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Nth Term

Linear & Quadratic Sequences: GCSE & IGCSE Practice Questions with Worked Solutions

GRADE 8 TARGETED

by Sir Faraz Hassan · IGCSE / GCSE Maths Specialist

13 exam-style questions, 59 marks, grades 4 to 8. Full worked solutions and mark schemes.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. The first four terms of a sequence are $4, 11, 18, 25, \ldots$

GRADE 6

(a) Find the nth term of the sequence. *[2 marks]*

(b) Using your answer to part (a), find an expression for the product of the nth and $(n+2)$th terms of the sequence. Simplify your answer as much as possible. *[2 marks]*

(a) ....

(b) ....

[Total 4 marks]

2. This question is about the sequence $2, 8, 14, 20, \ldots$

GRADE 4

(a) Find the nth term of the sequence. *[2 marks]*

(b) Explain why 300 cannot be a term in this sequence. *[2 marks]*

(a) ....

(b) ....

[Total 4 marks]

3. The first four terms in a sequence are $\sqrt{3}, 3, 3\sqrt{3}, 9, \ldots$

GRADE 5

(a) Find the next two terms in the sequence. *[2 marks]*

(b) Circle the expression for the nth term of the sequence. *[1 mark]*

$\sqrt{3n}$ $\quad n\sqrt{3}$ $\quad (\sqrt{3})^n$ $\quad n(\sqrt{3})^2$

(a) ....

(b) circle one: $\sqrt{3n}$ $\quad n\sqrt{3}$ $\quad (\sqrt{3})^n$ $\quad n(\sqrt{3})^2$

[Total 3 marks]

4. The term-to-term rule of a sequence is $u_{n+1} = 3u_n - 2$.

GRADE 5

(a) If $u_1 = 2$, find the values of the next two terms in the sequence. *[2 marks]*

(b) A different sequence has the same term-to-term rule, but $u_1 = 0.5$. Find u_2, u_3 and u_4. *[3 marks]*

(c) What do you notice if you start with $u_1 = 1$? *[1 mark]*

(a) ....

(b) ....

(c) ....

[Total 6 marks]

Finding the nth term of a quadratic sequence

A quadratic sequence has an nth term of the form $an^2 + bn + c$. Find the coefficients from these three relationships:

$2a = \text{second difference (always constant)}$

$3a + b = (\text{2nd term}) - (\text{1st term})$

$a + b + c = \text{1st term}$

5. A quadratic sequence begins $4, 10, 18, 28, \ldots$

GRADE 7

(a) Write down the next term in the sequence. *[2 marks]*

(b) Find an expression for the nth term of the sequence. *[3 marks]*

(a) ....

(b) ....

[Total 5 marks]

6. The term-to-term rule of a sequence is $u_{n+1} = \dfrac{1}{1-u_n}$.

GRADE 5

(a) If $u_1 = 3$, find the values of the next three terms in the sequence. *[2 marks]*

(b) Write down the value of u_{50}. *[1 mark]*

(a) ....

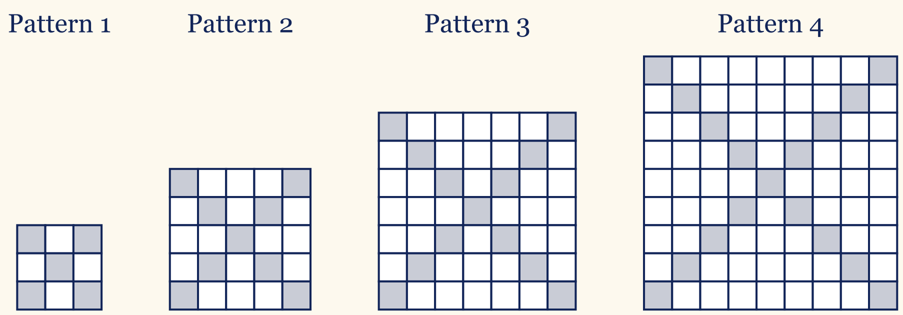
(b) ....

[Total 3 marks]

7. The diagrams show a sequence of patterns made from grey and white squares.

GRADE 7

Problem solving



(a) Find an expression, in terms of n, for the number of grey squares in Pattern n. *[2 marks]*

(b) Two consecutive patterns use a total of 206 grey squares. Find the pattern numbers. *[3 marks]*

(c) Find an expression, in terms of n, for the total number of squares in Pattern n. *[3 marks]*

(a) ....

(b) ....

(c) ....

[Total 8 marks]

8. The nth term of a sequence is $\left(\frac{1}{3}\right)^n$.

Find the difference between the 3rd and 5th terms in the sequence. *[2 marks]*

GRADE 5

....

[Total 2 marks]

9. Faraz and Jessica each think of a sequence. The sequence Faraz thinks of is arithmetic, with nth term $50 - 4n$. The sequence Jessica thinks of is quadratic and starts $10, 12, 16, 22, \ldots$

GRADE 8

Problem solving

(a) Find an expression for the nth term of the sequence Jessica thinks of. *[3 marks]*

(b) Find the only term that is the same in both sequences. *[3 marks]*

(a) ....

(b) ....

[Total 6 marks]

10. A quadratic sequence has nth term $2n^2 + n - 3$.

GRADE 8

Problem solving

The sum of two consecutive terms in the sequence is 301.

What are the two terms? *[5 marks]*

.... and ....

[Total 5 marks]

11. The first, second and third terms of an arithmetic sequence are $3x - 1$, $5x - 4$ and $8x - 13$, where x is an integer.

