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Simultaneous Equations

Linear & Non-Linear: GCSE & IGCSE Practice Questions with Worked Solutions

GRADE 8-9 TARGETED

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11 exam-style questions with full worked solutions and mark schemes.

Practice Questions

Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. Solve the simultaneous equations.

GRADE 5

$$2x + y = 8$$
$$x + 3y = 14$$

$x =$ .... , $y =$ ....

[Total 3 marks]

2. Solve the simultaneous equations.

GRADE 5

$$4x + 3y = 17$$
$$3x + 5y = 21$$

$x =$ .... , $y =$ ....

[Total 4 marks]

3. A cafe charges £7.40 for 3 teas and 2 coffees, and £13.40 for 5 teas and 4 coffees. Work out the cost of 2 teas and 3 coffees.

GRADE 6

£ ....

[Total 4 marks]

4. Solve the simultaneous equations.

GRADE 7

$$x^2 + y = 7$$
$$y = 2x + 4$$

$x =$ .... , $y =$ ....
$x =$ .... , $y =$ ....

[Total 5 marks]

5. Solve the simultaneous equations.

$$2x^2 + y^2 = 33$$
$$y = x + 3$$

GRADE 8

$x =$ .... , $y =$ ....
$x =$ .... , $y =$ ....

[Total 5 marks]

6. The line $y = 2x + 1$ and the curve $y = x^2 - 4x + 6$ meet at two points. The distance between the two points is $k\sqrt{5}$. Find the value of k.

GRADE 9

Problem solving

$k =$ ....

[Total 6 marks]

7. Solve the simultaneous equations.

$$x^2 + 2y^2 = 18$$
$$3x - 2y = x + 6$$

GRADE 7

$x =$ .... , $y =$ ....
$x =$ .... , $y =$ ....

[Total 5 marks]

8. The line $x + y = 1$ crosses the circle $x^2 + y^2 = 25$ at the points A and B. Work out the exact length of AB. Give your answer in its simplest surd form.

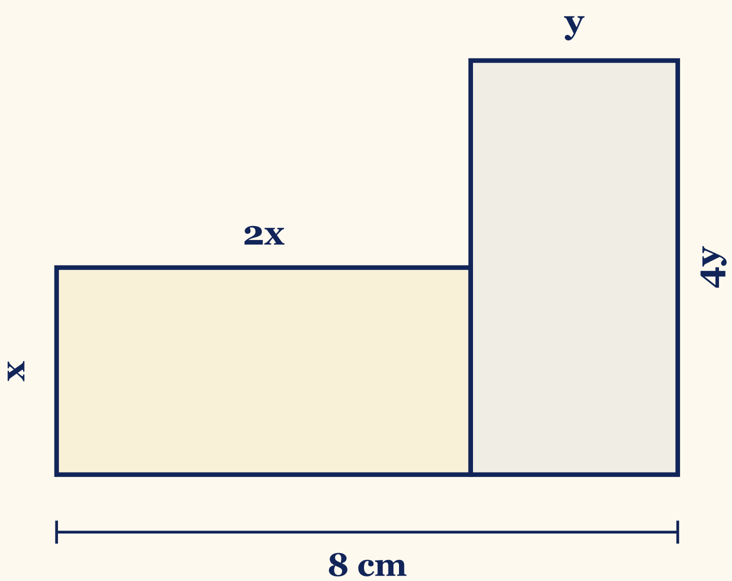
GRADE 9

$AB =$ ....

[Total 6 marks]

9.

GRADE 7



Not drawn accurately

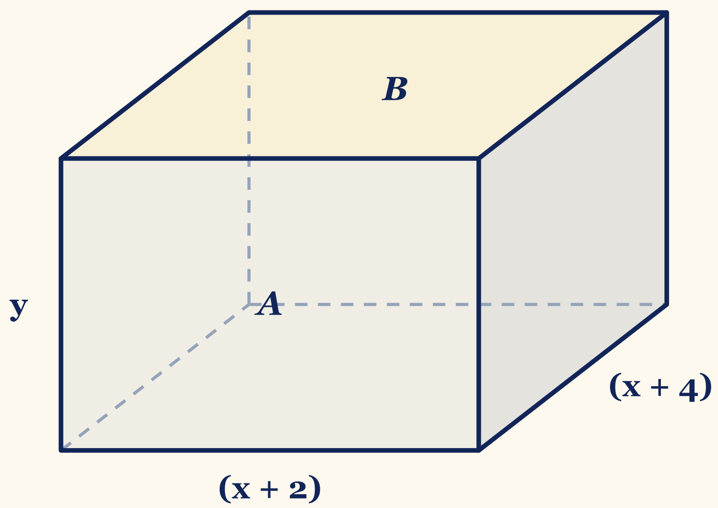
The diagram shows an L-shape made from two rectangles. The total area is 34 cm² and the total width along the base is 8 cm. Given that x and y are positive integers, work out the values of x and y.

