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Venn Diagrams

Set Notation, Shading, Probability and Algebra: GCSE & IGCSE Practice Questions with Worked Solutions

GRADE 8-9 TARGETED

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11 exam-style questions with full worked solutions and mark schemes.

Practice Questions

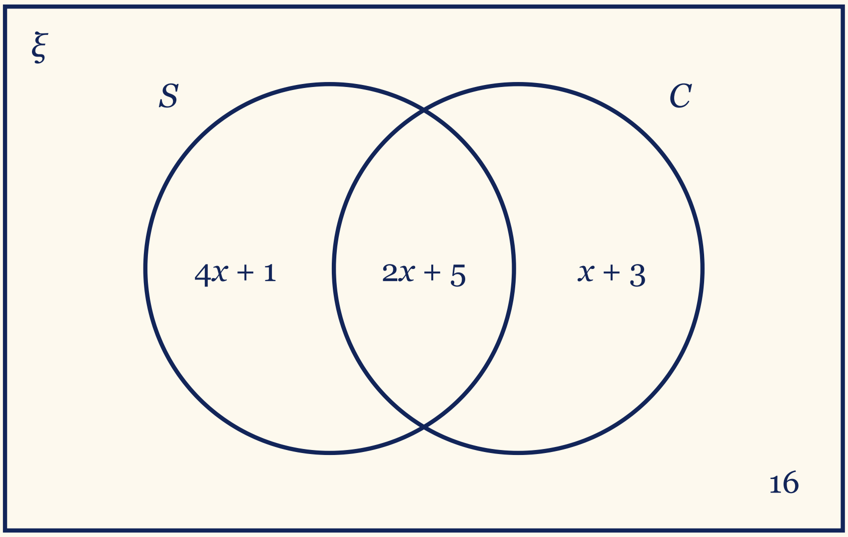
Try each question on paper first. Full worked solutions and mark schemes follow in the next section.

1. A garden centre recorded what its customers bought one Saturday morning.

GRADE 6

ξ is the set of all customers that morning.
S is the set of customers who bought seeds.
C is the set of customers who bought compost.

The Venn diagram shows the number of customers in each region.



(a) Given that $n(\xi) = 60$, find the value of x. *[2 marks]*

(b) Find $n(S \cup C)$. *[2 marks]*

(c) One of the customers is chosen at random. Find $P(S \cap C)$. Give your answer as a fraction in its simplest form. *[2 marks]*

(a) $x =$ ....

(b) $n(S \cup C) =$ ....

(c) $P(S \cap C) =$ ....

[Total 6 marks]

2. ξ is the set of integers from 1 to 30.

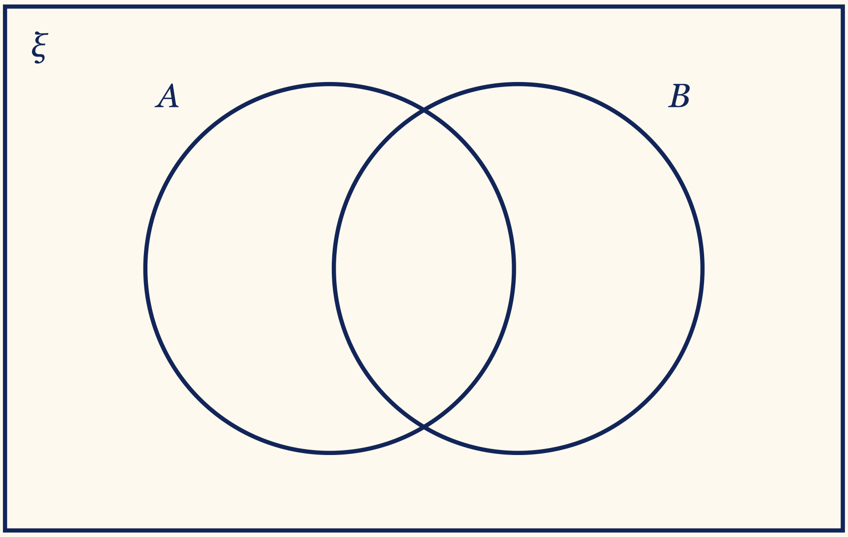
Set A is made up of the numbers generated by the sequence $5n - 2$, where n is a positive integer.

Set B is made up of the numbers generated by the sequence $n^2 + 2$, where n is a positive integer.

GRADE 6

(a) List the elements of set A and the elements of set B. *[2 marks]*

(b) Complete the Venn diagram to show the **number of elements** in each region.



[3 marks]

(c) Complete each statement using one of the symbols $\in$, $\notin$, $\subset$ or $\not\subset$.

(i) 18 ____ $A \cap B$

(ii) $\{3,\ 6,\ 8\}$ ____ B *[2 marks]*

(d) A number is chosen at random from ξ. Find $P(A \cap B)$. *[1 mark]*

(c) (i) .... (ii) ....

(d) $P(A \cap B) =$ ....

[Total 8 marks]

3. Each member of a hillwalking club was asked which of these three items they had with them.

GRADE 7

ξ is the set of all the members.

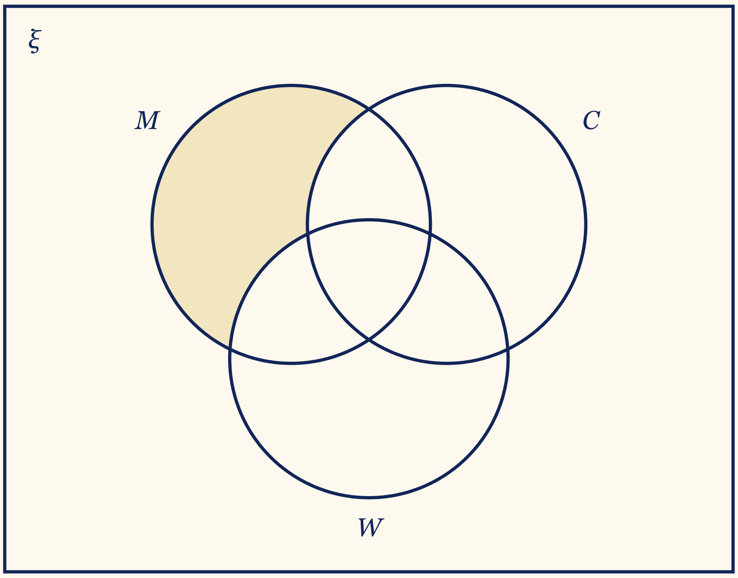
M is the set of members who had a **map**.

C is the set of members who had a **compass**.

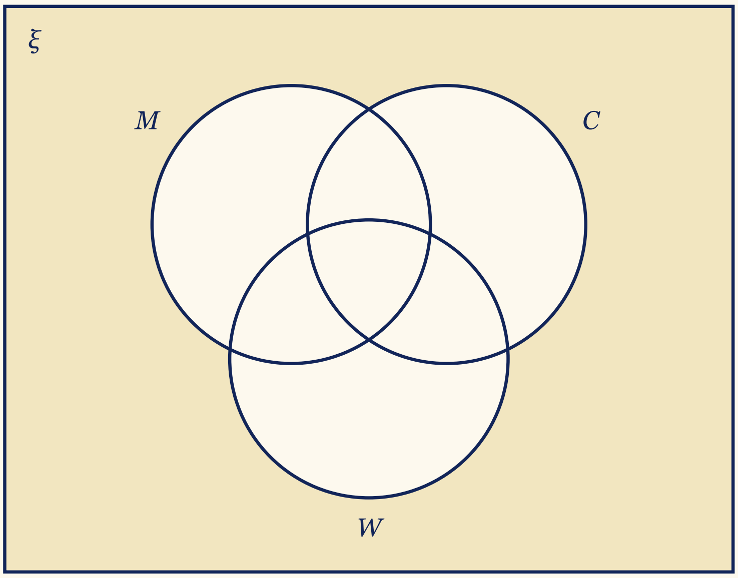
W is the set of members who had a **whistle**.

(a) Describe, in words, what each of the shaded areas represents.

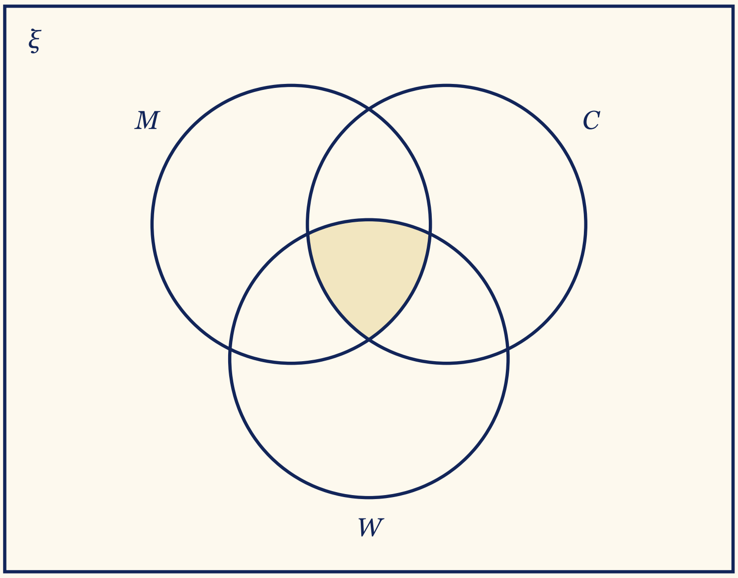
(i)



(ii)



(iii)

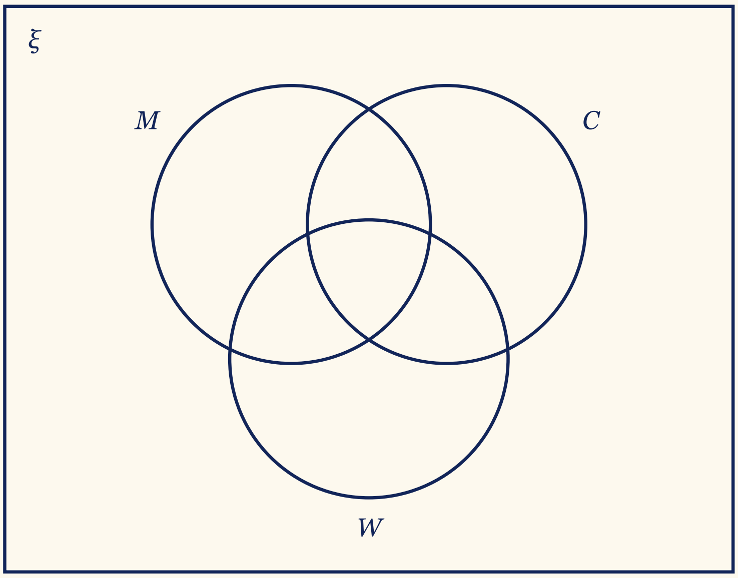


[3 marks]

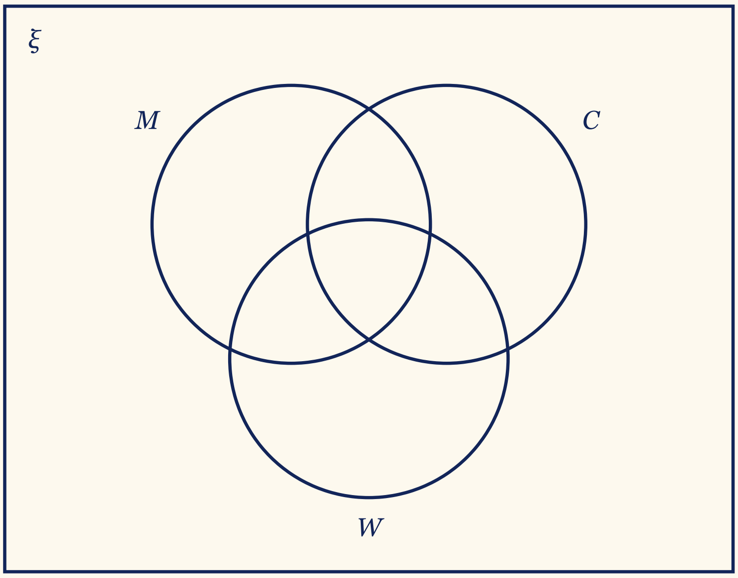
(b) Write each of the three shaded regions in part (a) using set notation.
[3 marks]

(c) Shade each Venn diagram to show:

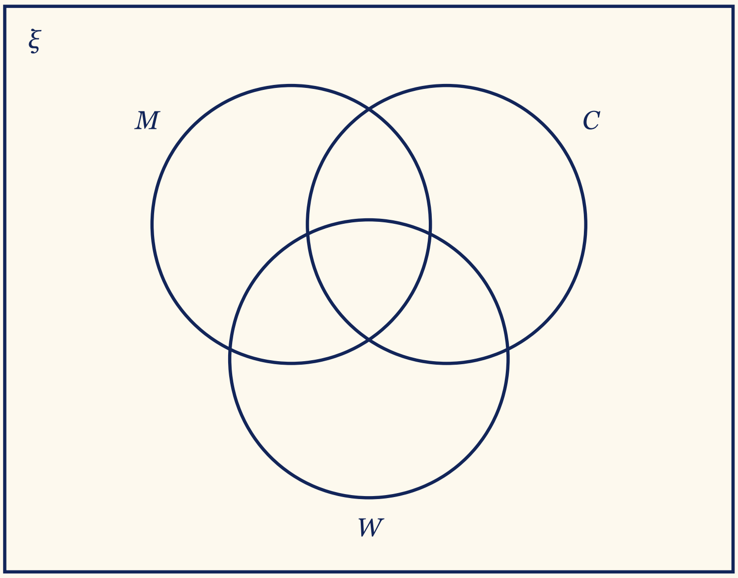
(i) the members who had a compass **only**



(ii) **all** the members who had a whistle



(iii) the members who had **exactly two** of the three items



[3 marks]

(b) (i) .... (ii) ....

(iii) ....

[Total 9 marks]

4. A cafe's menu has 50 dishes.

ξ is the set of all 50 dishes.

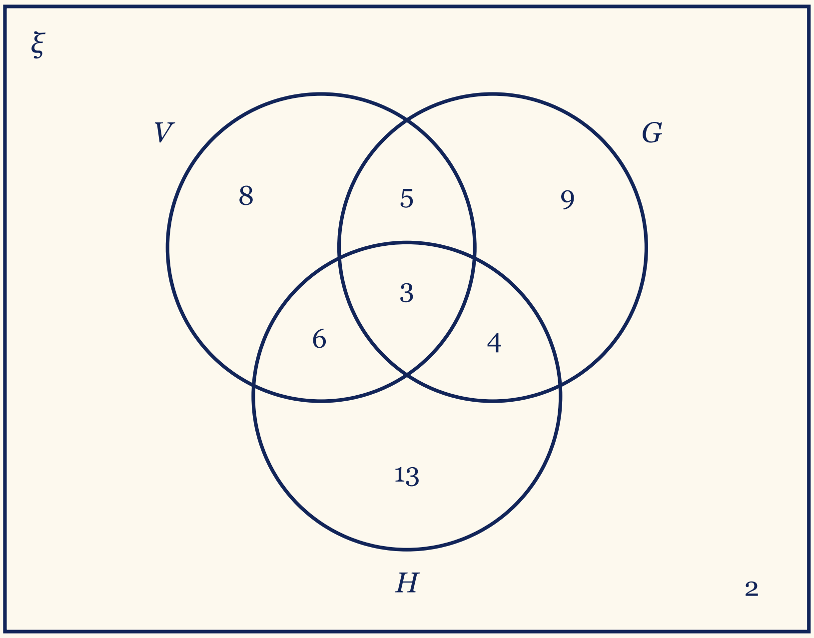
V is the set of dishes that are vegetarian.