GRADE 8

Problem solving

Find the 12th term in the sequence. *[6 marks]*

....

[Total 6 marks]

12. A sequence has nth term $72 - \frac{1}{2}n^2$.

GRADE 8

Problem solving

Find the value of the first term in the sequence that is less than 0. *[3 marks]*

Hint: set up and solve a quadratic inequality rather than using trial and error.

....

[Total 3 marks]

13. Leo and Nora each think of a sequence.

GRADE 8

Problem solving

Leo - geometric sequence

nth term $= (\sqrt{2})^n$

Nora - quadratic sequence

first three terms are $8, 9, 12$

Show that the sum of the 8th term of the sequence Leo thinks of and the 6th term of the sequence Nora thinks of is a square number. *[4 marks]*

Hint: you do not have to work out the nth term of the quadratic sequence, as you only need to go as far as the 6th term.

[Total 4 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

$4, 11, 18, 25, \ldots$

A linear (arithmetic) sequence: the terms rise by the same amount each time.

Find

The nth term of the sequence, then the simplified product of the nth and $(n+2)$th terms.

Plan the Solution

- Find the common difference; that is the coefficient of n in the formula.
- Adjust by a constant so that $n = 1$ gives the first term, 4.
- For part (b), substitute $n + 2$ into the nth-term formula to get the later term.
- Multiply the two expressions and expand carefully, all four products, then collect like terms.

Worked Solution [4 marks]

> **Rule - nth term of a linear sequence:** for a common difference d, the nth term is $dn + c$, where the constant c is chosen so that $n = 1$ gives the first term. Substituting $n + 2$ in place of n gives a parallel expression for the later term.

Step 1 (part a): Find the common difference, then the nth term.

$$d = 11 - 4 = 7$$

$$7(1) + c = 4 \implies c = -3$$

$$n\text{th term} = 7n - 3$$

(Reason: a linear sequence rises by the same amount each time, so its nth term is 7n plus a constant; the constant is fixed by making n = 1 give the first term, 4.)

Step 2 (part b): Write down the (n+2)th term.

$$7(n+2) - 3 = 7n + 14 - 3 = 7n + 11$$

(Reason: the (n+2)th term comes from substituting n+2 in place of n in the formula 7n - 3.)

Step 3 (part b): Multiply the two terms and simplify.

$$(7n - 3)(7n + 11)$$
$$= (7n)(7n) + (7n)(11) + (-3)(7n) + (-3)(11)$$
$$= 49n^2 + 77n - 21n - 33$$
$$= 49n^2 + 56n - 33$$

(Reason: multiply every term in the first bracket by every term in the second, all four products, then collect the two n terms: 77n - 21n = 56n.)

(a) $n\text{th term} = 7n - 3$

(b) $49n^2 + 56n - 33$

Verification

✓ **Check A (part a):** Substitute $n = 4$ into $7n - 3$: $7(4) - 3 = 28 - 3 = 25$, which is the fourth given term. The nth-term formula is correct.

✓ **Check B (part b):** At $n = 2$ the nth term is $7(2) - 3 = 11$ and the $(n + 2)$th term is $7(2) + 11 = 25$, so their product is $11 \times 25 = 275$. The expanded formula gives $49(2)^2 + 56(2) - 33 = 196 + 112 - 33 = 308 - 33 = 275$. The two agree, so the expansion is correct.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): use the common difference 7 to obtain 7n	M1	Method - coefficient of n	✓
Part (a): give the nth term 7n - 3	A1	Accuracy - part (a) answer	✓
Part (b): form (7n - 3)(7n + 11), or state the (n+2)th term 7n + 11	M1	Method - product of the two terms	✓
Part (b): expand and simplify to 49n squared + 56n - 33	A1	Accuracy - part (b) answer	✓

Full marks: 4/4

Question 2 - Exam Solution

Understanding the Question

Given

$2, 8, 14, 20, \ldots$

A linear (arithmetic) sequence: the terms rise by the same amount each time.

Find

The nth term of the sequence, then an explanation of why 300 is not a term.

Plan the Solution

- Find the common difference; that is the coefficient of n in the formula.
- Adjust by a constant so that $n = 1$ gives the first term, 2.
- For part (b), set the nth term equal to 300 and solve for n.
- If n is not a whole number, the value is not a term. Show this clearly.

Worked Solution [4 marks]

> **Rule - is a value a term?** a value is a term of a linear sequence only if setting the nth term equal to that value gives a positive whole-number n. If n comes out as a fraction, the value lies between two terms and is skipped.

Step 1 (part a): Find the common difference, then the nth term.

$$d = 8 - 2 = 6$$

$$6(1) + c = 2 \implies c = -4$$

$$n\text{th term} = 6n - 4$$

(Reason: a linear sequence rises by the same amount each time, so its nth term is 6n plus a constant; the constant is fixed by making n = 1 give the first term, 2.)

Step 2 (part b): Set the nth term equal to 300 and solve.

$$6n - 4 = 300$$

$$6n = 304$$

$$n = \frac{304}{6}$$

(Reason: if 300 were a term, this n would have to be a positive whole number, because n counts the position in the sequence.)

Step 3 (part b): Show 304 is not a multiple of 6.

$$6 \times 50 = 300$$

$$6 \times 51 = 306$$

$$300 < 304 < 306$$

(Reason: since 304 lies between two consecutive multiples of 6, it is not a multiple of 6, so n is not a whole number and 300 is skipped by the sequence.)

$$\textbf{(a)}\ n\text{th term} = 6n - 4$$

$$\textbf{(b)}\ \textbf{300 is not a term: } n = \frac{304}{6}, \textbf{ not a whole number.}$$

Verification

✓ **Check A (part a):** Substitute $n = 4$ into $6n - 4$: $6(4) - 4 = 24 - 4 = 20$, which is the fourth given term. The nth-term formula is correct.