$x =$ .... , $y =$ ....

[Total 5 marks]

10.

GRADE 8



Not drawn accurately

The cuboid shown has a weight of 48 N. When it rests on face A the pressure on the ground is 4 N/m², and when it rests on face B the pressure is 2 N/m². Work out the volume of the cuboid.

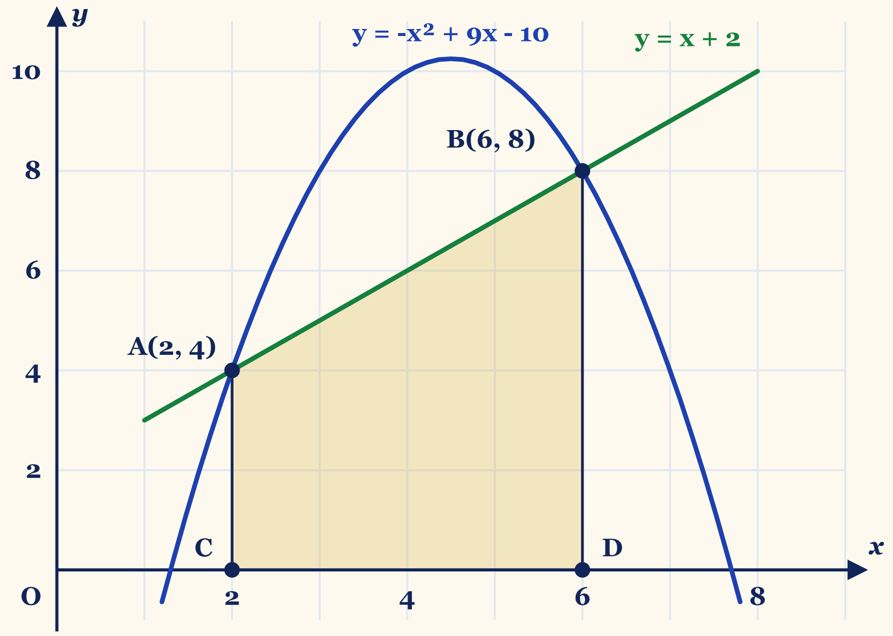
Volume = ... m^3

[Total 6 marks]

11.

GRADE 8

Problem solving



Not drawn accurately

The line $y = x + 2$ meets the curve $y = -x^2 + 9x - 10$ at the points A and B, as shown. Points C and D lie on the x-axis so that AC and BD are vertical. Work out the area of quadrilateral ABDC.

Area = ... units²

[Total 6 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

$2x + y = 8$

$x + 3y = 14$

Two linear equations, two unknowns.

Find

The values of x and y that satisfy both equations.

Plan the Solution

- Use elimination.
- Multiply (1) by 3 so both equations have $3y$, then subtract to eliminate y and find x.
- Substitute x back to find y.

Worked Solution [3 marks]

> **Rule - Elimination:** make one variable's coefficient equal in both equations, then add or subtract to eliminate that variable.

Step 1: Label the equations.

$2x + y = 8 \quad (1)$

$x + 3y = 14 \quad (2)$

(Reason: labelling keeps the working organised.)

Step 2: Match the y-coefficients - multiply (1) by 3.

$6x + 3y = 24 \quad (3)$

(Reason: Equations (2) and (3) both have $3y$, so subtracting removes y.)

Step 3: Subtract (2) from (3) to eliminate y.

$(6x + 3y) - (x + 3y) = 24 - 14$

$5x = 10$

$x = 2$

(Reason: $3y - 3y = 0$, so y is eliminated.)

Step 4: Substitute x = 2 into (1) to find y.

$2(2) + y = 8$

$4 + y = 8$

$y = 4$

(Reason: use the simplest equation to find the second variable.)

$$x = 2, \quad y = 4$$

Verification

✓ **Check 1:** $2(2) + 4 = 8$

✓ **Check 2:** $2 + 3(4) = 14$

✓ **Check 3:** both values are simple whole numbers, sensible for a non-calculator pair.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Scale one equation to match a coefficient (×3 to get 3y)	**M1**	Method - correct elimination setup	✓
Eliminate and solve for the first variable (x = 2)	**A1**	Accuracy - first value	✓
Substitute back and solve for the second (y = 4)	**A1**	Accuracy - second value	✓

Full marks: 3/3

Question 2 - Exam Solution

Understanding the Question

Given

$4x + 3y = 17$

$3x + 5y = 21$

Neither coefficient matches yet.

