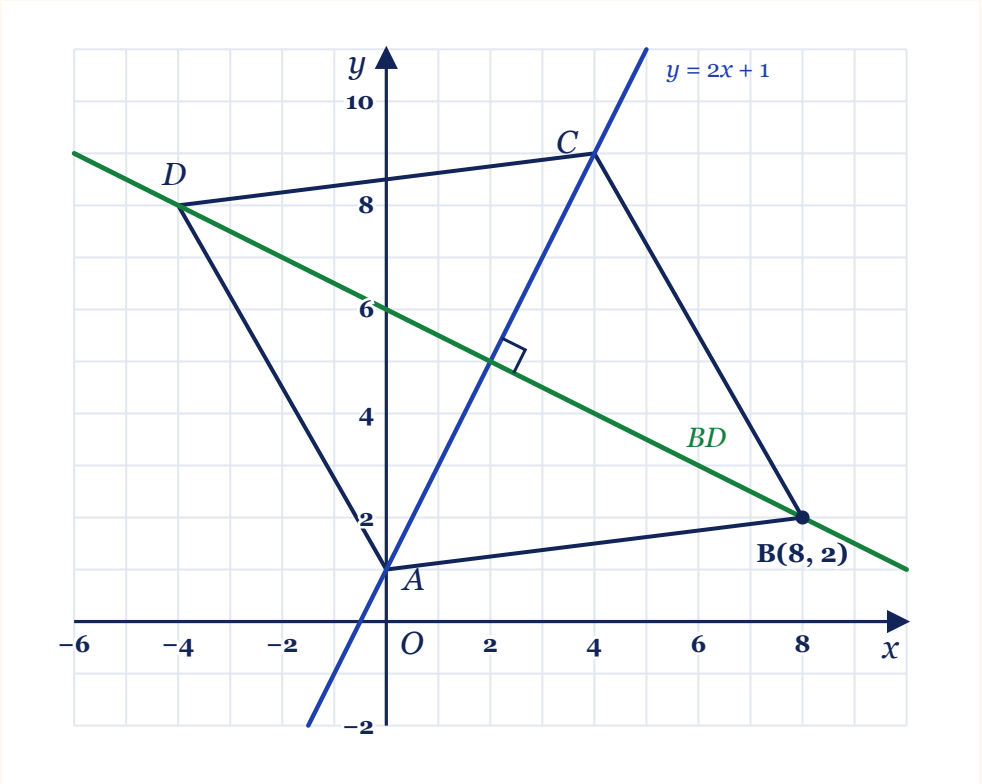
Step 3: Substitute B(8, 2) to find c.

$$y = -\frac{1}{2}x + c$$

$$2 = -\frac{1}{2}(8) + c$$

$$2 = -4 + c$$

$$c = 6$$



(Reason: B is one end of the diagonal BD, so its coordinates satisfy the equation of BD.)

$$y = -\frac{1}{2}x + 6$$

Verification

✓ **Check 1:** $m_{AC} \times m_{BD} = 2 \times \left(-\frac{1}{2}\right) = -1$, so the two diagonals are perpendicular.

✓ **Check 2:** Through $B(8, 2)$ with gradient $-\frac{1}{2}$, the line is $y - 2 = -\frac{1}{2}(x - 8)$. Multiplying both sides by 2 and collecting the terms on one side gives $(x - 8) + 2(y - 2) = 0$, which is $x + 2y = 12$, and this rearranges to $y = -\frac{1}{2}x + 6$, the same equation.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
States that the diagonals of a rhombus are perpendicular	**B1**	Independent - rhombus property	✓
Gradient $-\frac{1}{2}$, and substitute $B(8, 2)$ into $y = -\frac{1}{2}x + c$	**M1**	Method - perpendicular gradient and substitution	✓
$y = -\frac{1}{2}x + 6$ oe	**A1**	Accuracy - equation of BD	✓

Full marks: 3/3

Question 8 - Exam Solution

Understanding the Question

Given

$R(-6, 4)$ and $N(0, 1)$, where N is the midpoint of RS.

T lies on RS with $NT : TS = 2 : 1$.

A midpoint, then a point that divides NS in a given ratio.

Find

The coordinates of S, then the ratio $RT : RS$.

Plan the Solution

- For part (a), N is halfway from R to S, so the step from N to S is the same as the step from R to N.
- For part (b), $NT : TS = 2 : 1$ splits NS into 3 equal parts, so T is two thirds of the way from N to S.
- Find T, compare the x-steps from R to T and from R to S, and simplify.

Worked Solution [5 marks]

> **Rule - Midpoints and ratios on a line segment:** a midpoint is halfway, so the step from R to N is repeated from N to S. A point that divides a segment in the ratio $p : q$ is $\dfrac{p}{p+q}$ of the way along it, so add that fraction of each step to the starting point.

Step 1 (part a): Find the step from R to N, then repeat it from N.

$R \to N: \ (0-(-6),\ 1-4) = (6, -3)$

$S = (0+6,\ 1-3) = (6, -2)$

(Reason: N is the midpoint, so S is as far beyond N as N is beyond R.)

Step 2 (part b): Take two thirds of the step from N to S.

$N \to S: \ (6-0,\ -2-1) = (6, -3)$

$\dfrac{2}{3}$ of $(6, -3) = (4, -2)$

$T = (0+4,\ 1-2) = (4, -1)$

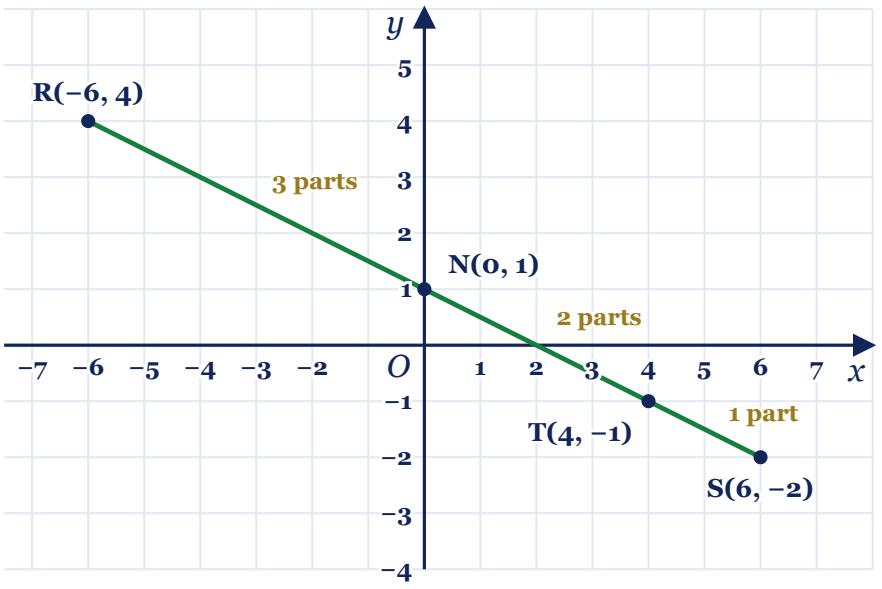
(Reason: NT : TS = 2 : 1 has 2 + 1 = 3 parts, and T is 2 parts along from N, so T is two thirds of the way from N to S.)

Step 3 (part b): Compare the x-steps from R to T and from R to S.

$R \to T: \ 4-(-6) = 10$

$R \to S: \ 6-(-6) = 12$

$RT : RS = 10 : 12 = 5 : 6$



(Reason: R, T and S lie on one straight line, so RT and RS are in the same ratio as their x-steps; dividing both by 2 gives the simplest form.)