✓ **Check B (part b):** The terms either side of 300 are $6(50) - 4 = 296$ and $6(51) - 4 = 302$. These are two consecutive terms, and 300 falls between them, so 300 is skipped by the sequence.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): use the common difference 6 to obtain 6n	**M1**	Method - coefficient of n	✓
Part (a): give the nth term 6n - 4	**A1**	Accuracy - part (a) answer	✓
Part (b): set 6n - 4 = 300 and reach 6n = 304, or show the terms are 4 less than a multiple of 6	**M1**	Method - test whether 300 is a term	✓
Part (b): explain that 304 is not a multiple of 6, so n is not a whole number and 300 is not a term	**A1**	Accuracy - complete explanation	✓

Full marks: 4/4

Question 3 - Exam Solution

Understanding the Question

Given

$\sqrt{3}, 3, 3\sqrt{3}, 9, \ldots$

Find

The next two terms of the sequence, then the correct nth-term expression from the

A geometric sequence: each term is the previous one multiplied by a fixed amount.

four options.

Plan the Solution

- Find what you multiply by to get from one term to the next (the common ratio).
- Multiply the fourth term by that ratio twice to get the next two terms.
- For part (b), see that each term is $\sqrt{3}$ raised to the power of its position.
- Test the options at a known term to confirm which one is correct.

Worked Solution [3 marks]

> **Rule - geometric sequence:** in a geometric sequence each term is the previous one multiplied by a fixed common ratio r. If the first term is r itself, the nth term is r^n.

Step 1 (part a): Find the common ratio.

$$\frac{3}{\sqrt{3}} = \sqrt{3}$$
$$\frac{3\sqrt{3}}{3} = \sqrt{3}$$

(Reason: the ratio between consecutive terms is the same each time, so the sequence is geometric and each term is the one before it times root 3.)

Step 2 (part a): Find the next two terms from the fourth term.

$$9 \times \sqrt{3} = 9\sqrt{3}$$
$$9\sqrt{3} \times \sqrt{3} = 9 \times 3 = 27$$

(Reason: multiply the fourth term, 9, by the common ratio to get the fifth term, then multiply again to get the sixth.)

Step 3 (part b): Identify the nth term.

$$(\sqrt{3})^1 = \sqrt{3}, \quad (\sqrt{3})^2 = 3, \quad (\sqrt{3})^3 = 3\sqrt{3}$$
$$n\text{th term} = (\sqrt{3})^n$$

(Reason: multiplying by root 3 each time means the nth term is root 3 raised to the power n; the first term is root 3 to the power one, and each step adds one to the power.)

(a) $9\sqrt{3}, \quad 27$

(b) $n\text{th term} = (\sqrt{3})^n$

Verification

✓ **Check A (part a):** Writing out the run gives $\sqrt{3}, 3, 3\sqrt{3}, 9, 9\sqrt{3}, 27$, so $9\sqrt{3}$ and 27 are the fifth and sixth terms. Continuing once more, $27 \times \sqrt{3} = 27\sqrt{3}$ would be the seventh term, which fits the pattern.

✓ **Check B (part b):** Substitute $n = 2$ into each option: $\sqrt{3n} = \sqrt{6}$, $n\sqrt{3} = 2\sqrt{3}$, $(\sqrt{3})^n = 3$, and $n(\sqrt{3})^2 = 2 \times 3 = 6$. Only $(\sqrt{3})^n$ gives the second term, 3, so it is the correct expression.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): 9 root 3 (the fifth term)	**B1**	Accuracy - fifth term	✓
Part (a): 27 (the sixth term)	**B1**	Accuracy - sixth term	✓
Part (b): circle root 3 to the power n	**B1**	Accuracy - correct nth-term expression	✓

Full marks: 3/3

Question 4 - Exam Solution

Understanding the Question

Given

$u_{n+1} = 3u_n - 2$

A term-to-term (recurrence) rule: each term is built from the one before.

Find

The requested terms for each starting value, and what happens when $u_1 = 1$.

Plan the Solution

- Apply the rule to the current term to get the next: multiply by 3, then subtract 2.
- Repeat for as many terms as each part asks for.
- Take care with negative and decimal values as they build up.
- For part (c), check whether the starting value maps to itself.

Worked Solution [6 marks]

Rule - term-to-term (recurrence): a term-to-term rule builds each new term from the previous one. Here, multiply the current term by 3 and subtract 2 to get the next. A value that maps to itself is a fixed point, and gives a constant sequence.

Step 1 (part a): First sequence, starting value 2.

$$u_2 = 3(2) - 2 = 6 - 2 = 4$$
$$u_3 = 3(4) - 2 = 12 - 2 = 10$$

(Reason: apply the rule to the first term to get the second, then to the second to get the third.)

Step 2 (part b): Second sequence, starting value 0.5.

$$u_2 = 3(0.5) - 2 = 1.5 - 2 = -0.5$$
$$u_3 = 3(-0.5) - 2 = -1.5 - 2 = -3.5$$
$$u_4 = 3(-3.5) - 2 = -10.5 - 2 = -12.5$$

(Reason: the same rule applies; watch the signs as the terms turn negative.)