G is the set of dishes that are gluten-free.

H is the set of dishes that are served hot.

The Venn diagram shows the number of dishes in each region.

GRADE 7



Find:

(a) $n(V)$ *[1 mark]*

(b) $n(V \cap G)$ *[1 mark]*

(c) $n(G \cup H)$ *[1 mark]*

(d) $n((V \cup G) \cap H)$ *[2 marks]*

(e) $n(V \cup (G \cap H))$ *[2 marks]*

(f) Parts (d) and (e) use the same three sets and the same two symbols. Only the bracket has moved. Explain why the answers are different. *[2 marks]*

(a) $n(V) =$ ....

(b) $n(V \cap G) =$ ....

(c) $n(G \cup H) =$ ....

(d) $n((V \cup G) \cap H) =$ ....

(e) $n(V \cup (G \cap H)) =$ ....

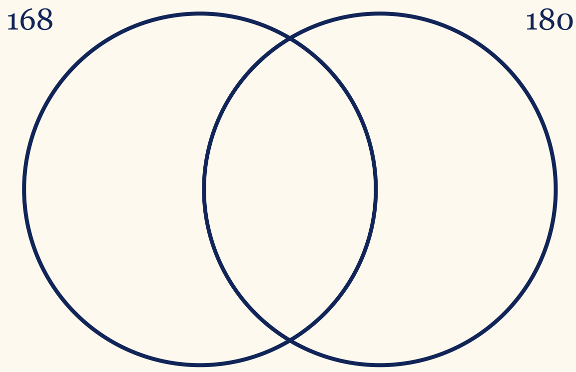
[Total 9 marks]

5. **(a)** Write 168 and 180 as products of their prime factors. *[2 marks]*

GRADE 7

Problem solving

(b) Complete the Venn diagram to show the prime factors of 168 and 180.



[2 marks]

(c) Use the Venn diagram to write down the highest common factor (HCF) of 168 and 180. *[1 mark]*

(d) Use the Venn diagram to work out the lowest common multiple (LCM) of 168 and 180. *[2 marks]*

(e) Show that $\text{HCF} \times \text{LCM} = 168 \times 180$. Explain, using the Venn diagram, why this will always be true for any two numbers. *[3 marks]*

(c) $\text{HCF} =$ .... (d) $\text{LCM} =$ ....

[Total 10 marks]

GRADE 8

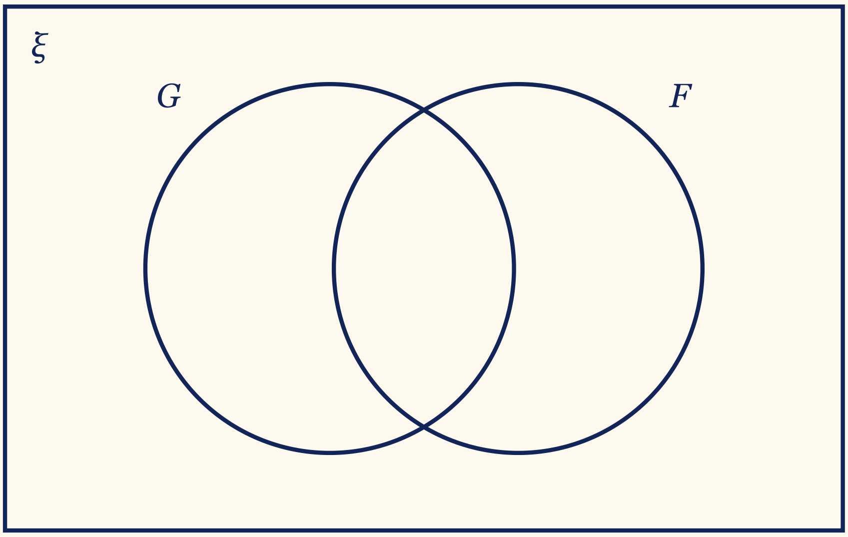
Problem solving

6. A borough council estimates that, for a randomly chosen household in the borough:

the probability that the household recycles glass is 0.45
the probability that the household recycles food waste is 0.58
the probability that the household recycles both is 0.31

(a) Find the probability that a randomly chosen household recycles neither glass nor food waste. *[2 marks]*

(b) There are 2000 households in the borough. Complete the Venn diagram to show the expected number of households that recycle glass only, food waste only, both, and neither.



[3 marks]

(c) A household is chosen at random. Find the probability that it recycles glass, **given that** it does not recycle food waste. Give your answer as a fraction in its simplest form. *[2 marks]*

(a) $P(\text{neither}) =$ ....

(c) $P(G \mid F') =$ ....

[Total 7 marks]

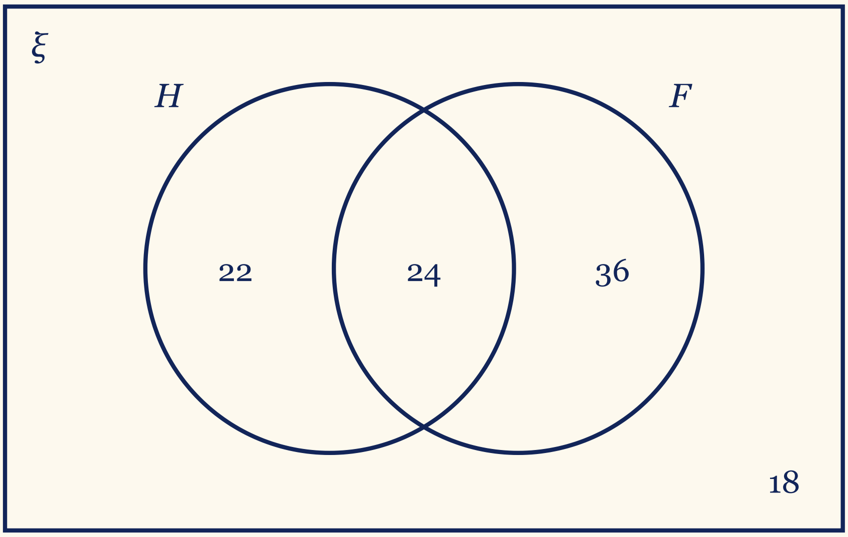
7. A bookshop surveys its stock. Every book is either a hardback or a paperback.

GRADE 8

Problem solving

ξ is the set of all the books in the shop.
H is the set of books that are hardbacks.
F is the set of books that are fiction.

The Venn diagram shows the **percentage** of the shop's books in each region.



(a) Find the probability that a randomly chosen book is a paperback, **given that** it is fiction. Give your answer as a fraction in its simplest form. *[2 marks]*

(b) The shop has 800 books in total. By working out the number of fiction books and the number of paperback fiction books, show that your answer to part (a) is unchanged. *[3 marks]*

(c) A customer claims that the probability a book is fiction, given that it is a paperback, must also equal the answer to part (a), because it involves the same two categories.

Show that the customer is wrong, and find the correct probability. *[3 marks]*

(a) $P(H' \mid F) =$ ....

(c) $P(F \mid H') =$ ....

[Total 8 marks]

8. A leisure centre has 200 members. Every member uses at least one of the swimming pool, the gym and the tennis courts.

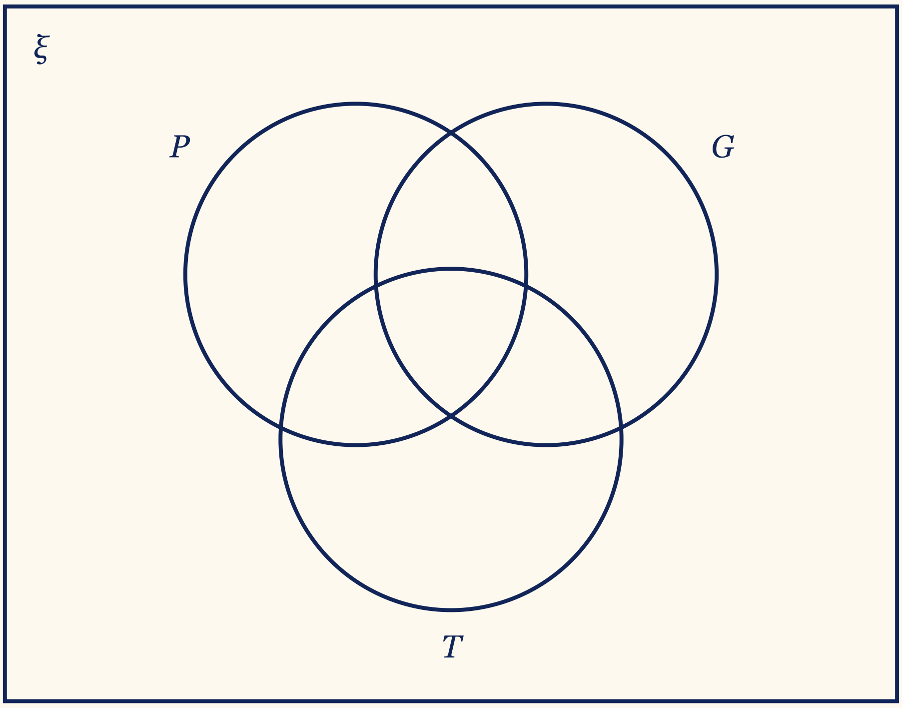
GRADE 8

Problem solving

ξ is the set of all 200 members.
P is the set of members who use the swimming pool.
G is the set of members who use the gym.
T is the set of members who use the tennis courts.

92 members use the gym. Of these, 32 also use the tennis courts.
96 members use the swimming pool. Of these, 28 also use the gym and 20 also use the tennis courts.
12 members use all three facilities.

(a) Complete the Venn diagram to show this information.



[3 marks]

(b) Find $n(P \cap T \cap G')$ and $n((P \cup G \cup T)')$. *[2 marks]*

(c) One member is chosen at random. Find $P(P \cup G)$. *[1 mark]*

(d) Given that a member uses the tennis courts, find the probability that this member also uses the swimming pool. Give your answer as a fraction in its simplest form. *[2 marks]*

(b) $n(P \cap T \cap G') =$ .... $n((P \cup G \cup T)') =$ ....

(c) $P(P \cup G) =$ ....

(d) $P(P \mid T) =$ ....

[Total 8 marks]

9. A museum recorded which galleries its visitors saw one Saturday.

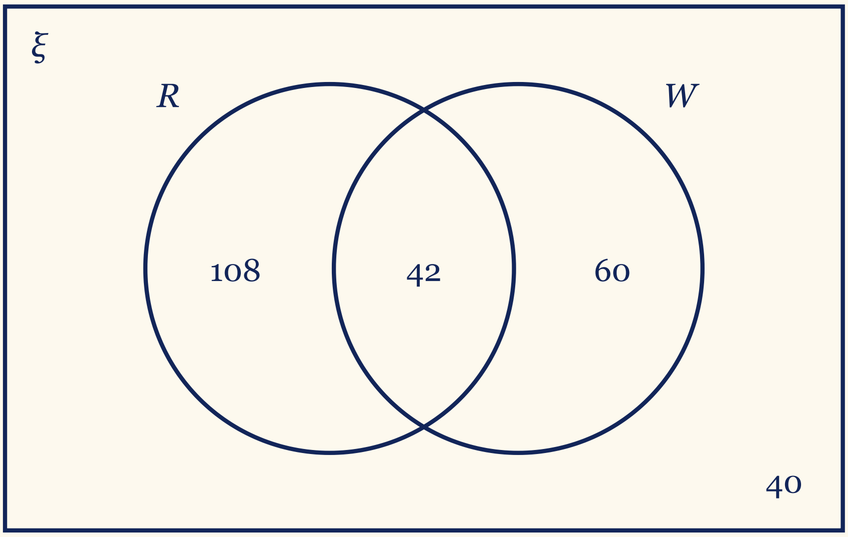
GRADE 8

Problem solving

ξ is the set of all the visitors that day.

R is the set of visitors who saw the Roman gallery.

W is the set of visitors who saw the wildlife gallery.



(a) Find the ratio of visitors who saw the Roman gallery to the total number of visitors. Give your answer in its simplest form. *[2 marks]*

(b) A visitor is chosen at random. Find the probability that they saw the wildlife gallery, **given that** they saw the Roman gallery. *[2 marks]*

(c) A visitor is chosen at random. Find the probability that they saw the Roman gallery, **given that** they saw **exactly one** gallery. *[2 marks]*

(d) Find $n(R \cup W)$, and explain why this is **not** the number of visitors who saw exactly one gallery. *[2 marks]*

(a) ....

(b) $P(W \mid R) =$ ....

(c) ....

(d) $n(R \cup W) =$ ....

[Total 8 marks]

10. A community allotment has 120 plot holders. Every plot holder grows at least one of potatoes, tomatoes and beans.

GRADE 8

Problem solving

ξ is the set of all 120 plot holders.

P is the set of plot holders who grow potatoes.

T is the set of plot holders who grow tomatoes.

B is the set of plot holders who grow beans.

Three fifths of the plot holders grow **exactly one** crop.

5% of the plot holders grow all three crops.

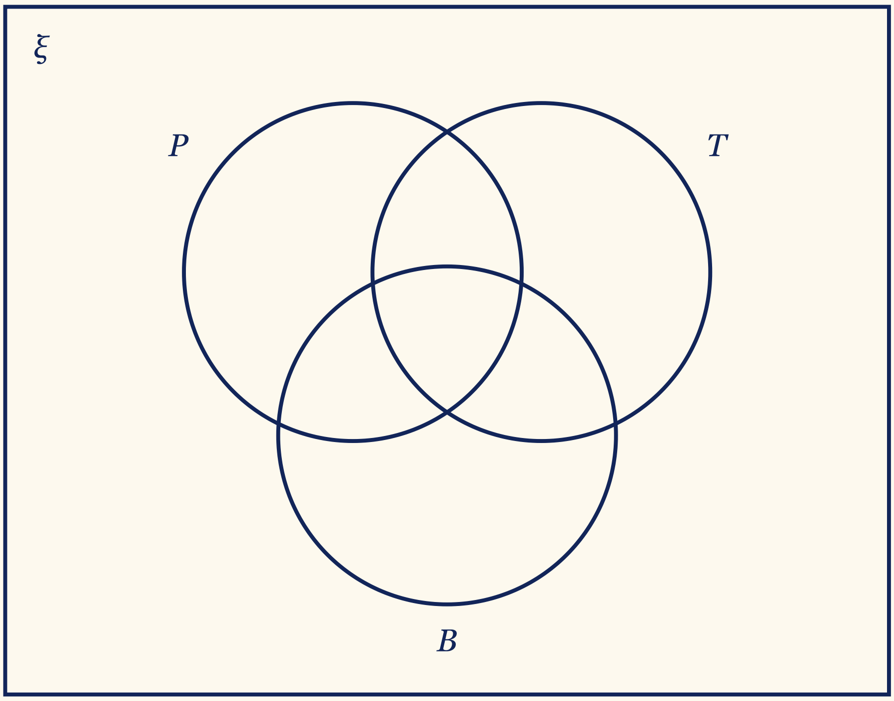
20 plot holders grow potatoes and tomatoes.