(a) $S(6, -2)$

(b) $RT : RS = 5 : 6$

Verification

✓ **Check 1 (part b):** Count equal parts. $RN = NS$ and NS is $2 + 1 = 3$ parts, so $RN = 3$, $NT = 2$ and $TS = 1$. Then $RT = 3 + 2 = 5$ parts and $RS = 3 + 2 + 1 = 6$ parts, giving $5 : 6$.

✓ **Check 2 (part b):** The y-steps give the same ratio: $R \to T:\ -1 - 4 = -5$ and $R \to S:\ -2 - 4 = -6$. Both steps go down, by 5 and by 6, so $RT : RS = 5 : 6$.

✓ **Check 3 (part a):** The midpoint of $R(-6, 4)$ and $S(6, -2)$ is $\left(\frac{-6+6}{2}, \frac{4+(-2)}{2}\right) = (0, 1)$, which is N.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): $x = 6$	B1	Independent - x-coordinate of S	✓

Step	Mark	Description	Got it?
Part (a): $y = -2$	**B1**	Independent - y-coordinate of S	✓
Part (b): a correct method for T, for example $(0, 1) + \frac{2}{3}(6, -3)$	**M1**	Method - find T	✓
Part (b): $T(4, -1)$	**A1**	Accuracy - coordinates of T	✓
Part (b): $RT : RS = 5 : 6$	**A1**	Accuracy - ratio in its simplest form	✓

Full marks: 5/5

Question 9 - Exam Solution

Understanding the Question

Given

Rectangle $WXYZ$, with $U(-3, -1)$ on WZ, $X(9, 8)$ and $Y(9, -4)$.

P lies on UX with $UP : PX = 1 : 2$.

A point that divides a line segment in a given ratio, then a distance.

Find

The length of PY.

Plan the Solution

- $UP : PX = 1 : 2$ has 3 parts, so P is one third of the way from U to X.
- Find the step from U to X, take one third of it, and add it to U.
- Use Pythagoras with the horizontal and vertical distances from P to Y.

Worked Solution [4 marks]

Rule - Dividing a line segment, then Pythagoras: for $UP : PX = 1 : 2$, P is $\frac{1}{3}$ of the way from U to X. The distance between two points is $\sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}$.

Step 1: Find the step from U to X.

$$U \to X: \ (9-(-3),\ 8-(-1)) = (12, 9)$$

(Reason: the differences in x and in y describe the move from U to X along the line.)

Step 2: Take one third of the step and add it to U.

$$\frac{1}{3} \text{ of } (12, 9) = (4, 3)$$

$$P = (-3+4,\ -1+3) = (1, 2)$$

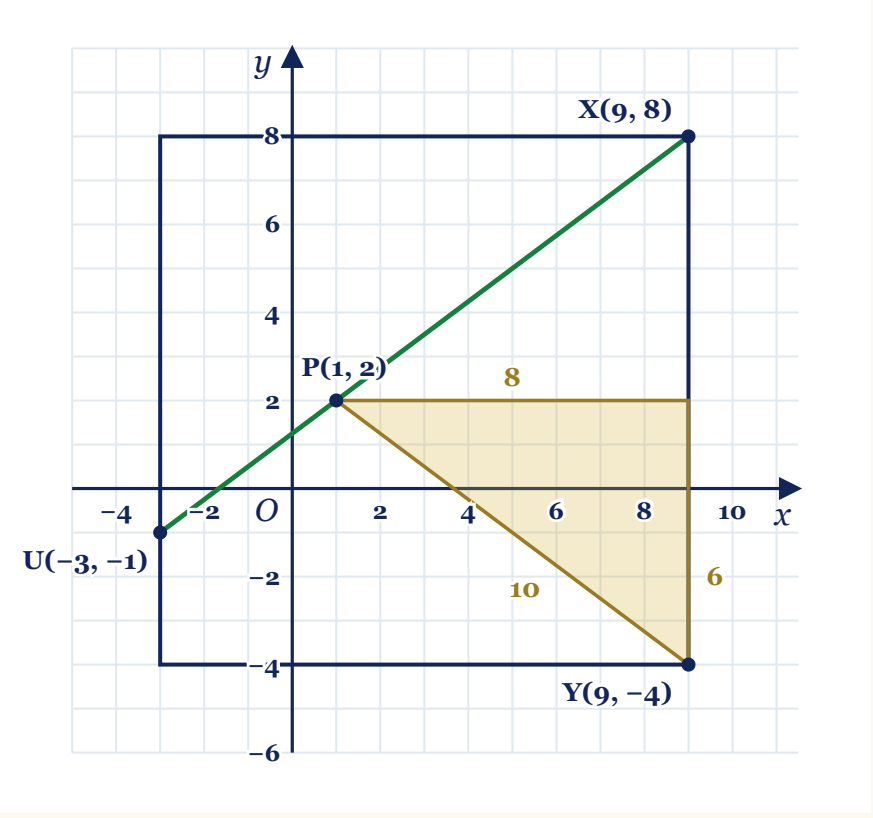
(Reason: UP : PX = 1 : 2 has 1 + 2 = 3 parts, and P is 1 part along from U, so P is one third of the way from U to X. The step (4, 3) is how far to move from U, so add it to the coordinates of U.)

Step 3: Use Pythagoras to find PY.

$$PY^2 = (9-1)^2 + (-4-2)^2$$

$$PY^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$$PY = \sqrt{100} = 10$$



(Reason: the horizontal distance 8 and the vertical distance 6 are the two shorter sides of a right-angled triangle, and PY is its hypotenuse.)

$$PY = 10 \text{ units}$$

Verification

✓ **Check 1:** The section formula gives the same point:

$$P = \left(\frac{2(-3)+9}{3}, \frac{2(-1)+8}{3}\right) = (1, 2).$$

✓ **Check 2:** P lies on UX: the gradient of UP is $\frac{2-(-1)}{1-(-3)} = \frac{3}{4}$ and the gradient of UX is $\frac{8-(-1)}{9-(-3)} = \frac{9}{12} = \frac{3}{4}$, the same.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Differences in x and in y from U to X: 12 and 9	**M1**	Method - steps from U to X	✓
$P(1, 2)$	**A1**	Accuracy - coordinates of P	✓
Pythagoras: $PY^2 = 8^2 + 6^2$	**M1**	Method - Pythagoras for PY	✓
$PY = 10$	**A1**	Accuracy - length of PY	✓

Full marks: 4/4

Question 10 - Exam Solution

Understanding the Question

Given

$A(-2, 7)$ and $B(6, 3)$.

The perpendicular bisector passes through the midpoint of AB, at right angles to AB.

Find

The equation of the perpendicular bisector of AB.

Plan the Solution

- Find the midpoint of AB. The bisector passes through it.
- Find the gradient of AB, then take the negative reciprocal to get the perpendicular gradient.
- Substitute the midpoint into $y = mx + c$ to find c.

Worked Solution [5 marks]

> **Rule - Perpendicular bisector:** it passes through the midpoint of the line segment and is perpendicular to it. Perpendicular gradients multiply to give -1, so the perpendicular gradient is the negative reciprocal.

Step 1: Find the midpoint of AB.

$$\left(\frac{-2+6}{2}, \frac{7+3}{2}\right) = (2, 5)$$

(Reason: a bisector cuts AB in half, so it passes through the midpoint of AB.)

Step 2: Find the gradient of AB.

$$m_{AB} = \frac{3-7}{6-(-2)}$$

$$m_{AB} = \frac{-4}{8} = -\frac{1}{2}$$

(Reason: the gradient is the change in y divided by the change in x.)