Step 3 (part c): Third sequence, starting value 1.

$$u_2 = 3(1) - 2 = 3 - 2 = 1$$

(Reason: 1 maps to itself under the rule, so every term equals 1 and the sequence never changes; this is a fixed point, which gives a constant sequence.)

$$\textbf{(a)}\ u_2 = 4, \quad u_3 = 10$$
$$\textbf{(b)}\ u_2 = -0.5, \quad u_3 = -3.5, \quad u_4 = -12.5$$
$$\textbf{(c)}\ \text{every term is 1 (the sequence stays constant at 1)}$$

Verification

✓ **Check A (parts a and b):** Re-substitute each term into the rule. For (a), $3(4) - 2 = 10$, so u_3 follows from u_2. For (b), $3(-0.5) - 2 = -3.5$ and $3(-3.5) - 2 = -12.5$, so the chain is consistent.

✓ **Check B (part c):** Substitute $u = 1$ into the rule: $3(1) - 2 = 1$, which equals the starting value, so 1 is a fixed point and the sequence is constant.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): u2 = 4	**B1**	Accuracy - second term	✓
Part (a): u3 = 10	**B1**	Accuracy - third term	✓
Part (b): u2 = -0.5	**B1**	Accuracy - second term	✓

Step	Mark	Description	Got it?
Part (b): u3 = -3.5	**B1**	Accuracy - third term	✓
Part (b): u4 = -12.5	**B1**	Accuracy - fourth term	✓
Part (c): notice every term is 1 (a constant sequence)	**B1**	Accuracy - constant sequence	✓

Full marks: 6/6

Question 5 - Exam Solution

Understanding the Question

Given

$4, 10, 18, 28, \ldots$

A quadratic sequence: the second difference is constant.

Find

The next term, then the nth term in the form $an^2 + bn + c$.

Plan the Solution

- Find the first and second differences; a constant second difference means the sequence is quadratic.
- The next term continues the first-difference pattern.
- Halve the second difference to get a, the coefficient of n^2.
- Subtract an^2 from the sequence; what remains is a linear sequence, which gives $bn + c$.

Worked Solution [5 marks]

> **Rule - nth term of a quadratic sequence:** it has the form $an^2 + bn + c$, where $2a$ equals the constant second difference. Subtracting an^2 from the sequence leaves a linear sequence, which gives $bn + c$.

Step 1 (part a): Find the differences and the next term.

$$\text{sequence: } 4, 10, 18, 28$$
$$\text{first differences: } 6, 8, 10$$
$$\text{second difference: } 2 \text{ (constant)}$$
$$\text{next first difference} = 10 + 2 = 12, \quad \text{next term} = 28 + 12 = 40$$

(Reason: the first differences go up by 2 each time, so the next one is 12; adding it to 28 gives 40.)

Step 2 (part b): Find a from the second difference.

$$2a = 2 \implies a = 1$$

(Reason: for a quadratic sequence the second difference equals 2a.)

Step 3 (part b): Subtract n squared and find the linear part.

$$\text{sequence: } 4, 10, 18, 28$$
$$n^2\text{: } 1, 4, 9, 16$$
$$\text{sequence} - n^2\text{: } 3, 6, 9, 12$$
$$\text{linear, common difference } 3 \implies 3n \quad (b = 3,\ c = 0)$$

(Reason: after removing n squared, the leftover 3, 6, 9, 12 is a linear sequence with nth term 3n.)

Step 4 (part b): Combine.

$$n\text{th term} = n^2 + 3n$$

(Reason: a = 1, b = 3 and c = 0 give n squared plus 3n.)

(a) 40

(b) $n\text{th term} = n^2 + 3n$

Verification

✓ **Check A (part a):** Substitute $n = 5$ into the nth term: $5^2 + 3(5) = 25 + 15 = 40$, which matches the next term found in part (a).

✓ **Check B (part b):** Substitute $n = 4$ into $n^2 + 3n$: $4^2 + 3(4) = 16 + 12 = 28$, the fourth given term. The expression is correct.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): find the differences (first differences 6, 8, 10, or second difference 2)	**M1**	Method - differences	✓
Part (a): next term 40	**A1**	Accuracy - next term	✓
Part (b): a = 1 (second difference divided by 2)	**M1**	Method - coefficient a	✓

Step	Mark	Description	Got it?
Part (b): linear part 3n after subtracting n squared	M1	Method - linear part	✓
Part (b): n squared + 3n	A1	Accuracy - nth term	✓

Full marks: 5/5

Question 6 - Exam Solution

Understanding the Question

Given

$$u_{n+1} = \frac{1}{1 - u_n}$$

A term-to-term rule: watch for the sequence returning to its start and repeating after a fixed number of terms.

Find

The next three terms of the sequence, then the value of u_{50}.

Plan the Solution

- Apply the rule to each term to get the next one.
- Watch for the sequence returning to its start, which reveals a repeating cycle.
- Once you know the cycle length, find where term 50 sits within a cycle.

Worked Solution [3 marks]

> **Rule - periodic (repeating) sequences:** some term-to-term rules produce a sequence that repeats after a fixed number of terms. If the cycle length is p, then terms whose positions leave the same remainder when divided by p are equal.

Step 1 (part a): Apply the rule to find the next three terms.

$$u_2 = \frac{1}{1-3} = \frac{1}{-2} = -\frac{1}{2}$$

$$u_3 = \frac{1}{1 - \left(-\frac{1}{2}\right)} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$u_4 = \frac{1}{1 - \frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3$$

(Reason: apply the rule to each term; note that u4 = 3 = u1, so the sequence repeats.)