Find

The values of x and y that satisfy both equations.

Plan the Solution

- Use elimination.
- Neither coefficient matches yet, so scale both equations.
- Multiply (1) by 3 and (2) by 4 so the x-terms both become $12x$, then subtract to eliminate x and find y.
- Substitute back to find x.

Worked Solution [4 marks]

> **Rule - Elimination:** when no coefficient matches, multiply each equation so one variable's coefficients become equal, then add or subtract to eliminate it.

Step 1: Label the equations.

$4x + 3y = 17 \quad (1)$

$3x + 5y = 21 \quad (2)$

(Reason: labelling lets us track each multiplication.)

Step 2: Match the x-coefficients - multiply (1) by 3 and (2) by 4.

$12x + 9y = 51 \quad (3)$

$12x + 20y = 84 \quad (4)$

(Reason: The LCM of 4 and 3 is 12, so both x-terms become $12x$.)

Step 3: Subtract (3) from (4) to eliminate x.

$(12x + 20y) - (12x + 9y) = 84 - 51$

$11y = 33$

$y = 3$

(Reason: $12x - 12x = 0$, so x is eliminated.)

Step 4: Substitute y = 3 into (1) to find x.

$4x + 3(3) = 17$

$4x + 9 = 17$

$4x = 8$

$x = 2$

(Reason: substitute the known value and rearrange to isolate x.)

$$x = 2, \quad y = 3$$

Verification

✓ **Check 1:** $4(2) + 3(3) = 8 + 9 = 17$

✓ **Check 2:** $3(2) + 5(3) = 6 + 15 = 21$

✓ **Check 3:** both values are simple whole numbers, sensible for a non-calculator pair.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Multiply BOTH equations to equate a coefficient (×3 and ×4 → 12x)	**M1**	Method - set up elimination	✓
Subtract to eliminate x: 11y = 33 → y = 3	**A1**	Accuracy - first value	✓
Substitute y = 3 back and rearrange (4x + 9 = 17)	**M1**	Method - find second variable	✓
Second value x = 2	**A1**	Accuracy - second value	✓

Full marks: 4/4

Question 3 - Exam Solution

Understanding the Question

Given

3 teas and 2 coffees cost £7.40
5 teas and 4 coffees cost £13.40

Find

The total cost of 2 teas and 3 coffees.

Plan the Solution

- Let t = cost of one tea and c = cost of one coffee (in £).
- Turn each fact into an equation and solve by elimination.
- Then use the prices to cost 2 teas and 3 coffees.

Worked Solution [4 marks]

> **Rule - Form and solve:** turn the words into two equations, solve them simultaneously, then answer the exact question asked.

Step 1: Define variables and write the equations.

$3t + 2c = 7.40 \quad (1)$

$5t + 4c = 13.40 \quad (2)$

(Reason: t and c are the unknown prices; each sentence gives one equation.)

Step 2: Match the c-coefficients - multiply (1) by 2.

$6t + 4c = 14.80 \quad (3)$

(Reason: Equations (2) and (3) both have $4c$, so subtracting removes c.)

Step 3: Subtract (2) from (3) to eliminate c.

$(6t + 4c) - (5t + 4c) = 14.80 - 13.40$

$t = 1.40$

(Reason: $4c - 4c = 0$, so c is eliminated.)

Step 4: Substitute t = 1.40 into (1) to find c.

$3(1.40) + 2c = 7.40$

$4.20 + 2c = 7.40$

$2c = 3.20$

$c = 1.60$

So tea = £1.40 and coffee = £1.60.

(Reason: substitute the known value and solve for c.)

Step 5: Answer the question - cost of 2 teas and 3 coffees.

$2(1.40) + 3(1.60) = 2.80 + 4.80 = 7.60$

(Reason: the question asks for a specific combination, not just the prices - always finish with what was asked.)

£7.60

Verification

✓ **Check 1:** $3(1.40) + 2(1.60) = 4.20 + 3.20 = 7.40$

✓ **Check 2:** $5(1.40) + 4(1.60) = 7.00 + 6.40 = 13.40$

✓ **Check 3:** £1.40 and £1.60 are realistic cafe prices, so the answer makes sense.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Form two correct equations from the words	**M1**	Method - translate the problem	✓
Solve simultaneously (correct elimination)	**M1**	Method - solve the pair	✓
Both prices correct: tea £1.40, coffee £1.60	**A1**	Accuracy - the two prices	✓
Cost of 2 teas and 3 coffees = £7.60	**A1**	Accuracy - final answer asked for	✓

Full marks: 4/4

Question 4 - Exam Solution

Understanding the Question

Given

$x^2 + y = 7$

$y = 2x + 4$

One curve, one line.