Step 3: Find the perpendicular gradient.

$$-\frac{1}{2} \times m = -1$$

$$m = 2$$

(Reason: the bisector is perpendicular to AB, so its gradient is the negative reciprocal of $-\frac{1}{2}$: turn it upside down and change its sign.)

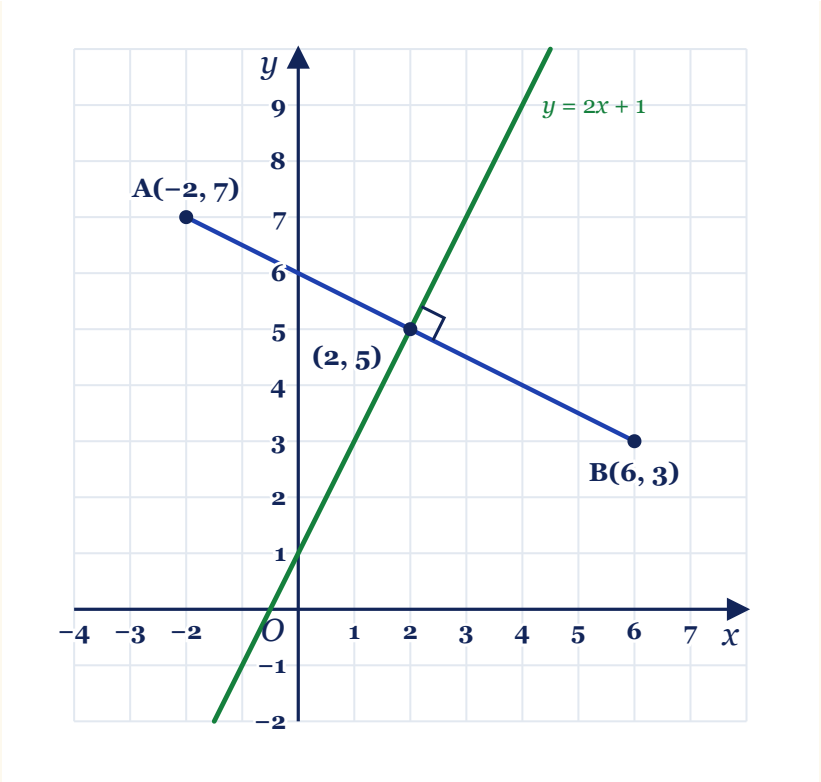
Step 4: Substitute the midpoint (2, 5) to find c.

$$y = 2x + c$$

$$5 = 2(2) + c$$

$$5 = 4 + c$$

$$c = 1$$



(Reason: the midpoint lies on the bisector, so its coordinates satisfy the equation.)

$$y = 2x + 1$$

Verification

✓ **Check 1:** Every point on the bisector is the same distance from A and B, so $(x+2)^2 + (y-7)^2 = (x-6)^2 + (y-3)^2$. Expanding gives $4x - 14y + 53 = -12x - 6y + 45$, which simplifies to $y = 2x + 1$.

✓ **Check 2:** For example, $(0, 1)$ is on the line. Its squared distance from A is $2^2 + 6^2 = 40$ and from B is $6^2 + 2^2 = 40$, the same.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Midpoint $(2, 5)$	**B1**	Independent - midpoint of AB	✓
Gradient of AB, $\dfrac{3-7}{6-(-2)}$	**M1**	Method - gradient of AB	✓
Perpendicular gradient 2	**M1**	Method - perpendicular gradient	✓

Step	Mark	Description	Got it?
Substitutes the midpoint, $5 = 2(2) + c$	M1	Method - substitutes the midpoint	✓
$y = 2x + 1$ oe	A1	Accuracy - equation of the perpendicular bisector	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

Line L through $(0, 2)$ and $(6, 5)$.

A parallel line through $(4, 9)$, and a perpendicular line through $(-4, 6)$.

(0, 2) sits on the y-axis, so it gives the y-intercept of L directly.

Find

The equation of L, the equation of the parallel line, and the equation of the perpendicular line.

Plan the Solution

- For part (a), find the gradient from the two points. Because $(0, 2)$ is on the y-axis, it gives c directly.
- For part (b), a parallel line has the same gradient, so substitute $(4, 9)$ to find its own value of c.
- For part (c), a perpendicular line has the negative reciprocal gradient. Find it, then substitute $(-4, 6)$ to find c.

Worked Solution [8 marks]

> **Rule - Gradients, parallel lines and perpendicular lines:** the gradient is $\frac{\text{change in } y}{\text{change in } x}$. Parallel lines have equal gradients, and perpendicular lines have gradients that multiply to give -1.

Step 1 (part a): Find the gradient of L from the two points.

$$m = \frac{5-2}{6-0} = \frac{3}{6} = \frac{1}{2}$$

(Reason: the gradient is the change in y divided by the change in x, with the two points taken in the same order on the top and on the bottom.)

Step 2 (part a): Read off c from (0, 2), then write the equation.

$$c = 2$$

$$y = \frac{1}{2}x + 2$$

(Reason: (0, 2) has x-coordinate 0, so it is the point where L crosses the y-axis, and its y-coordinate is the value of c.)

Step 3 (part b): Use the same gradient and substitute (4, 9).

$$y = \frac{1}{2}x + c$$

$$9 = \frac{1}{2}(4) + c$$

$$9 = 2 + c$$

$$c = 7$$

$$y = \frac{1}{2}x + 7$$

(Reason: parallel lines have equal gradients, and (4, 9) lies on the new line, so its coordinates satisfy the equation.)

Step 4 (part c): Find the perpendicular gradient.

$$\frac{1}{2} \times m = -1$$

$$m = -2$$

(Reason: the new line is perpendicular to L, so its gradient is the negative reciprocal of $\frac{1}{2}$: turn it upside down and change its sign.)

Step 5 (part c): Substitute (−4, 6) to find c.

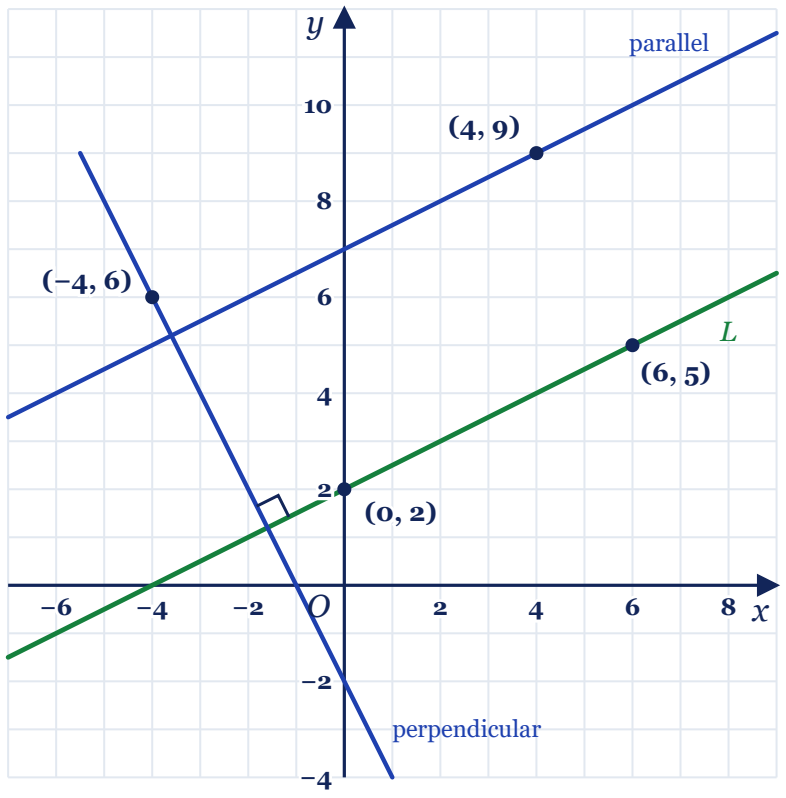
$$y = -2x + c$$

$$6 = -2(-4) + c$$

$$6 = 8 + c$$

$$c = -2$$

$$y = -2x - 2$$



(Reason: (−4, 6) lies on the perpendicular line, so its coordinates satisfy the equation.)