Step 2 (part b): Identify the cycle and locate term 50.

$$\text{cycle (period 3): } 3,\ -\frac{1}{2},\ \frac{2}{3},\ 3,\ -\frac{1}{2},\ \frac{2}{3}, \ldots$$

$$50 = 3 \times 16 + 2$$

$$\text{so } u_{50} \text{ is the 2nd term of a cycle} = -\frac{1}{2}$$

(Reason: the sequence has period 3, so terms in the same cycle position are equal; 50 leaves remainder 2 when divided by 3, so u50 equals the 2nd term of the cycle.)

$$\textbf{(a)}\ u_2 = -\frac{1}{2},\ u_3 = \frac{2}{3},\ u_4 = 3$$

$$\textbf{(b)}\ u_{50} = -\frac{1}{2}$$

Verification

✓ **Check A (part a):** Apply the rule once more: $u_5 = \frac{1}{1-3} = -\frac{1}{2}$, which equals u_2. So the pattern $3,\ -\frac{1}{2},\ \frac{2}{3}$ genuinely repeats.

✓ **Check B (part b):** The positions giving the 2nd term $-\frac{1}{2}$ are $n = 2, 5, 8, \ldots$, that is every n with remainder 2 when divided by 3. Since $50 = 3(16) + 2$, it is one of these, so $u_{50} = -\frac{1}{2}$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): two of the three next terms correct	**B1**	Accuracy - two of the three terms	✓
Part (a): all three next terms correct (minus one half, two thirds, three)	**B1**	Accuracy - all three terms	✓
Part (b): u50 = minus one half	**B1**	Accuracy - the 50th term	✓

Full marks: 3/3

Question 7 - Exam Solution

Understanding the Question

Given

grey: $5, 9, 13, 17, \dots$ total: $9, 25, 49, 81, \dots$

Grey squares sit on the two diagonals of a growing odd square grid, so the grey count grows linearly while the total grows quadratically.

Find

The nth term for the grey squares, the two consecutive pattern numbers, then the nth term for the total number of squares.

Plan the Solution

- Count the grey squares in each pattern and find the linear nth term.
- For part (b), add the grey counts of Pattern n and Pattern $(n+1)$, set the sum equal to 206, and solve for n.
- For part (c), count the total squares, find the constant second difference, and get the quadratic.

Worked Solution [8 marks]

Rule - patterns and nth terms: count the shape squares for each pattern to form a sequence; a constant first difference gives a linear nth term, a constant second difference gives a quadratic nth term.

Step 1 (part a): Count the grey squares and find the nth term.

grey: $5, 9, 13, 17$

first differences: $4, 4, 4 \implies 4n$

$4(1) + c = 5 \implies c = 1$

grey nth term $= 4n + 1$

(Reason: the grid grows by two each way, adding four grey squares, one per arm of the X; a constant first difference of 4 gives 4n, and the constant is fixed by making n = 1 give 5.)

Step 2 (part b): Add two consecutive grey counts and solve.

Pattern n: $4n + 1$

Pattern $(n+1)$: $4(n+1) + 1 = 4n + 5$

$(4n + 1) + (4n + 5) = 8n + 6$

$8n + 6 = 206 \implies 8n = 200 \implies n = 25$

(Reason: consecutive patterns are n and n+1, so add their grey counts and solve; n = 25 gives Patterns 25 and 26.)

Step 3 (part c): Count the totals and find the quadratic nth term.

$$\text{total: } 9, 25, 49, 81$$
$$\text{first differences: } 16, 24, 32$$
$$\text{second difference: } 8 \text{ (constant)}$$
$$2a = 8 \implies a = 4$$
$$\text{total} - 4n^2\text{: } 5, 9, 13, 17 \implies 4n + 1$$
$$\text{total } n\text{th term} = 4n^2 + 4n + 1$$

(Reason: a constant second difference means the sequence is quadratic; a is half the second difference, and the leftover after subtracting 4n squared is the linear part 4n + 1.)

$$\textbf{(a)}\ \text{grey } n\text{th term} = 4n + 1$$
$$\textbf{(b)}\ \textbf{Patterns 25 and 26}$$
$$\textbf{(c)}\ \text{total } n\text{th term} = 4n^2 + 4n + 1$$

Verification

✓ **Check A (part a):** Pattern 4 should have $4(4) + 1 = 17$ grey squares. The fourth diagram is a 9×9 grid whose two diagonals hold $9 + 9 - 1 = 17$ squares, the centre being shared. The expression is correct.

✓ **Check B (part b):** $\text{grey}(25) = 4(25) + 1 = 101$ and $\text{grey}(26) = 4(26) + 1 = 105$, so the total is $101 + 105 = 206$, as required. Patterns 25 and 26 are correct.

✓ **Check C (part c):** Pattern 3 total $= 4(3)^2 + 4(3) + 1 = 36 + 12 + 1 = 49$, and the third diagram is a 7×7 grid holding 49 squares. Correct.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): use the common difference 4 to obtain 4n	M1	Method - coefficient of n	✓
Part (a): grey nth term 4n + 1	A1	Accuracy - part (a) answer	✓
Part (b): form 8n + 6, or (4n + 1) + (4n + 5)	M1	Method - sum of two consecutive patterns	✓

Step	Mark	Description	Got it?
Part (b): solve 8n + 6 = 206 to reach n = 25	M1	Method - solve for n	✓
Part (b): Patterns 25 and 26	A1	Accuracy - part (b) answer	✓
Part (c): a = 4 (half the constant second difference 8)	M1	Method - coefficient a	✓
Part (c): leftover 4n + 1 after subtracting 4n squared	M1	Method - linear part	✓
Part (c): total nth term 4n squared + 4n + 1	A1	Accuracy - part (c) answer	✓

Full marks: 8/8

Question 8 - Exam Solution

Understanding the Question

Given

$$n\text{th term} = \left(\frac{1}{3}\right)^n$$

A geometric sequence with ratio one third: the terms shrink as n grows.