Find

The values of x and y; a quadratic appears, so expect two solution pairs.

Plan the Solution

- Use substitution (the line gives y).
- Put it into the quadratic; rearrange to a quadratic equal to 0 and factorise for the two x-values.
- Substitute each x back into the line for its y.

Worked Solution [5 marks]

> **Rule - Substitution (line into curve):** replace y in the quadratic with the linear expression, solve the resulting quadratic, then find each y.

Step 1: Label the equations.

$x^2 + y = 7 \quad (1)$

$y = 2x + 4 \quad (2)$

(Reason: (2) is already $y = \ldots$, which makes substitution natural.)

Step 2: Substitute (2) into (1).

$x^2 + (2x + 4) = 7$

$x^2 + 2x + 4 = 7$

(Reason: replacing y leaves one equation in x only.)

Step 3: Rearrange to a quadratic equal to 0.

$x^2 + 2x + 4 - 7 = 0$

$x^2 + 2x - 3 = 0$

(Reason: a quadratic must equal 0 before factorising.)

Step 4: Factorise and solve.

$(x + 3)(x - 1) = 0$

$x = -3$ or $x = 1$

(Reason: two numbers with product -3 and sum $+2$ are $+3$ and -1.)

Step 5: Find each matching y from the line (2).

When $x = 1$: $y = 2(1) + 4 = 6 \rightarrow (1, 6)$

When $x = -3$: $y = 2(-3) + 4 = -2 \rightarrow (-3, -2)$

(Reason: each x pairs with exactly one y on the line.)

$$x = 1, \quad y = 6$$
$$x = -3, \quad y = -2$$

Verification

✓ **Check 1 - (1, 6):** $1^2 + 6 = 7$

✓ **Check 2 - (-3, -2):** $(-3)^2 + (-2) = 9 - 2 = 7$

✓ **Check 3:** each y came from the line, and a quadratic gives two pairs - both consistent.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Substitute y = 2x + 4 into $x^2 + y = 7$	**M1**	Method - substitution	✓
Form the correct quadratic: $x^2 + 2x - 3 = 0$	**A1**	Accuracy - correct equation	✓
Factorise / solve: (x + 3)(x - 1) = 0	**M1**	Method - solve the quadratic	✓
Both x-values: x = 1 and x = -3	**A1**	Accuracy - the two roots	✓
Both complete pairs: (1, 6) and (-3, -2)	**A1**	Accuracy - matching y-values	✓

Full marks: 5/5

Question 5 - Exam Solution

Understanding the Question

Given

$2x^2 + y^2 = 33$

$y = x + 3$

A curve and a line.

Find

Values of x and y; expect two pairs.

Plan the Solution

- Substitute $y = x + 3$ into the curve.
- Expand the squared bracket, collect, and simplify to a quadratic equal to 0.
- Factorise for the two x-values; find each matching y.

Worked Solution [5 marks]

> **Rule - Substitution (line into curve):** replace y in the curve, expand and simplify, then solve the quadratic.

Step 1: Label the equations.

$2x^2 + y^2 = 33 \quad (1)$

$y = x + 3 \quad (2)$

(Reason: (2) gives y directly, so substitute into (1).)

Step 2: Substitute (2) into (1).

$2x^2 + (x + 3)^2 = 33$

(Reason: replacing y leaves an equation in x only.)

Step 3: Expand the bracket and collect like terms.

$(x + 3)^2 = x^2 + 6x + 9$

$2x^2 + x^2 + 6x + 9 = 33$

$3x^2 + 6x + 9 = 33$

(Reason: expand $(x + 3)^2$ carefully - the middle term $6x$ is the one most often missed.)

Step 4: Rearrange to = 0, then simplify.

$3x^2 + 6x - 24 = 0$

$x^2 + 2x - 8 = 0$ *(divide by 3)*

(Reason: subtract 33, then divide by the common factor 3 to make factorising easier.)

Step 5: Factorise and solve.

$(x+4)(x-2)=0$

$x=-4$ or $x=2$

(Reason: two numbers with product -8 and sum $+2$ are $+4$ and -2.)

Step 6: Find each matching y from the line (2).