22 plot holders grow tomatoes and beans.

62 plot holders grow potatoes.

58 plot holders grow beans.

(a) Complete the Venn diagram to show this information.



[5 marks]

(b) Find the number of plot holders who grow tomatoes. *[1 mark]*

(c) A plot holder is chosen at random. Find the probability that they grow potatoes, **given that** they grow **at least two** crops. Give your answer as a fraction in its simplest form. *[2 marks]*

(b) $n(T) =$ ....

(c) ....

[Total 8 marks]

11. A school orchestra has 55 members.

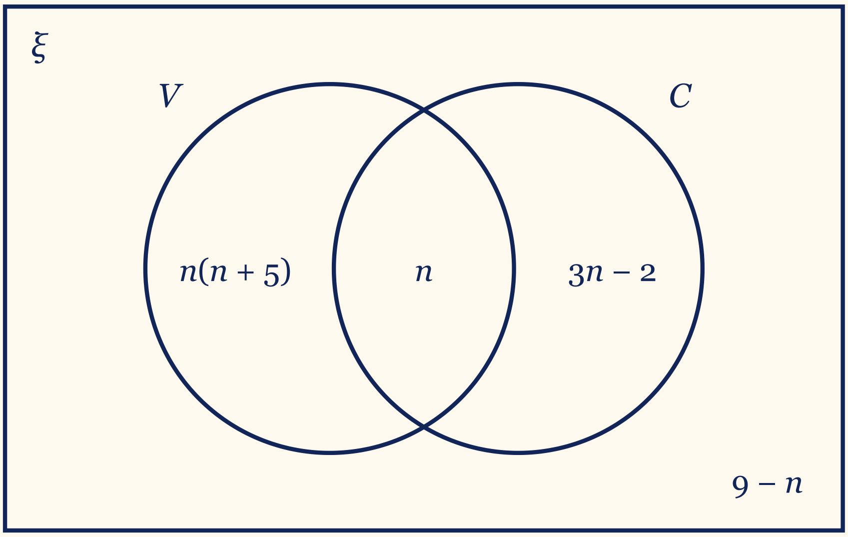
GRADE 9

Problem solving

ξ is the set of all 55 members.

V is the set of members who play the violin.

C is the set of members who play the cello.



(a) Show that $n^2 + 8n - 48 = 0$. *[3 marks]*

(b) Solve the equation to find the value of n, and write down the number of members in each region of the Venn diagram. *[3 marks]*

(c) Two **different** members of the orchestra are chosen at random. Find the probability that they both play the violin. *[3 marks]*

(b) $n =$ ....

(c) $P(\text{both play the violin}) =$ ....

[Total 9 marks]

Worked Solutions & Mark Schemes

Every mark (M for method, A for accuracy) is shown, just like a real examiner.

Question 1 - Exam Solution

Understanding the Question

Given

A two-set Venn diagram for a garden centre's customers, with S the customers who bought seeds and C the customers who bought compost.

Regions: S only $= 4x + 1$, $S \cap C = 2x + 5$, C only $= x + 3$, and 16 outside both circles.

$n(\xi) = 60$.

Find

(a) The value of x.

(b) $n(S \cup C)$.

(c) $P(S \cap C)$, as a fraction in its simplest form.

Plan the Solution

- The four regions of a two-set Venn diagram account for every element, so add all four and set the total equal to $n(\xi)$. That gives one equation in x.
- Solve it, then substitute x back to turn every region into a number.
- For the union, add the three regions the circles cover. For the probability, put the overlap over $n(\xi)$, not over the union.

Part (a): Find the value of x [2 marks]

Rule - The four regions fill the universal set: every element of ξ lies in exactly one region of a two-set Venn diagram, so the four values add up to $n(\xi)$.

Step 1: Add all four regions and set the total equal to n(ξ).

$$(4x + 1) + (2x + 5) + (x + 3) + 16 = 60$$

(Reason: The three regions inside the circles and the 16 outside them account for every customer, so together they make the whole of ξ. Forgetting the 16 is the most common error here.)

Step 2: Collect like terms.

$7x + 25 = 60$

(Reason: $4x + 2x + x = 7x$, and $1 + 5 + 3 + 16 = 25$.)

Step 3: Solve for x.

$7x = 35$

$x = 5$

(Reason: Subtract 25 from both sides, then divide both sides by 7.)

$$x = 5$$

MARK SCHEME

Step	Mark	Description	Got it?
$(4x + 1) + (2x + 5) + (x + 3) + 16 = 60$	**M1**	Method - all four regions add to n(ξ)	✓
$x = 5$	**A1**	Accuracy - correct value of x	✓

Full marks: 2/2

Part (b): Find $n(S \cup C)$ [2 marks]

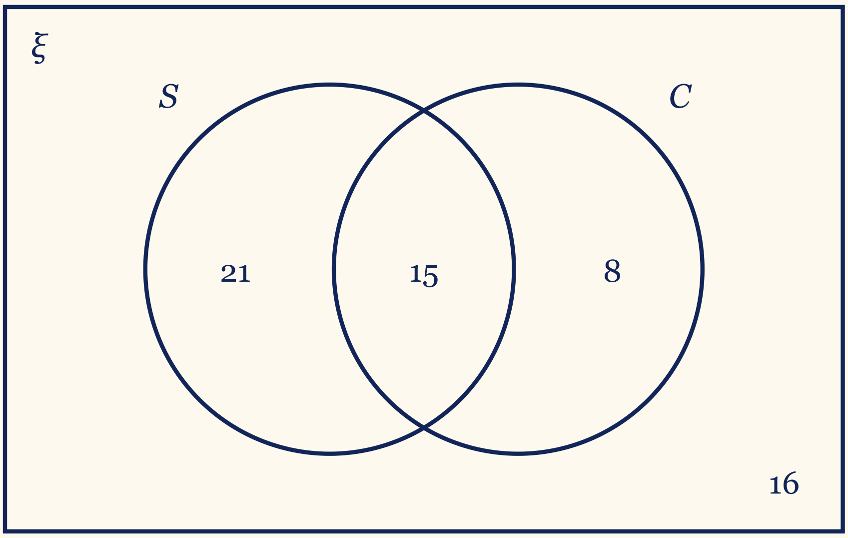
Rule - Union: $n(S \cup C)$ counts every element in S, in C, or in both. On the diagram that is everything the two circles cover.

Step 1: Substitute x = 5 into the three regions inside the circles.

S only $= 4(5) + 1 = 21$

$S \cap C = 2(5) + 5 = 15$

C only $= 5 + 3 = 8$



(Reason: With x known, each expression becomes a number of customers.)

Step 2: Add the three regions inside the circles.

$n(S \cup C) = 21 + 15 + 8 = 44$

(Reason: The union is everything inside either circle. The 16 outside is not part of the union, so it is not added.)

$$n(S \cup C) = 44$$

MARK SCHEME

Step	Mark	Description	Got it?
Substitute $x = 5$ to get 21, 15 and 8	**M1**	Method - evaluate the three regions	✓
$n(S \cup C) = 44$	**A1**	Accuracy - correct union	✓

Full marks: 2/2

Part (c): Find $P(S \cap C)$ [2 marks]

Rule - Probability: $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$, then simplify the fraction.

Step 1: Write down n(S ∩ C).

$n(S \cap C) = 2(5) + 5 = 15$

(Reason: $S \cap C$ is the overlap, the customers who bought both seeds and compost.)

Step 2: Divide by the total number of customers.

$$P(S \cap C) = \frac{15}{60}$$

(Reason: The customer is chosen at random from all of ξ, so the denominator is $n(\xi) = 60$, not 44. Using the union by mistake is the classic trap in this part.)

Step 3: Simplify the fraction.

$$\frac{15}{60} = \frac{1}{4}$$

(Reason: Divide the numerator and the denominator by 15.)

$$P(S \cap C) = \frac{1}{4}$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(S \cap C) = 15$, or the fraction $\frac{15}{60}$	**M1**	Method - correct numerator over n(ξ)	✓
$P(S \cap C) = \frac{1}{4}$	**A1**	Accuracy - simplified fraction	✓

Full marks: 2/2

Verification

- ✓ **Check 1:** The completed Venn accounts for everyone: $21 + 15 + 8 + 16 = 60 = n(\xi)$.
- ✓ **Check 2:** The union, found a different way: $n(S) = 21 + 15 = 36$ and $n(C) = 15 + 8 = 23$, so $n(S \cup C) = 36 + 23 - 15 = 44$. This agrees with part (b).
- ✓ **Check 3:** The union, found a third way: everything except the 16 outside, so $60 - 16 = 44$. Three independent routes, the same answer.
- ✓ **Check 4:** The probability is sensible: $\frac{1}{4}$ of 60 is 15, which is exactly $n(S \cap C)$.

Question 2 - Exam Solution

Understanding the Question

Given

ξ is the set of integers from 1 to 30, so $n(\xi) = 30$.

Set A is generated by $5n - 2$ and set B by $n^2 + 2$, where n is a positive integer.

Find

(a) The elements of A and of B.

(b) The completed Venn diagram of counts.

(c) The correct set symbols.

(d) $P(A \cap B)$.

Plan the Solution

- Substitute $n = 1, 2, 3, \ldots$ into each rule and stop as soon as the value goes past 30.
- Compare the two lists to find the elements in both. Every other element of A goes in A only, and every other element of B goes in B only.
- The outside region is everything in ξ that is in neither list, so count it by subtraction.

Part (a): List the elements of A and B [2 marks]

> **Rule - Substitute and stop:** put $n = 1, 2, 3, \ldots$ into the rule and keep going until the value leaves the universal set.

Step 1: Generate set A from 5n − 2.

$5(1) - 2 = 3, \quad 5(2) - 2 = 8, \quad 5(3) - 2 = 13$

$5(4) - 2 = 18, \quad 5(5) - 2 = 23, \quad 5(6) - 2 = 28$

$A = \{3,\ 8,\ 13,\ 18,\ 23,\ 28\}$

(Reason: The next term is $5(7) - 2 = 33$, which is bigger than 30, so it is not in ξ. A has 6 elements.)

Step 2: Generate set B from n² + 2.

$1^2 + 2 = 3, \quad 2^2 + 2 = 6, \quad 3^2 + 2 = 11, \quad 4^2 + 2 = 18, \quad 5^2 + 2 = 27$

$B = \{3,\ 6,\ 11,\ 18,\ 27\}$

(Reason: The next term is $6^2 + 2 = 38$, which is bigger than 30. B has 5 elements. Note that B grows much faster than A, because it is a quadratic rule.)

$$A = \{3, 8, 13, 18, 23, 28\} \qquad B = \{3, 6, 11, 18, 27\}$$

MARK SCHEME

Step	Mark	Description	Got it?
$A = \{3, 8, 13, 18, 23, 28\}$	**B1**	Accuracy - correct set A, stopping at 30	✓

Step	Mark	Description	Got it?
$B = \{3, 6, 11, 18, 27\}$	**B1**	Accuracy - correct set B, stopping at 30	✓

Full marks: 2/2

Part (b): Complete the Venn diagram [3 marks]

Rule - Overlap first: find the elements common to both lists, then everything else in A, then everything else in B, then subtract from $n(\xi)$ to get the outside.

Step 1: Find the elements in both lists.

$A \cap B = \{3,\ 18\}$

$n(A \cap B) = 2$

(Reason: 3 and 18 appear in both lists. No other value does.)

Step 2: Find the two "only" regions.

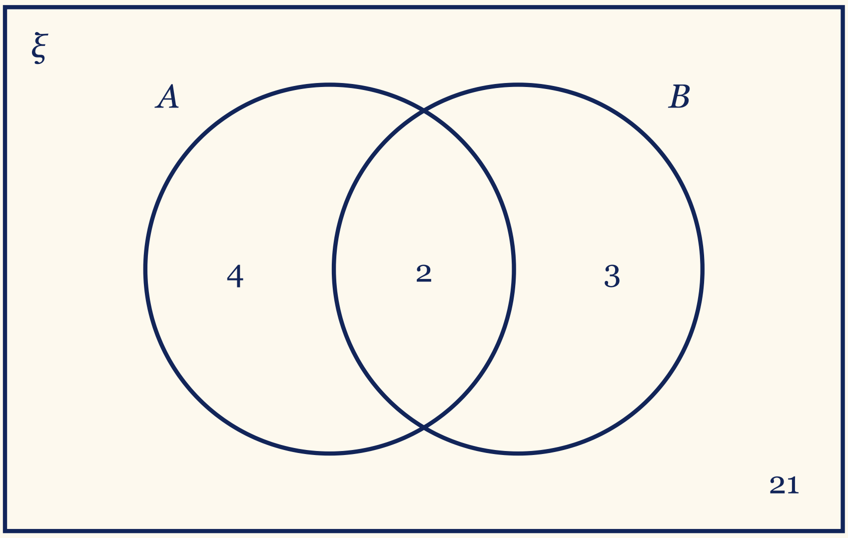
A only $= \{8,\ 13,\ 23,\ 28\} \quad (4)$

B only $= \{6,\ 11,\ 27\} \quad (3)$

(Reason: Remove the two shared elements from each list.)

Step 3: Find the outside region and place all four values.

$n((A \cup B)') = 30 - (4 + 2 + 3) = 30 - 9 = 21$



(Reason: 9 of the 30 integers are in at least one set, so the other 21 are in neither.)

The completed diagram: 4 in A only, 2 in the overlap, 3 in B only, and 21 outside both circles.

MARK SCHEME

Step	Mark	Description	Got it?
$A \cap B = \{3, 18\}$, so 2 in the overlap	**M1**	Method - identifies the shared elements	✓
$n((A \cup B)') = 30 - 9 = 21$	**M1**	Method - subtracts from n(ξ) to get the outside	✓
A completely correct Venn diagram	**A1**	Accuracy - all four regions correct and correctly placed	✓

Full marks: 3/3

Part (c): Complete each statement [2 marks]

Rule - Element or subset: $\in$ links a **single element** to a set. $\subset$ links a **set** to a set. A dash through the symbol reverses it.

Step 1: Part (i).