Find

The difference between the 3rd and 5th terms, as a fraction in its simplest form.

Plan the Solution

- Substitute $n = 3$ and $n = 5$ into the nth-term formula to get the two terms.
- Raising a fraction to a power raises both the numerator and the denominator.
- Write both terms over a common denominator and subtract.

Worked Solution [2 marks]

> **Rule - substituting into an nth term:** to find a specific term, substitute its position number for n. For a fraction raised to a power, raise the top and the bottom separately.

Step 1: Find the 3rd and 5th terms.

$$\text{3rd term} = \left(\frac{1}{3}\right)^3 = \frac{1}{3^3} = \frac{1}{27}$$

$$\text{5th term} = \left(\frac{1}{3}\right)^5 = \frac{1}{3^5} = \frac{1}{243}$$

(Reason: substitute n = 3 and n = 5 into the formula; raising a fraction to a power raises both the top and the bottom, so the top stays 1 and the bottom becomes a power of 3.)

Step 2: Subtract using a common denominator.

$$\frac{1}{27} = \frac{9}{243} \quad (243 \text{ divided by } 27 = 9)$$

$$\frac{9}{243} - \frac{1}{243} = \frac{8}{243}$$

(Reason: the two terms need the same denominator before subtracting; 243 is that common denominator, so the 3rd term is rewritten over 243 and the numerators are then subtracted.)

$$\frac{8}{243}$$

Verification

✓ **Check A:** $\frac{8}{243}$ is in its simplest form: $243 = 3^5$ and $8 = 2^3$, so the two share no common factor and the fraction cannot be reduced.

✓ **Check B:** Add the difference back to the smaller term: $\frac{1}{243} + \frac{8}{243} = \frac{9}{243} = \frac{1}{27}$, which is the 3rd term. The difference is correct.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Both terms correct: one over 27 and one over 243	B1	Accuracy - the 3rd and 5th terms	✓
Difference eight over 243	B1	Accuracy - the difference	✓

Full marks: 2/2

Question 9 - Exam Solution

Understanding the Question

Given

arithmetic: nth term $= 50 - 4n$

quadratic: $10, 12, 16, 22, \ldots$

One arithmetic sequence and one quadratic sequence: a common term is where the two nth terms are equal.

Find

The nth term of the quadratic sequence, then the single value common to both sequences.

Plan the Solution

- For the quadratic, find the constant second difference; half of it gives the coefficient of n^2.
- Subtract that n^2 term to leave a linear sequence for the rest.
- For part (b), the common term is where the two nth terms are equal: set them equal and solve.
- Reject any negative solution, since n must be a positive whole number.

Worked Solution [6 marks]

> **Rule - quadratic nth term and common terms:** a quadratic sequence has nth term $an^2 + bn + c$, with $2a$ equal to the constant second difference. Two sequences share a term where their nth terms are equal, found by setting the expressions equal and solving.

Step 1 (part a): Find the nth term of the quadratic sequence.

sequence: $10, 12, 16, 22$

first differences: $2, 4, 6$

second difference: 2 (constant)

$2a = 2 \implies a = 1$

sequence $- n^2$: $9, 8, 7, 6 \implies -n + 10$

nth term $= n^2 - n + 10$

(Reason: a constant second difference means the sequence is quadratic; a is half the second difference, and the leftover after subtracting n squared is the linear part.)

Step 2 (part b): Set the two nth terms equal.

$50 - 4n = n^2 - n + 10$

$0 = n^2 - n + 10 - 50 + 4n$

$0 = n^2 + 3n - 40$

(Reason: a shared term occurs where the two expressions are equal, so move everything to one side; the constants combine as 10 - 50 = -40.)

Step 3 (part b): Factorise and solve.

$0 = (n-5)(n+8)$

$n = 5$ or $n = -8$

n must be positive, so $n = 5$

(Reason: factorise the quadratic; reject the negative root because n is a positive whole number counting the position.)

Step 4 (part b): Find the value of the common term.

$50 - 4(5) = 50 - 20 = 30$

(Reason: substitute n = 5 into either sequence to get the common value.)

(a) $n\text{th term} = n^2 - n + 10$

(b) the common term is 30

Verification

✓ **Check A (part a):** Substitute $n = 4$ into $n^2 - n + 10$: $16 - 4 + 10 = 22$, which is the fourth given term. The expression is correct.

✓ **Check B (part b):** Substitute $n = 5$ into the quadratic: $25 - 5 + 10 = 30$, and into the arithmetic: $50 - 20 = 30$. Both give 30, so 30 is genuinely common to the two sequences.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Part (a): a = 1 (the constant second difference is 2)	M1	Method - coefficient a	✓
Part (a): linear part minus n plus 10 after subtracting n squared	M1	Method - linear part	✓
Part (a): nth term n squared minus n plus 10	A1	Accuracy - part (a) answer	✓
Part (b): set 50 minus 4n equal to n squared minus n plus 10 and rearrange to n squared plus 3n minus 40 = 0	M1	Method - form the equation	✓
Part (b): factorise and solve to n = 5, rejecting n = -8	M1	Method - solve for n	✓
Part (b): common term 30	A1	Accuracy - part (b) answer	✓

Full marks: 6/6

Question 10 - Exam Solution

Understanding the Question

Given

nth term $= 2n^2 + n - 3$

The sum of two consecutive terms is 301; the task is to find which two.