When $x=2$: $y=2+3=5 \rightarrow (2,5)$

When $x=-4$: $y=-4+3=-1 \rightarrow (-4,-1)$

(Reason: each x gives one y on the line.)

$$x=2, \quad y=5$$
$$x=-4, \quad y=-1$$

Verification

✓ **Check 1 - (2, 5):** $2(2)^2+5^2=8+25=33$

✓ **Check 2 - (-4, -1):** $2(-4)^2+(-1)^2=32+1=33$

✓ **Check 3:** both pairs also satisfy the line, and a quadratic gives two pairs - consistent.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Substitute $y = x + 3$ into $2x^2 + y^2 = 33$	M1	Method - substitution	✓
Expand $(x + 3)^2$ and collect to a quadratic = 0	M1	Method - expand and rearrange	✓
Correct quadratic: $x^2 + 2x - 8 = 0$	A1	Accuracy - correct equation	✓
Both x-values: $x = 2$ and $x = -4$	A1	Accuracy - the two roots	✓
Both complete pairs: (2, 5) and (-4, -1)	A1	Accuracy - matching y-values	✓

Full marks: 5/5

Question 6 - Exam Solution

Understanding the Question

Given

$y = 2x + 1$

$y = x^2 - 4x + 6$

A line and a curve; their gap is $k\sqrt{5}$.

Find

The value of k - so find the two crossing points first, then the distance.

Plan the Solution

- Set line = curve and solve the quadratic for the two x-values.
- Find each y from the line to get A and B.
- Use the distance formula for AB, then simplify the surd into $k\sqrt{5}$ form.

Worked Solution [6 marks]

Rule - Intersections, then distance: solve line = curve to find both points, then apply $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Step 1: Set the line equal to the curve (both equal y).

$2x + 1 = x^2 - 4x + 6$

(Reason: at a crossing point the line and curve share the same x and y.)

Step 2: Rearrange to a quadratic equal to 0.

$x^2 - 6x + 5 = 0$

(Reason: move every term to one side $(x^2 - 4x + 6 - 2x - 1)$.)

Step 3: Factorise and solve for the x-coordinates.

$(x - 1)(x - 5) = 0$

$x = 1$ or $x = 5$

(Reason: two numbers with product $+5$ and sum -6 are -1 and -5.)

Step 4: Find each point's y from the line.

When $x = 1$: $y = 2(1) + 1 = 3 \rightarrow A(1, 3)$

When $x = 5$: $y = 2(5) + 1 = 11 \rightarrow B(5, 11)$

(Reason: substitute each x into the simpler equation, the line.)

Step 5: Apply the distance formula to A(1, 3) and B(5, 11).

$AB = \sqrt{(5 - 1)^2 + (11 - 3)^2}$

$AB = \sqrt{4^2 + 8^2} = \sqrt{16 + 64}$

$AB = \sqrt{80}$

(Reason: horizontal gap = 4, vertical gap = 8; the distance is the hypotenuse.)

Step 6: Simplify the surd into k√5 form.

$\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

$k\sqrt{5} = 4\sqrt{5}$, so $k = 4$

(Reason: pull out the largest square factor, 16, to leave √5.)

$$k = 4$$

Verification

✓ **Check 1 - A(1, 3):** $1^2 - 4(1) + 6 = 3$

✓ **Check 2 - B(5, 11):** $5^2 - 4(5) + 6 = 11$

✓ **Check 3:** $4\sqrt{5} \approx 8.94$ and $\sqrt{80} \approx 8.94$, and $k = 4$ is a whole number, as expected.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Set line = curve and rearrange to $x^2 - 6x + 5 = 0$	**M1**	Method - form the quadratic	✓
Solve for both x: x = 1 and x = 5	**A1**	Accuracy - the two roots	✓
Coordinates of both points: A(1, 3), B(5, 11)	**A1**	Accuracy - both points	✓
Apply the distance formula with these coordinates	**M1**	Method - distance formula	✓
Correct distance: AB = √80	**A1**	Accuracy - unsimplified surd	✓
Simplify to 4√5, so k = 4	**A1**	Accuracy - final value of k	✓

Full marks: 6/6

Question 7 - Exam Solution

Understanding the Question

Given

$x^2 + 2y^2 = 18$

$3x - 2y = x + 6$

A curve and a linear equation that is not yet tidy.

Find

Values of x and y; expect two pairs.

Plan the Solution

- Tidy the linear equation first - collect the x terms and simplify to $x = \ldots$.
- Substitute into the curve, expand and simplify to a quadratic = 0.
- Factorise for the two y-values, then find each matching x.

Worked Solution [5 marks]

> **Rule - Rearrange, then substitute:** when the linear equation is untidy, simplify it to $x = \ldots$ (or $y = \ldots$) first, then substitute into the curve.

Step 1: Label the equations.

$x^2 + 2y^2 = 18 \quad (1)$

$3x - 2y = x + 6 \quad (2)$

(Reason: (2) has x on both sides, so it needs tidying before use.)

Step 2: Simplify the linear equation (2).