$18 \in A \cap B$

(Reason: 18 is a single number, so the symbol must be $\in$ or $\notin$. It appears in A (from $n = 4$) and in B (from $n = 4$), so it is in the overlap.)

Step 2: Part (ii).

$\{3,\ 6,\ 8\} \not\subset B$

(Reason: This is a set, so the symbol must be $\subset$ or $\not\subset$. $3 \in B$ and $6 \in B$, but $8 \notin B$, because $n^2 + 2 = 8$ would need $n^2 = 6$, and 6 is not a square number. One element outside is enough to break the subset.)

$$\text{(i) } 18 \in A \cap B \qquad \text{(ii) } \{3, 6, 8\} \not\subset B$$

MARK SCHEME

Step	Mark	Description	Got it?
$\in$	**B1**	Accuracy - element symbol, and 18 is in both sets	✓

Step	Mark	Description	Got it?
$\not\subset$	**B1**	Accuracy - subset symbol negated, because 8 is not in B	✓

Full marks: 2/2

Part (d): Find $P(A \cap B)$ [1 mark]

Rule - Probability: the number of favourable elements over $n(\xi)$.

Step 1: Divide and simplify.

$$P(A \cap B) = \frac{2}{30} = \frac{1}{15}$$

(Reason: 2 of the 30 integers are in both sets, and the number is chosen from all of ξ.)

$$P(A \cap B) = \frac{1}{15}$$

MARK SCHEME

Step	Mark	Description	Got it?
$\frac{2}{30} = \frac{1}{15}$	**B1**	Accuracy - correct probability, simplified	✓

Full marks: 1/1

Verification

- ✓ **Check 1:** The four regions account for all 30 integers: $4 + 2 + 3 + 21 = 30$.
- ✓ **Check 2:** The Venn rebuilds both sets: $n(A) = 4 + 2 = 6$, matching the 6 elements listed in part (a). $n(B) = 3 + 2 = 5$, matching the 5 elements listed.
- ✓ **Check 3:** Both lists really are complete: the next term of A is $5(7) - 2 = 33 > 30$, and the next term of B is $6^2 + 2 = 38 > 30$. Nothing was missed.
- ✓ **Check 4:** The probability works backwards: $\frac{1}{15}$ of 30 is 2, which is exactly $n(A \cap B)$.

Question 3 - Exam Solution

Understanding the Question

Given

A three-set Venn diagram for a hillwalking club, with M the members who had a map, C a compass and W a whistle.

Find

(a) A description in words of each shaded area.

(b) The same three regions in set notation.

(c) Three shaded diagrams.

Plan the Solution

- Read a shaded region one circle at a time: is the shading **inside** this circle, or **outside** it? Do that for all three.
- Then translate: inside is the set, outside is the complement, and "and" between them is $\cap$.
- To shade, do the reverse. Work out which regions the description covers, then shade **all** of them and **only** them.

Part (a): Describe each shaded area [3 marks]

Rule - Check each circle in turn: for each of the three circles, ask whether the shading lies inside it or outside it. Three answers give the region exactly.

Step 1: Diagram (i).

The shading is **inside M**, **outside C** and **outside W**.

(Reason: The members who had a map, but did not have a compass and did not have a whistle.)

Step 2: Diagram (ii).

The shading is **outside M**, **outside C** and **outside W**. It fills the rest of the rectangle.

(Reason: The members who had none of the three items.)

Step 3: Diagram (iii).

The shading is **inside M**, **inside C** and **inside W**.

(Reason: The members who had all three items: a map, a compass and a whistle.)

(i) the members who had a map only. (ii) the members who had none of the three items. (iii) the members who had all three items.

MARK SCHEME

Step	Mark	Description	Got it?
(i) map only, not compass and not whistle	**B1**	Accuracy - identifies all three conditions	✓
(ii) none of the three items	**B1**	Accuracy - recognises the region outside every circle	✓
(iii) all three items	**B1**	Accuracy - recognises the central region	✓

Full marks: 3/3

Part (b): Write each region in set notation [3 marks]

Rule - Translate one symbol at a time: inside a set is the letter, outside a set is the dash, and "and" is $\cap$. If the shading is outside a **combined** region, put the dash outside a bracket.

Step 1: Region (i).

$M \cap C' \cap W'$

(Reason: Inside M, outside C, outside W.)

Step 2: Region (ii).

$(M \cup C \cup W)'$

(Reason: Everything inside at least one circle is $M \cup C \cup W$, so the shading is what is left. This is the one to be careful with: shade everything, then remove the union.)

Step 3: Region (iii).

$M \cap C \cap W$

(Reason: Inside all three.)

(i) $M \cap C' \cap W'$ (ii) $(M \cup C \cup W)'$ (iii) $M \cap C \cap W$

MARK SCHEME

Step	Mark	Description	Got it?
$M \cap C' \cap W'$	**B1**	Accuracy - correct notation for "in one, out of the other two"	✓

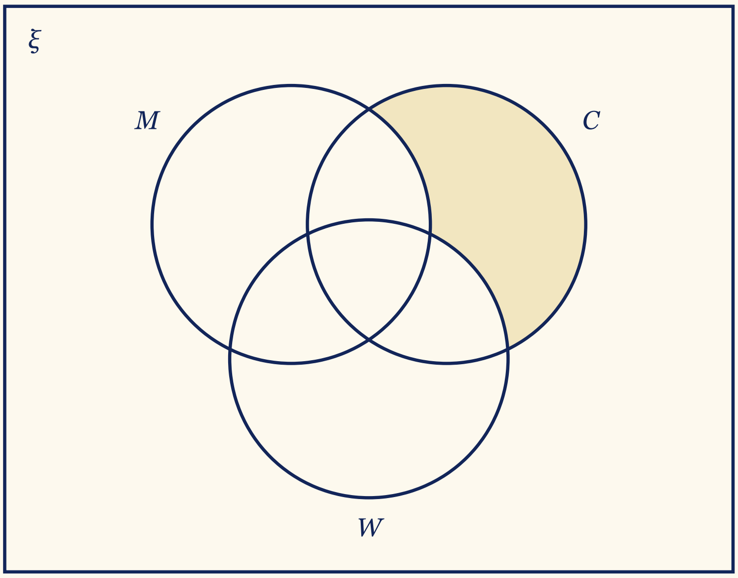
Step	Mark	Description	Got it?
$(M \cup C \cup W)'$	B1	Accuracy - the complement of the union	✓
$M \cap C \cap W$	B1	Accuracy - the triple intersection	✓

Full marks: 3/3

Part (c): Shade each diagram [3 marks]

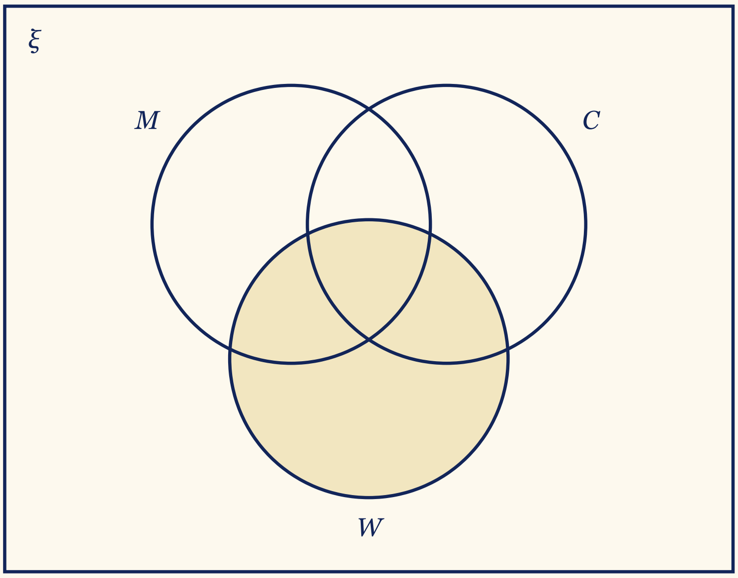
Rule - Shade every region the description covers, and nothing else: the commonest error is shading a whole circle when the word "only" appears, or shading the centre when the word "exactly" appears.

Step 1: (i) A compass only.



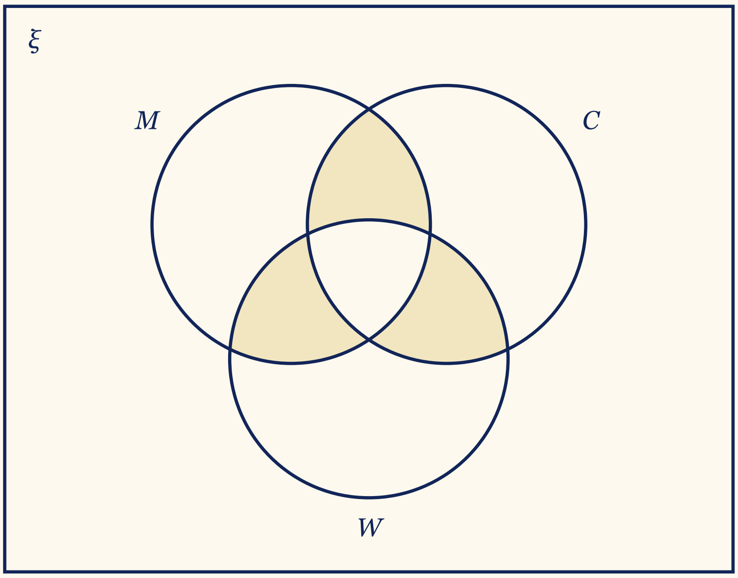
(Reason: Inside C, but outside both M and W. Only the right-hand crescent.)

Step 2: (ii) All those who had a whistle.



*(Reason: **All** of W, so every region inside the whistle circle, overlaps included. Compare this with (i): "only" means one region, "all" means the whole circle.)*

Step 3: (iii) Exactly two of the three items.



*(Reason: The three regions where exactly two circles overlap. The centre is **not** shaded: someone in the centre has **three** items, not two. This is the trap.)*

"Only" is a single region. "All" is the whole circle. "Exactly two" is the three pairwise overlaps with the centre left clear.

MARK SCHEME

Step	Mark	Description	Got it?
Only the C crescent shaded	B1	Accuracy - "only" means one region, not the whole circle	✓
The whole W circle shaded	B1	Accuracy - "all" includes every overlap	✓
The three pairwise regions shaded, centre left clear	B1	Accuracy - "exactly two" excludes the centre	✓

Full marks: 3/3

Verification

✓ **Check 1:** (a)(i) and (b)(i) agree: the words say "map, no compass, no whistle" and the notation says $M \cap C' \cap W'$. Same region.

✓ **Check 2:** (a)(ii) covers the rest of the diagram: every member is in exactly one region. The union $M \cup C \cup W$ covers everything inside at least one circle, so its complement is everything else, which is exactly what is shaded.

✓ **Check 3:** (c)(i) and (c)(ii) are different regions: "a compass only" is **one** region, and "all those with a whistle" is **four** regions. If your two diagrams look the same, one of them is wrong.

✓ **Check 4:** (c)(iii) leaves the centre clear: the centre is "all three", which is three items, not two. Shading it would count those members twice over.

Question 4 - Exam Solution

Understanding the Question

Given

A three-set Venn diagram of a cafe's 50 dishes, with V vegetarian, G gluten-free and H served hot.

The eight regions are 8, 5, 9, 6, 3, 4, 13 and 2.

Find

$n(V)$, $n(V \cap G)$, $n(G \cup H)$, $n((V \cup G) \cap H)$ and $n(V \cup (G \cap H))$.

Then why the last two differ.

Plan the Solution

- $\cap$ means "and", so it is the **overlap**. $\cup$ means "or", so it is **everything in either**.

- Always add the **regions**, not the set totals. A set total already contains its overlaps, so adding two of them double-counts.
- Where there is a bracket, **do the bracket first**. The bracket decides which set is combined with which.

Part (a): Find $n(V)$ [1 mark]

Rule - Add the regions, not the totals: a circle is made of several regions, and every one of them counts.

Step 1: Add the four regions inside V.

$n(V) = 8 + 5 + 6 + 3 = 22$

(Reason: The V circle covers four regions: V only, $V \cap G$ only, $V \cap H$ only, and all three.)

$$n(V) = 22$$

MARK SCHEME

Step	Mark	Description	Got it?
22	**B1**	Accuracy - all four regions of V	✓

Full marks: 1/1

Part (b): Find $n(V \cap G)$ [1 mark]

Rule - An overlap on a three-set Venn is TWO regions: the pairwise-only region, and the centre.

Step 1: Add both regions of the overlap.

$n(V \cap G) = 5 + 3 = 8$

*(Reason: The 3 in the centre are in V **and** in G, so they count. Forgetting the centre is the commonest error here.)*

$$n(V \cap G) = 8$$

MARK SCHEME

Step	Mark	Description	Got it?
8	**B1**	Accuracy - includes the centre	✓

Full marks: 1/1

Part (c): Find $n(G \cup H)$ [1 mark]

Rule - A union is everything inside either circle: or, faster, the whole diagram minus what is outside both.

Step 1: Add every region inside G or H.

$n(G \cup H) = 9 + 5 + 3 + 4 + 13 + 6 = 40$

(Reason: Quicker: everything except V only and the outside, so $50 - 8 - 2 = 40$.)

$$n(G \cup H) = 40$$

MARK SCHEME

Step	Mark	Description	Got it?
40	**B1**	Accuracy - correct union	✓

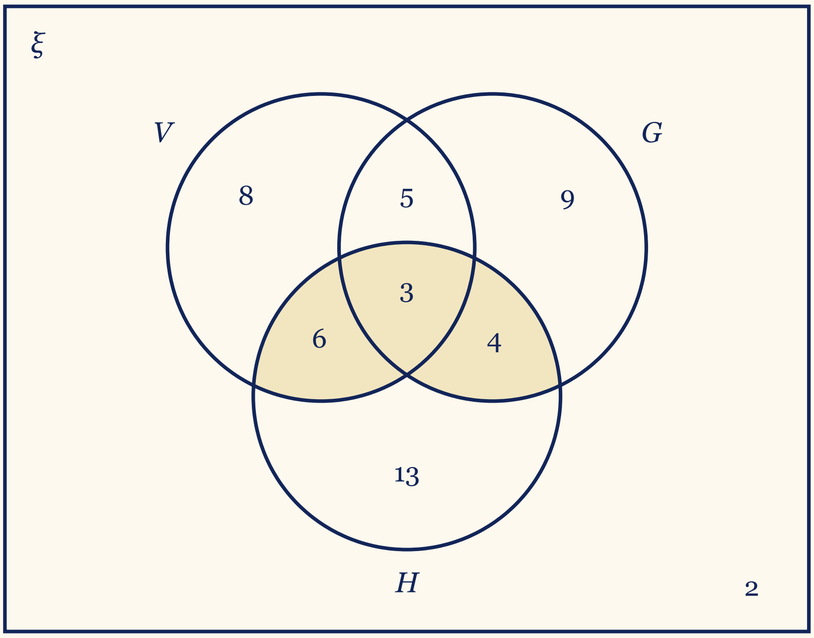
Full marks: 1/1

Part (d): Find $n((V \cup G) \cap H)$ [2 marks]

Rule - Do the bracket first: $V \cup G$ is everything vegetarian **or** gluten-free. Then $\cap H$ keeps only the part of that which is **also hot**. So every dish counted must be **hot**.