Find

The two consecutive terms whose sum is 301.

Plan the Solution

- Write the nth term and the $(n+1)$th term, expanding the second.
- Add them and set the sum equal to 301.
- Rearrange to a quadratic equation and solve.
- Reject any solution that is not a positive whole number, then evaluate the two terms.

Worked Solution [5 marks]

> **Rule - consecutive terms of a quadratic sequence:** the $(n+1)$th term is found by substituting $n+1$ into the nth-term formula. Setting the sum of consecutive terms equal to a value gives a quadratic equation in n to solve.

Step 1: Write the nth and (n+1)th terms.

$n\text{th term} = 2n^2 + n - 3$

$(n+1)\text{th term} = 2(n+1)^2 + (n+1) - 3 = 2n^2 + 4n + 2 + n - 2 = 2n^2 + 5n$

(Reason: substitute n+1 into the formula and expand to get the next term.)

Step 2: Add the two terms and set the sum equal to 301.

$(2n^2 + n - 3) + (2n^2 + 5n) = 4n^2 + 6n - 3$

$4n^2 + 6n - 3 = 301$

$4n^2 + 6n - 304 = 0$

(Reason: the two consecutive terms sum to 301; move everything to one side.)

Step 3: Simplify and factorise.

$2n^2 + 3n - 152 = 0 \quad \text{(divide by 2)}$

$(n-8)(2n+19) = 0$

$n = 8$ or $n = -\frac{19}{2}$

(Reason: divide by the common factor 2, then factorise; reject the negative fraction because n is a positive whole number, so n = 8.)

Step 4: Evaluate the two terms.

$$\text{8th term} = 2(8)^2 + 8 - 3 = 128 + 8 - 3 = 133$$
$$\text{9th term} = 2(9)^2 + 9 - 3 = 162 + 9 - 3 = 168$$

(Reason: substitute n = 8 and n = 9 into the nth-term formula.)

133 and 168

Verification

✓ **Check A:** Add the two terms: $133 + 168 = 301$, which matches the given sum. The pair is correct.

✓ **Check B:** Confirm $n = 8$ is a root by substituting into $2n^2 + 3n - 152$: $2(64) + 24 - 152 = 128 + 24 - 152 = 0$, so $n = 8$ is genuinely a solution.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The (n+1)th term expanded to 2n squared + 5n	M1	Method - the next term	✓
Form 4n squared + 6n - 3 = 301	M1	Method - sum of consecutive terms	✓
Reach 2n squared + 3n - 152 = 0, or 4n squared + 6n - 304 = 0	M1	Method - rearrange to a quadratic	✓
Factorise and solve to n = 8, rejecting the negative root	M1	Method - solve for n	✓
Both terms 133 and 168	A1	Accuracy - the two terms	✓

Full marks: 5/5

Question 11 - Exam Solution

Understanding the Question

Given

$3x - 1, \quad 5x - 4, \quad 8x - 13$

An arithmetic sequence: the differences between consecutive terms are equal.

Find

The 12th term of the sequence.

Plan the Solution

- In an arithmetic sequence the difference between consecutive terms is constant.
- Form the two differences and set them equal to find x.
- Substitute x back to get the first three terms and the common difference.
- Write the nth term, then evaluate it at $n = 12$.

Worked Solution [6 marks]

> **Rule - algebraic arithmetic sequences:** consecutive terms of an arithmetic sequence have a constant difference, so $(\text{2nd} - \text{1st}) = (\text{3rd} - \text{2nd})$. Solving this gives the unknown, and the nth term follows from the first term and the common difference.

Step 1: Form the two differences.

$$\text{2nd} - \text{1st} = (5x - 4) - (3x - 1) = 2x - 3$$
$$\text{3rd} - \text{2nd} = (8x - 13) - (5x - 4) = 3x - 9$$

(Reason: in an arithmetic sequence these two differences must be equal.)

Step 2: Set the differences equal and solve for x.

$$2x - 3 = 3x - 9$$
$$9 - 3 = 3x - 2x$$
$$x = 6$$

(Reason: equal differences give an equation in x; solve it.)

Step 3: Find the first three terms.

$$3(6) - 1 = 17$$
$$5(6) - 4 = 26$$
$$8(6) - 13 = 35$$
$$\text{sequence: } 17, 26, 35, \ldots \text{ (common difference 9)}$$

(Reason: substitute x = 6 into each expression; the differences confirm a common difference of 9.)

Step 4: Find the nth term, then the 12th term.

common difference 9 $\implies$ $9n$

$9(1) = 9,$ first term $17,$ so add 8

nth term $= 9n + 8$

12th term $= 9(12) + 8 = 108 + 8 = 116$

(Reason: the common difference gives 9n; adjust by a constant so n = 1 gives 17; then substitute n = 12.)

$$\text{12th term} = 116$$

Verification

✓ **Check A:** Confirm $x = 6$ gives a genuine arithmetic sequence: $26 - 17 = 9$ and $35 - 26 = 9$. The two differences are equal, so the sequence really is arithmetic.