(a) $y = \frac{1}{2}x + 2$

(b) $y = \frac{1}{2}x + 7$

(c) $y = -2x - 2$

Verification

✓ **Check 1 (part a):** $(6, 5)$ lies on L, because $\frac{1}{2}(6) + 2 = 5$.

✓ **Check 2 (part b):** Through $(4, 9)$ with gradient $\frac{1}{2}$, the line is $y - 9 = \frac{1}{2}(x - 4)$, which expands to $y = \frac{1}{2}x - 2 + 9 = \frac{1}{2}x + 7$, the same equation.

✓ **Check 3 (part c):** Through $(-4, 6)$ with gradient -2, the line is $y - 6 = -2(x + 4)$, which expands to $y = -2x - 8 + 6 = -2x - 2$, the same equation. The two gradients multiply to give $\frac{1}{2} \times (-2) = -1$, so the lines are perpendicular.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): gradient $\frac{5-2}{6-0}=\frac{1}{2}$	M1	Method - gradient of L	✓
Part (a): $c=2$	B1	Independent - y-intercept from (0, 2)	✓
Part (a): $y=\frac{1}{2}x+2$ oe	A1	Accuracy - equation of L	✓
Part (b): substitutes $(4,9)$ with gradient $\frac{1}{2}$, $9=\frac{1}{2}(4)+c$	M1	Method - parallel gradient and substitution	✓
Part (b): $y=\frac{1}{2}x+7$ oe	A1	Accuracy - equation of the parallel line	✓
Part (c): perpendicular gradient -2	M1	Method - perpendicular gradient	✓
Part (c): substitutes $(-4,6)$, $6=-2(-4)+c$	M1	Method - substitutes the point	✓
Part (c): $y=-2x-2$ oe	A1	Accuracy - equation of the perpendicular line	✓

Full marks: 8/8

Question 12 - Exam Solution

Understanding the Question

Given

The lines $3y-4x=6$ and $2y-x=14$.

They intersect at the point T.

Neither equation is written in the form y = mx + c.

Find

The gradient of the line $3y-4x=6$, then the coordinates of T.

Plan the Solution

- For part (a), rearrange $3y - 4x = 6$ to make y the subject, then read off the gradient.
- For part (b), T lies on both lines, so solve the two equations simultaneously.
- Make the coefficients of y the same, subtract to remove y, then substitute back to find the other coordinate.

Worked Solution [8 marks]

> **Rule - Gradient and intersection of lines:** the gradient can be read off only when the equation is in the form $y = mx + c$, so make y the subject first. The point where two lines intersect lies on both lines, so its coordinates satisfy both equations at the same time.

Step 1 (part a): Make y the subject.

$$3y - 4x = 6$$
$$3y = 4x + 6$$
$$y = \frac{4}{3}x + 2$$

(Reason: add 4x to both sides, then divide every term by 3.)

Step 2 (part a): Read off the gradient.

$$m = \frac{4}{3}$$

(Reason: in the form y = mx + c, the gradient is the number multiplying x.)

Step 3 (part b): Label the two equations.

$$3y - 4x = 6 \quad (1)$$
$$2y - x = 14 \quad (2)$$

(Reason: T lies on both lines, so its coordinates satisfy both equations at the same time.)

Step 4 (part b): Match the y-coefficients - multiply (1) by 2 and (2) by 3.

$$6y - 8x = 12 \quad (3)$$
$$6y - 3x = 42 \quad (4)$$

(Reason: Equations (3) and (4) both have $6y$, so subtracting removes y.)

Step 5 (part b): Subtract (4) from (3) to eliminate y.

$$(6y - 8x) - (6y - 3x) = 12 - 42$$
$$-5x = -30$$
$$x = 6$$

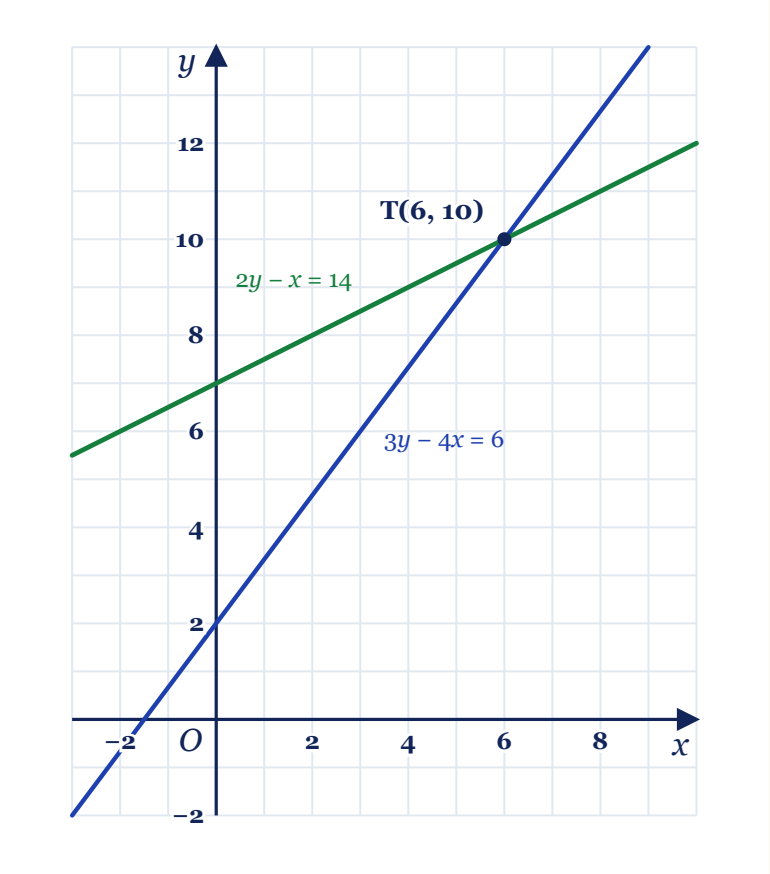
(Reason: $6y - 6y = 0$ and $-8x - (-3x) = -5x$, so only x is left.)

Step 6 (part b): Substitute x = 6 into (2) to find y.

$2y - 6 = 14$

$2y = 20$

$y = 10$



(Reason: use the simplest equation to find the second coordinate.)

(a) $m = \frac{4}{3}$

(b) $T(6, 10)$

Verification

✓ **Check 1 (part a):** $(0, 2)$ and $(3, 6)$ are both on the line, because $3(2) - 4(0) = 6$ and $3(6) - 4(3) = 6$. Between them the gradient is $\frac{6 - 2}{3 - 0} = \frac{4}{3}$.

✓ **Check 2 (part b):** $T(6, 10)$ satisfies both equations: $3(10) - 4(6) = 30 - 24 = 6$ and $2(10) - 6 = 20 - 6 = 14$.