Step 1: Identify the regions.

The regions inside H that are also inside V or G are: $V \cap H$ only, all three, and $G \cap H$ only.



*(Reason: H has four regions, but H only (the 13 dishes that are hot and nothing else) is **not** in $V \cup G$, so it is excluded.)*

Step 2: Add them.

$n((V \cup G) \cap H) = 6 + 3 + 4 = 13$

(Reason: Only the shaded band counts.)

$$n((V \cup G) \cap H) = 13$$

MARK SCHEME

Step	Mark	Description	Got it?
Identifies the three regions 6, 3 and 4	**M1**	Method - bracket first, then intersect with H	✓
13	**A1**	Accuracy	✓

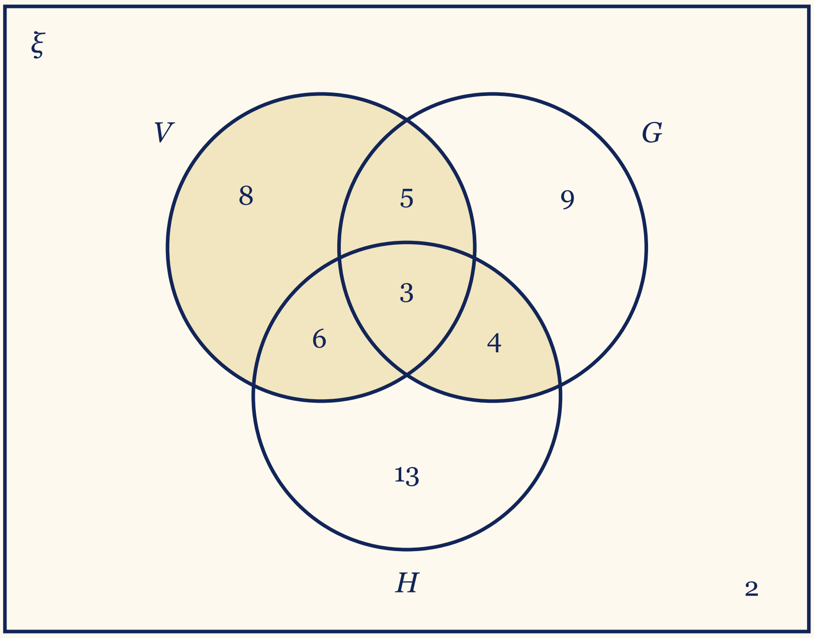
Full marks: 2/2

Part (e): Find $n(V \cup (G \cap H))$ [2 marks]

Rule - Do the bracket first: $G \cap H$ is the dishes that are gluten-free **and** hot. Then $V \cup$ that keeps **all** of V, plus that overlap. Nothing in V is excluded.

Step 1: Identify the regions.

All four regions of V (8, 5, 6, 3), plus $G \cap H$. But $G \cap H$ is $4 + 3$, and the 3 is already inside V, so only the **4** is new.



(Reason: V contributes all 22 of its dishes, hot or not.)

Step 2: Add them.

$n(V \cup (G \cap H)) = 22 + 4 = 26$

(Reason: Then the 4 that are gluten-free and hot but not vegetarian are added.)

$$n(V \cup (G \cap H)) = 26$$

MARK SCHEME

Step	Mark	Description	Got it?
All of V (22), plus the 4 in $G \cap H$ but not V	**M1**	Method - bracket first, then union with V	✓
26	**A1**	Accuracy	✓

Full marks: 2/2

Part (f): Explain why the answers differ [2 marks]

Rule - The bracket decides which set is on the outside: whichever set sits **outside** the bracket controls the answer.

Step 1: Look at what is outside the bracket in each.

In **(d)**, H is outside the bracket, and the symbol is $\cap$. So **every** dish counted must be **hot**. The answer can never be bigger than $n(H)$.

In **(e)**, V is outside the bracket, and the symbol is $\cup$. So **every** dish in V is counted, hot or not. The answer can never be smaller than $n(V)$.

(Reason: The two shaded regions above are completely different shapes. (d) is trapped inside H. (e) is not, because the 16 dishes in V that are not hot are still counted.)

The bracket changes which set is combined last. In (d) the final operation is $\cap H$, so everything must be hot, giving 13. In (e) the final operation is $\cup V$, so all 22 vegetarian dishes are included whether they are hot or not, giving 26. Moving a bracket in set notation changes the region, exactly as moving a bracket in arithmetic changes the answer.

MARK SCHEME

Step	Mark	Description	Got it?
States that in (d) everything must be in H, but in (e) all of V is included	**B1**	Reason - identifies the set outside the bracket	✓
Links this to the two different shaded regions, or to $13 \neq 26$	**B1**	Reason - explains the consequence	✓

Full marks: 2/2

Verification

- ✓ **Check 1:** Every dish is accounted for: $8 + 5 + 9 + 6 + 3 + 4 + 13 + 2 = 50$.
- ✓ **Check 2:** Part (c) three ways: adding the six regions gives 40; $50 - 8 - 2 = 40$; and inclusion-exclusion gives $n(G) + n(H) - n(G \cap H) = 21 + 26 - 7 = 40$. All three agree.
- ✓ **Check 3:** Part (d) must fit inside H: $13 \leq n(H) = 26$. It does, because everything counted is hot.
- ✓ **Check 4:** Part (e) must contain all of V: $26 \geq n(V) = 22$. It does, and the extra is exactly the 4 dishes that are gluten-free and hot but not vegetarian.

Question 5 - Exam Solution

Understanding the Question

Given

The numbers 168 and 180.

Find

(a) Their prime factorisations.

(b) The completed Venn diagram.

(c) The HCF. **(d)** The LCM.

(e) That $\text{HCF} \times \text{LCM} = 168 \times 180$, and why.

Plan the Solution

- Break each number down into primes and list **every copy** of each prime.
- A prime goes in the middle **once for each copy the two numbers share**. Any copies left over stay on their own side. Getting this wrong is where the marks go.
- The HCF is the middle. The LCM is everything.

Part (a): Write each as a product of prime factors [2 marks]

Rule - Divide by the smallest prime that goes in, and keep going: stop when you reach 1.

Step 1: Factorise 168.

$168 = 2 \times 84 = 2 \times 2 \times 42 = 2 \times 2 \times 2 \times 21 = 2 \times 2 \times 2 \times 3 \times 7$

$168 = 2^3 \times 3 \times 7$

(Reason: Keep dividing by 2 while it goes in, then move on to 3.)

Step 2: Factorise 180.

$180 = 2 \times 90 = 2 \times 2 \times 45 = 2 \times 2 \times 3 \times 15 = 2 \times 2 \times 3 \times 3 \times 5$

$180 = 2^2 \times 3^2 \times 5$

(Reason: The same method. 180 has only two 2s, but two 3s.)

$$168 = 2^3 \times 3 \times 7 \qquad 180 = 2^2 \times 3^2 \times 5$$

MARK SCHEME

Step	Mark	Description	Got it?
$168 = 2^3 \times 3 \times 7$	**B1**	Accuracy	✓

Step	Mark	Description	Got it?
$180 = 2^2 \times 3^2 \times 5$	**B1**	Accuracy	✓

Full marks: 2/2

Part (b): Complete the Venn diagram [2 marks]

Rule - Match the copies, not the primes: a prime goes in the middle **once for every copy the two numbers share**. Count the copies, take the smaller count, and put that many in the middle. Whatever is left stays on its own side.

Step 1: Match the 2s.

168 has **three** 2s. 180 has **two**. They share **two**, so **two 2s go in the middle**, and **one 2 is left over on the 168 side**.

(Reason: Writing "2" once in the middle because both numbers "have a 2" is the classic error. You must count the copies.)

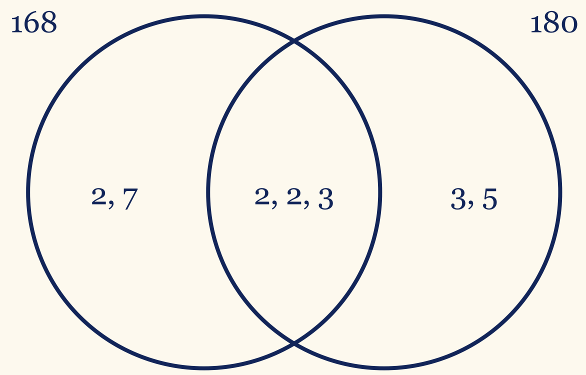
Step 2: Match the 3s.

168 has **one** 3. 180 has **two**. They share **one**, so **one 3 goes in the middle**, and **one 3 is left over on the 180 side**.

(Reason: The same rule, the other way round. Both repeated primes split.)

Step 3: Everything else is unshared.

The **7** belongs to 168 only. The **5** belongs to 180 only.



(Reason: Neither number shares them, so they stay outside the middle.)

168 only: 2 and 7. The middle: 2, 2 and 3. 180 only: 3 and 5.

MARK SCHEME

Step	Mark	Description	Got it?
Middle $= 2,\ 2,\ 3$	M1	Method - matches the number of copies, not just the primes	✓
168 only $= 2,\ 7$ and 180 only $= 3,\ 5$	A1	Accuracy - all three regions correct	✓

Full marks: 2/2

Part (c): Write down the HCF [1 mark]

Rule - The HCF is the middle: it is the biggest number that divides into both, so it is made of exactly the factors they share.

Step 1: Multiply the middle region.

$\text{HCF} = 2 \times 2 \times 3 = 12$

(Reason: Every factor in the middle divides into both numbers.)

$$\text{HCF} = 12$$

MARK SCHEME

Step	Mark	Description	Got it?
12	B1	Accuracy - the product of the middle region	✓

Full marks: 1/1

Part (d): Work out the LCM [2 marks]

Rule - The LCM is everything: it is the smallest number that both divide into, so it must contain every factor of both, and it must not contain any factor twice over.

Step 1: Multiply every factor in the diagram.

$\text{LCM} = (2 \times 7) \times (2 \times 2 \times 3) \times (3 \times 5)$

$\text{LCM} = 14 \times 12 \times 15$

*(Reason: The middle is used **once**, not twice. That is the whole point of the diagram: it stops you double-counting the shared factors.)*

Step 2: Evaluate.

$14 \times 12 = 168, \qquad 168 \times 15 = 2520$

(Reason: Multiply in any order that keeps the arithmetic easy.)

$$\text{LCM} = 2520$$

MARK SCHEME

Step	Mark	Description	Got it?
Multiplies all three regions together, using the middle once	M1	Method	✓
2520	A1	Accuracy	✓

Full marks: 2/2

Part (e): Show that $\text{HCF} \times \text{LCM} = 168 \times 180$, and explain why [3 marks]

Rule - Read the identity off the diagram: the LCM contains the middle once. The HCF **is** the middle. Multiplying them therefore uses the middle **twice**, which is exactly once for each number.

Step 1: Check the numbers.

$\text{HCF} \times \text{LCM} = 12 \times 2520 = 30240$

$168 \times 180 = 30240$

(Reason: The two agree, so the identity holds for this pair.)

Step 2: Explain it from the diagram.

Call the three regions **L** (168 only), **M** (the middle) and **R** (180 only). Then:

$168 = L \times M \qquad 180 = M \times R$

$\text{HCF} = M \qquad \text{LCM} = L \times M \times R$

$\text{HCF} \times \text{LCM} = M \times (L \times M \times R) = (L \times M) \times (M \times R) = 168 \times 180$

(Reason: Every factor is accounted for exactly twice on each side, so the two products must be equal. Nothing about this argument depends on the numbers being 168 and 180.)

The LCM uses the middle once, and the HCF is the middle. Multiplying them uses the middle twice, once for each number, which is exactly what 168×180 does. So $\text{HCF} \times \text{LCM}$ equals the product of the two numbers, for any pair.

MARK SCHEME

Step	Mark	Description	Got it?
$12 \times 2520 = 30240 = 168 \times 180$	B1	Accuracy - the calculation	✓
Writes $168 = L \times M$ and $180 = M \times R$ from the diagram	M1	Method - names the regions	✓
$M \times (L \times M \times R) = (L \times M) \times (M \times R)$	A1	Reason - the middle is counted once for each number	✓

Full marks: 3/3

Verification

- ✓ **Check 1:** Each circle rebuilds its own number: $(2 \times 7) \times (2 \times 2 \times 3) = 14 \times 12 = 168$ and $(3 \times 5) \times (2 \times 2 \times 3) = 15 \times 12 = 180$. If either fails, the Venn is wrong.
- ✓ **Check 2:** The HCF really is a common factor: $168 \div 12 = 14$ and $180 \div 12 = 15$. Both are whole numbers, and 14 and 15 share no factors, so 12 is the **highest**.
- ✓ **Check 3:** The LCM really is a common multiple: $2520 \div 168 = 15$ and $2520 \div 180 = 14$. Both whole. Notice that these are exactly the products of the two outer regions, swapped over.
- ✓ **Check 4:** The identity holds: $12 \times 2520 = 30240 = 168 \times 180$.

Question 6 - Exam Solution

Understanding the Question

Given

For a randomly chosen household: $P(G) = 0.45$, $P(F) = 0.58$ and $P(G \cap F) = 0.31$, where G is recycling glass and F is recycling food waste.

Find

(a) $P(\text{neither})$.

(b) The completed Venn diagram of expected numbers.

There are 2000 households in the borough.	**(c)** $P(G \mid F')$, as a fraction in its simplest form.

Plan the Solution

- For (a), use the addition rule to find $P(G \cup F)$, then take the complement.
- For (b), subtract the overlap from each set total to get the "only" regions, then multiply every region probability by 2000.
- For (c), the condition restricts the sample space to the households outside F. That restricted total becomes the denominator.

Part (a): Find $P(\text{neither})$ [2 marks]

Rule - The addition rule, then the complement: $P(G \cup F) = P(G) + P(F) - P(G \cap F)$, and $P(\text{neither}) = 1 - P(G \cup F)$.

Step 1: Apply the addition rule.

$P(G \cup F) = 0.45 + 0.58 - 0.31 = 0.72$

(Reason: Adding $P(G)$ and $P(F)$ counts the overlap twice, so the overlap is subtracted once.)

Step 2: Take the complement.

$P(\text{neither}) = 1 - 0.72 = 0.28$

(Reason: "Neither" is everything outside $G \cup F$, and all the probabilities add up to 1.)

$$P(\text{neither}) = 0.28$$

MARK SCHEME

Step	Mark	Description	Got it?
$P(G \cup F) = 0.45 + 0.58 - 0.31 = 0.72$	M1	Method - the addition rule, subtracting the overlap	✓
$P(\text{neither}) = 0.28$	A1	Accuracy - correct probability	✓

Full marks: 2/2

Part (b): Complete the Venn diagram [3 marks]

Rule - Subtract the overlap, then scale: each "only" region is the set total minus the overlap. The expected number in a region is its probability multiplied by the number of households.