$2x - 2y = 6$ *(collect the x terms)*

$x - y = 3$ *(divide by 2)*

$x = y + 3 \quad (3)$

(Reason: $3x - x = 2x$, then divide by 2 to make it clean.)

Step 3: Substitute (3) into (1).

$(y + 3)^2 + 2y^2 = 18$

(Reason: replacing x leaves an equation in y only.)

Step 4: Expand, collect, and rearrange to = 0.

$y^2 + 6y + 9 + 2y^2 = 18$

$3y^2 + 6y - 9 = 0$

$y^2 + 2y - 3 = 0$ *(divide by 3)*

(Reason: expand $(y+3)^2 = y^2 + 6y + 9$, then simplify and divide by 3.)

Step 5: Factorise and solve.

$(y+3)(y-1) = 0$

$y = -3$ or $y = 1$

(Reason: two numbers with product -3 and sum $+2$ are $+3$ and -1.)

Step 6: Find each matching x from (3).

When $y = 1$: $x = 1 + 3 = 4 \rightarrow (4, 1)$

When $y = -3$: $x = -3 + 3 = 0 \rightarrow (0, -3)$

(Reason: use the tidy linear $x = y + 3$ to pair each y with its x.)

$$x = 4, \quad y = 1$$
$$x = 0, \quad y = -3$$

Verification

✓ **Check 1 - (4, 1):** $4^2 + 2(1)^2 = 16 + 2 = 18$

✓ **Check 2 - (0, -3):** $0^2 + 2(-3)^2 = 0 + 18 = 18$

✓ **Check 3:** both pairs also fit the linear x = y + 3, and a quadratic gives two pairs - consistent.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Rearrange the linear to x = y + 3	**M1**	Method - tidy the linear first	✓
Substitute into the curve and simplify to a quadratic = 0	**M1**	Method - substitute and expand	✓
Correct quadratic: $y^2 + 2y - 3 = 0$	**A1**	Accuracy - correct equation	✓
Both y-values: y = 1 and y = -3	**A1**	Accuracy - the two roots	✓
Both complete pairs: (4, 1) and (0, -3)	**A1**	Accuracy - matching x-values	✓

Full marks: 5/5

Question 8 - Exam Solution

Understanding the Question

Given

$x + y = 1$
$x^2 + y^2 = 25$

A line cutting a circle at A and B.

Find

The exact length of AB in surd form.

Plan the Solution

- Rearrange the line to $y = \ldots$ and substitute into the circle.
- Solve the quadratic for the two x-values, then find each y to get A and B.
- Use the distance formula for AB, then simplify the surd.

Worked Solution [6 marks]

> **Rule - Intersections, then distance:** substitute the line into the circle to find both points, then apply $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Step 1: Label the equations.

$x + y = 1 \quad (1)$

$x^2 + y^2 = 25 \quad (2)$

(Reason: the line is easiest to rearrange and substitute.)

Step 2: Rearrange the line and substitute into (2).

$y = 1 - x \quad (3)$

$x^2 + (1 - x)^2 = 25$

(Reason: replacing y leaves an equation in x only.)

Step 3: Expand, collect, and rearrange to = 0.

$x^2 + 1 - 2x + x^2 = 25$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$ *(divide by 2)*

(Reason: expand $(1 - x)^2 = 1 - 2x + x^2$, simplify, then divide by 2.)

Step 4: Factorise and solve for the x-coordinates.

$(x - 4)(x + 3) = 0$

$x = 4$ or $x = -3$

(Reason: two numbers with product -12 and sum -1 are -4 and $+3$.)

Step 5: Find each point's y from the line (3).

When $x = 4$: $y = 1 - 4 = -3 \rightarrow A(4, -3)$

When $x = -3$: $y = 1 - (-3) = 4 \rightarrow B(-3, 4)$

(Reason: substitute each x into the line.)

Step 6: Apply the distance formula to A(4, -3) and B(-3, 4).

$$AB = \sqrt{(-3 - 4)^2 + (4 - (-3))^2}$$

$$AB = \sqrt{(-7)^2 + 7^2} = \sqrt{49 + 49}$$

$$AB = \sqrt{98}$$

(Reason: horizontal gap = 7, vertical gap = 7; the distance is the hypotenuse.)