✓ **Check 3 (part b):** Substitution gives the same point. Equation (2) rearranges to $x = 2y - 14$, and putting this into (1) gives $3y - 4(2y - 14) = 6$, so $-5y = -50$ and

$y = 10$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): adds $4x$ to both sides, $3y = 4x + 6$	M1	Method - adds 4x to both sides	✓
Part (a): divides by 3, $y = \frac{4}{3}x + 2$	M1	Method - divides by 3	✓
Part (a): $\frac{4}{3}$	A1	Accuracy - gradient of the line	✓
Part (b): multiplies the first equation by 2, $6y - 8x = 12$	M1	Method - multiplies the first equation	✓
Part (b): multiplies the second equation by 3, $6y - 3x = 42$	M1	Method - multiplies the second equation	✓
Part (b): $x = 6$	A1	Accuracy - value of x	✓
Part (b): $y = 10$	A1	Accuracy - value of y	✓
Part (b): $(6, 10)$	A1	Accuracy - coordinates of T	✓

Full marks: 8/8

Question 13 - Exam Solution

Understanding the Question

Given

L_1 has equation $x + 3y = 30$.

L_2 is perpendicular to L_1 and passes through $(2, -4)$.

H is the point where the two lines meet, so it lies on both of them.

Find

The coordinates of H.

Plan the Solution

- Rearrange $x + 3y = 30$ into the form $y = mx + c$ to read off the gradient of L_1.
- Take the negative reciprocal for the gradient of L_2, then substitute $(2, -4)$ to find the equation of L_2.
- H lies on both lines, so set the two expressions for y equal and solve for x. Then substitute back to find y.

Worked Solution [5 marks]

> **Rule - Perpendicular lines and where two lines meet:** the gradients of two perpendicular lines multiply to give -1, so each gradient is the negative reciprocal of the other. The point where two lines meet lies on both lines, so its coordinates satisfy both equations.

Step 1: Rearrange x + 3y = 30 to find the gradient of L1.

$$3y = -x + 30$$
$$y = -\frac{1}{3}x + 10$$
$$m_1 = -\frac{1}{3}$$

(Reason: subtract x from both sides, then divide every term by 3. The gradient is the number multiplying x.)

Step 2: Find the gradient of L2.

$$-\frac{1}{3} \times m_2 = -1$$
$$m_2 = 3$$

(Reason: perpendicular gradients multiply to give –1, so m_2 is the negative reciprocal of $-\frac{1}{3}$: turn it upside down and change its sign.)

Step 3: Substitute (2, –4) to find the equation of L2.

$$y = 3x + c$$
$$-4 = 3(2) + c$$
$$-4 = 6 + c$$
$$c = -10$$
$$y = 3x - 10$$

(Reason: L2 passes through (2, –4), so these coordinates satisfy its equation.)

Step 4: Set the two equations equal and solve for x.

$$3x - 10 = -\frac{1}{3}x + 10$$

$$\frac{10}{3}x = 20$$

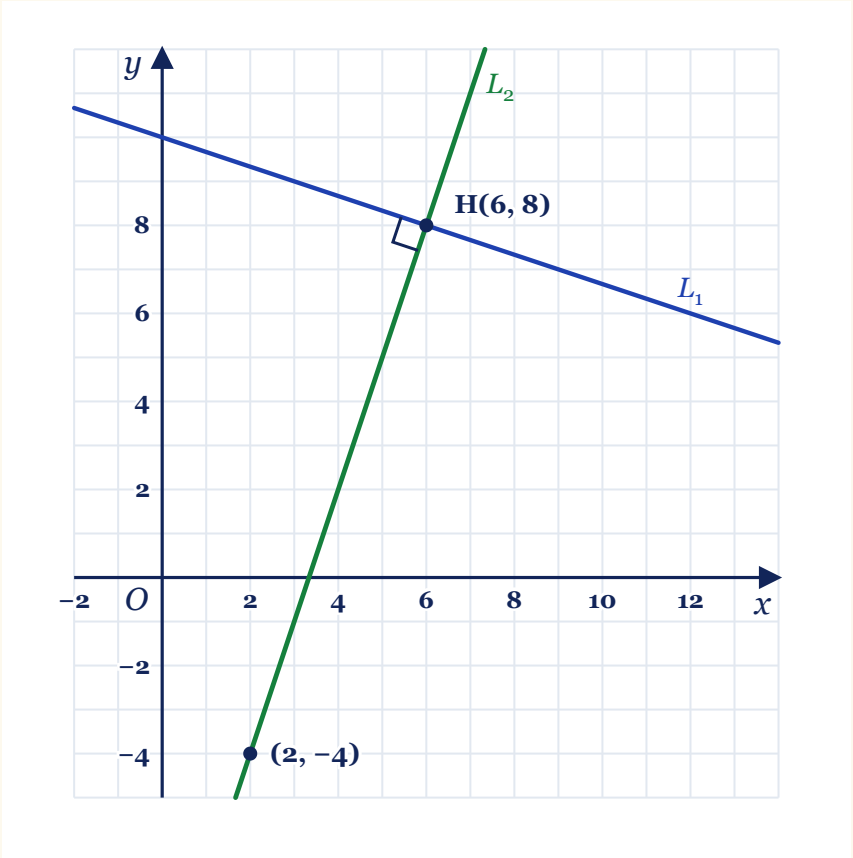
$$x = 6$$

(Reason: H lies on both lines, so the two expressions for y are equal at H. Add $\frac{1}{3}x$ and 10 to both sides, then multiply both sides by $\frac{3}{10}$.)

Step 5: Substitute x = 6 to find y.

$$y = 3(6) - 10 = 8$$

$$H(6, 8)$$



(Reason: either equation gives y. The equation of L2 has no fraction in it, so it is the quicker one to use.)

$$H(6, 8)$$

Verification

✓ **Check 1:** $H(6, 8)$ lies on L_1: $6 + 3(8) = 6 + 24 = 30$.

✓ **Check 2:** $m_1 \times m_2 = -\frac{1}{3} \times 3 = -1$, so the two lines are perpendicular.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $-\frac{1}{3}$	**M1**	Method - gradient of L_1	✓
Gradient of L_2: 3	**M1**	Method - perpendicular gradient	✓
Equation of L_2: $y = 3x - 10$	**M1**	Method - equation of L_2	✓
Sets $3x - 10 = -\frac{1}{3}x + 10$ and solves for x (or y)	**M1**	Method - sets the equations equal and solves	✓
$H(6, 8)$	**A1**	Accuracy - coordinates of H	✓

Full marks: 5/5

Question 14 - Exam Solution

Understanding the Question

Given

L_1 has equation $2y - x = 8$ and passes through $D(4, 6)$ and E.

L_2 is perpendicular to L_1, passes through D and meets the y-axis at F. FE is horizontal.

E has no coordinates in the question: it is the point of L1 level with F.

Find

The coordinates of E.

Plan the Solution

- Make y the subject of $2y - x = 8$ and read off the gradient of L_1.
- The gradient of L_2 is the negative reciprocal. Substitute $D(4, 6)$ to find the equation of L_2, whose y-intercept is F.

- E is level with F, so it has the same y-coordinate. Put that value into the equation of L_1 to find the x-coordinate of E.

Worked Solution [5 marks]

> **Rule - The y-intercept and a horizontal line:** a line crosses the y-axis where $x = 0$, so the line $y = mx + c$ crosses it at $(0, c)$. Every point on a horizontal line has the same y-coordinate.