Step 1: Find the probability of each of the four regions.

$P(G \text{ only}) = 0.45 - 0.31 = 0.14$

$P(F \text{ only}) = 0.58 - 0.31 = 0.27$

$P(G \cap F) = 0.31$

$P(\text{neither}) = 0.28 \text{ (from part (a))}$

(Reason: $P(G)$ counts every household that recycles glass, including those that also recycle food waste. Subtracting the overlap leaves glass only. Check: $0.14 + 0.31 + 0.27 + 0.28 = 1$.)

Step 2: Multiply each region probability by 2000.

$0.14 \times 2000 = 280$

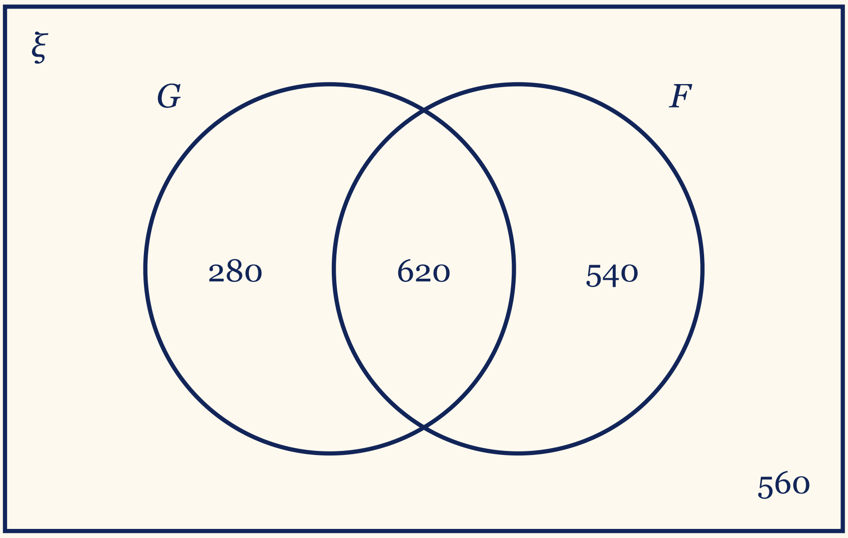
$0.31 \times 2000 = 620$

$0.27 \times 2000 = 540$

$0.28 \times 2000 = 560$

(Reason: The expected number in a region is its probability times the total number of households.)

Step 3: Place the four values on the diagram.



(Reason: 280 in G only, 620 in the overlap, 540 in F only, and 560 outside both circles.)

The completed Venn diagram is shown above: 280 households recycle glass only, 620 recycle both, 540 recycle food waste only, and 560

recycle neither.

MARK SCHEME

Step	Mark	Description	Got it?
$P(G \text{ only}) = 0.14$ and $P(F \text{ only}) = 0.27$	M1	Method - subtract the overlap from each set total	✓
Multiply each region probability by 2000	M1	Method - scale to expected frequencies	✓
A completely correct Venn diagram	A1	Accuracy - all four values correct and correctly placed	✓

Full marks: 3/3

Part (c): Find $P(G \mid F')$ [2 marks]

> **Rule - A condition shrinks the sample space:** "given that it does not recycle food waste" means only the households outside F are counted. That restricted total becomes the new denominator.

Step 1: Count the households that do not recycle food waste.

$$n(F') = 280 + 560 = 840$$

(Reason: Outside circle F there are two regions: the 280 who recycle glass only, and the 560 who recycle neither.)

Step 2: Of those, count the ones that recycle glass.

$$n(G \cap F') = 280$$

(Reason: Within F', only the "glass only" region also recycles glass.)

Step 3: Divide.

$$P(G \mid F') = \frac{280}{840} = \frac{1}{3}$$

*(Reason: The denominator is $n(F') = 840$, **not** 2000. This is the classic trap: once the condition is given, the sample space has shrunk.)*

$$P(G \mid F') = \frac{1}{3}$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(F') = 280 + 560 = 840$	M1	Method - correct denominator (the restricted sample space)	✓
$P(G \mid F') = \frac{1}{3}$	A1	Accuracy - correct simplified fraction	✓

Full marks: 2/2

Verification

✓ **Check 1:** The Venn accounts for every household: $280 + 620 + 540 + 560 = 2000$.

✓ **Check 2:** The Venn reproduces the given probabilities: $n(G) = 280 + 620 = 900$ and $\frac{900}{2000} = 0.45 = P(G)$. Likewise $n(F) = 620 + 540 = 1160$ and $\frac{1160}{2000} = 0.58 = P(F)$.

✓ **Check 3:** Part (a) agrees with the diagram: $\frac{560}{2000} = 0.28$.

✓ **Check 4:** The conditional answer works backwards: $\frac{1}{3}$ of 840 is 280, which is exactly $n(G \cap F')$.

Question 7 - Exam Solution

Understanding the Question

Given

A Venn diagram of a bookshop's stock, given as percentages, with H the hardbacks (so H' is the paperbacks) and F the fiction.

Regions: H only $= 22$, $H \cap F = 24$, F only $= 36$, and 18 outside both circles. These total 100.

Find

(a) $P(H' \mid F)$, as a fraction in its simplest form.
(b) That the answer is unchanged when the shop has 800 books.
(c) That $P(F \mid H')$ is not the same, and its value.

Plan the Solution

- The percentages already behave like counts out of 100, so they can be used directly.
- A condition shrinks the sample space. For (a) the denominator is the fiction total, not 100.

- For (b), scale every region by 800 and repeat. For (c), the condition changes, so the denominator changes.

Part (a): Find $P(H' \mid F)$ [2 marks]

Rule - A condition shrinks the sample space: "given that it is fiction" means only the fiction books are counted. The fiction total becomes the denominator.

Step 1: Find the fiction total.

$n(F) = 24 + 36 = 60$

(Reason: The fiction circle covers two regions: the 24 hardback fiction and the 36 paperback fiction.)

Step 2: Of the fiction books, count the paperbacks.

$n(H' \cap F) = 36$

(Reason: A paperback is a book that is not a hardback, so inside F but outside H. That is the F only region.)

Step 3: Divide and simplify.

$P(H' \mid F) = \frac{36}{60} = \frac{3}{5}$

*(Reason: The denominator is 60, **not** 100. The condition has already ruled out every non-fiction book.)*

$$P(H' \mid F) = \frac{3}{5}$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(F) = 24 + 36 = 60$	M1	Method - correct denominator (the restricted sample space)	✓
$P(H' \mid F) = \frac{3}{5}$	A1	Accuracy - correct simplified fraction	✓

Full marks: 2/2

Part (b): Show the answer is unchanged for 800 books [3 marks]

Rule - A percentage is a count out of 100: to convert to actual books, multiply each percentage by the total and divide by 100.

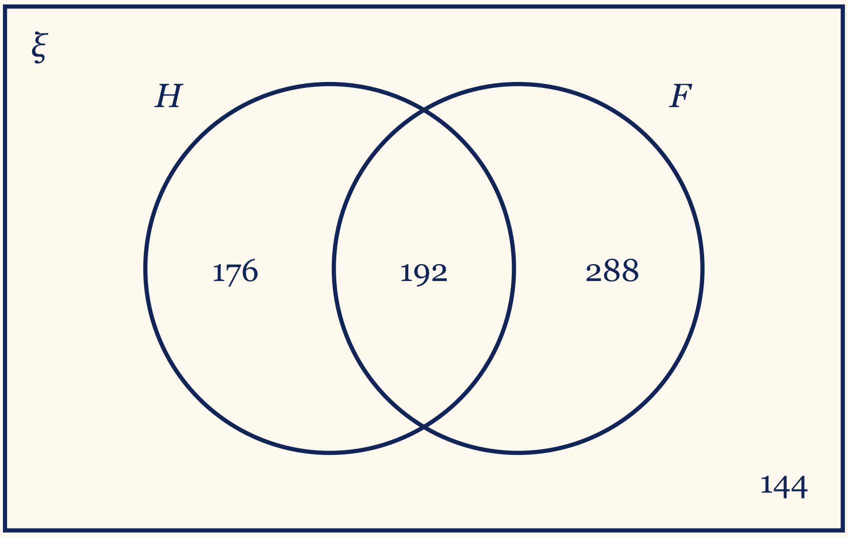
Step 1: Convert each region to a number of books.

22% of $800 = 176$

24% of $800 = 192$

36% of $800 = 288$

18% of $800 = 144$



(Reason: $176 + 192 + 288 + 144 = 800$, so every book is accounted for.)

Step 2: Find the fiction total and the paperback fiction total.

$n(F) = 192 + 288 = 480$

$n(H' \cap F) = 288$

(Reason: The same two regions as in part (a), now measured in books rather than percent.)

Step 3: Divide.

$$P(H' \mid F) = \frac{288}{480} = \frac{3}{5}$$

(Reason: 288 and 480 are both 4.8 times the earlier 36 and 60. The scale factor appears in the numerator ***and*** *the denominator, so it cancels.)*

The probability is $\frac{3}{5}$, exactly as in part (a). A conditional probability depends only on the ratio between two regions, so changing the size of the shop cannot change it.

MARK SCHEME

Step	Mark	Description	Got it?
36% of $800 = 288$	**M1**	Method - convert a percentage region to a number of books	✓
$n(F) = 480$	**M1**	Method - correct denominator in the new scale	✓
$\frac{288}{480} = \frac{3}{5}$, unchanged	**A1**	Accuracy - shows the probability is the same	✓

Full marks: 3/3

Part (c): Show the customer is wrong [3 marks]

Rule - Reversing a condition changes the denominator: $P(A \mid B)$ and $P(B \mid A)$ have the same numerator but different denominators, so they are generally **not** equal.

Step 1: Identify what has changed.

The condition is now "given that it is a paperback", so the denominator is the paperback total, not the fiction total.

(Reason: The numerator, paperback fiction, is the same region. Only the sample space has moved.)

Step 2: Find the paperback total.

$n(H') = 36 + 18 = 54$

(Reason: Outside the hardback circle there are two regions: the 36 paperback fiction and the 18 paperback non-fiction.)

Step 3: Divide and compare.

$$P(F \mid H') = \frac{36}{54} = \frac{2}{3}$$

$$\frac{2}{3} \neq \frac{3}{5}$$

(Reason: Same numerator (36), different denominator (54 instead of 60), so a different answer.)

The customer is wrong. $P(F \mid H') = \frac{2}{3}$, which is not equal to $\frac{3}{5}$. Swapping the two categories around swaps the denominator, so $P(A \mid B)$ and $P(B \mid A)$ are not the same.

MARK SCHEME

Step	Mark	Description	Got it?
$n(H') = 36 + 18 = 54$	M1	Method - correct denominator for the new condition	✓
$P(F \mid H') = \frac{2}{3}$	A1	Accuracy - correct simplified fraction	✓
States $\frac{2}{3} \neq \frac{3}{5}$, so the customer is wrong	A1	Reason - the two conditionals are not equal	✓

Full marks: 3/3

Verification

✓ **Check 1:** The percentages account for every book: $22 + 24 + 36 + 18 = 100$.

✓ **Check 2:** The 800 books account for every book: $176 + 192 + 288 + 144 = 800$.

✓ **Check 3:** Both scales give the same conditional: $\frac{36}{60} = \frac{3}{5}$ and $\frac{288}{480} = \frac{3}{5}$.

✓ **Check 4:** Part (c) also survives the change of scale: $n(H') = 288 + 144 = 432$, and $\frac{288}{432} = \frac{2}{3}$, matching $\frac{36}{54}$.

Question 8 - Exam Solution

Understanding the Question

Given

200 members, every one using at least one facility. P is the pool, G the gym and T the tennis courts.

$n(G) = 92$, of which $n(G \cap T) = 32$. $n(P) = 96$, of which $n(P \cap G) = 28$ and $n(P \cap T) = 20$. $n(P \cap G \cap T) = 12$.

Find

(a) The completed Venn diagram.
(b) $n(P \cap T \cap G')$ and $n((P \cup G \cup T)')$.
(c) $P(P \cup G)$.
(d) $P(P \mid T)$, as a fraction in its simplest form.

Plan the Solution

- Always fill a three-set Venn from the **centre outwards**. Start with the region in all three sets, then the pairwise-only regions, then the single-set regions, then whatever is left over.

- "Of these" always means an overlap that already contains the centre, so subtract the centre from it.
- Since every member uses at least one facility, the region outside all three circles is 0.

Part (a): Complete the Venn diagram [3 marks]

> **Rule - Work from the centre outwards:** every "of these" figure includes the members who use all three, so subtract the centre before placing a pairwise region. Then subtract everything already placed from each set total.

Step 1: Place the centre.

$$n(P \cap G \cap T) = 12$$

(Reason: This is given directly, and every other overlap contains it.)

Step 2: Place the pairwise-only regions.

$$G \cap T \text{ only} = 32 - 12 = 20$$
$$P \cap G \text{ only} = 28 - 12 = 16$$
$$P \cap T \text{ only} = 20 - 12 = 8$$

*(Reason: The 32 who use the gym and tennis includes the 12 who also use the pool, so 20 use the gym and tennis but **not** the pool. Same for the other two.)*

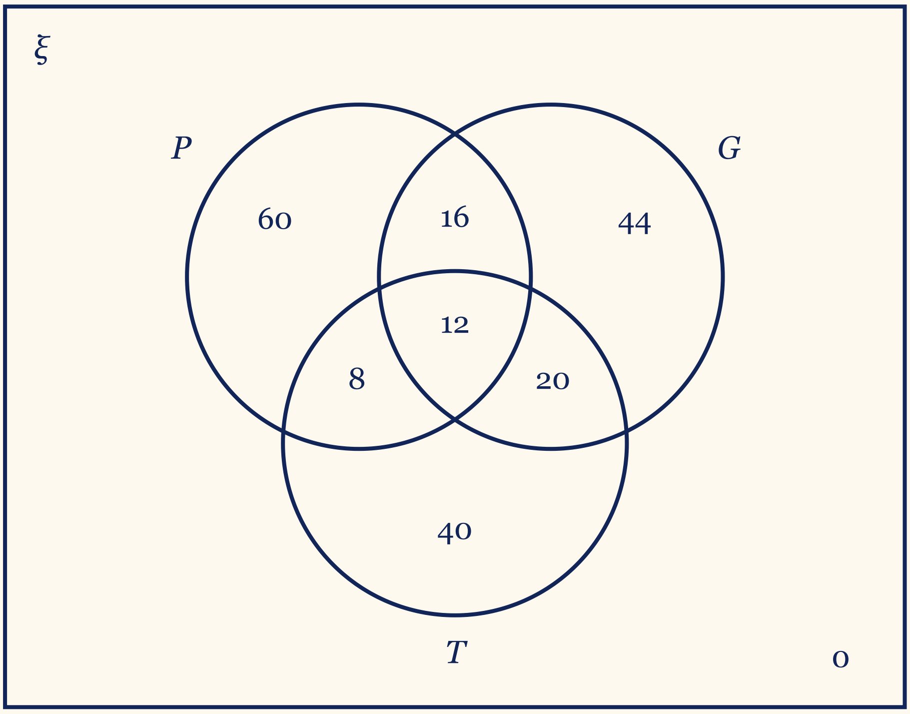
Step 3: Place the single-set regions.

$$P \text{ only} = 96 - 16 - 8 - 12 = 60$$
$$G \text{ only} = 92 - 20 - 16 - 12 = 44$$

(Reason: Subtract every region already inside P from $n(P) = 96$, and likewise for G.)