✓ **Check B:** Substitute $n = 3$ into $9n + 8$: $9(3) + 8 = 35$, which is the third term. The n th term is correct.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Both differences 2x - 3 and 3x - 9	**M1**	Method - consecutive differences	✓
Set the differences equal: 2x - 3 = 3x - 9	**M1**	Method - form the equation	✓
Solve to x = 6	**M1**	Method - solve for x	✓
First three terms 17, 26, 35	**M1**	Method - the numerical sequence	✓
nth term 9n + 8	**M1**	Method - the nth term	✓
12th term 116	**A1**	Accuracy - the 12th term	✓

Full marks: 6/6

Question 12 - Exam Solution

Understanding the Question

Given

Find

nth term $= 72 - \frac{1}{2}n^2$

The terms fall as n grows; we want the first one that is negative.

The value of the first term that is less than 0.

Plan the Solution

- Write the condition for a term to be negative as an inequality.
- Multiply through to clear the fraction, then factorise as a difference of two squares.
- Solve the inequality; keep only positive whole-number values of n.
- Substitute the first such n to find the term.

Worked Solution [3 marks]

> **Rule - quadratic inequalities from sequences:** to find where a term is negative, set the nth term less than 0 and solve the inequality. A difference of two squares factorises as $(a+n)(a-n)$, and the product is negative when the two factors have opposite signs.

Step 1: Set the term less than 0 and clear the fraction.

$$72 - \frac{1}{2}n^2 < 0$$

$$\text{multiply by 2:} \quad 144 - n^2 < 0$$

(Reason: a term is negative when it is less than 0; multiplying by 2 clears the fraction.)

Step 2: Factorise and solve the inequality.

$$(12+n)(12-n) < 0$$

$$n < -12 \text{ or } n > 12$$

(Reason: difference of two squares; the product is negative when the factors have opposite signs. Since n is a positive whole number, n > 12.)

Step 3: Identify the first term and evaluate it.

$$n = 13 \text{ is the first term below } 0$$

$$72 - \frac{1}{2}(13)^2 = 72 - \frac{169}{2} = 72 - 84.5 = -12.5$$

(Reason: the smallest positive whole number greater than 12 is 13; substitute it into the nth term.)

$$-12.5$$

Verification

✓ **Check A:** The 12th term is $72 - \frac{1}{2}(144) = 72 - 72 = 0$, which is not below 0, so 13 really is the first n giving a negative term.

✓ **Check B:** The 13th term is -12.5, which is below 0, while the 11th term is $72 - \frac{1}{2}(121) = 72 - 60.5 = 11.5$, still positive. The crossing happens at $n = 13$.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Set up 144 - n squared < 0, or 72 minus one half n squared < 0, and factorise to (12 + n)(12 - n) < 0	**M1**	Method - form and factorise the inequality	✓
n > 12, so n = 13 is the first term below 0	**M1**	Method - solve the inequality	✓
The 13th term, -12.5	**A1**	Accuracy - the first term below 0	✓

Full marks: 3/3

Question 13 - Exam Solution

Understanding the Question

Given

geometric: nth term $= (\sqrt{2})^n$

quadratic: $8, 9, 12, \ldots$

Add two specific terms, the 8th geometric and the 6th quadratic, then show the total is a square number.

Show

That the sum of the 8th geometric term and the 6th quadratic term is a square number.

Plan the Solution

- For the geometric sequence, substitute $n = 8$ into $(\sqrt{2})^n$ and simplify using powers.
- For the quadratic sequence, use the constant second difference to extend the terms to the 6th.
- Add the two terms and show the total is a perfect square.

Worked Solution [4 marks]

Rule - combining sequence terms: a geometric term $(\sqrt{2})^n$ simplifies to a power of 2; a quadratic sequence has a constant second difference, so its terms can be extended by increasing the first differences. A number is square if it equals an integer squared.

Step 1: Find the 8th term of the geometric sequence.

$$(\sqrt{2})^8 = 2^{8/2} = 2^4 = 16$$

(Reason: raising root 2 to the power n is the same as raising 2 to the power n over 2, so the 8th term is 2 to the power 4.)

Step 2: Find the 6th term of the quadratic sequence, using differences.

$$\text{terms so far: } 8, 9, 12$$
$$\text{first differences: } 1, 3 \implies \text{second difference } 2$$
$$\text{next first differences: } 5, 7, 9$$
$$8, 9, 12, 17, 24, 33$$

(Reason: with a constant second difference of 2, each first difference is 2 more than the last; this gives the terms up to the 6th without needing the nth term.)

Step 3: Add the two terms and show the total is square.

$$16 + 33 = 49 = 7^2$$

(Reason: the sum is 49, which equals 7 squared, so it is a square number.)

$$16 + 33 = 49 = 7^2, \text{ a square number}$$

Verification

✓ **Check A:** Confirm the geometric term another way: $(\sqrt{2})^8 = ((\sqrt{2})^2)^4 = 2^4 = 16$. The two routes agree, so the 8th term really is 16.

✓ **Check B:** The terms $12, 17, 24, 33$ have differences $5, 7, 9$, each increasing by 2, so the second difference stays 2 as a quadratic sequence requires. And $49 = 7 \times 7$, confirming the total is square.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
The 8th geometric term: root 2 to the power 8 = 16	M1	Method - the geometric term	✓

Step	Mark	Description	Got it?
Continue the quadratic first differences 5, 7, 9 to reach 17 and 24	M1	Method - extend the differences	✓
The 6th quadratic term = 33	M1	Method - the sixth term	✓
Sum 16 + 33 = 49 = 7 squared, stated as a square number	A1	Accuracy - the shown result	✓

Full marks: 4/4

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