Step 1: Rearrange 2y − x = 8 to find the gradient of L1.

$$2y = x + 8$$
$$y = \frac{1}{2}x + 4$$
$$m_1 = \frac{1}{2}$$

(Reason: add x to both sides, then divide every term by 2. The gradient is the number multiplying x.)

Step 2: Find the gradient of L2.

$$\frac{1}{2} \times m_2 = -1$$
$$m_2 = -2$$

(Reason: perpendicular gradients multiply to give −1, so m_2 is the negative reciprocal of $\frac{1}{2}$: turn it upside down and change its sign.)

Step 3: Substitute D(4, 6) to find the equation of L2.

$$y = -2x + c$$
$$6 = -2(4) + c$$
$$6 = -8 + c$$
$$c = 14$$
$$y = -2x + 14$$

(Reason: D lies on L2, so its coordinates satisfy the equation of L2.)

Step 4: Find F, then the y-coordinate of E.

$$F(0, 14)$$
$$y_E = 14$$

(Reason: F is on the y-axis, so its x-coordinate is 0 and its y-coordinate is the value of c for L2, which is 14. FE is horizontal, so E has the same y-coordinate as F.)

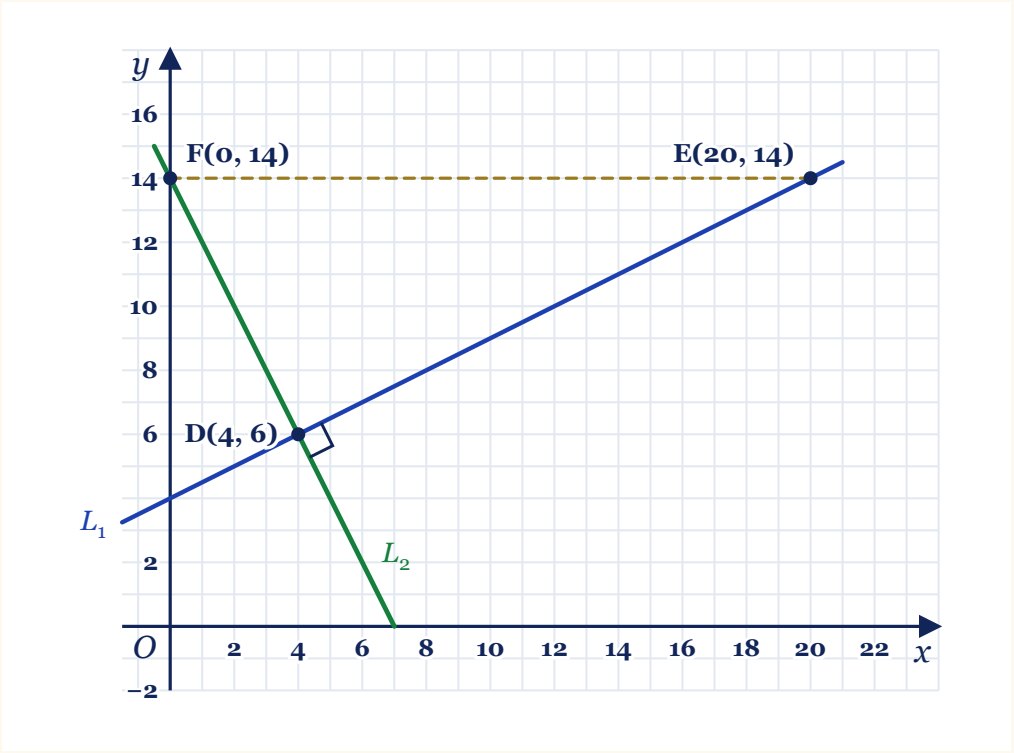
Step 5: Substitute y = 14 into the equation of L1.

$2(14) - x = 8$

$28 - x = 8$

$x = 20$

$E(20, 14)$



(Reason: E lies on L1, so its coordinates satisfy 2y − x = 8.)

$$E(20, 14)$$

Verification

✓ **Check 1:** $E(20, 14)$ lies on L_1: $2(14) - 20 = 28 - 20 = 8$.

✓ **Check 2:** $D(4, 6)$ lies on L_2: $-2(4) + 14 = -8 + 14 = 6$.

✓ **Check 3:** $m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$, so L_1 and L_2 are perpendicular.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $\frac{1}{2}$	M1	Method - gradient of L_1	✓

Step	Mark	Description	Got it?
Gradient of L_2: -2	M1	Method - perpendicular gradient	✓
Equation of L_2: $y = -2x + 14$, or $F(0, 14)$	M1	Method - equation of L_2 or the coordinates of F	✓
Uses $y = 14$ from F as the y-coordinate of E and substitutes into L_1: $2(14) - x = 8$	M1	Method - substitutes the y-coordinate of F into L_1	✓
$E(20, 14)$	A1	Accuracy - coordinates of E	✓

Full marks: 5/5

Question 15 - Exam Solution

Understanding the Question

Given

Parallelogram $PQRS$ with $P(-4, 7)$ and $S(-7, 1)$.

$M(2, 4)$ is the midpoint of PR, and N lies on PR with $PM : MN : NR = 3 : 2 : 1$.

An exact length is left as a simplified surd, not rounded to a decimal.

Find

The exact length of NQ.

Plan the Solution

- M is the midpoint of PR, so repeating the step from P to M leads from M to R.
- $MN : NR = 2 : 1$, so N is two thirds of the way from M to R.
- Opposite sides of a parallelogram are equal and parallel, so the step from S to R also leads from P to Q. Then use Pythagoras for NQ and simplify the surd.

Worked Solution [6 marks]

Rule - Parallelogram, ratio and Pythagoras: opposite sides of a parallelogram are equal and parallel, so the step from S to R is the same as the step from P to Q. For

> $MN : NR = 2 : 1$, N is $\frac{2}{3}$ of the way from M to R. The distance between two points is $\sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}$.

Step 1: Find the step from P to M, then R.

$$P \to M : \ (2 - (-4),\ 4 - 7) = (6, -3)$$

$$R = (2 + 6,\ 4 - 3) = (8, 1)$$

(Reason: M is the midpoint of PR, so the step from M to R is the same as the step from P to M. Add it to the coordinates of M.)

Step 2: Take two thirds of the step from M to R.

$$M \to R : \ (6, -3)$$

$$\frac{2}{3} \text{ of } (6, -3) = (4, -2)$$

$$N = (2 + 4,\ 4 - 2) = (6, 2)$$

(Reason: MN : NR = 2 : 1 has 2 + 1 = 3 parts, and N is 2 parts along from M, so N is two thirds of the way from M to R.)