Step 4: The last region, and the outside.

$$T \text{ only} = 200 - (60 + 44 + 16 + 8 + 20 + 12) = 40$$
$$n((P \cup G \cup T)') = 0$$



(Reason: Every member uses at least one facility, so nothing sits outside the circles, and the remaining members must be in T only.)

The completed diagram: 60 in P only, 44 in G only, 40 in T only, 16 in $P \cap G$ only, 8 in $P \cap T$ only, 20 in $G \cap T$ only, 12 in all three, and 0 outside.

MARK SCHEME

Step	Mark	Description	Got it?
Centre 12, then the pairwise-only regions 20, 16 and 8	M1	Method - subtract the centre from each pairwise overlap	✓
P only $= 60$ and G only $= 44$	M1	Method - subtract the placed regions from each set total	✓
A completely correct Venn diagram, including T only $= 40$	A1	Accuracy - all seven regions correct and correctly placed	✓

Full marks: 3/3

Part (b): Find $n(P \cap T \cap G')$ and $n((P \cup G \cup T)')$ [2 marks]

> **Rule - Read the notation one symbol at a time:** $\cap$ means "and", $\cup$ means "or", and a dash means "not". Translate the notation into a region, then read the number off the diagram.

Step 1: Translate the first expression.

$$n(P \cap T \cap G') = 8$$

*(Reason: "In P **and** in T **but not** in G" is the region where only the pool and tennis circles overlap.)*

Step 2: Translate the second expression.

$$n((P \cup G \cup T)') = 0$$

(Reason: $P \cup G \cup T$ is everything inside at least one circle, so its complement is everything outside all three. Every member uses at least one facility, so that region is empty.)

$$n(P \cap T \cap G') = 8 \qquad n((P \cup G \cup T)') = 0$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(P \cap T \cap G') = 8$	B1	Accuracy - correct region identified and read off	✓
$n((P \cup G \cup T)') = 0$	B1	Accuracy - recognises that nobody sits outside all three	✓

Full marks: 2/2

Part (c): Find $P(P \cup G)$ [1 mark]

> **Rule - Union:** $P \cup G$ is everything inside the pool circle **or** the gym circle, or both.

Step 1: Count the union.

$$n(P \cup G) = 60 + 44 + 16 + 8 + 20 + 12 = 160$$

(Reason: Every region except T only and the outside. Equivalently $200 - 40 - 0 = 160$.)

Step 2: Divide by the total.

$$P(P \cup G) = \frac{160}{200} = \frac{4}{5}$$

(Reason: The member is chosen at random from all 200.)

$$P(P \cup G) = \frac{4}{5}$$

MARK SCHEME

Step	Mark	Description	Got it?
$\frac{160}{200} = \frac{4}{5}$	**B1**	Accuracy - correct probability, simplified	✓

Full marks: 1/1

Part (d): Find $P(P \mid T)$ [2 marks]

Rule - A condition shrinks the sample space: "given that a member uses the tennis courts" restricts us to the T circle. $n(T)$ becomes the denominator, and it is **not** given: it must be read off the completed Venn.

Step 1: Read n(T) off the diagram.

$n(T) = 40 + 8 + 20 + 12 = 80$

(Reason: The tennis circle covers four regions: T only, $P \cap T$ only, $G \cap T$ only, and all three.)

Step 2: Of those, count the pool users.

$n(P \cap T) = 8 + 12 = 20$

(Reason: Inside T, the pool users are the $P \cap T$ only region and the all-three region.)

Step 3: Divide and simplify.

$P(P \mid T) = \frac{20}{80} = \frac{1}{4}$

*(Reason: The denominator is $n(T) = 80$, **not** 200.)*

$$P(P \mid T) = \frac{1}{4}$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(T) = 40 + 8 + 20 + 12 = 80$	**M1**	Method - correct denominator, read off the Venn	✓

Step	Mark	Description	Got it?
$P(P \mid T) = \frac{1}{4}$	**A1**	Accuracy - correct simplified fraction	✓

Full marks: 2/2

Verification

✓ **Check 1:** Every member is accounted for: $60 + 16 + 44 + 8 + 12 + 20 + 40 + 0 = 200$.

✓ **Check 2:** The completed Venn reproduces every given: $n(P) = 60 + 16 + 8 + 12 = 96$, $n(G) = 44 + 16 + 20 + 12 = 92$, $n(P \cap G) = 16 + 12 = 28$, $n(P \cap T) = 8 + 12 = 20$ and $n(G \cap T) = 20 + 12 = 32$. Every one matches, so the diagram is right.

✓ **Check 3:** Part (c) a second way: everything except T only, so $200 - 40 - 0 = 160$, giving $\frac{4}{5}$ again.

✓ **Check 4:** Part (d) backwards: $\frac{1}{4}$ of $n(T) = 80$ is 20, which is exactly $n(P \cap T)$.

Question 9 - Exam Solution

Understanding the Question

Given

A Venn diagram of a museum's 250 visitors, with R the Roman gallery and W the wildlife gallery.

Regions: R only $= 108$, $R \cap W = 42$, W only $= 60$, and 40 saw neither.

Find

(a) $n(R) : n(\xi)$, in its simplest form.
(b) $P(W \mid R)$.
(c) $P(R \mid \text{exactly one gallery})$.
(d) $n(R \cup W)$, and why it differs from "exactly one".

Plan the Solution

- Read $n(R)$ and $n(\xi)$ off the diagram, then cancel the ratio down.
- Every conditional restricts the sample space. For (b) the condition is a named set, so the denominator is $n(R)$.
- For (c) the condition is **not** a named set. "Exactly one" has to be built: it is the two crescent regions, and it excludes both the overlap and the outside.

Part (a): Find the ratio [2 marks]

Rule - A ratio is simplified like a fraction: divide both parts by their highest common factor.

Step 1: Read the two totals off the diagram.

$n(R) = 108 + 42 = 150$

$n(\xi) = 108 + 42 + 60 + 40 = 250$

(Reason: The Roman circle covers two regions. The universal set covers all four.)

Step 2: Write the ratio and cancel.

$150 : 250 = 3 : 5$

(Reason: The highest common factor of 150 and 250 is 50, and $150 \div 50 = 3$ *while* $250 \div 50 = 5$*.)*

$$n(R) : n(\xi) = 3 : 5$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(R) = 150$ and $n(\xi) = 250$, giving $150 : 250$	**M1**	Method - correct ratio before cancelling	✓
$3 : 5$	**A1**	Accuracy - fully simplified	✓

Full marks: 2/2

Part (b): Find $P(W \mid R)$ [2 marks]

Rule - A named condition shrinks the sample space to that set: "given that they saw the Roman gallery" means the denominator is $n(R)$.

Step 1: Identify the denominator.

$n(R) = 150$

(Reason: Only the Roman visitors are counted now, not all 250.)

Step 2: Of those, count the ones who also saw the wildlife gallery, then divide.

$$P(W \mid R) = \frac{n(R \cap W)}{n(R)} = \frac{42}{150} = \frac{7}{25}$$

(Reason: Inside the Roman circle, the wildlife visitors are the 42 in the overlap. Cancel by 6.)

$$P(W \mid R) = \frac{7}{25}$$

MARK SCHEME

Step	Mark	Description	Got it?
$\frac{42}{150}$	**M1**	Method - correct numerator over the correct denominator	✓
$\frac{7}{25}$	**A1**	Accuracy - simplified fraction	✓

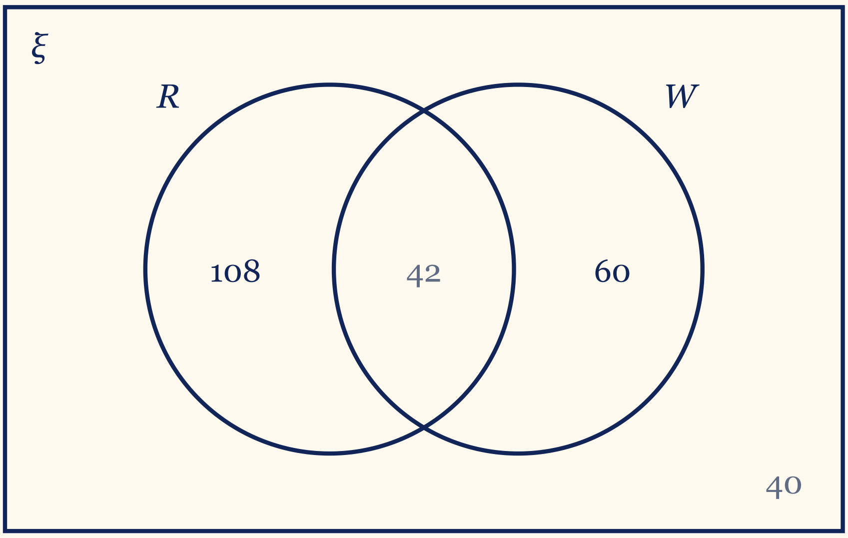
Full marks: 2/2

Part (c): Find P(R given exactly one gallery) [2 marks]

Rule - When the condition is not a named set, build it: "saw exactly one gallery" is not R, and it is not $R \cup W$. It is the two crescent regions only. The 42 who saw **both** are excluded, and so are the 40 who saw **neither**.

Step 1: Build the conditioning event.

exactly one $= 108 + 60 = 168$



(Reason: The two faded regions are excluded. Someone who saw both galleries did not see exactly one, and someone who saw neither did not see exactly one either.)

Step 2: Of those, count the Roman visitors.

$n(R \text{ only}) = 108$

(Reason: Within "exactly one", the visitors who saw the Roman gallery are precisely the 108 in the R-only crescent.)

Step 3: Divide and simplify.

$$P(R \mid \text{exactly one}) = \frac{108}{168} = \frac{9}{14}$$

*(Reason: Cancel by 12. The denominator is 168, **not** 250 and **not** 210.)*

$$P(R \mid \text{exactly one}) = \frac{9}{14}$$

MARK SCHEME

Step	Mark	Description	Got it?
Denominator $= 108 + 60 = 168$	M1	Method - builds "exactly one" and excludes both the overlap and the outside	✓
$\frac{9}{14}$	A1	Accuracy - simplified fraction	✓

Full marks: 2/2

Part (d): Find $n(R \cup W)$ and explain [2 marks]

Rule - "At least one" is not "exactly one": the union counts everyone inside either circle, **including** the overlap. "Exactly one" leaves the overlap out.

Step 1: Find the union.

$n(R \cup W) = 108 + 42 + 60 = 210$

(Reason: Everything inside either circle. Equivalently $250 - 40 = 210$, since only the 40 outside are excluded.)

Step 2: Compare it with "exactly one".

$210 - 168 = 42 = n(R \cap W)$

(Reason: The difference is exactly the overlap.)

$n(R \cup W) = 210$. This is "at least one gallery", so it includes the 42 visitors who saw both. "Exactly one gallery" is 168, because those 42

are left out. The two differ by precisely $n(R \cap W) = 42$.

MARK SCHEME

Step	Mark	Description	Got it?
$n(R \cup W) = 210$	B1	Accuracy - correct union	✓
Explains that the union includes the 42 who saw both, while "exactly one" excludes them	B1	Reason - the distinction between at least one and exactly one	✓

Full marks: 2/2

Verification

✓ **Check 1:** Every visitor is accounted for: $108 + 42 + 60 + 40 = 250$.

✓ **Check 2:** The ratio backwards: $\frac{3}{5}$ of 250 is 150, which is exactly $n(R)$.

✓ **Check 3:** Part (b) backwards: $\frac{7}{25}$ of $n(R) = 150$ is 42, which is exactly $n(R \cap W)$.

✓ **Check 4:** Part (c) backwards: $\frac{9}{14}$ of 168 is 108, which is exactly the R only region.

Question 10 - Exam Solution

Understanding the Question

Given

120 plot holders, all growing at least one crop. P is potatoes, T is tomatoes and B is beans.

Three fifths grow exactly one crop. 5% grow all three. $n(P \cap T) = 20$, $n(T \cap B) = 22$, $n(P) = 62$ and $n(B) = 58$.

Find

(a) The completed Venn diagram.
(b) $n(T)$.
(c) $P(P \mid \text{at least two crops})$.

Plan the Solution

- Start at the centre, then use each "and" figure to fill a pairwise region.
- One region cannot be reached that way. The clue "three fifths grow exactly one crop" is the way in: if 72 grow exactly one, the other 48 grow **at least two**, and those 48 are precisely

the four overlap regions.

- Then use $n(P)$ and $n(B)$ for two of the singles, and subtract from 120 for the last.

Part (a): Complete the Venn diagram [5 marks]

Rule - Turn every clue into a region: a percentage of the total gives the centre; each "and" figure gives a pairwise region once the centre is removed; and a statement about **how many crops** someone grows gives a whole band of the diagram.

Step 1: The centre.

$$n(P \cap T \cap B) = 5\% \text{ of } 120 = 6$$

(Reason: This is the one figure given as a percentage of the whole allotment.)

Step 2: Two pairwise regions.

$$P \cap T \text{ only} = 20 - 6 = 14$$

$$T \cap B \text{ only} = 22 - 6 = 16$$

(Reason: The 20 who grow potatoes and tomatoes includes the 6 who also grow beans.)