Step 7: Simplify the surd.

$$\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}$$

(Reason: pull out the largest square factor, 49, to leave √2.)

$$AB = 7\sqrt{2}$$

Verification

✓ **Check 1 - A(4, -3):** $4^2 + (-3)^2 = 16 + 9 = 25$

✓ **Check 2 - B(-3, 4):** $(-3)^2 + 4^2 = 9 + 16 = 25$

✓ **Check 3:** both points fit the line $x + y = 1$, and $7\sqrt{2} \approx 9.9 = \sqrt{98}$ - consistent.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Rearrange line to y = 1 - x and substitute into the circle	**M1**	Method - substitution	✓
Expand and simplify to $x^2 - x - 12 = 0$	**A1**	Accuracy - correct quadratic	✓
Coordinates of both points: A(4, -3), B(-3, 4)	**A1**	Accuracy - both points	✓
Apply the distance formula with these coordinates	**M1**	Method - distance formula	✓
Correct distance: AB = √98	**A1**	Accuracy - unsimplified surd	✓
Simplify to AB = 7√2	**A1**	Accuracy - exact surd form	✓

Full marks: 6/6

Question 9 - Exam Solution

Understanding the Question

Given

Left rectangle: $2x$ by x (area $2x^2$).

Right rectangle: y by $4y$ (area $4y^2$).

Total area 34 cm²; total base 8 cm; x and y are positive integers.

Find

The values of x and y.

Plan the Solution

- Write one equation for the total area and one for the total base.
- Because x and y must be positive integers, list the few pairs the base allows and test them in the area equation.
- This avoids solving a messy quadratic.

Worked Solution [5 marks]

> **Rule - Form and use the constraint:** turn the shape into two equations; when the unknowns must be positive integers, testing the few possible pairs is a complete method.

Step 1: Form the two equations from the shape.

$2x^2 + 4y^2 = 34 \quad (1)$ *(area)*

$2x + y = 8 \quad (2)$ *(base)*

(Reason: area = sum of the two rectangle areas; base = sum of the two widths.)

Step 2: List the possible integer pairs from (2).

With $2x + y = 8$ and positive integers, x can only be 1, 2 or 3, giving the pairs (1, 6), (2, 4), (3, 2).

(Reason: if $x \geq 4$ then $y \leq 0$, which is not allowed.)

Step 3: Test each pair in the area equation (1).

$2(1)^2 + 4(6)^2 = 2 + 144 = 146$ *(too big)*

$2(2)^2 + 4(4)^2 = 8 + 64 = 72$ *(too big)*

$2(3)^2 + 4(2)^2 = 18 + 16 = 34$ *(correct)*

(Reason: we need the area to equal 34 - only the last pair does.)

Step 4: Only (3, 2) satisfies both equations.

The pair (3, 2) fits both the base and the area, so it is the solution.

(Reason: the positive-integer constraint makes this pair unique.)

$$x = 3, \quad y = 2$$

Verification

✓ **Check 1 - base:** $2(3) + 2 = 8$

✓ **Check 2 - area:** $2(3)^2 + 4(2)^2 = 18 + 16 = 34$

✓ **Check 3:** both are positive whole numbers, exactly as the question requires.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Form the area equation: $2x^2 + 4y^2 = 34$	**M1**	Method - area from the shape	✓
Form the base equation: $2x + y = 8$	**M1**	Method - base from the shape	✓
Use positive integers to list the possible pairs	**M1**	Method - apply the constraint	✓
Test in the area equation - (3, 2) works, others rejected	**A1**	Accuracy - correct testing	✓
State $x = 3$ and $y = 2$	**A1**	Accuracy - final values	✓

Full marks: 5/5

Question 10 - Exam Solution

Understanding the Question

Given

Width $(x + 2)$, height y, depth $(x + 4)$.
Weight (force) = 48 N.
Face A (front) = $(x + 2)y$, pressure 4 N/m².
Face B (top) = $(x + 2)(x + 4)$, pressure 2 N/m².

Find

The volume of the cuboid.

Plan the Solution

- Use the pressure formula (force over area, as a fraction) to turn each pressure into a face area.
- Set each face expression equal to its area to get two equations.
- Face B involves only x, so solve that quadratic first, then find y, then the volume.

Worked Solution [6 marks]

Rule - Pressure: pressure = $\frac{\text{force}}{\text{area}}$, so rearranged, area = $\frac{\text{force}}{\text{pressure}}$.

Step 1: Turn each pressure into a face area.

Face A: area = $\frac{48}{4} = 12$

Face B: area = $\frac{48}{2} = 24$

(Reason: dividing the force, 48 N, by each pressure gives that face's area.)

Step 2: Form the two equations from the face expressions.

$(x + 2)y = 12 \quad (1)$ *(face A)*

$(x + 2)(x + 4) = 24 \quad (2)$ *(face B)*

(Reason: each face area equals its two dimensions multiplied together.)

Step 3: Solve equation (2) - it only involves x.

$x^2 + 6x + 8 = 24$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x = 2$ *(reject x = -8, a length cannot be negative)*

(Reason: expand the brackets, make it equal 0, then factorise.)