Step 3: Find Q from the step from S to R.

$$S \to R : \ (8 - (-7),\ 1 - 1) = (15, 0)$$

$$Q = (-4 + 15,\ 7 + 0) = (11, 7)$$

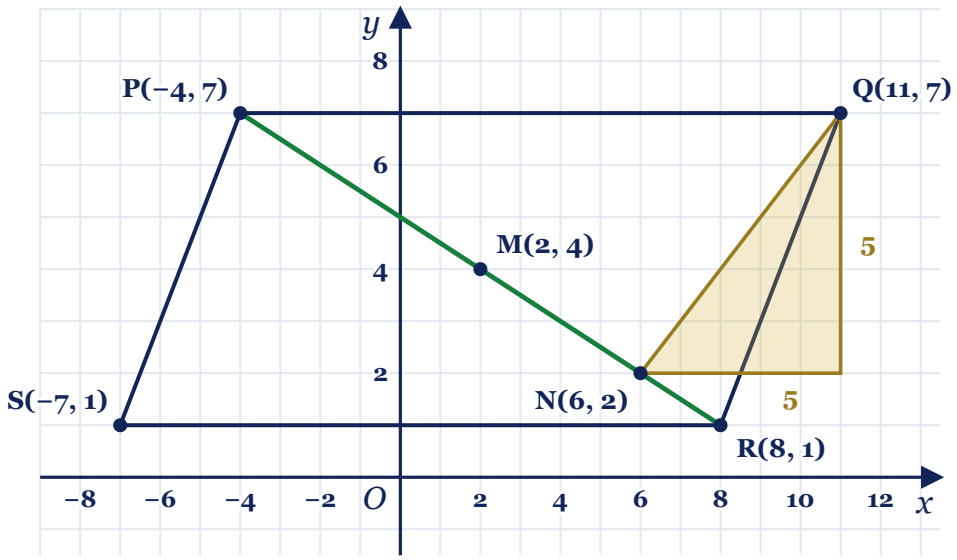
(Reason: PQ and SR are opposite sides of the parallelogram, so they are equal and parallel: the step from P to Q is the same as the step from S to R.)

Step 4: Use Pythagoras to find NQ.

$$NQ^2 = (11 - 6)^2 + (7 - 2)^2$$

$$NQ^2 = 5^2 + 5^2 = 25 + 25 = 50$$

$$NQ = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$



(Reason: the horizontal distance 5 and the vertical distance 5 are the two shorter sides of a right-angled triangle, and NQ is its hypotenuse. 25 is a square number and a factor of 50, so $\sqrt{50}$ simplifies to $5\sqrt{2}$.)

$$NQ = 5\sqrt{2} \text{ units}$$

Verification

✓ **Check 1:** The x-steps along PR are 6 from P to M, 4 from M to N and 2 from N to R, and $6 : 4 : 2 = 3 : 2 : 1$.

✓ **Check 2:** N is $3 + 2 = 5$ of the 6 parts from P to R, and the step from P to R is $(12, -6)$. So $N = \left(-4 + \frac{5}{6}(12),\ 7 + \frac{5}{6}(-6)\right) = (-4 + 10,\ 7 - 5) = (6, 2)$, the same point.

✓ **Check 3:** PQ and SR are both horizontal and both 15 long: from $P(-4, 7)$ to $Q(11, 7)$, and from $S(-7, 1)$ to $R(8, 1)$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
$R(8, 1)$	**B1**	Independent - coordinates of R	✓
$\frac{2}{3}$ of the step $(6, -3)$ from M towards R	**M1**	Method - correct method for N	✓

Step	Mark	Description	Got it?
$N(6, 2)$	**A1**	Accuracy - coordinates of N	✓
$Q(11, 7)$	**B1**	Independent - coordinates of Q	✓
Pythagoras: $NQ^2 = 5^2 + 5^2$	**M1**	Method - Pythagoras for NQ	✓
$5\sqrt{2}$ (accept $\sqrt{50}$)	**A1**	Accuracy - exact length of NQ	✓

Full marks: 6/6

Question 16 - Exam Solution

Understanding the Question

Given

Triangle ABC with $A(1, 2)$, $B(7, 5)$ and $C(5, 9)$.

Angle ABC is the angle at B, between the sides AB and BC.

Find

Proof that angle ABC is a right angle, then the area of triangle ABC.

Plan the Solution

- For part (a), find the gradients of AB and BC, the two sides that meet at B.
- If the two gradients multiply to give -1, the sides are perpendicular, so the angle at B is a right angle.
- For part (b), AB and BC are then the base and the height of the triangle. Find their lengths with Pythagoras, then use $\frac{1}{2} \times \text{base} \times \text{height}$.

Worked Solution [6 marks]

> **Rule - Perpendicular lines and the area of a right-angled triangle:** two lines are perpendicular when their gradients multiply to give -1. In a right-angled triangle, the two sides that meet at the right angle are the base and the height, so
> $$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}.$$

Step 1 (part a): Find the gradient of AB.

$$m_{AB} = \frac{5-2}{7-1} = \frac{3}{6} = \frac{1}{2}$$

(Reason: the gradient is the change in y divided by the change in x.)

Step 2 (part a): Find the gradient of BC.

$$m_{BC} = \frac{9-5}{5-7} = \frac{4}{-2} = -2$$

(Reason: take the two points in the same order on the top and on the bottom: C first, then B, both times.)

Step 3 (part a): Multiply the two gradients.

$$m_{AB} \times m_{BC} = \frac{1}{2} \times (-2) = -1$$

So AB is perpendicular to BC, and angle $ABC = 90°$.

(Reason: two lines are perpendicular when their gradients multiply to give –1, and AB and BC meet at B.)

Step 4 (part b): Find the lengths of AB and BC with Pythagoras.

$$AB = \sqrt{6^2 + 3^2} = \sqrt{45}$$

$$BC = \sqrt{2^2 + 4^2} = \sqrt{20}$$

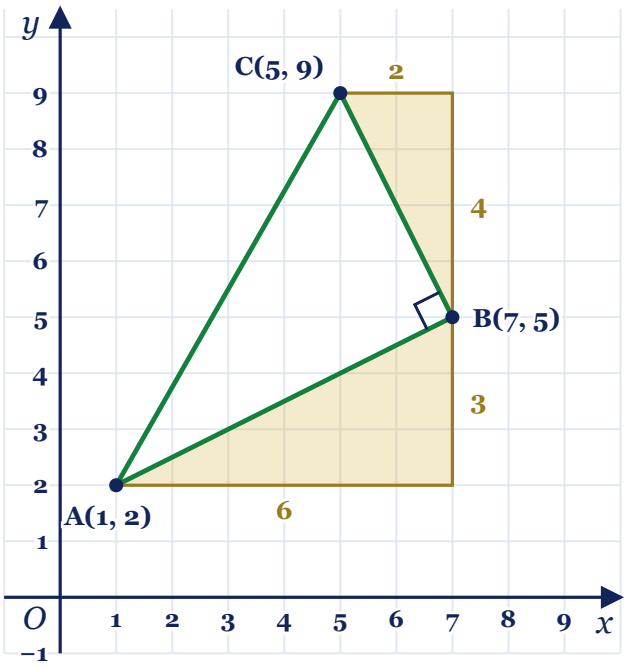
(Reason: from A to B is 6 across and 3 up, and from B to C is 2 to the left and 4 up. Each pair gives the two shorter sides of a right-angled triangle.)

Step 5 (part b): Use half of base times height.

$$\text{Area} = \frac{1}{2} \times \sqrt{45} \times \sqrt{20}$$

$$= \frac{1}{2} \times \sqrt{900}$$

$$= \frac{1}{2} \times 30 = 15$$



(Reason: angle ABC is a right angle, so AB and BC are the base and the height. Multiplying the two square roots gives the square root of 45 × 20 = 900, which is 30.)

(a) $\frac{1}{2} \times (-2) = -1$**, so angle** $ABC = 90°$

(b) 15 **square units**

Verification

✓ **Check 1 (part a):** The side lengths agree: $AB^2 + BC^2 = 45 + 20 = 65$, and $AC^2 = 4^2 + 7^2 = 65$. So Pythagoras holds with AC as the hypotenuse, and the right angle is at B.