Step 3: The key insight, and the third pairwise region.

$$\text{exactly one crop} = \frac{3}{5} \times 120 = 72$$

$$\text{at least two crops} = 120 - 72 = 48$$

$$P \cap B \text{ only} = 48 - (14 + 16 + 6) = 48 - 36 = 12$$

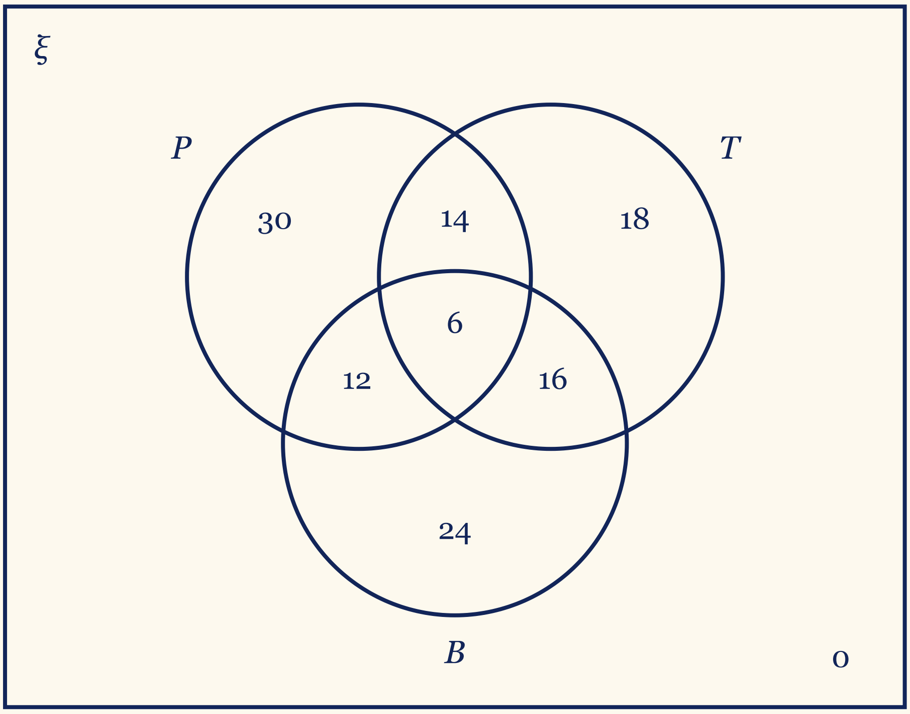
(Reason: Everyone grows at least one crop, so anyone not growing exactly one grows two or three. "At least two" is exactly the four overlap regions added together. Three of them are known, so the fourth falls out.)

Step 4: The single-crop regions.

$$P \text{ only} = 62 - (14 + 12 + 6) = 30$$

$$B \text{ only} = 58 - (12 + 16 + 6) = 24$$

$$T \text{ only} = 120 - (30 + 14 + 12 + 6 + 16 + 24) = 120 - 102 = 18$$



(Reason: Subtract every region already inside P from $n(P) = 62$, and likewise for B. The last region is whatever is left of the 120.)

The completed diagram: 30 in P only, 18 in T only, 24 in B only, 14 in $P \cap T$ only, 12 in $P \cap B$ only, 16 in $T \cap B$ only, 6 in all three, and 0 outside.

MARK SCHEME

Step	Mark	Description	Got it?
$n(P \cap T \cap B) = 6$	M1	Method - a percentage of the total gives the centre	✓
$P \cap T$ only $= 14$ and $T \cap B$ only $= 16$	M1	Method - subtract the centre from each pairwise overlap	✓
"at least two" $= 120 - 72 = 48$, so $P \cap B$ only $= 12$	M1	Method - uses the "exactly one" clue to reach the fourth overlap	✓
P only $= 30$ and B only $= 24$	M1	Method - subtract the placed regions from each set total	✓
A completely correct Venn diagram, including T only $= 18$	A1	Accuracy - all seven regions correct and correctly placed	✓

Full marks: 5/5

Part (b): Find $n(T)$ [1 mark]

Rule - The completed diagram knows more than the question told you: $n(T)$ was never given, but every region inside T now has a number.

Step 1: Add the four regions inside the tomato circle.

$n(T) = 18 + 14 + 16 + 6 = 54$

(Reason: The tomato circle covers T only, $P \cap T$ only, $T \cap B$ only, and all three.)

$$n(T) = 54$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(T) = 18 + 14 + 16 + 6 = 54$	B1	Accuracy - correct total read off the completed Venn	✓

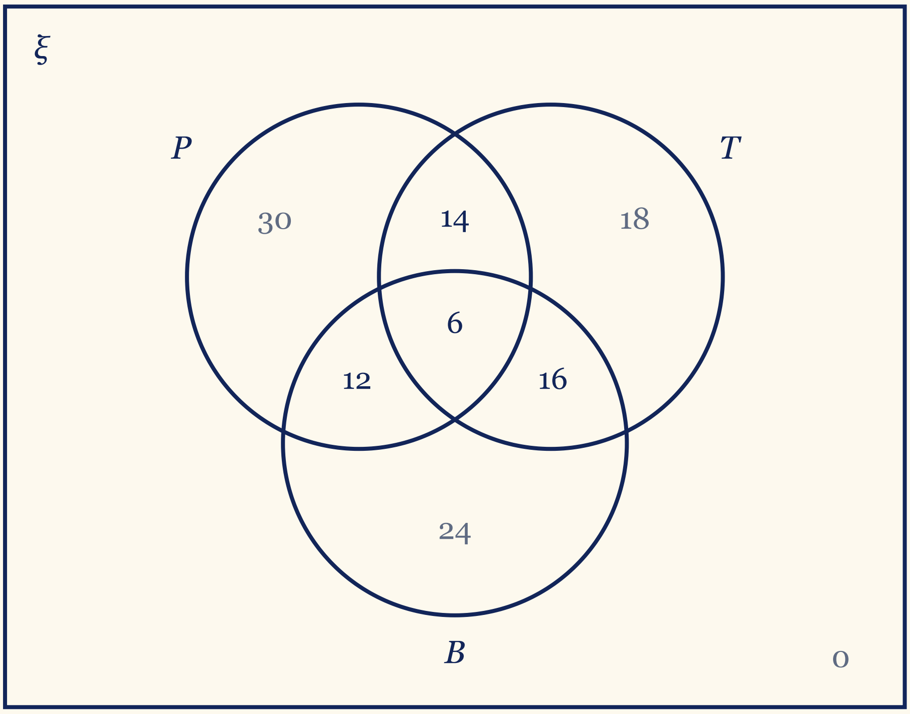
Full marks: 1/1

Part (c): Find P(P given at least two crops) [2 marks]

Rule - "At least two" is a band, not a set: on a three-set Venn it is every region where two or three circles overlap. That is **four** regions, not one.

Step 1: Build the denominator.

$\text{at least two} = 14 + 12 + 16 + 6 = 48$



*(Reason: The three faded single-crop regions are excluded, and so is the outside. The denominator is 48, **not** 120.)*

Step 2: Of those 48, count the ones who grow potatoes.

$14 + 12 + 6 = 32$

*(Reason: Inside the potato circle, the "at least two" regions are $P \cap T$ only, $P \cap B$ only and all three. The 16 who grow tomatoes and beans grow at least two crops but **not** potatoes, so they are in the denominator and not the numerator.)*

Step 3: Divide and simplify.

$$P(P \mid \text{at least two}) = \frac{32}{48} = \frac{2}{3}$$

(Reason: Cancel by 16.)

$$P(P \mid \text{at least two}) = \frac{2}{3}$$

MARK SCHEME

Step	Mark	Description	Got it?
Denominator $= 14 + 12 + 16 + 6 = 48$, numerator $= 14 + 12 + 6 = 32$	**M1**	Method - correct four-region denominator and three-region numerator	✓

Step	Mark	Description	Got it?
$\frac{2}{3}$	A1	Accuracy - simplified fraction	✓

Full marks: 2/2

Verification

✓ **Check 1:** Every plot holder is accounted for: $30 + 18 + 24 + 14 + 12 + 16 + 6 + 0 = 120$.

✓ **Check 2:** The clue that built the diagram is reproduced by it: the three single-crop regions total $30 + 18 + 24 = 72$, which is exactly $\frac{3}{5}$ of 120. The clue was used to derive the diagram, and the finished diagram gives it straight back.

✓ **Check 3:** The completed Venn reproduces every other given: $n(P) = 30 + 14 + 12 + 6 = 62$, $n(B) = 24 + 12 + 16 + 6 = 58$, $n(P \cap T) = 14 + 6 = 20$ and $n(T \cap B) = 16 + 6 = 22$. Every one matches.

✓ **Check 4:** Part (c) backwards: $\frac{2}{3}$ of 48 is 32, which is exactly the number who grow potatoes and at least two crops.

Question 11 - Exam Solution

Understanding the Question

Given

A Venn diagram for the 55 members of a school orchestra, with V the violinists and C the cellists.

Regions: V only $= n(n+5)$, $V \cap C = n$, C only $= 3n - 2$, and $9 - n$ outside both circles.

Find

(a) That $n^2 + 8n - 48 = 0$.

(b) The value of n, and every region.

(c) The probability that two different members both play the violin.

Plan the Solution

- The four regions fill the universal set, so add them and set the total equal to 55. Because one region is a product, this gives a **quadratic**, not a linear equation.
- A quadratic has two roots. Only one can be a number of people, so test both against the diagram.

- The two members are **different**, so the second is chosen from what is left. This is **without replacement**.

Part (a): Show that $n^2 + 8n - 48 = 0$ [3 marks]

Rule - The four regions fill the universal set: every member lies in exactly one region, so the four expressions add up to $n(\xi) = 55$.

Step 1: Add all four regions and set the total equal to 55.

$n(n+5) + n + (3n-2) + (9-n) = 55$

(Reason: The three regions inside the circles and the region outside them account for every member.)

Step 2: Expand the bracket and collect like terms.

$n^2 + 5n + n + 3n - 2 + 9 - n = 55$

$n^2 + 8n + 7 = 55$

(Reason: $n(n+5) = n^2 + 5n$. Then $5n + n + 3n - n = 8n$, and $-2 + 9 = 7$.)

Step 3: Rearrange so the quadratic equals zero.

$n^2 + 8n - 48 = 0$

(Reason: Subtract 55 from both sides, and $7 - 55 = -48$.)

Therefore $n^2 + 8n - 48 = 0$, as required.

MARK SCHEME

Step	Mark	Description	Got it?
$n(n+5) + n + (3n-2) + (9-n) = 55$	**M1**	Method - all four regions add to n(ξ)	✓
$n^2 + 8n + 7 = 55$	**M1**	Method - expands and collects correctly	✓
$n^2 + 8n - 48 = 0$	**A1**	Accuracy - reaches the required form	✓

Full marks: 3/3

Part (b): Solve, and complete the Venn diagram [3 marks]

Rule - A quadratic has two roots, but a region cannot: solve, then reject any root that would put a negative number of people in a region.

Step 1: Factorise.

$n^2 + 8n - 48 = 0$

$(n + 12)(n - 4) = 0$

(Reason: $12 \times (-4) = -48$ and $12 + (-4) = 8$.)

Step 2: Test both roots against the diagram.

$n = -12 \quad \text{or} \quad n = 4$

If $n = -12$, the overlap would hold -12 members and the cello-only region -38. **A region cannot hold a negative number of people**, so $n = -12$ is rejected.

$n = 4$

(Reason: It is not enough to say "n must be positive". The reason it must be positive is that every region counts people.)

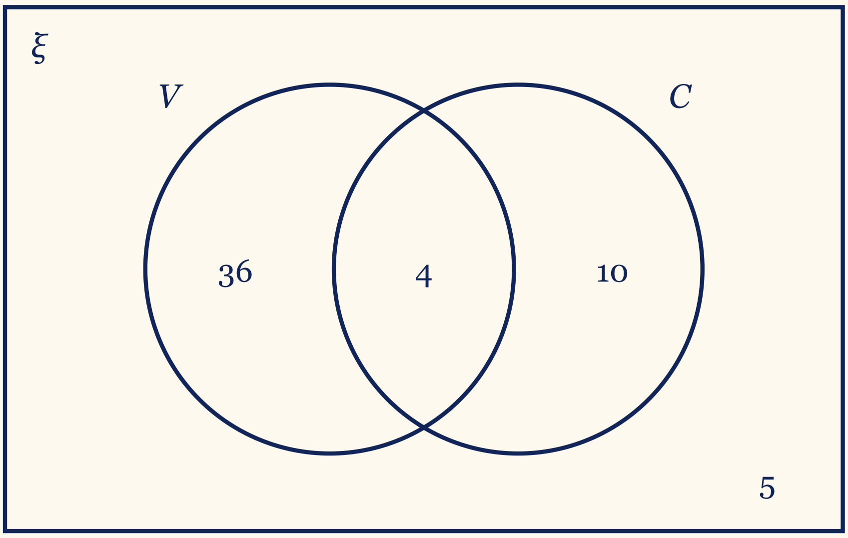
Step 3: Substitute n = 4 into each region.

$V \text{ only} = 4(4 + 5) = 36$

$V \cap C = 4$

$C \text{ only} = 3(4) - 2 = 10$

$\text{neither} = 9 - 4 = 5$



(Reason: $36 + 4 + 10 + 5 = 55$, so every member is accounted for.)

$n = 4$, and the completed Venn diagram is 36, 4, 10 and 5.

MARK SCHEME

Step	Mark	Description	Got it?
$(n + 12)(n - 4) = 0$	M1	Method - correct factorisation	✓
$n = 4$, rejecting $n = -12$ with a reason	A1	Accuracy - correct root, correctly justified	✓
Regions 36, 4, 10 and 5	A1	Accuracy - all four regions correct	✓

Full marks: 3/3

Part (c): Find P(both play the violin) [3 marks]

Rule - "Two different members" means WITHOUT replacement: the first member is not put back, so the second is chosen from one fewer person, and one fewer violinist.

Step 1: Find how many members play the violin.

$$n(V) = 36 + 4 = 40$$

(Reason: The violin circle covers V only and the overlap.)

Step 2: Write both probabilities.

$$P(\text{1st plays violin}) = \frac{40}{55}$$

$$P(\text{2nd plays violin}) = \frac{39}{54}$$

*(Reason: After a violinist is chosen, **39** violinists remain out of **54** members. Both the top and the bottom go down by one. This is the whole point of "two different members".)*

Step 3: Multiply and simplify.

$$P(\text{both}) = \frac{40}{55} \times \frac{39}{54} = \frac{8}{11} \times \frac{13}{18} = \frac{52}{99}$$

(Reason: Cancel each fraction first, then multiply.)

$$P(\text{both play the violin}) = \frac{52}{99}$$

MARK SCHEME

Step	Mark	Description	Got it?
$n(V) = 40$	**M1**	Method - correct number of violinists	✓
$\frac{40}{55} \times \frac{39}{54}$	**M1**	Method - without replacement, both numerator and denominator reduced by one	✓
$\frac{52}{99}$	**A1**	Accuracy - correct simplified fraction	✓

Full marks: 3/3

Verification

- ✓ **Check 1:** Substitute $n = 4$ back into the original expressions: $4(4+5) + 4 + (3(4) - 2) + (9-4) = 36 + 4 + 10 + 5 = 55$, which is $n(\xi)$.
- ✓ **Check 2:** The rejected root really is impossible: at $n = -12$ the overlap would be -12 and the cello-only region $3(-12) - 2 = -38$. Neither can be a number of people.
- ✓ **Check 3:** Counting check: 40 members play the violin, so 15 do not. The two regions outside the violin circle are 10 and 5, and $10 + 5 = 15$.
- ✓ **Check 4:** The probability is sensible: $\frac{52}{99} = 0.5253$, just **below** $\left(\frac{40}{55}\right)^2 = 0.5289$. Without replacement must give a slightly smaller answer than with replacement, and it does.

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