Step 4: Substitute x = 2 into (1) to find y.

$(2 + 2)y = 12$

$4y = 12$

$y = 3$

(Reason: now x is known, equation (1) gives y.)

Step 5: Work out the volume = width × height × depth.

$V = (x + 2) \times y \times (x + 4)$

$V = 4 \times 3 \times 6 = 72$ m³

(Reason: substitute the found dimensions (4, 3, 6) and multiply.)

Volume = 72 m³

Verification

- ✓ **Check 1 - face A:** area = $4 \times 3 = 12$, pressure = $\frac{48}{12} = 4$
- ✓ **Check 2 - face B:** area = $4 \times 6 = 24$, pressure = $\frac{48}{24} = 2$
- ✓ **Check 3:** dimensions 4, 3, 6 are all positive; the smaller face gives the higher pressure - consistent.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Use the pressure relationship (force over area) to get areas 12 and 24	M1	Method - areas from pressure	✓
Form both equations: (x + 2)y = 12 and (x + 2)(x + 4) = 24	M1	Method - two equations	✓
Expand and solve: $x^2 + 6x - 16 = 0$	M1	Method - solve the quadratic	✓
x = 2 (reject x = -8)	A1	Accuracy - value of x	✓
y = 3	A1	Accuracy - value of y	✓
Volume = (x + 2)(x + 4)y = 72 m³	A1	Accuracy - final volume	✓

Full marks: 6/6

Question 11 - Exam Solution

Understanding the Question

Given

Line $y = x + 2$.
Curve $y = -x^2 + 9x - 10$.
They meet at A and B; C and D lie on the x-axis with AC and BD vertical.

Since AC and BD are parallel, ABDC is a trapezium.

Find

The area of quadrilateral ABDC.

Plan the Solution

- Set line = curve and solve to find the two meeting points A and B.
- C and D sit directly below A and B on the x-axis, so AC and BD are just the y-values of A and B.
- Use the trapezium area formula with those two vertical sides and the gap between them.

Worked Solution [6 marks]

Rule - Trapezium: Area = $\frac{1}{2}(a+b)h$, where a and b are the parallel sides and h is the distance between them.

Step 1: Set the line equal to the curve.

$x + 2 = -x^2 + 9x - 10$

(Reason: at the meeting points the line and curve share the same x and y.)

Step 2: Rearrange to a quadratic equal to 0.

$x^2 - 8x + 12 = 0$

(Reason: move every term to one side and collect.)

Step 3: Factorise and solve for the x-coordinates.

$(x-2)(x-6) = 0$

$x = 2$ or $x = 6$

(Reason: two numbers with product $+12$ and sum -8 are -2 and -6.)

Step 4: Find each y from the line.

When $x = 2$: $y = 2 + 2 = 4 \rightarrow A(2, 4)$

When $x = 6$: $y = 6 + 2 = 8 \rightarrow B(6, 8)$

(Reason: substitute each x into the simpler equation, the line.)

Step 5: Drop C and D to the x-axis and find the sides.

$C(2,0)$ and $D(6,0)$

$AC = 4$ (the y of A)

$BD = 8$ (the y of B)

$CD = 6 - 2 = 4$ (the gap between them)

(Reason: AC and BD are vertical, so their lengths are the y-values of A and B.)

Step 6: Apply the trapezium area formula.

Area = $\frac{1}{2}(AC + BD) \times CD$

Area = $\frac{1}{2}(4 + 8) \times 4$

Area = $\frac{1}{2} \times 12 \times 4 = 24$ units²

(Reason: add the parallel sides, halve, then multiply by the gap.)

Area = 24 units²

Verification

✓ **Check 1 - A(2, 4):** $-(2)^2 + 9(2) - 10 = -4 + 18 - 10 = 4$

✓ **Check 2 - B(6, 8):** $-(6)^2 + 9(6) - 10 = -36 + 54 - 10 = 8$

✓ **Check 3:** both points are on the curve, and the trapezium has two vertical parallel sides - the method fits the shape.

Mark Scheme Breakdown

Step	Mark	Description	Got it?
Set line = curve and rearrange to $x^2 - 8x + 12 = 0$	M1	Method - form the quadratic	✓
Solve for both x: x = 2 and x = 6	A1	Accuracy - the two roots	✓
Coordinates: A(2, 4), B(6, 8)	A1	Accuracy - both points	✓
Find the sides: AC = 4, BD = 8, CD = 4	M1	Method - set up the trapezium	✓
Apply Area = ½(AC + BD) × CD	M1	Method - trapezium formula	✓
Area = 24 units²	A1	Accuracy - final area	✓

Full marks: 6/6

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