✓ **Check 2 (part b):** The rectangle drawn around the triangle is 6 by 7, with area 42. The three right-angled triangles in its corners have areas 9, 4 and 14, and $42 - 9 - 4 - 14 = 15$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): gradient of AB, $\frac{5-2}{7-1} = \frac{1}{2}$	M1	Method - gradient of AB	✓

Step	Mark	Description	Got it?
Part (a): gradient of BC, $\frac{9-5}{5-7}=-2$	M1	Method - gradient of BC	✓
Part (a): $\frac{1}{2}\times(-2)=-1$, so AB is perpendicular to BC and angle $ABC=90°$	A1	Accuracy - product −1 with the conclusion	✓
Part (b): Pythagoras for AB and BC, 6^2+3^2 and 2^2+4^2	M1	Method - lengths of AB and BC	✓
Part (b): $\sqrt{45}$ and $\sqrt{20}$	A1	Accuracy - both lengths	✓
Part (b): 15	A1	Accuracy - area of the triangle	✓

Full marks: 6/6

Question 17 - Exam Solution

Understanding the Question

Given

L_1 has equation $3x+2y=30$. L_2 is parallel to L_1 and passes through $(10,13)$.

L_3 is perpendicular to both lines and meets L_1 at $(6,6)$.

The point (6, 6) is on L1 and on L3, but not on L2.

Find

The coordinates of the point where L_2 and L_3 intersect.

Plan the Solution

- Rearrange $3x+2y=30$ to read off the gradient of L_1. L_2 is parallel, so it has the same gradient: substitute $(10,13)$ to find its equation.
- L_3 is perpendicular to both lines, so its gradient is the negative reciprocal. Substitute $(6,6)$ to find its equation.
- Set the equations of L_2 and L_3 equal and solve for x, then find y.

Worked Solution [6 marks]

> **Rule - Parallel and perpendicular lines:** parallel lines have equal gradients. The gradients of two perpendicular lines multiply to give -1, so each gradient is the negative reciprocal of the other. A line perpendicular to one of two parallel lines is perpendicular to both.

Step 1: Rearrange 3x + 2y = 30 to find the gradient of L1.

$$2y = -3x + 30$$

$$y = -\frac{3}{2}x + 15$$

$$m_1 = -\frac{3}{2}$$

(Reason: subtract 3x from both sides, then divide every term by 2. The gradient is the number multiplying x.)

Step 2: Substitute (10, 13) to find the equation of L2.

$$y = -\frac{3}{2}x + c$$

$$13 = -\frac{3}{2}(10) + c$$

$$13 = -15 + c$$

$$c = 28$$

$$y = -\frac{3}{2}x + 28$$

(Reason: L2 is parallel to L1, so it has the same gradient, and (10, 13) lies on L2, so its coordinates satisfy the equation.)

Step 3: Find the gradient of L3.

$$-\frac{3}{2} \times m_3 = -1$$

$$m_3 = \frac{2}{3}$$

(Reason: perpendicular gradients multiply to give –1, so m_3 is the negative reciprocal of $-\frac{3}{2}$: turn it upside down and change its sign.)

Step 4: Substitute (6, 6) to find the equation of L3.

$$y = \frac{2}{3}x + c$$

$$6 = \frac{2}{3}(6) + c$$

$$6 = 4 + c$$

$$c = 2$$

$$y = \frac{2}{3}x + 2$$

(Reason: L3 meets L1 at (6, 6), so this point lies on L3.)

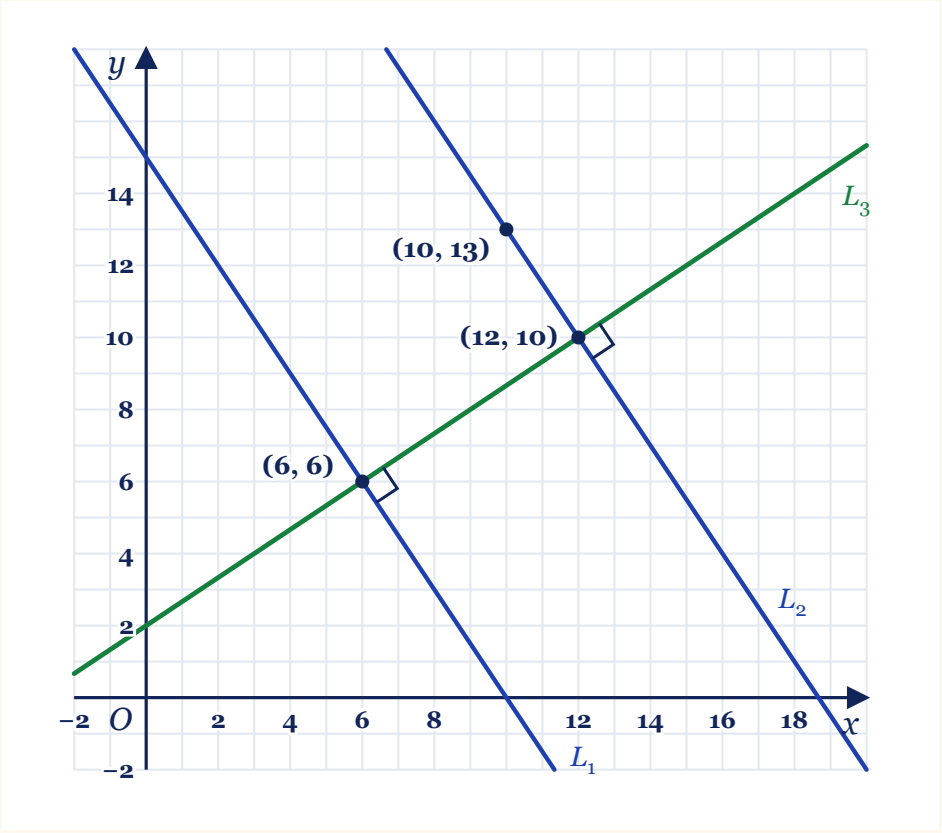
Step 5: Set the equations of L2 and L3 equal and solve.

$$-\frac{3}{2}x + 28 = \frac{2}{3}x + 2$$

$$26 = \frac{13}{6}x$$

$$x = 12$$

$$y = \frac{2}{3}(12) + 2 = 10$$



(Reason: the point lies on both lines, so the two expressions for y are equal there. Add $\frac{3}{2}x$ to both sides and subtract 2, using $\frac{2}{3} + \frac{3}{2} = \frac{13}{6}$, then multiply both sides by $\frac{6}{13}$.)

$$(12, 10)$$

Verification

✓ **Check 1:** $(12, 10)$ lies on L_2: $-\frac{3}{2}(12) + 28 = -18 + 28 = 10$.

✓ **Check 2:** Written without fractions, L_2 is $3x + 2y = 56$ and L_3 is $2x - 3y = -6$. Multiplying them by 3 and by 2 gives $9x + 6y = 168$ and $4x - 6y = -12$, which add to $13x = 156$, so $x = 12$ and $y = \frac{56 - 36}{2} = 10$, the same point.

✓ **Check 3:** $m_1 \times m_3 = -\frac{3}{2} \times \frac{2}{3} = -1$, so L_3 is perpendicular to L_1.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Gradient of L_1: $-\frac{3}{2}$	**M1**	Method - gradient of L_1	✓
Equation of L_2: $y = -\frac{3}{2}x + 28$	**M1**	Method - equation of L_2	✓
Gradient of L_3: $\frac{2}{3}$	**M1**	Method - perpendicular gradient	✓
Equation of L_3: $y = \frac{2}{3}x + 2$	**M1**	Method - equation of L_3	✓
Sets $-\frac{3}{2}x + 28 = \frac{2}{3}x + 2$ and solves	**M1**	Method - sets the equations equal and solves	✓
$(12, 10)$	**A1**	Accuracy - point of intersection	✓

Full marks: 6/6

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